Homework 4

(1) Consider the following simplified model of political opinion in a population put forth by Vasquez and Redner. The population is composed of the fraction of leftists x, rightists y, and centrists z. The leftists and rightists never talk to each other; they are too far apart to even consider a dialogue. But both leftists and rightists do talk to centrists, and this is how opinions can change. The parameter r is a real number (either positive or negative).

$$\frac{dx}{dt} = rxz$$

$$\frac{dy}{dt} = ryz$$

$$\frac{dz}{dt} = -rxz - ryz$$

(a) Show that $\frac{dx}{dt} + \frac{dy}{dt} + \frac{dz}{dt} = 0$ so that $x + y + z = 1$.

(b) Use part (a) to write the three differential equations as a pair of equations for x and y.

(c) Find the steady states of the system.

(d) For what values of r is the steady state $(x^*, y^*) = (0, 0)$ stable?

(e) Find the nullclines and draw an (x, y) phase plane assuming that $r > 0$.

(f) Discuss the meaning of the steady states and how you expect the final outcome to depend on the value of r.

(2) Rinalidi et al (2013) modeled the stormy love affair between Scarlett O'Hara and Rhett Butler with the system

$$\frac{dR}{dt} = -R + A_S + kSe^{-S}$$

$$\frac{dS}{dt} = -S + A_R + kRe^{-R}$$

Here R denotes Rhett's love for Scarlett, and S denotes Scarlett's love for Rhett. The parameters A_R, A_S, and k are all positive.

a) Interpret the three terms in each equation. What is there meaning romantically speaking? What do the functional forms of the third term in each equation signify about how Rheatt and Scarlett react to each other's endearments?

b)Show that all trajectories that begin with $R, S \geq 0$ stay in the first quadrant forever. Hint: Examine the signs of $\frac{dR}{dt}$ and $\frac{dR}{dt}$ at the boundary of the quadrant.

c) Characterize the stability of the fixed point $(R, S) = (0, 0)$ when $A_S = A_R = 0$.

d) Using R, analyze the phase plane when $A_S = 1.2$, $A_R = 1$, and $k = 15$. Assuming Rheatt and Scarlett are indifferent when they meet $(R(0) = S(0) = 0)$, plot the trajectory of their relationship. Provide an explanation of what you find.