

Use optimization methods such as Simplex, Newton's method, Steepest descent and conjugate gradient to find the (x,y) coordinates corresponding to the minimum of the functions

$$f(x, y, z, w) = \frac{1}{70} \left(21x^2 + 20y^2 + 19z^2 - 14xz - 20yz \right) + w^2 \quad (1)$$

$$f(x, y) = 100 \left(y - x^2 \right)^2 + (1 - x)^2 \quad (2)$$

Show the solution paths with different starting solutions and comment on the performance of the optimization algorithms.