**Problem 5.1:** Consider the following model equation

$$\begin{aligned} \frac{\partial y}{\partial t} &= D\frac{\partial^2 y}{\partial x^2} + \gamma y^3 \\ y(x,0) &= 0,\ 0 < x < \infty \\ y(0,t) &= y_0,\ t > 0,\ y(\infty,t) = 0,\ t > 0 \end{aligned}$$

Write the dimensions of each variable, parameter, and term in the equation.

**Solution:**

$$\begin{aligned} t &= [T] \\ x &= [L] \\ y &= [Y] \end{aligned}$$

Dimensions of Left Side:

$$\left[\frac{\partial y}{\partial t}\right] = \frac{[Y]}{[T]}$$

Dimensions of Right Side:

$$\left[D\frac{\partial^2 y}{\partial x^2} + \gamma y^3\right] = [D]\cdot\frac{[Y]}{[L]^2} + [\gamma]\cdot[Y]^3$$

Both right side terms must have the same dimensions as the left side term. Start with the first term:

$$\begin{aligned} \frac{[Y]}{[T]} &= [D]\cdot\frac{[Y]}{[L]^2} \\ [D] &= \frac{[L]^2}{[T]} \end{aligned}$$

Second term:

$$\begin{aligned} \frac{[Y]}{[T]} &= [\gamma]\cdot[Y]^3 \\ [\gamma] &= \frac{1}{[T][Y]^2} \end{aligned}$$

Summary of Dimensions:

$$\begin{aligned} [t] &= [T] \\ [x] &= [L] \\ [y] &= [Y] \\ [D] &= \frac{[L]^2}{[T]} \\ [\gamma] &= \frac{1}{[T][Y]^2} \end{aligned}$$