Statistical Inference: Problem 2

Sebastian Nilsson - 9003202158

February 2019

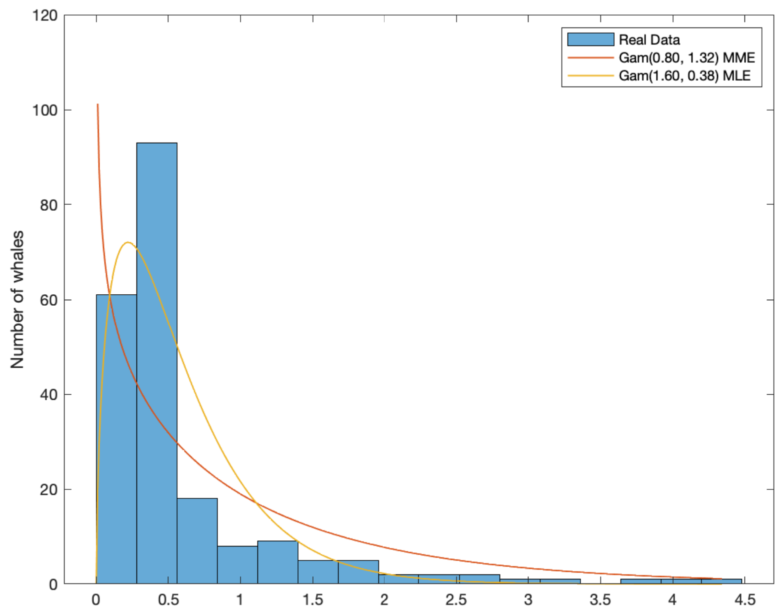

Figure 1: The sample data and the two fitted distributions

a)

Yes, it seems plausible. The sample data looks skewed to the the left, which can be modelled with the gamma distribution.

b)

The gamma distribution fitted with the method of moments can be seen as the red curve in figure 1. The following formula was used to find the parameters:

$$\mu_1 = E(X)$$
$$\mu_2 = E(X^2)$$
$$\tilde{\alpha} = \frac{\mu_1^2}{\mu_2 - \mu_1^2}$$
$$\tilde{\lambda} = \frac{\tilde{\alpha}}{\mu_1}$$

The resulting parameters found was $\tilde{\alpha} = 0.80, \tilde{\lambda} = 1.32$

c)

The formula used to find the most likelihood of alpha was

$$n \log \hat{\alpha} - n \log \bar{X} + \sum_{i=1}^{n} \log X_{\hat{\alpha}} - \frac{\Gamma'(\hat{\alpha})}{\Gamma(\hat{\alpha})} = 0$$

Mathematica was used to numerically find the most likelihood estimate of alpha:

```
n = 210;
FindRoot[n*Log[a] + 105.1841 - 177.6407 -
n*(Gamma'[a]/Gamma[a]) == 0, {a, 0.7992}]
```

Where the initial value of alpha , a, was the method of moments estimate of alpha, $\tilde{\alpha}$.

d)

The fits look reasonable (See figure 1), however they seem to cut off the second bin, which is very high.

e)

For task e) and f) the follwing formula was used to find the standard error, e, of the sampling distributions:

$$e = \frac{\sigma}{\sqrt{n}}$$

Where σ is the standard deviation of the sampling distribtuion.

Using 1500 samples of size 200 results in the following two sampling distributions:

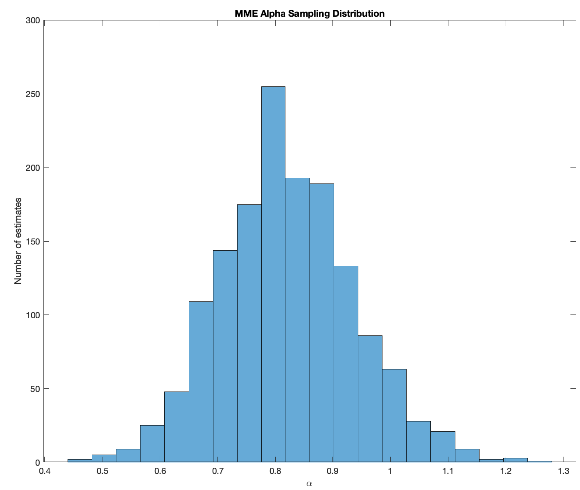

Figure 2: The sampling distributions for $\tilde{\alpha}$

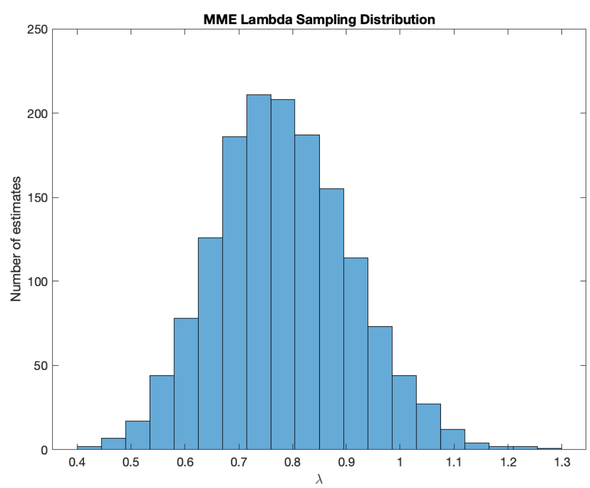

Figure 3: The sampling distributions for $\tilde{\lambda}$

The standard error for the $\tilde{\alpha}$ sampling distribution (Figure 2) was found to be approximately 0.008. The $\tilde{\lambda}$ sampling distribution (Figure 3) the standard error was found to be approximately 0.009.

f)

Again using 1500 samples of size 200 results in the following two sampling distributions:

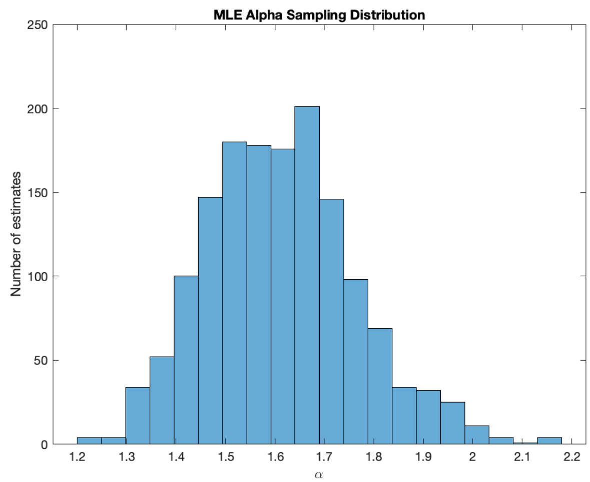

Figure 4: The sampling distributions for $\hat{\alpha}$

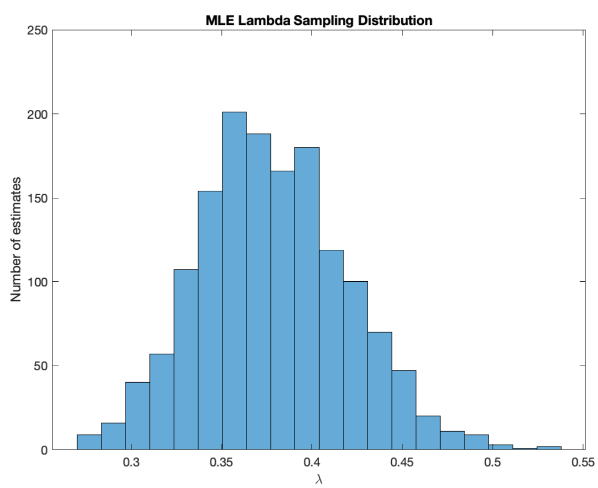

Figure 5: The sampling distributions for $\hat{\lambda}$

The standard error for the $\hat{\alpha}$ sampling distribution (Figure 4) was found to be approximately 0.010. The $\hat{\lambda}$ sampling distribution (Figure 5) the standard error was found to be approximately 0.002. Drawing from the distribution estimated by the maximum likelihood found that the standard error for alpha was slightly higher in comparison with method of moments. However, the standard error was significantly lower for $\hat{\lambda}$.

g)

We sort the bootstrap values, $\alpha_1^*, \alpha_2^*, ...\alpha_{1500}^*$, and calculate $\underline{\delta}$ and $\bar{\delta}$:

$$\underline{\delta}_{\hat{\alpha}} = \alpha_{75}^* - \hat{\alpha}$$
$$\bar{\delta}_{\hat{\alpha}} = \alpha_{1425}^* - \hat{\alpha}$$

We get the following 95 % confidence interval for $\hat{\alpha}$

$$(\hat{\alpha} - \bar{\delta}_{\hat{\alpha}}, \hat{\alpha} - \underline{\delta}_{\hat{\alpha}}) = (1.3032, 1.8020)$$

Repeating the same procedure for λ gets us the following interval for $\hat{\lambda}$

$$(\hat{\lambda} - \bar{\delta}_{\hat{\lambda}}, \hat{\lambda} - \underline{\delta}_{\hat{\lambda}}) = (0.3122, 0.4493)$$