Funding Value Adjustment and Incomplete Markets

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Abstract

Value adjustment of uncollateralized trades is determined within a risk–neutral pricing framework. When hedging such trades, investors cannot freely trade protection on their own name, thus facing an incomplete market. This fact is reflected in the non–uniqueness of the pricing measure, which is only constrained by the values of the hedging instruments tradable by the investor. Uncollateralized trades should then be considered not as derivatives but as new primary assets in the investor's economy. Different choices of the risk–neutral measure correspond to different completions of the market, based on the risk appetite of the investor, leading to different levels of value adjustments. We recover, in limiting cases, results well known in the literature.

Introduction

The value of uncollateralized trades must be undoubtedly adjusted in view of the credit merit of the trading parties. On the other hand, the magnitude of such adjustment is still source of debate in the financial community [12, 13, 14]. In particular, although credit and debt value adjustments (CVA/DVA) are rather clearly understood, less so is the adjustment due to different effective unsecured funding rates (FVA) faced by investors.

A variety of different approaches have been proposed in the literature [3, 4, 5, 6, 7, 8, 9, 10, 11, 18, 1, 2], often leading to different final expressions for the full value adjustment, even in the presence of identical assumptions on closeout amounts in case of default of one of the trading names. One very natural approach is based on the risk neutral valuation of all relevant cash flows. These flows must, in particular, include exogenous funding flows potentially faced by an investor when funding the full hedge [1, 2]. A different approach is based on the direct analysis of the hedge of the trade, with the relevant part being the hedge with funded instruments of the investor (typically bonds of different seniority). Using this reasoning, one notes that, in general, not all cash flows can be exactly hedged since an investor cannot, in general, trade collateralized

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protection on its own name. Different imperfect hedges are then possible, leading naturally to different values for the full adjustment [3, 4, 5, 6, 7, 8, 18].

In this quick note we wish to combine the two points of view above. The analysis of Burgard and Kjaer clearly shows that a general payoff, contingent on the default of the investor, cannot be hedged, in general, by the investor itself. This is due to the fact that, although external players can trade both funded and collateralized protection on the investor's name (bonds, CDS's etc.), this is not true from the point of view of the investor, which, in the most idealized setting, can only trade funded instruments (bonds of a given seniority). Based on the available hedging instruments, the investor then faces a market which is clearly incomplete and cash flows of a general trade, including closeouts at time of default, cannot be hedged exactly. In this sense, uncollateralized trades are not derivatives but become new primary assets in the economy, whose value must be determined on the basis of different reasoning, based on the risk appetite of the investor. As is well known in finance theory [17], incompleteness of the market is reflected directly in the non–uniqueness of the risk neutral bank account measure used to discount future cash flows. Different levels of risk appetite are then naturally parametrized by different choices of risk neutral measure. Each choice then corresponds to a different way of completing the market in a consistent arbitrage–free manner and leads to a different value adjustments for the full trade.

In the next sections, we analyze the idea outlined above in the simplest possible setting, where all market factors are considered as deterministic. This allows us to focus on the essentials, with the unique source of randomness coming from the default times of the trading names. More precisely, in section 1 we will consider only the investor as defaultable, with different risk neutral measures corresponding to different default intensities. This means that, for internal valuations, the investor is free to choose the level of CDS's on it's own name differently from the market level, adjusting the internal discount rate so as to keep fixed the values of all market instruments which are tradable by the investor, here chosen to be bonds of a given seniority. This leads to a simple expression for the full value adjustment parametrized by the default intensity of the investor. Section 2 extends the reasoning to the case of a defaultable counterparty. Again, the investor is free to choose, for internal valuations, the joint probability of default of the two trading names, keeping fixed available market instruments. These include CDS's on the counterparty's name, which fixes the default intensity of the counterparty to market levels, as well as bounds of the investor, potentially contingent on the counterparty's default event (credit linked notes, for instance). The discount rate must then also be changed, with the added feature that, in the presence of correlation between the default events, the dynamics of such rate, as seen by the investor, becomes contingent on the default of the counterparty, as we shall demonstrate in detail in the simplest case. We conclude with a brief description of potential future work.

1 Risk free counterparty

1.1 The market as seen by the external players

In order to keep the discussion simple and to focus on essential matters, we will consider all market factors as deterministic. The only random event will be the default time of the investor τ_I, with the counterparty considered as default free. Later in the discussion we will also model the default time of the counterparty τ_C.

All external market players (aside from the investor itself) have access and trade actively bonds and collateralized CDS's of the investor, and have access to the risk free money market. This determines, as usual, a market level for the

$r(t)$	riskfree funded cash rate
$r_X(t)$	collateral OIS rate
$\lambda_I(t)$	default intensity of the investor

The difference of the funded and the collateral rate gives rise to a liquidity basis

$$r(t) - r_X(t) \ .$$

As mentioned above, we consider the above market quantities as deterministic.

Basic bonds of the investor will be zero–coupon bonds with recovery R_I. These bonds pay principal at maturity T in the absence of default and a fraction R_I of their value in case of default before maturity. They have a value of

$$P_I(t,T) \cdot \mathbf{1}_{\tau_I > t}$$

determined by

$$dP_I(t,T) = r_F(t)\, P_I(t)\, dt$$

and $P_I(T,T) = 1$, where we denote with r_F the effective funding rate of the investor

$$r_F(t) = r(t) + (1 - R_I)\, \lambda_I(t) \ .$$

1.2 The market as seen by the investor

The market, as seen by the external players, is complete. Any payoff, deterministic or contingent to the default of the investor, can be perfectly replicated with the available hedging instruments and no ambiguity exists in the pricing of any stream of future dividends.

The situation is very different from the point of view of the investor. Although the investor has access to its own bond market, where new debt can be issued or bought–back, no direct access to the CDS market is available. The only tradable primary assets are therefore the investor's bonds, and the market, from this prospective, is now incomplete. Not all dividend streams are perfectly replicable and, for these payoffs, no unique price can be assigned. Finance theory [17] teaches us that this non–uniqueness corresponds to a non–uniqueness in the risk–neutral measure. Recalling that we are only modeling the default of the investor, we may then choose a different risk–free and default intensity

$\bar{r}(t)$	riskfree cash rate as seen by the investor
$\bar{\lambda}_I(t)$	default intensity of the investor as seen by the investor

with the unique constraint of keeping fixed the tradable assets in the economy – i.e. the investor's bonds. This can be easily achieved by keeping unaltered the funding rate, imposing

$$r_F(t) = r(t) + (1 - R_I)\lambda_I(t) = \bar{r}(t) + (1 - R_I)\bar{\lambda}_I(t) \ . \tag{1}$$

From the investor's point of view, different choices of $\bar{\lambda}_I$ corresponds to different internal quotes of the CDS market on it's own name, which is excluded and not directly tradable.

1.3 Value adjustment

Let us now consider a given contract which, in the absence of default events, pays a stream of dividends

$$dq(t)$$

to the investor. These dividends are considered as deterministic, not contingent to any default event.

First let us focus on the (perfectly) collateralized exchange of the above dividends. It is then well known that default events are immaterial and that the value of the collateral account $v_X(t)$ (also referred to, inaccurately, as the riskfree value of the contract) satisfies

$$\frac{dq}{dt} + \frac{dv_X}{dt} = r_X v_X \ . \tag{2}$$

Stated differently, future dividends are discounted at the collateral rate [15, 16].

We wish, on the other hand, to price the funded version of the above steam of dividends, keeping into account the effects of the default of the investor. Future dividends $dq(t)$ are then exchanged only if the investor has not defaulted. Moreover, in case of default, a deterministic closeout amount

$$k_I(t)$$

is exchanged as a potentially partial substitute of the missed future dividends. In formulae, the exact future dividends to be priced are given by

$$dQ(t) = dq(t) \cdot \mathbf{1}_{\tau_I > t+dt} + k_I(t) \cdot \mathbf{1}_{t<\tau_I<t+dt} \ .$$

We will also denote the full value of the contract by

$$V(t) = v(t) \cdot \mathbf{1}_{\tau_I > t} \ , \tag{3}$$

where $v(t)$ will again be a deterministic function to be determined.

As discussed above, we will be pricing under the risk–neutral measure $\bar{\mathbb{E}}_t$ corresponding to the internal choice of the default intensity $\bar{\lambda}_I$ of the investor. The value of the contract (3) will, in general, depend on this choice, signaling that no exact replica is available for the dividend stream $dQ(t)$. The basic pricing equation

$$\bar{\mathbb{E}}_t\left[dQ(t) + dV(t)\right] = \bar{r}(t)\, V(t)\ , \tag{4}$$

together with the simple facts

$$\begin{aligned}
\bar{\mathbb{E}}_t\left[dQ(t)\right] &= \mathbf{1}_{\tau_I > t} \cdot \left[dq(t) + k_I(t)\,\bar{\lambda}_I(t)\,dt\right] \\
\bar{\mathbb{E}}_t\left[dV(t)\right] &= \mathbf{1}_{\tau_I > t} \cdot \left[dv(t) - v(t)\,\bar{\lambda}_I(t)\,dt\right]\ ,
\end{aligned}$$

implies the fundamental equation

$$\frac{dq}{dt} + \frac{dv}{dt} = \left(\bar{r} + \bar{\lambda}_I\right) v - \bar{\lambda}_I k_I\ .$$

This equation should be compared with the corresponding one (2) for the riskfree value v_X. In particular, if we denote with

$$u(t) = v(t) - v_X(t)$$

the value adjustment due to the funded and default contingent nature of the full claim, we see that u must satisfy[1]

$$-\frac{du}{dt} + \left(\bar{r} + \bar{\lambda}_I\right) u = \bar{\lambda}_I\left(k_I - v_X\right) - \left(\bar{r} - r_X\right) v_X\ , \tag{5}$$

together with the boundary condition that $u(T) = 0$ at the maturity T of the contractual dividends dq. Two cases are often discussed in the literature

$$\begin{aligned}
-\frac{du}{dt} + r_F u &= -\left(r_F - r_X\right) v_X\ , \\
-\frac{du}{dt} + \left(r + \lambda_I\right) u &= \left(k_I - v_X\right)\lambda_I - \left(r - r_X\right) v_X\ .
\end{aligned}$$

The first case corresponds to a pure discounting internal view–point, with a vanishing intensity $\bar{\lambda}_I = 0$ of the investor's default[2] and $\bar{r} = r_F$. The opposite extreme case corresponds to $\bar{\lambda}_I = \lambda_I$ and $\bar{r} = r$, where the internal valuation of CDS's is equal to that of the market.

[1] We will write throughout value adjustment equations in differential form $-du/dt + \alpha u = \beta$, with $u(T) = 0$. The corresponding integral form

$$u(t) = \int_t^T ds\, \beta(s)\, e^{-\int_t^s \alpha(s')ds'}$$

leads to well known expressions for the adjustment.

[2] The case $\bar{\lambda}_I = 0$ does not correspond to an equivalent measure when $\lambda_I \neq 0$, and the corresponding prices allow arbitrage opportunities. One should consider this as a limiting case of the allowed market completions defined by $\bar{\lambda}_I = \varepsilon$:

To conclude, we consider the simplest case of closeout

$$k_I = v_X^+ - \mathcal{R}_I v_X^- ,$$

with $\mathcal{R}_I$ the closeout recovery. The pricing equation for the value adjustment then reads

$$-\frac{du}{dt} + \left(\bar{r} + \bar{\lambda}_I\right) u = \left(1 - \mathcal{R}_I\right) \bar{\lambda}_I v_X^- - \left(\bar{r} - r_X\right) v_X .$$

2 Defaultable counterparty

In this last section we will consider the case of a defaultable counterparty. All other market factors will be considered as deterministic, with the unique source of randomness coming from the default times of the investor and of the counterparty.

As before, external players face deterministic risk free and collateral rates $r(t)$ and $r_X(t)$ and value the probability of defaults of the investor I and of the counterparty C with survival probabilities

$$U_N(t_N) = \mathbb{E}\left[\mathbf{1}_{\tau_N > t_N}\right]$$

(in the sequel, N will denote the credit names I and C) and corresponding forward default intensities

$$\lambda_N(t_N) = -\partial_N \ln U_N(t_N) .$$

The correlation between the default events of the investor and of the counterparty is described by the joint cumulative

$$U(t_I, t_C) = \mathbb{E}\left[\mathbf{1}_{\tau_I > t_I} \mathbf{1}_{\tau_C > t_C}\right] .$$

In the sequel, we will also use the following quantities

$$\Lambda_N(t) = -\partial_N \ln U(t, t)$$

which satisfy $\Lambda_N = \lambda_N$ only in the uncorrelated case corresponding to a factorized joint cumulative $U = U_I \cdot U_C$.

Following the same line of reasoning as in the previous section, the investor is free to choose a different martingale measure and riskfree rate, keeping fixed the values of the subset of market instruments which are available to him. In particular, default probabilities will be different from the market ones, and we will denote with bars, as before, the quantities $\bar{U}_I, \bar{U}_C, \bar{U}, \cdots$ as seen from the investor's perspective. Corresponding to a change in the default probabilities, the investor sees a different short rate process

$$\bar{r}(t)$$

which, as we shall see later, cannot be modeled as deterministic in the presence of a defaultable counterparty and must be fixed by imposing the equality of the relevant bonds' values. Note, though, that the equality of collateralized credit products on the counterparty's name simply implies that

$$\bar{U}_C(t_C) = U_C(t_C) . \tag{6}$$

2.1 Value adjustment

Following closely the reasoning in section 1.3, we consider a stream of deterministic dividends $dq(t)$ exchanged between the counterparty and the investor, with collateralized value $v_X(t)$ given by (2). The uncollateralized position corresponds then to the following complete dividends[3]

$$\begin{aligned} dQ(t) \quad = \quad & dq(t) \cdot \mathbf{1}_{\tau_I > t+dt} \cdot \mathbf{1}_{\tau_C > t+dt} \\ & + k_I(t) \cdot \mathbf{1}_{t<\tau_I<t+dt} \cdot \mathbf{1}_{\tau_C > t+dt} \\ & + k_C(t) \cdot \mathbf{1}_{\tau_I > t+dt} \cdot \mathbf{1}_{t<\tau_C<t+dt} \, . \end{aligned}$$

As usual, contractual dividends $dq(t)$ are exchanged only if both parties are alive. In case of default of a given name N, a deterministic closeout amount $k_N(t)$ is exchanged, partially replacing lost future dividends. Denoting with

$$\tau = \tau_I \wedge \tau_C$$

the first to default time, it is immediate to show that

$$\bar{\mathbb{E}}_t \left[dQ(t) \right] = \mathbf{1}_{\tau > t} \left[\frac{dq(t)}{dt} + k_I(t)\, \bar{\Lambda}_I(t) + k_C(t)\, \bar{\Lambda}_C(t) \right] dt \, .$$

If we denote with

$$V(t) = \mathbf{1}_{\tau>t} v(t)$$

the value of the trade as seen by the investor, it is also simple to show that

$$\bar{\mathbb{E}}_t \left[dV(t) \right] = \mathbf{1}_{\tau > t} \left[\frac{dv(t)}{dt} - \left(\bar{\Lambda}_I(t) + \bar{\Lambda}_C(t) \right) v(t) \right] dt \, .$$

In order to apply the basic pricing equation (4), we must first recall that, as we shall demonstrate concretely in the next section in a specific case, the internal discount rate $\bar{r}(t)$ will be, in general, contingent on the default time of the counterparty. We can then write explicitly

$$\bar{r}(t) = \bar{r}'(t) \cdot \mathbf{1}_{\tau_C > t} + \cdots \, ,$$

where $\bar{r}'$ represents the value of the internal discount rate prior to the default of the counterparty and where $\cdots$ represents the realizations of $r(t)$ in states of the world when $\tau_C \leq t$. The pricing equation then implies

$$\frac{dq}{dt} + \frac{dv}{dt} = \left(\bar{r}' + \bar{\Lambda}_I + \bar{\Lambda}_C \right) v - \bar{\Lambda}_I k_I - \bar{\Lambda}_C k_C \, ,$$

or, in terms of $u = v - v_X$,

$$-\frac{du}{dt} + \left(\bar{r}' + \bar{\Lambda}_I + \bar{\Lambda}_C \right) u = \bar{\Lambda}_I (k_I - v_X) + \bar{\Lambda}_C (k_C - v_X) - (\bar{r}' - r_X) v_X \, , \quad (7)$$

[3] In this paper, we will consider probability measures such that joint defaults do not occur a.s.

which generalizes (5).

As before, consider the simplest case of closeouts

$$\begin{aligned} k_I &= v_X^+ - \mathcal{R}_I v_X^- \\ k_C &= \mathcal{R}_C v_X^+ - v_X^- \end{aligned}$$

in terms of closeout recoveries $\mathcal{R}_I$,$\mathcal{R}_C$. The value adjustment equation then reads

$$-\frac{du}{dt} + \left(\bar{r}' + \bar{\Lambda}_I + \bar{\Lambda}_C\right) u = \left(1 - \mathcal{R}_I\right) \bar{\Lambda}_I v_X^- - \left(1 - \mathcal{R}_C\right) \bar{\Lambda}_C v_X^+ - \left(\bar{r}' - r_X\right) v_X \,.$$

2.2 Interest rate model as seen by the investor

As described at the beginning of this section, once the internal default probability $\bar{U}\left(t_I, t_C\right)$ has been chosen (with the unique constraint (6)), the dynamics of the short rate $\bar{r}\left(t\right)$ must be determined in order to keep unaltered the values of the relevant investor's bonds. The rate $\bar{r}\left(t\right)$ will not be deterministic in general, but will be contingent on the default time of the counterparty.

More precisely, we will keep fixed the values of zero–coupon bonds which pay, at maturity T_I, the default contingent payoff

$$\mathbf{1}_{\tau_C > T_C} \cdot \mathbf{1}_{\tau_I > T_I} \,. \qquad \left(\text{payed at } T_I > T_C\right)$$

As before, these bonds pay, in case of default of the investor, a given recovery fraction R_I of their value.

2.2.1 Case of vanishing recovery and default–free investor

We will not consider in this paper the general case with arbitrary recovery R_I and choice of investor's default intensity $\bar{\lambda}_I\left(t\right)$. We will work here with the simpler case of vanishing bond recovery

$$R_I = 0$$

and default–free investor

$$\bar{\lambda}_I\left(t\right) = 0 \,.$$

This case has all the relevant qualitative features and, at the same time, is analytically easily tractable. Denoting with $D\left(t, T\right) = e^{-\int_t^T r(u)du}$ the discount factor (and similarly with $\bar{D}\left(t, T\right)$), we may equate the bond values within the two pricing schemes and immediately write

$$\begin{aligned} \bar{\mathbb{E}}\left[\mathbf{1}_{\tau_C > T_C}\, \bar{D}\left(0, T_I\right)\right] &= \mathbb{E}\left[\mathbf{1}_{\tau_C > T_C} \cdot \mathbf{1}_{\tau_I > T_I}\, D\left(0, T_I\right)\right] \\ &= P\left(0, T_I\right) U\left(T_I, T_C\right) \,, \end{aligned} \tag{8}$$

where $T_C < T_I$. If we then denote with $\bar{D}\left(0, T_I\right)\left[t_C\right]$ the bank account conditional to $\tau_C = t_C$, it then follows that

$$\bar{D}\left(0, T_I\right)\left[t_C\right] = P\left(0, T_I\right) \cdot \begin{cases} \frac{\partial_C U\left(T_I, t_C\right)}{\partial_C U_C\left(t_C\right)} & \left(t_C < T_I\right) \\ \\ \frac{U\left(T_I, T_I\right)}{U_C\left(T_I\right)} & \left(t_C > T_I\right) \end{cases}$$

The case $t_C < T_I$ is determined by direct differentiation of (8), whereas the case $t_C > T_I$ is fixed by using the limiting case of (8) for $T_C = 0$, given by

$$\bar{\mathbb{E}}\left[\bar{D}(0,T_I)\right] = P(0,T_I)\,U_I(T_I) \ .$$

Note that, if the default events of the investor and the counterparty are uncorrelated, then $\bar{D}(0,T_I)[t_C]$ is independent of t_C and hence deterministic. On the other hand, in the general case, rates $\bar{r}(t)$ do depend on the default event of the counterparty and we have correlation between rates and relevant market factors. In particular, the pre–default rate $\bar{r}'(t)$ relevant in (7) is given by $-\partial_t \ln \bar{D}(0,t)[\infty]$ or by

$$\bar{r}' = r + \Lambda_I + \Lambda_C - \lambda_C \ .$$

Using this fact, together with

$$\begin{aligned} \bar{\Lambda}_I &= \bar{\lambda}_I = 0 \, , \\ \bar{\Lambda}_C &= \bar{\lambda}_C = \lambda_C \, , \end{aligned}$$

the basic pricing equation finally reads

$$-\frac{du}{dt} + (r + \Lambda_I + \Lambda_C)\,u = \lambda_C\,(k_C - v_X) - (r + \Lambda_I + \Lambda_C - \lambda_C - r_X)\,v_X \ . \quad (9)$$

2.2.2 Independent default events

Finally, we briefly consider the case of independent default events, with $U = U_I \cdot U_C$ and $\bar{U} = \bar{U}_I \cdot \bar{U}_C$. This case is rather trivial and we can work with a general R_I and $\bar{\lambda}_I$. We just spell out the relevant results. The internal rate $\bar{r}$ is deterministic and such that the funding rate is invariant, as in (1). Moreover, the basic value adjustment equation 7 simply reads

$$-\frac{du}{dt} + \left(\bar{r} + \bar{\lambda}_I + \lambda_C\right) u = \bar{\lambda}_I\,(k_I - v_X) + \lambda_C\,(k_C - v_X) - (\bar{r} - r_X)\,v_X \ .$$

The usual two important limiting cases are $\bar{\lambda}_I = \lambda_I$, $\bar{r} = r$, with value adjustment in line with external market consensus and determined by

$$-\frac{du}{dt} + (r + \lambda_I + \lambda_C)\,u = \lambda_I\,(k_I - v_X) + \lambda_C\,(k_C - v_X) - (r - r_X)\,v_X \ ,$$

and the pure–funding point of view $\bar{\lambda}_I = 0$, $\bar{r} = r_F$, leading to

$$-\frac{du}{dt} + (r_F + \lambda_C)\,u = \lambda_C\,(k_C - v_X) - (r_F - r_X)\,v_X \ .$$

This last expression, when $R_I = 0$ and $r_F = r + \lambda_I$, should be compared with (9), which represents the extension to the correlated case.

3 Conclusions

We have analyzed value adjustment from the point of view of derivative pricing in incomplete markets, and we have done so in the simplest cases, when randomness comes uniquely from the default times of the trading parties. It is quite clear that the present approach can be extended to the case of stochastic market factors, and little changes will occur in final formulae in the absence of correlations of the investor's default time with relevant market factors. More work is needed, though, to handle correlations in a more general framework. As seen in section 2.2.1, the case of credit–credit correlation already poses some complexity and does not have a simple and analytic solution in the general case. It is then advisable to focus not on the general case, but to choose a specific model of correlation which retains the relevant and desired qualitative features and, at the same time, leads to analytic tractability of the problem. We leave such analysis to future work.

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