

Propagating state distributions: prop

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December 26th, 2011

Abstract

This is a short documentation of `prop.m`.

At time t the *state distribution* consists of two copies of the representation, the actual state, $s(t)$, and the *forward predicted belief state*, $r(t)$. These are jointly Gaussian

$$\begin{bmatrix} s(t) \\ r(t) \end{bmatrix} \sim \mathcal{N} \left(\begin{bmatrix} \mu_s \\ \mu_r \end{bmatrix}, \begin{bmatrix} \Sigma_s & \Sigma_c \\ \Sigma_c^\top & \Sigma_r \end{bmatrix} \right).$$

The belief state, $b(t)$, is computed by combining the noisy actual state $\mathcal{N}(\mu_s, \Sigma_y = \Sigma_s + \Sigma_n)$ with $r(t)$ using a convex combination where each mean is weighted by its precision. Note, that this only corresponds to the Bayesian posterior if $s(t)$ and $r(t)$ are uncorrelated.

$$\begin{aligned} \mu_b &= (\Sigma_s^{-1} + \Sigma_r^{-1})^{-1} (\Sigma_s^{-1} \mu_s + \Sigma_r^{-1} \mu_r) = Z_r \mu_s + Z_n \mu_r, \\ \Sigma_b &= Z_r (\Sigma_s + \Sigma_n) Z_r^\top + Z_n \Sigma_r Z_n^\top + Z_r \Sigma_c Z_n^\top + Z_n \Sigma_c^\top Z_r^\top, \end{aligned}$$

where we have defined $Z_r = \Sigma_r (\Sigma_s + \Sigma_r)^{-1}$ and $Z_n = \Sigma_n (\Sigma_s + \Sigma_r)^{-1}$.

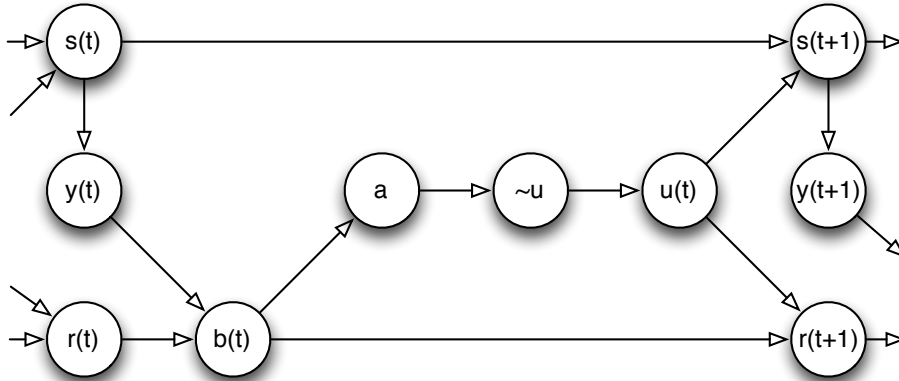


Figure 1: Graphical model representing the time evolution of the state s and belief-state b .

1	s(t)	c(t)										
D0	c'(t)	r(t)										
D1			b									
D2				a								
D3	trig s											
D4			trig b									
D5				trig a								
D6	~u											
D7	u											
D8										s(t+1)	c(t+1)	
D9										c'(t+1)	r(t+1)	
Da												
	1	D0	D1	D2	D3	D4	D5	D6	D7	D8	D9	Da

Figure 2: Diagram showing the memory layout for the covariance matrix associated with a single forward propagation. The mean vector uses a similar ordering. Shaded variables are never computed.