Problem 1: A ball is thrown directly upward from the surface of the Earth. Assuming that the maximum height reached is a monomial function of the acceleration due to gravity, the mass of the ball and the initial velocity, use dimensional analysis to approximate the expression for the maximum height reached.

Solution:

Gravity: $[LT^{-2}] = [\frac{L}{T^2}]$
Velocity: $[LT^{-1}] = [\frac{L}{T}]$
Mass: $[M]$
Height: $[L]$

$$[h] = [g^a v^b m^c]$$
$$[L] = \left[\frac{L}{T^2}\right]^a \left[\frac{L}{T}\right]^b [M]^c$$
$$[L] = [L^{a+b} \cdot T^{-2a-b} \cdot M^c]$$

This gives

$$\begin{aligned} a + b &= 1 \\ -2a - b &= 0 \\ c &= 0 \end{aligned}$$

Simplifying gives

$$b = 2$$

Plugging this into the first equation gives

$$a + 2 = 1 \implies a = -1$$

Therefore,

$$\begin{aligned} a &= -1 \\ b &= 2 \\ c &= 0 \end{aligned}$$

Therefore, the expression for the maximum height reached is

$$h = \frac{v^2}{g}$$