Solving $f' = f$ Using Completely Rigorous Methods

We want to solve the differential equation:

$$f' = f$$

First, rewrite this in terms of the integration operator:

$$f = \int f$$

Subtract $\int f$ from both sides and factor out f:

$$f - \int f = 0 \quad \Longrightarrow \quad \left(1 - \int\right) f = 0$$

I'm not allowed to do this, but I'm going to do it anyway.

$$f = (1 - \int)^{-1} 0$$

Using the geometric series formula:

$$(1 - x)^{-1} = 1 + x + x^2 + x^3 + \cdots$$

We apply this to our integration operator:

$$f = \left(1 + \int + \int\int + \int\int\int + \cdots\right) 0$$

Evaluating term by term:

$$1 \cdot 0 = 0$$
$$\int 0 = C$$
$$\int\int 0 = Cx$$
$$\int\int\int 0 = C\frac{x^2}{2!}$$
$$\int\int\int\int 0 = C\frac{x^3}{3!}$$
$$\vdots$$

Adding these up gives us:

$$f = 0 + C + Cx + C\frac{x^2}{2!} + C\frac{x^3}{3!} + \cdots$$

This is exactly the Taylor series for Ce^x! Therefore:

$$f(x) = Ce^x$$

No, this is not a rigorous proof. It is a fake proof that leads to the correct answer.