Homework 1

1. In each of the following systems determine the eigenvalues, find the solution of the system, and sketch the solution (by hand).
a)

$$\begin{aligned} x_{n+1} &= 3x_n + 2y_n \\ y_{n+1} &= x_n + 4y_n \end{aligned}$$

with $x_0 = 1$ and $y_0 = 3$.
b)

$$\begin{aligned} x_{n+1} &= -x_n + 3y_n \\ y_{n+1} &= y_n/3 \end{aligned}$$

with $x_0 = 2$ and $y_0 = 3$.

2. Consider the survival of a population of whales, and assume that if the number of whales falls below a minimum survival level m, then the species will become extinct. In addition assume that the population is limited by the carrying capacity M of the environment. That is, if the whale population is above M, then it will experience a decline because the environment cannot sustain that large a population level.

a) Let a_n represent the whale population after n years. Consider the model

$$a_{n+1} = a_n + k(M - a_n)(a_n - m),$$

where $k > 0$. Describe the model: what are the variables, the parameters, what does each term mean. Does this model fit the description above?

b) Find the fixed points of the model, and describe their stability via linearization. Assume that $M = 5000$, $m = 100$, and $k = 0.0001$.

c) Perform a graphical stability analysis using cobwebbing. Are your results consistent with the results from b)?

d) Sketch the graphs of a_n versus n for various representative initial conditions.

e) This model fails when the initial population size a_0 is too small or too large. Why?

3. Orcas are long-lived marine mammals that live in stable social groups called "pods." Brault and Caswell (1993) used the 1973-1987 data from the coastal waters of British Columbia and Washington state and a stage-structured matrix model to investigate several demographic questions concerning the orcas. They model the females with a mixed age-stage classification: yearlings, juveniles (past the first year, but not yet mature), mature, postreproductive. The projection matrix is given by

$$A = \begin{bmatrix} 0 & 0.0043 & 0.1132 & 0 \\ 0.9775 & 0.9111 & 0 & 0 \\ 0 & 0.0736 & 0.9534 & 0 \\ 0 & 0 & 0.0452 & 0.9804 \end{bmatrix}$$

Use R (or another program) to write a script file that a) computes the dominant eigenvalue λ and stable stage distribution $\mathbf{w}$ for the whale population;
b) projects the population dynamics for the next 50 years assuming that the current population

vector is $x_0 = (10, 60, 110, 70)$;
c) plots (using R or another program) on three separate graphs the projected changes over time in

- the total population $N(t)$ in year t.
- the annual population growth rate $\dfrac{N(t+1)}{N(t)}$.
- the proportion of individuals in each stage.

4. Consider a possible harvest from the orca population from problem (3) consisting of individuals from a single stage; for example, all juveniles or all reproductive adults. Suppose that the initial population structure is the stable distribution $\mathbf{w}$ with a total of 250 individuals. What is the maximum number of juveniles that can be taken each year such that the population is not driven to extinction? What is the maximum number of reproductive adults? Assume that the harvest will take place after the breeding season, so that the model becomes

$$\mathbf{x}(t+1) = \mathbf{A}\mathbf{x}(\mathbf{t}) - \mathbf{h}$$

where $\mathbf{h} = (h_1, h_2, h_3, h_4)$ is a vector of the number of individuals harvested from each stage in each year. Assume $\mathbf{h}$ is constant.