Week 8 Homework

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EENG 203 - Circuits and System Design

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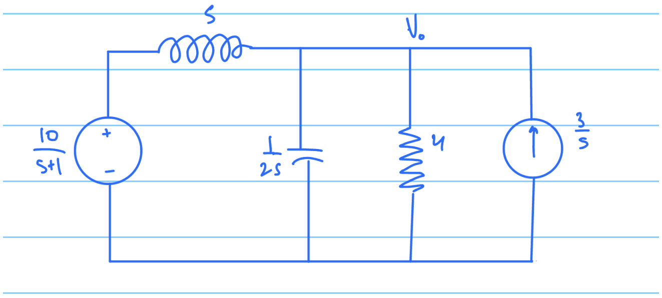
Homework for March 27, 2025

16.35

For $t < 0$, the capacitor and inducotr are in a steady state at 0 as the circuit is unexicted. Transforming the given circuit yeilds



$$\frac{V_o - \frac{10}{s+1}}{s} + \frac{2s(V_o - 0)}{1} + \frac{V_o - 0}{4} - \frac{3}{s} = 0$$

$$\left(\frac{1}{s} + 2s + \frac{1}{4}\right) V_o = \frac{10}{s(s+1)} + \frac{3}{s}$$

Solving for V_o yields

$$2\left(\frac{s^2 + 0.125s + 0.5}{s}\right) V_o = \frac{3s + 13}{s(s+1)}$$

$$V_o = \frac{1.5s + 6.5}{(s^2 + 0.125s + 0.5)(s+1)}$$

Find the roots of $s^2 + 0.125s + 0.5$,

$$s_{1,2} = \frac{-0.125 \pm \sqrt{0.015625 - 2}}{2} = \frac{-0.125 \pm \sqrt{-1.984375}}{2} = -0.0625 \pm j0.70435$$

Substituting those roots back into V_o yields

$$V_o = \frac{1.5s + 6.5}{(s+1)(s + 0.0625 + j0.70435)(s + 0.0625 - j0.70435)}$$

$$= \frac{A}{s+1} + \frac{B}{s + 0.0625 + j0.70435} + \frac{C}{s + 0.0625 - j0.70435}$$

where

$$A = \left.\frac{1.5s + 6.5}{(s + 0.0625 + j0.70435)(s + 0.0625 - j0.70435)}\right|_{s=-1} = \frac{5}{1.375} = 3.636$$

$$B = \frac{(1.5(-0.0625 + j0.70435) + 6.5)}{(-0.0625 + j0.70435 + 1)(-1.4087)} = \frac{6.40625 - j1.08625}{(0.9375 + j0.70435)(-1.4087)}$$

$$= \frac{(6.49272\angle -9.36497°)}{(1.17261\angle 36.9178°)(1.4087\angle -90°)} = 3.9306\angle 117.553°$$

$$C = \frac{(1.5(-0.0625 - j0.70435) + 6.5)}{(-0.0625 - j0.70435 + 1)(-1.4087)} = \frac{6.40625 + j1.056525}{(0.9375 - j0.70435)(-1.4087)}$$

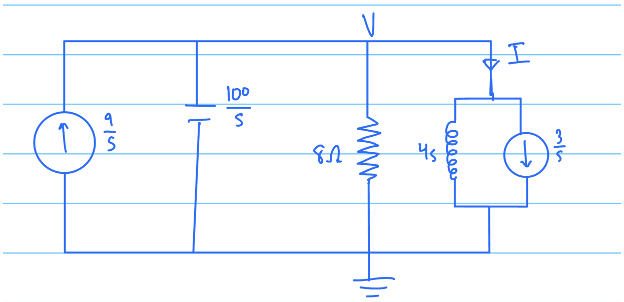
$$= \frac{6.49272\angle 9.36497°}{(1.17261\angle -36.9178°)(1.4087\angle 90°)} = 3.9306\angle -117.553°$$

Thus,

$$v_o(t) = (3.636e^{-t} + 3.9306e^{-0.0625t}e^{j117.559°}e^{-j0.7044t} + 3.9306e^{-0.0625t}e^{-j117.559°}e^{j0.7044t})u(t)\text{ V}$$

16.44

First note the initial conditions. For $t < 0$, the inductor functions as a short circuit, yielding $v_c(0) = 0$ and $i(0) = 3$ A. Also see that this circuit can be simplified and transformed into:



Solving for V and I in the above circuit yields the following:

$$-\frac{9}{s} + \frac{V}{\frac{100}{s}} + \frac{V}{8} + \frac{V}{4s} + \frac{3}{s} = 0$$

$$I = \frac{V}{4s} + \frac{3}{s}$$

$$\frac{6}{s} = \left[\frac{s}{100} + \frac{1}{8}\frac{1}{4s}\right]\text{ V} = \frac{s^2 + 12.5s + 25}{100s}\text{ V}$$

$$V(s) = \frac{600}{(s + 2.5)(s + 10)}$$

Therefore,

$$I(s) = \frac{150}{s(s + 2.5)(s + 10)} + \frac{3}{s} = \frac{150 + 3(s + 2.5)(s + 10)}{s(s + 2.5)(s + 10)}$$

Performing partial fraction expansion and reconverting back into the time domain yields:

$$I(s) = \frac{A}{s} + \frac{B}{s + 2.5} + \frac{C}{s + 10}$$

$$A = (s)I(s)|_{s=0} = \frac{150 + 75}{25} = 9$$

$$B = (s + 2.5)I(s)|_{s=-2.5} = \frac{150 + 0}{-2.5 * 7.5} = -8$$

$$C = (s + 10)I(s)|_{s=-10} = \frac{150 + 0}{-10 * -7.5} = 2$$

Therefore,

$$I(s) = \frac{9}{s} - \frac{8}{s + 2.5} + \frac{2}{s + 10}$$

Thus,

$$i(t) = \mathbb{L}\{I(s)\} = u(t)\left(9 - 8e^{-2.5t} + 2e^{-10t}\right)\text{ A}$$

Homework for April 1, 2025

16.96

Given that V_o is the voltage across R, KCL yields

$$I_s = \frac{V_o}{R} + sCV_o + \frac{V_o}{sL} = V_o\left(\frac{1}{R} + sC + \frac{1}{sL}\right)$$

Solving for V_o,

$$V_o = \frac{I_s}{\frac{1}{R} + sC + \frac{1}{sL}}$$

$$V_o = \frac{sLI_sR}{sL + R + s^2LRC}$$

Given that $I_o = \frac{V_o}{R}$,

$$I_o = \frac{sLI_s}{sL + R + s^2LRC}$$

$$H(s) = \frac{I_o}{I_s} = \frac{sL}{sL + R + s^2LRC} = \frac{\frac{s}{RC}}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$$

This yields the following roots:

$$s_{1,2} = \frac{-\frac{1}{RC} \pm \sqrt{\left(\frac{1}{RC}\right)^2 - 4\left(\frac{1}{LC}\right)}}{2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \left(\frac{1}{LC}\right)}$$

Given that R, L, and C are positive, the roots s_1 and s_2 must be negative. Therefore, the system is stable.