Week 5 Homework

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Homework for February 11, 2025

14.61

(a) Note that in this active filter,

$$V_o = V_- \text{ and } V_+ = V_i \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$$

Recall that $V_+ = V_-$. As such,

$$V_i \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = V_o$$

Thus,

$$H(\omega) = \frac{V_o}{V_i} = \frac{j\omega C}{j\omega C} \cdot \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{j\omega RC + 1}$$

(b) Note that in this active filter,

$$V_o = V_- \text{ and } V_+ = V_i \frac{R}{R + \frac{1}{j\omega C}}$$

Recall that $V_+ = V_-$. As such,

$$V_i \frac{R}{R + \frac{1}{j\omega C}} = V_o$$

Thus,

$$H(\omega) = \frac{V_o}{V_i} = \frac{j\omega C}{j\omega C} \cdot \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{j\omega RC + 1}$$

14.66

(a) Denote the non-inverting input as v_+ and the inverting input as v_-. From inspection,

$$v_+ = \frac{R_4}{R_3 + R_4} v_s$$

Recall that $v_- = v_+$. Thus,

$$v_- = \frac{R_4}{R_3 + R_4} v_s$$

Using KCL at the inverting input node,

$$\frac{v_- - v_s}{R_1} + \frac{v_- - v_o}{R_2} + Cs(v_- - v_o)) = 0$$

Substituting in v_-,

$$\frac{\frac{R_4}{R_3+R_4} v_s - v_s}{R_1} + \frac{\frac{R_4}{R_3+R_4} v_s - v_o}{R_2} + Cs(\frac{R_4}{R_3 + R_4} v_s - v_o)) = 0$$

Grouping like terms,

$$v_o(-\frac{1}{R_2} - sC) + v_s(-\frac{R_3}{R_1(R_3 + R_4)} + \frac{R_4}{R_2(R_3 + R_4)} + sC\frac{R_4}{R_3 + R_4}) = 0$$

$$v_o(\frac{1}{R_2} + sC) = v_s(-\frac{R_3}{R_1(R_3 + R_4)} + \frac{R_4}{R_2(R_3 + R_4)} + sC\frac{R_4}{R_3 + R_4})$$

$$\frac{v_o}{v_s} = \frac{\frac{1}{R_3+R_4}\left(-\frac{R_3}{R_1} + \frac{R_4}{R_2} + R_2 sC\right)}{\frac{1}{R_2} + sC}$$

Factoring out R_4C and multiplying parts by $\frac{1}{R_1C}$,

$$\frac{v_o}{v_s} = \frac{\frac{1}{R_3+R_4} \cdot R_4C\left(s + \frac{1}{R_1C}(\frac{R_1}{R_2} - \frac{R_3}{R_4})\right)}{\frac{1}{R_2} + sC}$$

Dividing the numerator and denominator by c,

$$\frac{v_o}{v_s} = \frac{R_4}{R_3+R_4}\frac{s + \frac{1}{R_1C}(\frac{R_1}{R_2} - \frac{R_3}{R_4})}{\frac{1}{R_2C} + s}$$

Recall that $H(s) = \frac{V_o}{V_s}$. Thus,

$$H(s) = \frac{R_4}{R_3+R_4}\frac{s + \frac{1}{R_1C}(\frac{R_1}{R_2} - \frac{R_3}{R_4})}{\frac{1}{R_2C} + s}$$

(b) Note that in order to operate as a high pass filter, $H(s)$ must take the form with some constant K,

$$H(s) = K \cdot \frac{sR_fC}{sR_iC + 1}$$

Rewriting $H(s)$,

$$H(s) = \frac{R_4}{R_3+R_4} \times \frac{R_2C}{R_2C} \times \frac{s + (1/R_1C)[R_1/R_2 - R_3/R_4]}{s + 1/R_2C} = \frac{R_4}{R_3+R_4} \times \frac{sR_2C + (R_2/R_1)[R_1/R_2 - R_3/R_4]}{sR_2C + 1}$$

When $\frac{R_1}{R_2} = \frac{R_4}{R_4}$, $H(s)$ takes the form,

$$H(s) = \frac{R_4}{R_3+R_4} \times \frac{sR_2C + 0}{sR_2C + 1} = \frac{R_4}{R_3+R_4} \times \frac{sR_2C}{sR_2C + 1}$$

Thus, in order for this circuit to act as a high pass filter, $\frac{R_1}{R_2} = \frac{R_3}{R_4}$. This can be confirmed as $H(0) = 0$ and $H(\infty) = 1$.

(c) Using the previous intermerdiate form,

$$H(s) = \frac{R_4}{R_3+R_4} \times \frac{sR_2C + (R_2/R_1)[R_1/R_2 - R_3/R_4]}{sR_2C + 1}$$

At $s = 0$,

$$H(0) = \frac{R_4}{R_3+R_4} \times \frac{0 + (R_2/R_1)[R_1/R_2 - R_3/R_4]}{0 + 1} = \frac{R_4}{R_3+R_4}(R_2/R_1)(R_1/R_2 - R_3/R_4)$$

Therefore in order to yield a nonzero DC gain at at $s = 0$,

$$\frac{R_1}{R_2} \neq \frac{R_3}{R_4}$$

In order for $H(\infty) = 0$, see that $R_3 \to \infty$, which would yield,

$$H(\infty) = 0$$

Thus, the circuit will act as a low pass filter.

Homework for February 13, 2025

14.95

(a) Recall that a circuit with a variable capacitor and antenna coil uses the formula,

$$f = \frac{1}{2\pi\sqrt{LC}}$$

At $C = 40$ pF,

$$f = \frac{1}{2\pi\sqrt{240\mu H \cdot 40pF}} = 1.6244 \text{ MHz}$$

At $C = 360$ pF,

$$f = \frac{1}{2\pi\sqrt{240\mu H \cdot 360pF}} = 0.5415 \text{ MHz}$$

Thus, the circuit can tune for any frequency between 0.5415 MHz and 1.6244 MHz.

(b) In order to calculate Q, recall that the formula for Q is,

$$Q = \frac{2\pi f L}{R}$$

At $f_1 = 0.5415$ MHz,

$$Q = \frac{2\pi \cdot 0.5415 \text{ MHz} \cdot 240\mu H}{12\Omega} = 68.047$$

At $f_2 = 1.6244$ MHz,

$$Q = \frac{2\pi \cdot 1.6244 \text{ MHz} \cdot 240\mu H}{12\Omega} = 204.128$$

14.96

First let us denote Z_1 as the node directly below C_1 and Z_2 as the node directly below C_2 Using nodal analysis, see that

$$Z_2 = \frac{R_L \cdot \frac{1}{sC_2}}{R_L + \frac{1}{sC_2}} = \frac{R_L}{R_L sL + 1}$$

and

$$Z_1 = \frac{1}{sL}||(sL + Z_2) = \frac{1}{sC}|| \left(\frac{sL + s^2 R_L C_2 L + R_L}{sR_L C} \right) = \frac{sL + s^2 R_L C_2 L + R_L}{1 + sR_L C_2 + s^2 LC_1 + s^3 R_L LC_1 C_2 + sC_1 R_L}$$

Analyzing V_1 and V_0, see that

$$V_1 = \frac{Z_1}{Z_1 + R_i} V_i$$

$$V_0 = \frac{Z_2}{Z_2 + sL} V_1 = \frac{Z_2}{Z_2 + sL} \frac{Z_1}{Z_1 + R_i} V_i$$

$$= \frac{sL + s^2 R_L LC_2 + R_L}{sL + s^2 R_L LC_2 + R_i(1 + sR_L C_2 + s^2 LC_1 + s^3 R_L LC_1 C_2 + sC_1 R_L)} \cdot \frac{R_L}{R_L + sL(1 + sR_L C_2)} V_i$$

$$\frac{V_o}{V_i} = \frac{R_L sL + s^2 R_L^2 LC_2 + R_L^2}{(sL + s^2 R_L LC_2 + R_i + R_i sR_L C_2 + R_i s^2 LC_1 + R_i s^3 R_L LC_1 C_2 + R_i sC_1 R_L)(R_L + sL + s^2 LR_L C_2)}$$

Thus,

$$H(\omega) = \frac{R_L}{sL + s^2 R_L LC_2 + R_i + R_i sR_L C_2 + R_i s^2 LC_1 + R_i s^3 R_L LC_1 C_2 + R_i sC_1 R_L}$$