Week 2 Homework

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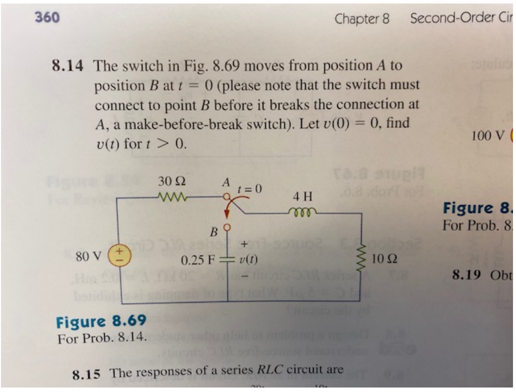
EENG 203 - Circuits and System Design

January 28, 2025

Homework for January 21, 2025

8.14



360 Chapter 8 Second-Order Cir

8.14 The switch in Fig. 8.69 moves from position A to position B at $t = 0$ (please note that the switch must connect to point B before it breaks the connection at A, a make-before-break switch). Let $v(0) = 0$, find $v(t)$ for $t > 0$.

Figure 8.69
For Prob. 8.14.

Figure 8.
For Prob. 8.

8.19 Obt

8.15 The responses of a series *RLC* circuit are

When moving the switch from position A to position B at $t = 0$, $v(0) = v(0^-) = 0$ since voltage across a capacitor cannot change instantaneously, and $i_L(0) = \frac{80V}{30\Omega+10\Omega} = 2A$.

After $t = 0$, the circuit is a source-free series RCL circuit.

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4H \cdot 0.25F}} = 1 \text{ Hz}$$

$$\alpha = \frac{R}{2L} = \frac{10\Omega}{1 \cdot 4H} = 1.25 \text{ Hz}$$

Since $\alpha > \omega_0$, the circuit is overdamped.
Therefore,

$$s_\pm = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -1.25 \pm \sqrt{1.25^2 - 1^2} = -1.25 \pm 0.75$$

$$s_+ = -0.5 \text{ and } s_- = -2$$

Thus,

$$v(t) = A_1^{-2t} + A_2 e^{-0.5t}$$

At $t = 0$, $v(0) = A_1 + A_2 = 0$ and $i_C(0) = C\frac{dv(0)}{dt} = -2$
Rewriting in terms of $\frac{dv}{dt}$,

$$\frac{dv(0)}{dt} = \frac{-2}{C} = \frac{-2}{0.25F} = -8 \text{ A}$$

Differentiating $v(t)$,

$$\frac{dv(t)}{dt} = -2A_1 e^{-2t} - 0.5A_2 e^{-0.5t}$$

At $t = 0$, $\frac{dv(0)}{dt} = -2A - 0.5B = -8$
Substituting $A_1 = 4 - 0.25A_2$ into $v(0) = A_1 + A_2 = 0$,

$$4 - 0.25A_2 + A_2 = 0$$

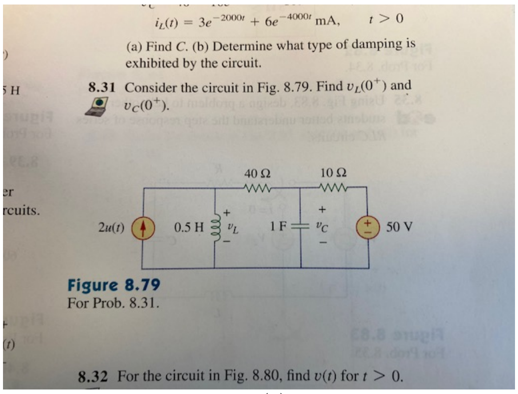
$$A_2 = -5.3333$$

$$A_1 = 5.3333$$

Thus,

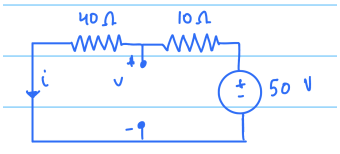
$$v(t) = 5.3333e^{-2t} - 5.3333e^{-0.5t} \text{ V}, \, , \forall t \geq 0$$

8.31

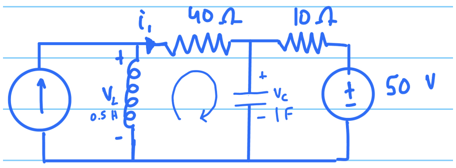


At $t = 0^-$, the $2u(t)$ current source acts as an open circuit, and the 1 F capcitor acts as an open circuit (via trivial properties of a step function and capacitor with a source, respectively). The inductor at steady state ($t = 0^-$) behaves as a short circuit.

The circuit at $t = 0^-$:



The circuit at $t = 0^+$:



The current through the entire circuit at $t = 0^-$,

$$i(0^-) = i(0^+) = \frac{V_{source}(0^-)}{R_{total}} = \frac{50V}{10\Omega + 40\Omega} = 1A$$

Calculating the voltage at the node with the open circuit capacitor (i.e. the 40 Ω resistor),

$$v(0^-) = v(0^+)_C = i \cdot R_1 = 1A \cdot 40\Omega = 40 \text{ V}$$

By KCL, the current at the node with the current source, 40 Ω resistor, and 0.5 H inductor is

$$i(0^+) + i_1 = 1 + i_1 = 2, \text{ so } i_1 = 1$$

Through KVL, the volage across the inner loop is

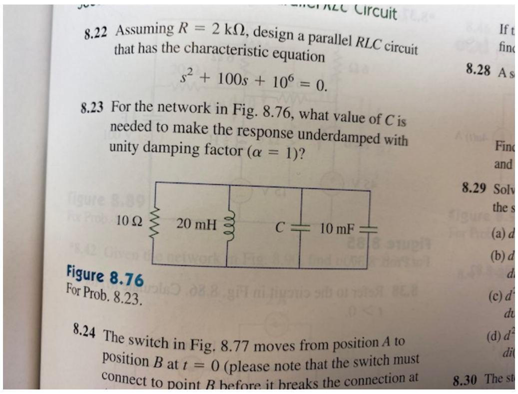
$$-v_L(0^+) + 40i_1 + v_C(0^+) = 0$$

$$v_L(0^+) = 40 \cdot 1 + 40 \text{ V} = 80 \text{ V}$$

Thus, $v_L(0^+) = 80$ V and $v_C(0^+) = 40$ V

Homework for January 23, 2025

8.23



8.22 Assuming $R = 2$ kΩ, design a parallel RLC circuit that has the characteristic equation

$$s^2 + 100s + 10^6 = 0.$$

8.23 For the network in Fig. 8.76, what value of C is needed to make the response underdamped with unity damping factor ($\alpha = 1$)?

Figure 8.76
For Prob. 8.23.

8.24 The switch in Fig. 8.77 moves from position A to position B at $t = 0$ (please note that the switch must connect to point B before it breaks the connection at

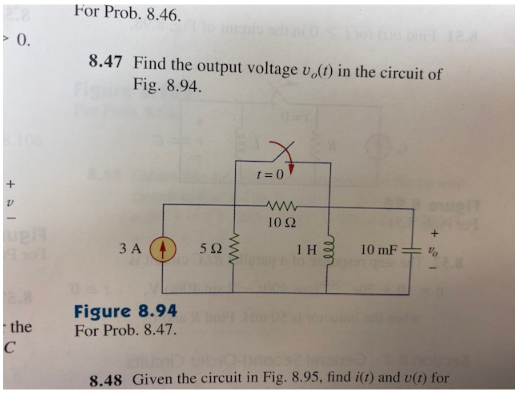
This is a source-free, parallel RCL circuit. Let the total capacitance be $C_{total} = C + 10$ mF . In a parallel circuit, $\alpha = \frac{1}{2RC_{total}}$ and $\omega_0 = \frac{1}{\sqrt{LC_{total}}}$.
Given that $\alpha = 1$,

$$\frac{1}{2RC_{total}} = 1$$

$$C_{total} = \frac{1}{2 \cdot 10\Omega} = 50 \text{ mF}$$

Thus, $C = C_{total} - 10 \text{ mF} = 40 \text{ mF}$.

8.47



For Prob. 8.46.

8.47 Find the output voltage $v_o(t)$ in the circuit of Fig. 8.94.

Figure 8.94
For Prob. 8.47.

8.48 Given the circuit in Fig. 8.95, find $i(t)$ and $v(t)$ for

At $t = 0^-$, the circuit is a RLC circuit with a current source, with

$$i_L(0^-) = i_{source}\frac{r_1}{r_1 + r_2}$$

where r_1 and r_2 are the 5Ω and 10Ω resistors, respectively.

$$i_L(0^-) = 3 \text{ V } \frac{5\Omega}{5\Omega * 10\Omega} = 1 \text{ A}$$

Since the 10 mF capacitor will function as an open circuit at $t = 0^-$,

$$v_o(0) = 0$$

At $t > 0$, the 10 Ω resistor will function as a short circuit, creating a parallel RLC circuit.

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \cdot 5\Omega \cdot 10\text{ mF}} = 10\text{ Hz}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1H \cdot 10mF}} = 10\text{ Hz}$$

Note that $\alpha = \omega_0$, so the circuit is critically damped.

$$s_{\pm} = -\alpha \pm \sqrt{\alpha^2 - \omega^2} = -10 \pm \sqrt{10^2 - 10^2} = 10$$

Therefore,

$$i(t) = i_s + (A_1 t + A_2)e^{-\alpha t} = 3 + (A_1 t + A_2)e^{-10t}$$

At $t = 0$, $i(0) = 3 + A_2 = 1$, so

$$A_2 = -2$$

$$v_o = L\frac{di}{dt} = L[A_1 e^{-10t} + (-10(A_1 t + A_2)e^{-10t})]$$

$$v_o(0) = A_1 - 10A_2 = 0$$

$$A_1 = -20$$

Thus,

$$v_o(t) = -20e^{-10t} + 200te^{-10t} + 20e^{-10t}$$

$$v_o(t) = 200te^{-10t}\text{ V}$$