

Week 5 Homework

Bryan SebaRaj

Professor Hong Tang

EENG 203 - Circuits and System Design

February 18, 2025

Homework for February 11, 2025

14.61

(a) Note that in this active filter,

$$V_o = V_- \text{ and } V_+ = V_i \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$$

Recall that $V_+ = V_-$. As such,

$$V_i \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = V_o$$

Thus,

$$H(\omega) = \frac{V_o}{V_i} = \frac{j\omega C}{j\omega C} \cdot \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{j\omega RC + 1}$$

(b) Note that in this active filter,

$$V_o = V_- \text{ and } V_+ = V_i \frac{R}{R + \frac{1}{j\omega C}}$$

Recall that $V_+ = V_-$. As such,

$$V_i \frac{R}{R + \frac{1}{j\omega C}} = V_o$$

Thus,

$$H(\omega) = \frac{V_o}{V_i} = \frac{j\omega C}{j\omega C} \cdot \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{j\omega RC + 1}$$

14.66

14.66 A “general” first-order filter is shown in Fig. 14.93.

(a) Show that the transfer function is

$$\mathbf{H}(s) = \frac{R_4}{R_3 + R_4} \times \frac{s + (1/R_1 C)[R_1/R_2 - R_3/R_4]}{s + 1/R_2 C},$$

$$s = j\omega$$

(b) What condition must be satisfied for the circuit to operate as a highpass filter?

(c) What condition must be satisfied for the circuit to operate as a lowpass filter?

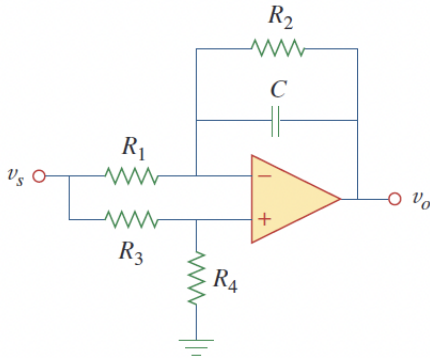


Figure 14.93

For Prob. 14.66.

(a) Let the node between R_3 and R_4 be denoted with a voltage v_a , the non-inverting input as v_+ and the inverting input as v_- . Trivially, $v_+ = v_a$

(b) Note that in order to operate as a high pass filter, $H(s)$ must take the form with some constant K ,

$$H(s) = K \cdot \frac{sR_f C}{sR_i C + 1}$$

Rewriting $H(s)$,

$$H(s) = \frac{R_4}{R_3 + R_4} \times \frac{R_2 C}{R_2 C} \times \frac{s + (1/R_1 C)[R_1/R_2 - R_3/R_4]}{s + 1/R_2 C} = \frac{R_4}{R_3 + R_4} \times \frac{sR_2 C + (R_2/R_1)[R_1/R_2 - R_3/R_4]}{sR_2 C + 1}$$

When $\frac{R_1}{R_2} = \frac{R_3}{R_4}$, $H(s)$ takes the form,

$$H(s) = \frac{R_4}{R_3 + R_4} \times \frac{sR_2 C + 0}{sR_2 C + 1} = \frac{R_4}{R_3 + R_4} \times \frac{sR_2 C}{sR_2 C + 1}$$

Thus, in order for this circuit to act as a high pass filter, $\frac{R_1}{R_2} = \frac{R_3}{R_4}$. This can be confirmed as $H(0) = 0$ and $H(\infty) = 1$.

(c) Using the previous intermediate form,

$$H(s) = \frac{R_4}{R_3 + R_4} \times \frac{sR_2 C + (R_2/R_1)[R_1/R_2 - R_3/R_4]}{sR_2 C + 1}$$

At $s = 0$,

$$H(0) = \frac{R_4}{R_3 + R_4} \times \frac{0 + (R_2/R_1)[R_1/R_2 - R_3/R_4]}{0 + 1} = \frac{R_4}{R_3 + R_4} (R_2/R_1)(R_1/R_2 - R_3/R_4)$$

Therefore in order to yield a nonzero DC gain at $s = 0$,

$$\frac{R_1}{R_2} \neq \frac{R_3}{R_4}$$

In order for $H(\infty) = 0$, see that $R_3 \rightarrow \infty$, which would yield,

$$H(\infty) = 0$$

Thus, the circuit will act as a low pass filter.

Homework for February 13, 2025

14.95

(a) Recall that a circuit with a variable capacitor and antenna coil uses the formula,

$$f = \frac{1}{2\pi\sqrt{LC}}$$

At $C = 40$ pF,

$$f = \frac{1}{2\pi\sqrt{240\mu H \cdot 40pF}} = 1.6244 \text{ MHz}$$

At $C = 360$ pF,

$$f = \frac{1}{2\pi\sqrt{240\mu H \cdot 360pF}} = 0.5415 \text{ MHz}$$

Thus, the circuit can tune for any frequency between 0.5415 MHz and 1.6244 MHz.

(b) In order to calculate Q , recall that the formula for Q is,

$$Q = \frac{2\pi f L}{R}$$

At $f_1 = 0.5415$ MHz,

$$Q = \frac{2\pi \cdot 0.5415 \text{ MHz} \cdot 240\mu H}{12\Omega} = 68.047$$

At $f_2 = 1.6244$ MHz,

$$Q = \frac{2\pi \cdot 1.6244 \text{ MHz} \cdot 240\mu H}{12\Omega} = 204.128$$

14.96

14.96
ed

The crossover circuit in Fig. 14.108 is a lowpass filter that is connected to a woofer. Find the transfer function $\mathbf{H}(\omega) = \mathbf{V}_o(\omega)/\mathbf{V}_i(\omega)$.

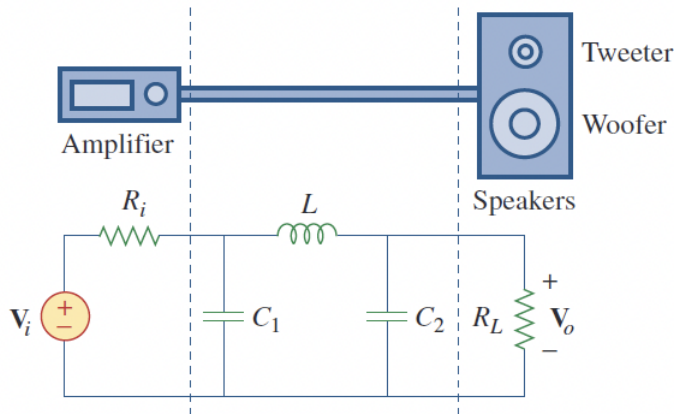


Figure 14.108

For Prob. 14.96.