Week 6 Homework

Bryan SebaRaj

Professor Hong Tang

EENG 203 - Circuits and System Design

February 25, 2025

Homework for February 18, 2025

15.5

The poles of $F(s)$ are (a) -4, (b) -3, and (c) -2.

15.8

Using the initial value theorem,

$$\lim_{s\to\infty} sF(s) = \lim_{s\to\infty} s\frac{s+1}{(s+2)(s+3)} = \lim_{s\to\infty} \frac{s^2+s}{s^2+5s+6} = 1$$

The initial value of $f(5)$ is (d) 1.

15.9

Given that $H(S) = \frac{s+2}{(s+2)^2+1}$, we can rewrite it as

$$H(S) = \frac{A(s+\alpha)}{(s+\alpha)^2+\beta^2} + \frac{B\beta}{(s+\alpha)^2+\beta}$$

where $\alpha = 2$, $\beta = 1$, $A = 1$ and $B = 0$. Thus,

$$\mathcal{L}^{-1}\{H(s)\} = e^{-2t}cos(t)u(t)$$

Homework for February 20, 2025

15.21

Let the period be $T = 2\pi$. From inspection,

$$f_1(t) = (1 - \frac{t}{2\pi})(u(t) - i(t-2\pi)) = u(t) - \frac{t}{2\pi}u(t) + \frac{t-2\pi}{2\pi}u(t-2\pi)$$

Using the common functions of a Laplace transform,

$$\mathcal{L}\{f_1(t)\} = F_1(s) = \frac{1}{s} - \frac{1}{2\pi s^2} + \frac{e^{-2\pi s}}{2\pi s^2} = \frac{2\pi s - 1 + e^{-2\pi s}}{2\pi s^2}$$

Projecting the Laplace transform from a single period to the entire function,

$$F(s) = \frac{F_1(s)}{1-e^{-2\pi s}} = \frac{2\pi s - 1 + e^{-2\pi s}}{2\pi s^2 - 2\pi s^2 e^{-2\pi s}}$$

15.22

(a) Let the period be $T = 1$. From inspection,

$$g_1(t) = 2t(u(t) - u(t-1)) = 2tu(t) - 2(tu(t-1)) = 2tu(t) + 2u(t-1) - 2(t-1)u(t-1)$$

Using the common functions of a Laplace transform,

$$\mathcal{L}\{g_1(t)\} = G_1(s) = \frac{2}{s^2} + \frac{2e^{-s}}{s} - \frac{2e^{-s}}{s^2} = \frac{2 + 2se^{-s} - 2e^{-s}}{s^2}$$

Projecting the Laplace transform from a single period to the entire function,

$$G(s) = \frac{G_1(s)}{1-e^{-s}} = \frac{2+2se^{-s}-2e^{-s}}{s^2-s^2e^{-s}}$$

(b) Let the period be $T = 2$. Define the periodic triangular wave as h_0, where $h = h_0 + u(t)$. From inspection,

$$h_1(t) = 2tu(t) + 2(1-2t)u(t-1) + 2u(t-1) + 2(t-2)u(t-2) = 2tu(t) + 4u(t-1) - 4tu(t-1) + 2(t-2)u(t-2)$$
$$= 2tu(t) - 4(t-1)u(t-1) + 2(t-2)u(t-2)$$

Using the common functions of a Laplace transform,

$$\mathcal{L}\{h_1(t)\} = H_1(s) = \frac{2}{s^2} - \frac{4e^{-s}}{s^2} + \frac{2e^{-2s}}{s^2} = \frac{2-4e^{-s}+2e^{-2s}}{s^2} = \frac{2(1-e^{-s})^2}{s^2}$$

Finding H_0,

$$H_0(s) = \frac{2(1-e^{-s})^2}{s^2(1-e^{-2s})}$$

Projecting the Laplace transform from a single period to the entire function,

$$H(s) = \frac{2(1-e^{-s})^2}{s^2(1-e^{-2s})} + \frac{1}{s}$$

15.27

(a) Trivially using the reverse transforms,

$$f(t) = \mathcal{L}^{-1}\{F(s)\} = u(t) + 2e^{-t}u(t)$$

(b) See that

$$G(s) = \frac{3(s+4)-11}{s+4} = 3 - \frac{11}{s+4}$$
$$g(t) = \mathcal{L}^{-1}\{G(s)\} = 3\delta(t) - 11e^{-4t}u(t)$$

(c) See that

$$H(s) = \frac{A}{s+1} + \frac{B}{s+3}, \quad \exists A, B \in \mathbb{Z}$$

The poles of $H(s)$ can be calculated as

$$A = (s+1)H(s)\big|_{s=-1} = 2$$
$$B = (s+3)H(s)\big|_{s=-3} = -2$$

Thus,

$$H(s) = \frac{2}{s+1} - \frac{2}{s+3}$$
$$h(t) = \mathcal{L}^{-1}\{H(s)\} = 2e^{-t}u(t) - 2e^{-3t}u(t)$$

(d) See that

$$J(s) = \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{C}{s+4}, \quad \exists A, B, C \in \mathbb{Z}$$

The poles of $J(s)$ can be calculated as

$$B = (s+2)^2 J(s)\big|_{s=-2} = 6$$
$$C = (s-4)J(s)\big|_{s=4} = 3$$

Note that A canot be determined in this manner, so we can multiply both sides of $J(s)$ by $(s+2)^2(s+4)$ and rewrite it as

$$12 = A(s+2)(s+4) + B(s+4) + C(s+2)^2$$

Substituting in B and C,

$$12 = A(s+2)(s+4) + 6(s+4) + 3(s+2)^2$$

Without expanding, we can see that $A = -3$. Thus,

$$J(s) = \frac{-3}{s+2} + \frac{6}{(s+2)^2} + \frac{3}{s+4}$$

$$j(t) = \mathcal{L}^{-1}\{J(s)\} = -3e^{-2t}u(t) + 6te^{-2t}u(t) + 3e^{-4t}u(t)$$

15.30

(a) See that we can split up $F_1(s)$ such that

$$F_1(s) = \frac{A}{s} + \frac{Bs + C}{s^2 + 2s + 5}$$

Multiplying both sides by $s(s^2 + 2s + 5)$, we get

$$6s^2 + 8s + 3 = A(s^2 + 2s + 5) + Bs^2 + Cs$$

Setting the coefficients of both sides equal to each other, see that

$$\text{for } s^2, 6 = A + B$$
$$\text{for } s^1, 8 = 2A + C$$
$$\text{for } s^0, 3 = 5A$$

Thus, $A = \frac{3}{5}$, $B = \frac{27}{5}$, and $C = \frac{34}{5}$. Substituting back into $F_1(s)$,

$$F_1(s) = \frac{3}{5s} + \frac{27s + 34}{5(s^2 + 2s + 5)} = \frac{3}{5s} + \frac{27(s + 1) + 7}{5[(s + 1)^2 + 4]}$$
$$f_1(t) = \frac{3}{5} + \frac{27}{5}e^{-t}cos(2t) + \frac{7}{5}s^{-t}sin(2t)$$

(b) Splitting up $F_2(s)$,

$$F_2(s) = \frac{A}{s + 1} + \frac{B}{(s + 1)^2} + \frac{C}{s + 4}$$

Multiplying both sides by $(s + 1)^2(s + 4)$, we get

$$6s^2 + 8s + 3 = A(s + 1)(s + 4) + B(s + 4) + C(s + 1)^2$$

Setting the coefficients of both sides equal to each other, see that

$$\text{for } s^2, 1 = A + C$$
$$\text{for } s^1, 5 = 5A + B + 2C$$
$$\text{for } s^0, 6 = 4A + 4B + C$$

Solving the system of equations,

$$14 = 16A + 7C$$
$$7 = 9A$$
$$A = \frac{7}{9}, C = \frac{2}{9}, B = \frac{2}{3}$$

Substituting back into $F_2(s)$,

$$F_2(s) = \frac{7}{9(s + 1)} + \frac{2}{3(s + 1)^2} + \frac{2}{9(s + 4)}$$
$$f_2(t) = \frac{7}{9}e^{-t}u(t) + \frac{2}{3}te^{-t}u(t) + \frac{2}{9}e^{-4t}u(t)$$

(c) Splitting up $F_3(s)$,

$$F_3(s) = \frac{A}{s + 1} + \frac{Bs + C}{s^2 + 4s + 8}$$

Multiplying both sides by $(s + 1)(s^2 + 4s + 8)$, we get

$$10 = A(s^2 + 4s + 8) + Bs^2 + Bs + Cs + C$$

Setting the coefficients of both sides equal to each other, see that

$$\text{for } s^2, 0 = A + B$$
$$\text{for } s^1, 0 = 4A + B + C$$
$$\text{for } s^0, 10 = 8A + C$$

Solving the system of equations,

$$3A + C = 0$$
$$5A = 10$$
$$A = 2, B = -2, C = -6$$

Substituting back into $F_3(s)$,

$$F_3(s) = \frac{2}{s + 1} - \frac{2s + 6}{s^2 + 4s + 8}$$
$$= \frac{2}{s + 1} - \frac{2(s + 1)}{(s + 1)^2 + 4} - \frac{4}{(s + 1)^2 + 4}$$
$$f_3(t) = 2e^{-t}u(t) - 2e^{-t}cos(2t)u(t) - 2e^{-t}sin(2t)u(t)$$