Problem 3: For a given model equation

$$\frac{dN(t)}{dt} = r_B N(t)\left(1 - \frac{N(t)}{K_B}\right) - B\frac{N(t)^2}{A^2 + N(t)^2}$$

perform non-dimensional analysis to reduce the equation to the form

$$\frac{du}{d\tau} = ru\left(1 - \frac{u}{q}\right) - \frac{u^2}{1+u^2}$$

Solution:

$$N = [N]u$$
$$t = [t]\tau$$

Then,

$$\frac{dN}{dt} = \frac{[N]}{[t]}\frac{du}{d\tau}$$

The model equation becomes

$$\begin{aligned}
\frac{[N]}{[t]}\frac{du}{d\tau} &= r_B[N]u\left(1 - \frac{[N]u}{K_B}\right) - B\frac{([N]u)^2}{A^2 + ([N]u)^2} \\
\frac{du}{d\tau} &= [t]r_B u\left(1 - \frac{[N]u}{K_B}\right) - [t]B\frac{[N]u^2}{A^2 + ([N]u)^2}
\end{aligned}$$

Let $[N] = A$

$$\begin{aligned}
\frac{du}{d\tau} &= [t]r_B u\left(1 - \frac{Au}{K_B}\right) - [t]B\frac{Au^2}{A^2 + A^2u^2} \\
&= [t]r_B u\left(1 - \frac{Au}{K_B}\right) - \frac{[t]B}{A}\frac{u^2}{1+u^2}
\end{aligned}$$

Let $[t] = \frac{A}{B}$

$$\frac{du}{d\tau} = \frac{r_B A}{B}u\left(1 - \frac{Au}{K_B}\right) - \frac{u^2}{1+u^2}$$

Define $r = \frac{r_B A}{B}$ and $q = \frac{K_B}{A}$

$$\frac{du}{d\tau} = ru\left(1 - \frac{u}{q}\right) - \frac{u^2}{1+u^2}$$