Problem 5.3: Consider the following model equation

$$\begin{aligned} \frac{\partial y}{\partial t} &= D\frac{\partial^2 y}{\partial x^2} + \gamma y^3 \\ y(x,0) &= 0,\ 0 < x < \infty \\ y(0,t) &= y_0,\ t > 0,\ y(\infty,t) = 0,\ t > 0 \end{aligned}$$

Analyze the non-dimensionalized parameters to find out what parameters to choose to make (relative) small diffusion or (relative) large diffusion.

Solution:

The non-dimensionalized parameters are:

$$\begin{aligned} [x] &= \sqrt{D} \\ [x^2] &= D \\ [y] &= \frac{1}{\sqrt{\gamma}} \\ [y^2] &= \frac{1}{\gamma} \\ [t] &= 1 \end{aligned}$$

D is the diffusion coefficient, so there is large diffusion when $D >> \gamma$.

Large diffusion: $[x]$ is big or [y] is big.
Small diffusion: $[x]$ is small or [y] is small.