Week 9 Homework

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16.102

From the given circuit, first define node 1 as the node between Y_1, Y_2, and Y_4, and node 2 as the node between Y_2, Y_3, and the op amp. Also see that $V_o = V_2$.
Starting with node 1,

$$(V_{\text{in}} - V_1)Y_1 = (V_1 - V_{\text{o}})Y_2 + (V_1 - V_{\text{o}})Y_4$$

$$V_{\text{in}}Y_1 = V_1(Y_1 + Y_2 + Y_4) - V_{\text{o}}(Y_2 + Y_4)$$

At Node 2,

$$(V_1 - V_{\text{o}})Y_2 = (V_{\text{o}} - 0)Y_3$$

$$V_1Y_2 = (Y_2 + Y_3)V_{\text{o}}$$

$$V_1 = \frac{Y_2 + Y_3}{Y_2}V_{\text{o}}$$

$$V_{\text{in}}Y_1 = \frac{Y_2 + Y_3}{Y_2}(Y_1 + Y_2 + Y_4)V_{\text{o}} - V_{\text{o}}(Y_2 + Y_4)$$

$$V_{\text{in}}Y_1Y_2 = V_{\text{o}}(Y_1Y_2 + Y_1Y_3 + Y_2^2 + Y_2Y_4 + Y_3Y_4 + Y_1Y_4 - Y_2^2 - Y_2Y_4)$$

$$\frac{V_{\text{o}}}{V_{\text{in}}} = \frac{Y_1Y_2}{Y_1Y_2 + Y_1Y_3 + Y_3Y_4 + Y_1Y_4}$$

Therefore, Y_1 and Y_2 are resistive, while Y_3 and Y_4 are capacitive. We can define $Y_1 = \frac{1}{R_1}, Y_2 = \frac{1}{R_2}$, $Y_3 = sC_1, Y_4 = sC_2$

$$\frac{V_{\text{o}}}{V_{\text{in}}} = \frac{\frac{1}{R_1R_2}}{\frac{1}{R_1R_2} + \frac{sC_1}{R_1} + \frac{sC_1}{R_2} + s^2C_1C_2}$$

$$\frac{V_{\text{o}}}{V_{\text{in}}} = \frac{\frac{1}{R_1R_2C_1C_2}}{s^2 + s\left(\frac{R_1+R_2}{R_1R_2C_2}\right) + \frac{1}{R_1R_2C_1C_2}}$$

Mapping the given transfer function onto this equation,

$$\frac{1}{R_1R_2C_1C_2} = 10^6$$

$$\frac{R_1 + R_2}{R_1R_2C_2} = 100$$

Solving for all possible combinations of C_1 and C_2 in terms of R_1 and R_2.

$$(R_1 + R_2) = 100 \cdot R_1R_2C_2$$

$$R_1R_2C_1C_2 = 10^{-6}$$

So,

$$C_1 = \frac{10^{-6}}{R_1R_2C_2}$$

$$\frac{R_1 + R_2}{R_1R_2C_2} = 100$$

$$R_1 + R_2 = 100 \cdot R_1R_2C_2$$

Solving for C_2 in terms of R_1 and R_2,

$$C_2 = \frac{R_1 + R_2}{100 \cdot R_1 R_2}$$

Now, let's find an expression for C_1 by substituting the expression for C_2:

$$C_1 = \frac{10^{-6}}{R_1 R_2 C_2}$$

$$= \frac{10^{-6}}{R_1 R_2 \cdot \frac{R_1+R_2}{100 \cdot R_1 R_2}}$$

$$= \frac{10^{-6} \cdot 100 \cdot R_1 R_2}{R_1 R_2 \cdot (R_1 + R_2)}$$

$$= \frac{10^{-4}}{R_1 + R_2}$$

Therefore,

$$C_1 = \frac{10^{-4}}{R_1 + R_2}$$

$$C_2 = \frac{R_1 + R_2}{100 \cdot R_1 R_2}$$

Finding a specific solution, suppose $R_1 = R_2 = 1\text{k}\Omega$. Therefore,

$$C_1 = 50 \text{ nF}$$

$$C_2 = 20 \ \mu\text{F}$$

16.103

Starting with the general formula for the transfer function using admittances,

$$\frac{V_o}{V_i} = \frac{-Y_1 Y_2}{Y_2 Y_3 + Y_4(Y_1 + Y_2 + Y_3)}$$

Note that the admittance of the input 0.5 μF capacitor is given by $Y_1 = sC_1$, the admittance of the two, 10 kΩ resistors are given by $Y_2 = Y_3 = \frac{1}{R_1}$, and the admittance of the 1 μF feedback capacitor is given by $Y_4 = sC_2$. Substituting these values into the transfer function,

$$\frac{V_o}{V_i} = \frac{-sC_1/R_1}{1/R_1^2 + sC_2(sC_1 + 2/R_1)} = \frac{-sC_1 R_1}{1 + sC_2 R_1(2 + sC_1 R_1)}$$

$$\frac{V_o}{V_i} = \frac{-sC_1 R_1}{s^2 C_1 C_2 R_1^2 + s \cdot 2C_2 R_1 + 1}$$

$$\frac{V_o}{V_i} = \frac{-s(0.5 \times 10^{-6})(10 \times 10^3)}{s^2(0.5 \times 10^{-6})(1 \times 10^{-6})(10 \times 10^3)^2 + s(2)(1 \times 10^{-6})(10 \times 10^3) + 1}$$

$$\frac{V_o}{V_i} = \frac{-100s}{s^2 + 400s + 2 \times 10,000}$$

Therefore,

$$a = -100, \quad b = 400, \quad c = 20,000$$