Bank Runs and Inequality

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Abstract

This article investigates the relationship between income inequality and bank runs and proposes a banking model to explain the causal link between these two. I develop a theoretical model that demonstrates that a rise in income inequality increases the likelihood of a bank run, thus establishing a causal relationship between the two. Increasing inequality drives changes in both relative risk aversion and marginal propensity to consume. In this way, the bank faces greater challenges and finds it more difficult to fulfill its role. Therefore, inequality brings financial instability. Using novel historical data for 17 countries, I find that increasing income inequality, in the form of an increase in the income share held by the top percentiles of the income distribution, correlates with an increased probability of bank runs of between 0.3 and 0.7 percentage points.

1 Introduction

Financial crises have, among others, adverse effects on consumption and output, investment, productivity, employment, and health (Chodorow-Reich, 2014, Cutler et al., 2002, Jensen and Johannesen, 2017, Romer and Romer, 2017). While Kirschenmann et al. (2016) and Paul (2022) identify income inequality as a significant predictor of financial crises in developed countries, they have not yet found a causal link between the two. I fill this gap in the literature, building on Friedman and Schwartz (1963), which recognized bank runs as triggers for financial crises, by studying the relationship between income inequality, bank runs, and financial instability.

This article studies another possible mechanism that may affect the probability of a bank run. Specifically, this paper looks at how increasing income inequality affects this probability. I extend the bank run model proposed by Diamond and Dybvig (1983) and Allen and Gale (1998) to accommodate heterogeneity in endowment levels between two groups of agents by having a mean-preserving distribution of endowment between groups. The wedge between both groups describes the income inequality of the economy. This model considers only fundamental

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bank runs (i.e., those not produced by sunspots or self-fulfilling prophecies as coordinating mechanisms). Furthermore, while it incorporates the assumptions on preferences and timing from Diamond and Dybvig (1983), it distinguishes from its model in two ways: (i) it does not assume sequential withdrawal from the depositors, and (ii) the illiquid assets held by the bank have different productivity between the states of the economy. In the model, depositor preferences exhibit decreasing relative risk aversion.[1]

The mechanism that mediates the relationship between income inequality and bank runs is that, by allowing ex ante differences in the endowment level, I can incorporate the impact of income inequality on both Relative Risk Aversion and Marginal Propensity to Consume (MPC) (Arrondel et al., 2019, Carroll et al., 2017, Fisher et al., 2020). As the outside option for the depositor (i.e., the utility of consuming its own endowment) goes on opposite directions for both income groups, the risk profile of each depositor changes given the type of Decreasing Relative Risk Aversion on their preferences. This also relates to the MPC. In this case, since the setting is an endowment economy, there is no monetary income. Hence, the MPC needs to be linked to the Marginal Rate of Substitution (MRS) if we want to know the fraction of an additional dollar of income spent on consumption. One can link MRS and MPC if one interprets the agent's endowment as an income where any trade between $t = 1$ and $t = 2$ can be considered as consumption from this endowment. If an individual is more willing to trade a significant portion of his endowment from $t = 1$, for consumption $t = 2$ (high MRS), this can be interpreted as a higher MPC for goods at $t = 1$.

Kaplan and Violante (2022) find that the MPC varies within the income distribution of the economy: Higher (lower) quintiles of the income distribution have lower (higher) MPC, leading to lower aggregate consumption. Additionally, income inequality affects the agent's risk profile through a decrease in relative risk aversion, so that low-income groups have less risk aversion than high-income groups, making the high-income group more willing to pool risk (Ogaki and Zhang, 2001). This also leads to changes in MPC (Carroll, 2009). Agents have preferences about liquidity and about their investment portfolio that satisfy their consumption demand. As there is a single bank in this economy, the bank has to address the different MPC of both groups of depositors by offering deposit contracts that (i) induce truth telling from the part of the depositors (i.e. reveal their liquidity preference), and (ii) are attractive enough so the depositors

[1] This is consistent with Ogaki and Zhang (2001) that found that relative risk aversion varies across the income distribution. This type of preference also allows for increasing risk sharing. These types of preferences are part of those represented by the hyperbolic absolute risk aversion (HARA), which includes those with constant and increasing relative risk aversion. See Appendix A for a quick review of these preferences.

are willing to accept the deposit contracts matching their portfolio preferences. However, as income inequality increases, there is a point where depositors will not be motivated to reveal their type, leading to a bank run.

Consequently, banks play a key role in protecting against liquidity shocks and aligning the demand for consumption allocations from deposit contracts with the preferences of the asset portfolio (Allen and Gale, 1998, Diamond and Dybvig, 1983, Farhi et al., 2009, Goldstein and Pauzner, 2005). Specifically, banks face a trade-off between risk sharing and the total surplus to be shared while fulfilling the consumption demand that comes from deposit contracts that are incentive compatible and promote depositor participation (Hellwig, 1998, Kaplan, 2006). In conclusion, inequality affects the fragility of the bank by amplifying the challenges it faces when fulfilling its role in the economy. The bank's intermediation role may fail due to low-state return realization (production side) or an unexpected liquidity shock (demand side) that may yield inadequate consumption allocations, ultimately producing bank runs.

In equilibrium, the probability of a bank run increases with income inequality. The bank's decision to allow this run stems from a rational cost analysis in which it is, in fact, optimal to offer deposit contracts that induce a bank run. Additionally, I find that the consumption allocations of the richer group increase with increasing inequality, while the opposite occurs to the poorer group. This can be attributed to the fact that increasing inequality raises the outside option for the latter, while it decreases for the former. Looking at the investment portfolio, the bank needs to balance it toward the illiquid assets. Once the income inequality reaches a point where a run is likely to occur, the bank adjusts the risk profile of the portfolio: reducing the amount of illiquid assets while still maintaining a balanced approach to that type of asset. These changes in the risk profile are consistent with the changing risk preferences of depositors reflected in their new consumption demands for deposit contracts.

Then, why does increasing income inequality lead to changes in both the relative risk aversion and the marginal propensity consume? Depositors from both groups show a shift in their risk behavior depending on their liquidity shock, where some become more risk averse and others become less risk averse. The bank must adjust its portfolio management strategy and act on its role as a liquidity provider to take into account these new risk profiles. Furthermore, the marginal propensity to consume changes with increasing inequality, making it difficult for banks to meet the deposit contract demands due to the availability of the consumption goods from their asset purchases. If the bank cannot meet the expected consumption demands, it is vulnerable to bank runs that lead to financial instability.

The main finding of the model was corroborated by examining the statistical correlation between income inequality and bank runs. The analysis suggests a positive correlation between increasing income inequality and the likelihood of bank runs. A standard deviation increase in income inequality is associated with a 0.3 to 0.7 percentage point increase in the probability of a bank run. Since the unconditional probability of a bank run was estimated to be around 4% per year, a percentage point increase is a significant increase.[2]

Literature Review. The literature has recently focused on the determinants of financial crises (see Baron et al. (2021), Gorton and Ordoñez (2020), Kirschenmann et al. (2016), Paul (2022)). Researchers have focused on a wider set of financial crises, following the definitions of Laeven and Valencia (2012), Schularick and Taylor (2012), and Reinhart and Rogoff (2009), among others. From the vast set of determinants, Kirschenmann et al. (2016) and Paul (2022) have found that income inequality has predictive power for financial crises, in a general sense, in developed countries. My contribution in this area is to document the particular correlation between income inequality (as measured by Paul (2022)) and bank runs (as defined by Baron et al. (2021)).[3]

The literature has also examined the theoretical channel between income inequality and financial fragility. Malinen (2016) provides a brief but interesting review of this channel, documenting that the relationship between income inequality and financial crises operates through the bank credit channel: An increase in income inequality leads to a rise in bank credit or leverage, amplifying credit cycles (which in turn can generate bank runs and then financial crisis). For example, Kumhof et al. (2016) finds that increasing income inequality leads to the accumulation of debt-to-income ratios, which results in a financial crisis. In summary, there is a relationship between income inequality and credit cycles in which credit accumulation plays a fundamental role. My contribution is to establish a channel between income inequality and fundamental bank runs in the form of riskier investment portfolios to cover aggregate consumption that increases with inequality. These riskier investment portfolios lead to financial fragility.

This paper contributes to the literature on income inequality and financial fragility (see Choi (2014) and Mitkov (2020)). More importantly, Garcia and Panetti (2022) looks into context similar to that in this paper. They investigated how wealth inequality makes financial crises

[2]This result holds for different covariates. A big caveat to this analysis is that with the available data, it is impossible to determine causality (that is, an increase in income inequality precedes a bank run).

[3]Baron et al. (2021) do not explicitly define bank runs as fundamental bank runs, they define them as bank panics. Thus, it is not possible to determine whether they are fundamental bank runs ex-ante.

more likely, finding that higher wealth inequality directly increases the incentives to run for the poor and indirectly for the rich through higher bank liquidity insurance. These incentives make bank runs more likely to be self-fulfilling. To achieve these results, they make use of two main assumptions. First, they have multiple balance sheets that ring-fence the asset investment by wealth level, acting as universal banks. Second, they have an investment externality assumption that accounts for the spread of wealth between wealth groups, ultimately leading to bank runs. The main difference in my contribution is that even in a model with a unique balance sheet and no investment externality, fundamental runs will happen if inequality is large enough.

Finally, this paper contributes to the literature on modeling bank runs from the seminal work of Diamond and Dybvig (1983). More precisely, it extends the model presented in Allen and Gale (1998) to accommodate ex ante identical agents with differing endowment levels to account for inequality.

This paper is divided as follows. Sections two, three, and four present the model, the numerical exercise, and the results and discussion of such results, respectively. Section five presents the correlation between income inequality and bank runs in the data. Section six concludes.

2 A Model of Income Inequality and Bank Runs

In this section, I elaborate on the banking model to present the mechanism underpinning the correlation between income inequality and bank runs.

2.1 Preliminaries, Preferences, and Endowments

There are three periods indexed to $t = 0, 1, 2$. Two possible states of nature $s = H, L$ occur with probability π_H and π_L such that $\pi_H + \pi_L = 1$. There is a continuum of agents with two masses composed of two groups of equal mass that differ only in their initial level of endowments. The two groups are indexed as $i = 1, 2$. Each depositor within a group is indexed to j. Without loss of generality, let the group of depositors $i = 1$ be endowed with ω_1 units of the final consumption good, and let the depositors in the group $i = 2$ be endowed with ω_2 units, where $\omega_1 > \omega_2 > 0$. These endowment levels are common knowledge. Depositors receive the endowment at $t = 0$ and do not receive any additional endowment at $t = 1, 2$. However, they want to consume $t = 1$ or $t = 2$.

These depositors are subject to a liquidity shock. That is, they are uncertain about the timing of their consumption. If the depositor j prefers to consume in $t = 1$, he is of type early, while if he prefers to consume in $t = 2$, he is of type late. These types are not common knowledge, but let the probability of being of the early type be $\lambda \in (0, 1)$, and consequently the probability of being of the late type be $1 - \lambda$, known to all agents. Given the equal mass of groups and the law of large numbers, the parameter λ can be interpreted as the proportion of agents that are of the early type.

The typical depositor j of the group i has preferences represented by a utility function $U(c_{ti})$ that is increasing, strictly concave, and differentiable twice continuously. Let the utility function be

$$U(c) = \frac{(c + \psi)^{1-\gamma}}{1 - \gamma} \tag{1}$$

This utility function in (1) represents Hyperbolic Absolute Risk Aversion (HARA) preferences. More importantly, let $-c < \psi < 0$ and $\gamma > 0$ so that (1) exhibits decreased relative risk aversion (DRRA).[4] Note that with this functional form for utility, the consumer will only have positive utility whenever $c > \psi$. The depositor will enjoy utility after a minimum consumption allocation is provided, also known as a subsistence level of consumption. The lower bound of ψ ensures that the utility is still a function of real value. Thus, consumption allocations live in $[-\psi, \infty)$.

The depositor j does not know if they are of type early or late until $t = 1$. He also does not know what would happen in $t = 2$. Hence, let the expected utility of the typical depositor j in group i be described by:

$$u(c_i, c_{2is}) = \sum_{s=H,L} \pi_s \left[\lambda U(c_i) + (1 - \lambda) U(c_{2is})\right] \tag{2}$$

Note that to truly reveal his type, the incentive compatibility constraint for agent j in group i is given by:

$$c_{1i} \leq c_{2i} \text{ for } i = 1, 2 \tag{3}$$

Finally, to ensure that the depositor accepts the deposit contract offered by the bank, this contract has to satisfy the participation constraint in the form of

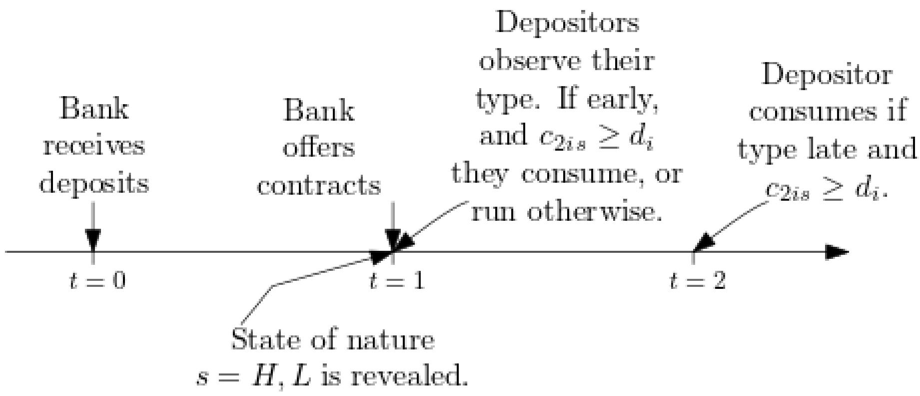
$$\mathrm{E}\left[\lambda U(c_{1i}) + (1 - \lambda) U(c_{2i})\right] \geq U(\omega_i) \text{ for } i = 1, 2 \tag{4}$$

The timing of the problem is presented in Figure 1. The bank receives the deposits at $t = 0$ and offers the deposit contracts at the end of that period. Most of the action occurs at $t = 1$. In this period, the depositor's type is revealed to the depositor (but not to the bank), and the state of nature is revealed to all economic agents. Then, if the incentive compatibility constraints hold,

[4] See Appendix A for a discussion of HARA preferences.

the depositors will withdraw at their respective periods and there will be no run. However, a run will occur if the incentive compatibility constraint is not satisfied.

Figure 1: The Timing of the Problem



2.2 Banks's Portfolio

There is a bank that takes the endowments of the depositors $\omega_1$and ω_2, and invests them in a portfolio made up of:

- A liquid asset (short-term) y with a constant return to scale technology that takes one unit of consumption good at t and transforms it into one unit of consumption good at $t+1$ for $t = 0, 1$. This technology can be thought of as a storage technology.

- An illiquid and risky asset x that has a constant return to scale technology that takes one unit of the consumption good in $t = 0$ and transforms it into R_H units of consumption good with probability $\pi_H \in (0, 1)$ in $t = 2$ or into R_L units of consumption good with probability $\pi_L \in (0, 1)$ in $t = 2$, where $\pi_H + \pi_L = 1$ and $R_H > R_L > 1$. Therefore, there are two possible states of nature $s = H, L$. In the early liquidation of this asset, the technology takes one unit of consumption good at $t = 0$ and transforms it into $1 > r > 0$ units of consumption good at $t = 1$.

Introducing this random asset return does not rule out bank runs that occur out of self-fulfilling prophecies or sunspots as a coordination mechanism. Thus, I am considering only essential bank runs (that is, bank runs that cannot be avoided).

Once the bank receives the endowments from the depositors at $t = 0$, it has to choose an investment portfolio (x, y) such that

$$x + y \leq \omega_1 + \omega_2 \tag{5}$$

This is a feasibility constraint for the bank. It suggests that the entire portfolio should be less than or equal to the total endowments in the economy. Now, let $\omega_1 \equiv 1+\tau$ and $\omega_2 \equiv 1-\tau$ for a $\tau \in (0,1)$ such that it complies with the assumption that $\omega_1 > \omega_2$ without increasing the size of the economy, that is, without making $\omega_1+\omega_2$ greater. Then, (5) becomes

$$x + y \leq 2 \equiv \omega \tag{6}$$

Note that the larger τ is, the greater inequality becomes. I am setting τ to move freely between 0 and 1. As the bank can only purchase assets with the aggregate level of endowment in the economy, it is not ring-fencing its services to attend a specific wealth group.

2.3 Bank's Possible Cases

Now, the bank cannot know which state of nature occurs. The late-type depositor in the group $i = 1, 2$ can run or not in the bank depending on whether the incentive compatibility constraint is satisfied or not. Therefore, the bank could potentially face up to 10 cases involving different maximization problems. Table 1 summarizes the possible cases.

Table 1: Possible Cases if Group $i = 1, 2$ Runs

State	**None**	**Both**	**group one**	**Group 2**
Low	Case 1	Case 2	Case 3	Case 4
High	Case 1	Case 5	Case 6	Case 7
Both	Case 1	Case 8	Case 9	Case 10

However, some cases are not optimal and can be discarded beforehand. The following propositions are aimed at discarding some nonoptimal cases.

Proposition 2.1. *There will never be a fundamental run in both states $s = H, L$ for both groups of agents.*

Proof. See Appendix B.1 □

The intuition behind this is that the contract that induces a fundamental run in both states for both groups of agents is dominated by a contract that offers at least the same amount as the previous contract in $t = 1$ and a positive amount in $t = 2$ because the return in either state $s = H, L$ is greater than 1. The social utility of the second contract is greater than that of the original contract. According to this proposition, Case 8, where the agents are running in both states, is not optimal.

Proposition 2.2. *It is never optimal to choose a contract that leads to a run in the high state for some group i but does not lead to a run for that group i in the low state.*

Proof. See Appendix B.2 □

The intuition is that this type of contract opens the possibility of a residual of the amount distributed at $t = 2$ that allows for a welfare-improving allocation where there is no run in the high state. It does not necessarily lead to a run in the low state for the group i. According to this proposition, the cases where groups 1 - case 6 - or 2 - case 7 - run in the high state and not in the low state cannot be optimal.

Proposition 2.3. *It is never optimal to choose a contract that leads to a run in **BOTH** states $s = H, L$ by some group i.*

Proof. See Appendix B.3. □

The intuition is that a contract that leads to a run in both states by a group i is dominated by a contract where the group $j \neq i$ is at least as well as before, and the group i is strictly better. This proposition suggests cases where group one, case 9 or group 2, case 10 cannot be optimal. Furthermore, the case where both groups run in the high state cannot be optimal by extension of propositions 2.2 and 2.3.

The remaining cases are case 1 (no group runs in any state), case 2 (both groups run in low state), case 3 (group one runs in low state), and case 4 (group 2 runs in low state).

2.3.1 Case 1: No agents run in both states

In the first case, neither agent runs in the bank in either or both states. Let d_i for $i = 1, 2$ be the face value of the deposit contract at $t = 1$. The bank's maximization problem is given by:

$$\max_{d_1, d_2, y, \{c_{2is}\}_{i=1,2,s=H,L}} \sum_{s=L,H} \pi_s \left\{ \lambda \left[U\left(d_1\right) + U\left(d_2\right) \right] + (1-\lambda) \left[U\left(c_{21s}\right) + U\left(c_{22s}\right) \right] \right\} \tag{7}$$

subject to

$$\lambda \left(d_1 + d_2\right) \leq y \tag{8}$$

$$(1-\lambda)\left(c_{21s} + c_{22s}\right) = R_s\left(\omega - y\right) + y - \lambda\left(d_1 + d_2\right) \text{ for } s = H, L \tag{9}$$

$$c_{2is} \geq d_i \text{ for } i = 1, 2, s = H, L \tag{10}$$

$$\lambda U(d_i) + (1 - \lambda) \left[\pi_H U(c_{2iH}) + \pi_L U(c_{2iL})\right] \geq U(\omega_i) \text{ for } i = 1, 2 \tag{11}$$

$$0 \leq y \leq \omega \tag{12}$$

Condition (8) suggests that the total face value of the deposit contracts offered to the λ share of early-type depositors in both groups should be less or equal to the amount invested in the liquid asset y. More importantly, Proposition 2.4 implies that this condition should be in equality, since it is never optimal to leave some investment in the liquid asset from $t = 1$ to $t = 2$ because if the same amount were invested in the illiquid asset, it would yield $R_s > 1$ units more at $t = 2$.

Proposition 2.4. *The condition $\lambda (d_1 + d_2) \leq y$ should hold with strict equality in optimum.*

Proof. See Appendix B.4 □

The condition (9) holds with equality since the bank will give back all of what is available to the depositors at $t = 2$, in any state $s = H, L$. By the result on the proposition (2.3), this condition can be rewritten as

$$(1 - \lambda)(c_{21s} + c_{22s}) = R_s (\omega - y) \tag{13}$$

Condition in (10) is the incentive compatibility constraint for both agents in both states if the bank offers a contract that leads to no runs by either agent in both states. These conditions imply that to induce truth-telling from the agents to reveal their type, those of the late type are provided with a consumption allocation greater than that of the early type.

The condition (11) is the participation constraint of both agents. These conditions imply that for the agent to take the deposit contract offered by the bank, the expected value of that contract should exceed the utility of consuming their endowment.

The condition (12) is a bank feasibility constraint that implies that the investment in the liquid asset should be greater than or equal to 0 and less than the total endowment of the economy. This condition will hold with strict inequality due to the non-negativity constraints of consumption allocations.

2.3.2 Case 2: Bank Offers a Contract such that All Run in the Low State

The second case is that both agents run on the bank in the low state. Let $\tilde{d}_{is}$ be the deposit contract that will be offered in case of a bank run in state L. The bank's maximization problem is given by

$$\max_{d_1,d_2,y,\{c_{2is}\}_{i=1,2}^{s=H,L}} \pi_H \sum_{i=1,2} \{\lambda [U(d_i) + (1-\lambda) U(c_{2iH})]\} + \pi_L \left\{U\left(\tilde{d}_{1L}\right) + U\left(\tilde{d}_{2L}\right)\right\} \tag{14}$$

subject to

$$d_1 + d_2 \geq [r(\omega - y) + y] \tag{15}$$

$$c_{2iH} > d_i, i = 1,2 \tag{16}$$

$$\tilde{d}_{iL} = \frac{d_i}{d_1 + d_2} [r(\omega - y) + y], i = 1,2 \tag{17}$$

$$\lambda (d_1 + d_2) = y \tag{18}$$

$$(1-\lambda)(c_{21H} + c_{22H}) = R_H (\omega - y) \tag{19}$$

$$\pi_H \{\lambda U(d_i) + (1-\lambda) U(c_{2iH})\} + \pi_L \left\{U\left(\tilde{d}_{is}\right)\right\} \geq U(\omega_i), s = 1,2 \tag{20}$$

$$0 \leq y \leq \omega \tag{21}$$

In this problem, the objective function changes. The portion for state $s = H$ is the same, since there is no run. However, for $s = L$, it changes to accommodate the fact that both agents are running in this state. In this case, the entire mass of both groups of agents is running. Thus, this portion does not depend on λ. In terms of the restrictions, first note that conditions (16), (18), (19) and (21) are similar to that in the case where no agent runs.

The incentive compatibility in state $s = L$ requires some additional explanation. This condition is to avoid the unilateral deviation of the late-type depositor. In this case, the deviation is that the late depositor does not run and waits to consume at $t = 2$. In case of a run, the bank has to liquidate their long-term asset at a fire sale rate of $r < 1$ and use it, in addition to whatever the bank has on the liquid asset, to pay for the consumption allocations of the agents that run. This is captured by the expression on the right-hand side of the condition (15).

Suppose condition (15) is violated such that the amount of liquidated assets is larger than the deposit contracts in $t = 1$. In this case, the bank can pay the deposit contracts to all agents that ran at their face value at $t = 1$. Given the assumption that the bank has to return whatever is left in $t = 2$, the agent that deviated would receive a large amount (infinity?) at $t = 2$. Since all agents of type late $(1-\lambda)$ are ex ante identical, all agents of type late would be incentivized to deviate and wait until $t = 2$ for consumption. In this case, a welfare-improving allocation would not be to liquidate the illiquid assets since $R_L > r$ and all agents would have been better off, and this case would not occur. It follows that to have a run from both groups at $s = H$ condition (15) should always be satisfied.

The condition (17) suggests how the liquidated value of the assets is distributed in the event of a run. In this case, it is distributed proportionally to the face value of the deposit contracts promised by the bank. Finally, the participation constraint (20) has the same intention as in the case of no run, but it is modified to accommodate run allocation at $s = L$.

2.3.3 Cases 3 and 4: Bank Offers A Contract Where One Group Runs and the Other Does Not.

First, note that cases 3 and 4 are identical with the changed subscripts of the group of agents. Then, let group 2 be the one that runs on the bank and group 1 does not run to the bank (i.e., case 9). The bank's maximization problem is given by:

$$\max_{d_1,d_2,y,\{c_{2is}\}_{i=1,2}^{s=H,L}} \pi_H \sum_{i=1,2} \{\lambda [U(d_i)(1-\lambda) U(c_{2iH})]\} + \pi_L \left\{U\left(\tilde{d}_{2L}\right) + \lambda U\left(\tilde{d}_{1L}\right) + (1-\lambda) U(c_{21L})\right\} \tag{22}$$

subject to

$$\begin{cases} c_{21L} \geq \tilde{d}_{1L} & \text{IC rich in low state} \\ \tilde{d}_{2L} \geq c_{22L} & \text{IC poor in low state} \end{cases} \tag{23}$$

$$c_{2iH} \geq d_i, i = 1, 2 \tag{24}$$

$$\lambda(d_1 + d_2) = y \tag{25}$$

$$\tilde{d}_{2L} = \begin{cases} d_2 & \lambda d_1 + d_2 \leq r(\omega - y) + y \\ \frac{d_2}{\lambda d_1 + d_2}[r(\omega - y) + y] & \text{otherwise} \end{cases} \tag{26}$$

$$\tilde{d}_{1L} = \begin{cases} d_1 & \lambda d_1 + d_2 \leq r(\omega - y) + y \\ \frac{\lambda d_1}{\lambda d_1 + d_2}[r(\omega - y) + y] & \text{otherwise} \end{cases} \tag{27}$$

$$(1-\lambda)(c_{21H} + c_{22H}) = R_H(\omega - y) + y - \lambda(d_1 + d_2) \tag{28}$$

$$(1-\lambda) c_{21L} = \begin{cases} 0 & \text{if } \lambda d_1 + d_2 > r(\omega - y) + y \\ R_L \left\{\omega - y - \left[\frac{\lambda d_1 + d_2 - y}{r}\right]_+\right\} + [y - (\lambda d_1 + d_2)]_+ & \text{otherwise} \end{cases} \tag{29}$$

$$\pi_H \{\lambda U(d_1) + (1-\lambda) U(c_{21H})\} + \pi_L \left\{\lambda U\left(\tilde{d}_{1L}\right) + (1-\lambda) U(c_{21L})\right\} \geq U(\omega_1) \tag{30}$$

$$\pi_H \{\lambda U(d_2) + (1-\lambda) U(c_{22H})\} + \pi_L \left\{U\left(\tilde{d}_{2L}\right)\right\} \geq U(\omega_2) \tag{31}$$

$$0 \leq y \leq \omega \tag{32}$$

where $[x]_+ = \max\{x, 0\}$

The objective function is different from the previous cases. The first portion is the sum of the utilities if the state $s = L$ occurs. The second portion is the sum of the utilities in the state $s = L$. In this case, a proportion of agents $(1 - \lambda)$ of group 1 will not run on the bank, while the entire mass of group 2 will run. Conditions (24), (25), (28), (30), and (31) are similar to those of the other cases.

The conditions in (23) are incentive compatibility constraints for both groups in state $s = L$. In this case, the incentive compatibility constraint for group one aims to deter the deviation of the late type to run on the bank, while for group 2 the objective is to deter the deviation of the late type to consume at $t = 2$ since, in this case, the bank offers a contract that induces a run in the low state for this group. Focus on the incentive compatibility for Group 2. The bank in this situation has to liquidate the illiquid asset in a fire sale at a rate $r < 1$ (in addition to the liquid asset) and use this to pay the consumption allocations promised in $t = 1$ for all the types early and late that run on the bank. That is, the bank has to pay $r(\omega - y) + y$.

2.3.4 Bank's Case Selection in Equilibrium

Let $W_k(d_1^k, d_2^k, c_{21H}^k, c_{22H}^k, c_{21L}^k, c_{22L}^k)$ be the social utility valued in the optimal allocations in case $k = 1, 2, 3, 4$. The bank will then choose case k over all other cases $-k$ whenever the social utility of case k is at least as good as the maximum social utility of case $-k$. That is,

$$W_k(\cdot) \geq \max\{W_{-k}(\cdot)\} \tag{33}$$

where $-k$ all the other cases but the k^{th}. By allowing the indifference to be solved in favor of the case with greater utility, I focus on the "best" equilibrium selection scenario.

3 Numerical Exercise

In this section, I present the results of a numerical exercise to demonstrate some of the properties of the model. The main goal is to look at how a change in τ, which implies changing the level of inequality, affects the various allocations of consumption, investment, the welfare function and, more importantly, the probability of a bank shutdown.

The parameters used in the numerical exercise are presented in Table 2. I set the parameter ψ at -0.4 so that the utility function presents a decreasing relative risk aversion. The parameters R_H and R_L imply that the risky asset pays two units of consumption goods per unit when it matures in the state H or 1.065 units in the state L. The parameter r implies that the recovery rate of the risky asset when liquidated early is about 80% the original investment value. I used

$\gamma = 3$ as standard. I set the parameter λ, the share of early-type depositors, at 15%. Finally, I set the probability of the low state at 15%.

Table 2: Set of parameters for numerical exercise

Parameter	λ	τ	R_H	R_L	γ	ψ	r	π_L
Value	0.15	[0.1,0.90]	2.0	1.065	3.0	-0.4	0.8	0.15

To estimate the model, I fixed the economic parameters and a level of τ and used the restrictions to bind grids of possible consumption allocations and investments. Given these consumption allocations and the investment level, the utility was estimated for each case. Then I proceeded to estimate the maximum utility given the fixed level of τ. Once I went through the entire grid of τ, I verified if the equilibrium conditions in (33) are satisfied by each level of τ.

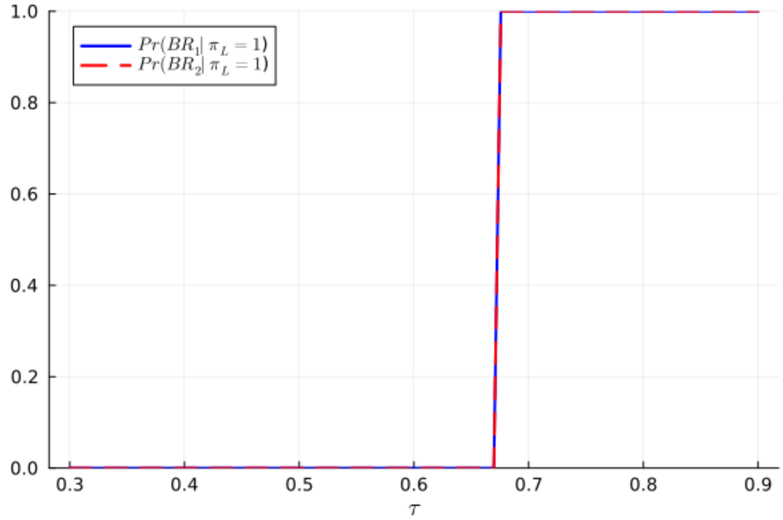
4 Results and Discussion

4.1 Results

The main result of the numerical estimation of the model is that the probability of a bank run increases with income inequality, measured by τ. This result is presented in Figure 2. Note that given the structure of the problem, runs can happen only in the low state. Thus, the probability of a bank run, conditional on being in the state L, is one once it reaches a sufficiently high τ (that is, τ^*). The unconditional probability is given by $Pr(BR|s = L) \times Pr(s = L) = \pi_L = 0.15$. The jump in the probability of a bank run is due to the discrete nature of the actions of either agent (that is, to run or not to run). This first result provides a causal relationship between income inequality and bank runs for the correlation found in the data.

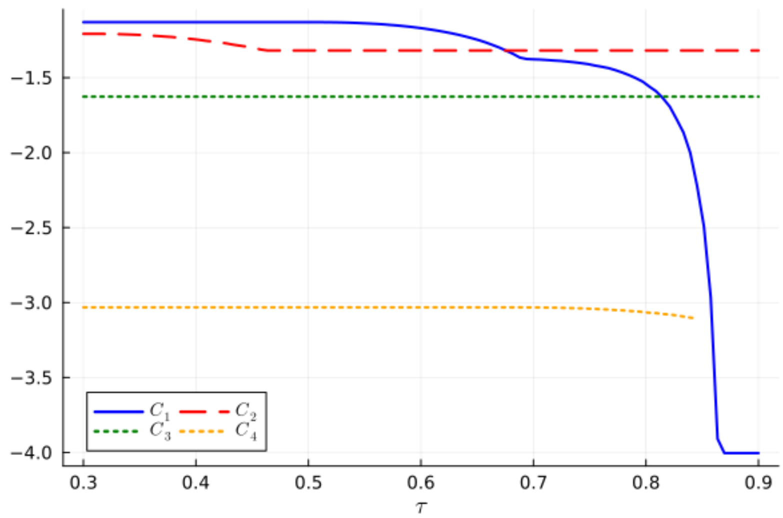
Second, we discuss the social utility function $W_k(\cdot)$ for each case $k = 1, 2, 3, 4$, presented in Figure 3. Focusing only on cases 1 and 2, one can see that the social utility function of case 1 (no bank run) intersects with that of case 2 (bank run) at τ^*. Given that the upper contour of the set of social utility functions gives the largest social utility, one can affirm that after τ^* it is socially optimal for the bank to commit to consumption allocations that induce a bank run. Note that the social utility functions that arise from Cases 3 and 4 are never optimal for the set of parameters used. The reason behind this is as follows. Assume case 3, where group 2 is given an allocation that induces a run, and group one does not. Since there is a run, the bank has to liquidate all its illiquid assets at $t = 1$ with a fire sale rate $r < 1$. This leaves the bank with $s^* \equiv r(\omega - y) + y$ units of the final consumption good to allocate in the λd_1 and d_2

Figure 2: Conditional Probability of Bank Run



deposit contracts. Suppose that s^* is just enough to cover the deposit contracts at $t = 1$. This, in turn, leaves $(1 - \lambda)$ depositors of the late type of group one with 0 consumption at $t = 2$ because there is no illiquid asset available to mature at $t = 2$. Hence, since $d_1 > 0 = C_{21L}$, the incentive compatibility constraint is not satisfied and late-type depositors of group one will run. A similar story will occur for Case 4.

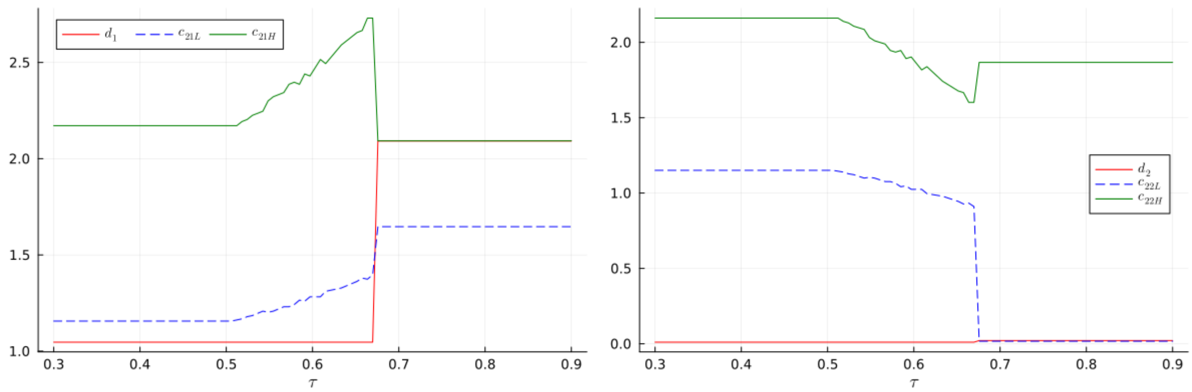
Figure 3: Utility Functions for Cases 1, 2, 3, and 4



In third place, I present the equilibrium consumption allocations for groups 1 and 2 in Figures 4a and 4b, respectively. Note that for group one (group 2), consumption allocations increase (decreasing) with τ. This trend in consumption allocations is because a higher income inequality (that is, an increase in τ) implies a larger outside option for group one, since their endowment increases with τ. On the contrary, the opposite happens for group 2. Then, the consumption allocation for each level of τ should be larger (smaller) for group 1 (group 2) to participate. Furthermore, depositors will only have positive utility when d_i and c_{2is} are greater than $-\psi$ for $i = 1, 2$ and $s = H, L$. One can understand the parameter ψ as the minimum consumption required for subsistence.

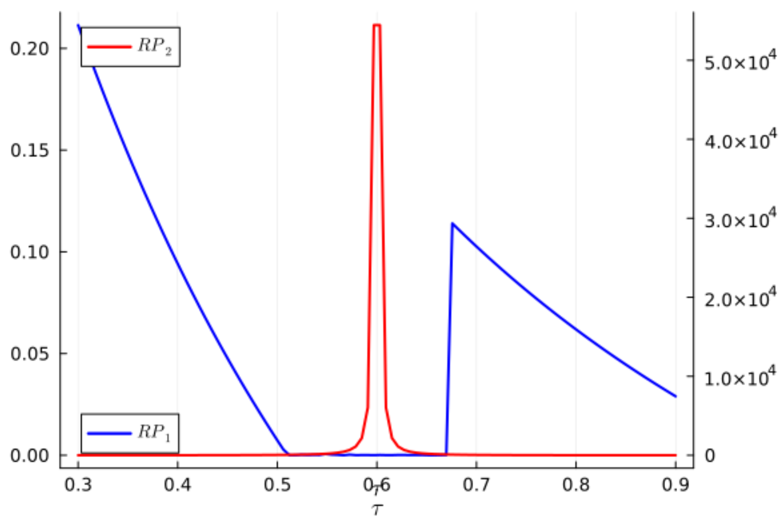
Figure 4: Equilibrium Consumption Allocations for Both Groups

(a) Group $i = 1$ (b) Group $i = 2$



The steplike consumption bundles can be due to i) grid restrictions on the numerical estimation and ii) how the participation constraint is satisfied. For the first reason, I estimated the model with different grid densities for consumption allocations, investment in the liquid asset, and the wedge between groups one and two. However, the estimated consumption allocations were identical. This would suggest that it is not a grid restriction.

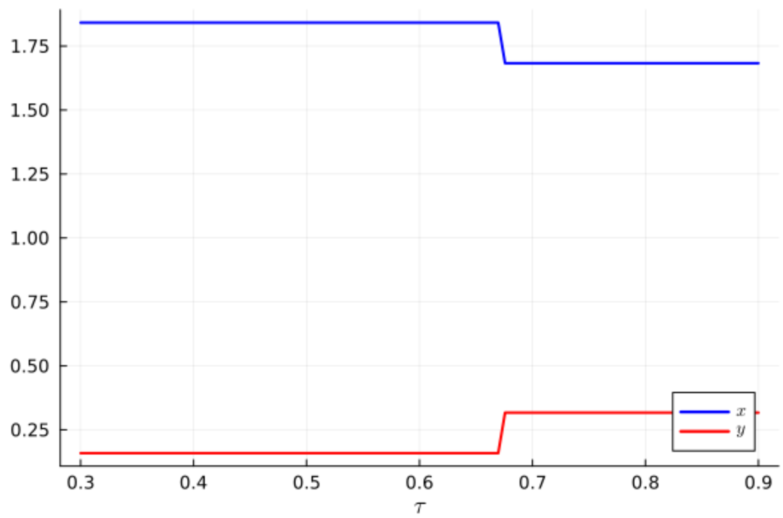
Figure 5: Participation Constraint in Equilibrium



For the second reason, I plot the participation constraint for both groups in Figure 5. The participation constraint behaves differently for both groups. The participation constraint of Group Two (red line) is constantly binding, except when τ is 0.6, which is satisfied but not binding. For the other group (blue line), it can be seen that it is satisfied and does not bond until an interval of τ between 0.5 and 0.7, where it is bound. Consequently, a plausible explanation for the step-like consumption schedules is that the bank constantly offers a constant consumption bundle that binds (satisfies) the participation constraint of group 2 (group one) until this constant bundle binds for both groups. From this point on, consumption allocations increase (decrease) for both groups while leaving the participation constraint binding for group one and satisfied for group 2 until τ^* where this relation reverses and replicates the same behavior: a constant bundle that satisfies the participation constraint for group one and binds for group 2.

In fourth place, the investment portfolio is presented in Figure 6. The portfolio must be balanced with the illiquid asset x to match the investment needs of the large proportion of late-type depositors in both groups. Once the level of inequality reaches τ^*, the portfolio must accommodate a lower risk by increasing the acquisition of liquid assets y. However, the total composition is still balanced toward the illiquid asset. This is because there is still the possibility of realizing the high state. Therefore, the bank must provide deposit contracts that cover this contingency while compensating for the loss of early liquidation in case the low state occurs, since the fire sale rate is $r < 1$.

Figure 6: Investment Portfolio



4.2 Mechanism Discussion

What is the mechanism behind the result that the probability of a bank run increases with increasing income inequality? The answer to this question is that increasing income inequality, where group one sees its endowment increase and group two sees it decrease, triggers changes in their relative risk aversion and marginal propensity to consume that jointly affect the bank's role as liquidity insurer and portfolio manager, thus altering the financial stability of the economy. I will explain each change separately below.

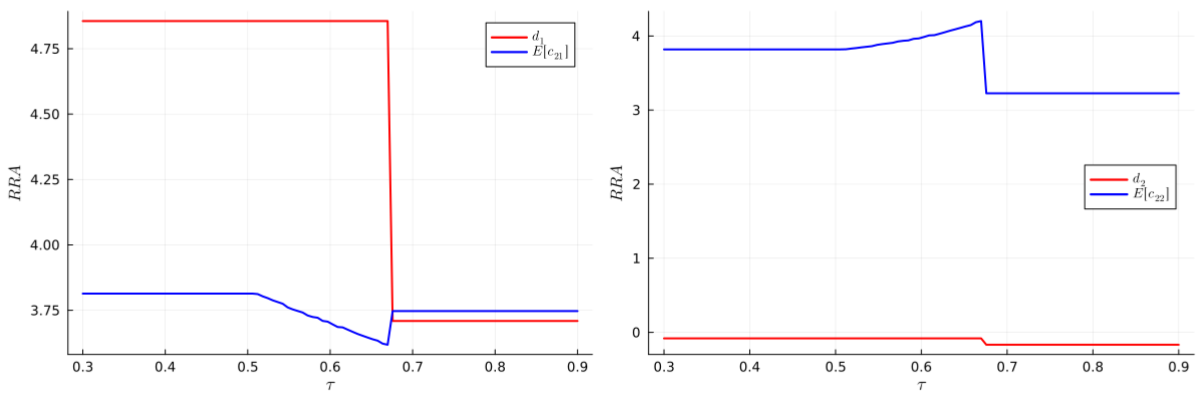
For the change in relative risk aversion, note that the preferences here exhibit decreasing relative risk aversion, so whenever income increases, the agent exhibits less risk aversion. As Appendix A explains, the sufficient condition for the preference to exhibit DRRA is $\psi < 0$. The relative risk aversion coefficient (RRA) is given by

$$RRA = \frac{\gamma c}{c + \psi} \tag{34}$$

Note that as long $c \neq \psi$, the RRA would be determined. The agent would exhibit risk-loving behavior if $c < \psi$, while $c > \psi$ would exhibit risk aversion. The RRA for each $i = 1, 2$ and for the allocation of consumption $(d_i, E[c_{2i}])$ where $E[c_{2i}] = \sum_{s=H,L} \pi_s c_{2is}$ in different τ is presented in Figures 7a and 7b.

Figure 7: Relative Risk Aversion Coefficient for Both Groups

(a) Group $i = 1$ (b) Group $i = 2$



The fraction λ of agents in the $i = 1, 2$ group that received a liquidity shock that compelled them to consume at $t = 1$ has a large RRA compared to the fraction $1 - \lambda$ of late-type depositors in the same group up to τ^*. Focusing on group 1, note that as shown in Figure 4a, consumption allocations increase in τ, which would reduce RRA (that is, become less risk-averse). This is consistent with the patterns reflected in the consumption allocations d_1 and $E[c_{21}]$. For group two, two situations occur. For the proportion $1-\lambda$ of late-type depositors, they exhibit greater risk aversion up to the point τ^* (blue line). After such a threshold, they reduce their risk aversion slightly, but still have a high relative risk aversion coefficient.

The situation differs slightly for the λ proportion of early-type depositors. Note that for such a group, the consumption bundles d_2 are less than $-\psi$; therefore, they would exhibit risk-loving behavior. This does not contradict the strictly concave requirement of the utility function since:

$$U'' = -\gamma(c+\psi)^{-\gamma-1} \tag{35}$$

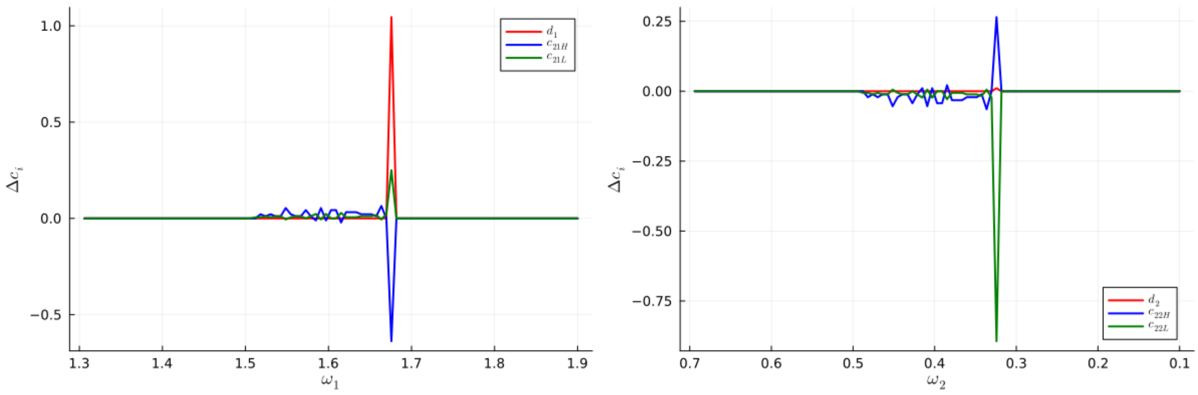
is still negative since $\gamma = 3$ in this parameterization. Therefore, early-type depositors in group 2 do not obtain positive utility from the consumption bundle. Therefore, their risk preference is that of a risk-loving one since equation (34) now becomes negative and this risk-loving behavior increases with increasing inequality, given the nature of the relative decreases in risk-aversion preferences. Bottom line, the risk profile for both groups changed and, therefore, the bank must account for this change in its role as portfolio manager, which explains the change in the portfolio risk balance in Figure 6 and its role as a liquidity shock insurer to cope with the consumption demanded from the deposit contracts shown in the consumption allocations in 4b.

Now, I will discuss the changes in the MPC. As discussed above, the MPC has been modified to accommodate the particulars of this economy. Given that there is no monetary income, one can assume that the income for each depositor is its exogenous endowment. Let the MPC be defined as $MPC = \frac{\Delta c_i}{\Delta \omega_i}$ for $i = 1, 2$. The definition suggests that the MPC could change by changes in either the consumption bundles or the endowment. In this case, changes in the denominator can be mapped directly to a change in income inequality. For some intervals of τ, the consumption allocations are constant; thus, in these intervals, the MPC will be 0, while for the remaining intervals, the MPC will vary for each group and each consumption allocation. The change in consumption allocation for each discrete jump in ω_i is presented in Figures 8a and 8b, for groups 1 and 2, respectively.

Focusing on the neighborhood of τ^*, one can see how increasing inequality affects consumption allocations differently for each group. For group 1, the marginal propensity to consume appears to increase for d_1 and c_{21L}, while it decreases for c_{21H} just before τ^*. For group two, the marginal propensity to consume increases for d_2 (although very slightly) and for c_{22H}, while it decreases drastically for c_{22L} just before τ^*. The bank has to match the consumption demands expected from the deposit contracts with the finite amount of final good available after the maturity of the assets it has purchased. Therefore, the large spikes and valleys in the MPC just

Figure 8: Marginal Propensity to Consume for Both Groups

(a) Group $i = 1$ (b) Group $i = 2$



before τ^* may make the bank unable to fulfill these deposit contracts and make it vulnerable to runs. The inability to provide the required deposit contracts generates financial instability in the economy.

In conclusion, increasing income inequality drives changes in both the relative risk aversion and the marginal propensity to consume, making it challenging for the bank to fulfill its role and inducing financial instability. For relative risk aversion, depositors exhibit changes in risk behavior between early and late types in both groups, with some becoming more risk averse, while others displaying risk-loving behavior. The bank must adjust its portfolio management strategies and its role as a liquidity insurer to account for these new risk profiles. Similarly, MPC changes with increasing inequality, making it difficult for banks to fulfill their deposit contracts around τ^*, given the constraints imposed by the availability of consumption goods arising from their asset purchases. If the bank cannot meet the expected consumption demands, it becomes susceptible to bank runs, which subsequently induce financial instability.

4.3 Comparative Statics

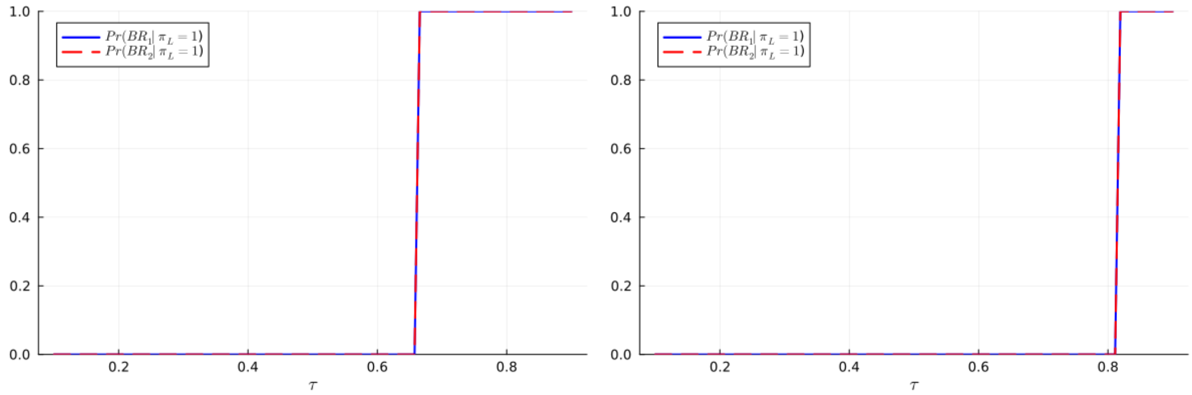
The rest of this section presents different comparative statics involving changes in the return of the illiquid asset in the low state R_L, changes in the liquidity preferences captured by λ, and changes in the fire-sale rate r. These parameters are more likely to affect the probability of a run, all else equal, because of its direct incidence in the low state.

Changes in the return of the return of the illiquid asset in the low state are presented in Figures 9a and 9b. Note that the decrease or increase in return shifts τ^* to the left or right, respectively. In return, this change affects the size of the final total good available for distribution in state

L. Thus, the increase in the return implies that the bank can pay for deposit contracts in cases with more inequality. The opposite occurs in the case where the return decreases.

Figure 9: Changes in $Pr(BR|s = L)$ with changes in R_L

(a) Decreasing R_L (b) Increasing R_L



Changes in liquidity preferences have big impacts on the probability of a run, all else equal. This result is presented in Figures 10a, 10b, and 10c. Liquidity preferences are captured by the parameter λ, which is the probability that a given depositor is of the early type. Then the changes in λ reflect whether a depositor is willing to wait longer (less) to consume in $t = 2$. The numerical results of a small decrease in λ suggest that, all else equal, the probability of a run is 0 in the domain of τ (Figure 10a). A small increase in λ moves τ^* to the right. This is because increasing λ reduces the mass of late-type depositors; thus, all else equal, the bank will be more likely to fulfill the deposit contracts at higher levels of inequality. Finally, note that if the increase changes to the point where the majority are early-type depositors, the probability of a run is now decreasing in τ.

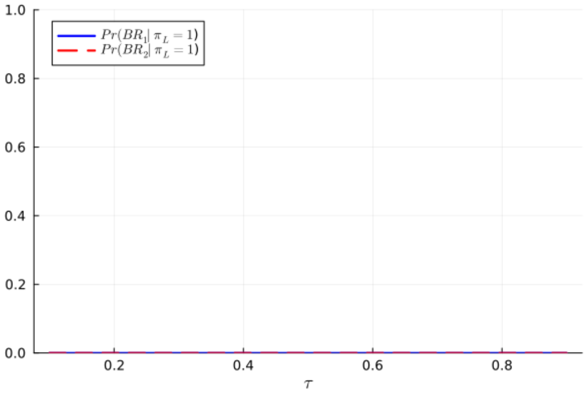
Finally, changes in the fire-sale rate emerge as a significant determinant in the probability of a bank run within our model. Specifically, increasing the fire-sale rate increases the return when banks are forced to liquidate their assets. This return, in turn, increases the incentive to accept deposit contracts for potential depositors available on a run. Consequently, it increases the probability of a bank run.

5 The Correlation Between Bank Runs and Income Inequality

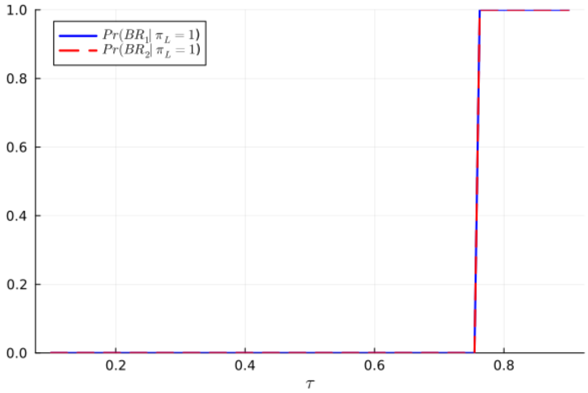
I use historical data in this section to discuss the correlation between income inequality and bank runs. First, I will describe the data used to look into this correlation. Second, I present evidence

Figure 10: Changes in $Pr(BR|s = L)$ with changes in λ

(a) Small Decrease in λ



(b) Small Increase in λ



(c) Large Increase in λ

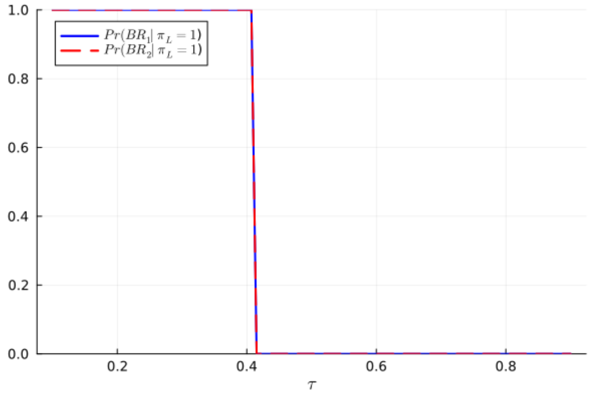
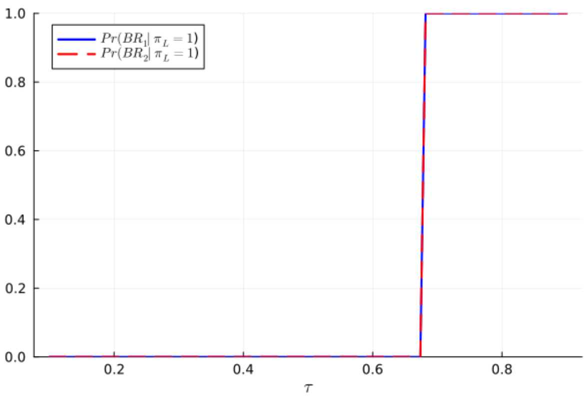
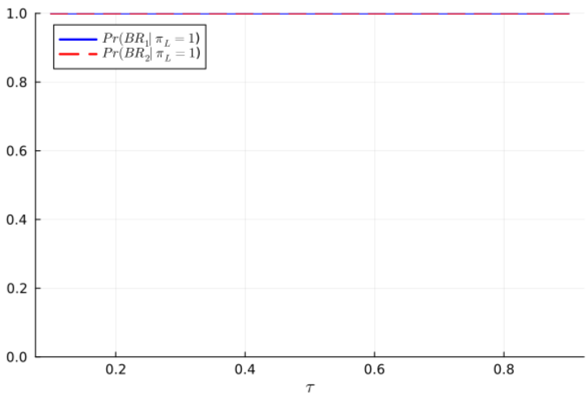


Figure 11: Changes in $Pr(BR|s = L)$ with changes in r

(a) Decreasing r



(b) Increasing r



on the prevalence of bank runs in advanced economies and how inequality behaves before such an event. Finally, I present the estimations that shed light on the correlation between bank runs and income inequality.

5.1 Data

I use two novel data sets. First, I use the data compiled by Paul (2022) on income inequality. This data set merges three long-term data sets from 1870 to 2013 for 17 countries.[5] The first long-term data set is from Òscar Jordà et al. (2016), which includes macro-financial variables for these 17 countries. The second long-term data set is from Bergeaud et al. (2016), which includes measures of TFP and labor productivity. The third long-term data set is the World Inequality Database, which includes measures of income shares held by various percentiles. The novel feature in the data set of Paul (2022) is that it includes income shares held by the upper percentiles, net of capital gains.[6]

In the second place, I use the data set for bank runs found in Baron et al. (2021). The authors collected information for 46 countries in a similar time frame as Paul (2022). More importantly, they collect bank run narratives under a common definition to capture bank runs.[7]

The final data set includes information for 17 countries from 1880 to 2013, accounting for 2,069 country-year observations of macroeconomic, income inequality, and bank run variables.

5.2 The Prevalence of Bank Runs and Inequality Trends

Figure 12 presents the prevalence of bank runs for these 17 countries between 1880 and 2013. According to Allen and Gale (2007), bank runs are nothing new and have not been restricted to emerging economies. Baron et al. (2021) suggests almost no evidence of nonfundamental runs occurring in this time frame. More importantly, bank runs are distributed across the period except for the post-WWII years (i.e., 1945-1970). Using the data set, I estimate that the unconditional probability of bank runs is around 4% per year.

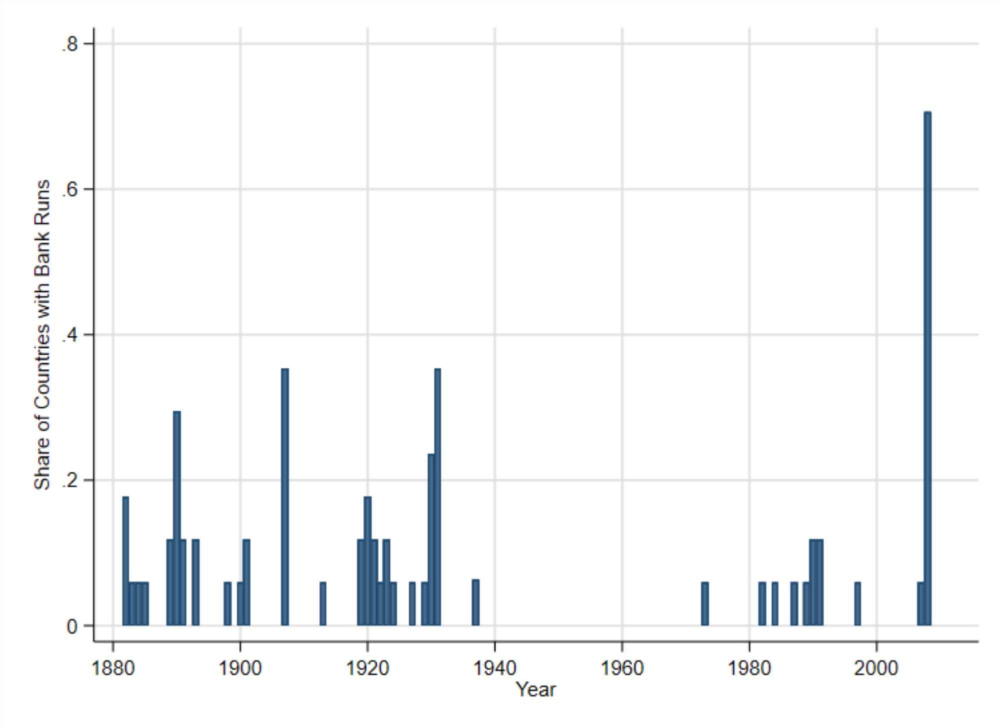
The measure of inequality I use is the share of income held by the top 0.1%, 1%, and 10% of

[5] The countries are Australia, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom, and the United States.

[6] The income share data is constructed using tax income data. The revenues of selling assets (i.e., stocks) are taxed depending on the tax system. The potential problem is that stock trading is typically concentrated among individuals in the upper percentile of the income distribution. This, in turn, makes capital gains available for reinvestment in the same assets, and the additional savings are not available for borrowing from other agents.

[7] Baron et al. (2021) describe bank runs as banking panics that are also bank equity crises (p. 102).

Figure 12: The Prevalence of Bank Runs



Source: Author estimation based on the data from Baron et al. (2021), Paul (2022).

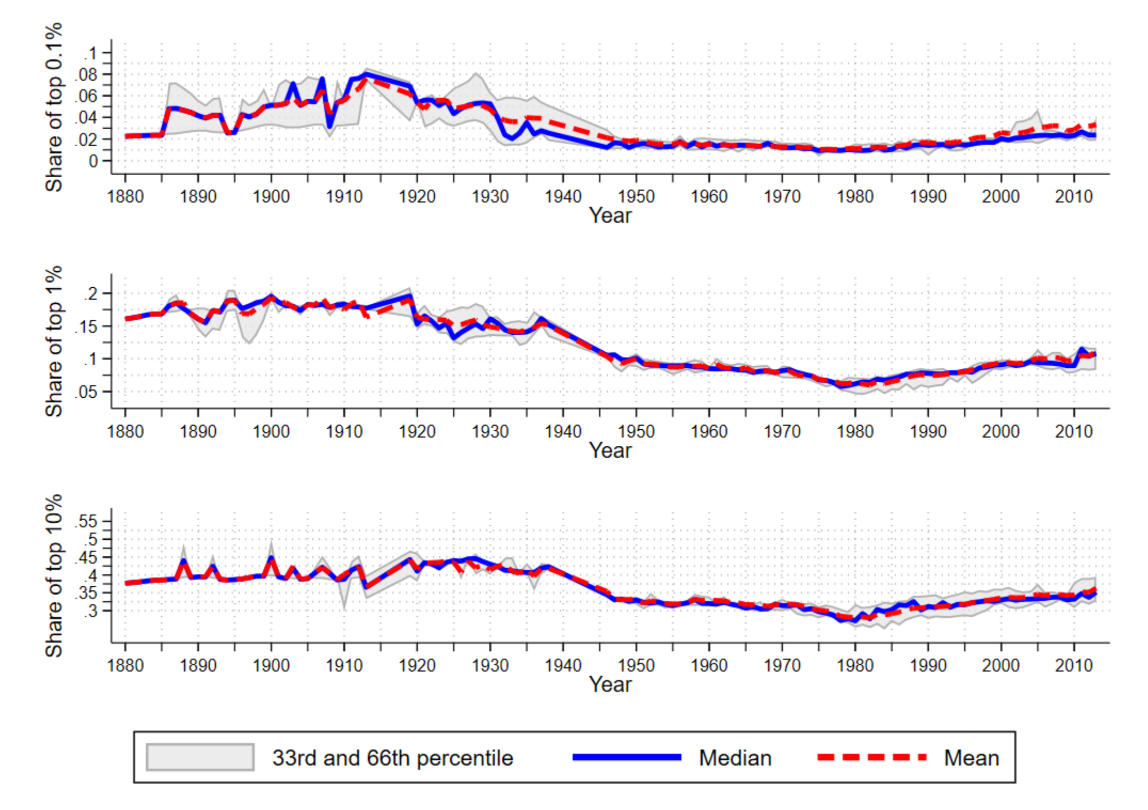
Note: The sample includes 17 advanced-economy countries described in footnote 5.

the income distribution net of capital gains found in Paul (2022). The trends of such inequality measures are presented in Figure 13. The dashed red line is the mean, while the solid blue line is the median. The borders of the gray area represent the 33rd and 66th percentiles of the inequality measures across the countries in the sample.

First, note that the inequality trends presented in Figure 13 have long cycles (that is, they oscillate very slowly). For instance, they increased steadily from the late 19th century until the 1930s. Then, they decreased similarly in the post-WWII period until the 1980s, when they increased again until the end of the sample's time frame. Second, the sample median for each of the three measures is 2.8%, 12.1%, and 35.7%, for the top 0.1%, 1%, and 10%, respectively. These suggest that between 1880 and 2013, in 50% of the countries in the sample, almost 35% of income was held by the top percentiles of the income distribution. Finally, the dispersion of such measures (i.e., the difference between the 33rd and 66th percentile) increases in increasing inequality, while it collapses in decreasing inequality.

These three features are by no means an exhaustive look at inequality trends around the world because they only apply to the countries in the sample. However, they account for the fact that

Figure 13: Share of Income Held by Top 0.1%, 1% and 10%.



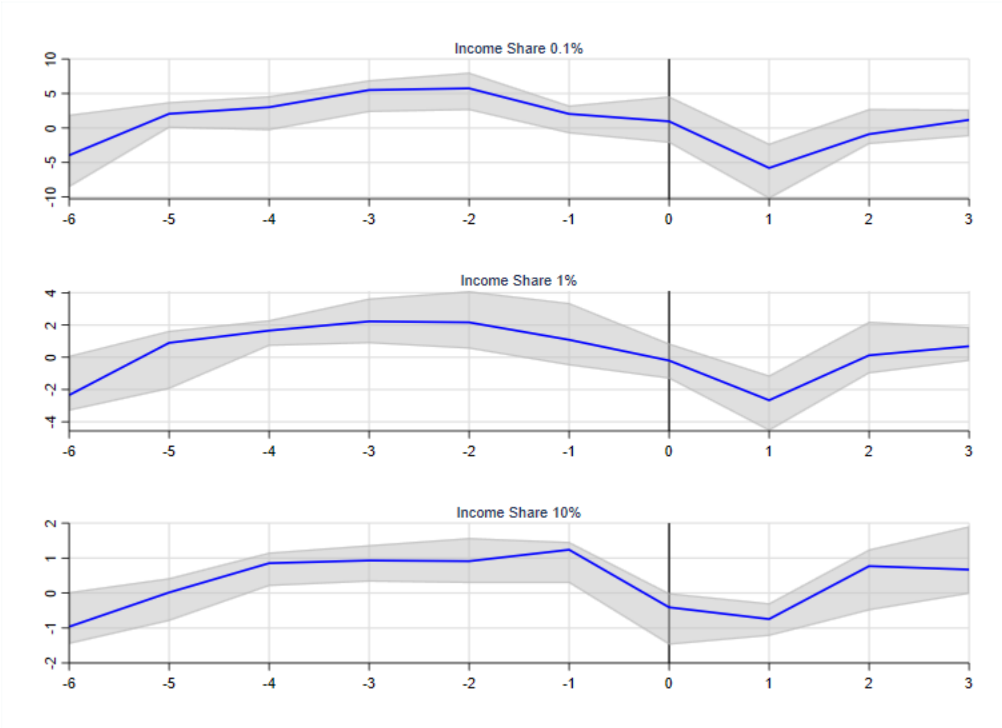
Source: Author's estimation using Paul (2022)

inequality has increased since 1980 to the present day (see Piketty and Saez (2003)).

5.3 Estimating the Correlation Between Bank Runs and Income Inequality

Does changes in the level of income inequality precede the occurrence of a bank run? The answer to this question is empirical. To achieve this goal, I perform two analyses. In the first place, I perform a statistical analysis that resembles a naive event study to provide evidence on how the share of the income held by the top percentiles of the income distribution behaves in the periods preceding a bank run. To do so, I estimate the median, the 33rd, and the 66th percentile of the annual percentage change seven periods before and after a given bank run in the sample. The results of this analysis are presented in Figure 14.

Figure 14: Median, 33rd and 66th Percentile of Income Shares



Source: Author estimation using the data from Paul (2022) and Baron et al. (2021).

The annual percentage changes in each income share measure increase drastically in the years before the bank run, followed by a drastic fall that lasts until after the bank run, and they start recovering after that. Furthermore, note the inverted U shape of the growth in the income inequality measures. This shape suggests that "rapid" inequality growth (i.e., changing from negative growth to positive growth from one period to another) precedes (or correlates with) an episode of bank runs. Finally, bank runs appear to reduce the income share held by the top percentiles of the income distribution (the 0. 1% and 1%), where the median reduction can go up to a 5% annual percentage change.

Second, I want to answer the following question: Is there a (positive or negative) correlation between income inequality and bank runs? Note that I am not implying any causality because, with the available data, it is impossible to discern any causal relationship between these two phenomena. The best thing that can be done with the available data is to establish the type of correlation that governs the relationship between income inequality and bank runs. To do so, let the probability of a bank run be described by:

$$Pr\left(BR_{j,t}=1|\,\mathrm{Ineq}_{j,t-1},X_{j,t-1};\boldsymbol{\beta}\right)=\frac{1}{1+\exp\left(-(\alpha_j+\beta_1\Delta_h\,\mathrm{Ineq}_{j,t-1}+\beta_2\Delta_h X_{j,t-1}+\varepsilon_{j,t})\right)} \tag{36}$$

where α_j is country-specific constant, $\Delta_h\,\mathrm{Ineq}_{j,t-1}$ is the change from period $t-1-h$ to $t-1$ of either measure of inequality or a vector that includes a combination of these measures for country j, $\Delta_h X_{j,t-1}$ is the change vector of controls X from $t-1-h$ to $t-1$, and $\varepsilon_{j,t}$ is the error term. Following Paul (2022), I normalize the variables in Ineq and X by their standard deviation. The selection of h is 4, following both Paul (2022) and Gorton and Ordoñez (2020).

The results of the estimate (36) are presented in Table 3. First, for the estimation of the results in columns (1), (5) and (9), I included the logarithmic change in the credit-to-GDP ratio as controls in addition to the country fixed effects. The first three columns are estimates for each inequality measure individually.[8] The table includes the point estimation of the odds ratio, the robust standard errors in parentheses, and the marginal effect in brackets for each explanatory variable.

The results in columns (1), (5) and (9) of Table 3 suggest that a standard deviation increase in the growth of the income share held by the top percentile is correlated with a 1 to 1.2 percentage point increase in the probability of a bank run. These percentage point increases in probability occur after controlling for the credit-to-GDP ratio, which has been deemed a determinant of the probability of financial crises in the literature (see Gorton and Ordoñez (2020), Paul (2022)). Remember that the unconditional probability of the bank run sample is 4%, so a percentage point increase is a fairly significant increase in the probability of bank runs.

Taking the previous results as benchmarks, I performed additional robustness checks to estimate the correlation between income inequality and bank runs. Robustness analysis follows that in Paul (2022), and is presented in the remaining columns of Table 3. Columns (2), (6), and (10) present the results, using as additional controls besides the 4-year change of credit-to-GDP ratio the following (in 4-year changes): investment-to-GDP ratio, public debt-to-GDP ratio, current account-to-GDP ratio, consumer price index, long- and short-term interest rates. The results suggest that even accounting for macroeconomic variables, increased inequality is correlated with an increased probability of a bank run of around one percentage point.

In columns (3), (7), and (11), the estimation also controls for changes in the domestic and

[8] Including any combination of income share in one estimation will produce a high correlation between explanatory variables, generating biased point estimates.

Table 3: Probability of Bank Runs

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
$\Delta_4 \log \text{Credit}_{t-1}$	0.559***	0.516**	0.648***	0.420**	0.560***	0.553**	0.650***	0.413**	0.358**	0.399*	0.504**	0.372*
	(0.146)	(0.232)	(0.231)	(0.207)	(0.129)	(0.220)	(0.212)	(0.195)	(0.163)	(0.238)	(0.231)	(0.212)
	[0.015]	[0.008]	[0.010]	[0.004]	[0.015]	[0.008]	[0.009]	[0.004]	[0.010]	[0.005]	[0.006]	[0.003]
Δ_4Income Share 0.1%$_{t-1}$	0.393***	0.578***	0.618***	0.590***								
	(0.120)	(0.181)	(0.178)	(0.201)								
	[0.010]	[0.009]	[0.009]	[0.005]								
Δ_4Income Share 1%$_{t-1}$					0.443***	0.650***	0.701***	0.517**				
					(0.155)	(0.223)	(0.204)	(0.234)				
					[0.012]	[0.009]	[0.009]	[0.005]				
Δ_4Income Share 10%$_{t-1}$									0.345*	0.585***	0.576***	0.454**
									(0.178)	(0.208)	(0.214)	(0.214)
									[0.009]	[0.008]	[0.007]	[0.004]
Set of controls		✓	✓	✓		✓	✓	✓		✓	✓	✓
GDP & Global GDP			✓	✓			✓	✓			✓	✓
Stock & House Prices				✓				✓				✓
Number of crises	28	27	27	25	34	33	33	30	26	26	26	26
Observations	808	758	758	690	952	891	891	841	812	768	768	754
Countries	15	15	15	15	17	17	17	17	17	17	17	17
Country FE	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p-value	0.337	0.484	0.377	0.861	0.380	0.643	0.606	0.620	0.283	0.531	0.494	0.731
Pseudo R^2	0.067	0.162	0.187	0.279	0.073	0.179	0.203	0.281	0.040	0.172	0.195	0.276

Source: Author's elaboration using Paul (2022) and Baron et al. (2021) data.

global real GDP. The results follow a similar trend as in columns (2), (6), and (8): Their significance level increases, and the point estimate of the marginal effects is smaller than those in the benchmark case. Finally, columns (4), (8) and (12) add real stock and house prices as additional controls. Note that its significance level remains relatively high for the change in the income share held by the top 0.1%. The significance level was reduced for the other two measures. At the same time, the point estimates of the marginal effects are almost identical between income shares but smaller than those from the benchmark case.

In conclusion, increasing income inequality, in the form of an increase in the income share held by the top percentiles of the income distribution, correlates with an increased probability of bank runs. This correlation suggests that an increase in one standard deviation in the growth of such shares is roughly correlated with an increase of one percentage point in the probability of a bank run. These results strongly motivate the study of the mechanism underpinning such a correlation in a theoretical model.

6 Conclusion

In conclusion, this article provides evidence of a positive correlation between income inequality and the probability of bank failure. The proposed banking model shows that increasing income inequality increases the probability of a bank run. In this sense, the model establishes a causal link between income inequality and financial instability. The widening gap between the rich and the poor has caused a shift in both the level of risk aversion and the amount of money people are willing to spend, making it difficult for banks to carry out their duties and leading to financial instability.

The findings of the paper have important implications for policymakers, suggesting that reducing income inequality can help prevent financial crises. In particular, policies that aim to redistribute wealth and reduce the concentration of wealth at the top can help stabilize the financial system and reduce the likelihood of bank runs. Moreover, the banking model provided in the article provides a framework for analyzing the effects of different policy interventions on the probability of a bank run.

Econometric analysis suggests that an increase in inequality by one standard deviation is associated with a 0.3 to 0.7 percentage point increase in the probability of a bank run. This result accounts for different covariates, such as GDP per capita, inflation, and financial development.

In general, the paper contributes to the growing literature on the relationship between income inequality and financial stability and highlights the need for policymakers to consider the distribution of wealth when designing policies to prevent financial crises.

Appendix

A Agents preferences and Relative Risk Aversion

The following utility function represents Hyperbolic Absolute Risk Aversion (HARA) preferences

$$U(c) = \frac{(c+\psi)^{1-\gamma}}{1-\gamma} \tag{37}$$

It follows that

$$U'(c) = (c+\psi)^{-\gamma} \tag{38}$$

$$U''(c) = -\gamma (c+\psi)^{-\gamma-1} \tag{39}$$

To satisfy the conditions described before for the problem, the second derivative must comply with

$$U''(c) < 0 \Leftrightarrow \begin{cases} \gamma > 1 \wedge c > 0 \wedge \psi > -c \\ \frac{c_1}{2} \in \mathbb{Z} \wedge c_1 \leq -2 \wedge c_1 = -1-\gamma \wedge c > 0 \wedge \psi < -c \end{cases} \tag{40}$$

The Relative Risk Aversion is given by:

$$RRA = -c\frac{U''(c)}{U'(c)} = -\frac{c(-\gamma)(c+\psi)^{-\gamma-1}}{(c+\psi)^{-\gamma}} = \frac{\gamma c}{c+\psi} \tag{41}$$

Given that $\gamma > 0$ the utility will display Increasing, Constant, or Decreasing Relative Risk Aversion if:

$$\frac{\partial RRA}{\partial c} = \frac{\psi\gamma}{(c+\psi)^2} \Rightarrow \begin{cases} \frac{\partial RRA}{\partial c} > 0 \Leftrightarrow \psi > 0 & \text{Increasing - IRRA} \\ \frac{\partial RRA}{\partial c} = 0 \Leftrightarrow \psi = 0 & \text{Constant - CRRA} \\ \frac{\partial RRA}{\partial c} < 0 \Leftrightarrow -c < \psi < 0 & \text{Decreasing - DRRA} \end{cases} \tag{42}$$

If $\psi > 0$, the depositor still enjoys some utility even if the consumption allocation is 0. On the contrary, if $\psi < 0$, the depositor faces a decrease in its utility level for any consumption allocation greater than 0. The case of $\psi = 0$ is the typical utility of constant relative risk aversion (CRRA).

B Proofs

B.1 Proof of Proposition 2.1

Suppose that the bank offers a deposit contract that leads to running for both types in both states. Given that all agents will run with probability one, the bank invests all its deposits in the liquid asset so that $y = \omega$ is optimal. Suppose that the deposit contracts have the form of $d_1 = d_2 = \frac{\omega}{2}$. The bank's utility under this contract is characterized by

$$W = 2U\left(\frac{\omega}{2}\right) \tag{43}$$

Now, suppose the following deposit contract:

$$d_i = \frac{\omega}{2} \text{ for } i = 1, 2 \tag{44}$$

$$y = \lambda\omega \tag{45}$$

Given that $\lambda \in (0, 1)$, then the amount invested in the illiquid asset is $x = \omega - \lambda\omega = \omega(1 - \omega)$. The return on this asset is given by $R_H > R_L > 1$. Thus, the amount available to distribute among depositors at $t = 2$ is $R_s\omega(1 - \lambda)$. The deposit contract for $t = 2$ is of the form (assuming an equal distribution among groups) as follows:

$$c_{2is} = \frac{R_s\omega(1-\lambda)}{2(1-\lambda)} = \frac{R_s\omega}{2} \text{ for } i = 1, 2, s = H, L \tag{46}$$

Then, this last contract will dominate the original contract whenever

$$c_{2is} \geq \frac{\omega}{2} \Leftrightarrow \frac{R_s\omega}{2} \geq \frac{\omega}{2} \Leftrightarrow R_s \geq 1 \text{ for } i = 1, 2, s = H, L \tag{47}$$

It follows that since $R_H > R_L > 1$, the last inequality in the previous condition will hold with strict inequality. Since it holds for any agent i and any state s, the bank will never find the optimal way to offer a contract that induces a run in both states $s = H, L$, for both groups of agents. $\square$

B.2 Proof of Proposition 2.2

Suppose that the banks offer a contract in which some type of agent i run in the high state but do not run in the low state that satisfies

$$c_{2iL} \geq d_i > c_{2iH} \tag{48}$$

where d_i is the face value of the deposit contract that induces the supposed behavior of group i. Given that there is no run in the state $s = L$ by the $(1 - \lambda)$ proportion of late-type agents of the group i, it must be true that $y \in (0, 1)$ to invest in the illiquid asset $x = \omega - y > 0$ so as to have a $R_L(\omega - y)$ units to distribute among depositors conditional on being in state L. Now, suppose a contract in the form of

$$\begin{cases} \tilde{c}_{2iH} = c_{2iL} & \text{if } s = H \\ \tilde{c}_{2iL} = c_{2iL} & \text{if } s = L \end{cases} \tag{49}$$

Under this contract, the $(1-\lambda)$ agents of late-type in group i are at least as good as they were in the previous contract. On the other hand, the deposit contract that the bank has provided at $t=2$ in this state H for both group i and j is given by

$$(1-\lambda)\left(\tilde{c}_{2iH}+\tilde{c}_{2jH}\right)=R_L\left(\omega-y\right) \text{ for } j\neq i \tag{50}$$

Note, however, that the return in state H is $R_H > R_L$, such that the bank will have an excedent of returns in $t=2$ given by

$$\left(R_H-R_L\right)\left(\omega-y\right)>0 \tag{51}$$

Hence, it is still possible to offer a welfare-improving allocation in which there is no run in the high state, and it does not necessarily lead to a run in the low state. □

B.3 Proof of Proposition 2.3

Suppose that the bank offers a contract that leads to a run for the group i in both states $s=H,L$. Note that the present contract could not lead to a run in both states for both groups (see Proposition 2.1) or that it leads to a run by group $j\neq i$ in state H but not in state L (see Proposition 2.2). It follows that the only two remaining scenarios are that this contract does (not) lead to a run in state L for group j, but never in state H.

First, assume that the present contract does not lead to a run for the group j in either state. This means that

$$\begin{cases} c_{2js}\geq d_j & \text{for states } s=H,L \\ d_i > c_{2is} & \text{for states } s=H,L \end{cases} \tag{52}$$

The bank has to invest in the liquid asset at least the amount to cover the deposit contracts of the group i that ran on the bank and the proportion λ of early types of group j. That is,

$$y\geq d_i+\lambda d_j>\lambda d_i+\lambda d_j \tag{53}$$

The second inequality characterizes the total face value of a contract that does not lead to a run for group i in either state and given that $\lambda\in(0,1)$. Under the present contract, given that $y\in(0,\omega)$ since it needs to be large enough to provide non-zero consumption at $t=1$ for either group and it needs to invest some in the illiquid asset since $R_s>1$ in either state $s=H,L$. Initially, assume that $y>d_i+\lambda d_j$, then the total amount that the bank would have to distribute at $t=2$ is given by

$$R_s\left(\omega-y\right)+y-d_i-\lambda d_j \text{ for } s=H,L \tag{54}$$

Now, suppose that the bank offers a contract such that group i does not lead to a run in state $s = H$ but leads to a run in state L (given proposition 2.1 and 2.2). Furthermore, assume that the contract is such that $\varrho d_i + \lambda d_j < y$ where $\varrho \in (0,1)$and $\varrho \leq \lambda$ is the proportion of agents that do not run on the bank of group i. This contract will dominate the original contract because, for any state $s = H, L$, the total amount that the bank can distribute will be greater. That is,

$$R_s\left(\omega - y\right) + y - d_i - \lambda d_j < R_s\left(\omega - y\right) + y - \varrho d_i - \lambda d_j \leq R_s\left(\omega - y\right) + y - \lambda d_i - \lambda d_j \tag{55}$$

Hence, the depositors will have deposit contracts at least as big as the original contract. Note that in the case of $y = d_i + \lambda d_j$, the pie in $t = 2$ in either state is $R_s\left(\omega - y\right)$ and it will be strictly smaller than $R_s\left(\omega - y\right) + y - d_i - \lambda d_j$. Consequently, the previous result holds for $y \geq d_i + \lambda d_j$.

Second, assume that the bank offers a contract that leads to a run in state L for group j, then the bank would have to invest in the liquid asset

$$y \geq d_i + d_j \text{ for } j \neq i \tag{56}$$

Note that this case will be dominated by the case where the bank offers a contract that does not lead to a run in state L for group j, given that

$$y \geq d_i + d_j > d_i + \lambda d_j > \lambda d_i + \lambda d_j \tag{57}$$

Consequently, it is never optimal to choose a contract that leads to a run in **BOTH** states $s = H, L$ by some group i. $\square$

B.4 Proof of Proposition 2.4

Suppose that it does not hold with equality. That is, $\lambda(d_1 + d_2) < y$. This suggests that some of the investment in the liquid asset is left over at date 1. The bank could reduce the amount invested in such an asset by $\epsilon > 0$ and invest in the illiquid asset with a return $R_s > 1$ for all $s = H, L$. The net change in the available return for consumption at $t = 2$ is $(R - 1)\,\epsilon > 0$. Hence, one could improve the consumption of the late-type consumer without affecting the consumption of the early-type. This cannot be optimal. It follows that in any optimal plan $\lambda(d_1 + d_2) = y$. $\square$

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