CVA and FVA with liability-side pricing

Central to the funding valuation adjustment (FVA) debate is the law of one price. Wujiang Lou finds the fair funding rate for an uncollateralised derivative's fair value is the liability side's rate. He presents a liability-side derivatives pricing theory, and a new definition of credit valuation adjustment and FVA conforming to the law

Banks are moving ahead with funding valuation adjustment (FVA), although it is far from a settled issue among academic researchers, accountants, quants, traders and regulatory stakeholders. Indeed, the complexity surrounding FVA has only increased as discussions have deepened. KPMG (2013), for example, highlights nine propositions for understanding and implementing FVA. While discussion is dominated by the accounting definition of fair value, the International Swaps and Derivatives Association's (ISDA) documentation of derivatives settlement and collateralisation, investment theory and practice, and banks' treasury and derivatives desk operations, a central difficulty is the lack of consensus on what economic principles should apply when incorporating funding costs into bilateral derivatives pricing.

In the existing literature, a dealer's own default risk is hedged by buying back their own bonds or multiple classes of their own debt (Burgard & Kjaer 2011, 2013), and the funding of an uncollateralised derivative is carried out on the dealer side. The resulting fair price is dealer dependent and thus violates the law of one price. We introduce a new idea that the party on the liability side is economically indifferent to depositing a cash amount with the other party to hedge counterparty exposure and fund the derivative, so long as the deposit earns the market interest rate of the depositor's debt. A Black-Scholes-Merton partial differential equation is rigorously derived to establish that the uncollateralised option fair value is the expectation of the liability-side discounted option payoff. This article contributes a model to the FVA debate that takes funding costs into account, but abides by the law of one price.

The new model has these additional appealing characteristics: the choice of discounting curves is made intuitive and explicit; fair value does not allow arbitrage on a credit support annex (CSA) or a non-CSA; total counterparty risk adjustment links to bond yield curves, which is critical for economies without credit default swap (CDS) markets; and there is no need to specify unobservable recovery rates. Total counterparty risk adjustment is coherently decomposed into credit valuation adjustment (CVA), the default risk component, and FVA, the funding risk component, corresponding to the CDS-implied default risk and the liquidity or funding basis,[1] respectively. Coherent CVA and FVA are accounting-rule compliant, with no double counting, and can be implemented together.

Economics of an uncollateralised bilateral trade

Party B (a hypothetical bank) and party C (a client institution) enter into an option trade under the ISDA agreements without a CSA. Both parties have access to a liquid and exogenous corporate bond market, primary or secondary.

[1] The negative basis for bank intermediaries, for example, has been quite significant since the financial crisis, typically in the range of 30–50bp.

Let us start by conducting an economic thought experiment. When the derivative is a receivable to party C, C has a positive credit exposure to party B. C asks B to deposit an amount of cash by promising to pay interest at the market rate r_b of B's debt. Party B could raise the cash by issuing in the bond market at the same rate. From the perspective of B's balance sheet, the new liability offsets its deposit (on the asset side) so there is no net balance sheet impact. From a cashflow perspective, B receives r_b from C and passes it to bond investors. No new cashflow is generated. B is therefore economically neutral. From C's perspective, the derivative asset is now financed by B's deposit. No other form of financing is necessary.

If the derivative is a receivable to B, B could do the same: that is, to ask C to make a deposit and pay its market rate r_c. Bilateral counterparties can therefore finance each other, as stated below.

Proposition 1: market funding of uncollateralised derivatives. *Parties in an uncollateralised bilateral derivative trade implicitly grant each other a funding obligation at their respective debt market rates.*

Lacking a principal derivatives financing market to observe market levels of derivatives funding, proposition 1 establishes a fair funding cost for uncollateralised derivatives. When evaluating a derivative as an investment – a contingent claim issued by party C – agents will consider their investment costs, including funding cost. If an agent's funding cost is lower than C's, they will analyse the gap in their profit margin and negotiate to realise the fullest extent possible. If the agent's level is higher than C's, C will turn down this inferior bid. If no lower-cost agent can be found, C could sign up a CSA to effectuate its own funding cost. In the end, market equilibrium will strike the derivative's embedded funding cost at C's funding level. This is effectively a no-funding arbitrage condition.

Proposition 1 departs from the existing literature in that both counterparties can enter the funding arrangement. Burgard & Kjaer (2011), for example, arrange funding solely from the dealer's perspective.

If B defaults, C has two transactions with B: the derivative as a receivable and the deposit as a payable. Although the deposit is not part of the derivative's netting set, by virtue of the set-off provision under the ISDA Master Agreement, the right to set-off would apply to the final settlement amount. If the deposit amount matches the derivative close-out amount, the derivative contract and the deposit set off completely. The net balance is zero and there is no default settlement cashflow. C suffers no loss and is fully protected against B's default.

Economically, C sees that the deposit, if made dynamically, constitutes a counterparty risk hedging strategy. Reciprocally, the defaulting party, B, has no default settlement cashflow, neither windfall nor shortfall; there is nothing to hedge upon its own default. C's proposal of a deposit from B serves as an exposure hedge for party B as well as party C.

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Furthermore, if C defaults instead, B would surrender the deposit in lieu of the payment under derivative settlement. Again, there is no default settlement cashflow.

We conclude the thought experiment with the following proposition.

PROPOSITION 2: mutual counterparty exposure hedge.
Parties in an uncollateralised bilateral derivative trade can hedge credit exposure mutually by making a cash deposit of the same amount as the derivative's close-out amount.

The deposit is effectively voluntary cash collateral. A critical difference from collateralisation under bilateral CSA exists, however, in that collateralisation under CSA is contractual and part of the derivative's netting set – that is, an integral part of the trade – while the deposit is non-contractual and is not part of the netting set but is economically justified and implied. The mitigation effect of the deposit goes by set-off rather than netting. This leads to different rates being applied to cash: a deposit rate (eg, a federal fund rate in US dollars) with a CSA and a debt rate without a CSA.

Proposition 2 removes the need to hedge a dealer's own default risk, which is often a controversial practice, whether CDSs or own bonds are employed, and it is thus distinct from the existing literature. Burgard & Kjaer (2013), for instance, show that own default risk cannot be hedged unless subordinated debt is enlisted in addition to senior debt.

Combining propositions 1 and 2, the cash deposit serves the purpose of both hedging counterparty credit exposure and financing the derivative. The total economic cost of hedging and funding is the market rate of the depositor's debt. These are the fundamental economics of an uncollateralised bilateral derivative trade in the absence of funding arbitrage.

Extended Black-Scholes-Merton model for uncollateralised trades

Bank B dynamically hedges the option with Δ_t shares of underlying stock. Let S_t denote the stock price, V_t the option fair price and π_t the wealth of the hedged option economy. M_t is the cash (or bank) account balance, which earns the risk-free rate r. N_t is B's debt account, which issues short-term rolled debt at par rate $r_N(t)$, $r_N(t) \geqslant r(t)$. The account could be secured by the remaining asset of the economy and may have recourse to the bank. It could also be thought of as a treasury funding account of a derivatives desk, where the treasury charges interest up to the firm's senior unsecured debt interest rate r_{b}.

The stock hedge is financed in repo or security lending markets. Let L^{s}_t be the cash amount posted to the stock lender, who in turn pays rebate interest at the rate of r_{s}. $r - r_{\mathrm{s}}$ is the stock borrowing cost. The borrower posts the stock short sale proceeds with the lender and may have to post additional cash with a haircut $h \geqslant 0$, so that:

$$L^{\mathrm{s}}_t = (1+h)\Delta_t S_t$$

To capture the deposit made under propositions 1 and 2, a voluntary collateral account is added to the economy, with its balance denoted by L_t. Write $L_t = L^+_t - L^-_t$, where L^+_t is the cash amount deposited or posted by party C to B, who pays C's cash rate $r_{\mathrm{c}}(t)$, and L^-_t is the amount posted by B to C, earning B's cash rate $r_{\mathrm{b}}(t)$.

Putting everything together, the wealth equation is:

$$\pi_t = M_t + (1-\Gamma_t)(V_t - L_t - N_t - \Delta_t S_t + L^{\mathrm{s}}_t)$$

where $1-\Gamma$ is the parties' joint survival indicator.

The self-financing equation, including default settlement, is written as:

$$\begin{aligned}
\mathrm{d}M = rM\,\mathrm{d}t &+ (1-\Gamma)[\mathrm{d}\Delta(S+\mathrm{d}S) - \Delta S q\,\mathrm{d}t + \mathrm{d}N \\
&\qquad - r_N N\,\mathrm{d}t + \mathrm{d}L - r_{\mathrm{c}} L^+\,\mathrm{d}t \\
&\qquad + r_{\mathrm{b}} L^-\,\mathrm{d}t - \mathrm{d}L^{\mathrm{s}} + r_{\mathrm{s}} L^{\mathrm{s}}\,\mathrm{d}t] \\
&+ \mathrm{d}\Gamma[-\Delta S + (V^+ - L^+)(R_{\mathrm{c}} + X(1-R_{\mathrm{c}})) \\
&\qquad - (N + V^- - L^-)(1 - X(1-R_{\mathrm{b}}))]
\end{aligned}$$

where R_{b} and R_{c} are B and C's recovery rates, q is the dividend yield and X is a random marker of the defaulting party; $X = 1$ if party B defaults. We have dropped all t subscripts and applied the pre-default market price as the close-out amount.

Enforcing a full exposure hedge under proposition 2, $L_t = V_t$, the differential wealth equation becomes:

$$\begin{aligned}
\mathrm{d}\pi - r\pi\,\mathrm{d}t = (1-\Gamma)[&\mathrm{d}V - rV\,\mathrm{d}t - \Delta(\mathrm{d}S - (r-q)S\,\mathrm{d}t) \\
&- (r_N - r)N\,\mathrm{d}t - (r_{\mathrm{c}} - r)V^+\,\mathrm{d}t \\
&+ (r_{\mathrm{b}} - r)V^-\,\mathrm{d}t + (r_{\mathrm{s}} - r)L^{\mathrm{s}}\,\mathrm{d}t] \\
&+ \mathrm{d}\Gamma X(1-R_{\mathrm{b}})N
\end{aligned}$$

Setting $\pi_t = 0$ leads to $N_t = h\Delta_t S_t$, showing the economy only needs to finance the residual stock-lending margin.

Assuming zero haircut, $N_t = 0$, the jump term disappears from the portfolio equation. The option is thus fully replicated. Now, by applying Ito's lemma, delta hedging ($\Delta = \partial V/\partial S$) under the usual geometric Brownian motion stock price of volatility σ, and setting the $\mathrm{d}t$ term to zero, we obtain the following partial differential equation (PDE):

$$\frac{\partial V}{\partial t} + (r_{\mathrm{s}} - q)S\frac{\partial V}{\partial S} + \tfrac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + r_{\mathrm{b}} V^- - r_{\mathrm{c}} V^+ = 0$$

For a pure derivative asset or receivable, $V^- = 0$, the r_{b} term drops out and V is governed by r_{c}, the bond curve of party C, who is on the liability side. A derivative asset's fair value therefore does not depend on the buyer's (party B's) funding cost.

For a pure derivative liability, such as a short call option, B is on the liability side, party C's funding curve drops out and V is governed by its own curve r_{b}. This is nothing unusual, as a liability such as debt is priced off its issuer's, rather than its buyer's, credit and liquidity. Whether party B is holding a receivable or a payable, the bilateral option is always priced on its liability side.

The PDE has the counterparty's funding rates accompanying the positive and negative parts of the derivative fair value, replacing the usual rV term in the Black-Scholes-Merton (BSM) equation. An intuitive explanation is that, with non-defaultable or fully collateralised counterparties, derivative fair value accrues at a risk-free rate; with defaultable and uncollateralised counterparties, the positive part (the receivable) will accrue at the counterparty's debt interest rate, while the negative part will accrue at its own rate.

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In fact, following Black & Scholes' (1973) original equilibrium approach, we write the hedge portfolio and finance equations as follows:

$$\begin{aligned}\pi_t &= V_t - \Delta_t S_t \\ \mathrm{d}\pi_t &= \mathrm{d}V_t - \Delta_t\,\mathrm{d}S_t \\ &= (r_c V^+ - r_b V^-)\,\mathrm{d}t - (r_s - q)\Delta_t S_t\,\mathrm{d}t\end{aligned}$$

The final line consists of a stock-financing cost at the repo rate and derivative financing at rates dictated by the liability side. Applying Ito's lemma, delta hedging and setting the $\mathrm{d}t$ term to zero leads to the same PDE.

In the classic BSM theory, a delta-hedged option performs like a risk-free portfolio. With defaultable counterparties, the same portfolio becomes a credit-risky debt of the party on the liability side, albeit with variable exposure. A plain corporate debt of issuer C can be considered as a special, but nonetheless trivial derivative between issuer C and its buyer. A structured note linked to equity, commodity or foreign exchange rates can be seen as a hybrid debt, which should be priced at the issuer's credit rate once the underlying risk is hedged. This is intuitive when the embedded derivatives are deep out-of-the-money and the notes are, economically, a plain debt of the issuing bank.

Let r_e denote the effective discount rate:

$$r_e(t) = r_b I(V_t < 0) + r_c I(V_t \geqslant 0)$$

where $I(\cdot)$ is the indicator function. Rewrite the PDE and apply the Feynman-Kac theorem to obtain the discounted payoff expectation formula for an option with payoff function $H(T)$, as follows.

PROPOSITION 3: liability-side pricing principle.
In the absence of funding arbitrage, an uncollateralised derivative exposure is priced at the market interest rate of the liability side's debt. In particular, an option's no-arbitrage price is the expected risky discount of the option payoff under the risk-neutral measure, Q:

$$V(t) = E_t^Q\left[\exp\left(-\int_t^T r_e\,\mathrm{d}u\right)H(T)\right]$$

This is the same as the risk-neutral pricing formula, except that the risk-free rate is replaced by the effective discount rate. The equivalent martingale measure for the underlying stock price remains the same. In general, the risky discount factor,

$$D(t) = \exp\left(-\int_0^t r_e\,\mathrm{d}u\right)$$

depends on the local fair value, so the formula is recursive.

■ **PDE with partial collateralisation.** Let W_t be the collateral amount posted under a weak CSA that pays interest at r_w. A general PDE can be derived:

$$\begin{aligned}\frac{\partial V}{\partial t} + (r_s - q)S\frac{\partial V}{\partial S} + \tfrac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial^2 S} - rV + f_b(V-W)^- \\ - f_c(V-W)^+ - (r_W - r)W - f_N N = 0\end{aligned}$$

where f_b and f_c are rates applicable to an unsecured portion of the fair value. The last four terms are adjustments to the BSM equation, including the benefit on own default and funding, the cost on the counterparty's default and funding, the extra cost paid to get cash collateral, and a charge from the treasury if N is the treasury funding amount.

There have been a number of efforts to extend the BSM PDE to study the impact of collateralisation and funding cost on derivatives pricing. Piterbarg (2010) considers an extra collateral return as a special case by setting:

$$f_N = f_b = f_c = 0$$

In the last term, both the funding amount N and the applicable rate f_N could vary, owing to different funding and hedging arrangements. Burgard & Kjaer (2011) have:

$$\begin{aligned}N &= (V - W)^+ \\ f_N &= s_F = r_b - r \\ f_b &= \lambda_b(1 - R_b) \\ f_c &= \lambda_c(1 - R_c)\end{aligned}$$

where λ_b and λ_c are B and C's zero-recovery, zero-coupon bond-yield spreads in the case of a risky close-out without a derivatives repo market. The fair value of the derivative asset then depends on buyers' funding costs.

Applying propositions 1 and 2 for the uncollateralised exposure $V - W$ results in $f_b = r_b - r$ and $f_c = r_c - r$, and, in particular, $N = 0$. The spreads f_b and f_c agree with Burgard & Kjaer (2011) if there is no funding basis such that λ_b and λ_c are the default intensities and:

$$\begin{aligned}r_b - r &= \lambda_b(1 - R_b) \\ r_c - r &= \lambda_c(1 - R_c)\end{aligned}$$

With a non-zero funding basis, however, $\lambda_b(1 - R_b)$ cannot be $r_b - r$, because the zero-recovery CDS spread can be scaled by $1 - R$ to arrive at the CDS spread of the fixed recovery rate CDS. It is, however, difficult to establish that a zero-recovery bond's basis is subject to the same loss given default scale when converted to a fixed recovery bond's basis. The fact that our result has no bearing on the recovery rate eliminates the need to assume and justify various non-market observable recovery rate values.

Obviously, $W = 0$ corresponds to non-CSA and $W = V$ corresponds to full CSA, where the PDE collapses to Piterbarg's (2010) PDE, and further to the BSM equation if $r_w = r$.

Fair value adjustments

As the fair value V derived from the extended PDE fully incorporates counterparty credit risk and derivative funding cost, the total counterparty risk adjustment (CRA) is trivially the difference between V and the (counterparty) risk-free value V^*. Let $U = \mathrm{CRA} = V^* - V$. Subtracting the BSM equation for V^* from the extended PDE leads to:

$$\frac{\partial U}{\partial t} + (r_s - q)S\frac{\partial U}{\partial S} + \tfrac{1}{2}\sigma^2 S^2\frac{\partial^2 U}{\partial S^2} - r_e U + (r_e - r)V^* = 0$$

Noting that $U_T = 0$, applying the Feynman-Kac theorem immediately leads to the CRA formula:

$$U = E_t\left[\int_t^T (r_e - r)V^*(s)\exp\left(-\int_t^s r_e\,\mathrm{d}u\right)\mathrm{d}s\right]$$

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Intuitively, the total adjustment made to the risk-free price is the sum of the liability-side discounted excess return $(r_{\mathrm{e}} - r)$ on a notional amount of V^*.

CRA links to the parties' credit spreads, which can be decomposed into a default risk component and a liquidity or funding basis component, reflecting other funding factors such as market liquidity, tax and accounting treatment. Most of the world's economies do not have CDS markets to trade the default risk component. In advanced economies, where there are developed (but shrinking) CDS markets, CRA can be further decomposed into a credit risk component, CVA, and a funding risk component, FVA, in parallel with the spread decomposition.

To carry out the decomposition, we define a synthetic funding curve with its short rate denoted by $r^{\sim}$, implied from a firm's CDS curve. It has a non-negative spread over the risk-free curve; that is, $r^{\sim} - r \geqslant 0$. Under the zero-recovery assumption, this corresponds to $r^{\sim} = r + \lambda$, where λ is the default intensity.

The risky price $V^{\sim}$ is solved from the PDE or computed from the expectation formula, when each counterparty's funding curve is assumed to be at its synthetic curve. This allows us to separate the credit component from the financing cost, so that it can be managed in the CDS market.

As an example, for a T maturity, zero-coupon, zero-recovery bond issued by C, assuming flat and deterministic continuously compounding rates r_{c} and r, and default intensity λ_{c}, the fair price, the risk-free price and the risky price are given by:

$$\begin{aligned} V_t &= \mathrm{e}^{-r_{\mathrm{c}}(T-t)} \\ V_t^* &= \mathrm{e}^{-r(T-t)} \\ V_t^{\sim} &= \mathrm{e}^{-(r+\lambda_{\mathrm{c}})(T-t)} \end{aligned}$$

For the buyers of C's bond, $V^* - V$ agrees with U, calculated from the CRA formula with $r_{\mathrm{e}} = r_{\mathrm{c}}$. CVA is $V^* - V^{\sim}$, capturing the default risk. The difference $V^{\sim} - V$ is an adjustment attributed to the funding basis $r_{\mathrm{c}} - r - \lambda_{\mathrm{c}}$. In parallel to CVA, we define credit funding adjustment as CFA $= V^{\sim} - V$. Now CRA decomposes into CVA and CFA; that is:

$$\mathrm{CRA} = V^* - V = \mathrm{CVA} + \mathrm{CFA}$$

CVA and CFA of a pure receivable. Here, $V \geqslant 0$, the r_{b} term drops out from the PDE, $r_{\mathrm{e}} = r_{\mathrm{c}}$, and B's own funding curve therefore has no impact. CVA is defined as the difference between the risk-free price and the synthetic price; that is:

$$\mathrm{CVA} = V^* - V^{\sim}$$

CFA is defined as the difference between the synthetic price and the cash price of the derivative, CFA $= V^{\sim} - V$. Or it can be computed from CRA by deducting CVA, CFA $= U - \mathrm{CVA}$.

Here, CVA is positive. CFA is assumed positive and therefore it is understood as a cost. Morini & Prampolini (2011) make the point that the FVA definition should allow the recovery of bond pricing; that is, $V^* - \mathrm{CVA} - \mathrm{CFA}$ should be the same as V, which is satisfied by design.

Now let us consider a fully collateralised but otherwise identical trade between the same counterparties under CSA. Party C would post cash collateral under the CSA in the amount of risk-free option price V^*, earning the risk-free rate. One question is how C comes up with the cash amount. If C has to raise the money in the debt market, then it incurs a funding cost at its own debt rate, r_{c}. The funding interest of $(r_{\mathrm{c}} - r)V^*\,\mathrm{d}t$ has to be discounted at C's curve, as the payment is its own liability. This is exactly what the CRA formula stands for. With this funding cost considered, party C's all-in economics of the trade agree with the uncollateralised trade fair value. In fact, we have just proven the following lemma.

LEMMA. *Under the liability-side pricing principle, a bilaterally cleared derivative does not admit arbitrage on collateral support choices.*

DVA and DFA of a pure payable. Here, $V \leqslant 0$ and $r_{\mathrm{e}} = r_{\mathrm{b}}$, so counterparty C's funding curve has no impact, and there is no CVA or CFA. Debit valuation adjustment (DVA) is defined as the difference between the risky price (from a party's own synthetic funding curve) and the risk-free price, DVA $= |V^*| - |V^{\sim}|$.

Debit funding adjustment (DFA) is the difference between the cash price and the synthetic price; that is:

$$\mathrm{DFA} = V - V^{\sim} = |V^{\sim}| - |V|$$

or:

$$\mathrm{DFA} = -\mathrm{DVA} - U$$

DVA and DFA are benefits:

$$V = V^* + \mathrm{DVA} + \mathrm{DFA}$$

By flipping the PDE with a negative sign, one can easily see that a party's CVA (CFA) is the other party's DVA (DFA).

Coherent CVA and FVA. In general, a derivative could be a switcher; that is, neither a pure payable nor a pure receivable during its life cycle. It must be split into an asset part and a liability part to be priced, and if CVA and DVA coexist, so will CFA and DFA.

To strictly attribute CVA to counterparty default risk, leaving DVA to its own default risk, we apply party C's synthetic curve while keeping its own curve at the risk-free curve to get a new price. The difference from V^* becomes the CVA. DVA, CFA and DFA can also be obtained with an incremental curve-shift scheme like this.

Write:

$$r_{\mathrm{e}}(f_{\mathrm{b}}, f_{\mathrm{c}}) = f_{\mathrm{c}} I(V \geqslant 0) + f_{\mathrm{b}} I(V < 0)$$

where f_{b} and f_{c} represent the curves used on behalf of parties B and C, respectively. From this we gain the following formula, which guarantees $V = V^* - \mathrm{CVA} + \mathrm{DVA} - \mathrm{CFA} + \mathrm{DFA}$:

$$\begin{aligned} \mathrm{CVA} = {} & E_t^Q\biggl[\exp\biggl(-\int_t^T r_{\mathrm{e}}(r, r)\,\mathrm{d}u\biggr) H(T)\biggr] \\ & - E_t^Q\biggl[\exp\biggl(-\int_t^T r_{\mathrm{e}}(r, r_{\mathrm{c}}^{\sim})\,\mathrm{d}u\biggr) H(T)\biggr] \\ \mathrm{DVA} = {} & E_t^Q\biggl[\exp\biggl(-\int_t^T r_{\mathrm{e}}(r_{\mathrm{b}}^{\sim}, r_{\mathrm{c}}^{\sim})\,\mathrm{d}u\biggr) H(T)\biggr] \\ & - E_t^Q\biggl[\exp\biggl(-\int_t^T r_{\mathrm{e}}(r, r_{\mathrm{c}}^{\sim})\,\mathrm{d}u\biggr) H(T)\biggr] \end{aligned}$$

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$$
\begin{aligned}
\mathrm{CFA} = {} & E_t^Q\left[\exp\left(-\int_t^T r_{\mathrm{e}}(r_{\mathrm{b}}^{\sim}, r_{\mathrm{c}}^{\sim})\,\mathrm{d}u\right)H(T)\right] \\
& - E_t^Q\left[\exp\left(-\int_t^T r_{\mathrm{e}}(r_{\mathrm{b}}^{\sim}, r_{\mathrm{c}})\,\mathrm{d}u\right)H(T)\right] \\
\mathrm{DFA} = {} & E_t^Q\left[\exp\left(-\int_t^T r_{\mathrm{e}}(r_{\mathrm{b}}, r_{\mathrm{c}})\,\mathrm{d}u\right)H(T)\right] \\
& - E_t^Q\left[\exp\left(-\int_t^T r_{\mathrm{e}}(r_{\mathrm{b}}^{\sim}, r_{\mathrm{c}})\,\mathrm{d}u\right)H(T)\right]
\end{aligned}
$$

Economically, hedging of counterparty exposure with bonds is fully funded, but buying CDS protection is unfunded and relies on the performance of the protection seller at the time of the obligor default. CVA is therefore the unfunded adjustment, while the sum of CVA and FVA becomes the funded adjustment. In this sense, CVA and FVA defined as such are coherent. In order to distinguish these terms from traditional definitions, we call the above coherent CVA and FVA.

This type of decomposition scheme is not unique, but in practice it should not be a concern. This is because, for valuation purposes, the total valuation matters, which is invariant; for hedging CVA purposes, once linearised for delta hedging, the order of decomposition does not matter.

Getting explicit and exact formulas for CVA/FVA is generally difficult, except in special cases. For a receivable with zero recovery rate, for instance, we have

$$
\begin{aligned}
r_{\mathrm{c}}^{\sim} - r &= \lambda_{\mathrm{c}} \\
\eta_{\mathrm{c}} &= r_{\mathrm{c}} - r - \lambda_{\mathrm{c}}
\end{aligned}
$$

where η_{c} is C's funding basis, and coherent CVA and FVA are listed below:

$$
\begin{aligned}
\mathrm{CVA} &= E_t\left[\int_t^T V^*(s)\exp\left(-\int_t^s r\,\mathrm{d}u\right)\exp\left(-\int_t^s \lambda_{\mathrm{c}}\,\mathrm{d}u\right)\lambda_{\mathrm{c}}\,\mathrm{d}s\right] \\
\mathrm{FVA} &= E_t\left[\int_t^T (\lambda_{\mathrm{c}} + \eta_{\mathrm{c}})V^*(s)\exp\left(-\int_t^s (r + \lambda_{\mathrm{c}} + \eta_{\mathrm{c}})\,\mathrm{d}u\right)\mathrm{d}s\right] \\
&\quad - \mathrm{CVA}
\end{aligned}
$$

This CVA formula has the usual look of a risk-free discounted, default-probability-weighted derivative exposure V^*, which can be interpreted as the price to buy protection on the derivative's counterparty exposure, and it is in fact the standard definition (Tang & Li 2007). A popular and seemingly intuitive expression for FVA then follows:

$$
E_t\left[\int_t^T V^*(s)\exp\left(-\int_t^s r\,\mathrm{d}u\right)\exp\left(-\int_t^s \lambda_{\mathrm{c}}\,\mathrm{d}u\right)\eta_{\mathrm{c}}\,\mathrm{d}s\right]
$$

which can be interpreted as the present value of funding a derivative exposure V^*, assuming party C does not default (thus survival probability), at the funding basis rate. The problem is that FVA so defined does not add CVA back to the total valuation adjustment, which is precise and indisputable for a simple receivable, such as a bond.

The coherent CVA and FVA method offers many advantages, as summarised below.

■ **Accounting rule compliant.** Even for a simple derivative receivable, practitioners are divided as to whether to apply banks' own funding rates or a proxy, such as an average of selected banks. Both are unsatisfactory from the International Financial Reporting Standards (IFRS) perspective, as the former is entity specific and the latter is not from a principal market. IFRS 13 Fair Value Measurement adopts an exit price standard and requires the use of 'non-entity'-specific market information. Liability-side pricing uses the debt market (a principal market) observed counterparty's (not a bank's) debt rate and is therefore IFRS compliant.

■ **Law of one-price conforming.** Calculating the FVA of a derivative asset using a bank's own funding rate results in different banks seeing different prices and as such violates the law of one price. With liability-side pricing, an asset is priced at its issuer's, not buyers', credit and commands only one price. For a derivative liability, fair value depends on the issuing party's own funding curve, in the same way a corporate bond's price depends on its issuer's credit. The assumption of a pure liability therefore demands different prices for each firm that assumes the liability. Of course, when the derivative is a switcher between asset and liability, its price may depend on both parties' funding curves. The law of one price, however, does not apply to this type of asset, but the principle is preserved.

■ **No overlap.** The overlapping issue is twofold. For an asset, charging FVA on the derivative mark-to-market ($V > 0$; that is, $N = V$ in our notation) implies funding the asset at its own rate, but the same funding would incur a DVA ('DVA2' in Hull & White (2012)) that would offset the FVA when there is no funding basis. On the other hand, for a liability $V < 0$, there is a DVA ('DVA1' in Hull & White (2012)). When a symmetric benefit is calculated on the same notional amount ($|V|$) and at own funding rate r_{b}, the benefit then overlaps with DVA1. Hull & White (2012) clarify that another DVA2 reduction would offset this benefit, and with that there is no double counting. Coherent FVA is attributed to the funding basis alone while DVA is attributed to default risk, so, by design, these types of overlaps are nonexistent.

■ **Link to discounting clarified.** The liability-side pricing formula formally establishes a direct and unequivocal link between the fair value and counterparties' discounting curves. For a derivative of only incoming net cashflow, the discounting curve is intuitively the counterparty's yield curve; for net outflows, the discounting curve is its own curve. For a swap-like switcher, the discounting curve is switching. This link provides support for popular cashflow discounting-based CVA calculation.

■ **Libor discounting of uncollateralised trades.** When the overnight indexed swap rate (OIS) was proposed as the discount curve for collateralised trades, most thought Libor could retain its role for uncollateralised trades. This can be partially justified using the liability-side pricing formula, if we assume the funding basis of the counterparties is the same as the basis of Libor to OIS. In other words, a near-Libor funding bank may consider FVA as a secondary matter for Libor-discounted trades that have calculated CVA/DVA.

Numerical results

Under a simple case, in which the dealer buys an option from party C, assuming all rates are flat, the CVA from Burgard & Kjaer (2011),

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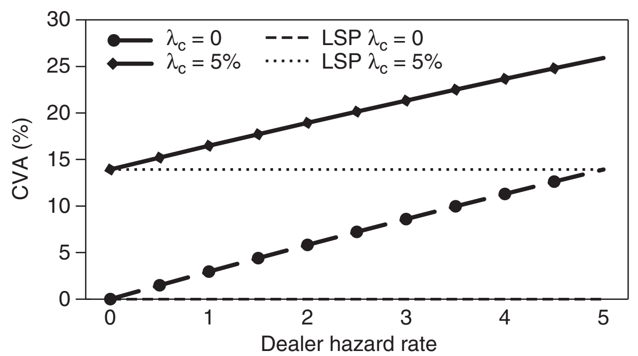
1 Comparisons of LSP CVA ratio to Burgard & Kjaer CVA ratio versus dealer's hazard rate (λ_b) under two counterparty hazard rates $\lambda_c = 0$ and $\lambda_c = 5\%$



denoted as $\mathrm{CVA_{BK}}$, is:

$$\mathrm{CVA_{BK}} = (1 - \exp(-(\lambda_c(1 - R_c) + \lambda_B(1 - R_B))^*(T - t)))V^*(t)$$

To highlight the difference between Burgard & Kjaer's CVA and coherent CVA, assuming zero basis, our CVA is given by:

$$\mathrm{CVA_{LSP}} = (1 - \exp(-\lambda_c(1 - R_c)^*(T - t)))V^*(t)$$

where LSP denotes liability-side pricing. Obviously, $\mathrm{CVA_{BK}}$ depends on the hazard rate of the dealer, λ_b, but $\mathrm{CVA_{LSP}}$ does not. The ratios of CVA to the risk-free option price are plotted in figure 1, with $T - t =$ 5 years and both recovery rates at 40%.

The extended BSM PDE under the liability-side pricing principle can be solved by developing a finite-difference scheme, in which an iterative procedure is necessary to determine the asset/liability boundary (Lou, 2015).

To demonstrate, we price a shifted stock forward, that is, an option trade that consists of a long 45 strike European call and a short 55 strike put of the same one-year expiry, with the spot price at 50. This trade has a positive payoff when stock price S_T is above 50 and is negative otherwise, so it involves true bilateral valuation adjustments.

At 50% volatility, a 5% risk-free rate and a 50 basis points repo or stock borrowing cost, the risk-free price is 1.6009. If we set bank B's zero recovery CDS at a flat 50bp and a liquidity basis of 20bp, and set C's CDS at a flat 300bp and a liquidity basis of 50bp, the fair price of the trade is 1.3577. The total valuation adjustment of 0.2432 is decomposed into CVA = 0.2501, DVA = 0.0342, CFA = 0.0410 and DFA = 0.0136.

Conclusion

Starting from an economic analysis of uncollateralised derivatives, we found the liability-side party can fund and credit hedge the other party by making a deposit that earns its debt interest rate. An option remains replicable, and the delta-hedged trade economically resembles an issued variable-funding note of the liability side and shall be priced at the market rate of the issuer's debt to avoid funding arbitrage. The newly derived PDE applies parties' funding curves separately to the derivative receivable part and payable part. A liability-side pricing formula is obtained that extends risk-neutral pricing theory and explicitly establishes the link between counterparties' discount curves and the fair value.

The extended PDE serves to define the total counterparty risk adjustment precisely by discounting the product of the risk-free price and the credit spread at the local liability curve. The adjustment can be broken coherently into CVA, a default risk component corresponding to CDS, and FVA, a component due to funding or liquidity basis. Specifically for FVA, we define a cost-credit funding adjustment and a benefit-debit funding adjustment, in parallel to CVA and DVA. This resolves a number of outstanding FVA debate issues, such as accounting fair value interpretation, double counting, the law of one price, the choice of discounting curves and the proper use of Libor discounting, thereby providing a platform for future research to extend the liability-side pricing to uncollateralised swaps and to implement Monte Carlo simulation to efficiently compute coherent CVA and FVA for large derivatives portfolios or netting sets. **R**

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