Problem 6: According to the radioactive decay law, the per capita decay rate of the amount $A(t)$ of C^{14} is $-\lambda$. Suppose that an archeologist excavates a bone and measures its content for C^{14} and finds it to be 25% of the carbon present in bones of a living organism. What can be said about the age of the bone? The half-life of C^{14} is 5730 years.

Solution:

The decay rate is given by:

$$\frac{dA}{dt} = -\lambda A$$

The solution to this differential equation is:

$$A(t) = A_0 e^{-\lambda t}$$

Given that:

$$\begin{aligned}
A(5730) &= 0.5A_0 \\
A_0 e^{-\lambda 5730} &= 0.5A_0 \\
e^{-\lambda 5730} &= 0.5 \\
-\lambda 5730 &= \ln(0.5) \\
\lambda &= \frac{\ln(0.5)}{-5730}
\end{aligned}$$

Want to find t when $A(t) = 0.25A_0$:

$$\begin{aligned}
A(t) &= 0.25A_0 \\
A_0 e^{-\lambda t} &= 0.25A_0 \\
e^{\frac{\ln(0.5)}{5730}t} &= 0.25 \\
\frac{\ln(0.5)}{5730}t &= \ln(0.25) \\
t &= \frac{5730\ln(0.25)}{\ln(0.5)} \\
t &= \frac{5730 \cdot 2\ln(0.5)}{\ln(0.5)} \\
t &= 2 \cdot 5730 \\
t &= 11460
\end{aligned}$$

Therefore, the age of the bone is 11460 years.