Week 3 Homework

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EENG 203 - Circuits and System Design

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Homework for January 28, 2025

14.3

14.3 Given the circuit in Fig. 14.70, $R_1 = 2\ \Omega$, $R_2 = 5\ \Omega$, $C_1 = 0.1$ F, and $C_2 = 0.2$ F, determine the transfer function $\mathbf{H}(s) = \mathbf{V}_o(s)/\mathbf{V}_i(s)$.

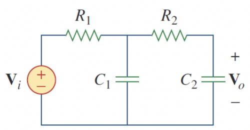


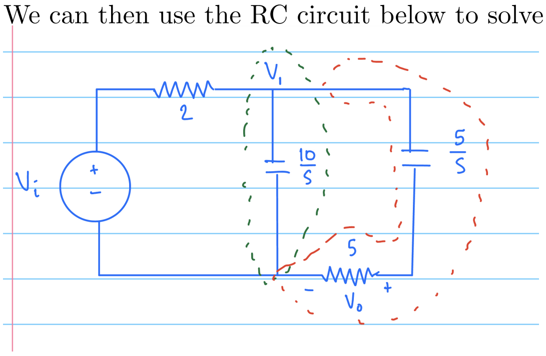
Figure 14.70
For Prob. 14.3.

Note that,

$$C_1 = 0.1F \rightarrow \frac{1}{j\omega C} = \frac{1}{s * 0.1F} = \frac{10}{s}$$

$$C_2 = 0.2F \rightarrow \frac{1}{j\omega C} = \frac{1}{s * 0.2F} = \frac{5}{s}$$

We can then use the RC circuit below to solve for the transfer function:



$$Z_{in}(\omega) = \frac{Z_1 Z_2}{Z_1 + Z_2} \text{ where } Z_1 = \frac{10}{s} \text{ and } Z_2 = \frac{5}{s} + 5$$

So,

$$Z_{in}(\omega) = \frac{\frac{10}{s}(\frac{5}{s} + 5)}{\frac{15}{s} + 5} = \frac{50 + 50s}{15s + 5s^2} = \frac{10 + 10s}{3s + s^2}$$

At V_1,

$$V_1 = \frac{Z_{in}(\omega)}{Z_{in}(\omega) + 2\Omega} V_i$$

$$V_o = \frac{5\Omega}{5\Omega + \frac{5}{s}} V_1 = \frac{5\Omega}{5\Omega + \frac{5}{s}} \frac{Z_{in}(\omega)}{Z_{in}(\omega) + 2\Omega} V_i$$

$$\frac{V_o}{V_i} = \frac{5\Omega}{5\Omega + \frac{5}{s}} \frac{Z_{in}(\omega)}{Z_{in}(\omega) + 2\Omega}$$

Solving for $H(s)$,

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{5}{5+\frac{5}{s}\frac{\frac{10+10s}{3s+s^2}}{\frac{10+10s}{3s+s^2}+2}} = \frac{s}{1+s}\frac{10(1+s)}{10+10s+6s+2s^2} = \frac{10s}{2s^2+16s+10} = \frac{5s}{s^2+8s+5}$$

$$H(s) = \frac{5s}{s^2+8s+5}$$

14.7

Section 14.3 The Decibel Scale

14.7 Calculate $|\mathbf{H}(\omega)|$ if H_{dB} equals

(a) 0.05 dB (b) −6.2 dB (c) 104.7 dB

Using the formula,

$$20\log_{10}|H(\omega)| = H_{dB} \rightarrow |H(\omega)| = 10^{H_{db}/20}$$

(a)

$$|H(\omega)| = 10^{H_{db}/20} = 10^{0.0025} = 1.00577$$

(b)

$$|H(\omega)| = 10^{H_{db}/20} = 10^{-0.31} = 0.48978$$

(c)

$$|H(\omega)| = 10^{H_{db}/20} = 10^{5.235} = 1.71791 \cdot 10^5$$

Homework for January 30, 2025

14.9

Section 14.4 Bode Plots

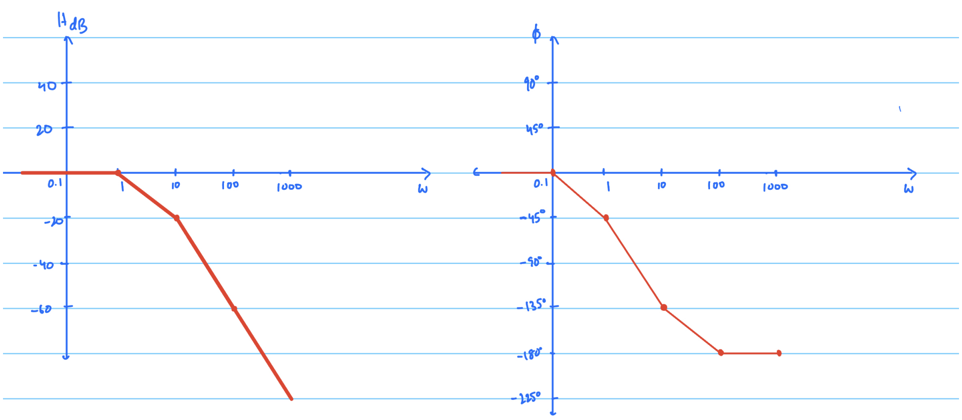
14.9 A ladder network has a voltage gain of

$$\mathbf{H}(\omega) = \frac{10}{(1+j\omega)(10+j\omega)}$$

Sketch the Bode plots for the gain.

$$H(\omega) = \frac{10}{10(1+j\omega)(1+\frac{j\omega}{10})} = \frac{1}{(1+j\omega)(1+\frac{j\omega}{10})}$$

Therefore, there are two poles at 1 and 10, and no zeros.



14.16

14.16 Sketch Bode magnitude and phase plots for

$$H(s) = \frac{1.6}{s(s^2 + s + 16)}, \quad s = j\omega$$

$$H(s) = \frac{1.6}{16s(\frac{s^2}{16} + \frac{s}{16} + 1)} = \frac{0.1}{s(1 + \frac{s^2}{16} + \frac{s}{16})}$$