

Problem 2.2: Non-dimensionalize the following equation:

$$\frac{dy}{dt} = sy(a - y)(y - b)$$

where s, a, b are constants.

Solution:

$$\begin{aligned} y &= [y]y^* \\ t &= [t]t^* \end{aligned}$$

From the last problem, we know that:

$$\frac{dy}{dt} = \frac{[y]}{[t]} \frac{dy^*}{dt^*}$$

Plug this in and simplify:

$$\begin{aligned} \frac{[y]}{[t]} \frac{dy^*}{dt^*} &= s[y]y^*(a - [y]y^*)([y]y^* - b) \\ \frac{1}{[t]} \cdot \frac{dy^*}{dt^*} &= sy^*(a - [y]y^*)([y]y^* - b) \\ \frac{dy^*}{dt^*} &= s[t]y^*(a - [y]y^*)([y]y^* - b) \end{aligned}$$

Let $[y] = a$. Then

$$\begin{aligned} \frac{dy^*}{dt^*} &= s[t]ay^*(a - ay^*)(ay^* - b) \\ &= s[t]a^2y^*(1 - y^*)(y^* - \frac{b}{a}) \end{aligned}$$

To eliminate the coefficient, set:

$$s[t]a^2 = 1 \quad \Rightarrow \quad [t] = \frac{1}{sa^2}$$

Then,

$$\frac{dy^*}{dt^*} = y^*(1 - y^*) \left(y^* - \frac{b}{a} \right)$$