Internet Appendix for "Funding Value Adjustments"

LEIF ANDERSEN, DARRELL DUFFIE, and YANG SONG[1]

This internet appendix provides supplementary results and proofs for Andersen, Duffie, and Song (2018, ADS). In Section I, we consider variation margin and margin value adjustments. In Section II, we generalize the basic model of ADS to multiple periods. Finally, in Section III, we consider the effect of netting with legacy positions.

I. Secured or Hedged Swaps

This section extends the results of Section III.A of ADS to consider cases involving variation margin, hedged swap positions, and margin value adjustments.

A. Variation Margin and Interdealer Hedging

When a dealer trades an unsecured swap with a client, the dealer is likely to combine the position with a suitable hedge. In practice, two separate hedges would typically be used. One hedge would mitigate the risk of default of the swap counterparty, for instance, using a credit default swap (CDS) referencing the counterparty. Another position would be taken as a hedge against the market risk exposure of the floating-side payment X.

Using the setup in Section III.A of ADS, we can incorporate the effect of hedging a swap by assuming that the hedge simply takes the form of an offsetting position paying $-Y$, where Y is the net payout given by equation (6) of ADS. As an abstract simplification, this covers both the counterparty risk and the underlying market risk X. The hedge is executed with another dealer, called the "hedge dealer." As is standard practice in interdealer transactions, the hedge requires the posting of variation margin, a running exchange of collateral that is sufficient to cover the entire present value of the transaction. In addition to providing default protection for both dealers, the variation margin mechanism provides an automatic source of cash funding of the hedge position, as we mentioned earlier.

In our one-period model, we can capture the effect of a running posting of variation margin in the following simplified way.

- At time 0, the dealer receives a cash payment from the hedge dealer equal to the market value $\delta E^*(Y)$. The dealer immediately posts this cash amount back to the hedge dealer

[1]Citation format: Andersen, Leif, Darrell Duffie, and Yang Song, Internet Appendix for "Funding Value Adjustments," *Journal of Finance,* doi:10.1111/jofi.12739. Please note: Wiley-Blackwell is not responsible for the content or functionality of any supporting information supplied by the authors. Any queries (other than missing material) should be directed to the authors of the article.

as a variation margin payment, earning the risk-free rate on the associated posting of collateral. As the two initial cash payments cancel, neither the dealer nor the hedge dealer needs any financing to instantiate the hedge transaction.

- At time 1, but before other cash flows at time 1 are paid, the collateral is refreshed. That is, the dealer receives $E^*(Y)$ back from the hedge dealer. (This is margin posted at time 0, plus the risk-free interest.) The dealer pays Y to the hedge dealer. The hedge dealer is assumed to be paid with priority over all other creditors.[2] As the swap itself pays Y, given this assumed priority, the dealer will always be able to make this payment. This abstracts from some potential loss of priority that might apply in extreme practical cases, for example, in an administrative failure resolution process that could override contractual termination rights.

Netting the cash flows, the total package consisting of an unsecured asset and the hedge will pay the dealer $E^*(Y)$ at time 1, an amount that is known at time 0. As desired, the hedge removes the variability of the payment Y, replacing it with its fair market forward value.

Assuming that the dealer finances the purchase of the client asset by issuing debt, we can now repeat the funding cost analysis shown in Section III.A. The results, found in Appendix B of ADS, are obvious. Because the hedge removes net payout variance, the covariance term disappears, and the FVA for the package consisting of the asset and its hedge is simply $g(v - d) = -\Phi$.

As we have explained, the assumption of a perfectly offsetting hedge payout of $-Y$ is an idealization. In practice, the risk associated with the client swap payoff is not completely extinguished. This allows small default covariance terms to creep back into the break-even price v^*. Further, interdealer hedge swaps are virtually always executed at par, that is, at a fixed rate of $\tilde{K} = E^*(X)$, rather than at an arbitrary rate of K. We deal with this minor complication in the next section.

B. Par Swaps and Forward Swap Rates Without Margin

In practice, the fixed swap rate K is typically negotiated so that there is no upfront payment. In this case, the swap is known as a "par-valued swap." The resulting fixed rate

[2] This effective priority over standard debt claims follows from exemptions for swaps from automatic stays in bankruptcy or other insolvency proceedings. Even under proposed methods for resolving the failure of a systemically important dealer that would apply the effect of an automatic stay on swap terminations, the dealer's swaps would likely retain priority over ordinary creditors, who would be "bailed in." This would fully prioritize swap counterparties except in the most extreme scenarios, in which even the cancellation of all debt subject to bail-in is insufficient to recapitalize the dealer.

K is often known as the "forward swap rate." In our setting, three different forward swap rates are of interest:

- The forward swap rate $\tilde{K}$ for a fully collateralized dealer-to-dealer swap. The swap has a market value of $\delta E^*(X - \tilde{K})$, so the fair forward swap rate $\tilde{K} = E^*(X)$ reflects no credit risk component. This is the benchmark forward swap rate typically shown on standardized trading screens. In practice, the risk-neutral probability measure P^* used by dealers for market valuation would typically be calibrated so as to match the risk-neutral expected payment $E^*(X)$ to the "screen rate" $\tilde{K}$, and likewise for other liquidly traded financial instruments.
- The forward swap rate $\hat{K}$ for an unsecured client swap that is executed at fair market pricing. If we express v in equation (7) of ADS as $v = \eta(K)$, then $\hat{K}$ is the solution in K of the equation $\eta(K) = 0$.
- The forward swap rate K' for an unsecured client swap that leaves shareholders indifferent to the trade. From equation (6) and equation (8) of ADS, K' is determined by the equation $E^*(1_{D^c} y(K')) = 0$.

Neither $\hat{K}$ nor K' depend on the financing strategy used by the dealer. Without an upfront payment, no financing is required, putting aside for now the issue of initial margin, which we get to later in this section. Here, $\hat{K}$ and K' differ only because the DVA benefit on the swap is excluded from K'.

LEMMA IA1: *Suppose that either (a) the dealer's default indicator 1_D is uncorrelated (under P^*) with the swap payment Y, or (b) the swap position is fully hedged by an interdealer swap. Then $K' \leq \hat{K}$ and $K' \leq \tilde{K}$.*

In a model with several time periods, even a position with no upfront cash payment may involve an FVA. For example, consider a position entered into at time 0 with no upfront payment, requiring a significant positive expected cash payment by the dealer at some intermediate date or dates, before compensating payments are later received by the dealer. A common example of this is a long-dated swap issued in an environment with a steeply sloped yield curve. As we explain in more detail in Section V of ADS, such a position can be associated with a substantial FVA.

C. *Par Swaps with Initial Margin and Margin Value Adjustment (MVA)*

Par-valued swaps require no upfront funding and therefore have no FVA in our one-period setting. This situation changes with the introduction of initial margin, whether on the client swap itself or on the hedge swaps. In fact, it is becoming increasingly common to encounter swap agreements that require one or both counterparties to post risk-based initial margin, providing an additional layer of credit risk protection beyond variation margin. For instance, such agreements are routinely required by CCPs and are now mandatory under the Dodd-Frank Act and European MiFID regulations. Because initial margin always implies a positive initial cash outlay, even for par-valued swaps, FVAs for margin inevitably result in costs to dealer shareholders.

To be concrete, we consider the funding cost impact on the shareholders of a swap dealer that hedges an unsecured par-valued swap with a par-valued hedge transaction that requires the dealer to post initial margin. In summary, the swaps dealer in question is contemplating a pair of transactions consisting of:

(i) An uncollateralized swap with a client, where the dealer pays a fixed rate K in exchange for a floating payment X, for a net contractual receivable at time 1 of $X - K$. We take K as given for now, and assume that the client swap terms involve no initial exchange of cash. The terms of trade for the swap are thereby captured entirely by the fixed-side payment K.

(ii) A hedge-motivated fully collateralized swap with another dealer or a central counterparty, where the dealer has a net receivable at time 1 of $\tilde{K} - X$, at the fair forward swap rate $\tilde{K} = E^*(X)$. As before, we suppose that the hedge swap involves variation margin and no net initial payment. In this case, however, the swap additionally requires the dealer to post a specified cash initial margin of $I > 0$. The recipient of the margin, typically either a CCP or a third-party custodian, invests the margin in risk-free assets, paying the dealer RI back at time 1 (unless the dealer defaults). As a simplification, we assume that the margin agreement is sufficient to ensure that both of the counterparties to the hedge swap are fully secured against loss.

The hedge swap payout $\tilde{K} - X$ is not an exact match for the client swap, except in the unlikely case that $K = \tilde{K}$. We do not consider a CDS hedge against default, but our results can be trivially extended to this case.[3] Our results are unaffected if the initial margin I_q for a position of size q is not necessarily proportional to q, provided that the per-unit margin has some limit $I \equiv \lim_{q \downarrow 0} I_q/q$. Likewise, our results remain as stated if the swap fixed-side terms K and $\tilde{K}$ depend on q, provided only that they converge with q to limits denoted K and $\tilde{K}$, respectively. These generalizations are avoided merely for notational simplicity.

[3] As we have already seen in Section I.A, adding a CDS hedge essentially removes the covariance effects in the CVA term. For instance, the term $\delta E^*[1_{D^c}\gamma(X-K)^+]$ in (IA1) would become $\delta P^*(D^c)E^*[\gamma(X-K)^+]$.

We carry over all notation from Section III.A of ADS. The effect of pre-existing positions between the swap counterparties is considered only in Section III of the Internet Appendix. We model variation margin in the same manner as in Section I.A of the Internet Appendix, so that the net payment at time 1 on the hedge swap is $E^*(X - \tilde{K}) - (X - \tilde{K}) = \tilde{K} - X$. Before considering the impact of dealer default, the package of swap transactions therefore has a per-unit cash flow to the dealer at time 1, including the return of the margin with interest, of

$$Y = RI + \tilde{K} - K - \gamma(X - K)^+.$$

The initial required per-unit cash investment u is merely the initial margin I, because the swaps themselves are all executed without upfront payments.

Assuming that the initial margin is funded by debt issuance, Proposition 1 of ADS implies that the marginal value of the transaction to the dealer's shareholders is

$$G = \delta P^*(D^c)(\tilde{K} - K) - \delta E^*[1_{D^c}\gamma(X - K)^+] - \Lambda, \tag{IA1}$$

where $\Lambda = \delta P^*(D^c)SI$ is the funding cost adjustment for the payment of initial margin, known in industry practice as the margin value adjustment (MVA). In this simplest of settings, the value adjustment Λ for initial margin is the initial market value of the component of net margin-funding interest expense SI that is borne by shareholders at time 1. The shareholders bear the entire expense SI if the dealer does not default, and bear none of the expense if the dealer defaults.

We also calculate the total market value of the package of swap transactions. For a position of q units, the initial margin payment generates cash flow of $-qI$ to the dealer at time 0. At time 1, the payment of the hedging swap, including the return of margin with interest, is $q(\tilde{K} - X) + qIR$. The payment of the client-to-dealer swap to the dealer is $q(X - K)$ before considering default. The cash flow $q(X - K)$ is not paid in full at time 1 in either of two events: *(i)* the client defaults and $q(X - K) > 0$, in which case the dealer receives $\beta q(X - K)^+$ from the client; and *(ii)* the dealer defaults and $q(X - K) < 0$, in which case the client is pari passu with the other creditors of the dealer, and the swap client receives $\mathcal{R}(q)q(X - K)^-$, where

$$\mathcal{R}(q) = \frac{\kappa(A + q(\tilde{K} - X) + qIR)}{L + q(X - K)^- + qI(R + s(q))}$$

is the fractional recovery of the dealer's assets in default on the event that $X - K < 0$. The numerator of $\mathcal{R}(q)$ is the amount of the dealer's assets that are recovered if the dealer defaults and $X - K < 0$. The denominator is the aggregate liabilities of the dealer, which include the legacy liabilities L, the liabilities due to financing the initial margin, which is $qI(R+s(q))$, and the liabilities to the swap client, which is $q(X-K)^-$. By assumption, $A+qIR+q\,(X-K)^+$ is always sufficient to pay the amount $q(\tilde{K}-X)^-$ due on the secured hedge.

Following the definitions of Section II.B of ADS, the net actual cash flow at time 1 of the package of swap transactions is

$$\hat{\mathcal{C}}(q) = q(\tilde{K}-X) + qIR + q(X-K) - q\gamma(X-K)^+ + q1_{\hat{\mathcal{D}}(q)}(1-\mathcal{R}(q))(X-K)^-,$$

where

$$\hat{\mathcal{D}}(q) = \{A + q(\tilde{K}-K) - q\gamma(X-K)^+ - L - qIs(q) < 0\}$$

is the event of the dealer's default.

The total market value of the package of transactions is

$$\mathcal{V}(q) = -qI + \delta E^*(\hat{\mathcal{C}}(q)).$$

One can see that the initial payment I of margin at time 0 and the return payment of RI at time 1 have offsetting impacts on the total market value of the swap. When considering the marginal value of the transaction to shareholders, however, the computation shows the crucial impact on shareholder value of of financing the initial margin.

Similar to the case of Proposition 2 of ADS, the marginal value of the swap,

$$v = \left.\frac{\partial \mathcal{V}(q)}{\partial q}\right|_{q=0} = \delta(\tilde{K}-K) - \delta E^*[\gamma(X-K)^+] + \delta E^*[\phi(X-K)^-], \tag{IA2}$$

is decomposed into the present value of the gross swap spread $\tilde{K}-K$, less the CVA, plus the DVA. As anticipated, the per-unit market value v of the combined swap position does not depend on the amount I of required initial margin, nor does v depend on how the margin was financed. As we have noted, however, this invariance of valuation to the financing of initial margin is contrary to current dealer valuation practice.

Appendix B of ADS calculates the impact of the value H of the package on the legacy creditors. If there are no default distress costs, we have the usual value-conservation identity $H + G = v$.

The fair market level of the spread $\tilde{K}-K$ between the two swap rates, obtained from (IA2) by setting v equal to zero, is

$$\mathcal{S} = E^*[\gamma(X-K)^+] - E^*[\phi(X-K)^-], \tag{IA3}$$

which is merely the net risk-neutral expected default loss on the client swap (loss from client default net of loss from dealer default). The swap spread $\mathcal{S}' = \tilde{K} - K$ that makes the dealer's shareholders indifferent to the trade is instead obtained from (IA1) by setting $G = 0$, leaving

$$\mathcal{S}' = SI + \frac{E^*[1_{D^c}\gamma(X-K)^+]}{P^*(D^c)}.$$

To generate positive shareholder returns in this setting, the dealer must be able to identify hedged swap positions at fixed swap rates that improve on fair market rates by $\mathcal{S}' - \mathcal{S}$. In gauging how difficult this may be for the dealer's swap desk, we suppose that the dealer's default event is uncorrelated under P^* with the client default loss $\gamma(X-K)^+$. The dealer must then be able to improve on fair market swap rates by at least

$$\mathcal{S}' - \mathcal{S} = SI + E^*(\phi)E[(X-K)^-].$$

For the typical (small) credit spreads of major dealers, and for small risk-free interest rates (that is, R near one), we have the Taylor approximation $S \simeq E^*(\phi)$, and thus

$$\mathcal{S}' - \mathcal{S} \simeq S\left(I + E^*[(X-K)^-]\right), \tag{IA4}$$

where the first term originates from the margin funding costs and the second from the DVA. This is the adjustment to the swap quote necessary to overcome effect of value impact on shareholders shown by equation (12) of ADS.

Because initial margins set by CCPs or in the interdealer swap market are standardized, the right-hand side of (IA4) is the dealer's credit spread S multiplied by some positive swap-specific amount that does not depend on the identity of the dealer.

II. Multiperiod Model

We generalize the basic model of Section III of ADS to two periods with three dates $t = 0, 1, 2$. New information is revealed at the interim date 1 through observation of a collection Z of random variables. All uncertainty is resolved at date 2. We let E_1^* denote expectation under P^* conditional on Z. We assume that the one-period gross risk-free returns are R_0 and R_1 at time 0 and 1, respectively. We don't require R_1 to be constant. Thus, the market value of the cash flows $\{C_t\}_{t=1}^2$ is defined as $\sum_{t=1}^2 E^*(\delta_t C_t)$, where $\delta_1 = 1/R_0$ and $\delta_2 = 1/(R_0 R_1)$.

We consider a dealer whose pre-existing assets have payoffs at time 2 given by some random variable A. The firm has short-term liabilities L_1 that expire at time 1 and longterm liabilities L_2 that expire at time 2. We assume that the dealer liquidates a portion of its legacy assets to pay back the maturing liabilities L_1 at time 1 and pay out dividend π_1, which is also a random variable. If the liquidation value of the asset is not enough to cover L_1, the dealer defaults, the event that we denote by D_1. We let W denote the payoff at time 2 of the liquidated assets. As a result, the firm defaults at time 2 in the event that $D_2 = \{A - W < L_2\}$. In the dealer's default events D_1 and D_2, we assume that all liabilities are pari passu with each other, and the recovery rates of assets are some constant κ_1 and κ_2, respectively. We let τ_D denote the dealer's default time. If the dealer survives at time 2, that is, $\tau_D = \infty$, the firm is liquidiated and the remaining cash flows are attributed to shareholders after paying back creditors. Thus, the total value of the firm's equity is $E^*[\delta_1 \mathbf{1}_{\{\tau_D>1\}}\pi_1] + E^*[\delta_2 \mathbf{1}_{\{\tau_D>2\}}(A - W - L_2)]$. The total value of the dealer's liabilities is

$$E^*[\delta_1 \mathbf{1}_{\{\tau_D>1\}} L_1 + \delta_1 \mathbf{1}_{\{\tau_D=1\}} \kappa_1 E_1^*(A)/R_1] + E^*[\delta_2 \mathbf{1}_{\{\tau_D>2\}} L_2 + \delta_2 \mathbf{1}_{\{\tau_D=2\}} \kappa_2 (A - W)].$$

We assume either *(i)* finite states of the world, or *(ii)* infinitely many states of the world with standard continuity conditions of (A, W, L_1, L_2) as in Section II of ADS. As in Section II of ADS, the dealer's marginal credit spread at time 0 for short-term (one-period) debt is

$$S_0 = \frac{E^*(\phi_1) R_0}{1 - E^*(\phi_1)},$$

where $\phi_1 = \mathbf{1}_{D_1}(L_1 + E_1^*(L_2)/R_1 - \kappa_1 E_1^*(A))/(L_1 + E_1^*(L_2)/R_1)$. If the dealer survives at time 1, the dealer's marginal credit spread at time 1 for one-period debt is

$$S_1 = \frac{E_1^*(\phi_2) R_1}{1 - E_1^*(\phi_2)},$$

where $\phi_2 = \mathbf{1}_{D_2}(L_2 - \kappa_2(A - W))/L_2$.

In this two-period setting, a swap is a contract promising *(i)* floating payment X_1 in exchange for fixed payment K_1 at time 1, and *(ii)* floating payment X_2 in exchange for fixed payment K_2 at time 2, before considering the effect of counterparty default. We let $C_1 \equiv X_1 - K_1$ and $C_2 \equiv X_2 - K_2$. We focus on the payer swap, that is, the positive cash flow of this contract is an asset to the dealer, whereas the negative cash flow is a contingent liability. A swap position of size q requires the dealer to make an upfront payment of $U(q)$. We assume that $u = \lim_{q \downarrow 0} U(q)/q$ exists. Results for the reverse case are obvious by analogy.

The supporting calculations for the following results are similar to Appendix B of ADS and are omitted for brevity.

A. Valuing Unsecured Swaps with Upfront Payment

In this section, we extend the results in Section III.A of ADS. That is, the client swap is assumed to be fully unsecured. For simplicity, we assume that at the interim period, swap counterparties default after the coupon payment.[4] We let τ_C denote the swap client's default time. At the client's default, the dealer recovers a fraction β_1 and β_2 of any remaining contractual amount due to the dealer at time 1 and time 2, respectively. We also suppose that there are no pre-existing positions between the swap client and the dealer. The effect of netting the new swap flows against those of the legacy positions with the same client is analyzed in Section III.

We have the following natural extension of the basic one-period swap valuation model in Section III.A of ADS.

PROPOSITION IA1: *Whether the dealer finances any net payments by issuing debt, issuing equity, or using existing cash on its balance sheet, the marginal market value of the swap is well defined by*

$$v = E^*\left(\sum_{t=1}^{2}\delta_t C_t - u\right) + E^*\left(\sum_{t=1}^{2}\delta_t \mathbf{1}_{\{\tau_D=t,\tau_C>t-1\}}\phi_t V_t^-\right) - E^*\left(\sum_{t=1}^{2}\delta_t \mathbf{1}_{\{\tau_C=t,\tau_D>t-1\}}(1-\beta_t)V_t^+\right), \tag{IA5}$$

where $V_1 = E_1^*(C_2)/R_1$ *and* $V_2 = C_2$.

As in the single-period model, the swap value (IA5) includes two credit-related adjustments for the default-free value, $V_0 = E^*(\delta_1 C_1) + E^*(\delta_2 C_2)$, for default. The CVA is $E^*\left[\sum_{t=1}^{2}\delta_t \mathbf{1}_{\{\tau_C=t,\tau_D>t-1\}}(1-\beta_t)V_t^+\right]$, and the DVA is $E^*\left[\sum_{t=1}^{2}\delta_t \mathbf{1}_{\{\tau_D=t,\tau_C>t-1\}}\phi_t V_t^-\right]$. The market value of the same swap from the viewpoint of the swap client is of course $-v$.

Now we analyze the marginal value of the new swap to shareholders of the dealer, and we assume that the positive financing requirement is financed by issuing short-term (one-period) debt. Likewise, any net positive cash flow to the dealer is used to retire short-term debt.

PROPOSITION IA2: *If the dealer issues debt to finance net payments and uses received cash to retire outstanding debt, then the marginal value of the swap to the dealer's shareholders is well defined by*

$$G = E^*\left[\mathbf{1}_{\{\tau_D>2\}}\left(\sum_{t=1}^{2}\delta_t C_t - u\right)\right] - E^*\left[\mathbf{1}_{\{\tau_D>2\}}\left(\sum_{t=1}^{2}\delta_t \mathbf{1}_{\{\tau_C=t\}}(1-\beta_t)V_t^+\right)\right] - \Phi(u), \tag{IA6}$$

[4]This assumption is valid for the purpose of marginal analysis.

where

$$\Phi(u) = E^* \left[\delta_1 \mathbf{1}_{\{\tau_D>1\}} u S_0 + \delta_2 \mathbf{1}_{\{\tau_D>2,\tau_C>1\}} u R_0 S_1\right] - E^* \left[\delta_2 \mathbf{1}_{\{\tau_D>2,\tau_C>1\}} C_1 S_1)\right]$$

is the debt FVA.

As in Section III.B of ADS, if the swap is executed at the "conventional" upfront payment,

$$u^* = V_0 - c^* = V_0 - E^* \left(\sum_{t=1}^{2} \mathbf{1}_{\{\tau_C=t\}} \delta_t (1-\beta_t) V_t^+ \right),$$

then the marginal value of the swap portfolio to the dealer's shareholders is

$$G = \operatorname{cov}\left(\mathbf{1}_{\{\tau_D>2\}}, \sum_{t=1}^{2} \delta_t C_t - \sum_{t=1}^{2} \mathbf{1}_{\{\tau_C=t\}} \delta_t (1-\beta_t) V_t^+ \right) - \Phi(u^*). \qquad \text{(IA7)}$$

In practice, c^* is often known as *Unilateral Credit Valuation Adjustments* (UCVA),[5] which is different from the CVA in (IA5) as it does not take into account the dealer's default. In the case that the dealer's default is independent of the swap cash flows, the shareholder value is

$$G = -\Phi(u^*).$$

By analogy with (12) of ADS, for a small spread S, the dealer's indifference quote is approximately $u^* - \Phi(u^*)$.

B. Interdealer Hedges, Initial Margin, and MVA

In this subsection, we consider the case in which a swap dealer hedges the unsecured swap with a fully collateralized interdealer swap that requires the dealer to post both initial margin and variation margin. We assume that the hedge-motivated collateralized swap with another dealer or a central counterparty has a net receivable of $-C_1 = K_1 - X_1$ at time 1 and a net receivable of $-C_2 = K_2 - X_2$ at time 2. The hedging swap requires the dealer to post both cash initial margin of I_0 and I_1, and variation margin M_0 and M_1 at time 0 and time 1, respectively. We follow the same variation margin mechanism as in Section I.A, and we assume that $M_0 = V_0 = E^* \left(\sum_{t=1}^{2} \delta_t (X_t - K_t)\right)$ and $M_1 = V_1 = E_1^*(X_2 - K_2)/R_1$, the standardized margin payment that is equal to the market value of the hedging swap. We assume that this hedging swap is transacted at the fullly collateralized value V_0.

We have the following natural extension of the basic one-period swap valuation model

[5] See Albanese and Andersen (2014) for details on UCVA.

with interdealer hedge.

PROPOSITION IA3: *If the dealer issues debt to finance margin payments and uses received margin to retire outstanding short-term debt obligations, then the marginal value of the swap portfolio to the dealer's shareholders is well defined by*

$$G = E^* \left[\mathbf{1}_{\{\tau_D>2\}} (V_0 - u)\right] - E^* \left[\mathbf{1}_{\{\tau_D>2\}} \left(\sum_{t=1}^{2} \delta_t \mathbf{1}_{\{\tau_C=t\}}(1-\beta_t)V_t^+\right)\right] - \Phi(u) - \Psi,$$

where

$$\Phi(u) = E^* \left[\delta_1 \mathbf{1}_{\{\tau_D>1\}} u S_0 + \delta_2 \mathbf{1}_{\{\tau_D>2,\tau_C>1\}} V_1 S_1\right] + E^* \left[\delta_2 \mathbf{1}_{\{\tau_D>2,\tau_C>1\}} (u - V_0)\right]$$

is the FVA and

$$\Psi = E^* \left(\delta_1 \mathbf{1}_{\{\tau_D>1\}} I_0 S_0\right) + E^* \left(\delta_2 \mathbf{1}_{\{\tau_D>2,\tau_C>1\}} I_1 S_1\right)$$

is the MVA.

In the special case in which the unsecured swap is executed at the default-free market value, that is, $u = V_0$, the FVA is

$$\Phi(V_0) = E^* \left[\delta_1 \mathbf{1}_{\{\tau_D>1\}} V_0 S_0 + \delta_2 \mathbf{1}_{\{\tau_D>2,\tau_C>1\}} V_1 S_1\right].$$

C. *Imperfect Variation Margin and FVA*

So far we have assumed that the client swap is fully unsecured. It is also of interest to consider the case in which the client swap requires both counterparties to post some variation margin. To be concrete, we assume that the client swap requires some "imperfect" variation margin, so that m_0 and m_1 are the amount of variation margin in the dealer's possession at time 0 and time 1, respectively. We assume that this client swap is hedged with the same fully collateralized interdealer swap in Section II.B.

By direct algebra, the FVA in this case is

$$\begin{aligned}\Phi(u) =& E^* \left[\delta_1 \mathbf{1}_{\{\tau_D>1\}} (V_0 - m_0) S_0 + \delta_2 \mathbf{1}_{\{\tau_D>2,\tau_C>1\}} (V_1 - m_1) S_1\right] \\ &+ E^* \left[\delta_1 \mathbf{1}_{\tau_D>1} (u - V_0)\right] + E^* \left[\delta_2 \mathbf{1}_{\{\tau_D>2,\tau_C>1\}} (u - V_0)\right].\end{aligned}$$

In the case in which the client swap is executed at the default-free market value V_0, the FVA is

$$\Phi(V_0) = E^* \left[\delta_1 \mathbf{1}_{\{\tau_D>1\}} (V_0 - m_0) S_0 + \delta_2 \mathbf{1}_{\{\tau_D>2,\tau_C>1\}} (V_1 - m_1) S_1\right].$$

If the "imperfect" margin becomes "perfect," that is, if $m_0 = V_0$ and $m_1 = V_1$, then the FVA $\Phi(V_0) = 0$.

D. Cash Management Strategy and Asymmetric FVA

Our definition of FVA is symmetric, in the sense that cash inflows and outflows are assumed to be financed or to reduce financings, respectively, at a spread of S. For the case of cash inflow, this implicitly assumes that there is always some short-term unsecured debt to roll over whose total amount can be reduced by swap cash inflows.

Now we consider the case in which the cash outflows are financed with unsecured debt and cash inflows are invested at the risk-free rate. All else is as in Section II.B. Correspondingly, we can calcuate the "asymmetric funding value adjustment" (AFVA) as

$$\tilde{\Phi}(u) = E^* \left[\delta_1 \mathbf{1}_{\{\tau_D > 1\}} u^+ S_0\right] + E^* \left[\delta_2 \mathbf{1}_{\{\tau_D > 2, \tau_C > 1\}} (V_1 + u - V_0)^+ S_1\right].$$

If the unsecured swap is executed at $u = V_0$, then the AFVA is

$$\tilde{\Phi}(V_0) = E^* \left[\delta_1 \mathbf{1}_{\{\tau_D > 1\}} V_0^+ S_0\right] + E^* \left[\delta_2 \mathbf{1}_{\{\tau_D > 2, \tau_C > 1\}} V_1^+ S_1\right].$$

III. The Effect of Netting with Legacy Positions

In this section, we extend the results in Section III.A of ADS to the case in which the dealer has a pre-existing swap position with the swap client.

The dealer purchases a new unsecured swap from a client, which is identical to that in Section III.A of ADS. This same client already has a legacy swap position with the dealer, whose contractually promised payment is c_0 and requires the dealer to make an upfront payment of u_0. As has been our convention, the positive cash flow of this contract is an asset to the dealer, whereas the negative cash flow is a contingent liability.

As in the main text, we characterize the marginal value of the new swap investment for the dealer's legacy shareholders and legacy creditors (excluding the swap counterparty). We also characterize the marginal market value of the new swap investment. As we have noted, this first-order valuation approach is sufficiently accurate to analyze the investment, except for in cases in which the size of the investment is large relative to the dealer's entire balance sheet. To this end, we compute the first-order valuation effects of the aggregate positions and the legacy swap with the client. The difference between the two is the first-order valuation of the new swap investment.

A. Market Value

As explained by Mengle (2010), standard ISDA agreements specify close-out netting at default of either counterparty. We let B denote the client's default event, which is assumed to be independent under P^* of the floating-side swap payment X. By direct analogy with calculations in Appendix B of ADS, the marginal market value of the new swap is well defined by

$$V = -u + \delta \left(E^*(X - K) + E^* \left[\phi \left((X - K + c_0)^- - c_0^- \right) \right] - E^* \left[\gamma \left((X - K + c_0)^+ - c_0^+ \right) \right] \right), \tag{IA8}$$

and V is invariant to whether the dealer finances the swap by issuing debt, issuing equity, or using existing cash on its balance sheet. That is, $\delta E^*[\gamma((X - K + c_0)^+ - c_0^+)]$ and $\delta E^*[\phi((X - K + c_0)^- - c_0^-)]$ are the incremental CVA and DVA due to the new swap position, respectively.

B. Shareholder Value

We focus on the case in which the dealer finances swap positions by issuing new debt. From Proposition 1 of ADS, the first-order valuation effect to shareholders of the swap portfolio is

$$G_a = \delta E^*[\mathbf{1}_{D^c}(X - K + c_0)] - \delta E^*[\mathbf{1}_{D^c}(u_0 + u)(R + S)] - \delta E^*[\mathbf{1}_{D^c}\gamma(X - K + c_0)^+].$$

Similarly, the first-order valuation effect of the legacy swap to shareholders is

$$G_0 = \delta E^*(\mathbf{1}_{D^c} c_0) - \delta E^*[\mathbf{1}_{D^c} u_0(R + S)] - \delta E^*(\mathbf{1}_{D^c}\gamma c_0^+).$$

Thus, the marginal value of the new swap to the shareholders is

$$G = G_a - G_0 = \delta E^*[\mathbf{1}_{D^c}(X - K)] - \delta E^*[\mathbf{1}_{D^c} u(R + S)] - \delta E^*[\mathbf{1}_{D^c}\gamma((X - K + c_0)^+ - c_0^+)].$$

C. Legacy Creditor Value

We also consider the marginal value of the new swap to the dealer's existing creditors (excluding the swap client). To this end, we characterize the first-order effect of the legacy swap, and we characterize the first-order effect of the swap portfolio. Thus, the marginal value of the new swap to the dealer's legacy creditors is

$$H = \delta E^*[\mathbf{1}_D(X-K)] - \delta E^*(\mathbf{1}_D uR) + \delta E^*(\mathbf{1}_{D^c} uS) + \delta E^*[\phi((X-K+c_0)^- - c_0^-)]$$
$$- \delta E^*[\gamma \mathbf{1}_D((X-K+c_0)^+ - c_0^+)] - \delta(1-\kappa)J,$$

where

$$J = \lim_{q \to 0} E^* \left(\frac{\mathcal{A}(q)\mathbf{1}_{\mathcal{D}(q)} - \mathcal{A}_0(q)\mathbf{1}_{\mathcal{D}_0(q)}}{q} \right),$$

and J is well defined by the same argument used in Appendix B of ADS.

In the special case of no distress costs ($\kappa = 1$), we have $V = G + H$.

REFERENCES

Albanese, Claudio, and Andersen, Leif, 2014, Accounting for OTC derivatives: Funding adjustments and the re-hypothecation option, Working paper, Cass School of Business.

Andersen, Leif, Darrell Duffie, and Yang Song, 2018, Funding value adjustments, *Journal of Finance*, Forthcoming.

Mengle, David, 2010, The importance of close-out netting, ISDA Research Notes.