Homework 3

1. (Ellner and Guckenheimer) An isolated village experiences an outbreak of influenza in which 812 of the 1100 residents contract the infection. Estimate R_0 assuming that the outbreak started with a single case contracted from outside the village, with all others susceptible at the start of the outbreak.

2. (Ellner and Guckenheimer) Starting with a constant population SIR model with births, suppose that newborns are vaccinated, with the result that a fraction $p \leq 1$ of the newborns are born as removed (or recovered) rather than susceptible.

a) Write down the resulting system of differential equations. Your population size should still be constant, and when $p = 0$ your equations should reduce to the original SIR model with births.

b) Show that as p is increased from 0, the number of infectives at the endemic steady state decreases until it eventually hits $I = 0$ at some value $p < 1$ (find this value). This is sometimes called herd immunity: even though some individuals have not been immunized by vaccination, the disease cannot sustain itself in the population as a whole.

3. (Ellner and Guckenheimer) a) Starting with a constant population SIR model with births, modify the model to assume that offspring of an infected parent have probability v of being born infected rather than susceptible.

b) Use linear stability analysis of the no disease steady state to study how vertical transmission affects conditions for persistence of the disease.

4. Suppose that you are on a cruise ship containing 1000 people very far from land and one individual is infected with a contagious virus. Everyone who gets the disease eventually recovers, and infected individuals remain infected for four days on average before recovering with full immunity.

a) Formulate an SIR model of this scenario.

b) For what parameter values does an epidemic occur.

c) Numerically solve your SIR model, and estimate the time of the peak of epidemic for β =0.003, 0.005, and 0.0125.