Bank Runs and Inequality

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Abstract

This study explores the link between income inequality and bank runs. Using data from 17 countries between 1880 and 2013, I found that when inequality rises by one percentage point, the risk of a bank run grows by 0.3 to 0.7 percentage points. This correlation is later corroborated using a modified bank run model that factors in different income levels. I found that as income inequality increases, the chance of bank runs also goes up. This result can be explained by the link between income inequality, marginal propensity to consume, and decreasing relative risk aversion. When income gaps widen, the spending habits of rich and poor groups change: the rich tend to spend more, while the poor spend less. These shifts are also influenced by how willing each group is to take financial risks. Faced with these changes, banks often adjust by reducing their riskier investments and holding onto more cash. My results show that growing income differences can destabilize the banking system, emphasizing the need for careful policies.

1 Introduction

Financial crises have adverse effects on consumption and output (Jensen and Johannesen, 2017, Romer and Romer, 2017), investment, productivity, employment (Chodorow-Reich, 2014), and health (Cutler et al., 2002), among others. Understanding what causes financial crises or how they can be anticipated can help policymakers act to prevent crises and their effects on economic and social outcomes. This paper examines the relationship between income inequality, bank runs, and financial crises.

In the 1960s, Friedman and Schwartz (1963) recognized bank runs as triggers for financial crises. More recently, Kirschenmann et al. (2016) and Paul (2022) identify income inequality as a significant predictor of financial crises in developed countries. In addition, Malinen (2016) documents that the relationship between income inequality and financial crises operates through the bank credit channel: An increase in income inequality leads to a rise in bank credit or leverage, amplifying

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credit cycles (which in turn can generate bank runs and then financial crisis). Understanding the correlation between income inequality and bank runs and describing the underlying mechanism can provide valuable insights into the determinants of the financial crisis and overall system fragility.

This paper addresses two questions. The first is related to the correlation between income inequality and bank runs. To answer this question, I conducted an econometric analysis that suggested a positive correlation between increasing income inequality and the likelihood of bank runs. Specifically, an increase in inequality by one standard deviation is associated with a 0.3 to 0.7 percentage point increase in the probability of a bank run. The unconditional probability of a bank run was estimated to be around 4% for the sample. Thus, a percentage point increase is a rather significant increase. This result holds accounting for different covariates.

The second question is focused on studying the mechanism that underpins the connection between income inequality and bank runs. Understanding how income inequality influences the occurrence of bank runs is crucial for developing targeted policies to address this issue. To answer this question, I extend the bank run model proposed by Allen and Gale (1998) to accommodate heterogeneity in the endowment levels between two groups of agents (otherwise, the groups are ex-ante identical in preferences and mass). More precisely, I have a mean preserving distribution of endowment between groups, where the wedge between endowment levels describes the income inequality of the economy. Additionally, I do not determine ex-ante that payouts are equal between groups of depositors. Still, nothing prevents the bank from offering this type of contract to the depositors in equilibrium. This model considers only fundamental bank runs (i.e., those not produced by sunspots or self-fulfilling prophecies as coordinating mechanisms). The model incorporates the assumptions on preferences and timing from Diamond and Dybvig (1983). Still, it distinguishes in two features: i) it does not assume a sequential withdrawal by the agents, and ii) illiquid assets held by the banks are risky and perfectly correlated across banks. Furthermore, I endow the depositors with preferences that exhibit decreasing relative risk aversion.

The intuition behind the mechanism that mediates the relationship between income inequality and bank runs is as follows. By allowing ex-ante differences in the endowment level, I can incorporate income inequality's impact on the Marginal Propensity to Consume (MPC) (Arrondel et al., 2019, Carroll et al., 2017, Fisher et al., 2020). More importantly, the MPC varies across the income distribution of the economy: higher (lower) quintiles of the income distribution have lower (higher)

MPC, which leads to lower aggregate consumption (Kaplan and Violante, 2022). Additionally, income inequality affects the risk profile of the agent via decreasing relative risk aversion, such that low-income groups have more risk aversion than high-income groups, making the high-income group more willing to risk pooling (Ogaki and Zhang, 2001). This leads also to changes in the MPC (Carroll, 2009). The agents in an economy have liquidity preferences (i.e., the timing of their consumption needs) and preferences on their portfolio of investments that satisfy their demand for consumption. Thus, banks play a pivotal role in insuring against liquidity shocks and aligning the consumption allocations demand from deposit contracts with asset portfolio preferences (Allen and Gale, 1998, Diamond and Dybvig, 1983, Farhi et al., 2009, Goldstein and Pauzner, 2005). Thus, the bank has a trade-off between risk sharing and the total surplus to be shared (Hellwig, 1998, Kaplan, 2006). In conclusion, inequality affects the bank's fragility by amplifying the challenges it faces when fulfilling its role in the economy. The bank's intermediation role may fail due to low-state return realization (production side) or an unexpected liquidity shock (demand) that may yield inadequate consumption allocations, ultimately producing bank runs.

I estimate my model numerically. The main finding is that the probability of a bank run increases with income inequality, given a set of standard economic parameters. This result speaks to the correlation found in the data where a one percentage point increase in income inequality represented a 0.3 to 0.7 percentage point increase in the probability of a bank run. Importantly, this result is an equilibrium result where the bank's decision to allow this run stems from a rational cost analysis, deducing that it is, in fact, more cost-effective to permit the bank run.

In the presented model, income inequality impacts consumption patterns across different income groups. As income inequality increases, the equilibrium consumption allocations for the high-income group rise, while the opposite occurs for the other group. This dynamic is explained by the fact that the outside options utility increases with income inequality for the high-income group, leading them to increase demand for consumption stemming from the deposit contract. Concurrently, it diminishes the outside options for the low-income group, resulting in their decreased consumption. A changed risk aversion further influences this shift in consumption: increased income inequality, along with the decreasing relative risk aversion, tends to make the high-income group more inclined towards risk-pooling, effectively subsidizing consumption alongside the low-income group. In contrast, the poor, faced with this widening economic gap, exhibit increased risk aversion, leading them to accept a reduced consumption bundle.

As income inequality increases, there is a shift in the bank's investment portfolio. The model suggests that when a bank faces a run, it adjusts the asset portfolio. Specifically, there's a reduction in the proportion of risky assets, serving as a countermeasure to offset potential financial setbacks from premature liquidations. At the same time, there is an increase in the acquisition of liquid assets.

In the model, the return on illiquid assets is a pivotal variable that can shift the threshold after which a bank run becomes probable. Specifically, when this return decreases, the threshold shifts to the right. This phenomenon is intrinsically linked to the behavior of the high-income group. A rightward shift implies that this group becomes less risk-averse with reduced returns on illiquid assets. Their inclination towards risk-pooling becomes more pronounced.

Furthermore, alterations in liquidity preferences can substantially impact the odds of a run occurring. Banks, in their central role as insurers against liquidity shocks, face these fluctuating preferences, and these challenges can exacerbate their financial instability. Similarly, another parameter that may affect the probability of a run is the fire-sale rate. Any modifications to this rate, especially an increase, translate to higher returns when the bank is compelled to liquidate assets. Consequently, this amplifies the allure of deposit contracts with higher returns, inadvertently increasing the probability of a bank run.

Literature Review. The literature has recently focused on the determinants of financial crises (see Baron et al. (2021), Gorton and Ordoñez (2020), Kirschenmann et al. (2016), Paul (2022)). Researchers have focused on a broader set of financial crises, following the definitions of Laeven and Valencia (2012), Schularick and Taylor (2012), and Reinhart and Rogoff (2009), among others. From the vast set of determinants, Kirschenmann et al. (2016) and Paul (2022) have found that income inequality has predictive power for financial crises, in a general sense, in developed countries. My contribution in this area is to document the particular correlation between income inequality (as measured by Paul (2022)) and fundamental bank runs (as defined by Baron et al. (2021)).

The literature has also examined the theoretical channel between income inequality and financial fragility. Malinen (2016) provides a brief but interesting review of this channel. For instance, Kumhof et al. (2016) finds that increasing income inequality leads to the accumulation of debt-to-income ratios, which results in a financial crisis. In summary, a relationship exists between income inequality and credit cycles in which an accumulation of credit plays a fundamental role.

My contribution is establishing a channel between income inequality, and fundamental bank runs in the form of riskier investment portfolios to cover aggregate consumption that increases with inequality. These riskier investment portfolios lead to financial fragility.

This paper contributes to the income inequality and financial fragility literature (see Choi (2014) and Mitkov (2020)). More importantly, Garcia and Panetti (2022) looks into context similar to this paper's. They investigate how wealth inequality makes financial crises more likely, finding that higher wealth inequality directly increases the incentives to run for the poor and indirectly for the rich through higher bank liquidity insurance. These incentives make self-fulfilling bank runs more likely. To reach these results, they make use of two main assumptions. First, they have multiple balance sheets that ring-fence the asset investment by wealth level, acting as universal banks. Second, they have an investment externality assumption that accounts for the contagion across wealth groups, ultimately leading to bank runs. The main difference in my contribution is that even in a model with a unique balance sheet and no investment externality, fundamental runs will happen if inequality is large enough.

Finally, this paper contributes to the literature on modeling bank runs from the seminal work of Diamond and Dybvig (1983). More precisely, it extends the model presented in Allen and Gale (1998) to accommodate ex-ante identical agents with differing endowment levels to account for inequality.

This paper is divided as follows. The next section elaborates on the correlation between bank runs and income inequality. Section three presents the theoretical model and discusses the different cases present in the model. Section four presents the numerical exercise, while section five presents and discusses the results of the numerical exercise. Section six concludes.

2 The Correlation Between Bank Runs and Income Inequality

I use historical data in this section to discuss the correlation between income inequality and bank runs. First, I will describe the data used to look into this correlation. Second, I present evidence on the prevalence of bank runs in advanced economies and how inequality behaves before such an event. Finally, I present the estimations that shed light on the correlation between bank runs and income inequality.

2.1 Data

I use two novel data sets. First, I use the data compiled by Paul (2022) on income inequality. This data set merges three long-term data sets from 1870 to 2013 for 17 countries.* The first long-term data set is from Òscar Jordà et al. (2016), which includes macro-financial variables for these 17 countries. The second long-term data set is from Bergeaud et al. (2016), which includes TFP and labor productivity measures. The third long-term data set is the World Inequality Database, which includes measures of income shares held by various percentiles. The novel feature in the data set of Paul (2022) is that he includes income shares held by the upper percentiles, net of capital gains.†

In the second place, I use the data set on bank runs found in Baron et al. (2021). The authors collect information for 46 countries in a similar time frame as Paul (2022). More importantly, they collect bank-run narratives under a common definition to capture fundamental bank runs.‡

The final data set includes information for 17 countries from 1880 to 2013, accounting for 2,069 country-year observations of macroeconomic, income inequality, and bank run variables.

2.2 The Prevalence of Bank Runs and Inequality Trends

Figure 1 presents the prevalence of bank runs for these 17 countries between 1880 and 2013. According to Allen and Gale (2007), bank runs are nothing new and have not been restricted to emerging economies. Baron et al. (2021) suggests almost no evidence of non-fundamental runs happening in this time frame. More importantly, bank runs are distributed across the period except for the post-WWII years (i.e., 1945-1970). Using the data set, I estimate that the unconditional probability of bank runs is around 4%.

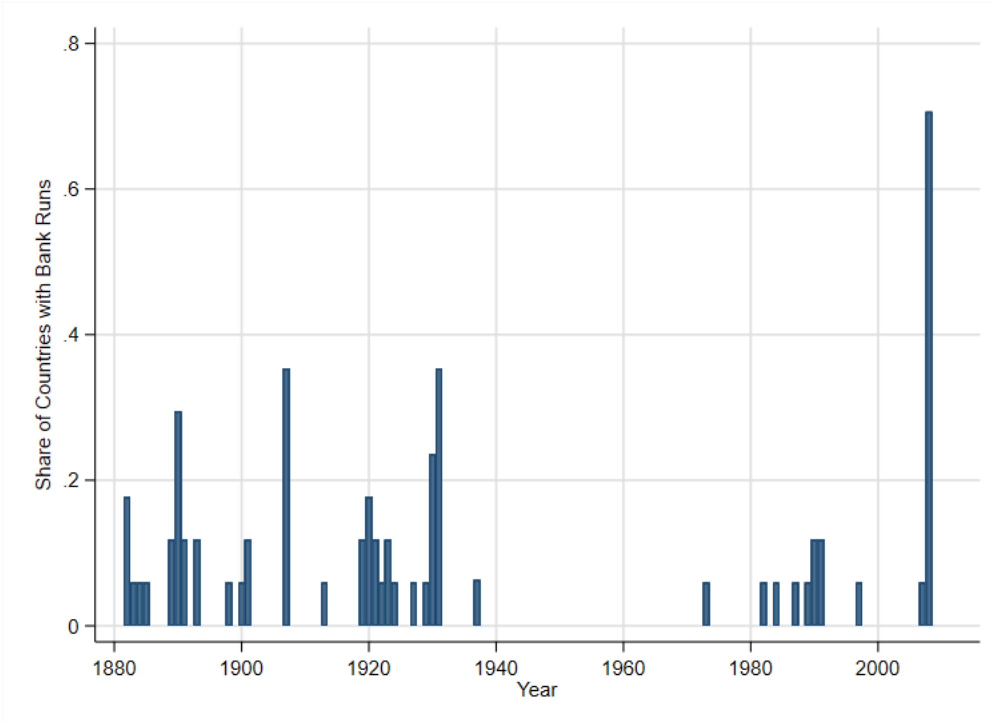
The measure of inequality I use is the share of income held by the top 0.1%, 1%, and 10% of the income distribution net of capital gains found in Paul (2022). The trends of such inequality measures are presented in Figure 2. The dashed red line is the mean, while the solid blue line is

*The countries are Australia, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom, and the United States.

†The income share data is constructed using tax income data. The revenues of selling assets (i.e., stocks) are taxed depending on the tax system. The potential issue is that stock trading is typically concentrated among individuals in the upper percentile of the income distribution. This, in turn, makes capital gains available for reinvestment on the same assets, and the additional savings are not available for borrowing for other agents.

‡Baron et al. (2021) describe bank runs as banking panics that are also bank equity crises (p. 102).

Figure 1: The Prevalence of Bank Runs



Source: Author estimation based on the data from Baron et al. (2021), Paul (2022).

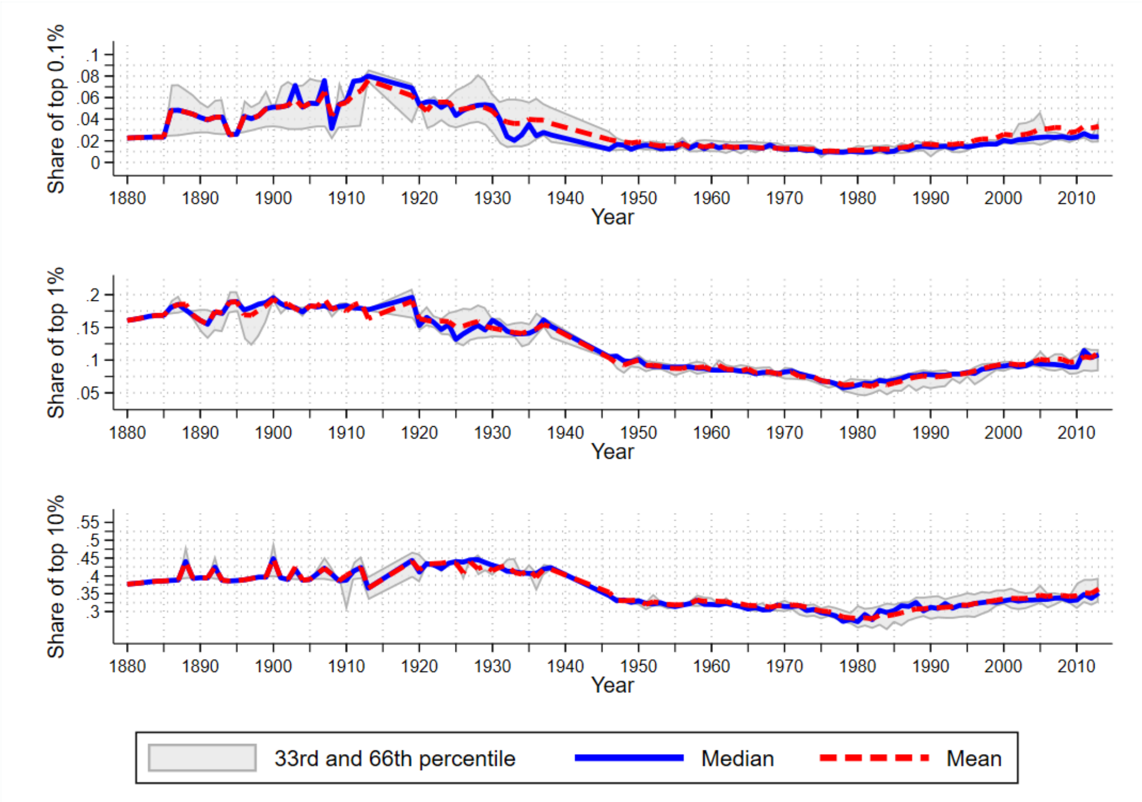
Note: The sample includes 17 advanced-economy countries described in footnote ∗.

the median. The borders of the grey area represent the 33rd and 66th percentiles of the inequality measures across the countries in the sample.

First, note that the inequality trends presented in Figure 2 have long cycles (i.e., they oscillate very slowly). For instance, they increased steadily from the late 19th century until the 1930s. Then, they decreased similarly in the post-WWII period until the 1980s, when they increased again until the end of the sample's time frame. Second, the sample median of each of the three measures is 2.8%, 12.1%, and 35.7%, for the top 0.1%, 1%, and 10%, respectively. These suggest that, between 1880 and 2013, in 50% of the countries in the sample, almost 35% of the income was held by the top percentiles of the income distribution. Finally, the dispersion of such measures (i.e., the difference between the 33rd and 66th percentile) increases in increasing inequality while it collapses in decreasing inequality.

These three features are by no means an exhaustive look into the inequality trends worldwide

Figure 2: Share of Income Held by Top 0.1%, 1% and 10%.



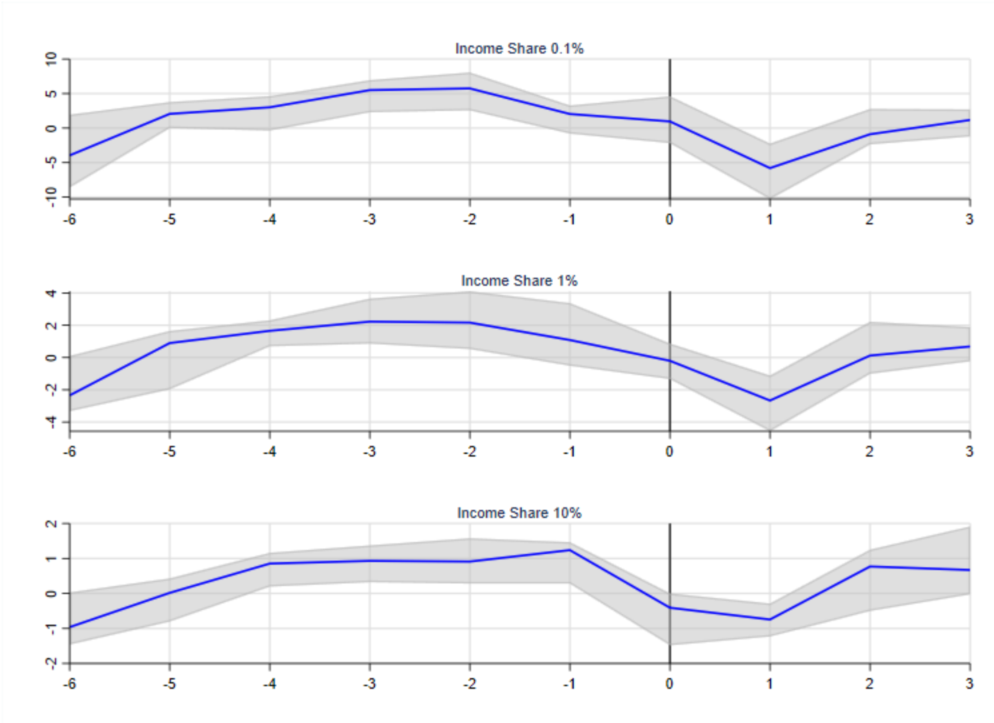
Source: Author's estimation using Paul (2022)

because they only apply to the countries in the sample. However, they account for the fact that inequality has increased since 1980 up to the present day (see Piketty and Saez (2003)).

2.3 Estimating the Correlation Between Bank Runs and Income Inequality

I will examine how the income shares held by the top percentiles behave around the time of a bank run. The results of this analysis are presented in Figure 3. The figure presents the median annual percentage change (in blue) and the 33rd and 66th percentile (in grey) before and after a bank run.

Figure 3: Median, 33rd and 66th Percentile of Income Shares



Source: Author estimation using the data from Paul (2022) and Baron et al. (2021).

The annual percentage changes in each income share measure increase drastically in the years before the bank run, followed by a drastic fall that lasts until after the bank run, and they start recovering after that. Furthermore, note the inverted U-shape of the growth in the income inequality measures. This shape suggests that “rapid” inequality growth (i.e., changing from negative growth to positive growth from one period to another) precedes (or correlates with) an episode of bank runs. Finally, bank runs seem to reduce the income share held by the top percentiles of the income distribution (the 0.1% and the 1%), where the median reduction can go up to a 5% annual percentage change.

I want to answer the following question: Is there a (positive or negative) correlation between income inequality and bank runs? Note that I am not implying any causality because, with the data available, it is impossible to discern any causal relationship between these two phenomena. The best that can be done with the data available is to establish the type of correlation that governs the relationship between income inequality and bank runs. To do so, let the probability of a bank run be described by:

$$Pr\left(BR_{j,t}=1|\,\mathrm{Ineq}_{j,t-1},X_{j,t-1};\boldsymbol{\beta}\right)=\frac{1}{1+\exp\left(-(\alpha_j+\beta_1\Delta_h\,\mathrm{Ineq}_{j,t-1}+\beta_2\Delta_h X_{j,t-1}+\varepsilon_{j,t})\right)} \quad (1)$$

where α_j is country-specific constant, $\Delta_h\,\mathrm{Ineq}_{j,t-1}$ is the change from period $t-1-h$ to $t-1$ of either measure of inequality or a vector that includes a combination of these measures for country j, $\Delta_h X_{j,t-1}$ is the change vector of controls X from $t-1-h$ to $t-1$, and $\varepsilon_{j,t}$ is the error term. Following Paul (2022), I normalize the variables in Ineq and X by their standard deviation. The selection of h is 4, following both Paul (2022) and Gorton and Ordoñez (2020).

The results of estimating (1) are presented in Table 1. First, for the estimation of the results in columns (1), (5), and (9), I included the log change in the credit-to-GDP ratio as controls in addition to the country fixed effects. The first three columns are the estimations of each inequality measure individually.[§] The table includes the point estimation of the odds ratio, the robust standard errors in parentheses, and the marginal effect in brackets for each explanatory variable.

The results in columns (1), (5), and (9) of Table 1 suggest that a standard deviation increase in the growth of the income share held by the top percentile is correlated with a 1 to 1.2 percentage point increase in the probability of a bank run. These percentage point increases in probability occur after controlling for the credit-to-GDP ratio, which has been deemed a determinant of the probability of financial crises in the literature (see Gorton and Ordoñez (2020), Paul (2022)). Remember that the sample's unconditional probability of bank runs is 4%, so a percentage point increase is a fairly significant increase in the probability of bank runs.

Taking the previous results as benchmarks, I perform additional robustness checks on estimating the correlation between income inequality and bank runs. The robustness analysis follows that in Paul (2022), and it is presented in the remaining columns of Table 1. Columns (2), (6), and (10) present the results, using as additional controls besides the 4-year change of credit-to-GDP ratio the following (in 4-year changes): investment-to-GDP ratio, public debt-to-GDP ratio, current account-to-GDP ratio, consumer price index, long and short-term interest rates. The results suggest that even accounting for macroeconomic variables, increased inequality is correlated with an increased probability of a bank run of around one percentage point.

[§]Including any combination of income share in one estimation will produce a high correlation between explanatory variables, generating biased point estimates.

Table 1: Probability of Bank Runs

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
$\Delta_4 \log \text{Credit}_{t-1}$	0.559***	0.516**	0.648***	0.420**	0.560***	0.553**	0.650***	0.413**	0.358**	0.399*	0.504**	0.372*
	(0.146)	(0.232)	(0.231)	(0.207)	(0.129)	(0.220)	(0.212)	(0.195)	(0.163)	(0.238)	(0.231)	(0.212)
	[0.015]	[0.008]	[0.010]	[0.004]	[0.015]	[0.008]	[0.009]	[0.004]	[0.010]	[0.005]	[0.006]	[0.003]
Δ_4Income Share 0.1%$_{t-1}$	0.393***	0.578***	0.618***	0.590***								
	(0.120)	(0.181)	(0.178)	(0.201)								
	[0.010]	[0.009]	[0.009]	[0.005]								
Δ_4Income Share 1%$_{t-1}$					0.443***	0.650***	0.701***	0.517**				
					(0.155)	(0.223)	(0.204)	(0.234)				
					[0.012]	[0.009]	[0.009]	[0.005]				
Δ_4Income Share 10%$_{t-1}$									0.345*	0.585***	0.576***	0.454**
									(0.178)	(0.208)	(0.214)	(0.214)
									[0.009]	[0.008]	[0.007]	[0.004]
Set of controls		✓	✓	✓		✓	✓	✓		✓	✓	✓
GDP & Global GDP			✓	✓			✓	✓			✓	✓
Stock & House Prices				✓				✓				✓
Number of crises	28	27	27	25	34	33	33	30	26	26	26	26
Observations	808	758	758	690	952	891	891	841	812	768	768	754
Countries	15	15	15	15	17	17	17	17	17	17	17	17
Country FE	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p-value	0.337	0.484	0.377	0.861	0.380	0.643	0.606	0.620	0.283	0.531	0.494	0.731
Pseudo R^2	0.067	0.162	0.187	0.279	0.073	0.179	0.203	0.281	0.040	0.172	0.195	0.276

Source: Author's elaboration using Paul (2022) and Baron et al. (2021) data.

In columns (3), (7), and (11), the estimation additionally controls for changes in domestic and global real GDP. The results follow a similar trend as that in columns (2), (6), and (8): Their significance level increases, and the point estimate of the marginal effects is smaller than those in the benchmark case. Finally, columns (4), (8), and (12) add real stock and house prices as additional controls. Note that their significance level remains relatively high for the change in the income share held by the top 0.1%. The significance level was reduced for the other two measures. At the same time, the point estimates of the marginal effects are almost identical between income shares but smaller than those from the benchmark case.

In conclusion, increasing income inequality, in the form of rising income share held by the top percentiles of the income distribution, correlates with an increased probability of bank runs. This correlation suggests that an increase of one standard deviation in the growth of such shares correlates roughly with an increase of one percentage point in the probability of a bank run. These results strongly motivate studying the mechanism underpinning such correlation in a theoretical model.

3 A Model of Income Inequality and Bank Runs

In this section, I elaborate on the banking model to present the mechanism underpinning the correlation between income inequality and bank runs.

3.1 Preliminaries, Preferences, and Endowments

There are three periods indexed to $t = 0, 1, 2$. Two possible states of nature $s = H, L$ occur with probability π_H and π_L such that $\pi_H + \pi_L = 1$. There is a continuum of agents with mass two composed of two groups with equal mass that differ only in their initial level of endowments. The two groups are indexed to $i = 1, 2$. Each depositor within a group is indexed to j. Without loss of generality, let the group of depositors $i = 1$ be endowed with ω_1 units of the final consumption good, and depositors in group $i = 2$ are endowed with ω_2 units, where $\omega_1 > \omega_2 > 0$. These endowment levels are common knowledge. The depositors receive the endowment at $t = 0$ and do not receive any additional endowment in $t = 1, 2$. However, they want to consume in either $t = 1$ or $t = 2$.

These depositors are subject to a liquidity shock. That is, they are uncertain about the timing of their consumption. If the depositor j prefers to consume in $t = 1$, he is from type early, whereas if he prefers to consume in $t = 2$, he is from type late. These types are not common knowledge, but let the probability of being early type be $\lambda \in (0,1)$, and consequently, the probability of being late type is $1-\lambda$), known to all agents. Given the equal mass of groups and by the law of large numbers, the parameter λ can be interpreted as the proportion of agents that are of type early.

The typical depositor j from group i has preferences represented by a utility function $U(c_{ti})$ that is increasing, strictly concave, and twice continuously differentiable. Let the utility function be

$$U(c) = \frac{(c+\psi)^{1-\gamma}}{1-\gamma} \tag{2}$$

This utility function in (2) represents Hyperbolic Absolute Risk Aversion (HARA) preferences. More importantly, let $\psi < 0$ and $\gamma > 0$ so the (2) exhibits Decreasing Relative Risk Aversion (DRRA).[¶] With this utility, the depositor gets a positive utility after a subsistence level of consumption is satisfied.

The depositor j does not know if they are from type early or late until $t = 1$. He also does not know what would happen in $t = 2$. Let d_i for $i = 1, 2$ be the face value of the deposit contract at $t = 1$. Hence, let the expected utility of the typical depositor j in group i be described by:

$$u(d_i, c_{2is}) = \sum_{s=H,L} \pi_s \left[\lambda U(d_i) + (1-\lambda) U(c_{2is})\right] \tag{3}$$

Note that to truly reveal his type, the incentive compatibility constraint for agent j in group i is given by:

$$c_{1i} \leq c_{2i} \text{ for } i = 1, 2 \tag{4}$$

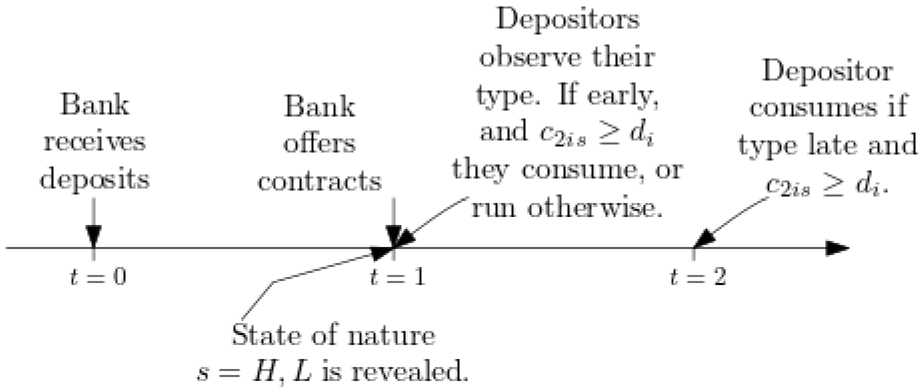
Finally, to ensure that the depositor accepts the deposit contract offered by the bank, this contract has to satisfy the participation constraint in the form of

$$\mathrm{E}\left[\lambda U(c_{1i}) + (1-\lambda) U(c_{2i})\right] \geq U(\omega_i) \;\text{ for } i = 1, 2 \tag{5}$$

The timing of the problem is presented in Figure 4. The bank receives the deposits at $t = 0$ and offers the deposit contracts by the end of that period. Most of the action occurs at $t = 1$. In this period, the depositor's type and the state of nature are revealed. Then, if the incentive compatibility constraints hold, the depositors will withdraw at their respective periods, and there will be no run. However, a run will occur if the incentive compatibility constraint is not satisfied.

[¶]See Appendix A for a discussion on HARA preferences.

Figure 4: The Timing of the Problem



3.2 Banks's Portfolio

There is a bank that takes the depositors' endowments $\omega_1$and ω_2, and invests them in a portfolio composed of:

- A liquid asset (short-term) y with a constant return to scale technology that takes one unit of consumption good at t and transforms it into one unit of consumption good at $t+1$ for $t = 0, 1$. This technology can be thought of as a storage technology.

- An illiquid and risky asset x that has a constant return to scale technology that takes one unit of consumption good at $t = 0$ and transforms it into R_H units of the consumption good with probability $\pi_H \in (0, 1)$ at $t = 2$ or into R_L units of the consumption good with probability $\pi_L \in (0, 1)$ at $t = 2$, where $\pi_H + \pi_L = 1$ and $R_H > R_L > 1$. Therefore, there are two possible states of nature $s = H, L$. In the early liquidation of this asset, the technology takes one unit of consumption good at $t = 0$ and transforms it into $1 > r > 0$ units of the consumption good at $t = 1$.

Introducing this random asset return does not rule out bank runs that occur out of self-fulling prophecies or sunspots as a coordination mechanism. Thus, I am considering only essential bank runs (i.e., bank runs that cannot be avoided).

Once the bank receives the endowments from the depositors at $t = 0$, it has to choose an investment portfolio (x, y) such that

$$x + y \leq \omega_1 + \omega_2 \tag{6}$$

This is a feasibility constraint for the bank. It suggests that the whole portfolio should be less or equal to the total endowments in the economy. Now, let $\omega_1 \equiv 1 + \tau$ and $\omega_2 \equiv 1 - \tau$ for a $\tau \in (0, 1)$ such that it complies with the assumption that $\omega_1 > \omega_2$ without increasing the size of the economy – that is, without making $\omega_1 + \omega_2$ greater. Then, (6) becomes

$$x + y \leq 2 \equiv \omega \tag{7}$$

Note that the larger τ is, the greater inequality becomes. I am setting τ to move freely between 0 and 1. As the bank can only purchase assets with the aggregate level of endowment in the economy, it is not ring-fencing its services to attend a specific wealth group.

3.3 Bank's Possible Cases

Now, the bank cannot know which state of nature occurs. The depositor of type late in group $i = 1, 2$ can run or not on the bank depending on whether the incentive compatibility constraint is satisfied or not. It follows that the bank could potentially face up to 10 cases that entail different maximization problems. Table 2 sums the possible cases.

Table 2: Possible Cases if Group $i = 1, 2$ Runs

State	**None**	**Both**	**Group 1**	**Group 2**
High	Case 1	Case 2	Case 5	Case 8
Low	Case 1	Case 3	Case 6	Case 9
Both	Case 1	Case 4	Case 7	Case 10

However, some cases are not optimal and can be discarded beforehand. The following propositions are aimed at discarding the non-optimal cases.

Proposition 3.1. *There will never be a fundamental run in both states $s = H, L$ for both groups of agents.*

Proof. See Appendix B.1 $\square$

The intuition behind this is that the contract that induces a fundamental run in both states for both groups of agents is dominated by a contract that offers at least the same amount as the previous contract at $t = 1$ and a positive amount in $t = 2$ because the return in either state $s = H, L$ is greater than 1. The social utility from the second contract is greater than that in the original contract. According to this proposition, case 4, where agents run in both states, is not optimal.

Proposition 3.2. *It is never optimal to choose a contract that leads to a run in the high state for some group i but does not lead to a run for that group i in the low state.*

Proof. See Appendix B.2 □

The intuition is that this type of contract opens the possibility to some leftover of the amount distributed at $t = 2$ that allows for a welfare-improving allocation where there is no run at the high state. It does not necessarily lead to a run in the low state for group i. According to this proposition, the cases where groups 1 - case 5 - or 2 - case 8 - run in the high state and not in the low state cannot be optimal.

Proposition 3.3. *It is never optimal to choose a contract that leads to a run in* ***BOTH*** *states $s = H, L$ by some group i.*

Proof. See Appendix B.3. □

The intuition is that a contract that leads to a run in both states by a group i is dominated by a contract where the group $j \neq i$ is at least as well as before, and group i is strictly better. This proposition suggests cases where group 1 - case 7- or group 2 - case 10 - cannot be optimal. Furthermore, the case where both groups run in the high state cannot be optimal by extension of propositions 3.2 and 3.3.

The remaining cases are case 1 (no group runs at any state), case 3 (both groups run at low state), case 6 (group 1 runs at low state), and case 9 (group 2 runs at low state).

3.3.1 Case 1: No Runs in Both States by Either Agent

In the first case, neither agent runs on the bank in either or both states. The bank's maximization problem is given by:

$$\max_{d_1,d_2,y,\{c_{2is}\}_{i=1,2,s=H,L}} \sum_{s=L,H} \pi_s \left\{\lambda \left[U\left(d_1\right) + U\left(d_2\right)\right] + (1-\lambda)\left[U\left(c_{21s}\right) + U\left(c_{22s}\right)\right]\right\} \tag{8}$$

subject to

$$\lambda\left(d_1+d_2\right) \leq y \tag{9}$$

$$(1-\lambda)\left(c_{21s}+c_{22s}\right)=R_s\left(\omega-y\right)+y-\lambda\left(d_1+d_2\right) \text{ for } s=H,L \tag{10}$$

$$c_{2is} \geq d_i \text{ for } i=1,2, s=H,L \tag{11}$$

$$\lambda U(d_i)+(1-\lambda)\left[\pi_H U(c_{2iH})+\pi_L U(c_{2iL})\right] \geq U\left(\omega_i\right) \text{ for } i=1,2 \tag{12}$$

$$0 \leq y \leq \omega \tag{13}$$

Condition (9) suggests that the total face value of the deposit contracts offered to the λ share of early-type depositors in both groups should be less or equal to the amount invested in the liquid asset y. More importantly, Proposition 3.4 implies that this condition should hold with equality since it is never optimal to leave some investment in the liquid asset from $t=1$ to $t=2$ because if that same amount were to be invested in the illiquid asset, it would yield $R_s > 1$ units more at $t=2$.

Proposition 3.4. *The condition $\lambda\left(d_1+d_2\right) \leq y$ should hold with strict equality in optimum.*

Proof. See Appendix B.4 □

The condition (10) holds with equality since the bank will give back all of what is available to the depositors at $t=2$, in any state $s=H,L$. By the result on proposition (3.3), this condition can be rewritten as

$$(1-\lambda)\left(c_{21s}+c_{22s}\right)=R_s\left(\omega-y\right) \tag{14}$$

Condition in (11) is the incentive compatibility constraint for both agents in both states if the bank offers a contract that leads to no runs by either agent in both states. These conditions imply that to induce truth-telling from the agents to reveal their type, those of the late type are provided with a consumption allocation greater than that of the early type.

Condition (12) is the participation constraint of both agents. These conditions imply that for the agent to take the deposit contract offered by the bank, the expected value of that contract should exceed the utility of consuming their endowment.

Condition (13) is a bank feasibility constraint that implies that the investment in the liquid asset should be greater than or equal to 0 and less than the total endowment of the economy. This condition will hold with strict inequality due to the non-negativity constraints of consumption allocations.

3.3.2 Case 3: Bank Offers a Contract such that All Run in the Low State

The second case is that both agents run on the bank in the low state. The bank's maximization problem is given by

$$\max_{d_1,d_2,y,\{c_{2is}\}_{i=1,2}^{s=H,L}} \pi_H \sum_{i=1,2} \{\lambda [U(d_i) + (1-\lambda) U(c_{2iH})]\} + \pi_L \left\{U\left(\tilde{d}_{1L}\right) + U\left(\tilde{d}_{2L}\right)\right\} \tag{15}$$

subject to

$$d_1 + d_2 \geq [r(\omega - y) + y] \tag{16}$$

$$c_{2iH} > d_i, i = 1,2 \tag{17}$$

$$\tilde{d}_{iL} = \frac{d_i}{d_1 + d_2}[r(\omega - y) + y], i = 1,2 \tag{18}$$

$$\lambda(d_1 + d_2) = y \tag{19}$$

$$(1-\lambda)(c_{21H} + c_{22H}) = R_H(\omega - y) \tag{20}$$

$$\pi_H \{\lambda U(d_i) + (1-\lambda) U(c_{2iH})\} + \pi_L \left\{U\left(\tilde{d}_{is}\right)\right\} \geq U(\omega_i), s = 1,2 \tag{21}$$

$$0 \leq y \leq \omega \tag{22}$$

In this problem, the objective function changes. The portion for state $s = H$ is the same since there is no run. However, for $s = L$, it changes to accommodate the fact that both agents are running in this state. In this case, the entire mass of both groups of agents is running. Thus, this portion does not depend on λ. In terms of the restrictions, first note that conditions (17), (19), (20) and (22) are similar to that in the case where no agent runs.

The incentive compatibility in state $s = L$ requires some additional explanation. This condition is to avoid the unilateral deviation of the late-type depositor. In this case, the deviation is that the depositor of type late does not run and waits to consume at $t = 2$. In case of a run, the bank has to liquidate their long-term asset at a fire sale rate of $r < 1$ and use it, in addition to whatever the bank has on the liquid asset, to pay for the consumption allocations of the agents that run. This is captured by the expression on the right-hand side of the condition (16).

Suppose condition (16) is violated such that the amount of the liquidated assets is larger than the deposit contracts in $t = 1$. In this case, the bank can pay the deposit contracts to all the agents that ran at their face value at $t = 1$. Given the assumption that the bank has to return whatever is left in $t = 2$, the agent that deviated would receive a large amount (infinity?) at $t = 2$. Since all $(1 - \lambda)$ agents of type late are ex-ante identical, all type late agents would be incentivized to deviate and wait until $t = 2$ for consuming. In this case, a welfare-improving allocation would be not to liquidate the illiquid assets since $R_L > r$ and all agents would have been better off, and this case would not occur. It follows that to have a run from both groups at $s = H$ condition (16) should always be satisfied.

The condition (18) suggests how the liquidated value of the assets is distributed in case of the run. In this case, it is distributed proportionally to the face value of the deposit contracts promised by the bank. Finally, the participation constraint (21) has the same intention as that in the case of no run, but it is modified to accommodate the run allocation at $s = L$.

3.3.3 Cases 6 and 9: Bank Offers A Contract Where One Group Runs and the Other Does Not.

First, note that cases 6 and 9 are identical with the group of agents subscripts changed. Then, let group 2 be the one that runs on the bank, and group 1 does not run to the bank (i.e., case 9). The bank's maximization problem is given by:

$$\max_{d_1,d_2,y,\{c_{2is}\}_{i=1,2}^{s=H,L}} \pi_H \sum_{i=1,2} \left\{\lambda \left[U\left(d_i\right)\left(1-\lambda\right) U\left(c_{2iH}\right)\right]\right\} + \pi_L \left\{U\left(\tilde{d}_{2L}\right) + \lambda U\left(\tilde{d}_{1L}\right) + \left(1-\lambda\right) U\left(c_{21L}\right)\right\} \tag{23}$$

subject to

$$\begin{cases} c_{21L} \geq \tilde{d}_{1L} & \text{IC rich in low state} \\ \tilde{d}_{2L} \geq c_{22L} & \text{IC poor in low state} \end{cases} \tag{24}$$

$$c_{2iH} \geq d_i, i = 1, 2 \tag{25}$$

$$\lambda \left(d_1 + d_2\right) = y \tag{26}$$

$$\tilde{d}_{2L} = \begin{cases} d_2 & \lambda d_1 + d_2 \leq r\left(\omega - y\right) + y \\ \frac{d_2}{\lambda d_1 + d_2}\left[r\left(\omega - y\right) + y\right] & \text{otherwise} \end{cases} \tag{27}$$

$$\tilde{d}_{1L} = \begin{cases} d_1 & \lambda d_1 + d_2 \leq r\left(\omega - y\right) + y \\ \frac{\lambda d_1}{\lambda d_1 + d_2}\left[r\left(\omega - y\right) + y\right] & \text{otherwise} \end{cases} \tag{28}$$

$$\left(1-\lambda\right)\left(c_{21H} + c_{22H}\right) = R_H\left(\omega - y\right) + y - \lambda\left(d_1 + d_2\right) \tag{29}$$

$$(1-\lambda)\,c_{21L} = \begin{cases} 0 & \text{if } \lambda d_1 + d_2 > r\,(\omega - y) + y \\ R_L \left\{ \omega - y - \left[\frac{\lambda d_1 + d_2 - y}{r} \right]_+ \right\} + \left[y - (\lambda d_1 + d_2) \right]_+ & \text{otherwise} \end{cases} \tag{30}$$

$$\pi_H \left\{ \lambda U\,(d_1) + (1-\lambda)\,U\,(c_{21H}) \right\} + \pi_L \left\{ \lambda U \left(\tilde{d}_{1L} \right) + (1-\lambda)\,U\,(c_{21L}) \right\} \geq U\,(\omega_1) \tag{31}$$

$$\pi_H \left\{ \lambda U\,(d_2) + (1-\lambda)\,U\,(c_{22H}) \right\} + \pi_L \left\{ U \left(\tilde{d}_{2L} \right) \right\} \geq U\,(\omega_2) \tag{32}$$

$$0 \leq y \leq \omega \tag{33}$$

where $[x]_+ = \max\{x, 0\}$

The objective function is different than that of the previous cases. The first portion is the sum of the utilities if state $s = L$ occurs. The second portion is the sum of the utilities in the state $s = L$. In this case, a proportion of agents $(1-\lambda)$ of group 1 will not run on the bank, while the whole mass of group 2 will run. Conditions (25), (26), (29), (31), and (32) are similar to those in the other cases.

The conditions in (24) are the incentive compatibility constraints for both groups in state $s = L$. In this case, the incentive compatibility constraint for group 1 aims to deter the deviation of the late-type to run on the bank, while for group 2 is to deter deviation of the late type to consume at $t = 2$ since, in this case, the bank offers a contract that induces a run in the low state for this group. Focus on the incentive compatibility for group 2. The bank in this situation has to liquidate the illiquid asset at a fire sale at a rate $r < 1$ (in addition to the liquid asset) and use this to pay the consumption allocations promised in $t = 1$ for all the type early and type-late that run on the bank. That is, the bank has to pay $r\,(\omega - y) + y$.

3.3.4 Bank's Case Selection in Equilibrium

Let $W_k(d_1^k, d_2^k, c_{21H}^k, c_{22H}^k, c_{21L}^k, c_{22L}^k)$ be the social utility valued at the optimal allocations in for case $k = 1, 3, 6, 9$. Then, the bank will choose case k over all the other case $-k$ whenever the social utility of case k is at least as good as the maximum social utility of case $-k$. That is,

$$W_k(\cdot) \geq \max\{W_{-k}(\cdot)\} \tag{34}$$

where $-k$ all the other cases but the k^{th}. By allowing the indifference to be solved in favor of the case with greater utility, I am focusing on the "best" equilibrium selection scenario.

4 Numerical Exercise

In this section, I present the results of a numerical exercise to showcase some of the properties of the model. The main goal is to look at how a change in τ, which implies changing the level of inequality, affects the various consumption allocations, the investment, the welfare function, and, more importantly, the probability of a bank run.

The parameters used in the numerical exercise are presented in Table 3. I set the parameter ψ at -0.4 so the utility function presents Decreasing Relative Risk Aversion. The parameters R_H and R_L imply that the risky asset pays two units of consumption goods per unit when it matures in the state H or 1.065 units in the state L. The parameter r implies that the recovery rate of the risky asset when liquidated early is about 80% the original investment value. I used $\gamma = 3$ as it is standard. I set the parameter λ, the share of type early depositors, at 15%. Finally, I set the probability of the low state at 15%.

Table 3: Set of parameters for numerical exercise

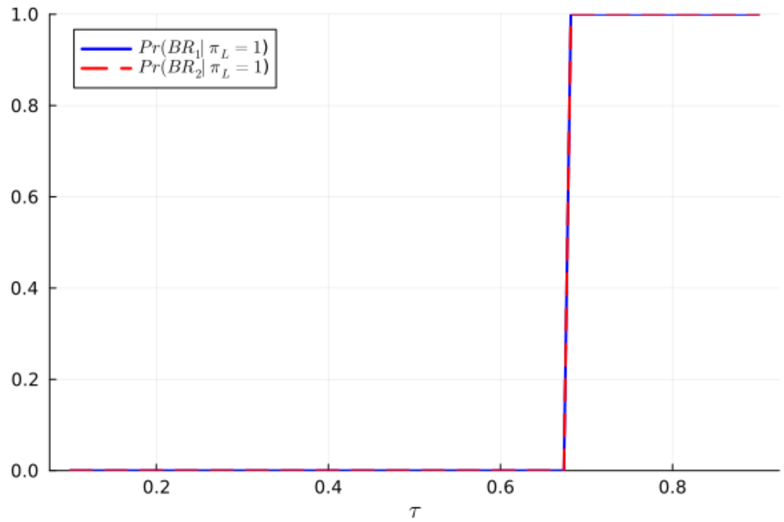
Parameter	λ	τ	R_H	R_L	γ	ψ	r	π_L
Value	0.15	[0.1,0.90]	2.0	1.065	3.0	-0.4	0.8	0.15

To estimate the model, I fixed the parameters of the economy and a level of τ and used the restrictions to bound grids of possible consumption allocations and investment. Given these consumption allocations and the investment level, I estimated the utility for each case. I then proceeded to estimate the maximum utility given the fixed level of τ. Once I went over the entire grid of τ, I verified if the equilibrium conditions in (34) are satisfied by each level of τ.

5 Results and Discussion

This section presents the main results of the numerical estimation of the model. The main result is that the probability of a bank run is increasing in τ. This result is presented in Figure 5. Note that given the structure of the problem, runs can only happen in the low state. Thus, the probability of a bank run, conditional on being in the state L, is one once it reaches a sufficiently high τ (i.e., τ^*). The unconditional probability is given by $Pr(BR|s = L) \times Pr(s = L) = \pi_L = 0.15$. The jump in the probability of a bank run is due to the discrete nature of the actions of either agent (i.e., to run or not to run). This first result confirms a correlation between income inequality and bank runs in the data. As inequality rises (i.e., larger τ), the probability of the bank run increases.

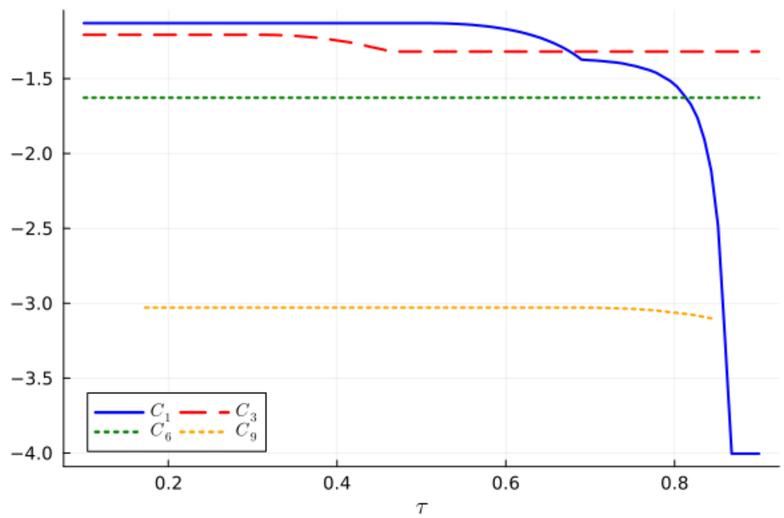
Figure 5: Conditional Probability of Bank Run



To explain why the probability of a run is increasing with inequality, first let us discuss the social utility function $W_k(\cdot)$ for each of the $k = 1, 3, 6, 9$ cases, presented in Figure 6. Note that the consumption allocations stemming from cases 6 and 9 are never optimal for the set of parameters used. The reason behind this is as follows. Assume case 9, where group 2 is given an allocation that induces a run and group 1 does not. Since there is a run, the bank has to liquidate all its illiquid assets at $t = 1$ at a fire sale rate $r < 1$. This leaves the bank with $s^* \equiv r(\omega - y) + y$ units of the final consumption good to allocate in the λd_1 and d_2 deposit contracts. Suppose s^* is just enough to cover the deposit contracts at $t = 1$. This, in turn, leaves the $(1 - \lambda)$ depositors of late-type of group 1 with 0 consumption at $t = 2$ because there is no illiquid asset available to mature at $t = 2$. Hence, since $d_1 > 0 = C_{21L}$, the incentive compatibility constraint is not satisfied, and the late-type depositors of group 1 will run. A similar story will occur for case 6.

Furthermore, note that a τ^* exists over which $W_3(\cdot) \geq W_1$. This suggests that once the economy reaches a certain level of inequality, it is optimal for the bank to choose deposit contracts that induce a bank run. The mechanism behind this is that increasing inequality, where group 1 sees their endowment increase and group 2 sees it decrease, triggers changes in their relative risk aversion and marginal propensity to consume. For the relative risk aversion, note that the preferences here exhibit decreasing relative risk aversion, so whenever income increases, the agent exhibits less risk

Figure 6: Utility Functions for Cases 1 and 3



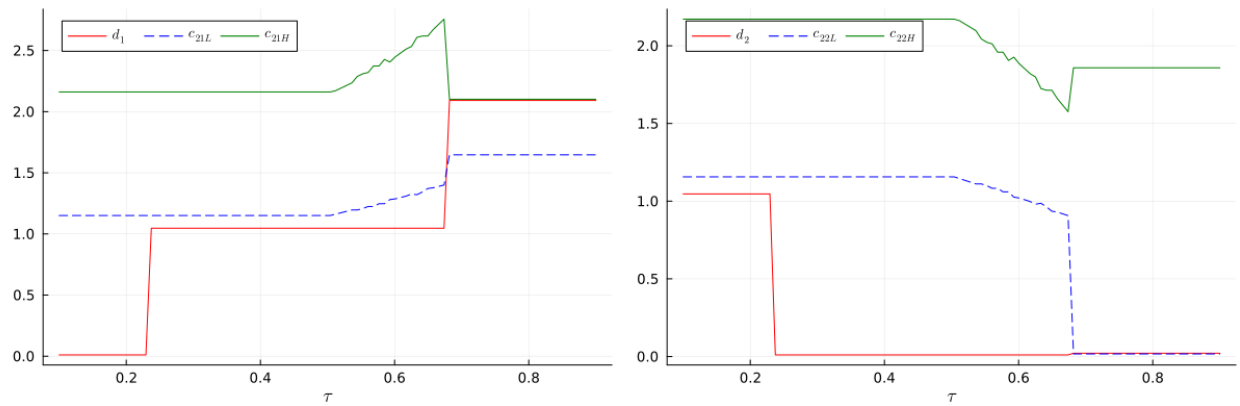
aversion. For the marginal propensity to consume, the group with the larger endowment exhibits a lower marginal propensity to consume, while the opposite happens for the group with a lower endowment. The bank maximizes the available surplus while aligning the consumption allocations demanded with the risk-sharing profile from the depositors. Thus, as the economy reaches and goes over τ^*, the bank finds that the consumption allocations of case 3 maximize the economy's surplus while fulfilling its role in the economy.

Figures 7a and 7b present the equilibrium consumption allocations for groups 1 and 2, respectively. The main takeaway is that the consumption allocations from the deposit contracts are increasing in τ for group 1 and decreasing for group 2. There are two forces at play here. First, a higher income inequality (i.e., an increase in τ) implies a larger outside option for group one since their endowment increases with τ, while the opposite happens for group 2. Then, the consumption allocation for each level of τ needs to be larger (smaller) for group 1 (group 2) to participate. Thus, this explains the increasing and decreasing nature of the consumption allocations for groups 1 and 2, respectively.

Second, a higher τ implies a change in the risk aversion and the marginal propensity to consume, as mentioned before. Thus, group 1 is more willing to risk pool (i.e., subsidize consumption for group 2), while group 2 becomes more risk averse, accepting a lower consumption bundle. This is possible because of the bank's unique balance sheet, which can redistribute in terms of consumption

Figure 7: Equilibrium Consumption Allocations for Both Groups

(a) Group $i = 1$ (b) Group $i = 2$



allocations, allowing group 2 access to the larger returns of the riskier asset while satisfying the demand of deposit contract for group 1.

Lastly, the investment portfolio is presented in Figure 8. The portfolio needs to be balanced towards the illiquid asset x to match the investment needs of the large proportion of late-type depositors from both groups. Once the inequality level reaches τ^*, the portfolio needs to accommodate a lower risk by increasing the acquisition of liquid asset y. However, the total composition is still balanced towards the illiquid asset. This is because there is still the possibility of realizing the high state. Thus, the bank needs to provide deposit contracts that cover that contingency while compensating for the loss of early liquidation in case the low state happens since the fire-sale rate is $r < 1$.

The rest of this section presents different comparative statics involving changes in the return of the illiquid asset in the low state R_L, changes in the liquidity preferences captured by λ, and changes in the fire-sale rate r. These parameters are more likely to affect the probability of a run, all else equal, due to its direct incidence in the low state.

The changes in the return of the illiquid asset return in the low state are presented in Figures 9a and 9b. Note that decreasing or increasing the return shifts τ^* to the left or the right, respectively. This change, in return, impacts the size of the total final good available for distribution at state L. Thus, the increase in the return implies that the bank can pay for deposit contracts in cases with more inequality. The opposite occurs in the case where the return decreases.

Figure 8: Investment Portfolio

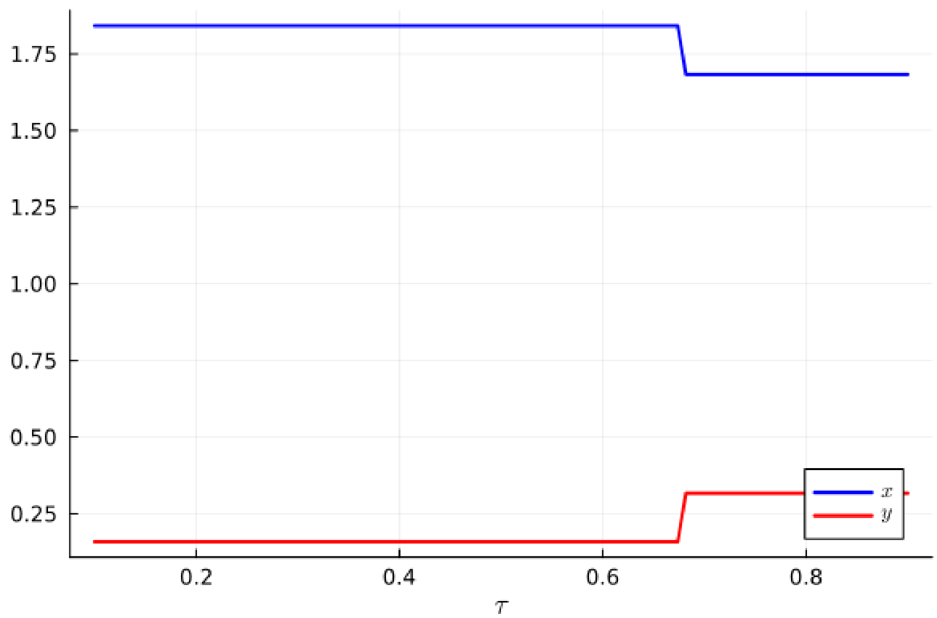
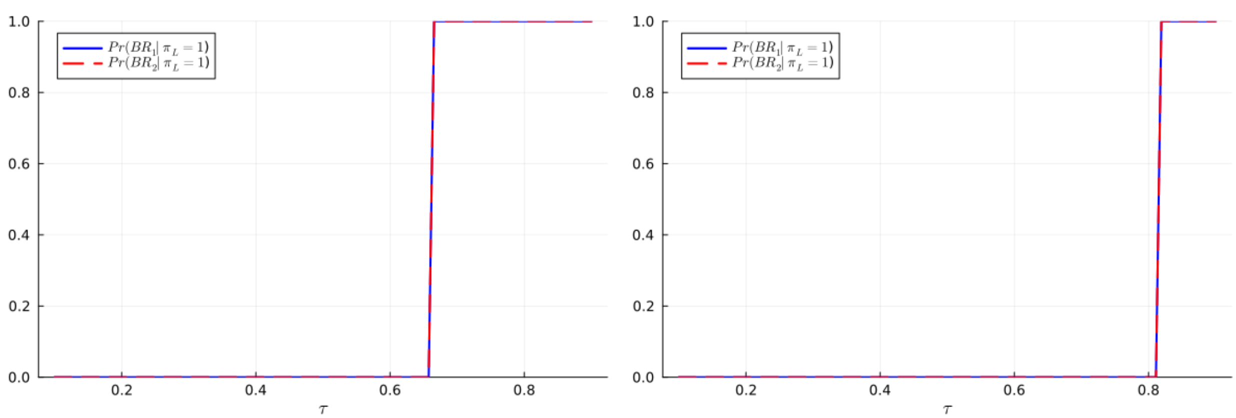


Figure 9: Changes in $Pr(BR|s = L)$ with changes in R_L

(a) Decreasing R_L

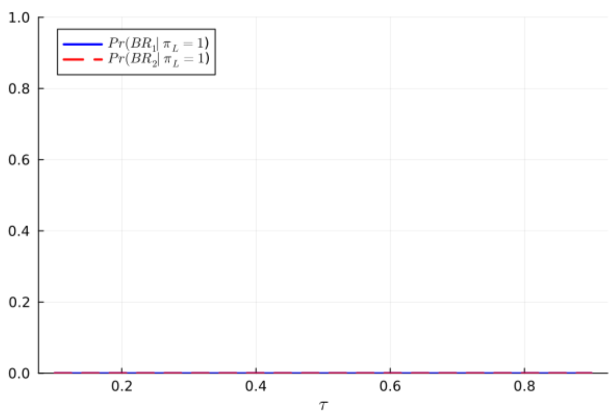
(b) Increasing R_L



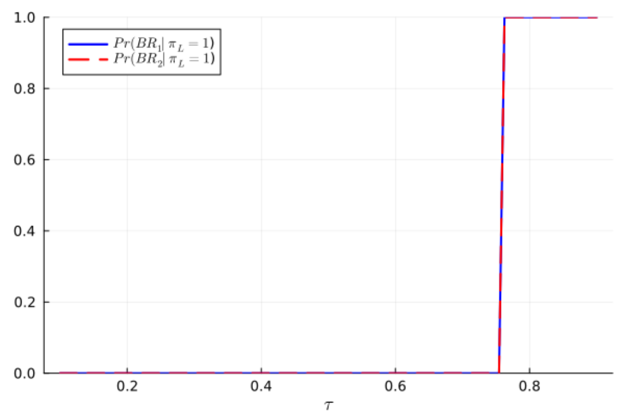
The changes in the liquidity preferences have big impacts on the probability of a run, all else equal. This result is presented in Figures 10a, 10b and 10c. The liquidity preferences are captured by parameter λ, which is the probability of a given depositor being early-type. Then, changes in λ reflect whether a depositor is willing to wait more (less) to consume in $t = 2$. The numeric results of a small decrease in λ suggest that all else equal, the probability of a run is 0 across the domain of τ (Figure 10a). A small increase in λ shifts τ^* to the right. This is because increasing λ reduces the mass of late-type depositors; thus, all else equal, the bank will be more likely to fulfill the deposit contracts at higher levels of inequality. Finally, note that if the increase were to change to the point where the majority are of early-type depositors, the probability of a run is now decreasing in τ.

Figure 10: Changes in $Pr(BR|s = L)$ with changes in λ

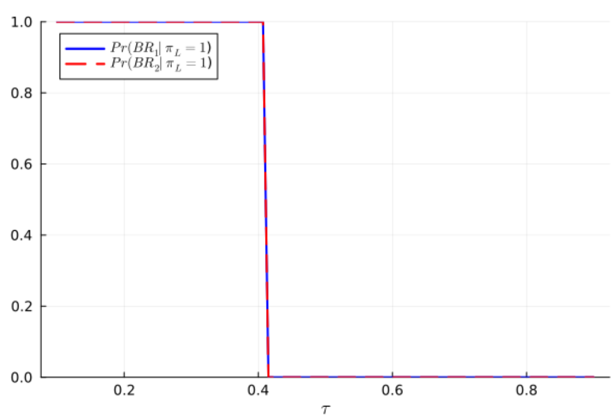
(a) Small Decrease in λ



(b) Small Increase in λ



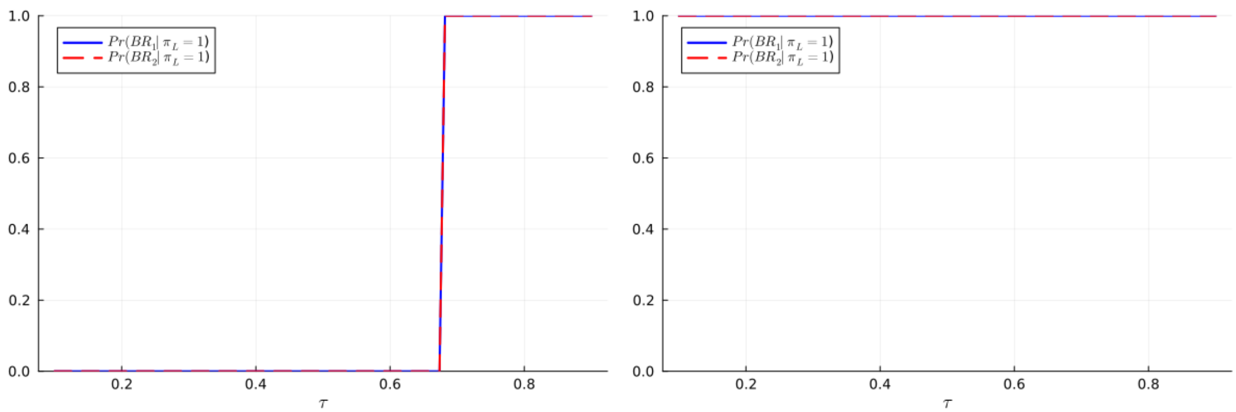
(c) Large Increase in λ



Finally, changes to the fire-sale rate emerge as a significant determinant in the probability of a bank run within our model. Specifically, increasing the fire-sale rate increases the return when banks are compelled to liquidate their assets. This return, in turn, increases the incentive for accepting deposit contracts for potential depositors available under a run. Consequently, it increases the probability of a bank run.

Figure 11: Changes in $Pr(BR|s = L)$ with changes in r

(a) Decreasing r (b) Increasing r



6 Conclusion

In conclusion, this paper provides evidence of a positive correlation between income inequality and the likelihood of bank runs. The econometric analysis suggests that an increase in inequality by one percentage point is associated with a 0.3 to 0.7 percentage point increase in the probability of a bank run. This result accounts for different covariates, such as GDP per capita, inflation, and financial development.

Furthermore, this paper proposes a banking model to explain this correlation, highlighting the role of income inequality in determining the probability of a bank run. Specifically, the model shows that increasing income inequality increases the probability of a bank run, as is shown in the data.

The paper's findings have important implications for policymakers, suggesting that reducing income inequality can help prevent financial crises. In particular, policies that aim to redistribute wealth and reduce the concentration of wealth at the top can help stabilize the financial system and reduce the likelihood of bank runs. Moreover, the paper's banking model provides a framework for analyzing the effects of different policy interventions on the probability of a bank run.

Overall, the paper contributes to the growing literature on the relationship between income inequality and financial stability and highlights the need for policymakers to consider the distribution of wealth when designing policies to prevent financial crises.

Appendix

A Agents preferences and Relative Risk Aversion

The following utility function represents Hyperbolic Absolute Risk Aversion (HARA) preferences

$$U\left(c\right)=\frac{\left(c+\psi\right)^{1-\gamma}}{1-\gamma} \tag{35}$$

It follows that

$$U'\left(c\right)=\left(c+\psi\right)^{-\gamma} \tag{36}$$

$$U''\left(c\right)=-\left(c+\psi\right)^{-\gamma-1} \tag{37}$$

To satisfy the conditions described before for the problem, the second derivative must comply with

$$U''\left(c\right)<0\Leftrightarrow\begin{cases}\gamma>1\wedge c>0\wedge\psi>-c\\ \frac{c_1}{2}\in\mathbb{Z}\wedge c_1\leq-2\wedge c_1=-1-\gamma\wedge c>0\wedge\psi<-c\end{cases} \tag{38}$$

The Relative Risk Aversion is given by:

$$RRA=-c\frac{U''(c)}{U'(c)}=\frac{c\left(c+\psi\right)^{-\gamma-1}}{\left(c+\psi\right)^{-\gamma}}=\frac{c}{c+\psi} \tag{39}$$

The utility will display Increasing, Constant, or Decreasing Relative Risk Aversion if:

$$\frac{\partial RRA}{\partial c}=\frac{\psi}{\left(c+\psi\right)^{2}}\Rightarrow\begin{cases}\frac{\partial RRA}{\partial c}>0\Leftrightarrow\psi>0 & \text{Increasing - IRRA}\\ \frac{\partial RRA}{\partial c}=0\Leftrightarrow\psi=0 & \text{Constant - CRRA}\\ \frac{\partial RRA}{\partial c}<0\Leftrightarrow-c<\psi<0 & \text{Decreasing - DRRA}\end{cases} \tag{40}$$

If $\psi>0$, the depositor still enjoys some utility even if the consumption allocation is 0. In contrast, if $\psi<0$, the depositor faces a decrease in their utility level for any consumption allocation greater than 0. The case of $\psi=0$ is the typical Constant Relative Risk Aversion (CRRA) utility.

B Proofs

B.1 Proof of Proposition 3.1

Suppose the bank offers a deposit contract that leads to running for both types in both states. Given that all agents will run with probability one, the bank invests all its deposits in the liquid asset such that $y = \omega$ is optimal. Suppose the deposit contracts have the form of $d_1 = d_2 = \frac{\omega}{2}$. The bank's utility under this contract is characterized by

$$W = 2U\left(\frac{\omega}{2}\right) \tag{41}$$

Now, suppose the following deposit contract:

$$d_i = \frac{\omega}{2} \text{ for } i = 1, 2 \tag{42}$$

$$y = \lambda\omega \tag{43}$$

Given that $\lambda \in (0, 1)$, then the amount invested in the illiquid asset is $x = \omega - \lambda\omega = \omega(1 - \omega)$. The return on this asset is given by $R_H > R_L > 1$. Thus, the amount available to distribute among the depositors at $t = 2$ is $R_s\omega(1 - \lambda)$. The deposit contract for $t = 2$ is of the form (assuming an equal distribution among groups) is

$$c_{2is} = \frac{R_s\omega(1-\lambda)}{2(1-\lambda)} = \frac{R_s\omega}{2} \text{ for } i = 1, 2, s = H, L \tag{44}$$

Then, this last contract will dominate the original contract whenever

$$c_{2is} \geq \frac{\omega}{2} \Leftrightarrow \frac{R_s\omega}{2} \geq \frac{\omega}{2} \Leftrightarrow R_s \geq 1 \text{ for } i = 1, 2, s = H, L \tag{45}$$

It follows that since $R_H > R_L > 1$, the last inequality in the previous condition will hold with strict inequality. Since it holds For any agent i and any state s, the bank will never find optimal to offer a contract that induces a run in both states $s = H, L$, for both groups of agents. $\square$

B.2 Proof of Proposition 3.2

Suppose that the banks offer a contract in which some type of agent i run in the high state but do not run in the low state that satisfies

$$c_{2iL} \geq d_i > c_{2iH} \tag{46}$$

where d_i is the face value of the deposit contract that induces the supposed behavior of group i. Given that there is no run in the state $s = L$ by the $(1 - \lambda)$ proportion of late-type agents of the group i, it must be true that $y \in (0, 1)$ to invest in the illiquid asset $x = \omega - y > 0$ as to have a $R_L(\omega - y)$ units to distribute among the depositors conditional on being in state L. Now, suppose a contract in the form of

$$\begin{cases} \tilde{c}_{2iH} = c_{2iL} & \text{if } s = H \\ \tilde{c}_{2iL} = c_{2iL} & \text{if } s = L \end{cases} \tag{47}$$

Under this contract, the $(1-\lambda)$ agents of late-type in group i are at least as good as they were in the previous contract. On the other hand, the deposit contract that the bank has provided at $t=2$ in this state H for both group i and j is given by

$$(1-\lambda)\left(\tilde{c}_{2iH}+\tilde{c}_{2jH}\right)=R_L\left(\omega-y\right) \text{ for } j\neq i \tag{48}$$

Note, however, that the return in state H is $R_H > R_L$, such that the bank will have an excedent of returns in $t=2$ given by

$$\left(R_H-R_L\right)\left(\omega-y\right)>0 \tag{49}$$

Hence, it is still possible to offer a welfare-improving allocation in which there is no run in the high state, and it does not necessarily lead to a run in the low state. □

B.3 Proof of Proposition 3.3

Suppose the bank offers a contract that leads to a run for group i in both states $s=H,L$. Note that the present contract could not lead to a run in both states for both groups (See Proposition 3.1) or that it leads to a run by group $j\neq i$ in state H but not in state L (See Proposition 3.2). It follows that the only two remaining scenarios are that this contract does (not) lead to run in state L for group j but never in the state H.

First, assume that the present contract does not lead to a run for group j in either state. This means that

$$\begin{cases} c_{2js}\geq d_j & \text{for states } s=H,L \\ d_i > c_{2is} & \text{for states } s=H,L \end{cases} \tag{50}$$

The bank has to invest in the liquid asset at least the amount to cover the deposit contracts of the group i that ran on the bank and the proportion λ of early types of group j. That is,

$$y\geq d_i+\lambda d_j > \lambda d_i+\lambda d_j \tag{51}$$

The second inequality characterizes the total face value of a contract that does not lead to a run for group i in either state and given that $\lambda\in(0,1)$. Under the present contract, given that $y\in(0,\omega)$ since it needs to be large enough to provide non-zero consumption at $t=1$ for either group and it needs to invest some in the illiquid asset since $R_s>1$ in either state $s=H,L$. Initially, assume that $y>d_i+\lambda d_j$, then the total amount that the bank would have to distribute at $t=2$ is given by

$$R_s\left(\omega-y\right)+y-d_i-\lambda d_j \text{ for } s=H,L \tag{52}$$

Now, suppose that the bank offers a contract such that group i does not lead to a run in state $s = H$ but leads to a run in state L (given proposition 3.1 and 3.2). Furthermore, assume that the contract is such that $\varrho d_i + \lambda d_j < y$ where $\varrho \in (0,1)$and $\varrho \leq \lambda$ is the proportion of agents that do not run on the bank from group i. This contract will dominate the original contract because, for any state $s = H, L$, the total amount for the bank to distribute will be bigger. That is,

$$R_s(\omega - y) + y - d_i - \lambda d_j < R_s(\omega - y) + y - \varrho d_i - \lambda d_j \leq R_s(\omega - y) + y - \lambda d_i - \lambda d_j \tag{53}$$

Hence, the depositors will have deposit contracts at least as big as the original contract. Note that in the case that $y = d_i + \lambda d_j$, the pie in $t = 2$ in either state is $R_s(\omega - y)$ and it will be strictly smaller than $R_s(\omega - y) + y - d_i - \lambda d_j$. Consequently, the previous result holds for $y \geq d_i + \lambda d_j$.

Second, assume that the bank offers a contract that leads to a run in state L for group j, then the bank would have to invest in the liquid asset

$$y \geq d_i + d_j \text{ for } j \neq i \tag{54}$$

Note that this case will be dominated by the case where the bank offers a contract that does not lead to a run in state L for group j, given that

$$y \geq d_i + d_j > d_i + \lambda d_j > \lambda d_i + \lambda d_j \tag{55}$$

Consequently, it is never optimal to choose a contract that leads to a run in **BOTH** states $s = H, L$ by some group i. $\square$

B.4 Proof of Proposition 3.4

Suppose that it does not hold with equality. That is, $\lambda(d_1 + d_2) < y$. This suggests that some of the investment in the liquid asset is left over at date 1. The bank could reduce the amount invested in such an asset by $\epsilon > 0$ and invest in the illiquid asset with a return $R_s > 1$ for all $s = H, L$. The net change in the available return for consuming at $t = 2$ is $(R - 1)\epsilon > 0$. Hence, one could improve the consumption of the late type consumer without affecting the consumption of the early type. This cannot be optimal. It follows that in any optimal plan $\lambda(d_1 + d_2) = y$. $\square$

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