1. $\frac{dS}{dt} = -\beta I S$

$\frac{dI}{dt} = \beta I S - \gamma I$

$\frac{dI}{dS} = \frac{\gamma}{\beta S} - 1 = \frac{N}{R_0 S} - 1$

$\int dI = \int \frac{N}{R_0 S} - 1 \, dS$

$I = \frac{N}{R_0} \ln S - S + C$ when $S = N$, $I = 0$

$0 = \frac{N}{R_0} \ln N - N + C$

$C = N - \frac{N}{R_0} \ln N$ solve for $I = 0$

$I = \frac{N}{R_0} \ln S - S + N - \frac{N}{R_0} \ln N = \frac{N}{R_0} \ln \frac{S}{N} + N - S$

$0 = \frac{N}{R_0} \ln \frac{S}{N} + N - S = \frac{N}{R_0} \ln \frac{S}{N} + R$

$R = -\frac{N}{R_0} \ln \frac{S}{N} = \frac{N}{R_0} \ln \frac{N}{S}$

$R_0 = \frac{N}{R} \ln \frac{N}{S}$

$R_0 = \frac{1100}{812} \ln \frac{1100}{288} \approx 1.815$

2. a) $\frac{dS}{dt} = \mu(N - pN - S) - \beta I S$

$\frac{dI}{dt} = \beta I S - \gamma I - \mu I$

$\frac{dR}{dt} = \mu(pN - R) + \gamma I$

b) Steady States when

$0 = \beta I S - \gamma I - \mu I$

$I = 0$ or $S = \frac{\gamma + \mu}{\beta}$

and

$0 = \mu(N - pN - S) - \beta I S$

$0 = \mu(N - pN - \frac{\gamma+\mu}{\beta}) - \cancel{\beta} \frac{\gamma+\mu}{\cancel{\beta}} I$

$I = \mu \frac{(N - pN - \frac{\gamma+\mu}{\beta})}{\gamma + \mu}$

$I = \mu N \frac{1-p}{\gamma+\mu} - \frac{\mu}{\beta}$ is an endemic steady state

$0 = \cancel{\mu}\left(\frac{N(1-p)}{\gamma+\mu} - \frac{1}{\beta}\right)$

$\gamma + \mu = \beta N(1-p)$

$p = 1 - \frac{\beta N}{\gamma + \mu}$ is the vaccination percentage required for herd immunity

3. a) $\frac{dS}{dt} = \mu(N - S - vI) - \beta I S$

$\frac{dI}{dt} = \beta I S - \gamma I - (1-v) I$

$\frac{dR}{dt} = \gamma I - \mu R$

b) steady states when $I = 0$

$0 = \mu(N - S - \cancel{vI}) - \cancel{\beta I S}$

$0 = N - S \quad S = N$

$J = \begin{bmatrix} -S - \beta I & -\mu v - \beta S \\ \beta I & \beta S - \gamma + \mu(v-1) \end{bmatrix}$

$J_{(N,0)} = \begin{bmatrix} -N & -\mu v - \beta N \\ 0 & \beta N - \gamma + \mu(v-1) \end{bmatrix}$

$\lambda_{1,2} = -N, \ \beta N - \gamma + \mu(v-1)$

$(N, 0)$ is stable when

$0 > \beta N - \gamma + \mu(v-1)$

or

$v < 1 - \frac{\beta N - \gamma}{\mu}$

4. a) $N = 1000 \quad \gamma = \frac{1}{4}$

$\frac{dS}{dt} = -\beta I S$

$\frac{dI}{dt} = \beta I S - \frac{I}{4}$

$\frac{dR}{dt} = \frac{I}{4}$

b) $R_0 = \frac{\beta N}{\gamma} = 4000\beta$

when $\beta > 2.5 \times 10^{-4}$

an epidemic can occur

c) Attached Seperately