Problem 2.1: Non-dimensionalize the following equation:

$$\frac{dy}{dt} = ry(1 - \frac{y}{K})$$

where r and K are constants.

Solution:

$$y = [y]y^*$$
$$t = [t]t^*$$

Find $\frac{dy}{dt}$:

$$\frac{d([y]y^*)}{dt} = [y]\frac{dy*}{dt} = [y]\frac{dy^*}{d[t]t^*} = \frac{[y]}{[t]}\frac{dy^*}{dt^*}$$

Substitute and simplify:

$$\frac{[y]}{[t]}\frac{dy^*}{dt^*} = r[y]y^*(1 - \frac{[y]y^*}{K})$$
$$\frac{1}{[t]} \cdot \frac{dy^*}{dt^*} = ry^*(1 - [y]\frac{y^*}{K})$$
$$\frac{dy^*}{dt^*} = r[t]y^*(1 - \frac{y^*}{K})$$

Let $[t] = \frac{1}{r}$: Then

$$\frac{dy^*}{dt^*} = y^*(1 - \frac{y^*}{K})$$

Let $[y] = K$: Then

$$\frac{dy^*}{dt^*} = y^*(1 - y^*)$$