The XVAs – An Illustrative Example

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Abstract

This paper considers the different valuation adjustments for derivatives (the *XVAs*) in the concrete context of a long-dated interest rate swap. I start by considering the economic origins of the different XVAs in the general case. I then put myself in the shoes of a bank and consider the swap under three representative scenarios regarding collateral: i) a completely uncollateralized swap (e.g. facing a corporate), ii) a swap collateralized with cash (e.g. facing another bank), and iii) a swap collateralized with non-cash securities (e.g. facing a pension fund).

1 Valuation Adjustments

The point of the so-called valuation adjustments (the *XVAs*) is to price all cashflows associated with a swap *in addition* to the cashflows specified in the term sheet. Roughly speaking, the division-of-labor in banks is such that the cashflows specified in the term sheet is priced and risk-managed by "the swap desk" (the interest rate swap desk, the cross-currency swap desk, ...) while everything else is priced and risk-managed by "the XVA desk". These "other" cashflows give rise to a number of valuation adjustments denoted by various three-letter acronyms, the most important of which are discussed below.

All expectations below are taken under the pricing measure using the appropriate money-market account $B(t) = e^{\int_0^t r_s ds}$ as numeraire.[1] We denote the random default times of the counterparty and the bank by τ_C and τ_B and the (fixed, by assumption) recoveries by R_C and R_B, respectively. $x^+ \equiv \max(x, 0)$ and $x^- \equiv \min(x, 0)$ denote the positive and negative elements of x.

1.1 Credit Value Adjustment

The credit value adjustment (*CVA*) is the present value of the (negative) cashflow that takes place if the counterparty to the swap defaults. Since the bank

[1] In applications, we proxy the short rate process $\{r_t\}$ by the interbank O/N rate (Eonia in EUR, Fed funds in USD, Cita in DKK, ...).

only incurs a loss if the counterparty defaults in a scenario where 1) the derivative is an asset,[2] and 2) the bank is still alive, the *CVA* at time 0 is given by:

$$CVA(0) \quad = \quad -(1-R_C)\mathbb{E}_0\left[\mathbf{1}_{\tau_C \le \tau_B} e^{-\int_0^{\tau_C} r_s ds} V(\tau_C)^+\right]$$

CVA is universally priced by all derivatives dealers and $CVA(0)$ is thought of as the value at time 0 of a hedging strategy that replicates the cash outflow (net of recovery) at counterparty default.

1.2 Debit Value Adjustment

The debit value adjustment (*DVA*) is simply the *CVA* as seen from the counterparty's point-of-view, i.e.

$$DVA(0) = (1-R_B)\mathbb{E}_0\left[\mathbf{1}_{\tau_B \le \tau_C} e^{-\int_0^{\tau_B} r_s ds} V(\tau_B)^-\right]$$

In theory, *DVA* is needed for the two parties in a derivatives transaction to agree on the price, and it is indeed an established part of Fair Value Accounting. However, it is never priced by derivative dealers as it is deemed impossible for shareholders to monetize pre-default. Thus, counterparties are effectively expected to "donate" the *DVA* to the dealer when transacting a derivative.

1.3 Funding Value Adjustment

The funding value adjustment (*FVA*) is the present value of funding the variation margin (*VM*) of the hedge strategy while the swap is alive. We let $s_F(t) \equiv r_F(t) - r(t)$ denote the banks' unsecured funding spread and we assume that the amount of *VM* at all times equals the market value of the derivative. In that case, we have:

$$FVA(0) = \mathbb{E}_0\left[\int_0^T \mathbf{1}_{t \le \min(\tau_C, \tau_B)} e^{-\int_0^t r_s ds} V(t) s_F(t) dt\right]$$

FVA is universally priced by all derivatives dealers (with the possible exception of a few small, regional players). The logic of the *FVA* is as follows.

If $V(t) > 0$ (client trade is in-the-money) we have to borrow *VM* equal to $V(t)$ at the unsecured funding rate $r_F(t)$ and give it to the fully-collateralized hedge counterparty where it earns the "risk-free" rate $r(t)$ (in practice, the O/N

[2] If the derivative is an asset the bank unwinds the hedge at $-V(\tau_C) < 0$ and recovers $R_C V(\tau_C)$ from the client netting a loss of $(1-R_C)V(\tau_C)$. If the derivative is a liability, the bank unwinds the hedge at $-V(\tau_C) > 0$ and turns over the proceeds over to the bankruptcy estate of the defaulted counterparty netting zero. This is of course an abstraction: the actual behaviour of banks when a derivative counterparty defaults is an interesting topic in itself, but outside the scope of this note.

rate). So, net we pay $V(t)\,(r_F(t) - r(t))\,dt \equiv V(t)s_F(t)dt$ over $[t, t+dt]$. Similarly, if $V(t) < 0$ we incur an inflow of $V(t)s_F(t)dt$ *either* because we can use the *VM* received from the hedge counterparty to retire existing unsecured debt at the funding spread $s_F(t)$ (unrealistic due to transaction costs and other frictions) *or* because we are overall short funding so the received *VM* can be used to fund other hedges reducing the need for unsecured funding elsewhere (may or may not be realistic depending on the bank's overall portfolio of uncollateralized trades).

1.4 Capital Value Adjustment

The capital value adjustment (*KVA*) is the present value of the required return to shareholders for commiting capital (at the level prescribed by regulators) while the swap is alive.[3] Assuming a capital requirement at time t of $K(t)$ and a cost-of-capital $\gamma(t)$, the *KVA* is given by:

$$KVA(0) = -\mathbb{E}_0\left[\int_0^T \mathbf{1}_{t\leq \min(\tau_C,\tau_B)} e^{-\int_0^t r_s ds} K(t)\gamma(t)dt\right]$$

The capital requirement $K(t)$ should in principle include all sources of risk that banks are required to capitalize – in a derivatives setting, the largest component is usually the counterparty credit risk (*CRR*) capital and this will be the focus here[4]. There are two types of *CRR* capital requirements:

- Capital must exceed a certain fraction of **risk-weighted assets** *RWA*:

  $$K(t) \geq \alpha_{RWA} RWA(t)$$

  For derivatives, risk-weigthed assets are computed as

  $$RWA(t) = RW(PD, LGD, M) \times EAD_x,\ x \in \{IMM, CEM\}$$

  where the risk weight depends on probability of default, loss-given-default and (effective) maturity (same as for the loan book from which this framework is adapted) and EAD is an "effective" notional of the derivative which may be simulated (internal model method, *IMM*) or looked up in a standardized schedule (current exposure method, *CEM*).

- Capital must exceed a certain fraction of **raw assets** A (refered to as "leverage ratio"):

  $$K(t) \geq \alpha_{LR} A(t)$$

  For derivatives, raw assets are computed simply as $A(t) = EAD_{CEM}$.

[3] We assume that the bank never decides to allocate more capital to a swap than is prescribed by regulators, i.e. the regulatory capital requirement in all scenarios exceed the economic capital requirement that an unconstrained bank would choose to allocate.

[4] Other components are market risk capital, capital for increases in *CVA* (short of outright default, which is covered by the *CRR*) and operational risk capital.

Naturally $\alpha_{RWA} > \alpha_{LR}$ and in any given scenario only one of these constraints will be strictly binding, i.e. $K(t) = \min(\alpha_{RWA}RWA(t), \alpha_{LR}A(t))$. Finally, we note that the *CEM* is to be replaced by a more risk-sensitive measure (the so-called *SA-CCR*) as part of the Fundamental Review of the Trading Book.

2 An Illustrative Example

We consider the concrete example of a 10Y interest swap where the bank receives fixed at the current market level, $K = 0.5700\%$. This specific example captures many of the issues around derivatives valuation adjustments that I want to emphasize in this note.

2.1 The Market

The yield curve is the official DKK swap curve as of 23Sep2016 published at http://www.finansraadet.dk, see Figure 1. For the swaption market, I assume a constant normalized volatility of 100 bps.

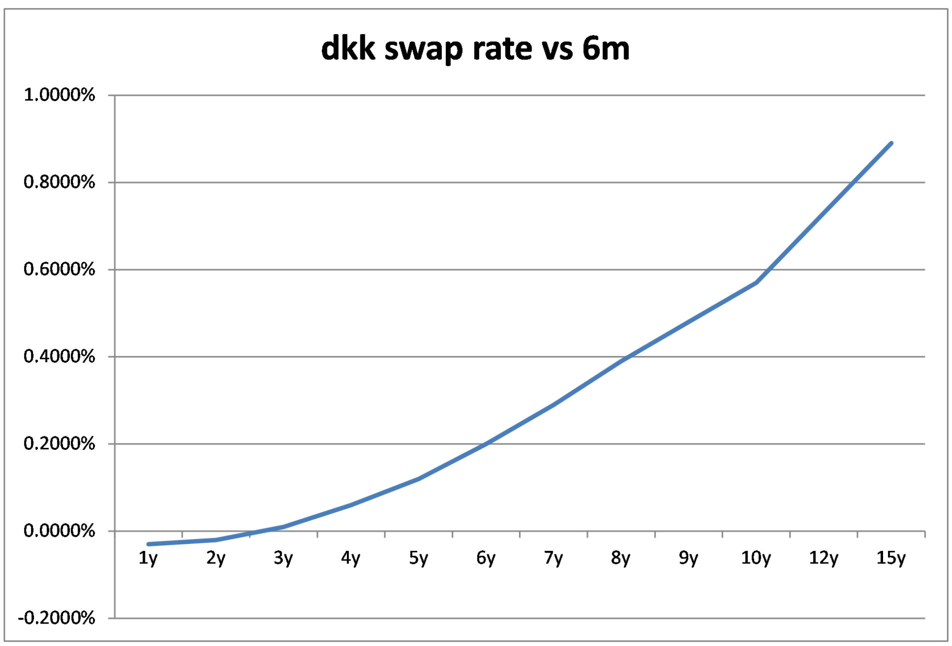

Figure 1: Official DKK swap curve as of 23Sep2016.

For the counterparty and the bank I make the following assumptions (all numbers are chosen to be within realistic ranges given my own experience). I assume a constant counterparty credit spread of $s_C(t) = 100$ bps, a constant own credit spread of $s_B(t) = 50$ bps and a constant own funding spread of

$s_F(t) = 50$ bps. I assume that the bank is allowed to compute EAD using the internal model method (IMM) and that the counterparty has a fixed risk weight of 50%. Finally, the bank has a capital requirement of 15% of risk-weighted assets (the leverage ratio is assumed to be non-binding throughout) and a required RoE of $\gamma(t) = 20\%$. Each unit of EAD thus has an annualized cost of $50\% \times 15\% \times 20\% = 150$ bps which we use for computing KVA.

2.2 Case I: No Collateral

Given the assumptions in 2.1 and assuming no collateral we have:[5]

$$\begin{aligned}
CVA(0) &= -100\,\text{bp} \times \int_0^{10Y} PV_{\text{ReceiverSwaption}}(0, t, 10Y, 0.5700\%)dt \\
DVA(0) &= +50\,\text{bp} \times \int_0^{10Y} PV_{\text{PayerSwaption}}(0, t, 10Y, 0.5700\%)dt \\
FVA(0) &= +50\,\text{bp} \times \int_0^{10Y} PV_{\text{FwdReceiverSwap}}(0, t, 10Y, 0.5700\%)dt \qquad (1) \\
KVA(0) &= -150\,\text{bp} \times \int_0^{10Y} \kappa PV_{\text{ReceiverSwaption}}(0, t, 10Y, 0.5700\%)dt
\end{aligned}$$

where $PV_x(0, t, T, K)$ is the time 0 price of instrument x struck at K, starting at time t and maturing at time T. $\kappa > 1$ in KVA corrects for various regulatory quirks in the EAD calculation – in practice, $\kappa \approx 1.5$, but we fix it at $\kappa = 1$ for simplicity below. The formula for KVA effectively assumes that at every future instant t we capitalize only the current value of the derivative (if positive, zero if negative) over the next infinitesimally small horizon $[t, t + dt]$. In reality, the EAD depends on the potential future exposure of the derivative over a one-year horizon, but incorporating this feature would dramatically increase the complexity of the calculation.

We implement Eqn. (1) by discretizing time into 6-month intervals and computing the corresponding prices for each t, i.e. I price the following swaptions and forward-starting swaps: 0m-into-120m, 6m-into-114m, 12m-into-108m, ..., 120m-into-0m each one struck at the current 10Y rate, $K = 0.5700\%$. Figure 2 illustrates the resulting profiles. We interpret the green curve as the "loan equivalent" of the derivative[6] and the red curve as the "deposit equivalent". Finally, the blue curve is interpreted as the "funding profile" – it is the expected amount of VM to be exchanged over the lifetime of the swap.

[5] We also ignore a survival probability $\mathbb{Q}(t \leq \min(\tau_C, \tau_B))$ in each of the integrals. This term can be quantitatively significant, but is immaterial to the intuition.

[6] Consistent with this interpretation, to hedge the credit risk of CVA we would buy CDS protection at time 0 with a notional profile given by the green curve.

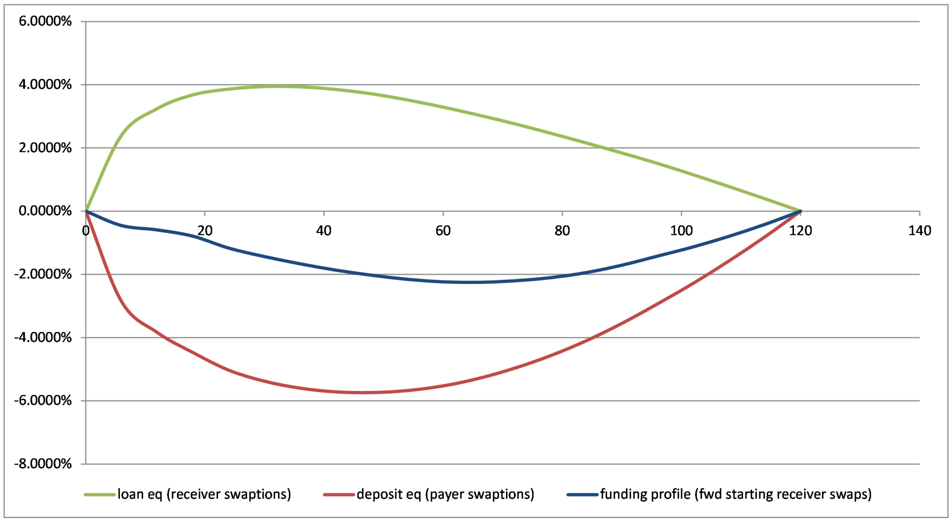

Figure 2: Loan equivalent, deposit equivalent and funding profile for the 10Y receiver swap.

It is worth thinking a bit about the shape of these curves. Consider first the loan equivalent of the derivative (green curve). It is zero at $t = 0$ since the swap is struck at par (the "trivial" 0m-into-120m swaptions has no intrinsic value) and it is also zero at $t = T$ since at maturity there are no more cashflows in the swap. For $0 < t < T$ the curve is in general the product of two opposing forces: **diffusion** which acts to increase the loan equivalent as there is more time for volatility to drive the derivative into positive territory, and **cashflows** which acts to decrease the loan equivalent as the duration of the underlying swap falls as cashflows are exchanged. The combined effect is a curve that starts at zero, initially increases and then decreases all the way down to zero.[7] Qualitatively, this is true for all swap products that does not include a final exchange of notional.

The intuition for the deposit equivalent (red curve) is identical except for a change of sign. The funding profile (blue curve) is just the sum of the loan equivalent and the deposit equivalent and by the put-call parity it is completely determined by the current forward curve, indeed it represents the future value of the swap assuming that the current forward curve is realized. Note, importantly, that the funding profile is negative meaning that the bank expects to be out-of-the-money against the client and therefore *receive* collateral on the hedge trade (this is because we, the bank, receive fixed on an upward-sloping yield curve), i.e. the funding component actually *increases* the value of the trade to the bank.

[7]Intuitively, the diffusion effect scales with $\sqrt{T}$ while the duration effect scales with $T - t$ giving a curve shape that resembles $\sqrt{T} \times (T - t)$.

Let us now evaluate Eqn. (1) numerically. Evaluating the integrals numerically we get:[8]

$$\begin{aligned} CVA(0) &= -0.2577\% \\ DVA(0) &= +0.1998\% \\ FVA(0) &= +0.0709\% \\ KVA(0) &= -0.3866\% \end{aligned}$$

Note that these numbers are PVs expressed as a percentage of notional. Traders always think in terms of *risk*, never in terms of notional, so before interpreting the numbers we divide through by the annuity (∼duration) of the 10Y swap, i.e. the PV of receiving one additional basis point on the fixed leg of the uncollateralized swap per unit notional. For simplicity, assume the risky annuity is $RA(0) = 9.00$. Hence, we have:

$$\begin{aligned} CVA_{\text{bp}}(0) &= -\frac{0.2577\%}{9.00 \times 1\,\text{bp}} = -2.86\,\text{bp} \\ DVA_{\text{bp}}(0) &= +\frac{0.1998\%}{9.00 \times 1\,\text{bp}} = +2.22\,\text{bp} \\ FVA_{\text{bp}}(0) &= +\frac{0.0709\%}{9.00 \times 1\,\text{bp}} = +0.79\,\text{bp} \\ KVA_{\text{bp}}(0) &= -\frac{0.3866\%}{9.00 \times 1\,\text{bp}} = -4.30\,\text{bp} \end{aligned}$$

Now, let us interpret the numbers in the context of an actual derivative transaction:

- The client calls his sales contact at the bank and asks for an offer in 10Y DKK
- The sales guy (or girl – traders are almost always guys, the sales force is more diverse) does two things immediately:
  - He asks the DKK swap desk for an offer in 10Y.[9]
  - He asks the XVA desk for an offer in 10Y against *this specific* client.
- After some time (∼30 sec for the swap desk, a few minutes for the XVA desk) the sales guy gets the response:
  - The swap desk says "1 bp from mid" meaning he would need to receive 1 bp above where he is marking his book right now (the sales guy can see the current DKK mid-curve on his own screen, so the trader just communicates the difference).

[8] CVA is computed as −100 bps times the (numerical) integral under the loan equivalent (green) curve and so on.

[9] Although the cashflows to the swap desk does *not* depend on the counterparty (they are insured by the XVA desk), the swap desk would typically want to know who the client is for the *informational* content of the trade (if it's Maersk hedging a bond issue, that's cool – if it's Brevan Howard putting on a speculative trade, that's not so cool).

  - The XVA desk in this case says "6.4 bp from mid" because $CVA_{\text{bp}}(0) + FVA_{\text{bp}}(0) + KVA_{\text{bp}}(0) = -6.4\,\text{bp}$.

- The sales guy combines the charges to arrive at a total charge of $1.0\,\text{bp} + 6.4\text{bp} = 7.4\,\text{bp}$.
- Suppose the current mid is 0.5700% as above. He then tells the client that his offer is 0.6440% (client pays) plus whatever additional sales margin he can get away with.
- If the client agrees, the trade ends up in the swap traders book with the agreed coupon and 6.4 bp (in PV terms) is transferred to XVA desk.
- At this point the sales guy's job is done and it is up to the swap desk and the XVA desk to hedge the risk as they see fit.

This example illustrates how the charges are used by the front office. Notice how DVA does not enter the process at all.

2.3 Case 2: Cash Collateral

We now assume that the swap is supported by a collateral agreement with cash collateral, daily variation margin calls and no initial margin. Economically, trading under such a collateral agreement is equivalent to unwinding the trade against cash at the end of every trading day and entering a new at-market trade instead. Thus, we can write the XVAs as follows:

$$\begin{aligned}
CVA(0) &= -100\,\text{bp} \times \int_0^{10Y} PV_{\text{ReceiverSwaption}}(0, t, 10Y, S_{t-\Delta})dt \\
DVA(0) &= +50\,\text{bp} \times \int_0^{10Y} PV_{\text{PayerSwaption}}(0, t, 10Y, S_{t-\Delta})dt \qquad (2) \\
FVA(0) &= +50\,\text{bp} \times \int_0^{10Y} PV_{\text{FwdReceiverSwap}}(0, t, 10Y, S_{t-\Delta})dt \\
KVA(0) &= -150\,\text{bp} \times \int_0^{10Y} \kappa PV_{\text{ReceiverSwaption}}(0, t, 10Y, S_{t-\Delta})dt
\end{aligned}$$

The only difference to Eqn. (1) is that we replace the fixed strike of the underlying $K = 0.5700\%$ with the prevailing par swap rate at the last margin call time $t - \Delta$. For bilateral swaps, Basel III requires that $\Delta = \min(x, \text{margin call frequency})$ when computing EAD, where x is 10 business days for bilateral swaps and 5 business days for swaps traded through a CCP. Thus, nontrivial valuation adjustments may remain even in the presence of strong collateral agreements. Initial margin is introduced precisely to address this so-called *margin-period-of-risk* (*MPoR*) – the period from your counterparty stops responding to margin calls and until the trade has been closed now.[10]

[10] It does so, however, at the cost of introducing additional funding costs (MVA, funding of intial margin).

To implement Eqn. (2) we need to compute values of *forward-starting swaptions* (i.e. swaptions whose strike is fixed in the future, in this case Δ days prior to expiry) which in general requires a dynamic model. Intuitively, the exposure is reset to zero immediately after every succesful margin call resulting in a "saw-tooth" shaped exposure profile – various approximations can be employed, but this it out of scope for this small note.

2.4 Case III: Non-cash Collateral

Let us finally assume that the swap is supported by a collateral agreement that allows posting of other securities besides cash, such as government bonds or covered bonds. This is common among pension funds and asset managers who holds these securities *qua* their investment mandates and for whom it is inconvenient to raise potentially large amounts of cash on short notice. The presence of non-cash collateral does not affect *CVA* or *DVA* (to a first order) as the riskiness of the collateral is assumed to be reflected through appropriate haircuts (interestingly, many sovereigns and quasi-sovereigns are allowed to post their own bonds as securities, underscoring the fact that collateral agreements are as much about funding as it is about credit risk mitigation). However, non-cash collateral has a potentially large effect through the *FVA* as explained below.

Basically, the impact of non-cash collateral depends on its liquidity. In one extreme, the collateral is completely illiquid – it cannot be turned into cash through the repo market at all – and thus has no funding value (think of a corporate pledging physical assets). In that case, we are back to the uncollateralized version of the *FVA* in Eqn. 1. In the other extreme, the collateral is super liquid and can be converted to cash at zero spread to the risk-free (O/N) rate. In that case, we are back to the cash-collateralized version of the *FVA* in Eqn. 2. In the intermediate cases where the collateral can be turned into cash at some (positive) spread to the O/N rate (think Danish MBS which may trade at, say, Cita + 30 bps) we are somewhere in between.

The most straightforward way to incorporate non-cash collateral into the valuation adjustments is to replace $s_F(t) \equiv r_F(t) - r(t)$ by $s_{Coll}(t) \equiv r_{Coll}(t) - r(t)$ where the $r_{Coll}(t)$ is the rate at which the collateral can be turned into cash through the repo market. In this case, banks often refer to the *FVA* as the Collateral Valuation Adjustment (*ColVA*), i.e.

$$ColVA(0) = \mathbb{E}_0 \left[\int_0^T \mathbf{1}_{t \leq \min(\tau_C, \tau_B)} e^{-\int_0^t r_s ds} V(t) s_{Coll}(t) dt \right]$$

3 Summary

In this short note I have defined the most important valuation adjustment for derivatives in the general case and I have applied them to the specific case of a long-dated interest rate swap under different assumptions regarding collateral.