Problem 5: The spruce budworm model

$$\frac{dN(t)}{dt} = r_B N(t)\left(1 - \frac{N(t)}{K_B}\right) - B\frac{N(t)}{A^2 + N(t)^2},$$

can be reduced to the following scaled equation

$$\frac{du}{d\tau} = ru(1 - \frac{u}{q}) - \frac{u^2}{1+u^2},$$

Perform the stability analysis and the bifurcation analysis with the parameter r fixed and the parameter q as a bifurcation parameter. Also, plot the bifurcation diagram.

Solution:

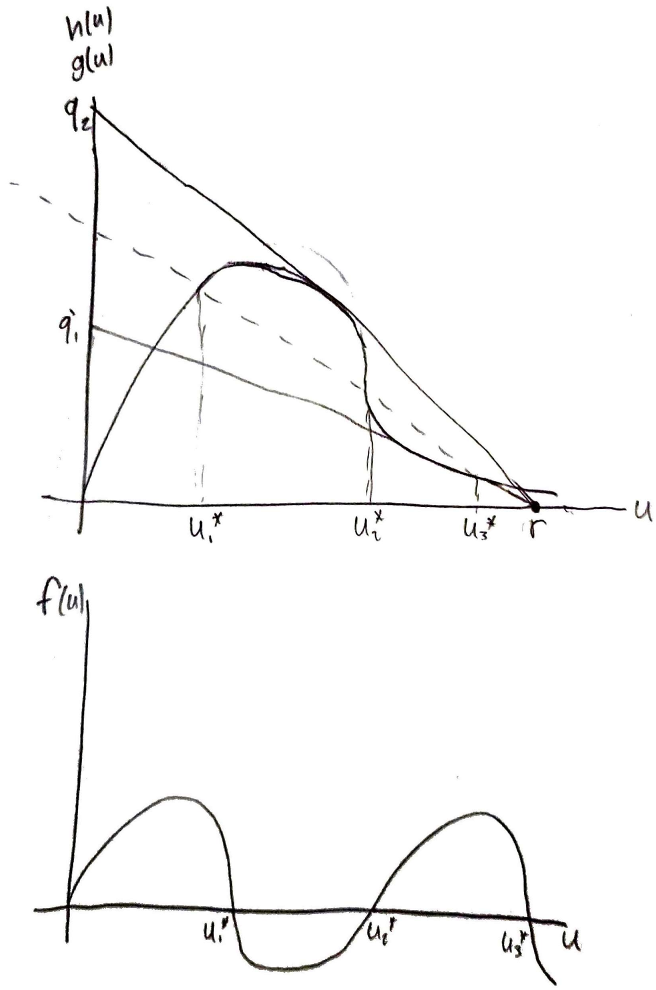
Set $\frac{du}{d\tau} = 0$ to find the equilibrium points.

$u^* = 0$ is always an equilibrium point.

From class, the other equlibirum happens when

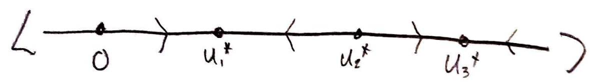
$$r\left(1 - \frac{u}{q}\right) = \frac{u^2}{1+u^2}$$
$$g(u) = h(u)$$

Let $f(u) = u(g(u) - h(u))$. Here are the graphs:

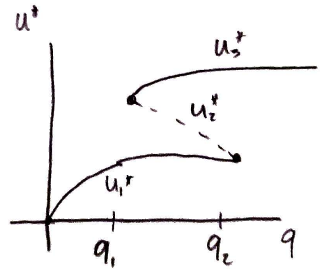


Here is the bifurcation diagram and phase plot:

Phase Plane



Bifurcation



u_1^* and u_3^* are asymptotically stable, and u_2^* and 0 are unstable.