VIHL Business Calculus Assignment 1

Due Date: May 6th at the beginning of class

Question 1 [**50pts.**] Leo is the manager of a weekend flea market by Terminal Avenue. He knows from past experience that if he charges x dollars for a rental space at the market, then the number y of available spaces for rent is described by the equation $y = 200 - 4x$.

a) Make a sketch of this relation. Is it linear? Should the domain of the function include negative numbers or not? [**5pts**]

b) Compute the value of the slope, the $y-intercept$ and the $x-intercept$. What are the units of these quantites? Based on the value of the slope, argue whether individuals are expected to rent a space for the whole month or for a weekend only. [**15pts**]

c) Based on the above relation, at what price charge per rental space will all available spaces be rented out? Calculate the revenue at this value. [**10pts**]

d) Suppose the owner of the space charges Leo $8000 per month for rent of the premises. If Leo rents out every space, what is his profit? [**10pts**]

e) Classify the cost of renting the premises from the owner as fixed or variable. Why would it make sense to consider this as an overhead cost? [**10pts**]

Question 2 [**40pts.**] If $C(x)$ is the cost of producing x units of a commodity, then we define the **average cost** per unit as $c(x) = C(x)/x$.

a) Show that if the average cost is at a minimum, then the marginal cost equals the average cost [**15pts**].

b) If $C(x) = 16,000 + 200x + 4x^{3/2}$ in dollars, find [**10pts**]:

(i) the cost, average cost, and marginal cost at a production level of 1000 units

(ii) the production level that will minimize the average cost;

(iii) the minimum average cost

c) Suppose that the cost function given above is measuring the daily cost for continued production. Based on the value that you obtained to minimize the average cost, would you say this function describes better a factory or a retail-business? Which sector of industry would you place this business in ? [**5pts**]

d) Suppose that the managers of the firm decide to switch from mass production to batch production, where products are completed in batches of 4. Assuming that the fixed cost remains unchanged, which of the following options would best describe the new cost function? **[5pts]**

(a) $C(x) = 64,000 + 800x + 16x^{3/2}$

(b) $C(x) = 16,000 + 800x + 32x^{3/2}$

(c) $C(x) = 4,000 + 50x + x^{3/2}$

(d) $C(x) = 16,000 + 800x + 16x^{3/2}$

e) For your choice in d), compute the new production level that will minimize the average cost. Based on your result, would you advice the managers to switch from mass production to batch production **[5pts]**?

Question 3 [10pts.] If $p(w)$ is the total value of the production when there are w workers in a plant,then the *average productivity* of the worforce at the plant is

$$A(w) = \frac{p(w)}{w}$$

a) Find $A'(w)$. Why does the company want to hire more workers if $A'(w) > 0$? **[4pts]**

b) Show that $A'(w) > 0$ if $p'(w)$ is greater than the average productivity. **[2pts]**

c) How does increase in productivity in this example relate to the factors of production? Do you think that productivity in the plant can increase indefinitely? Why or why not? Relate this to what you know about demand. **[4pts]**