Week 7 Homework

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EENG 203 - Circuits and System Design

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Homework for February 25, 2025

16.13

16.13 Using Fig. 16.36, design a problem to help other students better understand circuit analysis using Laplace transforms.

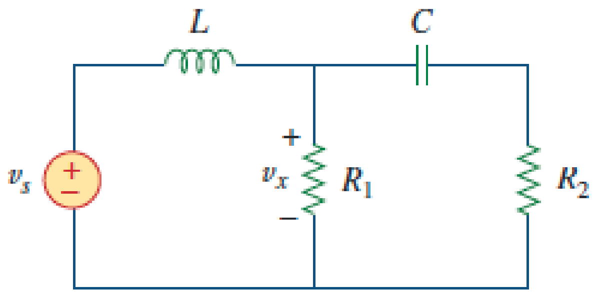

Figure 16.36
For Prob. 16.13.

Given the circuit show in Figure 16.36, suppose that $v_s = 4u(t)$, $L = 1$ H, the capacitor C is equivalent to $C = \frac{1}{8}$ H, $R_1 = 2\ \Omega$, and $R_2 = 4\ \Omega$. Find v_x

We first calculate

$$\frac{V_x - \frac{4}{s}}{s} + \frac{V_x}{2} + \frac{V_x}{4 + \frac{8}{s}} = 0$$

Multiplying by $s(4s+8)$, we get

$$V_x(4s+8) - \frac{16s+32}{s} + V_x(s(2s+4)) + V_x s^2$$

$$V_x(3s^2 + 8s + 8) = \frac{16s+32}{s}$$

$$V_x = \frac{16s+32}{s(3s^2+8s+8)}$$

Separating into partial fractions, we get

$$V_x = 16\left(\frac{A}{s} + \frac{B}{s + \left(\frac{4+j\sqrt{8}}{3}\right)} + \frac{C}{s + \left(\frac{4-j\sqrt{8}}{3}\right)}\right)$$

Solving for A, B, and C, we get

$$A = \frac{1}{4}$$

$$B = -\frac{1}{8}$$

$$C = -\frac{1}{8}$$

Therefore,

$$V_x = \frac{4}{s} - \frac{2}{s + \left(\frac{4+j\sqrt{8}}{3}\right)} - \frac{2}{s + \left(\frac{4-j\sqrt{8}}{3}\right)}$$

Taking the inverse Laplace transform, we get

$$v_x = 4u(t) - 4e^{-\frac{4}{3}t} \cos\left(\frac{2\sqrt{2}}{3}t\right) u(t) \text{ V}$$

16.15

16.15 For the circuit in Fig. 16.38, calculate the value of R needed to have a critically damped response.

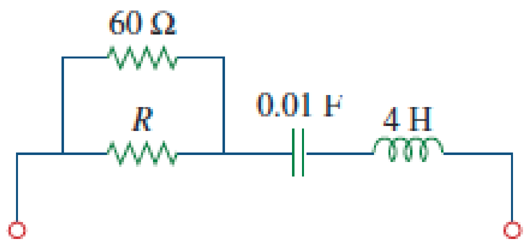

Figure 16.38
For Prob. 16.15.

Let $R_o = R||60\ \Omega$. Converting the circuit to the $s - domain$,

$$T(s) = R_o + \frac{1}{0.01s} + 4s = R_o + \frac{100}{s} + 4s = \frac{4s^2 + sR_o + 100}{s}$$

Therefore,

$$s_{\pm} = \frac{-R_o \pm \sqrt{R_o - 1600}}{2}$$

the system is critically damped when $R_o = 40$.

$$R_0 = \frac{R \times 60}{R + 60}$$

$$20R = 2400$$

$$R = 120\ \Omega$$