Homework 2

1 and 2. For each of the following differential equations:

$$\frac{dx}{dt} = 1 + \mu x + x^2$$

$$\frac{dx}{dt} = x - \mu x(1 - x)$$

a) Sketch all qualitatively different phase lines that occur as μ is varried.
b) Find the bifurcation value of μ analytically.
c) Sketch the bifurcation diagram with μ as the parameter.
d) Sketch representative solutions as a function of time for a variety of initial conditions.

3. Solve the logistic equation $\dfrac{dN}{dt} = rN(1 - N/K)$.

4. Some 90% of the world?s languages are expected to vanish by the end of this century. The following model by Abrams and Strogatz (2003) addresses the phenomenon of language extinction.

Let X and Y denote two languages competing for speakers in a given society. The proportion of populations speaking X is given by the variable $0 \le x \le 1$, and $1 - x$ is the fraction speaking Y . Let $\rho_{y,x}$ be the rate at which individuals switch from Y to X, and $\rho_{x,y}$ be the rate of switching from X to Y . We assume that the attractiveness of a language increases with its number of speakers and the socioeconomic opportunities given to its speakers according to the parameter $0 \le s \le 1$. Assume that $\rho_{y,x} = sx^a$ and $\rho_{x,y} = (1 - s)(1 - x)^a$ where $a > 1$ is an adjustable parameter. Then

$$dx/dt = s(1 - x)x^a - (1 - s)x(1 - x)^a$$

describes the evolution of the fraction of people speaking language X.

a) Show that this equation has three fixed points (Hint: $f(x) = s(1 - x)x^a - (1 - s)x(1 - x)^a$ is continuous.)
b) Show that the fixed points $x = 0$ and $x = 1$ are stable.
c) Show that the third fixed point is unstable.
d) What does this model say about the long-time fate of language X?