Yes, FVA is a Cost for Derivatives Desks

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A Note on "Is FVA a Cost for Derivatives Desks?" by Prof. Hull and Prof. White

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1 Introduction

A recent paper by Professors Hull and White [13] argues that the Funding Value Adjustment is not a cost for the derivative desks. They suggest that the correct discounting rate is always the risk-free rate, thus excluding any possible accounting for the actual funding rate paid by the bank.

The paper is short and it can be considered as a collection of a few statements. The *brevitas* and the authoritativeness of the statements make it resemble rather an ancient sacred sapience book than an academic work proving the thesis presented therein; besides their obscurity offers the reader a good chance to deeply think about serious issues affecting the activity in present financial markets.

Given that the authors are two esteemed Professors, we read carefully the paper and we pursued an accurate exegesis of the text. Their main statements and our thoughts on each one are shown in what follows, the result of our efforts (since we are strongly lacking the sense of suspense) is in the title of this paper.

2 Risk-Neutral and Discount Rate

Statement 1.

> *"It is important to avoid confusion about why the risk-free rate is used for discounting when derivatives are valued. It might be argued that the use of a risk-free discount rate indicates the valuation is only appropriate when the bank can fund the derivative at the risk-free rate. This is not*

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true. We discount at the risk-free rate because this is required by the risk-neutral valuation principle. Risk-neutral valuation is an artificial - but fantastically useful - tool that gives the correct economic valuation for a derivative, taking into account all its market risks."

We believe there is a (mainly conceptual) confusion in the statement and also a sort of inversion of the causal order of facts. Actually we discount at the risk-free rate because it is required by the risk-neutrality argument, provided we are operating in a given economy where credit (and hence funding) spreads do not exist. So the "economic valuation" (whatever it can possibly mean, and the expression will be more thoroughly specified below) derived by using only risk-free rate is correct only for a contract dealt in such economy.

In more detail, risk-neutral argument shows that, given a contract granting the claim to a certain pay-off, if an agent is able to achieve the same pay-off by a (possibly dynamic) replication strategy,[1] then at the inception the cost to enter into the contract is the same as the cost to set up the strategy.

Now, it is very true that in the "1970s, Fischer Black, Myron Scholes and Robert Merton (BSM) showed that, over any short period of time, an investment in an option could be replicated with a portfolio of stock and risk-free debt", so that the "economic" value of the option is that of the replication strategy, but it is also true that they consider a very specific economic environment, which for simplicity we name B&S economy, characterized by the following features:

1. there exists a bond that earns a risk-less instantaneous (possibly stochastic) interest rate at which agents can borrow and lend freely (basically default risk is excluded and agents can buy the bond to invest/lend money or they can short it to borrow money receiving or paying the same risk-free interest rate in both cases);

2. the asset underlying the contract (say, a stock) can be sold short with the the proceeds going to the seller;

3. there are no transactions costs or taxes;

4. agents cannot influence any price or rate, which is to say that the market is infinitely liquid.

Under these assumptions it is easy to show that all risk-less assets must earn the risk-less rate to prevent arbitrage.

As soon as one or more of these features are not met in the market, the replication is either impossible or its cost is different than that predicted in the B&S economy. The risk-free discounting is a result of the risk-neutral approach, but this outcome is possible only within the B&S economy, so it is fully justified only there. To put it in the Cox and Ross's [9] words (pg. 152, our integration or suppression in parenthesis):

[1]The strategy has also to fulfill the self-financing condition: no investment is required other than the initial one needed to start up the strategy.

"The fact that we could use a hedging argument to derive [...] [the fundamental option valuation equation] and the argument that [...] [the price of the derivative contract] exists uniquely means that given [the price of the underlying asset] S and t the value of the option, P, does not depend directly on the structure of investors preferences. Investors preferences and demand conditions in general enter the valuation problem only in so far as they determine the equilibrium parameter values. No matter what preferences are, as long as they determine the same relevant parameter values, they will also value the option identically. In the Black and Scholes case, for example, [the pricing PDE] does not depend on μ [the drift of S] and the only relevant parameters for the pricing problem are r [the constant instantaneous interest rate] and σ. To solve [...] [the fundamental option valuation equation], then, we need only find the equilibrium solution for P in some world where preferences are given and consistent with the specified parameter values; the solution obtained will then be preference free."

The risk-neutral argument is a consequence of the replication strategy. The replication of a derivative contract P with a portfolio comprising the underlying S and bonds B means that:

$$P_t = \alpha_t S_t + \beta_t B_t$$

The quantities α_t and β_t are chose so as to make the portfolio value and evolution equal to P:

$$\alpha_t = \Delta_t$$

$$\beta_t = \frac{P_t - \Delta_t S_t}{B_t}$$

so that:

$$dP_t = \Delta_t dS_t + (P_t - \Delta_t S_t)\widetilde{r}_t dt$$

where $\widetilde{r}_t = r_t \mathbf{1}_{\{\beta>0\}} + r_t^F \mathbf{1}_{\{\beta<0\}}$ and $\mathbf{1}_{\{\}}$ is the indicator function equal to 1 when the condition at the subscript is verified. In a B&S economy, risk-free rate is the only rate, so that $r^F = r$ and the derivative contract satisfies the standard PDE with only the risk-free rate appearing. In an economy where the funding rate r^F the bank has to pay when borrowing money is higher than r, if the quantity of bonds β is negative (implying borrowing money) the funding rate r^F applies, otherwise the purchase of bonds is remunerated at the risk-free rate. The resulting PDE may contain now the risk-free rate or the funding rate, or both[2] depending on the quantity β. For an agent operating in such economy, the correct evaluation of the contract is performed by solving this PDE, not the one resulting in a B&S world.

[2]Or even more rates, as when we include the possibility to repo the underlying asset: in this case either the risk-free or the funding rate can appear as discounting rates and the drift of the asset is the repo rate.

Risk-free discounting can be applied when the bank operates in a B&S economy, which is tantamount saying that the bank does not pay any funding spread over the risk-free rate and all the other features of the B&S economy are also prevailing in the market. If the bank has to pay a funding spread, this does not mean that the replication argument cannot be applied, provided that the other conditions are met. It is well known and investigated since early 80s (see *e.g.*: Cox and Rubenstein [10] and Bergman [3]) that the presence of a difference between lending and borrowing rates is not an impediment to the replication of the option, but for European options it only makes the value depending on the side (buy/sell) and the type (call/put) of the contract; closed-form valuation formulae are still available.[3] The implementation of the replication strategy implies that the drift of the underlying asset and the discounting rate can differ from the risk-free rate, the value they have in the B&S economy (*e.g.*: replication of a long call option has a cost derived by using the B&S formula with the drift of the asset and the discounting rate set equal to the borrowing rate).

We would tend to define still "risk-neutral" the option prices obtained in an economy with differential interest rates, since the price is not depending on any preference parameter,[4] yet the price is not that of B&S since the cost of the strategy is different. The price of the option will be different when the bank has to replicate a long or a short position and no more uniquely determined for both cases, but there is nothing preventing the set-up of the replication strategy.

So risk-neutrality is a powerful tool, as Professors Hull and White say, and it can work also in some circumstances when the B&S economy is not totally equal the actual economy, but the risk-free discounting is not a necessary implication of the approach and a discounting by a lending or a borrowing rate, in a way appropriate and consistent with the replication strategy, is the right choice to correctly evaluate the derivative contract. Discounting always and only by the risk-free rate does not produce correct results simply because it does not consider the actual costs related to the replication strategy, which the risk-neutrality argument hinges upon.

On the other hand we have elsewhere[5] shown that the risk-neutral approach can be naturally applied by discounting by the one risk-free rate all the components of the derivative contract's value, namely: the pay-off, the expected losses (*e.g.*: coun-

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[3]For more exotic options, a closed form formula may not be available, but the replication argument still works. See Castagna [7] for a discussion.

[4]This is not strictly true since the funding spread is an effect of the default risk of the bank and it depends on the probability of default, which ultimately is subjective.

[5]See Castagna [5].

terparty credit losses) and expected costs (*e.g.*: funding costs). Although the result achieved in this case would be the same as the one obtained by using an effective discounting rate (including all credit and funding effects) we believe that the decomposition of the value in all its building blocks is more useful for the management of risks within a dealing room.[6] In this case we agree with Professors Hull and White on the fact that the discounting rate should be the risk-free rate, but with disagree with them on that we add to the value of the contract all the components, amongst which we include the funding costs that they argue should not be considered.

We contend that the statement cannot be supported and that, in an economy where the bank has to pay a funding spread over the risk-free rate, the risk-free discounting is not always appropriate since it may produce risk-neutral values not corresponding to the costs of the replication strategy.

Statement 2.

> *"The funding of hedges is sometimes given as a reason for an FVA. However, trades in hedging instruments involve buying or selling assets for their market prices and are, therefore, zero net present value investments. As a result, the decision to hedge does not affect valuations."*

This statement is not really sensible: also buying an asset at its fair market price by borrowing money entails paying a funding interest rate. If assume that no repo market exists, the replication strategy involves unsecured funding that would require paying a funding spread over the risk-free rate. For example the bank sells a call option and it has to replicate it with the opposite sign (*i.e.*: hedge it): thus the bank needs to set up a portfolio that will surely contain a negative quantity of bonds (see equation above for β, the premium received is not enough to cover the Δ quantity of underlying asset) and it will need to borrow and pay its funding rate, that will then enter in the valuation formula.

If we consider that a repo market exists, also in this case the replication strategy may involve unsecured funding. For example if the bank sells a call option as above, it can buy at repo the Δ quantity of underlying, paying the repo rate (which can be safely assumed to be slightly above the risk-free rate) and it can invest in a risk-free asset the premium received, so actually the funding rate disappears from the valuation formula. But, as a counter-example, if the bank buys the same call option and it wants to hedge it, it has to sell at repo the Δ quantity, receiving the repo rate, and borrowing money paying the funding rate to fund the premium it has to pay. So the funding rate comes back into the valuation process.[7]

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[6] See Castagna [7].

[7] For contracts with more complex cash-flow patterns, including the funding costs is not as easy as using an effective discounting spread. For interest rate swaps, as an example, the inclusion of funding costs is not straightforward. See Castagna [8] for a possible method.

We can hardly find any type justification to the statement, so we still think that considering funding costs is a rational and economic sensible choice when they are relevant.

3 Separation Theorem and Investment Valuation

Statement 3.

> *"Another argument against FVA is a well-established principle in corporate finance theory that pricing should be kept separate from funding. (An exception arises because debt is taxed more favourably in many jurisdictions. If a new project is to be financed with additional debt, the additional tax benefit should be considered.) The discount rate used to value a project should depend on the risk of the project rather than the riskiness of the firm that undertakes it. For example, suppose a company that borrows at the risk-free rate plus 200bp has the opportunity to enter into a nearly risk-free project that returns the risk-free rate plus 80bp. Suppose the discount rate for the project, if considered as a stand-alone project, would be the risk-free rate plus 30bp. The project should be undertaken. Clearly, the project has a positive net present value when the appropriate discount rate (risk-free rate plus 30 bp) is used. The project will increase shareholder value because it reduces the risk of the company and, therefore, incrementally reduces its funding cost."*

Professors Hull and White refer surely to the Fisher-Hirshleifer's *Separation Principle* ([11] and [12]): the principle shows that investment decisions can be separated from financing decisions, although under some specific conditions that we do not need to repeat here. More generally, the independence of the capital structure of a company has been proved by the Modigliani-Miller Theorem ([15]): in an economy where

- there are no taxes;
- there is no separation between stockholders and bondholders;
- firms know their future financing needs;
- there are no bankruptcy costs;

the capital structure of a firm is irrelevant. The value of a firm is independent of its debt ratio and it will be determined by its projects' (*i.e.*: assets') cash flows. On the other hand the cost of capital (debt plus equity) of the firm will not change with leverage. As a firm increases its leverage, the cost of equity will increase just enough to offset any gains arising from the leverage. It is true that when taxes are considered, there could be a benefit from the leverage, but this can be easily accounted for and does not change the main result.

This is all very well known, but we would like to understand if it applies to the valuation of derivatives. We suspect that this is not the case and we try and show why.

Assume a bank wants to start a new business (or project, to use the corporate finance lore) and it is the following: provision of bid/offer prices for option contracts to its customers. Assume also that economic agents (*i.e.*: investors that can finance the bank via equity or bonds) decide to invest in this new project just considering its expected return, without taking into account the past projects the bank invested in. This business is the market-making activity of derivative contracts.

The bank adopts a technology to produce the options and sell them to customers[8] and it is the Black&Scholes model, which indicates two production factors:

1. the underlying asset of the options;

2. a risk-free bank account that can be sold or bought (alternatively said, borrowing and lending at the same risk-free rate).

If this two factors are available, and the bank operates in a B&S economy, the bank can deal option contracts at their B&S price, which is also the cost to produce them (*i.e.*: replicate them with the opposite sign via the replication strategy). It is very well known that the expected return on each dealt option, deducted the production costs (*i.e.*: the costs of the replication strategy) is nil:

$$dP_t - \Delta dS_t - (P_t - \Delta S_t)rdt = 0$$

One may wonder which is the investment needed to operate such a worthless (although not loosing) business. Well, it is zero: the bank does not need any money from investors because, at the start of each contract dealt with customers, no investment is needed. Cash-flows are provided by the premium received, by the underlying sold or by the money borrowed, and they are enough to grant a zero final P&L at the end of each contract. Clearly this is true because we are assuming that the bank deals with clients at the B&S price.

The capital structure, as we see, does not mean too much in this case. In theory a market-making business, if it could run alone by itself, can be started up without funding (either equity or debt) by the bank, although the production of the contracts entails borrowing money. Nevertheless, the money borrowed to replicate the contracts is a cost of a production factor and not debt used to fund a project.

If, for any reason (in practice always arising), the business requires some investments (*e.g.*: infrastructure, skilled traders, quants, etc.) then these can be funded via equity or debt and in this case the capital structure does not matter, as shown by Modigliani and Miller.[9] It is reasonable to expect that the market-making in this case is required to become a profitable business, whence the bid/ask prices and the

[8]Clearly also a bank's buy position in an option is a product sold to the customers.

[9]This is not the place to discuss all the real cases when the capital structure do matters, contrarily to the MM theorem.

mark-up's applied in the real market places by dealers. The return on the business depends on the preferences and risk-aversion of the investors for the market-making activity: the standard economic theory can be used to determine which is the its fair return and then the bank can decide the consistent levels for the prices dealt with customers to achieve this goal.

It is also clear that if the bank is not able to borrow money at the risk-free rate, the cost of the strategy is not the one predicted by B&S in all cases (sometimes it is, as noted above). If the bank dealt with the customers only at prices indicated by the replica working in a B&S economy, in some cases it would trade at levels not covering the costs of the actual replication strategy, that is operated at the risk-free and/or funding rates, depending on whether the bank is lending or borrowing money. Disregarding the differential between risk-free and funding rate would generate in some cases a loss[10] that would make the return of the business (the project) not zero, but negative. If a zero-investment zero-return project can still be worth being implemented, a zero-investment negative-return project is surely not.

The first conclusion is that the FVA should definitely be considered even if we assume that the Modigliani-Miller theorem is true: it is a blunder to apply the result of the theorem to a production factor contributing to the generation of cash-flows within the replication process.[11] This should be enough to invalidate the statement as far as the derivatives replication is concerned.

But we would like also stress the fact that the example proposed by Professors Hull and White, although not applicable to the derivatives replication, is anyway not really working in reality. In fact, it works only if the incremental cost for the capital needed to fund the (almost risk-free) project is actually the fair rate of return of the project (risk-free plus 30bps). If the bank raises funds at risk-free + 200bps, (not compensated by a decline of the return requested by shareholders, thus reducing the average cost of capital) the project is no more profitable so that it generates a loss that decreases, not increases, the value of the firm. The reduction of the riskiness of the total assets for the future is true, but unfortunately the loss generated by the project can more than counterbalance it. It should be noted that in reality it is quite unlikely to have an incremental cost of capital based on the return of last investment considered on a stand-alone basis. More frequently, the cost of capital will gradually update.[12]

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[10] *E.g.*: when selling a call option, the replication of a long position in the same contract costs more than the B&S replica if the borrowing rate is higher than the risk-free rate.

[11] See also Brealey and Mayers [4], ch. 17 and 18.

[12] A standard rule in microeconomics is that the corporate should keep on investing until its

It is true that in corporate finance text books it is acknowledged that the current WACC (Weighted Average Cost of Capital) of the corporate should be used to evaluate projects of similar risks as those comprised in the total assets, whereas one should use the proper fair rate of return (determined with the CAPM for example) for projects with different risks. The reason is quite clear, since using the WACC as a hurdle rate would make the corporate accept (reject) projects with risks higher (lower) than the average risk of the current asset. What is rarely indicated is the implied assumption that the corporate is able to finance the new project with a corresponding appropriate WACC, which is different from the current one: this implies also that investors are able to properly evaluate the difference between the new project and the old ones, and also that each project is implemented as it were a small independent firm.

In any case, when the corporate invests in some project with a return lower than the incremental WACC, the only way to really create value (*i.e.*: make enough money to pay the bondholders and the shareholders) is to charge the right mark-up over other products it sells, if this is possible, so as to compensate for the abnormal cost of capital funding the project. Otherwise the final result will be a loss although the riskiness of the assets declined, because the return generated is not enough to pay in a fair proportion the bondholders and the shareholders.[13]

On the other hand, since any kind of project in the end amounts to the production and the selling of a product,[14] the only way to create the appropriate return on the investment is to set a selling price incorporating all the production costs and the mark-up needed to grant the required remuneration of the capital.

Statement 4.

> *"Banks invest in Treasury instruments and other low-risk instruments that return less than their average cost of funding. They do not usually apply a funding cost to these investments. It would, therefore, seem in this context that they implicitly recognise that it is the risk of the project that matters."*

This statement is wrong under two perspectives. Firstly, if a bank invests in

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marginal cost of capital equates the marginal return on the investments.

[13] The fair return of the bonds and of the equity after the investment is not what has been requested by bond and share holders when financing the new project, if its rate of return is lower than the WACC.

[14] A project is not a metaphysical entity yielding a return by its attributes. In the real world the corporate evaluates a project by forecasting the cash-flow stream arising from the purchase of production factors and the selling of final product to clients.

Treasury and other low-risk assets for trading purposes, then the purchase is financed by a secured funding (*i.e.*: they buy them and then the use them as a collateral to fund the cost). In this case the funding cost is not the WACC, or the cost of unsecured funding, but it is a rate near (although slightly higher) than the risk-free rate.

Secondly, if low risk-assets are used as a liquidity buffer, so that they cannot be financed by secured funding[15] then the extra funding costs (WACC minus the expected return) paid by the bank to hold them is charged over other assets[16] generating the need of the buffer (as an example: a committed credit line is risk-managed by a liquidity buffer and the extra capital costs needed to fund it are charged in the proper way and typically included in the commitment fee). This is the way banks operate in practice because this is the best way they learnt form experience to run their business and make it fairly profitable, supported also by national and international supervising authorities (see, amongst others, EBA [2].)

4 DVA and FVA

Statement 5.

> *"FVA is closely related to debit valuation adjustment (DVA), but it is important to avoid confusing the two different types of DVA. One is the DVA arising because a dealer may default on its derivatives portfolio (we will refer to this as DVA_1). The other (DVA_2) is the DVA arising because a dealer may default on its other liabilities long-term debt, short-term debt, and so on. Both DVA_1 and DVA_2 are beneficial to the dealer's shareholder because they gain when the firm does not have to honour its obligations."*

FVA is related to DVA in the sense that for debt contracts the DVA **is** the FVA: it is the present value of the extra-costs, with respect to a risk-free counterparty, that a counterparty has to pay to compensate the other party for the default risk that the latter bears. For derivatives contracts the DVA is the compensation that a counterparty has to pay, when closing a contract, to the other party to remunerate the default risk that the latter bears and that is specularly measured as a credit value adjustment CVA. This compensation is the present value of the extra costs that the counterparty has to pay with respect to a risk-free counterparty. The funding cost related to a derivative contract is the sum of the funding costs that a counterparty

[15] Because otherwise they would not be unencumbered assets and hence they would not be suitable to form a liquidity buffer. See Basel Committee [16] or EBA [1].

[16] Traditional banking business has the peculiar feature that most of products sold to customers are actually assets bought by the financial institutions from them. So charging a mark-up over a product means in practice requiring a higher interest rate on one or more categories of assets.

has to pay on money it borrows to match negative cash flows generated by it.[17]

We agree on the distinction made by Professors Hull and White, although our definition of DVA and FVA offers in our opinion a more precise definition. On the other hand we still do not understand the ever too much cited, and never proved, benefit of the shareholders, arising from the default of the bank, mentioned also in the statement. We try and explain that there is no benefit whatsoever for anyone when the bank goes bankrupt, either for the bond or the share holders. To see this we recur to Merton's theory of the firm [14].

According to Merton, we can see the present value of the equity E_0 as the present value C of a call option on the assets V_0, with strike equal to the nominal value of the debt K, since (assuming that in T the bank stops the business activity):

$$E_T = \max(V_T - K, 0)$$

(the shareholders cannot loose more than the initial investment and the bank goes bankrupt if the terminal value of the assets is lower than the nominal value of the debt). On the other hand, the value of the debt in T is

$$B_T = \min(K, V_T) = K - \max(K - V_T; 0).$$

So the shareholders bought a call option on the value of the assets in T whereas the bondholders bought a bond with nominal value K and sold a put option whose present value is P. By put call parity

$$V_0 = C + Ke^{-rT} - P$$

so assets equate liabilities.

Now, if the bank goes bankrupt in T, the call option of the shareholders expires worthless. Would someone claim that they received any benefit? It would be as saying that an investor buying a call option on a stock would benefit from the fact that it expiries out of the money: a more sensible claim would be that the investor made a wrong investment since they lost the premium. It is exactly the same for the shareholder of the bank: when it goes burst they simply made a wrong investment but they did not benefit in any way from the default.

If the call option representing the equity expiries worthless, the put option P sold by the bondholders expiries in the money though. Who is exercising it? The bank, who exercises it against the bondholders so that they receive only the value of the assets V_T against a claim of nominal value $K > V_T$. So, do the shareholders benefit from this long put option? Not really since it is a covered long put, or a protective put, as an asset manager would say: the gains on the put are off-set by the losses on the assets. Affirming there is a benefit would be like affirming that someone is benefiting when their house goes on fire because they are receiving the compensation from an insurance contract. So even considering the put option

[17] See Castagna [5] for a discussion and a proof of these assertions

bought from the bondholders, shareholders are not benefiting. On the other hand bondholders are not happy at all either: default is no deal for anyone.

The premium for the put option is paid by shareholders, since it is clear that the return of bond holders (if the bank does not go defaulted) is

$$K - Ke^{-rT} + P = I + P$$

which is: the interest I plus the premium P. On the other hand the return on the equity is

$$V_T - K - (V_0 - Ke^{-rT} + P) = V_T - V_0 - I - P$$

which is the variation of the assets minus the interests I minus the premium P. So in the end the DVA, which is the same as the FVA for debt, is a cost for the shareholders to buy a protective put that is just covering the losses, not generating profits, when exercised. The conclusion is that there is not benefit linked to the DVA, since the shareholders loose the entire amount invested in the bank when it goes bankrupt.

Statement 6.

> *"We define $\Delta(DVA_2)$ as the increase in DVA_2 resulting from the funding requirements of a derivatives portfolio with a particular counterparty. The FVA for the portfolio, as usually calculated, is equal to the present value of the extra return required by lenders to compensate them for costs associated with possible defaults by the dealer on the funding. This is exactly equal to the benefit to the dealer from the possible defaults."*

Let us perform a thought experiment. Assume the bank, that can go bankrupt, has no assets and no liabilities (admittedly, it is a very "mental" experiment, but at least theoretically we can accept that): it sells a call option and we suppose that the replication can be operated with repo transactions, whose rate is equal to the risk-free rate. In this case it can be shown that the value of the option to the bank is the B&S price C and no FVA is operated.[18] The bank should give the contract a zero-NPV value only if it receives C as a premium.

Since the bank is defaultable for some external reason we do not need to specify here, for the buyer the option's economic value C^B is equal to the risk-free value C minus the expected losses on default, or the CVA: $C^B = C - CVA$. The buyer should give a zero-NPV to the contract only if it pays C^B.

There is a difference between the two values C^B and C, which means that the dealing price will be somewhere in between them. It should be noted that this price will cause a loss to both parties.

It is clear that in this case there is no FVA for the option, yet the bank trades at a price that is not the fair value to it. We know that the terminal P&L of the entire selling process would be zero if the bank sold the option at C, but if it sold, for example, at the value requested by the counterparty, C^B, the selling process would

[18] See Castagna [7].

end with a loss equal to $-DVAe^{rT}$. This cost is certain and so the bank needs at least an equity equal to DVA to cover it at the expiry of the option.

If the equity of the bank is zero at inception, it will surely go defaulted at the expiry of the option. On the other hand if the bank found a shareholder funding the activity for an amount equal to DVA, they would receive at the expiry 0, either if the bank will survive or if will go burst. But, if the shareholder will loose in any case the entire amount invested, why they should be happy with the bank accepting a reduction of the selling price of the option from C to C^B, justified by the "benefit on default" argument? Honestly we see no benefit here.

If the bank accepts to receive only C^B, then it has to find out another product to sell to some other counterparty upon which it can charge[19] a mark-up equal to DVA so as to make up for the certain loss suffered at the expiry on the call. Otherwise it will be quite difficult to convince any shareholder to fund the activity.[20]

Assume now that the replication is operated via borrowing and lending. It can be easily seen that if bank's borrowing rate is higher than the risk-free rate, the replication strategy suggested by B&S to produce (*i.e.*: hedge) the call option is the B&S formula where the discount and the drift rates are set equal to the borrowing rate. Denote by C^S this premium. The FVA is, according to our definition above, as $FVA = C^S - C$. The economic value to the bank of the sold call is C^S, not C, and it should give the contract a zero-NPV value only if it receives C^S as a premium.

On the other hand, for the buyer of the option, the economic value C^B is still equal to the risk-free value C minus the the CVA: $C^B = C - CVA$. There is an even bigger difference between the two values. If the bank accepts to trade at C^B it will make at the expiry a sure loss equal to $DVA + FVA$. By the same token as above, if the equity of the bank is $DVA + FVA$ the terminal value for the equity will be zero in any case, either if the bank goes bankrupt or it does not. No benefit, no compensation of the FVA with the DVA.

The statement cannot be supported. The bank must include FVA and it must not consider the DVA when determining the value to it of the contract: the DVA cannot be given up to the counterparty because there is no gain in case of default and the FVA and DVA cannot compensate each other in any reasonable way. If the dealing price is different from the economic value to the bank, the difference has to be covered by the profits generated somewhere else.

Statement 7.

> *FVA is a form of anti-EVA. It serves to adjust the economic value of the debt to a model price, and leads to poor choices if used for economic*

[19]We assume that in any case the selling price of this other product is such that it covers all production costs and yield a fair return on the capital invested.

[20]Without considering that shareholders will require also some fair return on the investment.

decision-making. To see why, consider the earlier example an FVA adjustment would incorrectly move the debt value from \$200 million to \$205 million. FVA adjustments should never be made because they move calculations away from the economic value. "

Since, as noted above, projects are not abstract entities living on their own and generating profits by themselves, the bank has to know how to set selling prices of products funded by the capital (equity plus debt). The FVA is a component of the mark-up needed to remunerate the debt; the bank has to consider also the return required by the shareholders to set an all-encompassing selling price.

Under an accountant perspective, one may reevaluate assets and liabilities by different criteria and decide to keep the FVA included in the value of the liabilities (so that the debt would be worth \$200 in the example of the statement); alternatively the FVA (and the DVA) could be considered as the present value of a cost born by the shareholders, given the order of participation to profits of the capital structure (the debt would be worth \$205).

In any case the FVA and the return on equity must be identified and the prices of products offered by the bank (amongst which derivatives contracts are included) must include both to grant the return required by the capital.

5 Conclusion

The analysis of the statements by Professors Hull and White confirms that the FVA is a cost of a production factor for derivative desks. As such it has to be included in the evaluation process to establish bid and offer prices. Besides it cannot be compensated in any case with the DVA, which is an additional cost[21] or the same thing as the FVA when referred to debt.

[21] Castagna [6] shows that the DVA is in practice not replicable by a dynamic strategy. Basel Committee (http://www.bis.org/press/p120725b.htm) accepted the thesis contained in this work.

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