Math524 Midterm Exam by Dr. Samuel Shen, FA2023

October 11, Wednesday, due 11:59PM the next day

This is an open-book exam. You can use Google or ChatGPT to search for any helpful information. You can use a calculator, R, computer, or experiment. However, you CANNOT ask natural person to help you. When done, please submit **a single pdf file and a single R file** *online via Canvas.*

Student Name:

Student ID Number:

1. [20 *points*] **R for space-time data matrix**
   Use R to read the `NOAAGlobalT.csv` data file, and Select the four grid boxes that cover the following four locations: San Diego (USA), New York (USA), Paris (France), and Tokyo (Japan). Generate a 4×7 space-time data matrix for the January mean surface air temperature anomaly data of the four selected grid boxes and the seven years from1981 to 1987.
   Hint: You can download the NOAA Global Surface Temperature (NOAAGlobalTemp.csv) dataset from this midterm site on Canvas. Use the homework solution as a reference.

2. [20 *points*] **R programming: Change a time-time data matrix to a data sequence, and then plot the data sequence**
   (a) [10 points] The data file `EarthTemperatureData.csv` contains the monthly and annual global average temperature anomalies from 1880 to 2015. Extract the monthly data from this data matrix and convert the monthly data into a sequence, i.e., a vector, according to time order: Jan 1880, Feb 1880, Mar 1880, ... , Dec 1880, Jan 1881, Feb 1881, ..., Nov 2015, Dec 2015. (b) [10 points] Plot the temperature data sequence from Jan 1880 to Dec 2015. The horizontal axis should be mark as Year from 1880 to 2015. The vertical axis is for the temperature anomaly data.

3. [20 *points*] **SVD calculation**
   The R command `svd(A)` gives the following results

```
svd(A)
$d
[1] 3 1

$u
            [,1]       [,2]
[1,] -0.7071068 -0.7071068
[2,] -0.7071068  0.7071068

$v
     [,1] [,2]
[1,]   -1    0
[2,]    0   -1
```

   (a) [10 points] What are A's SVD matrices U, D and V?
   (b) [10 points] Use hand-calculations for matrix multiplication: DV^t, then $U(DV^t)$ to approrimately recover the original matrix A. Your final result should be a 2-by-2 matrix A.

4. [20 *points*] **SVD theory**
   If A is an m-by-n matrix with $m > n$, and if

$$A = UDV^t \tag{0.1}$$

   is the SVD of A and

$$C = A^t A = QHQ^t \tag{0.2}$$

is the SVD of C, show that

$$D^2 = H \tag{0.3}$$

and

$$Q = V \tag{0.4}$$

assuming the SVD result for a matrix is unique.

5. $[20\ points]$ **Concept of independent vectors**
(a) [8 points] Show that the three column vectors of the following matrix A are not linearly independent

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}. \tag{0.5}$$

(b) [4 points] Use R to compute the determinant of this matrix A. Does this matrix A have an inverse?
(c) [4 points] Use R to compute the inverse matrix of the following matrix B:

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 1 & 9 \end{bmatrix}. \tag{0.6}$$

(d) [4 points] Use R to compute the determinant of the matrix B.