VIHL Business Calculus Assignment 02

Due Date: June 4th at the beginning of class

Question 1 [20 pts.] The manager of a 100-unit apartment complex knows from experience that all units will be occupied if the rent is \$800 per month. A market survey suggets that, on average, one additional unit will remain vacant for each \$10 increase in rent. What rent should the manager charge to maximize revenue?

Question 2 [25 pts.] The following citation is from chapter 7 of *Microeconomics: Theory and Applications with Calculus* by Jeffrey M. Perloff.

> "*In the (**short/long**) run, a firm cannot adjust the quantity of some inputs, such as capital. The firm varies its output by adjusting its variable inputs, such as (**capital/labor**). If all factors are fixed except labor, and a firm that was using very little labor increases its labor, its output may rise more than in proportion to the increase in labor because of greater specialization of workers. Eventually, however, as (**more/less**) workers are hired, the workers get in each other's way or wait to share equipment, so output (**increases/decreases**) by smaller and smaller amounts. This phenomenon is described by the law of diminishing marginal returns: The marginal product of an input–the extra output from the last unit of input–eventually (**increases/decreases**) as more of that input is used, holding other inputs fixed.*"

a) **[10 pts]** Select the correct term in each pair of boldfaced words so that the above description is consistent with the law of diminishing marginal returns. It may help to do some research on alternate statements of this law from various sources.

b) **[15 pts]** A mathematical statement of the law of diminishing marginal returns is given by

$$\frac{\partial \mathrm{MP}_L}{\partial L} = \frac{\partial(\partial q/\partial L)}{\partial L} = \frac{\partial^2 q}{\partial L^2} < 0 \tag{1}$$

Based on the above expression, what can you conclude about the concavitiy of q? What happens to productivity at the inflection point where $\partial q/\partial L = 0$? Sketch a graph of the function to justify your claims.

Question 3 [30 pts.] According to the analysis carried out by Hsieh (1995) of U.S manufacturing industries, the productivity of a firm in the paper industry can be modeled by a Cobb-Douglas production function of the form $q(L, K) = L^{0.44}K^{0.65}$.

a) **[10 pts]** In class we showed that given a multivariable function $f(x, y)$ for which $x(\lambda)$ and $y(\lambda)$ are functions of some parameter λ, the total derivative of $f(x, y)$ with respect to λ is given by

$$\frac{df(x,y)}{d\lambda} = \frac{\partial f}{\partial x}\frac{dx}{d\lambda} + \frac{\partial f}{\partial y}\frac{dy}{d\lambda} \tag{2}$$

This is a direct consequence of the **chain rule**. Now, returning to our production function, let $L(t)$ and $K(t)$ be functions of time measured in years. Use Equation (2) to obtain an expression for the total time derivative of q. What happens to the total derivative when the capital is fixed with respect to time at $K = \bar{K}$?

b) **[5 pts]** What is the average product of labor, AP_L, holding capital fixed at $K = \bar{K}$?

c) **[5 pts]** What is the marginal product of labor, MP_L ?

d) **[10 pts]** Does this production function have increasing, constant, or decreasing returns to scale? Discuss how this result relates to the concept of economies and diseconomies of scale.

Question 4 [25 pts.] The demand function for a certain commodity is $p(q) = 20 - 0.05q$. Find the consumer surplus when the sales level is 300. Illustrate your result by drawing the demand curve and identifying the consumer surplus as an area.