Week 8 Homework

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EENG 203 - Circuits and System Design

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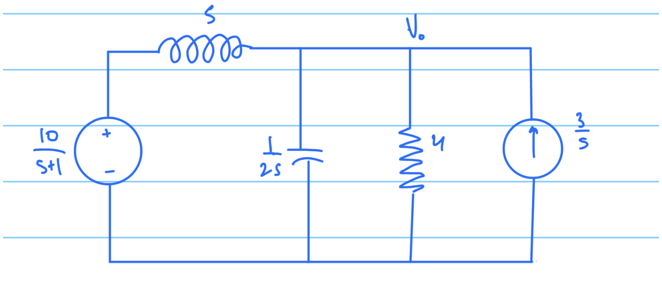
16.35

For $t < 0$, the capacitor and inducotr are in a steady state at 0 as the circuit is unexicted. Transforming the given circuit yeilds



The s-domain equation for KCL at the node v_o is:

$$\frac{v_o(s) - \frac{10}{s+1}}{s} + \frac{v_o(s)}{4} + 2s \cdot v_o(s) = \frac{3}{s}$$

$$\frac{v_o(s)}{s} - \frac{10}{s(s+1)} + \frac{v_o(s)}{4} + 2s \cdot v_o(s) = \frac{3}{s}$$

Multiply all terms by s,

$$v_o(s) - \frac{10}{s+1} + \frac{s \cdot v_o(s)}{4} + 2s^2 \cdot v_o(s) = 3$$

$$v_o(s) + \frac{s \cdot v_o(s)}{4} + 2s^2 \cdot v_o(s) = 3 + \frac{10}{s+1}$$

$$v_o(s)\left(1 + \frac{s}{4} + 2s^2\right) = 3 + \frac{10}{s+1}$$

$$1 + \frac{s}{4} + 2s^2 = \frac{4 + s + 8s^2}{4} = \frac{8s^2 + s + 4}{4}$$

Therefore,

$$v_o(s) = \frac{3 + \frac{10}{s+1}}{\frac{8s^2+s+4}{4}}$$

$$v_o(s) = \frac{4\left(3 + \frac{10}{s+1}\right)}{8s^2 + s + 4}$$

$$v_o(s) = \frac{12 + \frac{40}{s+1}}{8s^2 + s + 4}$$

$$\frac{12 + \frac{40}{s+1}}{8s^2 + s + 4} = \frac{12}{8s^2 + s + 4} + \frac{40}{(s+1)(8s^2 + s + 4)}$$

$\frac{12}{8s^2+s+4}$, can be expressed as

$$\frac{12}{8s^2+s+4} = \frac{Ds+E}{\left(s+\frac{1}{16}\right)^2+\frac{127}{256}}$$

$$\frac{40}{(s+1)(8s^2+s+4)} = \frac{A}{s+1} + \frac{Bs+C}{\left(s+\frac{1}{16}\right)^2+\frac{127}{256}}$$

Finding the denominator factors,

$$8s^2+s+4 = 8\left(s^2+\frac{s}{8}+\frac{1}{2}\right)$$

$$s^2+\frac{s}{8}+\frac{1}{2} = \left(s+\frac{1}{16}\right)^2+\frac{1}{2}-\frac{1}{256} = \left(s+\frac{1}{16}\right)^2+\frac{127}{256}$$

$$8s^2+s+4 = 8\left(\left(s+\frac{1}{16}\right)^2+\frac{127}{256}\right) = 8\left(s+\frac{1}{16}\right)^2+\frac{127}{32}$$

Solving for A, B, and C,

$$40 = A\left(8\left(-1+\frac{1}{16}\right)^2+\frac{127}{32}\right) = A\left(8\cdot\frac{225}{256}+\frac{127}{32}\right)$$

$$40 = A\left(\frac{225}{32}+\frac{127}{32}\right) = A\left(\frac{352}{32}\right) = 11A$$

$$A = \frac{40}{11}$$

$$B = -\frac{40}{11}$$

$$C = \frac{864}{11\sqrt{127}}$$

$$v_o(s) = \frac{40}{11}\frac{1}{s+1} + \frac{-40}{11}\frac{s}{\left(s+\frac{1}{16}\right)^2+\frac{127}{256}} + \frac{864}{11\sqrt{127}}\frac{1}{\sqrt{\left(s+\frac{1}{16}\right)^2+\frac{127}{256}}}$$

Find the inverse Laplace transform of each term:

$$\frac{40}{11}\frac{1}{s+1} \to \frac{40}{11}e^{-t}$$

$$\frac{-40}{11}\frac{s}{\left(s+\frac{1}{16}\right)^2+\frac{127}{256}}$$

$$\omega = \frac{\sqrt{127}}{16}, \text{ then:}$$

$$\frac{-40}{11}\frac{s}{\left(s+\frac{1}{16}\right)^2+\omega^2} \to \frac{-40}{11}e^{-\frac{t}{16}}\cos(\omega t)$$

$$\frac{864}{11\sqrt{127}}\frac{1}{\sqrt{\left(s+\frac{1}{16}\right)^2+\frac{127}{256}}}$$

$$\omega = \frac{\sqrt{127}}{16}$$

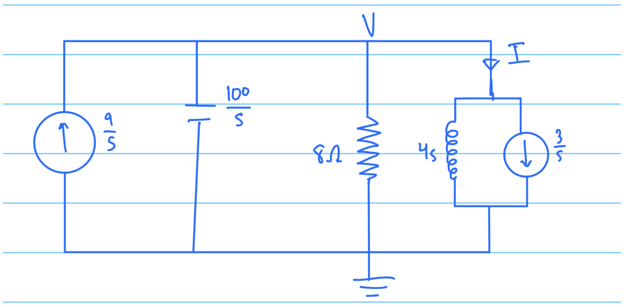
$$\frac{864}{11\sqrt{127}}\frac{1}{\sqrt{\left(s+\frac{1}{16}\right)^2+\omega^2}} \to \frac{864}{11\sqrt{127}}e^{-\frac{t}{16}}\sin(\omega t)$$

Combining all terms,

$$v_o(t) = \frac{40}{11}e^{-t} - \frac{40}{11}e^{-\frac{t}{16}}\cos\left(\frac{\sqrt{127}t}{16}\right) + \frac{864}{11\sqrt{127}}e^{-\frac{t}{16}}\sin\left(\frac{\sqrt{127}t}{16}\right)$$

16.44

First note the initial conditions. For $t < 0$, the inductor functions as a short circuit, yielding $v_c(0) = 0$ and $i(0) = 3$ A. Also see that this circuit can be simplified and transformed into:



Solving for V and I in the above circuit yields the following:

$$-\frac{9}{s} + \frac{V}{\frac{100}{s}} + \frac{V}{8} + \frac{V}{4s} + \frac{3}{s} = 0$$

$$I = \frac{V}{4s} + \frac{3}{s}$$

$$\frac{6}{s} = \left[\frac{s}{100} + \frac{1}{8}\frac{1}{4s}\right] \text{ V} = \frac{s^2 + 12.5s + 25}{100s} \text{ V}$$

$$V(s) = \frac{600}{(s+2.5)(s+10)}$$

Therefore,

$$I(s) = \frac{150}{s(s+2.5)(s+10)} + \frac{3}{s} = \frac{150 + 3(s+2.5)(s+10)}{s(s+2.5)(s+10)}$$

Performing partial fraction expansion and reconverting back into the time domain yields:

$$I(s) = \frac{A}{s} + \frac{B}{s+2.5} + \frac{C}{s+10}$$

$$A = (s)I(s)|_{s=0} = \frac{150+75}{25} = 9$$

$$B = (s+2.5)I(s)|_{s=-2.5} = \frac{150+0}{-2.5 * 7.5} = -8$$

$$C = (s+10)I(s)|_{s=-10} = \frac{150+0}{-10 * -7.5} = 2$$

Therefore,

$$I(s) = \frac{9}{s} - \frac{8}{s+2.5} + \frac{2}{s+10}$$

Thus,

$$i(t) = \mathbb{L}\{I(s)\} = u(t)\left(9 - 8e^{-2.5t} + 2e^{-10t}\right) \text{ A}$$

Homework for April 1, 2025

16.96

Given that V_o is the voltage across R, KCL yields

$$I_s = \frac{V_o}{R} + sCV_o + \frac{V_o}{sL} = V_o\left(\frac{1}{R} + sC + \frac{1}{sL}\right)$$

Solving for V_o,

$$V_o = \frac{I_s}{\frac{1}{R} + sC + \frac{1}{sL}}$$

$$V_o = \frac{sLI_sR}{sL + R + s^2LRC}$$

Given that $I_o = \frac{V_o}{R}$,

$$I_o = \frac{sLI_s}{sL + R + s^2LRC}$$

$$H(s) = \frac{I_o}{I_s} = \frac{sL}{sL + R + s^2LRC} = \frac{\frac{s}{RC}}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$$

This yields the following roots:

$$s_{1,2} = \frac{-\frac{1}{RC} \pm \sqrt{\left(\frac{1}{RC}\right)^2 - 4\left(\frac{1}{LC}\right)}}{2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \left(\frac{1}{LC}\right)}$$

Given that R, L, and C are positive, the roots s_1 and s_2 must be negative. Therefore, the system is stable.