Model Predictive Control
Part I – Introduction

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Main Idea

Objective:

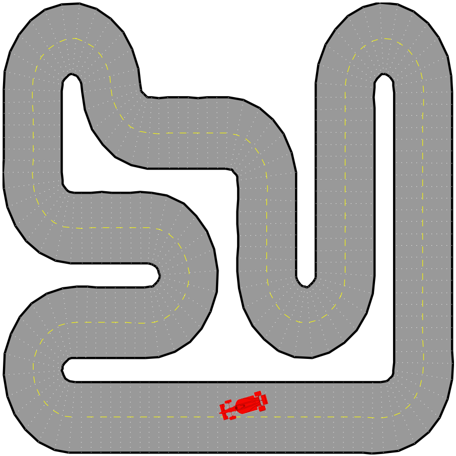
- Minimize lap time

Constraints:

- Avoid other cars
- Stay on road
- Don't skid
- Limited acceleration

Intuitive approach:

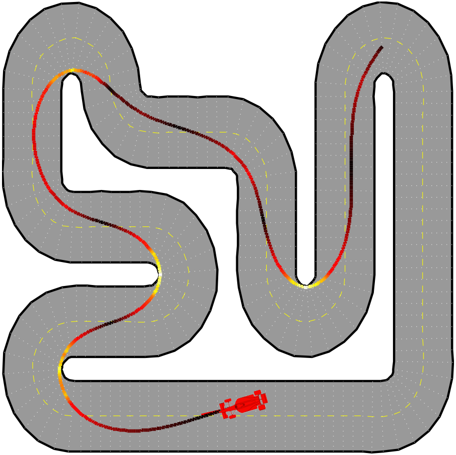
- Look forward and plan path based on
  - Road conditions
  - Upcoming corners
  - Abilities of car
  - etc...



Optimization-Based Control

Minimize	(lap time)
while	avoid other cars
	stay on road
	...

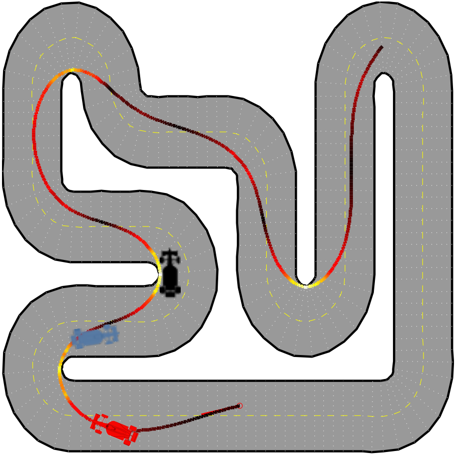
- Solve **optimization problem** to compute minimum-time path



Optimization-Based Control

Minimize	(lap time)
while	avoid other cars
	stay on road
	...

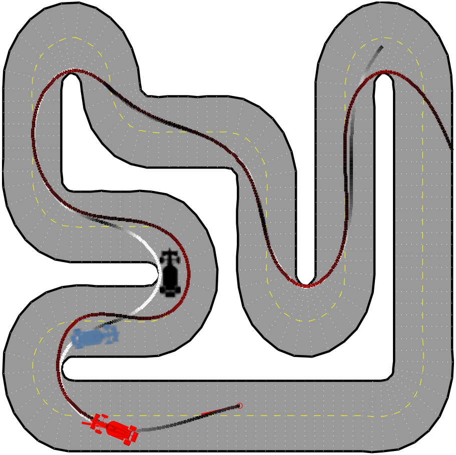
- Solve **optimization problem** to compute minimum-time path
- What to do if something unexpected happens?
  - We didn't see a car around the corner!
  - Must introduce *feedback*



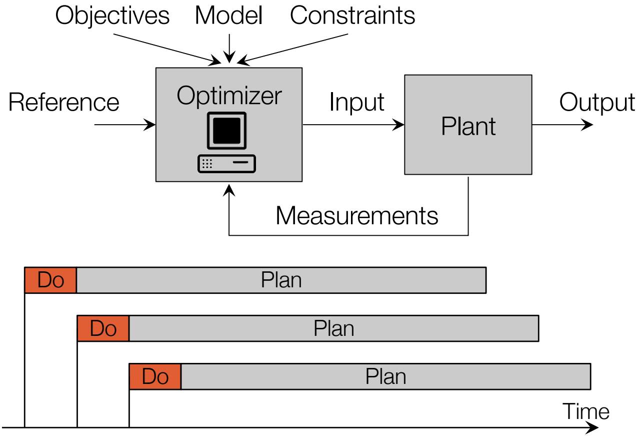
Optimization-Based Control

Minimize	(lap time)
while	avoid other cars
	stay on road
	...

- Solve **optimization problem** to compute minimum-time path
- Obtain series of planned control actions
- Apply *first* control action
- Repeat the planning procedure



Model Predictive Control



Receding horizon strategy introduces **feedback**.

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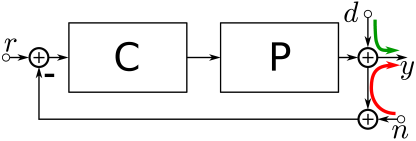
Two Different Perspectives

Classical design: design C

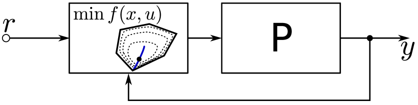


Dominant issues addressed

- Disturbance rejection $(d \to y)$
- Noise insensitivity $(n \to y)$
- Model uncertainty

(usually in *frequency domain*)

MPC: real-time, repeated optimization to choose $u(t)$



Dominant issues addressed

- Control constraints (limits)
- Process constraints (safety)

(usually in *time domain*)

Constraints in Control

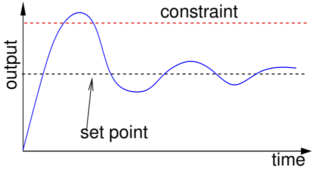
All physical systems have **constraints**:

- Physical constraints, e.g. actuator limits
- Performance constraints, e.g. overshoot
- Safety constraints, e.g. temperature/pressure limits

Optimal operating points are often near constraints.

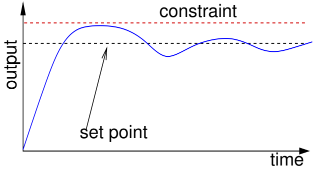
Classical control methods:

- Ad hoc constraint management
- Set point sufficiently far from constraints
- Suboptimal plant operation



Predictive control:

- Constraints included in the design
- Set point optimal
- Optimal plant operation



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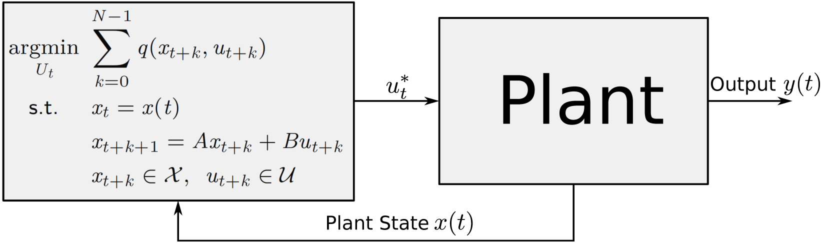
MPC: Mathematical Formulation

$$
\begin{aligned}
U_t^*(x(t)) := \operatorname*{argmin}_{U_t} \ & \sum_{k=0}^{N-1} q(x_{t+k}, u_{t+k}) & \\
\text{subj. to} \ & x_t = x(t) & \text{measurement} \\
& x_{t+k+1} = Ax_{t+k} + Bu_{t+k} & \text{system model} \\
& x_{t+k} \in \mathcal{X} & \text{state constraints} \\
& u_{t+k} \in \mathcal{U} & \text{input constraints} \\
& U_t = \{u_t, u_{t+1}, \ldots, u_{t+N-1}\} & \text{optimization variables}
\end{aligned}
$$

Problem is defined by

- **Objective** that is minimized,
  e.g., distance from origin, sum of squared/absolute errors, economic,...
- Internal **system model** to predict system behavior
  e.g., linear, nonlinear, single-/multi-variable, ...
- **Constraints** that have to be satisfied
  e.g., on inputs, outputs, states, linear, quadratic,...

MPC: Mathematical Formulation



At each sample time:

- Measure / estimate current state $x(t)$
- Find the optimal input sequence for the entire planning window N: $U_t^* = \{u_t^*, u_{t+1}^*, \ldots, u_{t+N-1}^*\}$
- Implement only the *first* control action u_t^*

Class Topics

- Week 1/2: Introduction and Fundamentals of Optimization
- Week 3: Reachability and Invariant Set Theory
- Week 4: Optimal Control and Dynamic Programming
- Week 5: Constrained Optimal Control
- Week 6: Predictive Control: Fundamentals
- Week 7: Predictive Control: Stability and Feasibility Theory
- Week 8: Integration with Loop-Shaping and Hybrid Systems

Some Remarks

- MPC name
- Continuous-Time versus Discrete-Time
- Theoretical, computation and practice
- MPC in Practice

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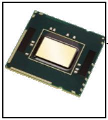
MPC: Applications



Computer control — ns

μs — Power systems

Traction control — ms

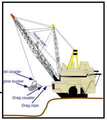
Seconds — Buildings

Refineries — Minutes

Hours — Nurse rostering

Train scheduling — Days

Weeks — Production planning



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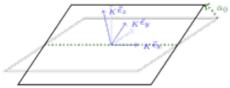
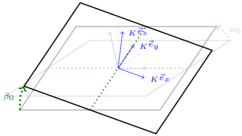
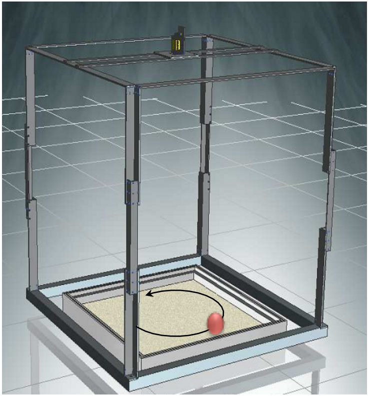
Ball on Plate

- **Movable plate** $(0.66\text{m} \times 0.66\text{m})$
- Can be revolved around two axis $[+17°; -17°]$ by two DC motors
- Angle is measured by potentiometers
- Position of the ball is measured by a camera
- *Model*: Linearized dynamics, 4 states, 1 input per axis
- *Input constraints*: Voltage of motors
- *State constraints*: Boundary of the plate, angle of the plate







[*R. Waldvogel. Master Thesis ETH, 2010*]

Ball on Plate

Controller comparison: LQR vs. MPC in the presence of input constraints

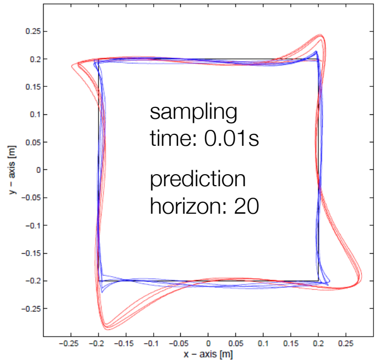


Figure: LQR (red) vs MPC (blue)

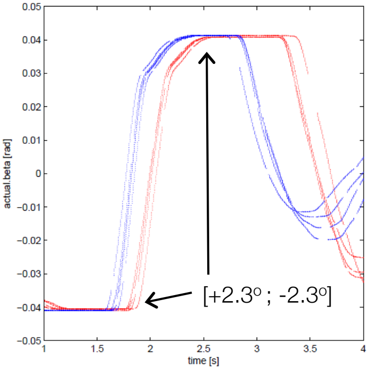


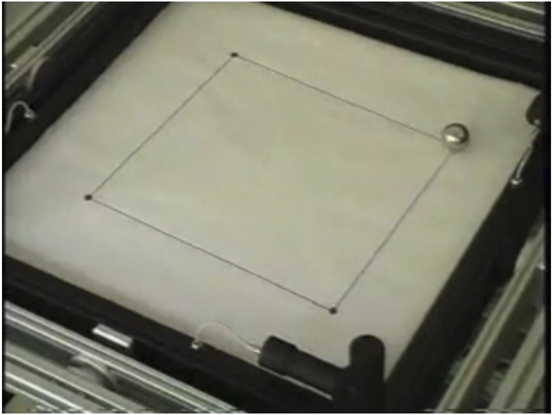
Figure: Input β for the upper left corner.

MPC introduces **preview** by predicting the state over a finite horizon

[*R. Waldvogel. Master Thesis ETH, 2010*]

Ball on Plate

MPC Control of a Ball and Plate System:



[*R. Waldvogel. Master Thesis ETH, 2010*]

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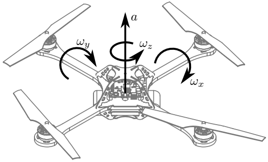
Autonomous Quadrocopter Flight

Quadrocopters:

- Highly agile due to fast rotational dynamics
- High thrust-to-weight ratio allows for large translational accelerations
- Motion control by altering rotation rate and/or pitch of the rotors
- High thrust motors enable high performance control

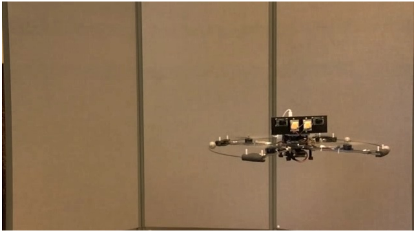


Control Problem:

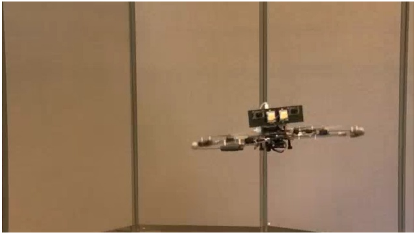
- *Nonlinear system* in 6D (position, attitude)
- *Constraints*: limited thrust, rates,...
- *Task*: Hovering, trajectory tracking
- *Challenges*: Fast unstable dynamics

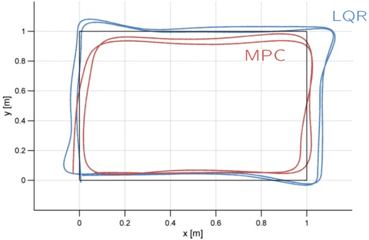
Autonomous Quadrocopter flight

LQR



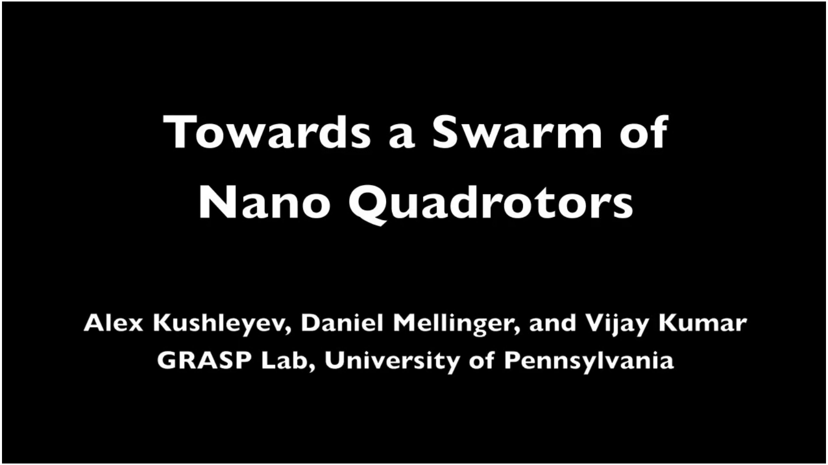
MPC





[*M. Burri. Master Thesis ETH, 2011*]

Autonomous Quadrocopter flight



[*GRASP Lab. University of Pennsylvania, 2012;* `http://www.grasp.upenn.edu/`]

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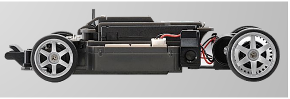
Autonomous dNaNo Race Cars

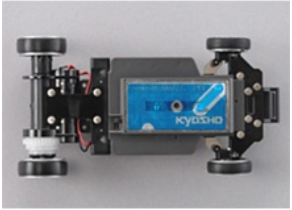
Race car:

- 1:43 scale, very light (50g) and fast
- Radio controlled
- 2.4GHz transmitter allows to run up to 40 cars

Control Problem:

- *Nonlinear model* in 4D (position, orientation)
- *Constraints:* acceleration, steering angle, race track, other cars...
- *Task:* Optimal path planning and path following
- *Challenges:* State estimation, effects that are difficult to model/measure, e.g. slip, small sampling times





Autonomous dNaNo Race Cars

[*ORCA Racer Project. ETH, 2011;* `http://orcaracer.ethz.ch/`]

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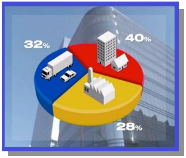
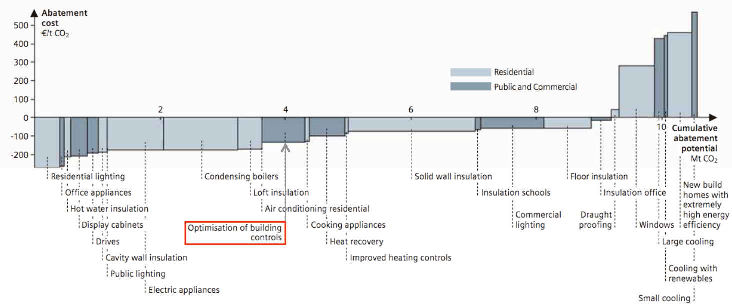
Energy Efficient Building Control

- Buildings account for approx. *40% of global energy use*
- Most energy is consumed during use of the buildings
- Building sector has large potential for cost-effective reduction of CO_2 emissions
- Most investments in buidlings are expected to pay back through *reduced energy bills*





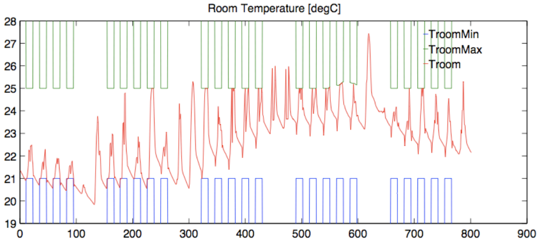
Greenhouse gas abatement cost curve for London buildings (2025, decision maker perspective)

Source: Watson, J. (ed.) (2008): Sustainable Urban Infrastructure, London Edition – a view to 2025. Siemens AG, Corporate Communications (CC) Munich, 71pp.

Energy Efficient Building Control

Integrated Room Automation:
Integrated control of heating, cooling, ventilation, electrical lighting, blinds,... of a single room/zone

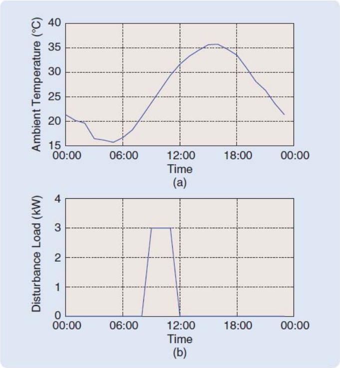
Control Task: Use minimum amount of energy (or money) to keep room temperature, illuminance level and CO_2 concentration in *prescribed comfort ranges*



[*OptiControl Project, ETH. 2010;* `http://www.opticontrol.ethz.ch/`]

Energy Efficient Building Control

Load Shifting:
Use Weather and Occupancy Forecast to Minimise Energy Bill



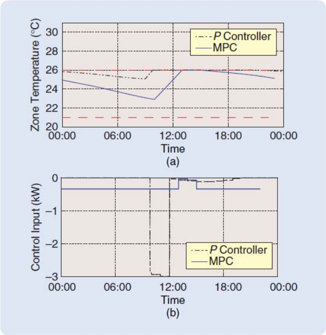
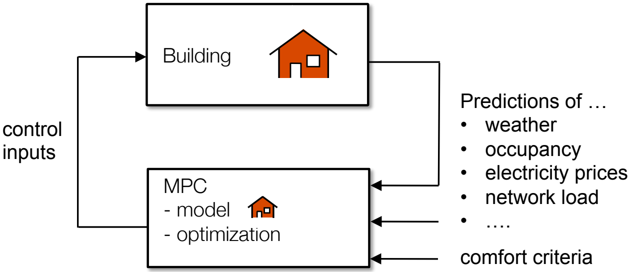


FIGURE 3 Closed-Loop simulations with model predictive control (MPC) and proportional control.

[*MPC Lab Project, UC Berkeley. 2010-2014;* http://www.mpc.berkeley.edu/]

Energy Efficient Building Control



MPC opens the possibility to

- exploit building's *thermal storage capacity*
- use *predictions* of future disturbances, e.g. weather, for better planning
- use forecasts of electricity prices to shift electricity demand for grid-friendly behavior
- offer grid-balancing services to the power network
- ...

while respecting requirements for building usage (temperature, light, ...)

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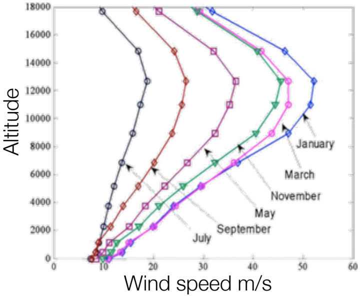
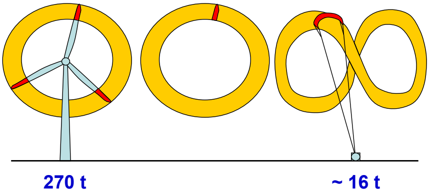
Kite Power

- Wind energy has potential to supply global energy need.
- Current wind technology is not able to exploit the potential
  - Traditional inland wind turbines are close to scaling limits
  - Economic operation only possible at a limited number of locations



Idea: Exploit the energy of high-altitude wind by means of light tethered wings (kites)
Goal: Wind power at lower cost than coal

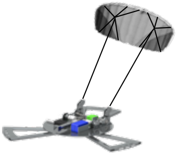


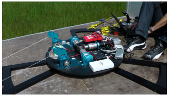
Exploit that

- Wind speed at 800m $=$ 1.5 x speed at 80m
- Power density $=$ (wind speed)^3

Kite Power

- Different kites proposed: flexible vs. rigid wings (different models, nonlinear)
- On board vs. ground level generator
- Ground level seems to be more viable for large-scale
- Number of lines?

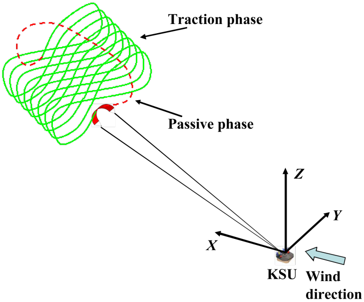




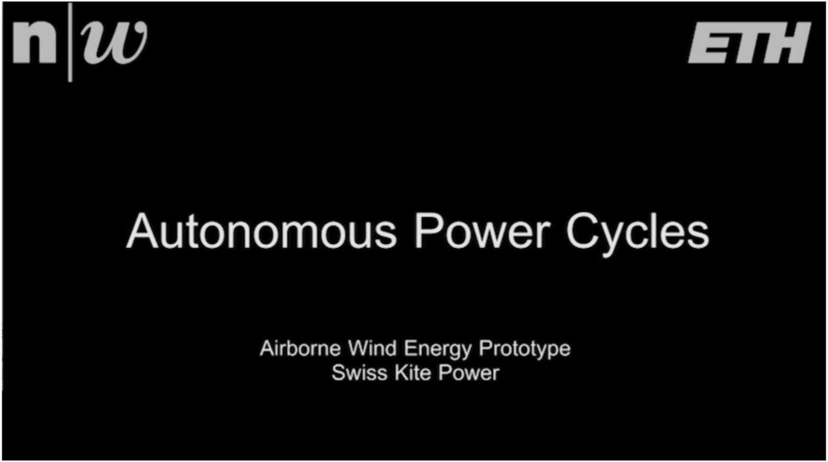
[A. Zgraggen, ETH, 2011]

Kite control problem:

- Maximize the net generated energy
- Maintain stability of the wing
- Exploit crosswind, i.e. kites fly transverse to wind at high speed
- Satisfy physical constraints: keep the kite far away from the ground, avoid line wrapping...
- Each configuration and working phase has its own performance goal



Kite Power


ETH
Autonomous Power Cycles
Airborne Wind Energy Prototype
Swiss Kite Power

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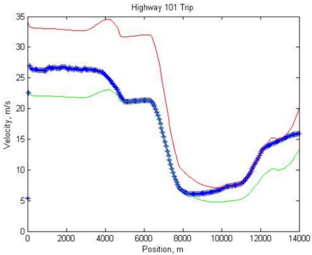
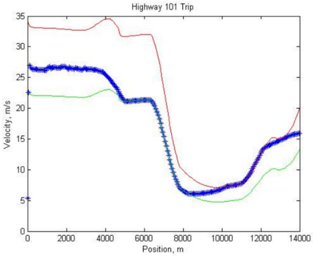
Audi Smart Engine



- **Fact:** Do not accelerate if there is a traffic jam, you will only waste fuel.
- **Idea:** Use traffic forecast to regulate the speed of a car to save fuel while getting to destination on time.

[*Khout, Borrelli and Hedrick. 15th World Congress on ITS, 2008*]

Audi Smart Engine



- MPC regulates the desired speed (through an Automatic Cruise Control) in order to reach the destination in the most fuel-efficient way, given a not-to-exceed arrival time.
- Min and Max traffic speed forecast and road grade used in the MPC constraints and model.
- Min and Max traffic speed forecast obtained from sensors embedded in the highway on each lane. (Available in the Bay Area, California).

[*Khout, Borrelli and Hedrick. 15th World Congress on ITS, 2008*]

Ford Autonomous Driving on Ice

- Autonomous double-lane change.
- Road forecast and nonlinear vehicle model (driving on ice) used in MPC.
- MPC controls differential braking and steering.
- Experimental results @ 72 km/h on ice.

[*Falcone, Borrelli et al. International Journal Vehicle Autonomous Systems, 2009*]

Volvo

- Autonomous lane keeping (minimally invasive).
- Road forecast and vehicle model used in MPC.
- MPC controls braking and steering.

[*Gray, Ali, Gao, Hedrick and Borrelli. IEEE Transactions on Intelligent Transportation Systems, 2013*]

Hyundai

- Autonomous Driving
- Road forecast and vehicle model used in MPC.
- MPC controls braking and steering.

[C:/Research/Courses/2014Fall/MPC_Course_Material/ME231A/MPC-Introduction/videos_mpg/media2.mp4]

[*MPC Lab Project, UC Berkeley. 2010-2014;* http://www.mpc.berkeley.edu/]

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Robotic Chameleon

- Tracking an object (point in video) using two independent cameras.
- MPC controls cameras pan tilt and zoom to keep object in a given field of view (constraints).
- MPC uses cameras models and forecast the object position (assuming moving at constant acceleration over the prediction horizon).
- Experimental results with MPC solved at 100 Hz.

[*Avin, Borrelli et al. Autonomous Robots, 2008*]

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Catalytic Cracker

[C:/Research/Courses/2014Fall/MPC_Course_Material/ME231A/MPC-Introduction/videos_mpg/catalytic_cracker.mpg]

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Predictive Control in NeuroScience

16:05 – [C:/Research/Courses/2014Fall/MPC_Course_Material/ME231A/MPC-Introduction/videos_mpg/CharlieRoseBrain.m4v]

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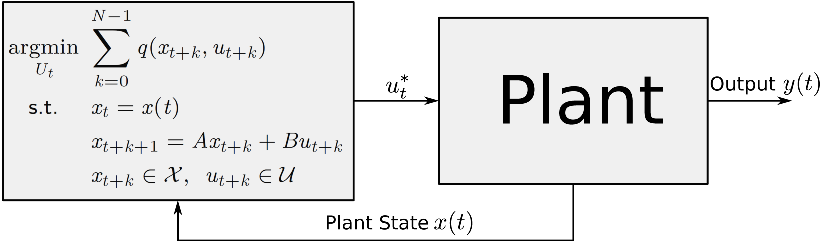
Summary: MPC



At each sample time:

- Measure /estimate current state $x(t)$
- Find the *optimal input sequence* for the entire planning window N: $U_t^* = \{u_t^*, u_{t+1}^*, \dots, u_{t+N-1}^*\}$
- Implement only the *first* control action u_t^*

Summary

- Obtain a model of the system
- Design a state observer
- Define optimal control problem
- Describe optimization problem in a modeling language
- Solve optimization problem to get optimal control sequence
- Verify that closed-loop system performs as desired,
  e.g., check performance criteria, robustness, real-time aspects,...

Important Aspects of Model Predictive Control

Main advantages:

- Systematic approach for handling *constraints*
- High *performance* controller

Main challenges:

- *Implementation*
  MPC problem has to be solved in real-time, i.e. within the sampling interval of the system, and with available hardware (storage, processor,...).
- *Stability*
  Closed-loop stability, i.e. convergence, is not automatically guaranteed
- *Robustness*
  The closed-loop system is not necessarily robust against uncertainties or disturbances
- *Feasibility*
  Optimization problem may become infeasible at some future time step, i.e. there may not exist a plan satisfying all constraints

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Literature

Model Predictive Control:

- Predictive Control for linear and hybrid systems, F. Borrelli, A. Bemporad, M. Morari, 2013 Cambridge University Press [`http://www.mpc.berkeley.edu/mpc-course-material`]
- Model Predictive Control: Theory and Design, James B. Rawlings and David Q. Mayne, 2009 Nob Hill Publishing
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Optimization:

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