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Studying Higgs production at the LHC and future colliders

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Abstract. In this project we study the production mechanisms of the Higgs boson. After a small theoretical introduction to the Higgs mechanism, the amplitude and the cross-section of the gluon fusion process was computed at leading order. Next, an effective field theory for gluon fusion was set up and numerical results were obtained for higher order terms, using ihixs2, a program for computing Higgs cross sections, for different collision energies. Finally, the expected number of $H \to \gamma\gamma$ events were estimated for the LHC and FCC colliders, concluding that the FCC should be able to produce about 1000 times more events than the LHC.

KEYWORDS: LHC, Higgs, gluon fusion,cross section,ihixs2

1 Introduction

The final missing piece of the Standard Model of Particle Physics (SM), the Higgs boson, was finally discovered in 2013[1]. The Higgs mechanism, which includes the Higgs boson, is a key part of model, explaining how particles obtain their masses without destroying the gauge principles that give rise to the fundamental interactions, and it's discovery was one of the key goals of the LHC programme. Since the experimental observation of the Higgs boson, the focus of particle physics has shifted from discovery to detailed measurements of its properties and their interpretation in light of the SM and theories beyond it.

Many of the current open questions in physics, such as dark matter and neutrino masses, should, given our current understanding of the universe, be related to the Higgs field, as they are related to massive particles that should interact with the Higgs boson.

Therefore, an important property of the Higgs to study is it's production processes at our colliders, since undiscovered massive particles could cause deviations from our predictions, which would be a key indicator of new physics to be discovered.

Currently, he most significant method for Higgs production is the gluon-gluon fusion process $gg \to H$, (ggF), accounting for almost 90% of the Higgs production cross section at the LHC [2]. This makes it the most important process to study, and will therefore be the focus of this project.

Gluons do not couple to the Higgs boson, as they do not have mass, they do, however, couple to quarks. This makes the ggF process possible through a quark loop, where the gluons split into a quark anti-quark pair, who then annihilate each other to create a Higgs boson. Since the coupling of the Higgs to fermions, the Yukawa coupling, is proportional to the mass of the fermions, this means that the quark loop is dominated by the top quark, since it is the most massive of all the quarks. The goal of this project is to compute the cross section of the ggF process at leading order (LO) first, then at next leading order (NLO), and discuss the result and it's implications for physics at the LHC and future colliders.

This report is organized as follows. In section 2, we will do a theoretical introduction to the Higgs mechanism and QCD. In section 3 we will go over the ggF process and manually compute it's cross section at (LO), and discuss the implications of the result. In section 4 we will use a program to compute the cross section numerically and comment on the results. Finally, in section 5, we will make our conclusions and discuss the future outlook of this report.

2 Theoretical Framework

Gauge Symmetries

One of the key concepts of the SM is the concept of gauge symmetries, based on Noether's theorem, which states that every differentiable symmetry of the action has an associated conservation law, applied to the internal symmetries of quantum fields. Say we have a Dirac field Ψ, which obeys with the Lagrangian

$$\mathcal{L} = \bar{\Psi}\left(i\gamma^\mu\partial_\mu - m\right)\Psi \tag{1}$$

It is easy to see that this Lagrangian is invariant to the global $U(1)$ transformation $\Psi(x) \to e^{-i\alpha}\Psi(x)$, and therefore, according to Noether's theorem, we have a conserved current, given by

$$j^\mu = \bar{\Psi}\gamma^\mu\Psi \tag{2}$$

However, if we want this transformation to be local, or, in other words, to depend on the coordinates x^μ, then $\Psi(x) \to e^{-i\alpha(x)}\Psi(x)$, and we find that it is no longer invariant, as the derivatives introduce new terms to the Lagrangian,

$$\mathcal{L} = \bar{\Psi}\left(i\gamma^\mu\partial_\mu - m\right)\Psi + \bar{\Psi}\gamma^\mu\left(\partial_\mu\alpha\right)\Psi. \tag{3}$$

To fix this, we want to redefine the derivative operator, $\partial_\mu \to D_\mu$, such that the derivatives transform like the fields,

$$D_\mu\Psi \to e^{-i\alpha(x)}D_\mu\Psi(x). \tag{4}$$

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In order to achieves this, we must introduce a new vector field, B^μ, and define the covariant derivative

$$D_\mu \Psi = \left[\partial_\mu + igB_\mu\right]\Psi \tag{5}$$

To ensure this, then the B^μ field must transform as

$$B^\mu \to B^\mu + \frac{\partial^\mu \alpha(x)}{g}. \tag{6}$$

And, lastly, we must introduce the kinetic terms for this new field, we can add the field strength tensor, $F^{\mu\nu} = (\partial^\mu B^\nu + \partial^\nu B^\mu)$, to the Lagrangian and write

$$\mathcal{L} = i\bar{\Psi}\gamma^\mu \partial_\mu \Psi - m\bar{\Psi}\Psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - gB_\mu \gamma^\mu \bar{\Psi}\Psi. \tag{7}$$

So what this all means is, by imposing a local $U(1)$ symmetry on our initial Lagrangian, we have to introduce a new gauge field, which in turn mediates a new interaction between the fermions of the Dirac field. In the SM, all fundamental interactions arise out of imposing different internal symmetries on the Lagrangian, $U(1)$ for electromagnetism, $SU(2)$ for the weak force and $SU(3)$ for the strong force.

However, these new fields are massless, and, in fact, if we tried to introduce a mass term for the B field, such as $m^2 B_\mu B^\mu$, the Lagrangian would no longer be gauge invariant. This is a problem, as we know that the weak interaction bosons, the W and Z bosons, are massive. In order to introduce the boson masses into the SM we are going to have to, once again, introduce a new field, the Higgs field.

Spontaneous Symmetry Breaking

Spontaneous symmetry breaking (SSB), happens when a Lagrangian is exactly symmetric, but the ground state is not, for example, take a real scalar field ϕ, with the Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial^\mu \phi)\left(\partial_\mu \phi\right) - V(\phi) \quad ; \quad V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4, \tag{8}$$

with $\lambda > 0$. This potential is symmetric under the $\mathbb{Z}_2$ symmetry $\phi \to -\phi$. If $\mu^2 > 0$, then the potential has only one minimum $\phi = 0$, and the symmetry is unbroken. However, if we take $\mu^2 < 0$, now the potential has an unstable configuration at $\phi = 0$, and the true minima of $V(\phi)$ are at $\phi = \pm v$, with $v = \sqrt{\frac{-\mu^2}{\lambda}}$. If we introduce perturbations around the minima, defining

$$\phi = v + h(x) \quad \text{with} \quad \langle h \rangle = 0. \tag{9}$$

Reintroducing these perturbations back into the Lagrangian, and simplifying, we get

$$\mathcal{L} = \frac{1}{2}(\partial^\mu h)\left(\partial_\mu h\right) - \frac{1}{2}\left(2\lambda v^2\right)h^2 - \lambda v h^3 - \frac{1}{4}\lambda h^4 + \frac{1}{4}\lambda v^4, \tag{10}$$

The scalar perturbations $h(x)$ have acquired mass, $m_h = \sqrt{2\lambda v^2}$, while the new potential is no longer invariant under $\mathbb{Z}_2$ transformations, and we say that the symmetry has been spontaneously broken.

The U(1) Higgs Mechanism

Let us now take a complex scalar field ϕ and reintroduce the covariant derivative from Eq. (5) and field tensors into the Lagrangian, writing

$$\mathcal{L} = (D^\mu \phi)^* \left(D_\mu \phi\right) - \mu^2|\phi|^2 - \lambda|\phi|^4 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \tag{11}$$

where $\mu^2 < 0$ and $\lambda > 0$. Once again, the vacuum state breaks the symmetry, and we find that there are infinite possible ground states, all with $|\phi| = \sqrt{\frac{-\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}$.. We can choose the vacuum state to be $\phi = \frac{v}{\sqrt{2}}$, and do small perturbations around this minimum, writing them as $\phi = \frac{1}{\sqrt{2}}(v + \eta(x) + i\chi(x))$. After reintroducing the perturbations back into Eq. (11), expanding the kinetic terms for ϕ and simplifying, we end up with

$$\begin{aligned}\mathcal{L} = \frac{1}{2}(\partial^\mu \chi)\left(\partial_\mu \chi\right) + \frac{1}{2}(\partial^\mu \eta)\left(\partial_\mu \eta\right) - \lambda v^2 \eta^2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \\ + \frac{1}{2}g^2 v^2 B^\mu B_\mu + gvB_\mu \partial^\mu \chi - V_{int},\end{aligned} \tag{12}$$

where V_{int} includes higher order terms corresponding to interactions between η and B . This Lagrangian can be further reduced by performing the gauge transformation $B_\mu \to B_\mu + \frac{1}{gv}\partial_\mu \chi$, completely eliminating χ from the Lagrangian, leaving us with

$$\mathcal{L} = \frac{1}{2}(\partial^\mu \eta)\left(\partial_\mu \eta\right) - \lambda v^2 \eta^2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}g^2 v^2 B^\mu B_\mu - V_{int} \tag{13}$$

We are now left with a massive scalar η, while the χ field has been gauged away, but, most importantly, we see that the B boson now has mass $m_B = g^2 v^2$. Working in the unitary gauge, where χ vanishes, we can write the perturbations simply as $\phi = \frac{1}{\sqrt{2}}(v + h(x))$. Writing now the Lagrangian with all the interaction terms

$$\begin{aligned}\mathcal{L} = \frac{1}{2}(\partial^\mu h)\left(\partial_\mu h\right) - \lambda v^2 h^2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}g^2 v^2 B^\mu B_\mu \\ + g^2 v B_\mu B^\mu h + \frac{1}{2}g^2 B_\mu B^\mu h^2 - \lambda v h^3 - \frac{1}{4}\lambda h^4.\end{aligned} \tag{14}$$

This is the Lagrangian for the $U(1)$ Higgs mechanism, and shows us how we can introduce particle masses into the SM without plugging them directly into the Lagrangian, and, more importantly, preserving gauge symmetry. Besides giving us the boson masses, the second line also tells us how the Higgs interacts with the massive bosons and with itself.

In the reality, the SM Higgs mechanism is more complex than this. In order to give the weak bosons mass, it must break the $SU(2)_L \times SU(1)_Y$ symmetry the SM is built on, and it therefore must be a $SU(2)$ scalar doublet, making the real Higgs much more complicated. Despite this, the $U(1)$ mechanism still gives us a very clear understanding of how the SM Higgs works.

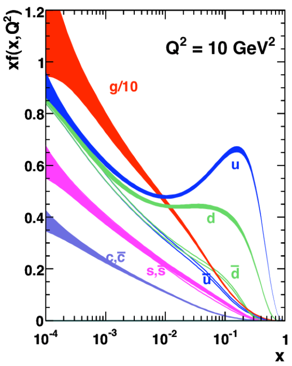

Figure 1. Example of a PDF

QCD and Proton Structure

Gluon fusion, as the name implies, involves gluons. However, due to colour confinement, independent gluons cannot be found in nature, as there exist no neutral colour gluons[3]. Gluons can, however, be found in hadrons, such as protons and neutrons.

One might be inclined to think that protons are somewhat simple structures, with two up quarks and one down quark, each one with about one third of the total momentum of the proton. In reality, protons are much more complex objects. Firstly, these quarks are constantly exchanging gluons with each other, which is how the proton stays together in the first place; and secondly, these gluons can sometimes split into $q\overline{q}$ pairs, complicating their structure even more. The three main quarks are usually called the valence quarks, while the rest of the quarks are usually referred to as the quark sea.

This means that, whenever we look inside a proton, we don't find just the three quarks we would expect, each with one third of the proton's total momentum, but we can actually find other quarks, anti-quarks and gluons. These particles, called partons, have a momentum distribution given by a parton distribution function (PDF), as seen in figure 1. So, when computing the cross section of a hadronic process, we must first integrate the amplitude in the partonic phase space, then integrate that result over the PDFs to obtain the total cross section of the process. For ggF, where we have two gluons in the initial state, we have,

$$\sigma_{total} = \int\int dx_1 dx_2 f(x_1) f(x_2) \sigma_{part}, \tag{15}$$

where x_1 and x_2 are the momentum fractions of each gluon, and $f(x)$ is the chosen PDF for gluons.

3 Gluon Fusion at LO

Out of the 4 main Higgs production methods at the LHC, ggF has by far the highest cross section, accounting for almost 90% of the total cross section. This process relies on a quark loop, as the gluons do not directly couple to the Higgs, so it's total amplitude $\mathcal{M}_{total}$ is given by

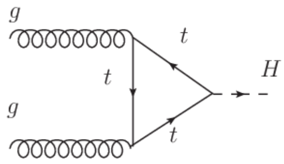

Figure 2. The first Feynman diagram for the ggF process using top quarks

$$\mathcal{M}_{total} = \sum_{quarks} \mathcal{M}^q, \tag{16}$$

as we must sum the amplitudes for all the quarks, furthermore, each quark has two possible diagrams, $\mathcal{M}_1^q$ and $\mathcal{M}_2^q$, one found in figure 2, the other being the same, but swapping the gluon lines. Since gluons are bosons, the relative sign between the diagrams is positive, and the diagrams end up having the same amplitude, so

$$\mathcal{M}^q = 2\mathcal{M}_1^q. \tag{17}$$

Lastly, since gluon fusion is dominated by the top channel, as we will soon see, we can write

$$\overline{|\mathcal{M}_{total}|^2} \simeq \overline{|\mathcal{M}^t|^2} = 4\overline{|\mathcal{M}_1^t|^2}. \tag{18}$$

For now, we will compute the amplitude for a single diagram of an arbitrary quark of mass m, which we will call $\mathcal{M}$.

3.1 Computing the ggF Amplitude

The Feynman rules for this process are :

1. Incoming gluons: $\varepsilon_1^\mu\, \varepsilon_2^\nu$;
2. Outgoing scalar: 1;
3. Fermion loop: -1 and integrate over the loop momentum;
4. Fermion propagators: $i\frac{(\not{q}+m)}{q^2-m^2}$, $i\frac{(\not{q}+\not{k}_1+m)}{(q+k_1)^2-m^2}$ and $i\frac{(\not{q}+\not{k}_1+\not{k}_2+m)}{(q+k_1+k_2)^2-m^2}$;
5. Quark-Gluon vertices: $\left(-ig_s\gamma_\mu T^a_{jk}\right)$ and $\left(-ig_s\gamma_\nu T^b_{kl}\right)$;
6. Yukawa vertex: $\left(-i\frac{g}{2}\frac{m}{m_W}\right)$;
7. Multiply by δ^{jl} to ensure the quark and anti-quark have the same color;
8. Finally, multiply by i;

where q is the quark loop momentum, k_1 and k_2 are the incoming gluons momenta and m is the quark's mass. So now, writing the full matrix element, we get

$$\mathcal{M} = -i\int \frac{d^4q}{(2\pi)^4}\varepsilon_1^\mu \left(ig_s\gamma_\mu T^a_{jk}\right) i\frac{(\not{q}+\not{k}_1+m)}{(q+k_1)^2-m^2}\left(-ig_s\gamma_\nu T^b_{kl}\right)\varepsilon_2^\nu$$
$$i\frac{(\not{q}+\not{k}_1+\not{k}_2+m)}{(q+k_1+k_2)^2-m^2}\left(-i\frac{g}{2}\frac{m}{m_W}\right) i\frac{(\not{q}+m)}{q^2-m^2}\delta^{jl}. \tag{19}$$

We can simplify this to

$$\mathcal{M} = \frac{1}{4\pi}\left(\sqrt{2}G_F\right)^{\frac{1}{2}} m\alpha_s Tr\left(T^a T^b\right)\varepsilon_1^{\mu}\varepsilon_2^{\nu}\int \frac{d^4q}{i\pi^2}\frac{T_{\mu\nu}}{D_0 D_1 D_2}, \tag{20}$$

where $\alpha_S = \frac{g_S^2}{4\pi}, G_F = \frac{\sqrt{2}g^2}{8m_W^2}$, $D_0 = \left(q^2 - m^2\right)$, $D_1 = \left((q+k_1)^2 - m^2\right)$, $D_2 = \left((q+k_1+k_2)^2 - m^2\right)$. If we explicitly write the indices of the Dirac matrices, we also find that

$$T_{\mu\nu} = \mathrm{Tr}\left[(\not{q}+m)\gamma_{\mu}(\not{q}+\not{k}_1+m)\gamma_{\nu}(\not{q}+\not{k}_1+\not{k}_2+m)\right]. \tag{21}$$

Since gluons are massless, they have only transverse components, therefore we can introduce the transverse projector, $P_{T\mu\nu} = \eta_{\mu\nu} - \frac{k_{1\mu}k_{2\nu}}{k_1.k_2}$, without losing any information, so now we can write the amplitude as

$$\mathcal{M} = \frac{1}{4\pi}\left(\sqrt{2}G_F\right)^{\frac{1}{2}} m\alpha_s Tr\left(T^a T^b\right)\varepsilon_1^{\mu}\varepsilon_2^{\nu} F P_{T\mu\nu}, \tag{22}$$

where

$$F = \frac{1}{4}P_T^{\mu\nu}\int \frac{d^4q}{i\pi^2}\frac{T_{\mu\nu}}{D_0 D_1 D_2}. \tag{23}$$

The spin averaged sum for this diagram is

$$|\overline{\mathcal{M}}|^2 = \frac{1}{2.2.8.8}\frac{1}{16\pi^2}\alpha_S^2\left(\sqrt{2}G_F\right)m^2\sum_{pol.}\sum_{a,b=1}^{8}\left|\frac{\delta^{ab}}{2}\varepsilon_1^{\mu}\varepsilon_2^{\nu}FP_{T\mu\nu}\right|^2. \tag{24}$$

The second half of the expression reduces to

$$\sum_{pol.}\sum_{a,b=1}^{8}\left|\frac{\delta^{ab}}{2}\varepsilon_1^{\mu}\varepsilon_2^{\nu}FP_{T\mu\nu}\right|^2 = \sum_{a=1}^{8}\left(\frac{2}{4}F^2\right) = 4F^2, \tag{25}$$

so now the final expression for the total amplitude squared is given by

$$|\overline{\mathcal{M}}|^2 = \frac{\sqrt{2}G_F\alpha_S^2}{32^2\pi^2}m^2F^2, \tag{26}$$

now we just need to compute F. First we have to compute the trace in Eq. 21, obtaining

$$\begin{aligned} T^{\mu\nu} &= 4m\left[4q^{\mu}q^{\nu} + 2q^{\mu}k_1^{\nu} + 4q^{\nu}k_1^{\mu} + 2q^{\nu}k_2^{\mu}\right. \\ &\left.+2k_1^{\mu}k_1^{\nu} + k_1^{\mu}k_2^{\nu} + k_1^{\nu}k_2^{\mu} + \eta^{\mu\nu}\left(m^2 - q^2 - 2q.k_1 - k_1.k_2 - k_1^2\right)\right]. \end{aligned} \tag{27}$$

Contracting $T^{\mu\nu}$ with the transverse projector, keeping in mind that $k_1^2 = k_2^2 = 0$ and $2k_1k_2 = m_H^2$, we get

$$P_{T\mu\nu}T^{\mu\nu} = 4m\left(3m^2 - m_H^2 - \frac{8}{m_H^2}k_1.q\ k_2.q - 2k_1.q + q^2\right). \tag{28}$$

To compute this integral, we need to use a technique called Passarino Veltman reduction, doing so gives us

$$F = \frac{1}{4}P_T^{\mu\nu}\int \frac{d^4q}{i\pi^2}\frac{T_{\mu\nu}}{D_0 D_1 D_2} = 2m + m(4m^2 - m_H^2)C_0(k_1,k_2,m), \tag{29}$$

where C_0 is the 3 point scalar function

$$C_0(k_1,k_2,m) = \begin{cases} -\frac{2}{m_H^2}\arcsin^2\left(\sqrt{\frac{1}{\rho}}\right), & \rho > 1 \\ \frac{1}{m_H^2}\left(\log\left(\frac{1+\sqrt{1-\rho}}{1-\sqrt{1-\rho}}\right) - i\pi\right)^2, & \rho < 1, \end{cases} \tag{30}$$

with $\rho = \frac{4m^2}{m_H^2}$.

This gives us two different amplitude expressions, one for quarks with $\rho < 1$, and one for quarks with $\rho > 1$. These expressions can be simplified to be

$$|\overline{\mathcal{M}}|^2 = \frac{\sqrt{2}G_F\alpha_S^2 m_H^4}{64^2\pi^2}A(\rho), \tag{31}$$

where $A(\rho)$ is given by

$$A(\rho) = \begin{cases} \rho^2\left(1 + (1-\rho)\arcsin^2\left(\sqrt{\frac{1}{\rho}}\right)\right)^2, & \rho > 1 \\ \rho^2\left(1 - \frac{1}{4}(1-\rho)\left|\log\left(\frac{1+\sqrt{1-\rho}}{1-\sqrt{1-\rho}}\right) - i\pi\right|^2\right)^2, & \rho < 1. \end{cases} \tag{32}$$

Since the quarks in this process are all virtual, they have no effect on the phase space of the problem, so the cross sections for different quarks are directly proportional to the amplitude, which in turn is proportional to $A(\rho)$.

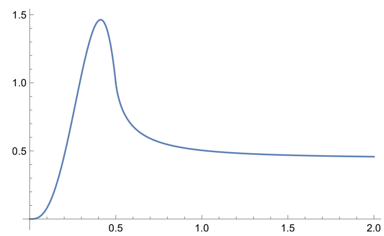

Figure 3. Plot of $A(\rho)$ for quark masses from 0 to $2m_H$

From this plot, we can clearly see that light quarks contribute much less to the cross section than more massive quarks. In fact, computing the ratio between the top quark and bottom quark, we get

$$\frac{|\overline{\mathcal{M}^t}|^2}{|\overline{\mathcal{M}^b}|^2} \simeq 150, \tag{33}$$

while the remaining quarks contribute even less than the bottom quark. This confirms our assumption that all other quarks can be safely ignored, therefore, the total amplitude for gluon fusion at LO becomes

$$\begin{aligned} |\overline{\mathcal{M}_{total}}|^2 &\simeq |\overline{\mathcal{M}^t}|^2 = 4|\overline{\mathcal{M}_1^t}|^2 \\ &= \frac{\sqrt{2}G_F\alpha_S^2 m_H^4}{32^2\pi^2}\rho^2\left(1 + (1-\rho)\arcsin^2\left(\sqrt{\frac{1}{\rho}}\right)\right)^2. \end{aligned} \tag{34}$$

We can also see what would happen if we introduced a new, fourth generation pair of quarks. Experiments from the LHC show that a new generation of quarks, that behave like the other SM quarks, should have a mass upwards of 1 TeV [4]. Therefore, these new quarks, q_1 and q_2, are much heavier than the Higgs boson, and we can study what happens to the amplitude in the case that $\rho \to \infty$. Taking the limit, we get

$$\lim_{\rho\to\infty} \rho \left(1 + (1-\rho)\arcsin^2\left(\sqrt{\frac{1}{\rho}}\right)\right)$$
$$= \lim_{\rho\to\infty} \rho \left[1 + (1-\rho)\left(\frac{1}{\rho} + \frac{1}{3\rho^2}\right)\right] + O\left(\frac{1}{\rho}\right) = \frac{2}{3}, \quad (35)$$

therefore,

$$\lim_{\rho\to\infty} A(\rho) = \frac{4}{9}. \quad (36)$$

If we now compared the amplitude for these quarks to the top quark, we find that they are quite similar, with $|\overline{\mathcal{M}^{q_1}}|^2 = |\overline{\mathcal{M}^{q_2}}|^2 \simeq 0.94|\overline{\mathcal{M}^{t}}|^2$. If we make the approximation that these three quarks have the same amplitude, then the total amplitude would now become

$$\mathcal{M}'_{total} = \sum_{quarks} \mathcal{M}^q \simeq \mathcal{M}^t + \mathcal{M}^{q_1} + \mathcal{M}^{q_2} \simeq 3\mathcal{M}^t = 3\mathcal{M}_{total}, \quad (37)$$

which results in an amplitude squared equal to

$$|\overline{\mathcal{M}_{total}}|^2 = 9|\overline{\mathcal{M}_{total}}|^2. \quad (38)$$

This, since gluon fusion currently makes up 90% of the total Higgs production cross section, would result in a total cross section about eight times larger than what we currently measure, seemingly ruling out the existence of these quarks. However, it is still possible to introduce fourth generation quarks while still complying with this constraint imposed by ggF.

The Yukawa coupling to up-like quarks is known to be positive [5] , however, it is not known whether it is positive or negative for down-like quarks. If this coupling were to be negative, then we would have that $\mathcal{M}^{q_1} = -\mathcal{M}^{q_2}$, meaning that

$$\mathcal{M}'_{total} = \sum_{quarks} \mathcal{M}^q \simeq \mathcal{M}^t + \mathcal{M}^{q_1} + \mathcal{M}^{q_2} = \mathcal{M}^t = \mathcal{M}_{total}, \quad (39)$$

and the ggF cross section would remain unchanged despite the existence of the new quarks.

3.2 Cross Section at LO

What we have computed so far is only for the partonic process. In reality, at the LHC, we have two protons colliding, and inside these protons is where we find the gluons. These gluons can carry varying portions of the total proton momentum. The total hadronic cross section is given by

$$\sigma_{total} = \int\int dx_1 dx_2 f(x_1) f(x_2)\, \sigma_{part}, \quad (40)$$

where x_1, x_2 are momentum fractions carried by each gluon, and $f(x)$ is the gluon parton distribution function. If we write each gluons' momentum as $k = xP$, P being the protons momentum, we find that

$$s = (k_1+k_2)^2 = (x_1P_1+x_2P_2)^2 = 4x_1x_2E^2 = x_1x_2S, \quad (41)$$

S being the center of mass energy of the proton collision. Here we have ignored the proton's mass, as its energy in colliders is always much larger than its mass.

The parton cross section is

$$\sigma_{part} = \frac{1}{2s}\int \frac{d^3p}{(2\pi)^3 2p_0}(2\pi)^4|\overline{\mathcal{M}_{total}}|^2 = \frac{\pi}{m_H^2}\delta\left(s - m_H^2\right)|\overline{\mathcal{M}_{total}}|^2. \quad (42)$$

From now, it will be useful to do a variable change from x_1 and x_2, to $x = x_1x_2$ and $y = \frac{1}{2}\log(\frac{x_1}{x_2})$. Now, $x_1 = \sqrt{x}\exp(y)$, $x_2 = \sqrt{x}\exp(-y)$, $dx_1dx_2 = dxdy$, and $s = xS$. The total hadronic cross section now becomes

$$\sigma_{total} = \frac{\pi}{m_H^2}|\overline{\mathcal{M}_{total}}|^2 \int\int dx\, dy\, f\left(\sqrt{x}\exp(y)\right)$$
$$\times f\left(\sqrt{x}\exp(-y)\right)\delta\left(xS - m_H^2\right). \quad (43)$$

Using the identity $\delta(\alpha x) = \frac{1}{|\alpha|}\delta(x)$ and simplifying, we arrive at

$$\sigma_{total} = \frac{\sqrt{2}G_F\alpha_S^2 m_H^2}{32^2\pi S}A(\rho)\int dy\, f\left(\sqrt{x_0}\exp(y)\right) f\left(\sqrt{x_0}\exp(-y)\right), \quad (44)$$

with $x_0 = \frac{m_H^2}{S}$.

In order to obtain a numerical value for the cross section, we now have to integrate over the PDFs. This, however, cannot be done by hand, and we must use a program to do this calculation for us, for this project, we chose to use Ihixs2 [6]. This program uses an effective field theory in order to speed up its calculations.

3.3 Effective Field Theory

In order to build the effective field theory, we can take the limit as the quark mass approaches infinity, where this interaction would behave as a point interaction. We can write the point interaction amplitude like this

$$\mathcal{M} = \mathcal{K}\, Tr\left(T^aT^b\right)\varepsilon_1^\mu\varepsilon_2^\nu P_{T\mu\nu}, \quad (45)$$

where we need to find what $\mathcal{K}$ is. Squaring and summing this amplitude we get

$$|\overline{\mathcal{M}}|^2 = \frac{1}{2.2.8.8}\mathcal{K}^2 \sum_{pol.}\sum_{a,b=1}^{8}\left|\frac{\delta^{ab}}{2}\varepsilon_1^\mu\varepsilon_2^\nu P_{T\mu\nu}\right|^2 = \frac{1}{64}\mathcal{K}^2, \quad (46)$$

now we just need to make sure it is equal to equation 31 in the large mass limit, with $A(\rho) = \frac{4}{9}$, so

$$\frac{1}{64}\mathcal{K}^2 = \frac{\sqrt{2}G_F\alpha_S^2 m_H^4}{64^2\pi^2}\frac{4}{9} \implies \mathcal{K} = \frac{(\sqrt{2}G_F)^{\frac{1}{2}}}{12\pi}m_H^2\alpha_S. \tag{47}$$

And the vertex for the effective field theory can be written by removing the incoming gluons from equation 45, obtaining

$$\sim \frac{(\sqrt{2}G_F)^{\frac{1}{2}}}{12\pi}m_H^2\alpha_S Tr\left(T^aT^b\right)P_{T\mu\nu}. \tag{48}$$

Using this vertex, we can compute the ggF amplitude in the large mass limit, simplifying higher order calculations. But as we have seen before, the amplitude for the top quark diagrams is slightly higher than the large mass limit, with $|\overline{\mathcal{M}^q}|^2 \simeq 0.94|\overline{\mathcal{M}^t}|^2$, meaning that, after performing the calculation using the effective field theory, we must then multiply our result by $\sim \frac{1}{0.94} \approx 1.06$ in order to obtain the correct result for the top quark loop amplitude. This is also already implemented into the ihixs2 code.

4 Numerical Results

Before computing the numerical results, we must define some constants when running the program. The mass of the Higgs boson we simply set to 125GeV, but we must also set μ_R and μ_F, these are the energy scales at which we will measure the strong coupling constant α_S, and the PDFs, respectively. Each of these quantities will vary depending on the energy level at which they are measured, for this project, we chose to set both of them to 62.5GeV, half of the mass of the Higgs, as a good middle ground for the energy level of the processes involved in ggF.

There are many PDF sets, all developed by different teams around the world. Ihixs2 lets us to choose the PDF we want, allowing us to compare results with different PDFs.

The first set of results uses the same PDF, NNPDF40, computing the cross section at increasing energy values, from 7TeV to 100TeV.

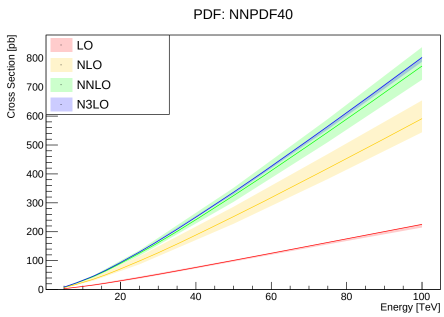

Figure 4. Plot of the cross section [pb] as a function of collision energy [TeV]

From figure 4, two things stand out. Firstly, the cross section increases almost linearly with the collider energy, being around twenty times larger at 100TeV than it is at the current 13TeV in the LHC.

Secondly, the LO approximation is very inaccurate, as it seems to only account for about one quarter of the total cross section. This is the result of two facts. The first being higher order processes in QCD open a lot of new channels, meaning that there are many new diagrams to take into account. Second, the strong coupling constant α_S is relatively large compared to the other interactions, being about ~ 0.1 at these energy scales, making it so it cannot cancel out the large number of extra diagrams that come with the NLO processes, and so we need to go up to N3LO in order to obtain an accurate result.

We can do a second plot which shows this more clearly.

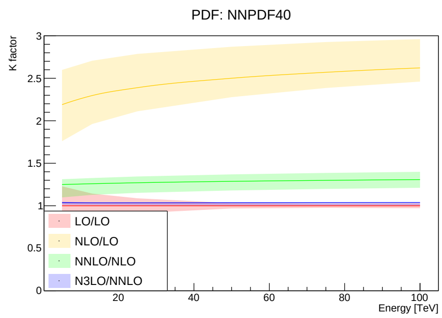

Figure 5. Plot of the relative contributions of each order computation as a function of collision energy [TeV]

In figure 5 we plot the relative contributions of each successive computational order compared to the previous. Here we can see more clearly that the NLO result dominates all others, being around 2.5 times larger than the LO cross section. We also see that N3LO contributes almost nothing compared to NNLO, meaning that any higher order terms can be safely ignored.

We can also compare the cross section for the same energy level with different PDFs

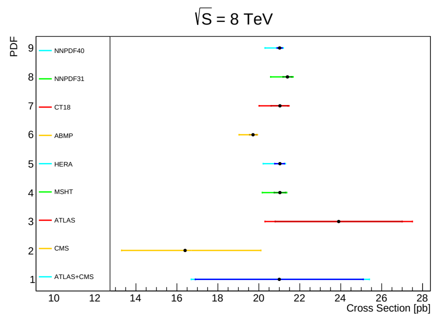

Figure 6. Plot of the cross section [pb] for various PDFs and experiments at 8TeV

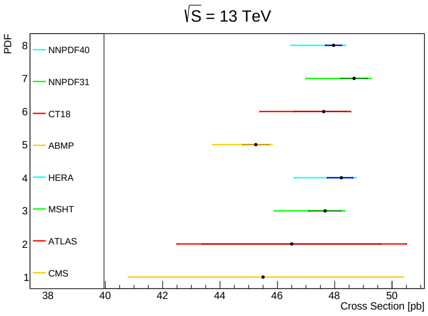

Figure 7. Plot of the cross section [pb] for various PDFs and experiments at 13TeV

In figures 6 and 7, we can compare various PDFs with each other, as well as the ATLAS and CMS experimental results from the LHC. We find that they mostly agree with each other, the only outlier being ABMP, which is a slightly outdated PDF. We also see that all the PDF agree with the experimental results, mostly due to the fact that the error bars on the experimental results are still quite large.

Finally, we can also compare how the Future Circular Collider (FCC) compares to the LHC in terms of producing Higgs bosons. During the first run of the LHC, it ran at an energy of 7TeV with an integrated luminosity of 4.8 fb^{-1}, and and an energy of 8TeV, with a luminosity of 20.7 fb^{-1}. The main way of detecting Higgs bosons is by through the $H \to \gamma\gamma$ decay channel, with a branching ratio of 0.2%, this mean we can calculate the number of expected events during the first run

$$N_{H\to\gamma\gamma} = BR_{H\to\gamma\gamma}\times\left[\sigma_{7TeV} \times 4.8 fb^{-1} + \sigma_{8TeV} \times 20.7 fb^{-1}\right] \approx 1000$$

As for the FCC, the estimated integrated luminosity is of $0.2 - 2$ ab^{-1} [7], doing the same calculation as before, we get

$$N_{H\to\gamma\gamma} = BR_{H\to\gamma\gamma}\times\left[\sigma_{100TeV} \times (0.2 - 2)ab^{-1}\right] \approx (0.32-3.2)\times 10^{6}$$

That is potentially one thousand times more $H \to \gamma\gamma$ events, per year! Such a collider would definitely reveal many more insights into the behaviour of the Higgs boson.

5 Conclusions

We have studied the production process of the Higgs boson, being able to obtain an expression for the gluon fusion cross section, as well as obtain numerical results using Ihixs2.

Starting from the ggF LO diagram, we were able to solve the three point integral for the quark loop, and obtain an analytical expression for the squared amplitude of the ggF process, obtaining a general function capable of computing the amplitude for a general quark with any given mass. From this, we were able to prove that the top quark is the single most important contributor, allowing us to reach a simple expression for the total amplitude.

Using the expression for the amplitude, we were able to obtain the partonic cross section and, from this, and expression the total cross section for the ggF process.

We were able to formulate an effective field theory, which simplifies the computation necessary to obtain the cross section, which was then calculated using the program Ihixs2. The numerical results obtained allowed us to see how the cross section evolves with the collision energy, while also agreeing with the currently available experimental data.

Finally, a simple calculation was made, showing how the proposed FCC should be able to produce around 1000 times more Higgs bosons per year than what was needed to originally discover the particle, which would surely boost the research efforts into this very important piece of the standard model.

Although the gluon fusion is the most important contribution to the Higgs cross section, in the future, a more detailed study should also focus on the other production processes, as they still account for around 10% of the total cross section. Additionally, the bottom quark's amplitude term is negligible, being about 150 times smaller than the top quark's, but the interference term between the top quark's and bottom quark's amplitude should also be studied, as it could also be of some significance.

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