Studying Higgs production at the LHC and future colliders

LIP summer internship

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Project supervisor: João Pires

08/09/2023

Gauge Symmetries

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$$\implies \mathcal{L} = \bar{\Psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\Psi + \bar{\Psi}\gamma^{\mu}\left(\partial_{\mu}\alpha\right)\Psi$$

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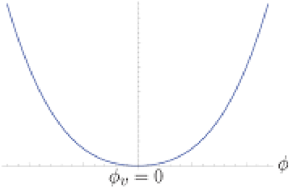
But mass terms for the fields, such as $m^2 B_\mu B^\mu$, break gauge symmetry.

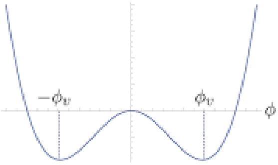
$$m^2 B^\mu B_\mu \to m^2 B^\mu B_\mu + \frac{2}{g} B^\mu \partial_\mu \alpha(x) + \frac{1}{g^2}\partial^\mu \alpha(x) \partial_\mu \alpha(x)$$

Spontaneous Symmetry Breaking (SSB)

$$\mathcal{L} = \frac{1}{2}\left(\partial^{\mu}\phi\right)\left(\partial_{\mu}\phi\right) - V(\phi) \quad ; \quad V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$$



$\mu^2 > 0$



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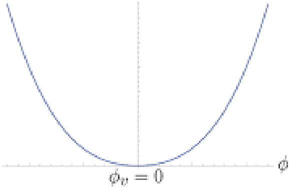

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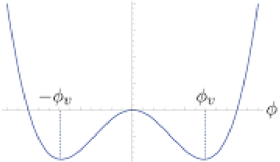

$\mu^2 < 0$

$$\phi(x) = v + h(x) \implies \mathcal{L} = \frac{1}{2}\left(\partial^\mu h\right)\left(\partial_\mu h\right) - \boxed{\frac{1}{2}\left(2\lambda v^2\right) h^2} - \lambda v h^3 - \frac{1}{4}\lambda h^4 + \frac{1}{4}\lambda v^4,$$

The $h(x)$ perturbations have mass $m = \sqrt{2\lambda} v$

The U(1) Higgs Mechanism

$$\mathcal{L} = (D^\mu \phi)^* (D_\mu \phi) - \mu^2 |\phi|^2 - \lambda |\phi|^4 - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

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Introducing perturbations $\phi(x) = \frac{1}{\sqrt{2}} (v + h(x) + i\chi(x))$.

$$\mathcal{L} = \frac{1}{2} (\partial^\mu \chi) (\partial_\mu \chi) + \frac{1}{2} (\partial^\mu h) (\partial_\mu h) - \lambda v^2 h^2 + \frac{1}{2} g^2 v^2 B^\mu B_\mu + g v B_\mu \partial^\mu \chi + (...)$$

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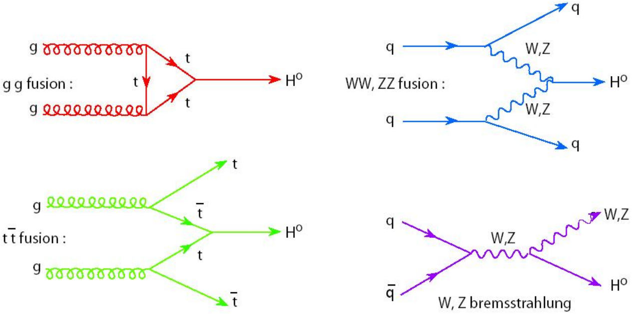
Performing the gauge transformation $B_\mu = B_\mu + \frac{1}{gv} \partial_\mu \chi$.

$$\mathcal{L} = \frac{1}{2} (\partial^\mu h)(\partial_\mu h) - \lambda v^2 h^2 + \boxed{\frac{1}{2} g^2 v^2 B^\mu B_\mu} + (...)$$

The B field now has mass $m_B = gv$.

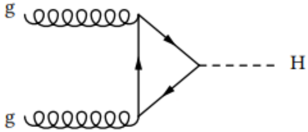
Higgs Production Processes

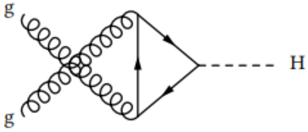
There are four main production processes for the higgs: gluon fusion, top anti-top fusion, vector boson fusion, and vector boson *bremsstrahlung*.
Of these, gluon fusion (ggF) is by far the most important, and it will be the focus of this project



Gluon Fusion at LO

There are two Feynman diagrams at leading order, both with the same amplitude





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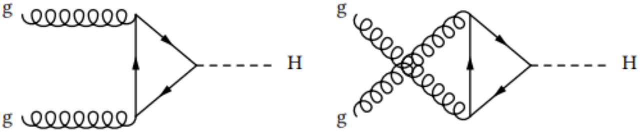

$$\mathcal{M} = -i \int \frac{d^4 q}{(2\pi)^4} \varepsilon_1^{\mu} \left(i g_s \gamma_\mu T^a_{jk} \right) i \frac{(\not{q} + \not{k}_1 + m)}{(q+k_1)^2 - m^2} \left(-i g_s \gamma_\nu T^b_{kl} \right) \varepsilon_2^{\nu} i \frac{(\not{q} + \not{k}_1 + \not{k}_2 + m)}{(q+k_1+k_2)^2 - m^2} \left(-i \frac{g}{2} \frac{m}{m_W} \right) i \frac{(\not{q} + m)}{q^2 - m^2} \delta^{jl}$$

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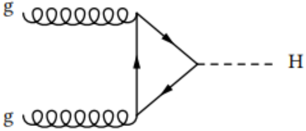

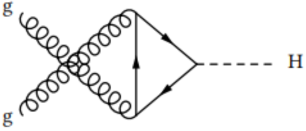

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$$\mathcal{M} = \frac{1}{4\pi}\left(\sqrt{2}G_F\right)^{\frac{1}{2}} m\alpha_s Tr\left(T^aT^b\right)\,\varepsilon_1^{\mu}\varepsilon_2^{\nu}\int\frac{d^4q}{i\pi^2}\frac{T_{\mu\nu}}{D_0D_1D_2}$$

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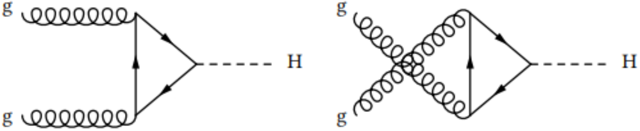

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$$T_{\mu\nu} = \mathrm{Tr}\left[(\not{q}+m)\gamma_\mu(\not{q}+\not{k}_1+m)\gamma_\nu(\not{q}+\not{k}_1+\not{k}_2+m)\right]$$

Gluon Fusion at LO (cont.)

Since gluons are massless, they only have transversal components, and we can introduce the transverse projector, $P_{T\mu\nu} = \eta_{\mu\nu} - \frac{k_{1\mu}k_{2\nu}}{k_1.k_2}$, without losing any information.

$$\Longrightarrow \ \varepsilon_1^{\mu}\varepsilon_2^{\nu} F P_{T\mu\nu} \quad \text{, with} \quad F = \frac{1}{4} P_T^{\mu\nu} \int \frac{d^4 q}{i\pi^2} \frac{T_{\mu\nu}}{D_0 D_1 D_2}.$$

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Now square the amplitude and sum over all the initial states of the gluons.

$$|\overline{\mathcal{M}}|^2 = \frac{1}{2.2.8.8} \frac{1}{16\pi^2} \alpha_S^2 \left(\sqrt{2} G_F\right) m^2 \sum_{pol.} \sum_{a,b=1}^{8} \left| \frac{\delta^{ab}}{2} \varepsilon_1^\mu \varepsilon_2^\nu F P_{T\mu\nu} \right|^2$$

$$\sum_{a=1}^{8} \left(\frac{P_{T\mu\nu} \eta^{\mu\alpha} \eta^{\nu\beta} P_{T\alpha\beta}}{4} F^2 \right) = 8 \left(\frac{P_T^{\mu\nu} P_{T\mu\nu}}{4} F^2 \right) = 4F^2$$

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So the expression for total amplitude squared is

$$\Longrightarrow \quad |\overline{\mathcal{M}}|^2 = \frac{\sqrt{2} G_F \alpha_S^2}{32^2 \pi^2} m^2 F^2$$

Result for the Amplitude

$$F = 2m + m\left(4m^2 - m_H^2\right) C_0\left(k_1, k_2, m\right)$$

$$C_0(k_1, k_2, m) = \begin{cases} -\frac{2}{m_H^2}\arcsin^2\left(\sqrt{\frac{1}{\rho}}\right), & \rho = \frac{4m^2}{m_H^2} > 1 \\ \frac{1}{m_H^2}\left(\log\left(\frac{1+\sqrt{1-\rho}}{1-\sqrt{1-\rho}}\right) - i\pi\right)^2, & \rho = \frac{4m^2}{m_H^2} < 1 \end{cases}$$

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$$\boxed{|\overline{\mathcal{M}}|^2 = \frac{\sqrt{2}G_F\alpha_S^2 m_H^4}{64^2\pi^2} A\left(\rho\right)}$$

with

$$A\left(\rho\right) = \begin{cases} \rho^2\left(1 + \left(1-\rho\right)\arcsin^2\left(\sqrt{\frac{1}{\rho}}\right)\right)^2, & \rho > 1 \\ \rho^2\left(1 - \frac{1}{4}\left(1-\rho\right)\left|\log\left(\frac{1+\sqrt{1-\rho}}{1-\sqrt{1-\rho}}\right) - i\pi\right|^2\right)^2, & \rho < 1. \end{cases}$$

Amplitude Plot

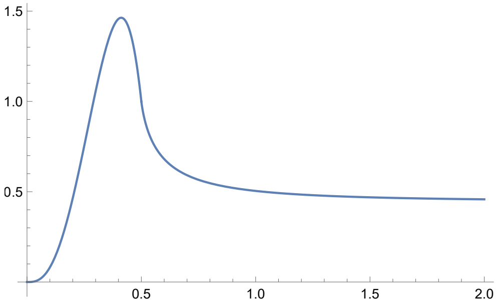

Figure: Plot of $A(\rho)$ for quark masses from 0 to $2m_H$

$$|\overline{\mathcal{M}^t}|^2 \simeq 150|\overline{\mathcal{M}^b}|^2$$

Cross Section at LO

$$\mathcal{M}_{total} = \sum_{quarks} \mathcal{M}^q \implies |\overline{\mathcal{M}_{total}}|^2 \simeq |\overline{2\mathcal{M}^t}|^2 = 4|\overline{\mathcal{M}^t}|^2$$

$$|\overline{\mathcal{M}_{total}}|^2 = \frac{\sqrt{2}G_F\alpha_S^2 m_H^4}{32^2\pi^2}\rho^2\left(1 + (1-\rho)\arcsin^2\left(\sqrt{\frac{1}{\rho}}\right)\right)^2$$

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The cross section is

$$\sigma_{part} = \frac{1}{2s} \int \frac{d^3 p}{(2\pi)^3 2p_0} (2\pi)^4 |\overline{\mathcal{M}_{total}}|^2 = \frac{\pi}{m_H^2} \delta\left(s - m_H^2\right) |\overline{\mathcal{M}_{total}}|^2$$

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$$\sigma_{total} = \frac{\sqrt{2}G_F\alpha_S^2 m_H^2}{32^2\pi S}A(\rho)\int dy\, f\left(\sqrt{\frac{m_H^2}{S}}\exp(y)\right) f\left(\sqrt{\frac{m_H^2}{S}}\exp(-y)\right)$$

Computational Results

Ihixs 2 - configuration

$m_H = 125\text{GeV}, \mu_R = 62.5 \text{ GeV}, \mu_F = 62.5 \text{ GeV}$

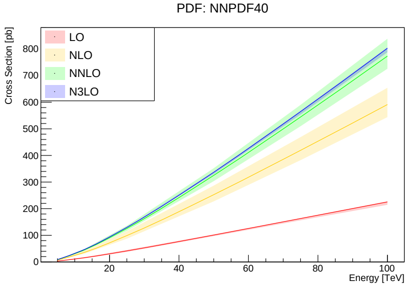


Figure: Plot of the cross section as a function of the collision energy

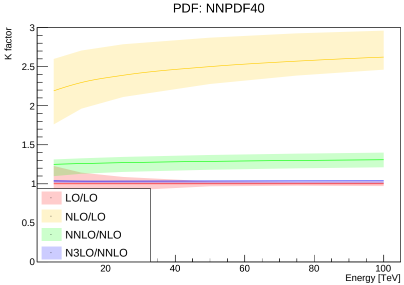


Figure: Plot of the relative contributions of each order as a function of the collision energy

Computational Results (cont.)

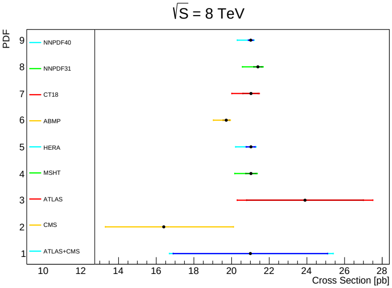


Figure: Plot of the cross section result for various PDFs at 8 TeV

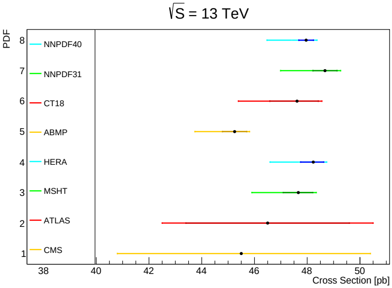
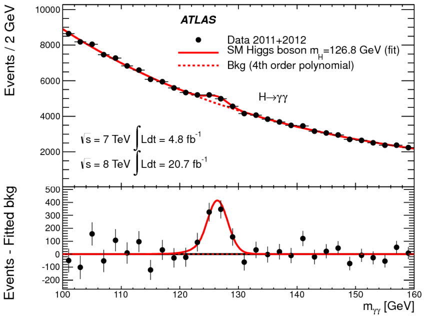


Figure: Plot of the cross section result for various PDFs at 13 TeV

Conclusions

$$N_{H\to\gamma\gamma} = BR_{H\to\gamma\gamma} \times \left[\sigma_{7TeV} \times 4.8fb^{-1} + \sigma_{8TeV} \times 20.7fb^{-1}\right] \approx 1000$$



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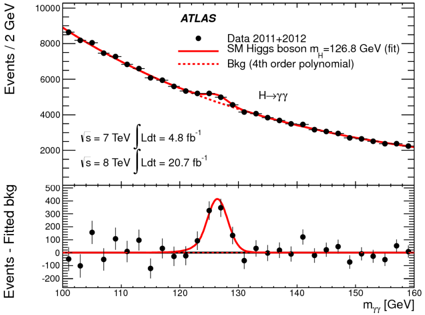

Future Circular Collider (FCC)

$$N_{H\to\gamma\gamma} = BR_{H\to\gamma\gamma} \times \left[\sigma_{100TeV} \times (0.2-2)ab^{-1}\right] \approx (0.32-3.2)\times 10^{6} \text{ per year!}$$

Acknowledgements

Thank you!