Week 4 Homework

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EENG 203 - Circuits and System Design

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Homework for February 4, 2025

14.35

In a parallel RLC circuit,

(a) $\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{8mH \cdot 60\mu F} = 1443.38$ rad/s

(b) $B = \frac{1}{RC} = \frac{1}{5k\Omega \cdot 60\mu F} = 3.33$ rad/s

(c) $Q = \omega_o RC = 1443.38$ rads/s $\cdot 5k\Omega \cdot 60\mu F = 433.01$

14.40

In a parallel resonance circuit,

(a) From $B = \frac{1}{RC}$,

$$C = \frac{1}{BR} = \frac{1}{(\omega_2 - \omega_1)R} = \frac{1}{2\pi(f_2 - f_1)R} = \frac{1}{2\pi(4kHz) \cdot 2k\Omega} = 19.894 \text{ nF}$$

(b) From $\omega_0 = \frac{1}{\sqrt{LC}}$,

$$L = \frac{1}{\omega_o^2 C} = \frac{1}{\left(\frac{\omega_1+\omega_2}{2}\right)^2 \cdot C} = \frac{1}{(2\pi f_0 \cdot)^2 \cdot C} = 164.42\mu H$$

(c) $\omega_0 = \frac{\omega_1+\omega_2}{2} = 2\pi \frac{f_1+f_2}{2} = 2\pi \cdot 88kHz = 552.92$ krad/s

(d) $B = \omega_2 - \omega_1 = 2\pi(f_2 - f_1) = 8\pi$ krad/s $= 25.13$ krad/s

(e) $Q = \frac{\omega_0}{B} = \frac{176\pi}{8\pi} = 22$

Homework for February 6, 2025

14.50

Note that $H(\omega) = \frac{V_o}{V_i} = \frac{j\omega L}{R + j\omega L}$

$H(0) = \frac{0}{R+0} = 0$ and $H(\infty) = \frac{j\infty L}{R+j\infty L} = 1$. Therefore, the given circuit is a high pass filter.

Calculating the corner frequency, using $H(\omega_c) = \frac{1}{\sqrt{2}}$,

$$H(\omega_c) = \frac{1}{\sqrt{1 + \left(\frac{R}{\omega_c L}\right)^2}}$$

$$\sqrt{1 + \left(\frac{R}{\omega_c L}\right)^2} = \sqrt{2}$$

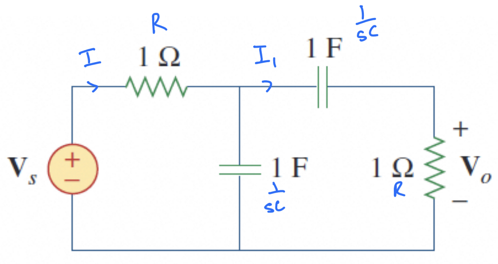
$$\frac{R}{\omega_c L} = \sqrt{1} = 1$$

$$\omega_c = \frac{R}{L}$$

$$2\pi f_c = \frac{R}{L}$$

$$f_c = \frac{R}{2\pi L} = 318.3 \text{ Hz}$$

14.57



(a)

First, the input impedance can be calculated as

$$Z(s) = R + \frac{\frac{1}{sC}\left(R + \frac{1}{sC}\right)}{\frac{1}{sC} + R + \frac{1}{sC}}$$

$$Z(s) = R + \frac{RsC + 1}{sC \cdot sC \cdot (R + \frac{2}{SC})} = R + \frac{RsC + 1}{2sC + s^2RC^2}$$

Recall that

$$I = \frac{V_s}{Z}$$

$$I_1 = \frac{\frac{1}{sC}}{\frac{1}{sC} + R + \frac{1}{sC}} I = \frac{V_s}{Z(sRC + 2)} = \frac{V_s}{\frac{1 + 3RsC + s^2R^2C^2}{sC}} = \frac{V_s sC}{1 + 3RsC + s^2R^2C^2}$$

$$V_o = I_1 R = \frac{RV_s sC}{1 + 3RsC + s^2R^2C^2}$$

$$\frac{V_o}{V_s} = H(s) = \frac{RsC}{1 + 3RsC + s^2R^2C^2}$$

Dividing the numerator and denominator by R^2C^2,

$$H(s) = \frac{\frac{s}{RC}}{\frac{1}{R^2C^2} + \frac{3s}{RC} + s^2}$$

Rewriting to balance the $\frac{s}{RC}$ coefficient,

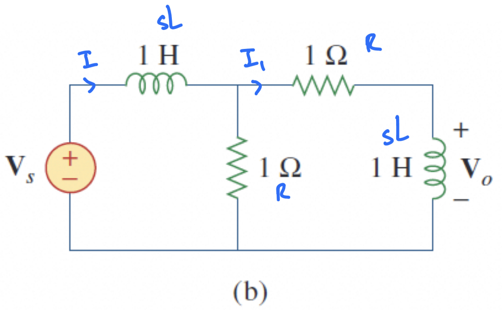
$$H(s) = \frac{1}{3}\left(\frac{\frac{3s}{RC}}{\frac{1}{R^2C^2} + \frac{3s}{RC} + s^2}\right)$$

Therefore, $\omega_0^2 = \frac{1}{R^2C^2}$, so

$$\omega_0 = \frac{1}{RC} = 1 \text{ rad/s}$$

$$B_{\text{bandpass filter}} = \frac{3}{RC} = 3 \text{ rad/s}$$

(b)



$$Z(s) = sL + \frac{R(R+sL)}{R+sL+R} = \frac{R^2+3sRL+s^2L^2}{2R+sL}$$

Recall that

$$I = \frac{V_s}{Z}$$

$$I_1 = \frac{R}{sL+2R}I = \frac{V_sR}{Z(sL+2R)} = \frac{V_sR}{R^2+3sRL+s^2L^2}$$

$$V_o = I_1 sL = \frac{V_sRL}{R^2+3sRL+s^2L^2}$$

$$\frac{V_o}{V_s} = H(s) = \frac{R}{R^2+3sRL+s^2L^2}$$

Dividing the numerator and denominator by L^2,

$$H(s) = \frac{\frac{sR}{L}}{\frac{R^2}{L^2}+\frac{3sR}{L}+s^2}$$

Rewriting to balance the $\frac{sR}{L}$ coefficient,

$$H(s) = \frac{1}{3}\left(\frac{\frac{3sR}{L}}{\frac{R^2}{L^2}+\frac{3sR}{L}+s^2}\right)$$

Therefore, $\omega_0^2 = \frac{R^2}{L^2}$, so

$$\omega_0 = \frac{R}{L} = 1 \text{ rad/s}$$

$$B_{\text{bandpass filter}} = \frac{3R}{L} = 3 \text{ rad/s}$$