1. Converting to a sinusoidal model

Consider a real-valued signal represented as a sum of exponentials

$$s\left(t\right)=\sum_{n=1}^{n_r} d_n e^{\tau_n t}+\sum_{m=1}^{M} c_m e^{\eta_m t}+\sum_{m=1}^{M} \overline{c_m} e^{\overline{\eta_m} t}, \tag{1.1}$$

where d_n and τ_n are real valued and c_m and η_m are complex valued and appear in complex conjugate pairs.

Writing c_n in polar form and using the real and imaginary parts of η_n,

$$\begin{aligned} c_m &= \frac{1}{2}A_m e^{i\varphi_m} \\ \eta_n &= a_m + i\omega_m \end{aligned}$$

we have

$$\begin{aligned} c_m e^{\eta_m t}+\overline{c_m} e^{\overline{\eta_m} t} &= A_m e^{a_m t}\cos\left(\omega_m t+\varphi_m\right) \\ &= A_m e^{a_m t}\sin\left(\omega_m t+\phi_m\right) \end{aligned}$$

for phase shifts $\phi_m = \varphi_m + \frac{\pi}{2}$, real-valued amplitudes A_m, frequencies ω_m, and real-valued exponents a_m (which determine the potential rate of growth or decay present in the signal).

For current and voltages measured at the power grid, we do not expect purely real exponential terms for the first sum in (1.1) except for the case of a zero exponent $\tau_1 = 0$, corresponding to a potential dc term. Hence, our sinusoidal model for current and voltages is

$$s(t)=\sum_{m=1}^{M} A_m e^{a_m t}\sin\left(\omega_m t+\phi_m\right)+dc+\text{noise}. \tag{1.2}$$

where we sort terms by decreasing amplitudes, $A_1 \geq A_2 \geq \cdots \geq A_M$. For $M = 1$ and $a_1 = 0$, we recover the familiar representation of a single component

$$s\left(t\right)=A\sin\left(\omega t+\phi\right)+dc+\text{noise},$$

where ω is the fundamental frequency. The representation (1.2) is an extension of the one typically used in power systems, where only harmonics may be present and there are no exponents a_m.

Under normal operation, all exponents a_m are zero and a number of terms $M > 1$ indicates the presence of harmonics. These harmonics have amplitudes much smaller than the fundamental.

2. Data sets

2.1. Samples from a SEL - 700 G protection relay, located at the utility point of interconnection and pulled at raw (32 samples per cycle) resolution.

The three currents (I) and voltages (V) are at tng/testing/arpa-e/data/DG_POI_DG_Disconnect_2021-02-22a.csv