Case 1: Interest rate swaps and liability hedging

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1 The Term Structure of Interest Rates (30 points)

During the first week of the course, we discussed the financial position of defined benefit pension funds. Typically, we use the nominal funding ratio to measure the financial healthiness of a pension fund. The nominal funding ratio FR at time t is defined as:

$$FR_t = \frac{A_t}{L_t},$$

where A_t represents the market value of all assets and L_t the value of the nominal liabilities. The value of the nominal liabilities is determined by the expected cash-flows and the discount rate:

$$L_t = \sum_{j=0}^{N} \frac{CF_{t+j}}{(1 + y_{t:t+j})^j}.$$

There are many ways to construct the term structure of discount rates. Before 2007 pension funds were allowed to use a fixed rate of 4%. As of the beginning of 2007 the regulation changed. From that moment pension funds had to use market interest rates rates for the valuation of nominal liabilities. Because of the lack of supply of long term bonds, market swap rates were chosen as basis for the zero nominal term structure.

The Excel-file provides the closing quotes of nominal interest rate swap rates in the Euro swap market on January 31, 2025 (tab Question 1, column B). In our notation January 31, 2025 stands for time $t = 0$. Based on these swap rates an interest rate curve of nominal "risk-free" zero rates and the forward curve needs to be constructed. For the purpose of this exercise you can assume that the floating rate of the swaps is the 1-year risk free rate. Fixed and floating payments take place once per year.

(a) Calculate the annually compounded per annum 1-yr, 2-yrs and 3-yrs nominal zero rates by using the 1-yr, 2-yrs and 3-yrs swap rates. Hint: as you know from the lectures, a nominal interest rate swap can be decomposed in a par fixed coupon bond and a par floating rate bond. The swap rate is the coupon of the fixed leg in the swap.

The Excel-file does not show swap rates for the 4-yrs and 7-yrs interest rate swaps. Hence, we need to make additional assumptions in order to be able to complete the term structure of nominal zero rates (implied by market quotes of swap rates). The assumption we make is:

$$f_{0:3:4} = f_{0:4:5},$$

where $f_{0:3:4}$ is the annually compounded per annum forward rate observed at time $t = 0$ and applicable to the fourth year.

(b) Under this assumption, calculate the annually compounded per annum 4-yrs and 5-yrs nominal zero rates. Hint: you can write the valuation formula for the fixed leg of the 5-yrs swap as an equation that has just one unknown.

Next step is to find the remaining zero rates until the 30-yrs point on the curve. We need to make an additional assumption to complete the term structure. The assumption we make is the following:

$$f_{0:5:6} = f_{0:6:7} = f_{0:7:8},$$

(c) Calculate the annually compounded per annum nominal zero rates for maturities 6-30 years using the provided swap rates and the answers to the previous questions. If you were not able to solve questions (a) and (b) you can assume that the **sum of discount factors** for maturities 1-5 years is equal to 4.6744. Please report your results by means of a table.

Final step is to complete the curve beyond the 30-years point. In order to be able to discount expected cash-flows for maturities longer than 30 years, we need to make additional assumptions. We assume in this exercise that the forward rate between maturities 29 and 30 years, can be extrapolated to all maturities beyond 30 years. To be specific: the forward rate is assumed to be constant after 30 years.

(d) Verify that the annually compounded (per annum) forward rate between maturities 29 and 30 equals 1.5033%. Use this forward rate to construct the nominal zero curve for maturities 31 years until 60 years. Provide a graph that shows the full term structure of nominal zero rates. Also provide the numerical value of the 60-yrs nominal zero rate (per annum, with annual compounding).

The regulator in the Netherlands changed the methodology for calculating the discount curve several times. In 2012, for instance, the regulator introduced the so-called Ultimate Forward Rate (UFR)-methodology (also used in the world of insurance companies). In this methodology, the forward rate in the long run ultimately converges to a value of 4.0%. Speed of convergence can be defined in many different ways, but let us assume that **per annum forward rates (with annual compounding)** can be determined as follows:

$$f_{0:30:30+h} = 4.0\% + (f_0^* - 4.0\%)B(h),$$

where f_0^* is defined as:

$$f_0^* = \frac{4}{7}\left(f_{0:29:30} + \frac{1}{2}f_{0:28:29} + \frac{1}{4}f_{0:27:28}\right).$$

and $B(h)$ is defined as:

$$B(h) = \frac{1 - e^{-0.4h}}{0.4h}.$$

Note that the forward rates (and zero rate) between year 1 and year 30 are not affected by this methodology.

(e) Using the UFR-methodology construct the UFR forward curve (per annum, with annual compounding). Provide a table which shows your results for maturities 1, 10, 30 and 50 years.

(f) Construct the UFR nominal zero curve using the forward curve from the previous question and plot both the UFR nominal zero curve and the market nominal zero curve in one graph. Provide a possible reason for using the UFR curve instead of using the market curve.

2 Liability hedging (35 points)

In the Excel-file (tab Question 2, column B) you find a nominal cash-flow pattern from pension fund Blue Sky. These cash-flows are the future expected payments of the pension fund. Pension fund Blue Sky is in good shape: the nominal funding ratio of the fund is 130%. You can find the zero curve (**per annum, with annual compounding**) in column C of tab Question 2 in the Excel file. The present value of all future expected cash-flows equals EUR 9,410,773,504.

(a) Verify that the modified duration and the modified DV01 of the nominal liabilities are given by 17.98 years and EUR 16.92 mln, respectively. Construct five maturity buckets: 1-10 years, 11-20 years, 21-30 years, 31-40 years and 41-60 years. Provide a (bar) graph that shows the contribution of each maturity bucket to total modified duration (in duration years). For the sake of clarity: the sum of the contribution of all maturity buckets to total modified duration equals 17.98 years.

(b) Shock the zero curve with -80bp, recalculate the value of the liabilities (exactly) using the zero rates after the shock and show that the difference between the new and the old value of the liabilities can be (almost) fully explained by modified DV01 and modified convexity. Would you judge the impact of convexity as economically relevant? Please motivate your answer. Moreover, provide the nominal funding ratio after the shock in interest rates under the assumption that assets do not change in value after the shock in interest rates. Reflect on the question whether interest rate risk hedging is needed or not.

Suppose the pension fund wants to match the interest rate sensitivity of the **funding ratio** by means of swaps and cash. Suppose that the pension fund just entered into a 5-years nominal interest rate swap with notional amount equal to EUR 6,000,000,000. The swap rate of this 5-years swap equals 2.2990%. The cash amount is equal to the total value of the assets. Assume that cash is invested in a 1-year zero coupon bond with a yield equal to 1-year Euribor (i.e., the 1-year nominal risk-free zero rate as we assume the absence of counterparty credit risk). As in Exercise 1, assume that the floating rate of the swaps is 1-year Euribor. Fixed and floating payments take place once per year. The 5-years swap is not sufficient to match the interest rate sensitivity of the funding ratio. Therefore, the pension fund wants to add a 30-years nominal interest rate swap to the liability hedging portfolio.

(c) Calculate the notional amount in the 30-yrs swap that is needed to meet the objective of the interest rate risk hedging strategy. Hint: first, calculate the modified DV01 of EUR 1 notional in the 30-yrs swap. Then, calculate how many EUR notional you need in order to get the target modified DV01. You can assume that the 30-yrs swap rate equals 2.2160%.

(d) Construct a similar (bar) graph as in (a) but now show the contribution of each maturity bucket to total modified duration for both liabilities and hedging portfolio.

(e) Now the zero curve changes to the zero curve as shown in column D of tab Question 2 in the Excel file (a non-parallel shock to the initial zero curve is applied). Calculate the impact of this shock on the value of the liabilitity hedging portfolio (i.e. cash, 5-yrs swap and 30-yrs swap) and the nominal funding ratio. Is the hedge effective in your opinion? Please motivate your answer and provide an explanation for the difference between the funding ratio after the shock and the intial funding ratio of 130%. If you were not able to solve question (c) you can use a notional amount of EUR 9,000,000,000 for the 30-yrs swap.

(f) Now apply the same shock to the initial curve as in question (b) and calculate the impact of this shock on the value of the liabilitity hedging portfolio (i.e. cash, 5-yrs swap and 30-yrs swap) and the nominal funding ratio. We can compare the result with the outcomes of question (e). Provide an explanation for the difference in results.

Pension fund Blue Sky is not fully satisfied with the properties of the liability hedging portfolio and therefore does not like to trade the notional amount in the 30-yrs swap that you calculated in question (c). Instead, the fund considers to enter into a 20-yrs nominal interest rate swap (swap rate: 2.4150%), a 30-yrs nominal interest rate swap and a 40-yrs nominal interest rate swap (swap rate: 2.0841%). The fund needs to make a choice on the notional amounts for these swaps (for the avoidance of doubt: the notional amount of the three swaps can be different). The positions in cash and the 5-yrs swap remain unchanged.

(g) Calculate the notionals that are needed in the 20-yrs, 30-yrs and 40-yrs swap to match the interest rate sensitivity of the funding ratio and assess the effectiveness of your new hedging portfolio by calculating the nominal funding ratio after applying the same shock to the initial zero curve as in (e).

3 New Pension Act (25 points)

Under the new Dutch Pension Act the concepts of promises and liabilities will disappear. Pension rights that have been accumulated in the past will be converted into a personal pension capital. For pensioners the amount of personal capital and the term structure of interest rates are used to determine what the benefit level is that can be paid from the amount of personal capital.

Consider a 70-years old pensioner who has an accumulated pension capital of EUR 1,000,000. Survival probabilities of the population are given in the Excel-sheet (tab Question 3, column A): the probability that someone with age 0 reaches age 1 equals 99.935% and the probability of someone of age 70 reaches age 71 is equal to 98.975%. Assume that pensions are paid once a year and that a benefit has just been paid (the next payment will be in exactly one year).

(a) Calculate for the 70-years old pensioner the probability that she will pass away at a certain age. Provide the results in a graph. Moreover, calculate (and provide) the expected age at which she will pass away.

(b) Calculate the lifelong benefit that can be paid to the 70-years old pensioner from her accumulated pension capital. You can use the zero curve of Exercise 2. Provide a graph with the expected cash-flows for the 70-years old pensioner.

The benefits of the pensioners will be fully protected against changes in interest rates. In addition, pensioners will be exposed to to equity markets for 40%.

(c) Provide an investment portfolio that the pension fund will possibly enter into for the 70-years old pensioner. Provide as much detail as possible on the fixed income / interest rate risk hedging part of the portfolio.

Suppose that after one year the pensioner is still alive (age 71 now), all interest rates went down with 1%-point, all equities increased with 4%, inflation over the past year was 2%, and the survival probabilities, as given in the Excel-sheet (tab Question 3, column A), are still applicable.

(d) Calculate the new level of pension capital for this pensioner and the new benefit that can be paid from this capital. Elaborate on the change in benefit and explain if the pensioner will be happy or not with the new pension benefit.

Requirements

- Groups of max four students
- Deadline for this case is **Friday March 7 at 23:59 CET**
- Final reports should be uploaded on Canvas
- Reports should be written in English. This means that you also should use English decimal notation in the text and in graphs
- There is no maximum to the number of words or pages but please try to be concise
- It is not allowed to just give answers to the questions. It should be clear to the reader how you have come to the answer
- Don't refer to cells of your own spreadsheets, I don't plan to read spreadsheets or other kind of computer source code
- Figures and graphs should have captions such that they can be read independently from the text
- Use page numbers
- Formulas are part of the sentence
- Put **names** and **VU student numbers** of all group members on the front page.

And last but not least...ENJOY!!