

COPENHAGEN BUSINESS SCHOOL

MASTER THESIS

CAND.MERC(MAT.)

Default risk and funding costs in derivatives valuation

Authors:

Amanda Langgaard-Sørensen

Nicoline Maibomm Nielsen

Supervisor:

David Lando

Submitted on October 10, 2014

No. of pages (characters): 105 (172,486)

Abstract

The financial crisis and regulation following the crisis led institutions to consider the impact of counterparty credit risk, funding costs and collateral in the derivatives valuation. This thesis aims to examine these changes to practice when valuing a single derivative on a single underlying asset.

We will examine credit and debt value adjustments (CVA and DVA) to the value of a derivative and show how the original Black Scholes and Merton arguments can be extended to incorporate counterparty credit risk. Furthermore we show that the OIS rate is the best proxy currently available for the risk-free interest rate, both when valuing collateralized and non-collateralized derivatives.

We also examine whether a bank should make a funding value adjustment (FVA) when valuing derivatives. We show how and when making an adjustment leads to double counting and arbitrage opportunities. We conclude that whether to make a FVA depends on if the funding costs is only due to default risk or not.

Finally we will look into the regulatory changes and how they incentivise and require the use of collateral and clearing through central counterparties. Furthermore we will show how to make a collateral rate adjustment to the value of a derivative, using the Black Scholes and Merton arguments.

Contents

Abstract	i
1 Introduction	1
1.1 Problem statement	5
1.2 Structure	6
2 The Derivatives Discounting Dilemma	7
2.1 LIBOR and OIS rate	10
2.1.1 Choice of discount rate	14
2.2 Credit and Debt Value Adjustment	15
2.2.1 Credit exposure	15
2.2.2 Credit Value Adjustment (CVA)	17
2.2.3 Debt Value Adjustment (DVA)	20
2.3 Default risk in derivatives pricing	22
2.3.1 Original BSM Argument	22
2.3.2 Derivative is an asset for the institution	25
2.3.3 Derivative is a liability for the institution	29
2.3.4 Derivative can be an asset or liability for the institution	32
2.4 Adjusting the portfolio value	34
2.4.1 Netting	34
2.4.2 Collateral	38
2.5 Adjusted value with the OIS and LIBOR	46
2.5.1 Adjustment with a risky discount rate	47
2.5.2 An example	49
2.6 Concluding remarks	57
3 Funding Value Adjustment	59
3.1 Funding and funding costs	62

CONTENTS

3.1.1	Funding costs as discount rate	64
3.2	Funding Value Adjustment	71
3.2.1	BSM with FVA	71
3.3	Adjusting for default risk and funding costs	76
3.3.1	A hedging argument	77
3.3.2	An economic perspective	79
3.3.3	FVA and liquidity considerations	83
3.4	Implications of FVA	86
3.4.1	Double counting of default risk	86
3.4.2	Arbitrage opportunities	89
3.5	The future of Counterparty Credit Risk	93
3.5.1	Regulation	94
3.5.2	Central counterparties	95
3.5.3	Collateral Rate Adjustment	96
4	Conclusion	103
	Appendix	105
	Default probability from credit spreads on bonds	105
	References	108

Chapter 1

Introduction

Before the financial crisis that started in 2007 many large financial institutions such as banks were considered to be almost default free. With the collapse of Lehman Brothers in September 2008 and other financial institutions, bailout of banks from national governments and the downturn in the stock market, the financial markets became aware that credit risk of large financial institutions should also be considered. These events have brought changes in market practices.

One change is accounting for counterparty credit risk in pricing of derivatives. Before the crisis derivatives have been priced based on the underlying asset and its risk and not dependent on the parties in the contract. Derivatives have been priced as if these parties could not default. The financial crisis has resulted in a growing importance of counterparty credit risk which has become a significant issue on the over-the-counter (OTC) market.

Large financial institutions are now focused on expanding their derivatives pricing models so they are able to incorporate and quantify default risk for both the institution and the counterparty. The default risk for the institution is accounted for by a credit value adjustment (CVA) which is the expected loss caused by a default by the counterparty. The default risk for the counterparty is accounted for by a debt value adjustment (DVA) which is the expected gain caused by a default by the institution itself.

As a direct response to the financial crisis, The Basel Committee on Banking Su-

pervision (BCBS) published, in December 2010, two Basel III documents. Basel III is built on Basel II and is a global regulatory framework for banks, which purpose is to strengthen the regulation, supervision and risk management of the banking sector. Basel III is recommendations and the actual regulations are implemented by local regulators. In relation to counterparty credit risk Basel III introduces a CVA capital charge.

The CVA capital charge is designed to capture the risk of loss on OTC derivatives and securities financing transactions due to changes in counterparties' credit spread caused by a change in its creditworthiness. So the CVA capital charge is a capital charge on the variation in CVA, which were not captured within Basel II¹.

The calculation of CVA capital charge depends on whether the institution has an Internal Method Model (IMM) approval or not. For institutions without IMM approval Basel III presents a Standard Approach to calculate the CVA capital charge. Whereas institutions with IMM approval uses the Advanced Approach where the CVA capital charge is calculated using internal models. Once they are approved to the Advanced Approach, they cannot just use the Standard Approach. Furthermore whether an institution has IMM approval or not, differentiate between security types. Since an institution can have more advanced models for some securities than others, it can be approved to use an Advanced Approach for some products and has to use the Standard Approach for others.

Besides the Basel III global regulatory framework, other local regulations have been developed. The US Dodd-Frank Wall Street Reform and Consumer Protection Act 2009 and European Market Infrastructure Regulation (EMIR) are aimed at increasing the financial stability in the OTC derivatives market in US and Europe respectively.

At the same time other changes in the market were taking place. In many years the London Interbank Offered Rate (LIBOR) was used as an appropriate proxy for the risk-free rate when valuing derivatives. However, LIBOR increased relative to the Overnight Indexed Swap (OIS) rate, and the spread between the two interest rates

¹Basel II captured losses due to counterparty default but not the MtM losses due to change in credit value adjustments. During the financial crisis roughly two-thirds of losses were due to CVA losses but only one-third were due to actual defaults.

(the LIBOR-OIS spread) rose to a record of 364 basis points in October 2008. The increase in LIBOR relative to the OIS rate was problematic and indicated that the credit risk premium in LIBOR could no longer be ignored. The OIS rate is the best proxy for the risk-free rate.

The risk-free interest rate is central in valuation of assets under the risk neutral measure. Therefore it is important that the used interest rate is a good estimate of the risk-free rate. Besides defining the expected growth rates of asset prices in a risk-neutral world the risk-free rate is also used to determine the discount rate for expected payoffs in this world.

It has been practice, in derivatives valuation, for the market participants to choose a discount rate which reflects their funding costs. In this way it is possible to account for the cost of funding in the valuation. Because LIBOR reflected the funding rate and was considered a good proxy for the risk-free rate, LIBOR was used as the discount rate in risk-neutral valuation. However this is no longer the case since LIBOR is no longer a good proxy for the risk-free rate.

When valuing derivatives we differentiate between whether a transaction is collateralized or non-collateralized. The OIS rate is used as discount rate to value collateralized transactions whereas some market participants continue to use LIBOR as discount rate for non-collateralized transactions. The use of OIS in some circumstances and LIBOR in others have raised the dilemma of which interest rate to use as discount rate. The dilemma is known as the "Derivatives Discounting Dilemma". But regardless whether the transaction is collateralized or not, it is most correct to use the "risk-free" OIS rate as discount rate.

As the OIS rate has been accepted as the discount rate, funding costs are taken into account by adjusting the derivative value with a funding value adjustment (FVA). A FVA represents the difference in the net present value of the derivative when the risk-free rate is used for discounting compared to when the funding cost is used for discounting. Using a FVA has the consequence that market participants no longer agree on a price, because the funding cost is individual for each participant. Furthermore it raises the debate whether the price of a derivative should depend on it's risk or on it's

costs.

The nature of funding costs is relevant for whether or not a FVA makes economic sense. One way to achieve funding is to issue bonds and then the spread between the bond yield and the risk-free rate reflects the institution's funding costs. This yield spread has been subject to a lot of research which shows that default risk is considered to be the main influence but liquidity also may have a small impact.

When the bond yield spread is due to default risk, making CVA, DVA and FVA will result in double counting of default risk, as FVA will not bring any information that is not already covered by CVA and DVA. Furthermore this double counting can lead to incorrect pricing and arbitrage opportunities. Besides this fact, funding costs exist for non-collateralized portfolios and because regulations aim to collateralize OTC derivatives, a FVA seems less important in the future.

When a derivative is collateralized and the collateral posted is cash, it is necessary to make a collateral rate adjustment (CRA). The purpose of a CRA is to account for the difference between the contractually defined rate paid on cash collateral and the rate that reflects the risk of the collateral. With the move towards a more collateralized and cleared OTC market, a CRA seems more important in the future.

1.1 Problem statement

The financial crisis has resulted in a growing importance of counterparty credit risk. The credit risk premium in LIBOR has become more visible which has influenced the choice of the risk-free rate. Choosing OIS as the risk-free rate no longer reflects an institutions funding costs, and this creates a problem whether funding costs should be accounted for in valuation.

We seek to analyse and understand the relevance of funding costs in derivatives valuation. In addition to this we seek to understand the shift there has been in the discount rate. When the discount rate no longer reflects the funding rate, funding costs are taken into account by a funding value adjustment (FVA). To analyse FVA we need to be able to account for counterparty credit risk in derivatives valuation. Therefore we seek to understand how a credit value adjustment (CVA) and a debt value adjustment (DVA) is incorporated into the derivative value.

1.2 Structure

The structure of this thesis is as follows. Chapter 2 analyses the Derivatives Discounting Dilemma. Starting with a description of LIBOR and the OIS rate in section 2.1. Section 2.2 presents the credit and the debt value adjustments, thereafter showing how to incorporate these credit adjustments in the Black Scholes and Merton framework in section 2.3. Section 2.4 shows how to adjust the portfolio value and presents netting and collateral. Finally section 2.5 adjusts the portfolio value if another discount rate than the risk-free is used for discounting. This is done by using an example.

Chapter 3 analyses the impact of making a funding value adjustment to the derivative price. Section 3.1 starts by discussing the need for funding and funding costs and presents 3 special cases where the funding cost can be used as the discount rate. The funding value adjustment is presented in section 3.2 and it is showed how this adjustment is incorporated in the Black Scholes and Merton framework. Section 3.3 adjusts the portfolio value for both default risk and funding costs and uses a hedging argument to show how the funding value adjustment can be eliminated. The funding value adjustment will be discussed from an economic perspective and furthermore liquidity considerations are presented. In section 3.4 the implications of including funding costs will be discussed and double counting and arbitrage opportunities will be presented. Finally this chapter looks into the future of counterparty credit risk in section 3.5. The most important regulations related to counterparty credit risk and the use of central counterparties is presented and how to account for collateral with a collateral rate adjustment is analysed.

Chapter 2

The Derivatives Discounting Dilemma

In the pricing of derivatives one of the key inputs is the risk-free term structure of interest rates. The risk-free interest rate serves two purposes, it is used to define the expected growth rates of asset prices in a risk-neutral world, and to determine the discount rate for expected payoffs in this world.

The Derivatives Discounting Dilemma represents the dilemma of which interest rate that should be used as the discount rate for the expected payoffs. Two rates come into question, the risk-free rate or the bank's funding rate. In reality no interest rate is truly risk free, but some are close enough to work as an approximation for a risk free rate.

Before the financial crisis began in 2007 derivatives dealers used London Interbank Offered Rate (LIBOR) as a proxy for the risk-free rate. Furthermore, LIBOR is considered to be a good estimate of the bank's funding rate and therefore no dilemma existed since LIBOR was used both as risk-free and as discount rate.

LIBOR has been preferred as discount rate partly because it is a good estimate of the bank's funding rate but also because it reflects the credit risk of the two parties in the derivatives contract. If the yield spread, by which we mean the excess rate compared to the risk-free rate, is entirely due to credit risk, then these two arguments for using LIBOR are the same. This also raises the question whether an institution's

own funding costs should be considered in pricing of derivatives. We will look more into this in section 3.

As the financial crisis unfolded the credit risk premium in LIBOR became too big to ignore and it became necessary to find another term structure of interest rates to use as a proxy for the risk-free. The Overnight Indexed Swap (OIS) has been pointed out as the best proxy for the risk-free rate when valuing collateralized transactions. One reason often given is that the derivative is funded by collateral and the interest rate most commonly paid on collateral is OIS (the effective federal funds rate). On the other hand, when valuing non-collateralized derivatives transactions, LIBOR is still used as the discount rate.

The purpose of this section is to examine the error that arises if another discount rate than the risk-free interest rate is used to value a non-collateralized transaction. The analysis is based on LIBOR as discount rate and OIS as the risk-free interest rate, but any other rate can be used instead of LIBOR.

In the determination of the value of a derivative (or a portfolio of derivatives) we will account for the counterparty credit risk that arises if either the institution or the counterparty defaults. The expected loss due to a possible default by the counterparty is referred to as the credit value adjustment (CVA). The expected gain due to a possible default by the institution is referred to as the debt value adjustment (DVA). The usual approach when dealing with counterparty credit risk contains determining/calculating the no-default value, i.e. the value of a derivative if none of the parties default. CVA is then the reduction in the value of a derivative and DVA is the increase in the value of a derivative.

Since we focus on finding the value of a non-collateralized transaction, we will leave out for now to consider collateral. In a collateralized portfolio a further adjustment for the interest paid on cash collateral may be necessary. This adjustment is referred to as a collateral rate adjustment (CRA). We will examine a CRA in section 3.5.3.

The analysis of OIS and LIBOR will show that even in a non-collateralized transaction, OIS is the best interest rate to use both as a proxy for the risk-free interest rate and as the discount rate.

The structure of this chapter is as follows. In section 2.1 we will start by understanding LIBOR and OIS and the risk they carry. Before we can calculate the value of a derivative where default risk is incorporated we need to have some formulas for CVA and DVA. These will be presented in section 2.2. Thereafter we will show how the same formulas for CVA and DVA appear in the Black, Scholes and Merton model in section 2.3. In section 2.4 we look at how to adjust the portfolio value of derivatives for default risk and will explain the effect of collateral and netting agreements. All these sections are necessary before we can establish an example showing the error of using LIBOR as the discount rate instead of OIS. This will be the purpose of section 2.5.

2.1 LIBOR and OIS rate

The LIBOR and OIS interest rate are well known in finance and are some of the most used interest rates when valuing derivatives. This section aims to get a better understanding of how they are determined and what risk they carry. This is important to know when we later in this chapter will show how to adjust the value of a non-collateralized derivatives portfolio when LIBOR is used as the discount rate instead of OIS.

The London Interbank Offered Rate (LIBOR) is estimated by British Bankers Association based on quotes from a panel of contributor banks (Thomson Reuters is designated to calculate LIBOR¹). LIBOR is a benchmark rate that gives an indication of the average interest rate that a LIBOR contributor bank would be charged to obtain unsecured funding on the London Interbank market, for a given period, in a given currency. LIBOR is calculated in 10 currencies and in 15 maturities, lasting from overnight to 12 months and thus 150 rates are produced every business day. The most commonly used is the 3-month US dollar LIBOR interest rate. LIBOR's primary function is to serve as the benchmark rate for debt instruments, including government and corporate bonds, mortgages, derivatives such as currency and interest rate swaps and many other financial products.

The LIBOR/swap is one of the most widely traded derivatives. The LIBOR/swap rate is the fixed rate you have to pay in a swap agreement where the floating leg received is LIBOR (for example 3-month or 6-month LIBOR). The fixed rate is determined between the parties entering the contract, and calculated as in standard swap valuation.

A basic and necessary tool in the valuation of derivatives and other securities is the zero curve. A zero curve is the graphical representation of zero coupon rates which is the rate/return paid on a bond with only one single payment at maturity. A zero curve therefore represents interest rates with different times to maturity. The LIBOR zero curve is the zero curve that represents LIBOR interest rates. Because LIBOR only comes in maturities lasting one year and below, the LIBOR/swap can

¹[19] British Bankers Association

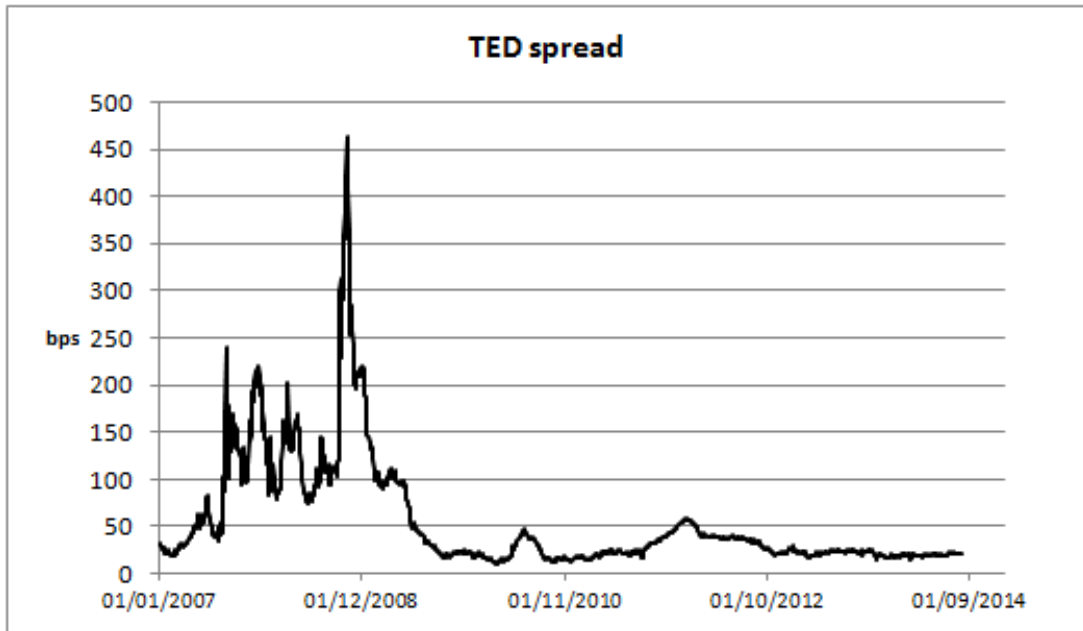


Figure 2.1.1: Shows the spread between 3-month USD LIBOR and 3-month Treasury rate (T-bill). Notice how it peaks at about 450 basis points. Source: Bloomberg

be used to bootstrap the LIBOR zero curve. This is because the LIBOR/swaps are traded in maturities of 1 year and upwards. In this way the LIBOR zero curve can be extrapolated into maturities lasting longer than a year.

LIBOR reflects the credit risk in lending to commercial banks. It has been shown that LIBOR/swap rates carry the same risk as a series of short term loans to financial institutions that are rated AA at the start of each loan².

The TED spread represents the spread between 3-month U.S. dollar LIBOR and the 3-month U.S. Treasury rate (T-bill). Since LIBOR reflects the credit risk whereas T-bills are considered risk-free, the spread between the two rates are an indication of credit risk. An increase in the TED spread reflects that the risk of default has increased. A decrease in the spread reflects a decrease in the credit risk. In normal market conditions the TED-spread is less than 50 basis points but in October 2008 the TED-spread peaked at over 450 basis point. Figure 2.1.1 shows the TED spread from January 2007 to August 2014.

²Collin-Dufresne and Solnik [5]

2.1. LIBOR AND OIS RATE

The Overnight Indexed Swap (OIS) interest rate is the fixed rate you have to pay in an interest rate swap agreement where the floating rate that is exchanged (received) is based on an overnight rate. The floating rate is refreshed daily and is therefore subject to less counterparty credit risk than LIBOR. This is one reason why OIS is considered to be a good proxy for the risk-free rate. In U.S. dollars the overnight rate used is the effective federal funds rate and in Euros it is the Euro Overnight Index Average (EONIA). The overnight rate is calculated as a volume-weighted average of traded unsecured lending rates.

The maturity of OIS swaps is relatively small (often 3 month or less), but transactions lasting as long as five to ten years are becoming more common. Similar to the LIBOR zero curve the OIS zero curve can be bootstrapped. Since the maturities of overnight indexed swaps are not as long as LIBOR/swaps, One approach is to assume that the spread between the OIS zero curve and the LIBOR/swap zero curve is the same at the long end as it is at the longest OIS maturity. Subtracting this spread from the LIBOR zero curve and adding it at the end of the OIS zero curve creates the long end of the OIS zero curve.

In an OIS there are two types of credit risk. There is credit risk from the US federal funds rate which can be argued to be small³. Second, there is the credit risk that arises from a possible default from one of the two swap counterparties. This lead to an adjustment to the fixed rate. The size of the adjustment depends on different factors such as the volatility of interest rates, the maturity of the swap, the probability of a default by a counterparty, the slope of the term structure and whether collateral should be posted or not.

Figure 2.1.2 shows the spread between the 3-month LIBOR and the 3-month OIS rate. As the figure shows it rose to a record of 364 basis points during the financial crisis in October 2008 whereas under normal market conditions it is about 10 basis points. The 3-month LIBOR-OIS spread reflects the credit risk between a 3-month loan to a bank and the credit risk in continually-refreshed one-day loans to banks, in both cases the banks are considered to be of acceptable credit quality. Besides the

³Hull & White: The Derivatives Discounting Dilemma [10]

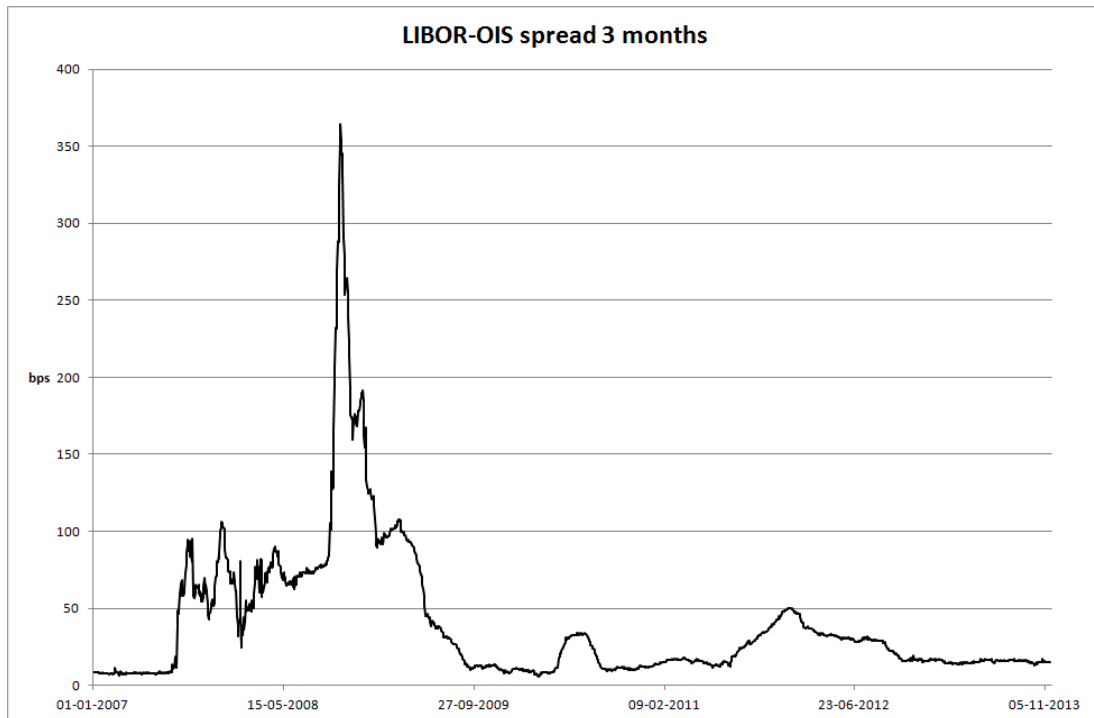


Figure 2.1.2: Shows the spread between 3-month LIBOR and 3-month OIS. Notice how it peaks at 364 basis points. Source: Bloomberg

record in October 2008 the spread also rose to about 30 basis points in June 2010 and 50 basis points in the end of 2011 because of European sovereign debt concerns.

The LIBOR/OIS spread and the TED-spread indicate that in stressed market conditions LIBOR may not be a good proxy for the risk-free rate.

Banks can borrow money in the overnight market on a secured or unsecured basis. The overnight LIBOR and OIS are both rates on unsecured borrowing and as such are not totally risk free. Since a secured loan is subject to less credit risk than an unsecured loan, it would be a better estimate of the risk free rate. An overnight repurchase agreement⁴ represents a secured loan since the borrowing is collateralized. However given the fact that there does not appear to be any way to determine a complete term structure for repos i.e. there does not exist a zero curve for repos, the repo rate cannot be used as a proxy for the risk-free rate. One of the reasons is that the cross sectional

⁴A repurchase agreement is a sale of a security with the agreement to repurchase it later. The repo rate is the price for borrowing the security.

2.1. LIBOR AND OIS RATE

variation in repo rates depends on the type of collateral posted which makes it difficult to determine a complete term structure for repos.

A portfolio of derivatives can be collateralized or non-collateralized. When valuing collateralized portfolios of derivatives institutions usually use the OIS interest rate in the discounting. One argument supporting this fact is that the derivatives are funded by collateral and the interest rate most commonly paid on collateral is the effective federal funds rate (in US), which is linked to the OIS rate.

Given this fact, the most correct argument for using the OIS interest rate as a proxy for the risk-free interest rate is simply that it is the best estimate since it is refreshed daily and therefore is subject to less credit risk than LIBOR. However in the valuation of non-collateralized portfolios some institutions continue to use LIBOR as the discount rate.

2.1.1 Choice of discount rate

The choice of discount rate depends on whether you want it to reflect the institution's funding rate or the risk-free rate. Before the financial crisis LIBOR was a good proxy for both the risk-free rate and the institution's funding rate.

In risk-neutral valuation the discount rate is the risk-free rate. This raises the question of what is the consequence of using another discount rate than the risk-free. We will examine this consequence and to do this we will look at the price difference in derivatives.

Post crisis there has been a change in derivatives valuation and default risk is accounted for by a credit and debt value adjustment. We will now introduce these adjustments and show how the original derivatives valuation framework can be extended to incorporate default risk.

2.2 Credit and Debt Value Adjustment

Until recent years derivatives have been priced based on the underlying asset and its risks, but the pricing did not depend on the counterparty credit risk associated with a default by one of the parties in the derivative contract. It meant that the derivative was priced as if the parties in the contract could not default. We will refer to this price as the no-default value and denote it by f_{nd} .

The financial crisis has left its mark and the awareness of counterparty credit risk has increased. Now it has become more important to account for the default risk between the parties in the contract when valuing derivatives. We will refer to the two parties in the contract as the institution and the counterparty. The counterparty's default risk is taken into account by a credit value adjustment (CVA) and the institution's own default risk is taken into account by a debt value adjustment (DVA). CVA is defined as the reduction in the value caused by counterparty default risk. DVA is defined as the addition to the value caused by the institution's own default risk. Throughout the thesis we will consider the value of a derivative from the institution's point of view, unless otherwise specified.

The credit adjustments depend on the exposure to the counterparty and is important in the valuation of derivatives. Furthermore netting and collateral agreements are important when adjusting the portfolio value of derivatives. We will look more into this in section 2.4.1 and 2.4.2 respectively.

This section aims to introduce the formula for a credit value adjustment (CVA) and the formula for a debt value adjustment (DVA) from an intuitive approach. Thereafter we will see how these formulas also appear when default risk is incorporated into the Black, Scholes and Merton (BSM) arguments.

But before we turn to the credit adjustments, let us look at the credit exposure.

2.2.1 Credit exposure

Credit exposure is the total amount of credit that a lender (the institution) is exposed to by a borrower (the counterparty). In the event where a counterparty defaults the

2.2. CREDIT AND DEBT VALUE ADJUSTMENT

institution may close out the relevant contracts and cease any future contractual payments with the counterparty. After the close out the net amount owed between them is determined (this of course requires a netting agreement between the two counterparts) and any collateral that may have been posted is taken into account. Credit exposure depends on whether the net value of the contracts, from the institution's point of view, is positive or negative. We will define this valuation of the relevant contracts at the default time as value V which includes the impact of netting and collateral.

In practice this value V can be difficult to find and the institution has to agree with the counterparty on the valuation. According to the ISDA documentation when determining the close out amount one should "*act in good faith and use commercially reasonable procedures in order to produce a commercially reasonable result*".

Positive exposure

If the derivative is an asset to the institution and if the counterparty defaults, the value V will be positive but the counterparty will be unable to undertake future commitments. This means that the institution will have a claim corresponding to the value at the time of default. When the value is positive the institution is exposed to the counterparty, or said in another way there is positive exposure which we will define as

$$X^+ = \max(V, 0)$$

The institution will expect to get some recovery of their claim. We will assume that this recovery is a fraction of their claim and denote it R_c .

Negative exposure

If the derivative is a liability to the institution and if the institution itself defaults, the value V will be negative and the institution is in debt to its counterparty. The negative exposure is

$$X^- = \min(V, 0)$$

Legally the institution will still be obliged to settle the amount, but the counterparty will get some recovery of their claim. Again we will assume that this recovery is a fraction of their claim and denote the fraction R_I .

Notice that the value V depends on the future payoffs from the derivative. So the expected exposure at time t depends on the value of the portfolio between t and maturity T .

We will, throughout the thesis, assume that the value V can be found by no-default valuation. If the derivatives contract follows the ISDA 2002 Master Agreement then in the case of default, the value is determined by a dealer poll with no reference to the counterparties. So this seems like an acceptable assumption.

2.2.2 Credit Value Adjustment (CVA)

Credit Value Adjustment (CVA) is the price of default risk for a derivative or a portfolio of derivatives with a particular counterparty. CVA can also be described as the price one would pay to hedge a derivative instrument or a portfolio of instruments against counterparty credit risk. CVA is found as the expected present value of exposure to the counterparty in the event of default.

$$\text{CVA} = E \left[\exp \left(- \int_0^{\tau_c} r_s ds \right) X^+(\tau_c, T) (1 - R_c(\tau_c)) 1_{\{\tau_c \leq T\}} \right]$$

Where r_t is the risk free interest rate at time t used for discounting, $X^+(t, T)$ is the exposure to the counterparty at time t and $R_c(\tau_c)$ is the counterparty's recovery rate at the time of default τ_c .

To calculate CVA we need to consider how we wish to model the interest rate r , the exposure X^+ , the recovery rate in case of default R_c and the default event τ_c .

We can let the risk free interest rate r be stochastic and follow a mean reverting process

$$dr_t = \kappa(\bar{r} - r)dt + \sigma_r \sqrt{r_t} dz_t$$

2.2. CREDIT AND DEBT VALUE ADJUSTMENT

Where z_t is a Brownian motion. Let $P(0, t)$ denote the price of a risk free zero coupon bond at time 0 with maturity t and remember

$$P(0, t) = E \left[\exp \left(- \int_0^t r_s ds \right) \right]$$

The exposure X^+ depends on the derivative contract entered by the institution and the counterparty, so we will not specify it now. As for the recovery rate we can assume it is deterministic. So when we integrate the recovery rate over the time step we are interested in, the entire path in that time span is covered and the value is known.

A possible way to model default events is with an intensity model⁵. In this setup we let the counterparty have a default intensity λ_t and define the default time as

$$\tau = \inf \left\{ t : \int_0^t \lambda_s ds \geq E_1 \right\}$$

Where E_1 is an exponential random variable with mean 1. When defining the default this way we avoid specifying events which cause default.

We will let the intensity be stochastic with the dynamics

$$d\lambda_t = \gamma(\bar{\lambda} - \lambda)dt + \sigma_\lambda \sqrt{\lambda} dz_t$$

Where z_t is a Brownian motion (not the same as the interest rate dynamics even though we use the same notation).

Within this setup the probability that default happens after the contract has matured is

$$S_c(0, t) \equiv \text{Prob}(\tau_c > T) = E \left[\exp \left(- \int_0^t \lambda_s ds \right) \right]$$

The probability of the event $(\tau_c > T)$ can be thought of as the probability of survival

⁵David Lando, Credit Risk Modelling [14]

to time T . And we have

$$Q_c(0, t) \equiv \text{Prob}(\tau_c = t) = E \left[\lambda_t \exp \left(- \int_0^t \lambda_s ds \right) \right]$$

so for small Δt , $Q_c(t)\Delta t$ can be seen as the probability of default by the counterparty between time t and $t + \Delta t$.

Using an intensity model to model default time we can rewrite CVA as

$$\begin{aligned} \text{CVA} &= E \left[\int_0^T \exp \left(- \int_0^t r_s ds \right) X^+(t, T) (1 - R_c(t)) \lambda_t \exp \left(- \int_0^t \lambda_s ds \right) dt \right] \\ \text{CVA} &= \int_0^T E \left[\exp \left(- \int_0^t r_s ds \right) X^+(t, T) (1 - R_c(t)) \lambda_t \exp \left(- \int_0^t \lambda_s ds \right) \right] dt \end{aligned}$$

We do not look into wrong-way and right-way risk. Wrong-way risk is when there is a positive dependence between the institution's exposure and the counterparty's probability of default. Right-way risk is when there is a negative dependence between the two. We will for simplicity assume that they are independent which seems like a reasonable assumption⁶. We can then reduce the expression to

$$\begin{aligned} \text{CVA} &= \int_0^T E \left[\exp \left(- \int_0^t r_s ds \right) X^+(t, T) \right] (1 - R_c(t)) E \left[\lambda_t \exp \left(- \int_0^t \lambda_s ds \right) \right] dt \\ \text{CVA} &= \int_0^T E \left[\exp \left(- \int_0^t r_s ds \right) X^+(t, T) \right] (1 - R_c(t)) Q_c(t) dt \end{aligned}$$

Define $f_{nd}^+(t, T)$ as the value today of the institution's exposure to the counterparty at time t , so $f_{nd}^+(t, T) = E \left[\exp \left(- \int_0^t r(s) ds \right) X^+(t, T) \right]$. CVA can then be written as

$$\text{CVA} = \int_0^T (1 - R_c(t)) f_{nd}^+(t, T) Q_c(t) dt$$

⁶It is only in the most extreme cases where a counterparty will default because of its derivatives positions with their counterparty. Often the event of default comes from a shock in the market. Therefore it is reasonable to assume independence between the shock and the interest rate on the derivative.

2.2. CREDIT AND DEBT VALUE ADJUSTMENT

Furthermore define $L_c(t)$ as the loss rate for the counterparty given by

$$L_c(t) \equiv Q_c(t)(1 - R_c(t))$$

Then CVA can be reduced to

$$\text{CVA} = \int_0^T f_{nd}^+(t, T) L_c(t) dt \quad (2.2.1)$$

In Appendix we show how the loss rate can be estimated from the bond credit spread. This reduced representation will be useful in the example where we adjust for LIBOR as the discount rate.

For most of the thesis we will have a very simplified setup where we for instance will assume a constant risk free rate to make notation easier and keep it simple. However, this setup can usually be extended to the modeling suggestions above. Furthermore to ease notation we will leave out the time, for instance by f_{nd}^+ we actually mean $f_{nd}^+(t, T)$.

2.2.3 Debt Value Adjustment (DVA)

DVA is the value of the expected gain if the institution itself defaults. It can also be seen as the addition to the derivatives value caused by the possibility of default by the institution. This gain might seem controversial since it will only be realized if the institution defaults (we will talk more of this later).

DVA is found as the expected present value of negative exposure to the counterparty in the event of default by the institution.

$$\text{DVA} = E \left[\exp \left(- \int_0^{\tau_I} r_s ds \right) X^-(\tau_I, T) (1 - R_I(\tau_I)) 1_{\{\tau_I \leq T\}} \right]$$

Where r_t is the risk free interest rate at time t used for discounting, $X^-(t, T)$ is the counterparty's exposure to the institution at time t and $R_I(\tau_I)$ is the institution's recovery rate at time of default τ_I .

If we use the same setup as we did with CVA, only this time we have that; for small Δt , then $Q_I(t)\Delta t$ can be seen as the probability of default by the institution between

time t and $t + \Delta t$, and we assume independence between the discounted exposure and the intensity for the institution. Then we have

$$\text{DVA} = \int_0^T E \left[\exp \left(\int_0^t r_s ds \right) X^-(t) \right] (1 - R_I(t)) Q_I(t) dt \quad (2.2.2)$$

Let us define $f_{nd}^-(t, T)$ as the value today of the counterparty's exposure to the institution at time t , this is exactly $E \left[\exp \left(- \int_0^t r(s) ds \right) X^-(t) \right]$. DVA is then

$$\text{DVA} = \int_0^T (1 - R_I(t)) f_{nd}^-(t, T) Q_I(t) dt$$

Define $L_I(t)$ as the loss rate of the institution,

$$L_I(t) \equiv Q_I(t)(1 - R_I(t))$$

DVA can then be reduced to

$$\text{DVA} = \int_0^T f_{nd}^-(t) L_I(t) dt \quad (2.2.3)$$

Just like for CVA, Appendix shows how we can estimate the institution's loss rate from their bond credit spread.

In the above we have shown the formulas for CVA and DVA. Next we will show how these formulas appear if we extend the well known Black Scholes and Merton approach to incorporate the risk of default.

2.3 Default risk in derivatives pricing

In the 1970's Black, Scholes and Merton (BSM) introduced an approach based on risk-neutral valuation for valuing derivatives. The approach (or model) is well known in finance and economics and has been the core of all derivatives pricing ever since.

The model was first used to value European stock options but is through the years extended to value many different types of derivatives on many different underlying assets.

Since the onset of the financial crisis in 2008 where the importance of credit risk increased, it has been natural to try to extend the BSM model to incorporate the risk of default.

The analysis we will present is for a single derivative that depends on a single underlying asset which provides a payoff at a particular future time. This gives a simple representation of the BSM model. The model can be extended to a portfolio that involves multiple derivatives but this will of course increase the complexity.

A simplifying assumption when incorporating default risk in derivatives pricing is that the impact of default is to reduce the value of a derivative and a bond by known proportional amounts. Furthermore we assume that no collateral is posted.

We will start by presenting the original argument of BSM which is the basis for the extension of the model. Thereafter we will show how the model can be extended with a credit value adjustment (CVA) and a debt value adjustment (DVA). Important to notice is that we look at the two components in isolation.

2.3.1 Original BSM Argument

We can use the original argument of BSM to find the no-default value (f_{nd}) of a derivative. When we wish to find the derivative value adjusted for, for instance, counterparty credit risk, it will depend on f_{nd} . In the risk-neutral valuation approach, Black Scholes and Merton derived a partial differential equation (PDE) where the solution is the price of the derivative. The PDE can be derived using either equilibrium arguments presented by Black and Scholes or no-arbitrage arguments presented by Merton. The

equilibrium arguments are built on the capital asset pricing model (CAPM) and are important when unhedgable risks are considered (for example the institution's own default risk which we will try to hedge later). Merton's no-arbitrage arguments are built on delta hedging. Both arguments result in the same PDE. We will use Merton's hedging argument to create a riskless portfolio by delta hedging⁷.

Suppose the price of the derivative is f and the underlying asset is a non-dividend paying stock S . Furthermore assume that the (risk-free) interest rate r is constant. Let the underlying asset S follow

$$dS = \mu S dt + \sigma S dz \quad (2.3.1)$$

where μ is the expected return on the stock, σ is the volatility of the stock and dz is a Wiener process. By using Ito's lemma the price of the derivative satisfies

$$\begin{aligned} df &= \mu_f dt + \sigma S \frac{\partial f}{\partial S} dz \\ \mu_f &= \frac{1}{f} \left[\frac{\partial f}{\partial t} + \mu S \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} \right] \end{aligned} \quad (2.3.2)$$

Let us consider the case where an institution has sold a derivative (a short position). The short position in the derivative exposes the institution to market risk because of the uncertainty in the price of the underlying stock. This market risk can be hedged by buying shares. Therefore we have a portfolio, Π that consists of a short position in the derivative and a position of $\partial f / \partial S$ in the underlying asset (the stock). This gives the portfolio value,

$$\Pi = -f + \frac{\partial f}{\partial S} S$$

The change in the portfolio value can be found using an application of Ito's lemma which means that the process for S in equation (2.3.1) and the process for f in equation

⁷We use this argument because the same argument is used when we in the next chapter look into a funding value adjustment (FVA).

2.3. DEFAULT RISK IN DERIVATIVES PRICING

(2.3.2) are inserted into the portfolio value, Π :

$$\begin{aligned} d\Pi &= - \left[\left(\frac{\partial f}{\partial t} + \frac{\partial f}{\partial S} \mu S + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 \right) dt + \sigma S \frac{\partial f}{\partial S} dz \right] + \frac{\partial f}{\partial S} (\mu S dt + \sigma S dz) \\ d\Pi &= - \frac{\partial f}{\partial t} dt - \frac{\partial f}{\partial S} \mu S dt - \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 dt - \sigma S \frac{\partial f}{\partial S} dz + \frac{\partial f}{\partial S} \mu S dt + \frac{\partial f}{\partial S} \sigma S dz \\ d\Pi &= - \frac{\partial f}{\partial t} dt - \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 dt \end{aligned}$$

The portfolio Π is risk-free and therefore the portfolio should earn the risk-free rate r , so the change in the portfolio value can also be written as

$$\begin{aligned} d\Pi &= r\Pi dt \\ - \frac{\partial f}{\partial t} dt - \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 dt &= \left[-rf + rS \frac{\partial f}{\partial S} \right] dt \end{aligned}$$

Hence the partial differential equation (PDE) is

$$\frac{\partial f}{\partial t} + rS \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf \quad (2.3.3)$$

The general solution to the PDE can be found by discounting the expected risk-neutral payoffs on the derivative back to the present time with the risk-free rate r . If the derivative provides a single positive payoff at maturity T , the solution to the PDE is

$$f_{nd}(S_t, t) = e^{-r(T-t)} E_r[f(S_T, T)] \quad (2.3.4)$$

where S_t denotes the stock price at time t and E_r represents the expectation taken over all paths that the stock might follow when the stock's expected return is r .

The solution to the PDE represents the no-default price of the derivative i.e. the price where it is assumed that neither the institution or the counterparty will default. This result is useful in the following sections.

2.3.2 Derivative is an asset for the institution

We wish to incorporate the risk of default by the counterparty into the valuation of a derivative. One way to do this is by extending the original BSM arguments and in this way derive the formula for the credit value adjustment (CVA).

Assume that the derivative is always an asset to the institution (it always has a positive value from the institution's point of view) and let us again use Merton's hedging argument to derive the partial differential equation (PDE).

Let the process for the derivative f include the possibility of a counterparty default by a jump process, hence

$$df = (\mu_f f)dt + \sigma S \frac{\partial f}{\partial S} dz - \gamma_c f dq_c$$

The event of default is denoted by the jump process dq_c . The size of the jump is one and the expected proportional reduction in the value of the derivative if a counterparty defaults is given by γ_c (which means that $\gamma_c = 1 - R_c^d$, where R_c^d is the recovery rate on the derivative when the counterparty defaults).

Suppose that the counterparty has an outstanding discount bond, B_c . The process for the bond is

$$dB_c = r_c B_c dt - \eta_c B_c dq_c$$

where r_c is the instantaneous return earned by the bondholder as long as the bond does not default and if the counterparty does default η_c represents the expected proportional reduction in the value of the bond ($\eta_c = 1 - R_c^B$ where R_c^B is the counterparty's recovery rate on the bond). The bond is used to hedge the counterparty credit risk.

We have a long position in the derivative f and wish to create a risk free portfolio. As in the original framework we can hedge the market risk by shorting $-\partial f / \partial S$ of the underlying asset. The derivative is always an asset to the institution, so if the counterparty defaults the institution will suffer a loss. In order to hedge this default risk the institution can enter a short position of n units of discount bonds issued by

2.3. DEFAULT RISK IN DERIVATIVES PRICING

the counterparty. The number of bonds must be set so

$$nB_c\eta_c = \gamma_c f$$

i.e. the loss of the bonds must equal the loss on the derivative in the case of default by the counterparty. The total portfolio value is

$$\Pi = f - \frac{\partial f}{\partial S}S - nB_c = f \left(1 - \frac{\gamma_c}{\eta_c}\right) - S \frac{\partial f}{\partial S}$$

The change in the portfolio value ($d\Pi$) can be derived using an application of Ito's lemma. Insert the dynamic for the derivative f , the dynamic for the asset S (equation (2.3.1)) and the dynamic for the bond B_c into the portfolio value Π .

$$\begin{aligned} d\Pi &= \left[\left(\frac{\partial f}{\partial t} + \frac{\partial f}{\partial S}\mu S + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 \right) dt + \sigma S \frac{\partial f}{\partial S} dz - \gamma_c f dq_c \right] \\ &\quad - \frac{\partial f}{\partial S} (\mu S dt + \sigma S dz) - n(r_c B_c dt - \eta_c B_c dq_c) \\ d\Pi &= \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial S} \mu S dt + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 dt + \sigma S \frac{\partial f}{\partial S} dz - \gamma_c f dq_c \\ &\quad - \frac{\partial f}{\partial S} \mu S dt - \frac{\partial f}{\partial S} \sigma S dz - nB_c r_c dt + nB_c \eta_c dq_c \\ d\Pi &= \frac{\partial f}{\partial t} dt + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 dt - \gamma_c f dq_c - nB_c r_c dt + nB_c \eta_c dq_c \end{aligned}$$

Use the fact that $nB_c = \frac{\gamma_c f}{\eta_c}$

$$\begin{aligned} d\Pi &= \frac{\partial f}{\partial t} dt + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 dt - \gamma_c f dq_c - \frac{\gamma_c f}{\eta_c} r_c dt + \frac{\gamma_c f}{\eta_c} \eta_c dq_c \\ d\Pi &= \frac{\partial f}{\partial t} dt + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 dt - r_c \frac{\gamma_c}{\eta_c} f dt \end{aligned}$$

The last term $r_c \frac{\gamma_c}{\eta_c} f$ is the interest that must be paid on the short position in the counterparty's debt. The portfolio Π is risk-free and should therefore earn the risk-free

rate r , so the change in the portfolio value can also be written as

$$\begin{aligned}
 d\Pi &= r\Pi dt \\
 \frac{\partial f}{\partial t} dt + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 dt - r_c \frac{\gamma_c}{\eta_c} f dt &= r f \left(1 - \frac{\gamma_c}{\eta_c} \right) dt - r S \frac{\partial f}{\partial S} dt \\
 \frac{\partial f}{\partial t} dt + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 dt + r S \frac{\partial f}{\partial S} dt &= r f \left(1 - \frac{\gamma_c}{\eta_c} \right) dt + r_c \frac{\gamma_c}{\eta_c} f dt \\
 \frac{\partial f}{\partial t} + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 + r S \frac{\partial f}{\partial S} &= r f \left(1 - \frac{\gamma_c}{\eta_c} \right) + r_c \frac{\gamma_c}{\eta_c} f
 \end{aligned}$$

Define $r_c^* = r + (r_c - r) \frac{\gamma_c}{\eta_c}$ which can be regarded as an adjusted rate of interest on the bond that allows for differences between recovery rates on the bond and the derivative. The PDE is

$$\frac{\partial f}{\partial t} + r S \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = r_c^* f \quad (2.3.5)$$

If the bond and the derivative are treated in the same way in the case of default i.e. $\gamma_c = \eta_c$, then $r_c^* = r_c$. The general solution to the PDE is found by discounting the expected risk-neutral payoffs with r_c^* ⁸. If the derivative provides a single positive payoff at maturity T then the price adjusted for counterparty credit risk is

$$f(S_t, t) = e^{-r_c^*(T-t)} E_r[f(S_T, T)] \quad (2.3.6)$$

Where S_t denotes the stock price at time t and E_r is the expectation of the stock price when the stock's expected return is r .

⁸Note that this is the same result as we would achieve in a Cox setting with recovery of market value (See David Lando: Credit Risk Modelling [14]). In this setup, the value of a defaultable contingent claim on counterparty c is found as discounting the expected payoff with the discount rate $r + (1 - \delta)\lambda_c$ where δ denotes the fractional recovery. In this setup, the value of a defaultable bond issued by the counterparty is found as discounting the expected payoff with the discount rate $r + \eta_c \lambda_c$. So $r_c = r + \eta_c \lambda_c$ and if we insert this in r_c^* we find that

$$r_c^* = r + (r + \eta_c \lambda_c - r) \frac{\gamma_c}{\eta_c} \Rightarrow r_c^* = r + \gamma_c \lambda_c$$

Since γ_c denotes the fractional loss on the derivative in the event of default, r_c^* is the same discount rate as in a Cox setting. The derivative price in equation (2.3.6) is therefore the same as in Cox setting with recovery of market value.

2.3. DEFAULT RISK IN DERIVATIVES PRICING

We wish to get an expression for the derivative value that depends on the no-default value from equation (2.3.4). With a little rewriting this is what we get

$$\begin{aligned} f(S_t, t) &= e^{-r_c^*(T-t)} E_r[f(S_T, T)] \frac{e^{-r(T-t)}}{e^{-r(T-t)}} \\ f(S_T, t) &= e^{-r(T-t)} E_r[f(S_T, T)] e^{-r_c^*(T-t)} e^{r(T-t)} \\ f(S_T, t) &= f_{nd} e^{-(r_c^*-r)(T-t)} \end{aligned} \tag{2.3.7}$$

With this expression of the solution we can find the formula for CVA. Remember that CVA is defined as the reduction in the value of a derivative due to the counterparty credit risk, which means $f = f_{nd} - \text{CVA}$ so this gives us

$$\begin{aligned} f_{nd} e^{-(r_c^*-r)(T-t)} &= f_{nd} - \text{CVA} \\ \text{CVA} &= f_{nd} - f_{nd} e^{-(r_c^*-r)(T-t)} \\ \text{CVA} &= f_{nd} (1 - e^{-(r_c^*-r)(T-t)}) \end{aligned}$$

Appendix shows how the loss rate $L_c(t) = (1 - R(t))Q(t)$ can be estimated from the bond credit spread based on a bond issued by the counterparty. If the default is treated equally for the bond and the derivative, $r_c^* = r_c$ and if we are looking for the value today $t = 0$ then

$$\begin{aligned} \text{CVA} &= f_{nd} (1 - e^{-(r_c^*-r)T}) \\ \text{CVA} &= f_{nd} \int_0^T L_c(t) dt \\ \text{CVA} &= \int_0^T f_{nd} L_c(t) dt \end{aligned}$$

And we have the same representation of CVA as in section 2.2.2 because when the derivative always is an asset to the institution then $f^+ = f_{nd}$. This can be done for more than a single payoff, but for now we will leave it at this.

2.3.3 Derivative is a liability for the institution

In the previous section we showed how the formula for the CVA component can be derived using the BSM arguments. Now we wish to consider the case of default by the institution itself and use the BSM arguments to derive the formula for the debt value adjustment (DVA). This can be done by assuming that the derivative is always a liability for the institution (it always have a positive value to the counterparty and a negative value to the institution).

Let the process for the derivative f in equation (2.3.2) include the possibility for a default by the institution by a jump process, so

$$df = (\mu_f f)dt + \sigma S \frac{\partial f}{\partial S} dz - \gamma_I f dq_I$$

Where dq_I denotes the jump process. The size of the jump is one and the expected proportional reduction in the value of the derivative if the institution defaults is given by γ_I (with R_I^d being the institution's recovery rate on the derivative when the institution defaults, we have $\gamma_I = 1 - R_I^d$).

Suppose that the institution has an outstanding discount bond, B_I . The process for the bond is

$$dB_I = r_I B_I dt - \eta_I B_I dq_I$$

where r_I is the instantaneous return earned by the bondholder as long as the bond does not default and if the institution defaults η_I represents the expected proportional reduction in the value of the bond ($\eta_I = 1 - R_I^B$ when the institution's recovery rate on the bond is R_I^B).

Again we will use Merton's hedging argument where the bond is used to hedge the institution's own default risk. We have a long position in the derivative f and wish to create a risk free portfolio. The market risk can be delta hedged by shorting $-\partial f / \partial S$ of the underlying asset. The derivative is (always) a liability to the institution, so if the institution defaults it will make a gain. In order to hedge this default risk

2.3. DEFAULT RISK IN DERIVATIVES PRICING

the institution can purchase (buy back) n units of the discount bonds issued by the institution itself. The number of bonds must be set so

$$nB_I\eta_I = \gamma_I f$$

i.e. the loss of the bonds must equal the loss on the derivative in the case of default by the institution. The total portfolio value is

$$\Pi = f - \frac{\partial f}{\partial S}S - nB_I = f \left(1 - \frac{\gamma_I}{\eta_I}\right) - S \frac{\partial f}{\partial S}$$

The change in the portfolio value can be derived using an application of Ito's lemma. Insert the process for the derivative f , the process for the stock S (equation (2.3.1)) and the process for the bond B_I into the portfolio value Π .

$$\begin{aligned} d\Pi &= \left[\left(\frac{\partial f}{\partial t} + \frac{\partial f}{\partial S}\mu S + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 \right) dt + \sigma S \frac{\partial f}{\partial S} dz - \gamma_I f dq_I \right] \\ &\quad - \frac{\partial f}{\partial S} (\mu S dt + \sigma S dz) - n(r_I B_I dt - \eta_I B_I dq_I) \\ d\Pi &= \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial S} \mu S dt + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 dt + \sigma S \frac{\partial f}{\partial S} dz - \gamma_I f dq_I \\ &\quad - \frac{\partial f}{\partial S} \mu S dt - \frac{\partial f}{\partial S} \sigma S dz - nB_I r_I dt + nB_I \eta_I dq_I \\ d\Pi &= \frac{\partial f}{\partial t} dt + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 dt - \gamma_I f dq_I - nB_I r_I dt + nB_I \eta_I dq_I \end{aligned}$$

Use the fact that $nB_I = \frac{\gamma_I f}{\eta_I}$

$$\begin{aligned} d\Pi &= \frac{\partial f}{\partial t} dt + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 dt - \gamma_I f dq_I - \frac{\gamma_I f}{\eta_I} r_I dt + \frac{\gamma_I f}{\eta_I} \eta_I dq_I \\ d\Pi &= \frac{\partial f}{\partial t} dt + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 dt - r_I \frac{\gamma_I}{\eta_I} f dt \end{aligned}$$

Since the portfolio Π is risk-free it should earn the same rate of return, r as other

risk-free assets, so the change in the portfolio value can also be written as

$$d\Pi = r\Pi dt$$

$$\frac{\partial f}{\partial t} dt + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 dt - r_I \frac{\gamma_I}{\eta_I} f dt = r f \left(1 - \frac{\gamma_I}{\eta_I} \right) dt - r S \frac{\partial f}{\partial S} dt$$

$$\frac{\partial f}{\partial t} dt + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 dt + r S \frac{\partial f}{\partial S} dt = r f \left(1 - \frac{\gamma_I}{\eta_I} \right) dt + r_I \frac{\gamma_I}{\eta_I} f dt$$

Define $r_I^* = r + (r_I - r) \frac{\gamma_I}{\eta_I}$ where γ_I and η_I are the expected proportional impact on the value of its derivative and its bonds of a default by the institution. Because the derivative is a liability for the institution it will gain γ_I times the value of the derivative. With a little rewriting we get the following PDE

$$\frac{\partial f}{\partial t} + r S \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = r_I^* f \quad (2.3.8)$$

If the bond and the derivative are treated in the same way in the case of default, then $\gamma_I = \eta_I$ which mean that $r_I^* = r_I$. The general solution to the PDE is found by discounting the expected risk-neutral payoffs with r_I^* .

If the derivative provides a single negative payoff at maturity T then the price adjusted for the institution's own default risk is

$$f(S_t, t) = e^{-r_I^*(T-t)} E_r[f(S_T, T)]$$

Our intention is to derive a formula for DVA. To do this we need to get the solution represented by the no-default solution in equation (2.3.4). With this solution in mind and with some rewriting we get

$$f(S_t, t) = e^{-r_I^*(T-t)} E_r[f(S_T, T)] \frac{e^{-r(T-t)}}{e^{-r(T-t)}}$$

$$f(S_T, t) = e^{-r(T-t)} E_r[f(S_T, T)] e^{-r_I^*(T-t)} e^{r(T-t)}$$

$$f(S_T, t) = f_{nd} e^{-(r_I^* - r)(T-t)} \quad (2.3.9)$$

Remember that DVA is defined as the addition to the derivative value due to the

2.3. DEFAULT RISK IN DERIVATIVES PRICING

institution's own default risk, i.e. $f = f_{nd} + \text{DVA}$

$$\begin{aligned} f_{nd}e^{-(r_I^*-r)(T-t)} &= f_{nd} + \text{DVA} \\ \text{DVA} &= f_{nd}e^{-(r_I^*-r)(T-t)} - f_{nd} \\ \text{DVA} &= -f_{nd}(1 - e^{-(r_I^*-r)(T-t)}) \end{aligned}$$

Appendix shows how the loss rate $L(t) = (1 - R(t))Q(t)$ can be estimated from the bond credit spread based on a bond issued by the institution. If the bond and the derivative are treated equally, $r_I^* = r_I$ and if we are looking for the value today $t = 0$ then

$$\begin{aligned} \text{DVA} &= -f_{nd}(1 - e^{-(r_I^*-r)T}) \\ \text{DVA} &= -f_{nd} \int_0^T L_I(t)dt \\ \text{DVA} &= \int_0^T -f_{nd}L_I(t)dt \end{aligned}$$

And we have the same representation of DVA as in section 2.2.3 because when the derivative always is a liability to the institution then $f^- = -f_{nd}$. This can be done for more than a single payoff, but for now we will leave it at this.

The hedging argument rely on the possibility of the institution to purchase own debt. This might seem weird. However, if the funds from selling the derivatives can be used as a source of finance to the bank and thereby reduce the need from external funding, then the effective rate is then the same as if the bond has been purchased. If we use the CAPM model we would end up with the same PDE as above and this does not depend on the possibility to purchase debt.

2.3.4 Derivative can be an asset or liability for the institution

When the derivative can be an asset or a liability to the institution, the value of the derivative can be either positive or negative but not both at the same time. So when you look at a specific time and if the value of the derivative is positive from

the institution's point of view, then the Merton hedging argument used in section 2.3.2 applies. If the value of the derivative is negative for the institution, the hedging argument used in section 2.3.3 applies.

The partial differential equation (PDE) becomes

$$\frac{\partial f}{\partial t} + rS \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = r_C^* \max(f, 0) + r_I^* \min(f, 0) \quad (2.3.10)$$

The above results for CVA and DVA can be used to adjust the total value of the portfolio which we will look more into in the next section.

2.4 Adjusting the portfolio value

In the previous section we have shown how we can incorporate default risk in derivatives pricing based on the Black Scholes and Merton arguments. When incorporating both CVA and DVA into the price of a derivative or a portfolio of derivatives, the total adjustment is

$$f = f_{nd} - \text{CVA} + \text{DVA}$$

Where CVA is a decrease in the portfolio value and DVA is an increase. This adjustment is also known as a bilateral adjustment of the portfolio value. An important and attractive implication of this formula is the symmetry. If two counterparties agree on the approach and parameters in the calculation, they will also agree on a price. If the institution calculates a loss then the counterparty will calculate the same but as a gain and vice versa.

However, CVA and DVA are usually not additive across transactions because of netting agreements. This means CVA and DVA has to be calculated on a portfolio basis for each counterparty. Besides netting agreements, collateral also has an effect on CVA and DVA since it mitigates the credit exposure to the counterparty. By regulation collateral agreements are becoming more and more used, for instance regulators request that all parties trading derivatives on the interbank market must post two-sided collateral. Netting and collateral agreements are often bilateral and thereby aim to reduce the risk for both parties in the contract. We therefore find it important to analyse the impact of netting agreements and collateral and will do so in section 2.4.1 and 2.4.2 respectively.

2.4.1 Netting

Netting is an agreement between an institution and their counterparty to offset the value of their transactions and in this way mitigate the exposure. There are two types of netting, payment netting and closeout netting. Payment netting gives an institu-

tion the ability to net cash flows occurring on the same day. Whereas closeout netting relates to the netting of transactions in the case where either the institution or the counterparty is defaulting. Netting distinguish between two types of risk, settlement and pre-settlement risk. Settlement risk is the risk of a counterparty defaulting during the settlement process, whereas pre-settlement risk is the risk of a counterparty defaulting prior to the expiration of the contract. Payment netting reduces the settlement risk, whereas closeout netting is relevant for counterparty risk and reduces the pre-settlement risk.

In the following we will discuss payment and close out netting and thereafter the impact netting has on exposure.

Payment netting

Counterparties often have multiple cash flows with each other during a given day and it is preferable if these cash flows can be netted into one single payment. This is exactly the purpose of payment netting as it nets the cash flows in the same currency during a given day. A simple example is an institution which has a floating swap payment of \$310 and receives a fixed payment of \$300 on the same day. The net payment will be \$10 and the risk associated with the fixed payment is deleted. In other words it is when an institution combines same-day cash flows in one single payment.

As described payment netting reduces the settlement risk and also enhances the operational efficiency, since only one payment should be made during a day. However payment netting does give rise to operational risk which is the risk that arises from people, systems, internal and external events. The case of Lehman Brothers and KfW Bankengruppe illustrates operational risk. Lehman and KfW Bankengruppe had a currency swap transaction with euros being paid to Lehman and dollars being paid to KfW Bankengruppe. Just prior to the default of Lehman Brothers, KfW Bankengruppe made an "automated transfer" where they paid Lehman 300 million euros despite the fact that Lehman would not be making the opposite dollar transfer. This mistake was blamed on two of the bank's board members and the head of risk control department and they were suspended after this transfer.

Closeout netting

Closeout netting applies to transactions between a defaulting and a non-defaulting institution. It is a process involving termination of obligations with the defaulted counterparty and subsequently the involved parties combine the transactions into one single net payable or receivable. Closeout netting consists of two components:

- 1) Closeout: First the non-defaulting institution terminates the transactions with the defaulted counterparty and all contractual payments cease.
- 2) Netting: Second is the valuation process where the replacement cost of each transaction is determined. Finally the net balance which is the sum of the positive (those owed to the non-defaulting party) and negative (those owed by the non-defaulting party) transaction values are determined and netted against each other to determine the final closeout amount.

Let us make an example to see one of the consequences if netting is not used. We consider two swap transactions between a defaulting and a non-defaulting party. Transaction 1 where the non-defaulting party owes \$1,000,000 (negative value) and transaction 2 where the non-defaulting party is owed \$800,000 (positive value). If the parties have an agreement of closeout netting the non-defaulting party will have to make a net payment to the defaulting party at \$200,000. If there is no netting agreement the non-defaulting party is obliged to make a payment of \$1,000,000 to the defaulting party and since the party is in default the non-defaulting party now has to wait months, maybe years before they will receive whatever amount of the \$800,000 is left from the bankruptcy. The example is illustrated in figure 2.4.1. The result of closeout netting is a reduction in the credit exposure from gross to net exposure.

The impact of netting

Netting is an essential component for institutions and other users of derivatives, together with collateral when they wish to hedge.

Suppose an institution wants to trade out of a position and because OTC derivatives

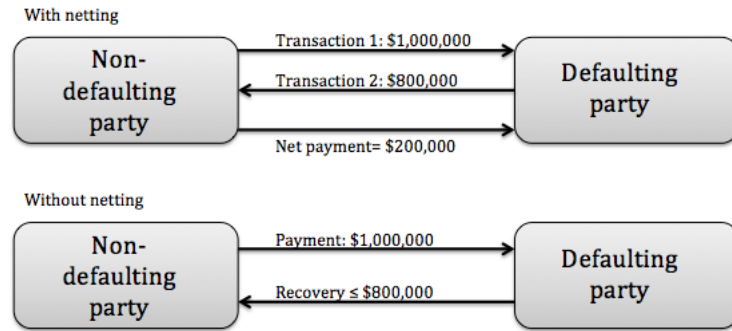


Figure 2.4.1: Illustration of how netting affects a transaction

often can not be traded readily the institution needs to hedge out of the position. If the institution executes an offsetting position with another counterparty, the market risk will be removed as required but there is still counterparty risk with respect to the original counterparty and the new counterparty. Whereas if the institution executes the reverse position with the original counterparty and there is an agreement of netting, then both market risk and counterparty risk will be offset.

The institution wishes to maintain a balanced book but every time it enters into a new transaction with a counterparty the institution takes on exposure to the transferred risk, to offset this exposure it enters into offsetting hedge transactions. In this way the institution avoids unwanted exposure (for example from currencies, interest rates and other sources of market risk).

When institutions hedge transactions it involves many counterparties all of which pose some risk of default and if a counterparty defaults they can no longer assume that the exposure is hedged and the dealer is now exposed to unanticipated market movements. In these situations netting is useful as it reduces the exposure that needs to be rebalanced. The result of these hedging and rebalancing transactions is a larger derivatives market

Netting reduces the overall exposure, but since CVA is non-additive for a portfolio of derivatives, it has to be calculated for the entire portfolio with the counterpart.

2.4.2 Collateral

A collateral agreement is an agreement between two counterparties to post collateral. An agreement is typically entered if the exposure is large and there is a clear risk that one of the counterparties can default. Typically collateral agreements are two-way which means that each counterparty is obliged to post collateral if the mark-to-market value (from their point of view) is negative.

The collateral agreement will have some predefined rules on when to post collateral. The *threshold* is the level the exposure has to exceed before any collateral is posted. A *minimum transfer amount* is the smallest amount of collateral that can be transferred. The minimum transfer amount is used to avoid the workload of small insignificant amounts of collateral and the size of the amount needs to represent a balance between risk mitigation and operational workload. Typically the posting of collateral is not continuous. The timescale on when collateral may be called and returned is denoted the *margin call frequency*. Some larger counterparties require daily margin calls.

The threshold and the minimum transfer amount are additive in the sense that the exposure must exceed the sum of the two before any collateral can be called.

Let us consider figure 2.4.2 which illustrates the impact of collateral on exposure. There are mainly three considerations. Threshold and minimum transfer amount have a negative granularity effect. Because of these parameters it is not always possible to call for the collateral required. The figure shows how this can lead to overcollateralization (where the collateral is higher than the exposure). Furthermore there is a time-delay in receiving the collateral. The figure also shows that if the collateral posted is not cash but instead a security, there will be some volatility in the collateral and there may be discussions about the value of the security. The basic idea of collateralization can simply be described with the following example. Party A makes a transaction with party B, then party A makes a mark-to-market (MtM) profit while party B makes a corresponding MtM loss. To mitigate the credit exposure due to the positive MtM party B posts some form of collateral to party A. This collateral can be cash or other forms of securities. Since collateral agreements often are bilateral (two-

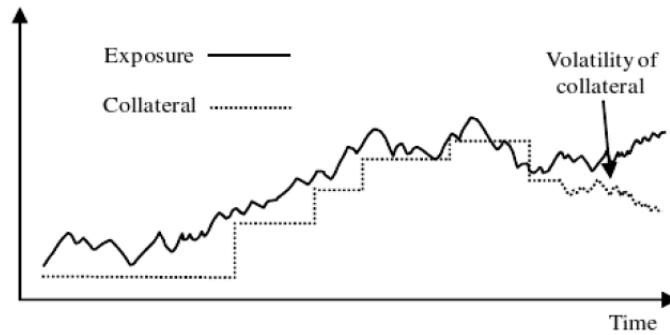


Figure 2.4.2: Illustration of the impact of collateral on exposure

way) it must be returned or posted in the opposite direction if the exposure decreases. So if the counterparty has a positive MtM value they call some collateral and if the counterparty has a negative MtM value they have to post collateral.

A collateral agreement can be compared to a mortgage loan in many ways. When a mortgage provider (the lender) gives a loan to a client (the borrower) to purchase a property then there is a risk that the borrower fails to make future mortgage payments (the principal and interest). Here the property will serve as collateral so if the borrower defaults the ownership of the property will be transferred to the mortgage provider (typically a bank or a mortgage institution).

Types of collateral

The major form of collateral is cash but the ability to post other forms of collateral is also preferable for liquidity reasons. Securities is another form of collateral. Before the credit crisis assets with minimal price volatility were assumed to be high quality assets but this is not the case any more. On the other hand in extreme market conditions cash tends to be in limited supply.

According to ISDA Margin Survey 2014⁹ cash represented 74.9% of the total collateral received in non-cleared OTC derivatives. Collateral in USD and EUR being the major currencies (representing together 91% of the cash collateral). Government securities by issuer represented 14.8% and other securities 10.3% of the total collateral received. Equities and corporate bonds are an example of other securities.

⁹ISDA Margin Survey 2014 [21]

Types of collateral agreements (CSA)

In the OTC derivatives market there are different types of collateral agreements. The International Swaps and Derivatives Association¹⁰ (ISDA) has within an ISDA Master agreement published a Credit Support Annex (CSA) which is one type of a collateral agreement. The CSA permits the parties entering the contract to mitigate their counterparty credit risk by agreeing to post various types of collateral. In April 2014 about 87% of collateral agreements on the non-cleared OTC derivatives market were ISDA agreements¹¹.

ISDA published in June 2013 a Standard Credit Support Annex¹² (SCSA). The SCSA deviates from the CSA as it seeks to standardize market practices in collateral management. Some of the differences between the two contracts is that the SCSA removes the embedded optionality in the existing CSA, supports the adoption of using OIS discounting and seeks to create a homogeneous valuation framework that aligns the mechanics and economics of collateralization between the bilateral and cleared OTC derivatives market¹³.

ISDA supports both CSA and SCSA and market participants have the option to decide whether to use the existing CSA or adopting the new SCSA. In general there are three types of CSAs:

- No CSA: When there is no collateral agreement between two counterparties. One reason could be if the credit quality for one of the institutions is far superior to the other. Another is if they can not commit to the operational and liquidity requirements that arise to a CSA.
- Two-way CSA: When there is a commitment for both counterparties to post collateral. A two-way CSA is typically beneficial for both counterparties and is the one most used.

¹⁰ISDA is a trade organisation for OTC derivatives practitioners which is designed to eliminate legal uncertainties and provide mechanisms on how to mitigate counterparty credit risk. It contains terms and conditions in an agreement between parties such as netting, collateral, definition of default and other termination events. It is a bilateral framework.

¹¹ISDA margin survey 2014 [21]

¹²ISDA Published 2013 Standard Credit Support Annex [22]

¹³These OTC derivatives are cleared through a central counterparty (CCP)

- One-way CSA: Some agreements are only one-way which are only beneficial for the collateral receiver. A one-way CSA can be worse than no CSA for the collateral giver because there is some additional risk.

Credit exposure with collateral

There are different scenarios where the institution has credit exposure when collateral is posted. Suppose that one of the parties in the derivatives contract defaults and that the non-defaulting party holds collateral posted by the defaulting party. The collateral can be used to compensate for the losses that might occur for the non-defaulting party. However the non-defaulting party might want to replace the outstanding transactions. The ISDA master agreement allows the non-defaulting party to claim the cost associated with the replacement. The cost is the mid-market value of the transactions at the time of the default (adjusted for the bid/offer spread that the non-defaulting party might incur when making the replacement with a third party). The cost of replacing the outstanding transactions can be either positive or negative.

Consider first the case where the replacement cost is positive (the portfolio has a positive value/positive exposure to the non-defaulting party). If the replacement cost is less than the posted collateral the non-defaulting party will suffer no loss and is required to turn back any excess collateral. Whereas if the replacement cost is greater than the posted collateral the non-defaulting party is an unsecured creditor for the collateral shortfall.

Next consider the case where the replacement cost is negative (the portfolio value of outstanding transactions is positive to the defaulting party, i.e. negative exposure to the non-defaulting party). If the collateral posted by the non-defaulting party is less the value of the outstanding transactions then the non-defaulting party is required to pay the collateral shortfall to the defaulting party. Whereas if the collateral posted by the non-defaulting party is greater than the value of the outstanding transactions from the point of view of the non-defaulting party then the non-defaulting party is an unsecured creditor for the excess collateral.

Let us focus on the case where the replacement cost is positive, this corresponds to

2.4. ADJUSTING THE PORTFOLIO VALUE

a positive credit exposure, X^+ . At time t the positive credit exposure where there is accounted for collateral is

$$X_C^+(t) = \max \{V(t) - C(t), 0\} \quad (2.4.1)$$

Where $C(t)$ represents the collateral and $V(t)$ is the value of the derivative (or portfolio of derivatives) at time t . Furthermore define the threshold for the institution as H_I and for the counterparty as H_c and the minimum transfer amount as MTA . Let the effective thresholds be defined as

$$H_I^e = H_I - MTA \text{ and } H_c^e = H_c + MTA$$

The collateral posted depends on whether it is a one-way (unilateral) or two-way (bilateral) collateral agreement. Consider first a unilateral agreement in the institution's favour, and assume the collateral is posted immediately. The effective threshold $H_c^e > 0$ and, the collateral is

$$C(t) = \max \{V(t) - H_c^e, 0\}$$

And the collateralized exposure is

$$X_C^+(t) = \max \{V(t) - C(t), 0\} = \begin{cases} 0 & \text{if } V(t) < 0 \\ V(t) & \text{if } 0 < V(t) < H_c^e \\ H_c^e & \text{if } V(t) > H_c^e \end{cases}$$

In a bilateral agreement where both parties have to post collateral, the collateral must be considered between time t and $t + \Delta t$. The institution's effective threshold is negative ($H_I^e < 0$) from the institution's point of view. The institution must post collateral when the portfolio value is below the threshold ($V(t) < H_I$). The collateral is

$$C(t) = \max \{V(t - \Delta t) - H_c^e, 0\} + \min \{V(t - \Delta t) - H_I^e, 0\}$$

The collateral $C(t)$ describes two future scenarios. The scenario described by the first term is where the institution receives collateral ($C(t) > 0$) and the scenario described in the last term is where the institution posts collateral ($C(t) < 0$). Note that both terms cannot be non-zero at the same time. The collateralized exposure is still given by (2.4.1).

Collateral example

Let us use an example to illustrate the amount of collateral that should be called in a two-way collateral agreement where we have specified the minimum transfer amount to \$50,000 and the threshold to \$500,000 and a rounding of \$5,000. Table 2.1 contains two examples. Example 1 gives a collateral calculation without any collateral held in the past and example 2 with collateral held in the past. The required collateral is calculated using this formula:

$$\text{Required collateral} = \max(V - H_I, 0) - \max(-V - H_c, 0) - C$$

V represents the current mark-to-market value of the relevant trades (the value of the portfolio), H_I and H_c is threshold for the institution and the counterparty respectively and C is the collateral that is already held. Example 1 has a portfolio value $V = \$653,167$, the required collateral is calculated as

$$\max(653,167 - 500,000; 0) - \max(-653,167 - 500,000; 0) - 0 = 153,167$$

which is above the minimum transfer amount ($153,167 > 50,000$), and after rounding this gives a collateral call at \$155,000. The institution therefore has to receive collateral from the counterparty.

Example 2 illustrates a collateral calculation where collateral has been held in the

2.4. ADJUSTING THE PORTFOLIO VALUE

Example 1: Collateral calculation

Portfolio value	\$653,167
Collateral held	-
Required collateral	\$153,167
Is the amount above the minimum transfer amount?	YES
Rounded amount that should be called	\$155,000

Example 2: Collateral calculation with existing collateral

Portfolio value	\$603,456
Collateral held	\$155,000
Required collateral	-\$51,544
Is the amount above the minimum transfer amount?	YES
Rounded amount	-\$50,000

Table 2.1: Examples of collateral calculations with and without collateral held already.

past and where the portfolio value has fallen to $V = \$603,456$. The required collateral is:

$$\begin{aligned} \max(603,456 - 500,000; 0) - \max(-603,456 - 500,000; 0) \\ - 155,000 = -51,544 \end{aligned}$$

In this example the institution has received collateral from the counterparty and since the exposure of the institution has dropped (the portfolio value has fallen) we are in the opposite case where the institution must return some collateral to the counterparty. Note that the institution is still exposed to the counterparty since the net exposure now is $603,456 - 155,000 = 448,456$ but must return collateral since it is lower than the threshold ($448,456 < 500,000$).

The effect of collateral on creditors

When collateral is posted in a derivatives transaction it has the advantage of mitigating the counterparty credit risk to the parties involved in this derivative transaction. However consider a situation where a defaulting party also has a liability to a loan

creditor. In this case there will be a risk transfer which we can illustrate with an example.

Example 1

A Defaulting Party has an outstanding derivatives transaction with a counterparty, which we will refer to as the Derivative Creditor with a liability equal to 50. Furthermore the defaulting party has a loan with a Loan Creditor, where the liability is equal to 100. This gives total liabilities equal to 150. Finally the total assets of the Defaulting Party is 100.

When there is no collateral posted in the derivatives transaction, the Derivative Creditor will receive $50/150 = 33\%$ in recovery and the Loan Creditor will receive $100/150 = 66\%$ in recovery.

If however there is full collateralization in the derivatives transaction. The Derivative Creditor will receive 100% (50/50) in recovery and left to the Loan Creditor is only $50/100=50\%$ in recovery.

This situation illustrates that in the event of default, when there is collateral posted, the Derivatives Creditor will receive a better recovery than the Loan Creditor. So the risk is just relocated.

In the following sections we suppose that no collateral is posted, to ease the calculations. We will turn back and look at collateral from another point of view and analyse the impact it has on the value of a derivative in section 3.5.3.

2.5 Adjusted value with the OIS and LIBOR

This section aims to analyse the consequences of choosing a different discount rate than the risk-free interest rate when valuing a derivatives transaction. The price of the derivative will be accounted for default risk by adjusting the value with CVA and DVA. Therefore recall the value of a derivatives portfolio

$$f = f_{nd} - \text{CVA} + \text{DVA} \quad (2.5.1)$$

When two different interest rates is used in the calculation of the portfolio value, it will be necessary to make adjustments to the components. Roughly this means that:

- 1) The no-default value f_{nd} will be calculated by using the risk-free rate and contains an adjustment to account for another discount rate
- 2) The credit value adjustment, CVA is calculated as $\text{CVA}_1 - \text{CVA}_2$
- 3) The debt value adjustment, DVA is calculated as $\text{DVA}_1 - \text{DVA}_2$

The purpose of the following subsection is to introduce and define these components. With these adjustments the portfolio value is

$$f = f_l - (\text{CVA}_1 - \text{CVA}_2) + (\text{DVA}_1 - \text{DVA}_2) \quad (2.5.2)$$

where f_l is the no-default value of a derivative when the risk-free rate is used to define the expected growth rates of asset prices, but is not used as the discount rate for the expected payoffs in this world.

In section 2.5.2 we will present an example and calculate the value of a derivatives transaction. We assume that the discount rate is LIBOR and the risk-free interest rate is the OIS rate. However there is nothing special about using LIBOR as the discount rate and any other yield curve can be used instead.

2.5.1 Adjustment with a risky discount rate

We will start by introducing a general approach of how an adjustment can be made if we use another discount rate than the risk-free interest rate.

When the LIBOR-OIS spread is the adjusted credit spread for a company then let $L_l(t)$ be the loss rate at time t and $s_l(t)$ be the zero-coupon LIBOR-OIS spread for maturity t . This means that in the general situation we have

$$1 - \int_0^T L_l(t)dt = \exp(-s_l(T)T)$$

and

$$\int_{t_1}^{t_2} L_l(t)dt = \exp(-s_l(t_1)t_1) - \exp(-s_l(t_2)t_2)$$

If LIBOR is used to define the loss rate for both the institution and the counterparty then the value of the portfolio is

$$\begin{aligned} f_l(0, T) &= f_{nd}(0, T) - \int_0^T f^+(t, T)L_l(t)dt + \int_0^T f^-(t, T)L_l(t)dt \quad \Leftrightarrow \\ f_{nd}(0, T) &= f_l(0, T) + \int_0^T f^+(t, T)L_l(t)dt - \int_0^T f^-(t, T)L_l(t)dt \end{aligned}$$

where the CVA and DVA components are represented by

$$\text{CVA} = \int_0^T f^+(t, T)L_l(t)dt \text{ and } \text{DVA} = \int_0^T f^-(t, T)L_l(t)dt$$

For the rest of the section we will ease notation by letting $f = f(0, T)$ and $f(t) = f(t, T)$.

Remember how we rewrote the bilateral adjusted value of the derivative in equation (2.5.1) and insert the above equation of f_{nd} , then we have

$$\begin{aligned}
f &= f_{nd} - \int_0^T f^+(t)L_c(t)dt + \int_0^T f^-(t)L_I(t)dt \\
f &= f_l + \int_0^T f^+(t)L_l(t)dt - \int_0^T f^-(t)L_l(t)dt - \int_0^T f^+(t)L_c(t)dt + \int_0^T f^-(t)L_I(t)dt \\
f &= f_l - \int_0^T f^+(t)[L_c(t) - L_l(t)]dt + \int_0^T f^-(t)[L_I(t) - L_l(t)]dt
\end{aligned}$$

Which is the value of the derivatives portfolio when it is adjusted for another discount rate. Let us make some definitions based on the above equations:

- $CVA1 = \int_0^T f^+(t)L_c(t)dt$ and represents the actual expected loss to the institution if their counterparty defaults (using OIS as discount rate)
- $CVA2 = \int_0^T f^+(t)L_l(t)dt$ and is the expected loss if the chosen discount curve (LIBOR) defined the counterparty's borrowing rates
- $DVA1 = \int_0^T f^-(t)L_I(t)dt$ which represents the actual expected loss to the counterparty from a default by the institution (using OIS as discount rate)
- $DVA2 = \int_0^T f^-(t)L_l(t)dt$ and is the counterparty's expected loss if the chosen discount curve (LIBOR) defined the institution's borrowing rates.

The second line in the above derivation of the portfolio value can therefore be rewritten as

$$\begin{aligned}
f &= f_l + CVA2 - DVA2 - CVA1 + DVA1 \\
f &= f_l - (CVA1 - CVA2) + (DVA1 - DVA2)
\end{aligned}$$

When making these adjustments, none of the three calculated values are correct (f_{nd} , CVA and DVA) but the net value f is. The next section introduces an example to illustrate the adjustments defined above. Important to note is that LIBOR is only used for discounting the expected derivative payoffs in the adjustment and we assume that the (risk-free) OIS rate is used to determine the risk-neutral expected payoffs.

Using LIBOR as the benchmark risk-free rate in some circumstances and OIS in other is liable to cause confusion.

2.5.2 An example

We will use a simple example to illustrate that the choice of discount rate has an effect on the derivatives price. Let us consider the case where the institution's portfolio contains a non-collateralized short position in a one-year forward contract to buy a non-dividend paying stock. The payoff to the buyer of the forward contract at maturity is

$$\text{Payoff} = S(T) - K$$

Where $S(T)$ denotes the stock price at time T and K denotes the delivery price specified in the contract. The institution's exposure is

$$X^+(t, T) = E \left[\exp \left(- \int_t^T r_s ds \right) \max\{S(T) - K, 0\} \right]$$

which remind us of a European call option. The counterparty's exposure to the institution is

$$X^-(t, T) = E \left[\exp \left(- \int_t^T r_s ds \right) \max\{K - S(T), 0\} \right]$$

which remind us of a European put option.

We wish to analyse the effect of using LIBOR as discount rate instead of the risk free rate (OIS). So we will distinguish between the discount rate r_{disc} and the risk-free rate r used for risk neutral valuation.

2.5. ADJUSTED VALUE WITH THE OIS AND LIBOR

The European call and put prices are calculated using the Black Scholes and Merton model¹⁴. With that in mind, let us find expressions for the no-default value, where f_t represents the no-default value when LIBOR is used as the discount rate and f_{nd} when the risk-free OIS rate is used as the discount rate.

No-default value

The value today of the institution's exposure at time t is

$$\begin{aligned} f_{nd}^+(t, T) &= \exp\left(-\int_0^t r_s ds\right) X^+(t, T) \\ f_{nd}^+(t, T) &= \exp\left(-\int_0^t r_s ds\right) E\left[\exp\left(-\int_t^T r(s) ds\right) \max\{S(T) - K, 0\}\right] \\ f_{nd}^+(t, T) &= E\left[\exp\left(-\int_0^T r(s) ds\right) \max\{S(T) - K, 0\}\right] \\ f_{nd}^+(t, T) &= Call^{BS}(T, r, S, K, \sigma_s) \end{aligned}$$

With the same approach we find that the value today of the counterparty's exposure at time t is

$$\begin{aligned} f_{nd}^-(t, T) &= \exp\left(-\int_0^t r_s ds\right) X^-(t, T) \\ f_{nd}^-(t, T) &= Put^{BS}(T, r, S, K, \sigma_s) \end{aligned}$$

¹⁴The Black Scholes price of a European call option is:

$$Call^{BS}(T, r, S, K, \sigma_s) = \Phi(d_1)S - \Phi(d_2)Ke^{-rT}$$

And the put price can be determined using the call price:

$$Put^{BS}(T, r, S, K, \sigma_s) = Ke^{-rT} - S + Call^{BS}(T, r, S, K, \sigma_s)$$

Where

$$\begin{aligned} d_1 &= \frac{1}{\sigma\sqrt{T}} \left[\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma_s^2}{2}\right)T \right] \\ d_2 &= d_1 - \sigma_s\sqrt{T} \end{aligned}$$

The inputs are the maturity date T , the risk-free rate r , the spot price of the underlying asset S , the strike price K and the volatility of the underlying asset σ_s .

We can find the no-default value of the derivative by remembering

$$\begin{aligned} f_{nd}(0, T) &= f_{nd}^+(0, T) - f_{nd}^-(0, T) \\ f_{nd}(0, T) &= Call^{BS}(T, r, S, K, \sigma_s) - Put^{BS}(T, r, S, K, \sigma_s) \end{aligned} \quad (2.5.3)$$

Equation (2.5.3) is the no-default value when the risk-free interest rate is used as the discount rate. Let us now find the no-default value if we use a different discount rate than the risk-free. The value today of the institution's exposure at time t is

$$f_l^+(t, T) = \exp\left(-\int_0^t r_{disc}(s)ds\right) E\left[\exp\left(-\int_t^T r_{disc}(s)ds\right) \max\{S(T) - K, 0\}\right]$$

With some rewriting we can use the standard Black Scholes formula for a call option

$$\begin{aligned} f_l^+(t, T) &= E\left[\exp\left(-\int_0^T r_{disc}(s)ds\right) \max\{S(T) - K, 0\}\right] \frac{\exp\left(-\int_0^T r(s)ds\right)}{\exp\left(-\int_0^T r(s)ds\right)} \\ f_l^+(t, T) &= \exp\left(-\int_0^T (r_{disc}(s) - r(s))ds\right) E\left[\exp\left(-\int_0^T r(s)ds\right) \max\{S(T) - K, 0\}\right] \\ f_l^+(t, T) &= \exp\left(-\int_0^T (r_{disc}(s) - r(s))ds\right) Call^{BS}(T, r, S, K, \sigma_s) \end{aligned}$$

With the same approach we find that the value today of the counterparty's exposure at time t is

$$\begin{aligned} f_l^-(t, T) &= \exp\left(-\int_0^t r_{disc}(s)ds\right) E\left[\exp\left(-\int_t^T r_{disc}(s)ds\right) \max\{K - S(T), 0\}\right] \\ f_l^-(t, T) &= \exp\left(-\int_0^T (r_{disc}(s) - r(s))ds\right) Put^{BS}(T, r, S, K, \sigma_s) \end{aligned}$$

2.5. ADJUSTED VALUE WITH THE OIS AND LIBOR

In the same way as before we can find the adjusted no-default value of a derivative

$$\begin{aligned}
f_l(0, T) &= f_{nd}^+(0, T) - f_{nd}^-(0, T) \\
f_l(0, T) &= \exp\left(-\int_0^T (r_{disc}(s) - r(s))ds\right) Call^{BS}(T, r, S, K, \sigma_s) - \\
&\quad \exp\left(-\int_0^T (r_{disc}(s) - r(s))ds\right) Put^{BS}(T, r, S, K, \sigma_s) \\
f_l(0, T) &= \exp\left(-\int_0^T (r_{disc}(s) - r(s))ds\right) \\
&\quad (Call^{BS}(T, r, S, K, \sigma_s) - Put^{BS}(T, r, S, K, \sigma_s))
\end{aligned} \tag{2.5.4}$$

Note that the risk-free rate (OIS) is used in the Black Scholes call and put prices and that the use of another discount rate lies in the adjustment

$\exp\left(-\int_0^T (r_{disc}(s) - r(s))ds\right)$. In the following to ease notation we will leave out most of the parameters in the call and put prices so, $Call^{BS}(T) = Call^{BS}(T, r, S, K, \sigma_s)$ and $Put^{BS}(T) = Put^{BS}(T, r, S, K, \sigma_s)$.

The credit risk adjustments

Let us turn to the adjustments CVA and DVA. With CVA_1 defined as the actual expected loss to the institution, we have

$$\begin{aligned}
CVA_1 &= \int_0^T f_{nd}^+(t, T) L_c(t) dt \\
&= \int_0^T Call^{BS}(T) L_c(t) dt
\end{aligned}$$

We will assume that the interest rates r_{disc} and r and the counterparty's credit spread s_c are constant. Then CVA_1 can be found by the following approximation

$$CVA_1 \approx \sum_{t=\Delta t}^T Call^{BS}(T) (\exp(-s_c(t - \Delta t)) - \exp(-s_c t)) \tag{2.5.5}$$

If the discount rate is the risk-free rate then $CVA = CVA_1$. However, if LIBOR is used as discount rate then $CVA = CVA_1 - CVA_2$ where CVA_2 is the expected loss if LIBOR

is the institution's borrowing rate, i.e.

$$\begin{aligned} \text{CVA}_2 &= \int_0^T f_{nd}^+(t, T) L_l(t) dt \\ &= \int_0^T \text{Call}^{BS}(T) L_l(t) dt \end{aligned}$$

We can find CVA_2 using the following approximation

$$\text{CVA}_2 \approx \sum_{t=\Delta t}^T \text{Call}^{BS}(T) (\exp(-s_l(t - \Delta t)) - \exp(-s_l t)) \quad (2.5.6)$$

where s_l is the spread between LIBOR and the risk-free rate.

Likewise for the DVA adjustment. With DVA_1 as the actual expected loss to the counterparty caused by default of the institution, we have

$$\begin{aligned} \text{DVA}_1 &= \int_0^T f_{nd}^-(t, T) L_I(t) dt \\ &= \int_0^T \text{Put}^{BS}(T) L_I(t) dt \\ &\approx \sum_{t=\Delta t}^T \text{Put}^{BS}(T) (\exp(-s_I(t - \Delta t)) - \exp(-s_I t)) \end{aligned} \quad (2.5.7)$$

where s_I is the institution's credit spread, and likewise this is assumed to be constant. If the discount rate is the risk-free rate then $\text{DVA} = \text{DVA}_1$. However, if LIBOR is used as discount rate then $\text{DVA} = \text{DVA}_1 - \text{DVA}_2$ where DVA_2 is the expected loss if LIBOR is the institutions borrowing rates.

$$\begin{aligned} \text{DVA}_2 &= \int_0^T f_{nd}^-(t, T) L_l(t) dt \\ &= \int_0^T \text{Put}^{BS}(T) L_l(t) dt \\ &\approx \sum_{t=\Delta t}^T \text{Put}^{BS}(T) (\exp(-s_l(t - \Delta t)) - \exp(-s_l t)) \end{aligned} \quad (2.5.8)$$

With equation (2.5.3), (2.5.4), (2.5.5), (2.5.6), (2.5.7) and (2.5.8) we have all the formulas we need to calculate the value of the institution's portfolio that contains a

	OIS discounting	LIBOR discounting
Call	13.283	13.270
Put	10.328	10.318
No-default value, f_{nd}	2.955	2.952
CVA ₁	0.263	0.263
DVA ₁	0.052	0.052
CVA ₂		0.013
DVA ₂		0.010
CVA	0.263	0.250
DVA	0.052	0.041
Net portfolio value, f	2.744	2.744

Table 2.2: Normal market conditions, input parameters are $S = 100$, $K = 100$, $\sigma_s = 0.3$, $r = 0.03$, $r_{disc} = 0.031$, $s_c = 0.02$ and $s_d = 0.005$

non-collateralized short position in a one-year forward contract to buy a non-dividend paying stock. The portfolio value is calculated using equation (2.5.1) when OIS is the discount rate and equation (2.5.2) when LIBOR is the discount rate.

Numerical results

The calculations are based on a stock price (S) and a strike price (delivery price) (K) that both equals 100, the volatility of the stock (σ_S) is 30% and the risk-free OIS rate (r) is 3% in both cases. The maturity of the forward contract is one year, so $T = 1$.

Consider case 1 which corresponds to normal market conditions where the discount rate (LIBOR) is $r_{disc} = 3.1\%$ for all maturities. The institution's and the counterparty's credit spread are $s_d = 0.5\%$ and $s_c = 2.0\%$ respectively, for all maturities.

Case 2 on the other hand corresponds to stressed market conditions where the discount rate (LIBOR) is $r_{disc} = 4.5\%$ for all maturities. The institution's and the counterparty's credit spread are $s_d = 2\%$ and $s_c = 3\%$ respectively, for all maturities.

It is assumed that the credit spread is adjusted for any difference that might be in recovery rates on derivatives and bonds. Furthermore all rates will be continuously compounded and we use 200 iterations ($\Rightarrow \Delta t = 0.005$) to calculate the values (the one year life of the forward contract is divided into 200 equal time steps). The no-default value f_{nd} , CVA and DVA are calculated with and without the adjustment.

Table 2.2 considers case 1 with normal market conditions. In both columns the

risk-free OIS rate is used to define the risk-neutral growth rate whereas the discount rate varies. In the column *OIS discounting* the OIS rate is used as the discount rate and no adjustment is made. In the column *LIBOR discounting* the discount rate used is LIBOR and therefore the adjustment CVA2/DVA2 is made.

Table 2.3 considers case 2 with stressed market conditions and the columns are defined in the same way as in table 2.2.

The first thing to notice in both table 2.2 and 2.3 is that when LIBOR is used as the discount rate, the values of f_{nd} , CVA and DVA are incorrect compared to the case where OIS is used as the discount rate. However the net portfolio value f is correct. So if the adjustment to the calculation of CVA and DVA in the LIBOR column is not made, the portfolio value f will be incorrect.

	OIS discounting	LIBOR discounting
Call	13.283	13.086
Put	10.328	10.174
No-default value, f_{nd}	2.955	2.911
CVA ₁	0.393	0.393
DVA ₁	0.205	0.205
CVA ₂		0.198
DVA ₂		0.154
CVA	0.393	0.195
DVA	0.205	0.051
Net portfolio value, f	2.767	2.767

Table 2.3: Stressed market conditions, input parameters are $S = 100$, $K = 100$, $\sigma_s = 0.3$, $r = 0.03$, $r_{disc} = 0.045$, $s_c = 0.03$ and $s_d = 0.02$

In table 2.4 we look at the case where LIBOR is used both as the discount rate and the risk-neutral growth rate when calculating the no-default value. The OIS rate is only used to calculate CVA₂ and DVA₂ where the credit spread is calculated as LIBOR-OIS. The *Error* represents the percentage price error in the calculated net portfolio value to the institution.

The column *Case 1* represents the case with normal market conditions and the *Error* row is calculated compared to the OIS discounting net portfolio value in table 2.2. The column *Case 2* represents the case with stressed market conditions and the

2.5. ADJUSTED VALUE WITH THE OIS AND LIBOR

Error is calculated compared to the *OIS discounting* net portfolio value in table 2.3. It is easy to note that the percentage price error is larger in case 2 with stressed market conditions. Partly because the institution's and the counterparty's credit spread is larger than in case 1 and partly because the discount rate, LIBOR is 4.5% where it is only 3.1% in case 1, so there is a harder discounting. However both cases result in large price errors.

	Case 1	Case 2
Call	13.330	13.991
Put	10.277	9.591
No-default value, f_{nd}	3.052	4.400
CVA ₁	0.264	0.414
DVA ₁	0.051	0.190
CVA ₂	0.013	0.208
DVA ₂	0.010	0.143
CVA	0.251	0.205
DVA	0.041	0.047
Net portfolio value, f	2.843	4.242
Error	3.60%	53.29%

Table 2.4: Case 1: $S = 100$, $K = 100$, $\sigma_s = 0.3$, $r = 0.03$, $r_{disc} = 0.031$, $s_c = 0.02$ and $s_d = 0.005$ and Case 2: $S = 100$, $K = 100$, $\sigma_s = 0.3$, $r = 0.03$, $r_{disc} = 0.045$, $s_c = 0.03$ and $s_d = 0.02$

This example shows that if we use a different discount rate than the risk-free rate, we can still obtain the correct value f by adjusting the no-default value with CVA1-CVA2 and DVA1-DVA2.

It also shows that if another interest rate is used both as discount rate and as risk-free rate this will lead to a large price error. However, these errors will not be as large if forward prices observed in the market is used in pricing instead of spot prices. Then the risk-neutral growth rate will not be an input to the pricing model.

2.6 Concluding remarks

The overnight indexed swap (OIS) rate is the best proxy at the moment for a risk-free rate. But some institution's continue to use LIBOR as the discount rate for non-collateralized transactions. LIBOR is used as discount rate because it reflects the credit risk and borrowing rates of the institution and not because it is a good proxy for the risk-free rate. The derivatives discounting dilemma has not been present until the financial crisis because LIBOR used to be an acceptable proxy for the risk-free rate. However for collateralized transactions, the OIS rate is accepted as discount rate because the most common rate paid on collateral is the OIS rate.

We can conclude that the LIBOR zero curve (or any other zero curve) can be used to define the discount rates when the no-default value of a non-collateralized portfolio is calculated. However if LIBOR is used as discount rate this leads to necessary adjustments in the calculations of CVA and DVA. Using another discount rate than the risk-free rate makes the calculations unnecessarily complicated and is liable to cause confusion.

As shown in an example, if LIBOR is used both as discount rate and as risk neutral growth rate the valuation of derivatives can lead to large pricing errors.

Furthermore the use of LIBOR for non-collateralized transactions and OIS for collateralized transactions does not make sense and is a violation of the law of one price. Since from an economic perspective the no-default value of a collateralized transaction should be the same as the no-default value of a non-collateralized transaction.

In practice most have now accepted OIS both as discount rate and as risk neutral growth rate. But they still feel the need to take funding costs into account. This has led to incorporating a funding value adjustment, which we will look into in the next chapter.

2.6. CONCLUDING REMARKS

Chapter 3

Funding Value Adjustment

As the discount rate in derivatives valuation has shifted from LIBOR to OIS, the discount rate no longer reflects the institution's funding cost. The funding cost is relevant for derivative traders in a financial institution, since they measure the profitability of a project or derivative as profit less expenses. Here the funding cost is included in expenses. When a trader needs funding he turns to the funding desk and is charged the (average) funding cost by the treasuries desk.

This way to measure performance has the consequence that for derivatives with an adequate low return, trades that require funding show a cost whereas trades that generate funding show a benefit. When the risk free rate is used for discounting, the traders way to account for this are with a funding value adjustment (FVA) to the derivative price.

A funding value adjustment has the consequence that participants no longer agree on a price; the funding cost is individual for each participant. It also raises a debate for whether the derivative price should depend on it's risks or it's costs.

The nature of funding costs is relevant for whether or not a FVA makes economic sense. One way to achieve funding is to issue bonds, then the bond yield spread¹ reflects the funding costs. The bond yield spread is and has been subject to a lot of research. A result of this research is that default risk is considered the main influence on the

¹By bond yield spread we mean the spread between the excess rate and the risk-free rate. This spread is often referred to as the credit spread because credit risk is considered the main reason why there is a spread.

yield spread. The intuition is that, if the institution has a high possibility to default it has to pay a high yield to borrow money. Another factor that can explain the yield spread is liquidity risk with the intuition that, if the bond issued by the institution is illiquid they might have to pay a higher yield. However, liquidity is considered to have a small influence on the yield spread.

The reason why the nature of funding costs is relevant is that it is a general principle in finance that the valuation of an investment should depend on its risk and not on its costs. If funding costs are caused by default risk, adjusting the derivatives price with a FVA only brings information that is already covered by a credit and debt value adjustment. This can lead to double counting of default risk and possibly arbitrage opportunities. If however, a significant amount of the funding costs is caused by liquidity risk, adjusting the derivatives price with a FVA brings the price closer to the economic value.

From an economic point of view an institution's funding costs exist as a compensation for the fact that in the event of default the institution experiences a gain. If funding costs are caused by default risk, the funding costs and gain in the event of default offset each other (they break even). However, if the institution does not acknowledge the gain in the event of default and wish to break even while being alive a FVA is necessary.

Accounting is based on a principle of fair value of assets. Accounting standards has not accepted FVA and FVA is therefore not part of the reported result. If the institution incorporates FVA in derivatives pricing, there is a difference between the internal performance measure and reported results. This difference is a problem.

Funding costs are only present when the derivative or portfolio is non-collateralized² since the rate paid on collateral is close to the risk-free rate. Regulation developed post crisis such as EMIR and the Dodd-Franc Act (and Basel III) aim to collateralize OTC derivatives either through collateral agreements or central clearing (CCP). This development will make an FVA unnecessary. However, when collateral is posted another adjustment to the derivative price needs to be taken into account. A collateral

²Assuming that if it is collateralized, it is a bilateral agreement

rate adjustment (CRA) accounts for the difference between the contractually defined rate paid on collateral and the rate that reflects the risk of the collateral.

The chapter is structured as follows. Section 3.1 describes the need for funding and funding costs and analyses the three special cases where funding costs can be used as discount rate. Then we will turn to the funding value adjustment and how it can be incorporated in the BSM arguments in section 3.2. Section 3.3 adjusts the price of a derivative with both default risk and funding costs using a hedging argument. Furthermore, we look at the economic perspective and include liquidity considerations. Double counting leading to arbitrage opportunities will be discussed in section 3.4. Finally section 3.5 describes the future of counterparty credit risk by looking into regulation, central counterparties and CRA.

3.1 Funding and funding costs

Since the financial crisis it is no longer possible for financial institutions to borrow (and lend) close to the risk-free rate, the credit risk premiums of large financial institutions have become too big to ignore. To obtain funding has become more difficult and you might have to post collateral in order to receive it.

The cost of funding depends on the attainability of the asset which needs to be funded. If the asset we wish to fund can be repoed, the funding cost is the repo rate which is close to the OIS rate. With derivatives it is often the case that the underlying asset can be repoed. But the derivative itself can usually not be repoed. So the institution face two different funding rates:

- r_s , funding of the underlying asset which can be repoed
- r_d , funding of derivatives

Let us bring an example to illustrate when there is a need of funding.

Example 2 (When there is funding costs)

Consider an institution trading an ATM swap with a fully collateralized counterpart (counterpart A). To hedge this swap transaction the institution makes a transaction in a reverse swap with another fully collateralized counterpart (counterpart B). In this situation all the cash flows from the swaps will match and so will the collateral. For example if the swap with counterpart A is negative from the point of view of the institution, the institution must post collateral to A. The institution gets the collateral from its hedged swap transaction with counterpart B and vice versa.

Now suppose that in the swap transaction with counterpart A only the institution has to post collateral and that the swap transaction with counterpart B still is fully collateralized. If the swap with counterpart A is negative for the institution then it has to post collateral which it will get from the hedged counterpart B. Whereas if the swap is positive with A then it is negative with its hedged counterpart B and the institution then has to post collateral to B. Since the institution does not get any collateral from

A it has to borrow the collateral and as an example this can be done at $r_d = r + 1\%$. This situation describes the need for funding.

A third scenario is the situation where there is no collateral agreement with counterpart A and the institution is still fully collateralized in the hedge with counterpart B. If the swap with A is negative for the institution it will be positive in the hedge with B where the institution receives collateral and since it does not have to post collateral to A the institution will have a surplus of funds. Whereas in the opposite situation the institution will have a deficit.

The above example clearly shows when there is a need for funding in the case where either one or both of the counterparts have to post collateral. Let us look at another reason why traders are motivated to take their funding costs into account when determining the prices they are willing to trade. The next example also illustrates the two different funding rates.

Example 3 (How funding costs can affect derivatives prices)

*This example illustrates how funding costs might effect the way derivatives are priced by institutions. An institution's client wants to enter a forward contract to buy a non-dividend-paying stock with a maturity of one year. Consider the transaction from the institution's point of view. The institution sells a forward contract and has to deliver a stock in one year. To hedge that position the institution will buy the stock today so that she is able to deliver it to her client in one year. Suppose that the current stock price is $S = 100$ and to fund this equity purchase the institution must pay a funding rate at $r_s = 4\%$. If the institution has to make a profit the delivery price in one year must be higher than the current stock price compounded forward at the funding rate which is $100 * 1,04 = 104$. This means that the delivery price at which the trader is willing to sell the forward contract in one year reflects the way the institution has to fund the underlying asset.*

Typically the institution assumes that the underlying asset can be funded through a repurchase agreement (and therefore the repo rate). A repurchase agreement is an agreement where the institution sells a security (here the stock) to a purchaser and

3.1. FUNDING AND FUNDING COSTS

then has to buy the security back at a specified price at a future date in time. The price the institution pays is represented by the repo rate and therefore the stock is said to be funded at the repo rate (in this example $r_s = 4\%$).

Suppose now that the delivery price of the stock is 106 and let X be the price of the forward contract. This cash outflow must also be funded by borrowing. Let us set the funding rate of the derivative (the forward contract) at $r_d = 5\%$. The year-end profit will be calculated as the [Delivery price]-[Forward compounded price of the underlying asset]-[Forward compounded price of the derivative]. In this example this gives $106 - 104 - 1.05X$ which means that the price of the forward contract will be no more than $2/1.05 = 1.905$. The present value of the forward contract is therefore determined by discounting the payoff on the contract at the rate at which the position in the derivative is funded.

The example illustrates that there exist two rates in the valuation of derivatives, the rate to fund the underlying asset, r_s and the rate to fund the derivative, r_d .

Let us, for now, assume that the funding costs are caused by default risk. Then there exist some cases where the funding rate can be used as discount rate in derivatives valuation and thereby account for the default risk of parties entering the contract.

3.1.1 Funding costs as discount rate

There exist three special cases where the funding rate can be used as a discount rate and thereby account for the counterparty credit/default risk in derivatives valuation. We have actually already seen two of them when we showed how the BSM arguments can be used when accounting for default risk in derivatives valuation in section 2.3.

1. The portfolio is an asset to the institution

Let us start by looking at a portfolio which promises a single positive payoff V to the institution at time T (and a negative payoff to the counterparty). This is the case where an institution buys a discount bond issued by the counterparty and where the portfolio always is an asset to the institution. We assume that in the event of default,

the bond is treated like a derivative, and the amount claimed on an uncollateralized derivative exposure is the no-default value. This means that the value today of the counterparty's exposure to the institution at time t , $f^-(t) = 0$ and the value today of the institution's exposure to the counterparty at time t is the no-default value, $f^+(t) = f_{nd}$. Based on the bilateral adjustment, we then have

$$\begin{aligned} f &= f_{nd} - \text{CVA} + \text{DVA} \\ f &= f_{nd} - \int_0^T f^+(t) L_c(t) dt + \int_0^T f^-(t) L_I(t) dt \\ f &= f_{nd} - \int_0^T f_{nd} L_c(t) dt \\ f &= f_{nd} \left(1 - \int_0^T L_c(t) dt \right) \end{aligned}$$

which is the value of the derivative expressed in terms of the no-default value and the loss rate for the counterparty ($L_c(t) = Q_c(t)(1 - R_c(t))$). In the appendix we show how this loss rate can be estimated from the bond credit spread,

$$1 - \int_0^T L_c(t) dt = \exp(-s_c(T)T)$$

If we use this specification of the loss rate the value of the derivative is

$$f = f_{nd} \exp(-s_c(T)T)$$

where the bond credit spread is defined as $s_c(t) = r_c(t) - r(t)$, r_c represents the counterparty's funding rate and r is the risk-free rate. f_{nd} is the expected payoff V under the risk neutral measure, discounted by the risk-free rate. Therefore, inserting $(V \exp(-r(T)T))$ instead of f_{nd} we get

$$\begin{aligned} f &= V \exp(-s_c(T)T) \exp(-r(T)T) \\ f &= V \exp(-r_c(T)T) \end{aligned}$$

which is the price of the derivative discounted with the counterparty's funding rate.

3.1. FUNDING AND FUNDING COSTS

This shows that the counterparty's funding rate from issuing bonds can be used as discount rate when the derivative is an asset to the institution and thereby adjust for default risk.

This result can be generalized to a portfolio consisting of a series of expected positive payoffs v_1, v_2, \dots, v_n . Let us define the no-default value of the i th payoff as

$$f_{nd}^i(t) = v_i \exp(-r_i(t_i - t)) \quad \text{for } t_i \geq t$$

and 0 for $t_i < t$. Then the no default value of the portfolio at time t is

$$f_{nd}(t) = \sum_{i=1}^n f_{nd}^i(t)$$

Again from the bilateral adjustment of the portfolio value,

$$\begin{aligned} f(0) &= f_{nd}(0) - \int_0^T f^+(t) L_c(t) dt + \int_0^T f^-(t) L_I(t) dt \\ f(0) &= f_{nd}(0) - \int_0^T \exp(-rt) f_{nd}(t) L_c(t) dt \\ f(0) &= f_{nd}(0) - f_{nd}(0) \int_0^T L_c(t) dt \\ f(0) &= \sum_{i=1}^n f_{nd}(0)^i \left(1 - \int_0^T L_c(t) dt \right) \\ f(0) &= \sum_{i=1}^n f_{nd}(0)^i \exp(-s_c(t_i) t_i) \\ f(0) &= \sum_{i=1}^n v_i \exp(-[r + s_c(t_i)] t_i) \end{aligned}$$

Hence with a portfolio consisting of a series of expected positive payoffs, the price adjusted for counterparty credit risk can be found by using the funding rate as discount rate.

2. The portfolio is a liability to the institution

Consider now a portfolio which promises a single negative payoff V to the institution at

time T (and a positive payoff to the counterparty). This corresponds to the case where a counterparty buys a discount bond issued by the institution. When the portfolio always is a liability to the institution, and the bond is treated like a derivative in the event of default, we then have $f^-(t) = -f_{nd}$ and $f^+(t) = 0$. Again based on the bilateral adjustment of the portfolio,

$$\begin{aligned}
 f &= f_{nd} - \text{CVA} + \text{DVA} \\
 f &= f_{nd} - \int_0^T f^+(t) L_c(t) dt + \int_0^T f^-(t) L_I(t) dt \\
 f &= f_{nd} - \int_0^T f_{nd} L_I(t) dt \\
 f &= f_{nd} \left(1 - \int_0^T L_I(t) dt \right)
 \end{aligned}$$

which is the price of the derivative expressed in terms of the no-default value and the loss rate for the institution ($L_I(t) = Q_I(t)(1 - R_I(t))$). The appendix shows how this loss rate can be estimated by the bond credit spread, and therefore we get

$$f = f_{nd} \exp(-s_I(T)T)$$

where the credit spread is defined as $s_I(t) = r_I(t) - r(t)$, represented by the institution's funding rate r_I and the risk-free rate r .

The no-default value, f_{nd} is the expected payoff V under the risk neutral measure, discounted by the risk-free rate, this gives

$$\begin{aligned}
 f &= V \exp(-r(T)T) \exp(-s_I(T)T) \\
 f &= V \exp(-r_I(T)T)
 \end{aligned}$$

which is the price of the derivative discounted by the institution's funding rate. When the derivative is a liability to the institution the institution's funding rate from issuing bonds can be used as discount rate and thereby adjust for default risk.

This result can be generalized to a portfolio consisting of a series of expected

3.1. FUNDING AND FUNDING COSTS

negative payoffs v_1, v_2, \dots, v_n . Let us define the no-default value of the i th payoff as

$$f_{nd}^i(t) = v_i \exp(-r_i(t_i - t)) \quad \text{for } t_i \geq t$$

and 0 for $t_i < t$ and the no default value for the portfolio at time t is $f_{nd}(t) = \sum_{i=1}^n f_{nd}^i(t)$. From the bilateral adjusted portfolio value we get

$$\begin{aligned} f(0) &= f_{nd}(0) - \int_0^T f^+(t) L_c(t) dt + \int_0^T f^-(t) L_I(t) dt \\ f(0) &= f_{nd}(0) - \int_0^T \exp(-rt) f_{nd}(t) L_I(t) dt \\ f(0) &= f_{nd}(0) - f_{nd}(0) \int_0^T L_I(t) dt \\ f(0) &= \sum_{i=1}^n f_{nd}(0)^i \left(1 - \int_0^T L_I(t) dt \right) \\ f(0) &= \sum_{i=1}^n f_{nd}(0)^i \exp(-s_I(t_i) t_i) \\ f(0) &= \sum_{i=1}^n v_i \exp(-[r + s_I(t_i)] t_i) \end{aligned}$$

This means that for a portfolio consisting of a series of expected negative payoffs, the price adjusted for counterparty credit risk can be found by using the funding rate as discount rate.

3. The institution and counterparty has identical loss rates

In this case we will assume that the portfolio can be an asset or a liability to the institution, but the institution and the counterparty have identical loss rates $L_c(t) = L_I(t) = L(t)$. As with the other two cases, we will start by a portfolio which consists of a single expected payoff V at time T .

Since we have defined the credit exposure as $X^+(t) = \max(V(t), 0)$ and $X^-(t) = \min(V(t), 0)$ the respective prices of the exposure are

$$f^+ = \exp(-rT)E[\max\{V, 0\}] \quad \text{and}$$

$$f^- = \exp(-rT)E[-\min\{V, 0\}]$$

With f_{nd} as the expected payoff V under the risk neutral measure, discounted by the risk-free rate, $f_{nd} = \exp(-rT)E[V]$, we have $f_{nd} = f^+ - f^-$. A bilateral adjustment of the portfolio is then

$$f = f_{nd} - \int_0^T f^+(t)L_c(t)dt + \int_0^T f^-(t)L_I(t)dt$$

$$f = f_{nd} - \int_0^T f^+(t)L(t)dt + \int_0^T f^-(t)L(t)dt$$

$$f = f_{nd} - \int_0^T (f^+(t) - f^-(t))L(t)dt$$

$$f = f_{nd} - \int_0^T f_{nd}L(t)dt$$

$$f = f_{nd} \left(1 - \int_0^T L(t)dt \right)$$

And with the loss rates estimated from the bond credit spreads

$$1 - \int_0^T L(t)dt = 1 - \int_0^T L_C(t)dt = \exp(-s_c(T)T) \quad \text{and}$$

$$1 - \int_0^T L(t)dt = 1 - \int_0^T L_I(t)dt = \exp(-s_I(T)T)$$

This implies that if the loss rates for the institution and the counterparty are identical, so are the credit spreads $s_c(t) = s_I(t) = s(t)$. We then get

$$f = f_{nd} \exp(-s(T)T)$$

$$f = V \exp(-r(T)T) \exp(-s(T)T)$$

$$f = V \exp(-r^*(T)T)$$

Just like the previous two cases, the (common) funding rate r^* from issuing bonds can

3.1. FUNDING AND FUNDING COSTS

be used as discount rate and thereby adjust for default risk.

This result can be generalized to a portfolio consisting of a series of expected payoffs v_1, v_2, \dots, v_n . Let us define the no-default value of the i th payoff as

$$f_{nd}^i(t) = v_i \exp(-r_i(t_i - t)) \quad \text{for } t_i \geq t$$

and 0 for $t_i < t$. The no-default value of the portfolio at time t is $f_{nd} = \sum_{i=1}^n f_{nd}^i(t)$.

And the bilateral adjusted portfolio value is

$$\begin{aligned} f(0) &= f_{nd}(0) - \int_0^T f^+(t) L_c(t) dt + \int_0^T f^-(t) L_I(t) dt \\ f(0) &= f_{nd}(0) - \int_0^T (f^+(t) - f^-(t)) L(t) dt \\ f(0) &= f_{nd}(0) - \int_0^T \exp(-rt) f_{nd}(t) L(t) dt \\ f(0) &= f_{nd}(0) - f_{nd}(0) \int_0^T L(t) dt \\ f(0) &= \sum_{i=1}^n f_{nd}(0)^i \left(1 - \int_0^T L(t) dt \right) \\ f(0) &= \sum_{i=1}^n f_{nd}(0)^i \exp(-s(t_i)t_i) \\ f(0) &= \sum_{i=1}^n v_i \exp(-[r + s(t_i)]t_i) \end{aligned}$$

So with a portfolio consisting of a series of expected payoffs with identical loss rates for the institution and the counterparty, the price adjusted for default risk can be found by using the common funding rate as discount rate.

3.2 Funding Value Adjustment

In derivatives valuation, the purpose of a funding value adjustment (FVA) is to adjust the value of a portfolio when the trader has a funding rate that is higher than the risk-free rate, $r_d > r$.

The funding value adjustment is the difference between the net present value when the risk-free rate is used for discounting and the net present value when the funding cost is used for discounting. The value of the derivatives portfolio is reduced by the adjustment caused by funding costs, so excluding the credit adjustments we have

$$f = f_{nd} - \text{FVA} \quad (3.2.1)$$

If funding costs (and benefits) are caused by default risk only, we will show that the funding value adjustment is

$$\text{FVA} = E \left[\exp \left(- \int_0^{\tau_I} r(s) ds \right) X(\tau_I) (1 - R_I(\tau_I)) 1_{\{\tau_I \leq T\}} \right] \quad (3.2.2)$$

Where X denotes the institution's exposure, R_I denotes the institution's recovery rate and τ_i denotes the the institution's default time. Notice that X also contains the negative exposure, so when the exposure is negative FVA represents a funding benefit. When the exposure is positive FVA represents a funding cost.

However, how the funding costs are incorporated in the above formula is not so clear. It will be easier to see when we incorporate funding costs into the BSM arguments and derive the FVA formula from this. So for now we will leave it at the above representation.

3.2.1 BSM with FVA

Let us extend the original BSM argument from section 2.3.1 and incorporate the institution's funding cost of a derivative. The funding cost will apply in the hedging portfolio.

For simplicity we will abstract from counterparty credit risk and assume that nei-

3.2. FUNDING VALUE ADJUSTMENT

ther the counterparty nor the institution can default (no CVA or DVA) and that the funding rate is not the risk-free for some unknown reason that is not default risk. Furthermore to simplify the analysis we will assume that interest rates and credit spreads are constant.

As in the original BSM the process assumed for the stock S is

$$dS = \mu S dt + \sigma S dz$$

And the process for the derivative f is

$$\begin{aligned} df &= \mu_f f dt + \sigma S \frac{\partial f}{\partial S} dz \\ \mu_f &= \frac{1}{f} \left[\frac{\partial f}{\partial t} + \mu S \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} \right] \end{aligned}$$

We consider the case where an institution has sold a derivative (a short position). This position in the derivative exposes the institution to market risk because of the uncertainty in the underlying stock. This market risk can be hedged by buying shares. So the portfolio, Π consists of a short position in the derivative and a position of $\partial f / \partial S$ in the stock,

$$\Pi = -f + \frac{\partial f}{\partial S} S$$

The change in the portfolio value is found by an application of Ito's lemma (as we did in section 2.3.1), so

$$d\Pi = -\frac{\partial f}{\partial t} dt - \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} dt \quad (3.2.3)$$

We distinguish between two kinds of funding rates. The rate at which the derivative is funded, denoted by r_d and the rate at which the stock (the underlying asset) is funded, denoted by r_s . Therefore instead of the portfolio earning the risk-free rate it earns r_d on the derivative and r_s on the stock. The change in the portfolio value becomes

$$d\Pi = \left[-r_d f + r_s S \frac{\partial f}{\partial S} \right] dt$$

It is usually assumed that the underlying asset (the stock S) can be repoed, but that the derivative itself cannot. This means that S is funded at the repo market with the repo rate which reasonably can be assumed to be the risk-free rate r . So $r_s = r$ and this gives

$$d\Pi = \left[-r_d f + r S \frac{\partial f}{\partial S} \right] dt \quad (3.2.4)$$

Combining equation (3.2.3) and (3.2.4) gives the following PDE

$$\begin{aligned} -\frac{\partial f}{\partial t} dt - \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} dt &= -r_d f dt + r S \frac{\partial f}{\partial S} dt \\ \frac{\partial f}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} + r S \frac{\partial f}{\partial S} &= r_d f \end{aligned} \quad (3.2.5)$$

The risk-neutral valuation argument shows that the solution of this partial differential equation is obtained by assuming that the expected return on the stock is $r_s = r$ and the expected payoff is discounted at r_d . So when the derivative provides a single possible payoff only at time T the solution to the PDE is

$$f^*(t, S_t) = e^{-r_d(T-t)} E_r [f(T, S_T)] \quad (3.2.6)$$

where S_t is the stock price at time t and E_r is the expectation taken over all paths that the stock price may follow when the stock's expected return is r . The price $f^*(t, S_t)$ can be seen as the institution's no default benchmark price. By this we mean that if the trader sells the derivative at this price, and there is no default risk, then the revenues generated are adequate to cover all the hedging costs including the funding costs.

Let us assume that the institution achieves funding by issuing bonds so that the bond yield reflects the funding rate. The bond yield spread to the risk free rate is then

3.2. FUNDING VALUE ADJUSTMENT

$s_d = r_d - r$. Using this spread we can rewrite so the funding adjusted price depends on the no-default price,

$$\begin{aligned} f^*(t, S_t) &= e^{-r_d(T-t)} E_r[f(T, S_T)] \frac{e^{-r(T-t)}}{e^{-r_d(T-t)}} \\ f^*(t, S_t) &= e^{-(r_d-r)(T-t)} f_{nd}(T, S_T) \\ f^*(t, S_t) &= e^{-s_d(T-t)} f_{nd}(T, S_T) \end{aligned}$$

As already mentioned, FVA is the reduction in the no-default value caused by funding costs, i.e. $f = f_{nd} - \text{FVA}$. Using this, FVA is

$$\begin{aligned} \text{FVA} &= f_{nd}(t, S_t) - f^*(t, S_t) \\ \text{FVA} &= f_{nd}(t, S_t) - f_{nd}(t, S_t) e^{-s_d(T-t)} \\ \text{FVA} &= f_{nd}(t, S_t) (1 - e^{-s_d(T-t)}) \end{aligned} \tag{3.2.7}$$

Up to now we have assumed that the funding costs were caused by something besides default risk. The reason for this was to simplify and clarify the effect of taking funding costs into account. In fact FVA has the same representation as above if funding costs were caused by default risk.³ If we assume that the bond yield spread is caused by default risk, we can use this spread to estimate the institution's loss rate (as shown in the appendix)

$$L(t) = (1 - e^{-s_d(T-t)})$$

where the loss rate at time t is defined as $L(t) \equiv Q(t)(1 - R(t))$. Furthermore let $f_{nd}(t, S_t)$ represent the value today of the exposure (both positive and negative exposure) at time t , i.e.

$$f_{nd}(t, S_t) = E \left[\exp \left(- \int_0^t r(s) ds \right) X(t) \right]$$

³In [4] Burgaard and Kjaer find the adjusted price when taking the institution's default risk, the counterparty's default risk and funding costs into account

Then when funding costs are caused by default risk, FVA is

$$\text{FVA} = \int_0^T E \left[\exp \left(- \int_0^t r(s) ds \right) X(t) \right] Q(t)(1 - R(t)) dt \quad (3.2.8)$$

The incorporation of a funding value adjustment has the inconvenient property that market participants no longer agree on a price. This can lead to some unexpected results, as illustrated by the next example.

Example 4 (How FVA can affect derivatives prices)

Suppose that a trader buys a one-year European call option on a non-dividend paying stock. The strike price $K = 100$, the stock price $S = 100$ and the volatility $\sigma_s = 30\%$. The payoff is discounted using the funding cost for the derivative $r_d = 5\%$ and equals the bank's average funding costs. The expected return on the stock is calculated using the funding rate for the stock which is assumed to be $r_s = r = 2\%$ (is equal to the risk-free rate as it is assumed that the stock can be funded using a repo transaction).

The European call option price is calculated using the standard Black Scholes as in section 2.5.2. The FVA adjustment leads to the price being calculated as,

$$e^{-(r_d-r)} \text{Call}^{BS}(T, r, S, K, \sigma_s)$$

This price represents the no-default value of the option adjusted for FVA, but before CVA and DVA adjustments. This gives a FVA-adjusted option price equal to 12.44.

In the interbank market most transactions need to be fully collateralized. The rate used to discount the payoff is now the OIS rate which is close to the repo rate. This means that the funding cost for the derivative equals the funding cost for the underlying asset, $r = r_d = 2\%$ and the Black Scholes price in this example will be 12.82. As you can see the FVA-adjusted option price at which the trader would sell to an end user is less than the price that would be offered to another institution in the interbank market. If it was possible to make a repo transaction on an option, 12.82 would also be the price offered in an uncollateralized market.

3.3 Adjusting for default risk and funding costs

Let us look at the derivatives price when we adjust for both default risk and funding costs. With the presence of funding costs there exist another element of DVA, which we will present next.

DVA is concerned with an institution's own default risk and FVA is concerned with funding. Accounting for funding costs when we wish to find the value of a derivative (or derivatives portfolio) leads to some changes in DVA; DVA then contains two components, DVA1 and DVA2. The value to an institution that might default on its derivatives obligations is defined as DVA1. Whereas we will refer to the value to an institution that might default on the funding required for the derivatives portfolio as DVA2. The benefit that DVA2 represents, arises because the counterparty absorbs some of the costs of the institution's poor performance instead of the institution itself. The total debt value adjustment is

$$\text{DVA} = \text{DVA1} + \text{DVA2}$$

DVA1 describes what we, until the introduction of funding costs, has known as DVA. We have actually already seen this decomposition of DVA when we showed how to calculate DVA when the discount rate is not the risk free rate in section 2.5.1.

If we account for both default risk and funding costs, the derivatives price is

$$f = f_{nd} - \text{CVA} + \text{DVA1} + \text{DVA2} - \text{FVA}$$

As you might expect, DVA and FVA are strongly related. We will look more into this in the following sections.

3.3.1 A hedging argument

It is possible to hedge FVA based on the relation between DVA2 and FVA and thereby avoid adjusting for funding costs. This is possible if the institution's credit spread is compensation for default risk, so for now we will assume this is the case.

It is a general finance valuation principle that a risk-free portfolio should earn the risk-free rate. When we included funding costs in Merton's hedging argument we ended up with a risk free portfolio not earning the risk free rate (see equation 3.2.4). If the derivative is a liability to the institution, the institution will earn a rate with a positive spread to the risk free rate. If the derivative is an asset to the institution, the institution will earn a rate with a negative spread to the risk free rate. From an economic perspective this spread is offset by the fact that the funding costs compensate for the possibility that the institution might default. DVA2 captures this compensation. The compensation or benefit for the institution is a cost for the counterparty since it absorbs some of the institution's poor performance.

When the derivative requires funding, FVA represents the compensation to lenders for the possibility that the institution defaults and this compensation is offset by the expected benefit an institution has from defaulting on its funding, represented by DVA2. However, if the derivative provides funding, DVA2 represents the compensation to lenders for the possibility that the institution defaults and this compensation is offset by the expected benefit an institution has from defaulting on its funding, represented by FVA.

Let the rate used to fund the derivative be r_d and the risk-free be r , and the institution's credit spread be $s_d = r_d - r$. Again we let the portfolio consist of a sold (stock) option and a delta hedge of the underlying asset. With a sold option the derivative is a liability to the institution and the counterparty has absorbed some of the institution's poor performance. This expected cost to the counterparty who is financing the derivative if the institution defaults, can be included in the hedge portfolio Π . The institution's benefit over a period from t to $t + \Delta t$ is Δt times the expected benefit amount given by $s_d f$ where f denotes the value of the derivative.

3.3. ADJUSTING FOR DEFAULT RISK AND FUNDING COSTS

Adding this benefit, the change in portfolio value becomes

$$d\Pi = \left[-r_d f + s_d f + rS \frac{\partial f}{\partial S} \right] dt$$

And if we insert the credit spread, we get

$$d\Pi = \left[-r_d f + (r_d - r)f + rS \frac{\partial f}{\partial S} \right] dt = \left[-rf + rS \frac{\partial f}{\partial S} \right] dt \quad (3.3.1)$$

which is a change in the value of the portfolio only represented by the risk-free rate.

The PDE is therefore the original BSM PDE:

$$\frac{\partial f}{\partial t} + rS \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf \quad (3.3.2)$$

This shows that when both DVA2 and FVA is included in the pricing of derivatives they actually cancel each other out and the price adjusted for FVA is equal to the price that is not adjusted for FVA.

In the case of a derivative that provides a single payoff at time T, we have that (same as equation (3.2.7))

$$\text{FVA} = \text{DVA2} = f_{nd}(t, S_t) - f_{nd}(t, S_t)e^{-s_d(T-t)}$$

So when we adjust for both default risk and funding costs, through CVA, DVA and FVA, the value of a portfolio is

$$f = f_{nd} - \text{CVA} + \text{DVA1} + \text{DVA2} - \text{FVA} \quad (3.3.3)$$

Because we have concluded that FVA=DVA2 the value of the portfolio is the bilateral adjusted portfolio value from section 2.4

$$f = f_{nd} - \text{CVA} + \text{DVA1}$$

What might not be so clear is how the added benefit that hedges the funding costs,

can be obtained in practice. As we have already mentioned, the benefit stems from a risk transfer. Suppose the institution has issued senior and junior bonds. When the counterparty absorbs some of the poor performance, the benefit can be obtained by buying some of the junior bonds back and issuing senior bonds instead⁴. However to actively trade own positions in such a way is difficult in practice.

Example 5 (Both FVA and DVA2 adjustments)

Continue example 4 where we have an option price that is not adjusted for FVA at 12.82 and a FVA-adjusted option price at 12.44 (both prices are before CVA and DVA adjustments). The FVA is the difference between the prices $12.82 - 12.44 = 0.38$ and since $FVA = DVA2$ they both equal 0.38 for a long option position and -0.38 for a short option position. When the institution buys the option, it has to fund it and in this case FVA reduces the value of the institution's portfolio by 0.38. Meanwhile because the institution might default on the funding for the option, DVA2 increases the option price with 0.38, which mean they have cancelled each other out. In the opposite case where the institution sells the option, the option provides funding and FVA increases the price with 0.38. Whereas DVA2 reduces the value by 0.38 because the benefits of a possible default are reduced.

We have seen how, over the entire life time, the institution's funding cost and benefit it has from a possible default break even. As long as the institution does not default it experiences a cost through funding, but in the event of default it experiences a gain. If the institution does not acknowledge this gain an FVA is necessary if it wish to account for funding costs. But from an economic point of view the gain should also be accounted for.

3.3.2 An economic perspective

From a general finance valuation perspective the value of a project should depend on the risk of the project and not its costs. As an example consider an institution that has

⁴In [2] Burgaard and Kjaer have suggested a strategy for a perfect hedge which also specifies the amount of senior and junior bonds

3.3. ADJUSTING FOR DEFAULT RISK AND FUNDING COSTS

a funding rate of 4.5% and the risk-free rate is 3%. Suppose the institution makes a risk-free investment earning 4%. Even though the earnings are lower than the funding rate, the institution should undertake the investment as long as it makes a positive value.

However, usually the institution's funding costs are more or less fixed, since the bonds have already been issued and the costs are not likely to change much. Both risky projects and almost risk free projects contribute to an average funding cost. In this way the projects offset each other so that different types of projects are enabled. If the average funding costs is more or less fixed it is tempting to use the same discount rate for all projects. But then it would be hard to distinguish the actual risk of a project since all projects do not carry the same risk. The consequence is that risk-free (or almost risk-free) projects will seem unattractive whereas the projects that are riskier than average will seem more attractive, which will lead to confusion in the risk of the investments.

Even though the return on the very-low-risk projects typically is less than the average funding costs, banks and other financial institutions do undertake these types of projects (for example those in government securities). Again the key point of this argument is that an investment should not depend on the funding costs but on the risk of the investment, risk-free or not.

When an institution obtains funding by issuing bonds, the bond yield spread reflects the institution's funding costs. In the case where the bond yield spread is due to default risk, a FVA will not add any new information to the economic value which is not already covered by CVA and DVA. Therefore CVA and DVA are referred to as economic value adjustments (EVAs) because they move the model value of a transaction closer to its economic value. Whereas FVA is referred to as an anti-EVA because it moves the calculations away from the economic value.

If however the bond yield spread reflects risks that are not already accounted for, it can be reasonable to take the funding costs into account. This could be because of liquidity risk which we will look more into in the next section.

Accounting is based on a concept on fair value. Furthermore in accounting, it is

required for institutions to make CVA and DVA adjustments to the reported values of their portfolios. However accounting standards do not accept FVA, because it does not reflect the economic "fair value" of a portfolio. We will look into how funding costs can influence whether a market participant wants to buy or sell at the fair value or not.

Fair Value

The fair value of an asset is the price where supply equals demand, i.e. where the market participants that want to buy the asset agree on a price with the market participants that want to sell it. IFRS 13 define fair value as: *the price that would be received to sell an asset or paid to transfer a liability in an orderly transaction between market participants at the measurement date.*

The question is if funding costs affect the decision of whether a market participant wants to buy/sell the asset or not. As an example take a share and consider two individuals A and B who want to buy this share. To fund the same share A can borrow at a rate of 2% and B can borrow at 6%. Because B's funding cost is higher than A's it is possible that this will influence their decision on whether to buy the share or not, but there is only one fair value of the share and that is its market price. Liquidity constraints, regulatory capital constraints and the ability to hedge the underlying risk are examples of other specifications that might effect the decision to buy or sell the asset, but not the fair value.

Accounting standards have accepted CVA and DVA, but not FVA. When counterparty credit risk (CVA and DVA) is taken into account in the valuation of derivatives the risk reflects the parties involved in the contract. Consider the equation where counterparty credit risk is adjusted for:

$$f = f_{nd} - \text{CVA} + \text{DVA}$$

Institutions with the same information and the same models should agree on a fair market price for a portfolio of derivatives entered into between two parties. The es-

3.3. ADJUSTING FOR DEFAULT RISK AND FUNDING COSTS

essential thing about the equation is that the no-default value of the portfolio to one party is the others negative value of the portfolio and the one party's CVA is the other party's DVA and therefore they should agree on a fair market price.

The problem can arise if one party novates a transaction to a new counterparty since the transaction will then reflect the new counterparty's default risk. Let us illustrate this point by an example.

Example 6 (Fair market price)

Consider a single transaction between two parties, A and B. The no-default value is 100, CVA is 5 and DVA is 10 from A's point of view. This gives a value to A on $100 - 5 + 10 = 105$ and by symmetry the value B's point of view is -105 . Assume A wants to novate the transaction to a new counterparty C, which has no default risk, this means C's DVA is zero. The value of the portfolio from C's point of view is $100 - 5 + 0 = 95$ and from B's point of view it is now -95 . The value of the transaction has dropped 10 and B is the one who gains from this. But if all costs and benefits will be taken into account B has to, at least in principle, pay A 10 when it announces it novates to C. So A will receive 10 from B and 95 from C. Both A and B therefore end up with the original portfolio value of 105 and -105 respectively.

The conclusion is that in the valuation of derivatives where counterparty credit risk is accounted, then as long as all costs and benefits are taken into account there will still be a fair market price between the parties in the contract.

When funding costs are on the other hand, included in the valuation and the portfolio value is found using the following equation:

$$f = f_{nd} - \text{CVA} + \text{DVA} - \text{FVA}$$

then it is possible that the parties in the contract might have different funding costs and they will no longer agree on a fair value or if they have the same funding costs they will continue to agree on a fair value. This is a troubling effect of FVA.

3.3.3 FVA and liquidity considerations

If the funding costs are not only caused by default risk, making a FVA can have economic meaning. Studies⁵ show that the non-default component of the yield spread is strongly related to measures of bond liquidity which is why we will continue to talk a little about liquidity.

Liquidity refers to how quickly an asset can be converted into cash with a little or no loss on the asset. Take for example a bond that is less liquid. An institution will require a higher return on this bond because it is not so tradable. Whereas if the bond was highly liquid the required return would be smaller. The liquidity varies from institution to institution which means that the liquidity premium will not be the same for different institution's. This can be a reason why there are different prices among institutions.

Suppose that the yield spread is represented by a default component and a non-default component. The default component α ($\alpha \leq 1$) is due to default risk and the non-default component $(1 - \alpha)$ is some non-economic factors (which could be because of liquidity risk). Let us examine this case using the BSM hedge argument.

The institution's benefit from a possible default during time Δt is $\alpha s_d \Delta t$ times the value of the derivative f , where s_d is the institution's spread. The institution's funding rate r_d is therefore represented by the risk-free rate plus the institution's spread ($r_d = r + s_d$).

When the institution has a short position in the derivative which is hedged by a position of $\partial f / \partial S$ in the stock, and the benefit from a possible default ($\alpha s_d f$) is included, the risk-free portfolio from equation (3.2.4) becomes

$$\begin{aligned} d\Pi &= \left[-r_d f + \alpha s_d f + r S \frac{\partial f}{\partial S} \right] dt \\ &= \left[-(r + s_d) f + \alpha s_d f + \frac{\partial f}{\partial S} \right] dt \\ &= \left[-(r + (1 - \alpha) s_d) f + \frac{\partial f}{\partial S} \right] dt \end{aligned}$$

⁵Longstaff, F., S. Mithal and E. Neis [16]

3.3. ADJUSTING FOR DEFAULT RISK AND FUNDING COSTS

By equating this change in the risk-free portfolio, with the change obtained by Ito's Lemma in equation 3.2.3, the PDE becomes

$$\frac{\partial f}{\partial t} + rS \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = [r + (1 - \alpha)s_d]f$$

In this case FVA is not equal to DVA2. This means that in the case where not all of the institution's spread is due to default risk, but only some fraction α , it will be correct to include an "FVA-DVA2" adjustment that is based on a spread of $(1 - \alpha)s_d$. From equation (3.2.7), the solution to a derivative that promises a single payoff at time T becomes

$$\text{FVA} - \text{DVA2} = f_{nd} [1 - e^{s_d(1-\alpha)(T-t)}]$$

From the institution's perspective the adjustment can be seen as a cost of doing business since they will not benefit for $(1 - \alpha)$ of the spread.

To make this adjustment it is necessary to estimate α at a point in time. One way to estimate the default component is by assuming that the CDS spread is equal to the default component of an institution. By making this assumption the non-default component is represented by the bond yield spread minus the corresponding CDS spread. This is the negative of the CDS basis,

$$\text{Non-default component} = \text{Bond yield spread} - \text{CDS spread}$$

Illiquidity does not have the same effect on CDS spreads as it has on the bond yield spread. When a bond becomes less liquid an investor will be less willing to buy the bond and then prices will decrease and the bond yield spread will increase which means that the size of the non-default component will increase. Whereas if a CDS become less liquid both buyers and sellers of protection will be less willing to trade CDSs and this effect will be insignificant.

There are different aspects of using the CDS basis as a measure of the non-default component in the yield spread but until now it is the best measure available. The

conclusion is that the "FVA-DVA2" adjustment reflects the non-default component of the yield spread and should be included if the yield spread is not only due to default risk.

3.4 Implications of FVA

It has implications to account for funding costs and default risk when the funding costs are caused by default risk. Because in this case both FVA and DVA1 is concerned with default risk. This can lead to double counting of default risk and arbitrage opportunities.

3.4.1 Double counting of default risk

We will analyse the situation where a portfolio value is adjusted for both FVA and DVA1. We base the analysis on two cases.

In case 1 we consider a simple transaction between an institution and a counterparty where an option is the only derivative traded. If the institution buys the option it will be an asset to the institution and never a liability, this means that $DVA1 = 0$.

In case 2 we consider the situation where the institution sells the option and will show that $DVA1 = FVA^*$, where $FVA^* = -FVA$.

Let us consider an institution that has m uncollateralized transactions with an end user and T represents the maturity of the longest transaction. We have that⁶

$$DVA1 = \int_{t=0}^T P(t)Q_I(t)[1 - R(t)]E \left[\max \left(\sum_{j=1}^m v_j(t), 0 \right) \right] dt$$

where $P(t)$ is the value of 1 received at time t (the discounting), $Q_I(t)$ is the probability of a default by the institution at time t , $R(t)$ is the recovery rate at time t , E denotes the risk-neutral expectation and $v_j(t)$ represents the value to the end user of the j th transaction at time t .

⁶Recall how we defined DVA in the derivatives valuation setup in equation 2.2.2

$$\begin{aligned} DVA &= \int_0^T E \left[\exp \left(\int_0^t r_s ds \right) X^-(t) \right] (1 - R_I(t))Q_I(t)dt \\ &= \int_0^T E [P(0, t)X^-(t)] (1 - R_I(t))Q_I(t)dt \end{aligned}$$

Recall how FVA is defined⁷ and let $FVA^* = -FVA$, i.e.

$$FVA^* = \int_{t=0}^T P(t)Q_I(t)[1 - R(t)]E\left(\sum_{j=1}^m v_j(t)\right) dt$$

hence FVA^* represents the benefit provided by FVA rather than the cost of the funding adjustment.

It always applies that

$$\max\left(\sum_{j=1}^m v_j(t), 0\right) \geq \left(\sum_{j=1}^m v_j(t)\right)$$

and because of this we can state that $DVA1 \geq FVA^*$. When the $v's$ are always positive, for example in our case 2 where the institution sells an option to an end user, $DVA1 = FVA^*$. A DVA1 adjustment in this case has the same effect as a FVA and they are both a benefit to the institution which makes the liability smaller. To avoid double counting the correct pricing will be to include only one of the components.

When the $v's$ are not always positive we have that $DVA1 > FVA^*$. Suppose an institution wants to make a transaction with a new end user (there are no existing transactions with this user). If the institution makes a FVA but not a DVA1 it will not price correctly because it will not fully reflect the DVA1 which is the end user's CVA. The institution will in this case be uncompetitive.

When an institution includes both adjustments in the valuation of a derivative (or derivatives portfolio) this can be seen as double counting. Let us use an example to illustrate the consequences of making both FVA and DVA1 adjustments when determining the price of a derivative.

⁷Remember how FVA is defined in equation 3.2.8:

$$\begin{aligned} FVA &= \int_0^T E\left[\exp\left(-\int_0^t r(s)ds\right)X(t)\right]Q(t)(1 - R(t))dt \\ &= \int_0^T E[P(0, t)X(t)](1 - R(t))Q(t)dt \end{aligned}$$

3.4. IMPLICATIONS OF FVA

Example 7 (Both FVA and DVA1 adjustments)

Let us continue example 4 with the European call option. In the case where the repo (close to the risk-free) rate of interest is 2% the Black Scholes price is calculated at 12.82. This is the price a trader would be willing to buy or sell the option at if she could fund at the "risk-free" rate.

Let us expand the example with both FVA and DVA1 adjustments to see how it affects the result. Start by considering an institution that wants to sell the option to a new end user (there are no outstanding transactions with the user), so seen from the other point of view the end user wants to buy the option. The price quoted will be $f_{nd} - DVA1 - FVA = 12.82 - 0.38 - 0.38 = 12.06$. In this price the CVA adjustment has not yet been taken into account. Because of the symmetry in CVA and DVA, the end user's CVA is equal to the institution's DVA1 which is 0.38. Therefore the price adjusted for CVA, DVA1 and FVA is $12.06 + 0.38 = 12.44$ (note that CVA is added since it is calculated from the end user's point of view). Since this price is less than 12.82, the price the institution will quote (if the end user were buying the option) is therefore favourable to the end user.

If a new end user instead wants to sell an option, the price will be uncompetitive for the institution. This is because the institution's FVA is positive which reduces the price it is prepared to pay and its DVA1 is zero because the option is an asset for the institution. The institution's price is $f_{nd} - CVA - FVA$ where CVA is a measure of the end user's default risk which is the same for all institutions. This means that a high-funding cost institution will be more uncompetitive than a low-funding cost institution if FVA are incorporated in the prices because the price the institution are prepared to buy the option at will be lower.

The conclusion from this example is that high-funding cost institutions will tend to sell options to end users, when FVA adjustments are made (whether or not DVA1 adjustments are made) and not buy options from them. An effect of this is that their book will not be balanced between long and short positions.

Example 7 showed that in the case where an end user wants to buy the option and when both FVA and DVA1 adjustments is made, the price the institution will quote

will be favourable, i.e. there exists an arbitrage opportunity. Therefore let us turn to explain this next.

3.4.2 Arbitrage opportunities

An arbitrage opportunity arises when it is possible to obtain a positive payoff without exposing yourself to risk. We have shown that accounting for both DVA and FVA can lead to double counting and in some cases this creates an arbitrage opportunity.

We will make an example to illustrate this arbitrage opportunity by an end user and thereafter present the situation where an institution incorporates FVA but not DVA1 in its pricing. Finally we will discuss how regulations try to prevent arbitrage opportunities.

Example 8 (Arbitrage opportunity)

Consider the results from example 7. The arbitrage opportunity will be straightforward in the case where an institution makes both FVA and DVA1 adjustments. We will consider two banks, Bank A and Bank B. Bank A funds itself at the risk-free rate of 2% and Bank B has a funding rate of 5%. If Bank A is interested in buying the option the Black Scholes price will be 12.82 minus the CVA for the counterparty which is $12.82 - 0.38 = 12.44$. Bank B on the other hand wants to sell the option and here the price is 12.06 (Bank B has no other transaction with the counterparty).

The new regulations do not allow institutions to sell on an uncollateralized basis which means all transactions between financial institutions must be fully collateralized. If Bank B could sell the option to Bank A on an uncollateralized basis there would be an arbitrage opportunity. For example if Bank B sold the option for 12.25 they would both make a profit of 0.19 ($12.44 - 12.25$ or $12.25 - 12.06$).

Instead the arbitrage opportunity exists if a high quality end user bought the option from Bank B and sold a similar to Bank A. Suppose the end user buys the option from Bank B at 12.18 and sells a similar option to Bank A for 12.70. If the end user wants to hedge against the counterparty's credit risk (CVA) it could for example short sell some of Bank B's debt at a net cost of 0.38 (which is Bank B's DVA1). The end user's end

3.4. IMPLICATIONS OF FVA

profit will be $12.70 - 12.06 - 0.38 = 0.14$. Because Bank A is willing to pay up to 12.82 for the option it will make a profit of $12.82 - 12.70 = 0.12$. Whereas because Bank B sold the option for 12.18 and actually could have sold it for $12.82 - 0.38 = 12.44$ (the price including CVA) Bank B actually on the one hand suffers a loss on $12.44 - 12.18 = 0.26$ and on the other makes a profit on $12.18 - 12.06 = 0.12$ (but the profit perspective is illusory).

Since institutions cannot trade on an uncollateralized basis⁸ with each other, it has to be a highly creditworthy end user making the trade if Bank A's default risk when buying from the end user should be avoided, and also to take advantage of Bank B's FVA. There are some possibilities for a highly creditworthy end user, for example a sovereign wealth fund or a corporate in deep understanding of derivatives. A cooperation between a hedge fund and a corporate where the corporate has to make the trade on an uncollateralized basis is also a possibility.

Let us present an arbitrage opportunity in the case where an institution incorporates FVA but not DVA1 in its pricing. Continue to use the setup used in section 3.4.1 and let us consider the effect of adding a new transaction to the portfolio. The value of the new transaction at time t is $u(t)$ and can be positive or negative. The incremental FVA* is

$$\Delta(\text{FVA}^*) = \int_{t=0}^T P(t)q(t)[1 - R(t)]E[u(t)]dt$$

When determining the incremental DVA1 the sum of $v_j(t)$ is replaced by the difference when the new transaction is added and when it is not added, this gives

$$\begin{aligned} \Delta(\text{DVA1}) = & \int_{t=0}^T P(t)q(t)[1 - R(t)] \\ & \left(E \left[\max \left(\sum_{j=1}^m v_j(t) + u(t), 0 \right) \right] - E \left[\max \left(\sum_{j=1}^m v_j(t), 0 \right) \right] \right) dt \end{aligned}$$

⁸Due to regulation (see section 3.5.1)

Consider the case where a sold option is added to the portfolio which means that $u(t) \leq 0$. In this case

$$E \left[\max \left(\sum_{j=1}^m v_j(t) + u(t), 0 \right) \right] - E \left[\max \left(\sum_{j=1}^m v_j(t), 0 \right) \right] \leq u(t)$$

And then the incremental FVA* is larger than or equal to the incremental DVA1 ($\Delta FVA^* \geq \Delta DVA1$). In the case where a bought option is added to the portfolio $u(t) \geq 0$ and we have that

$$E \left[\max \left(\sum_{j=1}^m v_j(t) + u(t), 0 \right) \right] - E \left[\max \left(\sum_{j=1}^m v_j(t), 0 \right) \right] \geq u(t)$$

And in this case it turns out that the incremental FVA* is smaller than or equal to the incremental DVA1 ($\Delta FVA^* \leq \Delta DVA1$). If an institution includes FVA but not DVA1 in its pricing there will be situations where the price of the derivatives portfolio will be favourable and others where it is uncompetitive. If prices are favourable then the end user can use the opportunity to enter into a transaction with a high-funding-cost institution and enter into an offsetting transaction with a low-funding-cost institution and in this way make a gain. The conclusion is that when $\Delta FVA^* \geq \Delta DVA1$ there will be potential arbitrage opportunities and when $\Delta FVA^* \leq \Delta DVA1$ there will be a tendency for the institution to be uncompetitive.

Continue example 8 to show how the potential profit from an arbitrage opportunity increases as the life of the option increases. In the example, the life of the option is 1 year. If we instead make the calculations over 10 years, Bank A's price will be 42.91 and Bank B's price will be 31.79 (after FVA and before CVA/DVA) and the potential profit will be greater.

3.4. IMPLICATIONS OF FVA

In practice, regulation and the fact that arbitrage opportunities require a lot of capital, can make it harder to take advantage of the arbitrage opportunities. But still, as example 8 showed, it is straightforward that an arbitrage opportunity exists if an institution makes both FVA and DVA1 calculations. Whereas it is more complex in the case where FVA is made but not DVA1. However in practice⁹ some institutions choose to incorporate CVA and FVA, but not DVA1 which leads to incorrect pricing.

⁹Hull & White (2013) [13]

3.5 The future of Counterparty Credit Risk

Since the global financial crisis, counterparty credit risk has been in the spotlight and so has the regulation of it. It has been of high importance to make changes to the regulation, some rules needed improvement and new ones have been introduced. One of the purposes of changing the regulation is to try to prevent repetition of the scenarios where large financial institutions collapsed and had to be bailed out by government.

A key part of regulation is to determine the minimum amount of capital that a given bank must hold. The capital works as a buffer that must absorb losses during turbulent periods and besides that it is significant when defining the creditworthiness of a bank. However defining the capital requirements for a bank is a difficult task, as they must be large enough to contribute to a very low possibility of default but not so high as to unfairly penalise the bank. Besides that the financial market cannot be predicted by models and historical experience.

A problem is that regulations are not always consistent globally because they differ by region. The Basel III covers globally but the precise implementation is decided by local regulators. The Dodd Frank Act, EMIR and CRD IV are just some of the important local regulations, all of them defining some kind of requirements. Capital requirements are split into various areas such as market, credit, liquidity and operational risk, however a large part of the new capital requirements under Basel III are due to counterparty credit risk.

Besides defining new capital requirements, regulation may also incentivise or require changes to market practices. Since the crisis one regulatory focus area has been the credit derivatives (ended up covering all OTC derivatives). This has led to a change where the OTC derivative market is moving towards collateralization or clearing through central counterparties (CCP). With the shift towards collateralized trades and since funding costs exist for non-collateralized derivatives, the adjustment for the rate paid on collateral (CRA) will most likely be more relevant in the future than FVA.

The structure of this section is as follows. We will start by introducing some of the most important regulations when dealing with OTC derivatives. Thereafter we will

introduce central counterparties and finally we will analyse when to make a collateral rate adjustment (CRA) to the derivative value.

3.5.1 Regulation

Let us start with an overview of the regulatory initiatives regarding OTC derivatives post crisis. Basel III¹⁰ is a global regulatory framework and its purpose among others is to strengthen the requirements for the management and capitalization of counterparty credit risk. In December 2010 the Basel Committee published two documents, "Basel III: A global regulatory framework for more resilient banks and banking systems" (a revised version was published in June 2011) and "Basel III: The Liquidity Coverage Ratio and liquidity risk monitoring tools" (a revised version was published in January 2013). Basel III is built on the Basel II document and the changes are scheduled to be implemented from January 2013 and with a transition period until 2019. In Europe the Capital Requirements Directive (CRD) IV transposes the global standards in Basel III into EU law. Furthermore, other local regulations have been developed.

In September 2009 the G20¹¹ made a commitment to regulate the OTC derivatives market. The commitment included that all standardized OTC derivatives contracts should be cleared through central counterparties and that OTC derivatives contracts should be reported to trade repositories. Furthermore, non-centrally cleared contracts should be subject to higher capital requirements. The purpose of these changes in the OTC derivatives market are to achieve a more transparent market and mitigate systemic risk¹². In the US the Securities Exchange Commission (SEC) as well as the Commodities Futures Trading Commission (CFTC) decide which derivatives are eligible and when the clearing obligation applies, whereas in Europe the European Securities and Markets Authority (ESMA) is responsible for this. In both the US and Europe new regulations have been developed. In US the Dodd-Frank Wall Street Reform and

¹⁰Bank for International Settlement [18]

¹¹Group of Twenty Finance Ministers and Central Bank Governors

¹²Systemic risk does not have a clear definition but it is the risk of a collapse of an entire financial system which in the context of counterparty credit risk could be triggered by a failure of one of the large financial institutions.

Consumer Protection Act (The Dodd Frank Act) was signed into law of the United States in July 2010 and SEC and CFTC are responsible for supervising this regulation. Whereas in Europe ESMA has been developing the European Markets Infrastructure Regulation (EMIR) which came into force in August 2012. The Dodd Frank Act and EMIR are very much alike in relation to the regulation of OTC derivatives markets.

In Europe EMIR applies to a broad range of OTC derivatives (interest rate, commodity, equity, credit and FX swaps and derivatives). EMIR contains two counterparty classifications, financial counterparties (FCs) and non-financial counterparties (NFCs). One consequence of EMIR is that all OTC derivatives created or changed after the 12th February 2014 must be reported to trade repositories (TR). Another is the obligation to centrally clear the derivatives through a central counterparty (CCP). The last part is still in development. EMIR does not cover all derivatives classes and neither does it cover non-financial institutions with a small engagement in derivatives. By the 18th September 2014 ESMA is to submit a draft on this clearing obligation.

Even though EMIR is not finished yet, it is clear from the regulations that ESMA wish to move towards a centrally cleared derivatives market where collateral will be posted. As far as it is possible this will also cover non-financial institutions/counterparties since EMIR wishes to treat financial and non-financial institutions as equally as possible.

3.5.2 Central counterparties

The regulatory interest has expanded the role of central counterparties (CCPs) which has become more important when managing and mitigating counterparty credit risk. When a transaction/contract between two counterparties is cleared through a CCP, the original contract between the two counterparties is novated into two new transactions, one transaction between the CCP and the seller, and one transaction between the CCP and the buyer. So a CCP interposes itself between counterparties to a derivative contract, in order to assume their rights and obligations, and in this way it becomes the buyer to every seller, and the seller to every buyer. This means that a CCP has a "matched book" since if it has one position with one institution it will have the

3.5. THE FUTURE OF COUNTERPARTY CREDIT RISK

opposite position with another institution. A CCP therefore bears no market risk but the counterparty credit risk is transferred to the CCP.

A CCP is a clearing member. The term *clearing* is used to describe what takes place between trade execution and trade settlement.

One of the advantages of a CCP is that they increase the transparency so that regulators can more easily see the positions being taken. Furthermore a CCP reduces some of the counterparty credit risk in the derivatives market. However it is important to note that a CCP will not remove counterparty credit risk completely, but reduce it.

The failure of a CCP is clearly extremely problematic but it can happen. It is therefore important for a CCP to have a fine-tuned structure with respect to collateralization, settlement and risk management.

Initial and variation margin

One way a CCP can mitigate the counterparty risk is by the use of collateral, referred to as margin. *Variation margin* is the collateral that is required to cover the exposure of the relevant positions. Furthermore a CCP will require an *initial margin* to account for a worst case scenario movement in exposure during the margin period of risk. An initial margin can therefore overcollateralize the counterparty credit risk. Initial margin can be increased or reduced depending on market conditions and may change through time.

3.5.3 Collateral Rate Adjustment

In section 2.4.2 we explained collateral and the effect it has on exposure. The purpose of this section is to expand our knowledge about collateral and the impact of collateral agreements on the value of a derivative.

A collateral rate adjustment (CRA) is an adjustment for the interest rate on collateral. The rate of return on the collateral is specified in the collateral agreement. If this rate does not reflect the risk of the collateral then a collateral requirement in a derivatives agreement will have a non-zero value and this leads to an adjustment. We will refer to the rate that reflects the risk of the collateral as the economic rate. The

CRA can also be referred to as the cost of a collateral agreement. This means that the value of the derivatives portfolio is reduced by the adjustment for the interest rate on collateral, so excluding the credit adjustments, the value of the derivatives portfolio is

$$f = f_{nd} - \text{CRA}$$

An advantage of CRA is that it is additive and symmetric so the institution and the counterparty agree on a price. Let us use the Black Scholes and Merton arguments to derive a formula for the CRA.

BSM with CRA

Let us consider a simplified situation. Suppose that we have a collateral agreement between an institution and their counterparty with the following specifications,

- They both have to post collateral (a two-way agreement) in the form of cash.
- The collateral is posted continuously up to the time of default (this means that there are no threshold and no minimum transfer amount).
- The collateral one of the parties has to post is equal to $\max(X, 0)$ where X represents the no-default value of the derivatives to the other party.
- The contractually defined interest rate that must be paid on cash collateral is equal to the risk-free rate plus a spread s , so $r_{col} = r + s$.

The spread s can be positive, negative or zero. When s is different from zero the interest rate paid on cash collateral is different from the risk-free rate r . This case where collateral is taken into consideration is the same as the no-default case (the original BSM) only with the exception that $(r + s)$ is paid on the value of the derivative at all times.

The process for the stock S is the same as in the original BSM framework,

$$dS = \mu S dt + \sigma S dz$$

3.5. THE FUTURE OF COUNTERPARTY CREDIT RISK

The institution has an investment in the derivative (f) and borrowings of f where an interest of $(r + s)$ is paid, so the process for the derivative is

$$\begin{aligned} df &= \mu_f f dt + \sigma S \frac{\partial f}{\partial S} dz \\ \mu_f &= \frac{1}{f} \left[\frac{\partial f}{\partial t} + \mu S \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} \right] - (r + s) \end{aligned}$$

In the case where f is negative, the derivative will be a liability and the borrowings are instead an investment earning $(r + s)$.

Consider Merton's hedging argument. We have a short position in the derivative which is hedged by a position of $\partial f / \partial S$ in the stock, the portfolio value is

$$\Pi = -f + \frac{\partial f}{\partial S} S$$

By inserting the dynamics for S and f , the change in the hedge portfolio becomes

$$\begin{aligned} d\Pi &= - \left[\frac{\partial f}{\partial t} dt + \mu S \frac{\partial f}{\partial S} dt + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} dt - (r + s) f dt + \sigma S \frac{\partial f}{\partial S} dz \right] \\ &\quad + \frac{\partial f}{\partial S} (\mu S dt + \sigma S dz) \\ d\Pi &= - \frac{\partial f}{\partial t} dt - \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} dt + (r + s) f dt \end{aligned}$$

The last term $(r + s) f dt$ reflects the income on the collateral part of the hedged portfolio. Since the portfolio is riskless it should earn the risk-free rate of return, so the change in the portfolio can also be described as

$$d\Pi = r\Pi dt = \left[-rf + rS \frac{\partial f}{\partial S} \right] dt + rf dt$$

where the last term reflects the expected return on the collateral part of the hedge portfolio in equilibrium. By combining the two equations, the PDE becomes

$$\frac{\partial f}{\partial t} + rS \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = (r + s) f \quad (3.5.1)$$

If $s = 0$ collateral arrangements do not affect the value of the derivative and the original BSM PDE apply.

The general solution to the PDE can be found by discounting risk-neutral payoffs with the interest rate on collateral, $r_{col} = r + s$. For a derivative that provides a payoff at time T the solution to the PDE is

$$f(S_t, t) = e^{-(r+s)(T-t)} E_r[f(S_T, T)]$$

Remember the no-default solution ($f_{nd}(S_t, t)$) from equation (2.3.4), with a little rewriting we get

$$\begin{aligned} f(S_t, t) &= e^{-(r+s)(T-t)} E_r[f(S_T, T)] \frac{e^{-r(T-t)}}{e^{-r(T-t)}} \\ f(S_t, t) &= e^{-r(T-t)} E_r[f(S_T, T)] e^{-(r+s)(T-t)} e^{r(T-t)} \\ f(S_t, t) &= f_{nd} e^{-s(T-t)} \end{aligned}$$

CRA is the reduction in the no-default value caused by an adjustment for the interest rate on collateral, $f = f_{nd} - \text{CRA}$ so

$$\begin{aligned} \text{CRA} &= f_{nd}(S(t), t) - f(S(t), t) \\ \text{CRA} &= f_{nd}(S(t), t) - f_{nd}(S(t), t) e^{-s(T-t)} \\ \text{CRA} &= f_{nd}(S(t), t) (1 - e^{-s(T-t)}) \end{aligned}$$

Using how we are able to estimate the loss rate from the bond credit spread from the appendix and if $t = 0$ we get

$$\text{CRA} = \int_0^T f_{nd} L_I(t) dt$$

CRA is additive and symmetric and the institution and the counterparty agree on a price. In the case where all collateral is cash, an approximation to CRA is to value the derivatives using a discount rate equal to $(r + s)$ instead of the risk-free rate r .

3.5. THE FUTURE OF COUNTERPARTY CREDIT RISK

Consider the case where the market participant expects to be a net payer of collateral. CRA will be positive when the rate paid on collateral r_{col} is less than the economic rate and negative in the case where r_{col} is greater than the economic rate.

The above situation is a simplified case and in practice collateral agreements are not this simple. Collateral can be one-way or two-way and comes in the form of cash or liquid securities. If securities are posted as collateral the return (income and capital gains) on the security belongs to the party posting the securities as collateral (this is by definition an economic return). The result is that $CRA=0$ and this means that a collateral rate adjustment is not necessary when securities are posted as collateral. It will be optimal to post cash as collateral when $s > 0$ since the economic rate for cash is r and thereby $r + s > r$. When $s < 0$ it will be optimal to post securities since it will be costly to post cash because $r + s < r$, so CRA is only necessary when $s > 0$.

If we also account for default risk, the adjusted value of a derivatives portfolio becomes

$$f = f_{nd} - CVA + DVA - CRA$$

In the event of default neither the institution or their counterparty will in theory suffer a loss because collateral is posted right up to the time of default from both sides. So in a two-sided zero threshold collateral agreement, we can think of collateral as a perfect hedge for losses due to default, hence it can be assumed that CVA and DVA both are zero. However in practice this assumption cannot be made because of the margin period of risk. This period begins when the defaulting party stops posting collateral and stops returning any excess collateral and it ends when the early termination of the outstanding derivatives are in effect. During this period the market can either move in favor of the non-defaulting party or against it. If the market moves in favor, the non-defaulting party will suffer a loss and be an unsecured creditor for the amount that reflects the movement. Whereas if the market moves against the non-defaulting party, it will be required to return any excess collateral to the defaulting party, and thereby not make a gain.

CRA relative to CVA/DVA

In section 3.1.1 we explained 3 situations where the funding rate can be used as the discount rate. We will use the BSM framework to expand case 3 where the institution and counterparty have identical loss rates. Remember the results from section 2.3.2 and 2.3.3. When the loss rates are identical, then $r_C^* = r_B^* = r^*$ and the PDE simplifies to

$$\frac{\partial f}{\partial t} + rS \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = r^* f \quad (3.5.2)$$

The value of the derivative can be obtained by discounting the risk-neutral payoffs at the rate r^* . For a derivative that provides a single payoff at time T the solution is

$$f(S_t, t) = f_{nd} e^{-(r^* - r)(T - t)}$$

Remember the bilateral adjustment, $f(S_t, t) = f_{nd} - \text{CVA} + \text{DVA}$, so

$$\text{CVA} - \text{DVA} = f_{nd} - f_{nd} e^{-(r^* - r)(T - t)}$$

which we will refer to the net credit adjustment.

This result can be used to explain CRA relative to CVA/DVA. Consider the partial differential equations in (3.5.1) and (3.5.2) which have similar forms, it is only the right-hand sides that are different. In the PDE for the net credit adjustment, CVA-DVA the right-hand side is

$$r^* f \text{ where } r^* = r + (r^* - r) \frac{\gamma_*}{\eta_*}$$

The credit spread is the difference between r^* and the risk-free rate r , so $s = r^* - r$. If we let the credit spread be $s = x$ (which is adjusted for differences between recovery rates on bond and derivatives) then $r^* = r + x$. Compare this to the right hand side in equation (3.5.1) and see that they are the same. This means the net credit adjustment, CVA-DVA when no collateral is posted, is the same as CRA in a two-

3.5. THE FUTURE OF COUNTERPARTY CREDIT RISK

sided zero threshold collateral agreement.

If collateral is posted in the case considered above, then CVA and DVA will be smaller compared to the no-collateral case. As a result CRA will be larger than the net credit adjustment. Furthermore new regulations have made collateral requirements more stringent between financial institutions, this will also have an effect of increasing CRA and decreasing the net credit adjustment in the case of a two-sided zero-threshold collateral agreement.

Differences between CVA, DVA and CRA

There are some differences worth noting between CVA, DVA and CRA. First is that in a two-way collateral agreement where all collateral is cash, CRA is additive across transactions and can be calculated on a transaction-by-transaction basis. Whereas CVA and DVA are usually not additive because of netting agreements and must be calculated on a portfolio-by-portfolio basis.

Another difference is that CRA depends on the rate that is defined in the contract whereas CVA and DVA depend on the credit spreads between the two parties and other market variables. If credit spreads and market variables change then CVA and DVA changes and this can be seen in the reportable income. If the rate on CRA is the risk-free plus a contractual spread ($r + s$) then there will be no income effects if market variables change. Whereas if the rate on CRA is specified as a fixed rate, income effects can occur.

Chapter 4

Conclusion

This thesis has analysed the choice of discount rate and the relevance of funding costs in derivatives valuation.

To do the analysis, we have showed how to extend the Black Scholes and Merton (BSM) arguments to incorporate default risk by the institution and the counterparty, funding costs and collateral agreements. We assume that the underlying asset follows a lognormal diffusion process, however other processes can be assumed. Furthermore the results are on a single derivative that depends on a single underlying asset providing no income. The results can be extended to apply to a single derivative or a portfolio of derivatives dependent on many underlying assets.

We have argued that the OIS rate is the best proxy for the risk-free rate regardless of the derivative is collateralized or not. By using the extended BSM arguments where we incorporate default risk for a non-collateralized derivative, we have showed that choosing a different discount rate than the risk-free will lead to a significant error in valuation.

We have showed, that in three special cases choosing a different discount rate than the risk-free can make sense. If funding costs are caused by default risk then the funding rate can be chosen as discount rate (in the original BSM setup) and thereby account for default risk. This only applies when the derivative is an asset to the institution, when the derivative is a liability to the institution or when the institution and the counterparty have identical funding rates.

We have realised that the institution's funding costs are compensation for the fact that in the event of default the institution experiences a gain. This gain is captured in the debt value adjustment (DVA). With the presence of funding costs we have separated the cause of default into two components: derivatives obligations and funding requirements for the derivative. If the funding costs are caused by default risk, we have shown that a FVA and an adjustment for the default on the funding requirements offset each other. Furthermore we have shown that accounting for both the default on derivatives obligations and FVA will lead to double counting of default risk, and this creates arbitrage opportunities.

From a general finance valuation perspective the value of a derivative should depend on its risk and not its costs. When funding costs are caused by default risk they bring no further information to the derivative that is not already covered by the credit and debt value adjustment.

If however a significant amount of the funding costs is not due to default risk, we have showed that a [FVA-DVA2] adjustment should be incorporated into the derivative valuation. This could be because of liquidity risk.

If the institution does not acknowledge the gain in the event of default, a FVA has to be incorporated to account for the cost the institution experiences while being alive because of funding. But from an economic perspective the gain the institution has in the event of default is real.

Regulatory initiatives indicate that a funding value adjustment seems less relevant in the future. This is because the regulation seeks towards a more collateralized and cleared derivatives market. Likewise credit and debt value adjustments will have less influence on derivatives valuation since collateral reduces the exposure. In situations where the rate paid on cash collateral is different from the risk-free rate, a collateral rate adjustment might on the other hand be more important in the future.

Appendix

Default probability from credit spreads on bonds

It is possible to estimate the loss rate from the bond credit spread. This depends on how derivatives are treated in default compared to bonds. Here we will assume that they are treated equally.

Recall the value adjusted for both CVA and DVA under the assumption that the intensity is deterministic. For small Δt , let $Q_c(t)\Delta t$ be the probability of default by the counterparty between time t and $t + \Delta t$. We will assume that the intensity is deterministic, and let $h_c(t)$ be the hazard rate for the counterparty up to time t and then $Q_c(t) = h_c(t) \exp \left[- \int_0^t h_c(s) ds \right]$. Then,

$$\begin{aligned} f &= f_{nd} - \text{CVA} + \text{DVA} \\ &= f_{nd} - \int_0^T (1 - R_c(t)) f_{nd}^+(t, T) Q_c(t) dt + \int_0^T (1 - R_I(t)) f_{nd}^-(t, T) Q_I(t) dt \end{aligned}$$

$f_{nd}^+(t, T)$ is the value today of the institution's exposure to the counterparty at time t and $f_{nd}^-(t, T)$ is the value today of the counterparty's exposure to the institution at time t . The loss rate for the counterparty is defined as

$$L_c(t) = Q_c(t)(1 - R_c(t))$$

And likewise the loss rate for the institution

$$L_I(t) = Q_I(t)(1 - R_I(t))$$

Then we can rewrite the bilateral adjusted value today of the derivatives position to the institution as

$$f(0, T) = f_{nd}(0, T) - \int_0^T f_{nd}^+(t, T) L_c(t) dt + \int_0^T f_{nd}^-(t, T) L_I(t) dt$$

Now let us consider the institution's portfolio which only consists of a zero-coupon bond issued by the counterparty and which promises a payoff of \$1 at time T . As we have assumed, in the event of default the zero-coupon bond is treated like a derivative and the amount claimed on an uncollateralized derivative exposure is the no-default value. This means that $f_{nd}^-(t, T) = 0$ and $f_{nd}^+(t, T) = f_{nd}(0, T)$ and we can reduce to

$$f(0, T) = f_{nd}(0, T) - \int_0^T f_{nd}(0, T) L_c(t) dt$$

Since the bond only is an asset to the institution, the adjusted value can be found as the no-default value discounted with the credit spread, i.e.

$$f(0, T) = f_{nd}(0, T) \exp(-s_c(T)T)$$

where $s_c(t)$ is the credit spread for a zero-coupon bond issued by the counterparty. Using the value can be found by discounting the no-default price with the credit spread and the reduced bilateral adjusted value, we get

$$\begin{aligned} f_{nd}(0, T) - \int_0^T f_{nd}(0, T) L_c(t) dt &= f_{nd}(0, T) \exp(-s_c(T)T) \\ 1 - \int_0^T L_c(t) dt &= \exp(-s_c(T)T) \end{aligned}$$

And if we consider two time steps where $t_1 < t_2$

$$\left(1 - \int_0^{t_2} L_c(t) dt\right) - \left(1 - \int_0^{t_1} L_c(t) dt\right) = \exp(-s_c(t_2)t_2) - \exp(-s_c(t_1)t_1)$$

and thereby

$$\int_{t_1}^{t_2} L_c(t)dt = \exp(-s_c(t_1)t_1) - \exp(-s_c(t_2)t_2)$$

This shows how we can estimate the loss rate from the credit spreads on bonds between two time steps.

Using the same procedure, but now instead the portfolio consists of a zero-coupon bond issued by the institution we have that $f^-(t, T) = -f_{nd}(0, T)$ and $f^+(t, T) = 0$. The bilateral adjusted portfolio can now be reduced to

$$f(0, T) = f_{nd}(0, T) - \int_0^T f_{nd}(0, T) L_I(t) dt$$

When the portfolio is always a liability to the institution, the value can be found as the no-default value discounted with the credit spread. The adjusted value can then be found as

$$f(0, T) = f_{nd}(0, T) \exp(-s_I(T)T)$$

Where $s_I(t)$ is the credit spread for a zero-coupon bond issued by the institution. This gives us

$$\begin{aligned} f_{nd}(0, T) - \int_0^T f_{nd}(0, T) L_I(t) dt &= f_{nd}(0, T) \exp(-s_I(T)T) \\ 1 - \int_0^T L_I(t) dt &= \exp(-s_I(T)T) \end{aligned}$$

and between two time steps $t_1 < t_2$ we get,

$$\int_{t_1}^{t_2} L_I(t) dt = \exp(-s_I(t_1)t_1) - \exp(-s_I(t_2)t_2)$$

Credit spreads on bonds are very used in finance and it is therefore very useful to estimate the loss rate from these spreads. These results of the loss rate will be important in the calculation of derivatives prices using credit spreads on bonds.

References

- [1] Brigo, Morini and Pallavicini (2013), "Counterparty Credit Risk, Collateral and Funding", John Wiley and Sons
- [2] Burgard, Cristoph and Mats Kjaer (2012), "Generalized CVA with Funding and Collateral via Semi-replication", Quantitative Analytics, Barclays, Version 8.0 August 2013
- [3] Burgard, Cristoph and Mats Kjaer (2011a), "In the balance", *Risk*, November 2011
- [4] Burgard, Cristoph and Mats Kjaer (2011b), "PDE representations of derivatives with bilateral counterparty risk and funding costs", *The Journal of Credit Risk*
- [5] Collin-Dufresne and Bruno Solnik (2001), "On the Term Structure of Default Premia in the Swap and Libor Market", *The Journal of Finance*, Vol. 56, June, page 1095-1115
- [6] Fares, Ziad and Benoit Genest (2013), "CVA capital charge under Basel III standardised approach: An explanation document", Global Research & Analytics, April 2013
- [7] Filipovc, Damir and Anders B. Trolle, "The term structure of interbank risk", ssrn.com/abstract=1923696
- [8] Gregory, Jon (2012), "Counterparty Credit Risk and Credit Value Adjustment: A Continuing Challenge for Global Financial Markets" Second Edition, John Wiley and Sons
- [9] Hull, John and Alan White (2012a), "Collateral and Credit Issues in Derivatives Pricing", Forthcoming: *Journal of Credit Risk*, October 2012 (Revised June 2014)
- [10] Hull, John and Alan White (2012b), "LIBOR vs. OIS: The Derivatives Discounting Dilemma", *Journal of Investment Management*
- [11] Hull, John and Alan White (2012c), "The FVA debate", *Risk*, July 2012
- [12] Hull, John and Alan White (2012d), "The FVA debate continues: Hull and White respond to their critics", Published in *Risk Magazine*, October 2012

REFERENCES

- [13] John Hull and Alan White (2013), "Valuing Derivatives: Funding Value Adjustment and Fair Value", *Financial Analyst Journal*, November 2013
- [14] Lando, David (2004), "Credit Risk Modelling: Theory and Applications", Princeton University Press
- [15] Laughton, Stephen and Aura Vaibrot (2012), "In defence of FVA - a response to Hull and White", Published in *Risk Magazine*, September 2012
- [16] Longstaff, F., S. Mithal and E. Neis (2005), "Corporate Yield Spreads: Default risk or liquidity? New evidence from the credit default swap market", *Journal of Finance*, October 2005
- [17] Basel III Handbook, www.accenture.com
- [18] Bank for International Settlement, www.bis.org/bcbs/basel3.htm
- [19] British Bankers Association, www.bbalibor.com
- [20] EMIR regulation: REGULATION (EU) No 648/2012 OF THE EUROPEAN PARLIAMENT AND OF THE COUNCIL, of 4 July 2012, on OTC derivatives, central counterparties and trade repositories
- [21] ISDA Margin Survey 2014, April 2014, www.isda.org
- [22] ISDA Published 2013 Standard Credit Support Annex, www2.isda.org/news/isda-publishes-2013-standard-credit-support-annex-scsa
- [23] Social Science Research Network, ssrn.com/abstract=2314998