Übersicht Kapitel 4: Leitungscodierung und Modulation

Datenübertragung auf einer Trägerfrequenz

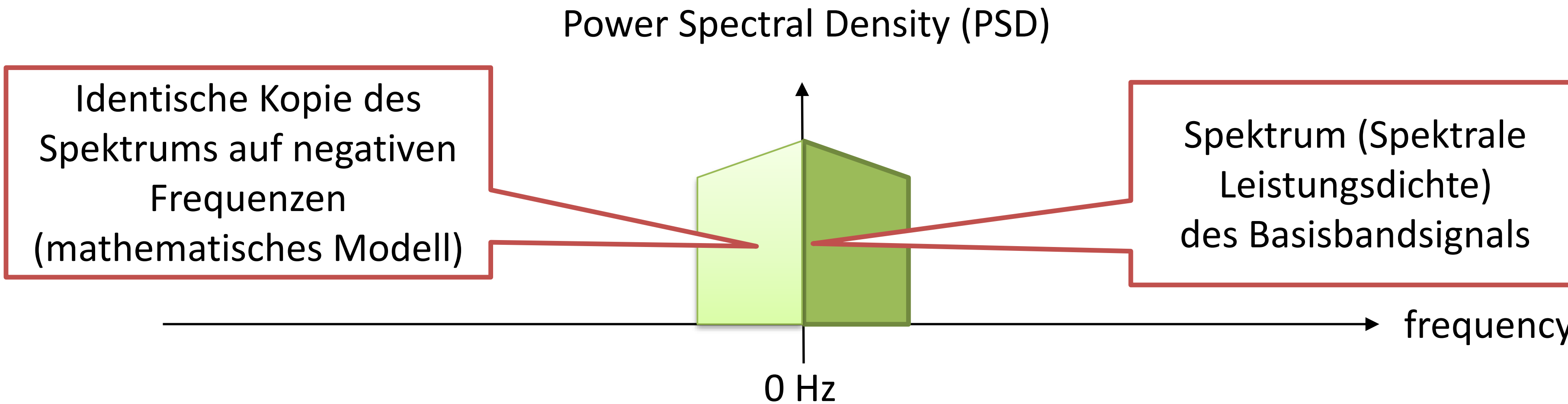
- Die Übertragung eines Signals im Basisband bedeutet, dass ein Signal in seinem natürlichen Spektrum übertragen wird.
  - menschliche Stimme liegt im Bereich von 80Hz bis 12kHz, Übertragung des Bereichs von 300Hz bis 3,4kHz ausreichend
  - MLT-3 Signal mit 125Mb/s benötigt den Frequenzbereich bis 31,5MHz
- Alle Basisbandsignale liegen im gleichen Spektrum ab 0Hz und würden sich bei gleichzeitiger Übertragung auf einem Medium überlagern
  - nur eine Übertragung pro Kabel
  - nur eine Übertragung über Funkwellen in räumlicher Umgebung
- Um gleichzeitige Übertragungen auf einem Medium zu ermöglichen, werden mehrere Basisbandsignale auf unterschiedliche **Trägerfrequenzen** verschoben, die weit genug auseinander liegen, dass sich die Signale nicht stören. Diese Technik nennt sich **Frequenzmultiplex**.
- Die Technik, ein Basisbandsignal auf den Frequenzbereich um eine Trägerfrequenz zu schieben, nennt sich **Modulation**.

Einsatz von Frequenzmultiplex

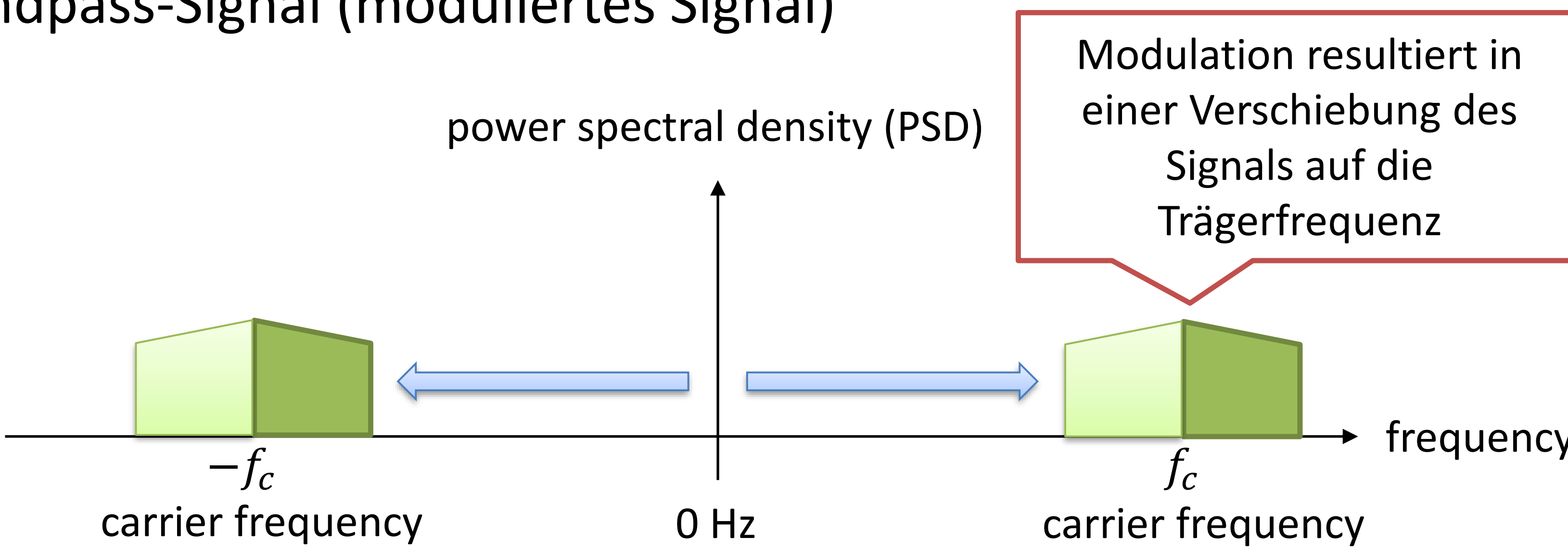
- Funk: Verschiedene Frequenzbereiche werden für verschiedene Funkanwendungen und -technologien verwendet
  - Radio (AM, FM):
    - nutzt einen Frequenzbereich um 100MHz
    - Einstellen der richtigen Frequenz für einen Radiosender
  - Mobilfunk:
    - nutzt verschiedene Frequenzbereiche von 900MHz für GSM bis 2,6GHz für LTE
    - weitere Unterteilung der Frequenzbereiche für verschiedene Technologien, Betreiber und schließlich Basisstationen und Handys
  - WLAN:
    - 20MHz Kanäle im 2.4GHz Band und 5GHz Band
- Kabel:
  - Kabelfernsehen: unterschiedliche Sender auf verschiedenen Frequenzen
  - Internet über Kabel:
    - alle Teilnehmer in einem Gebiet nutzen das gleiche Kabel
    - Nutzung unterschiedlicher Frequenzen für gemeinsame Fernsehübertragung und individuelle Datenübertragung

Datenübertragung auf einer Trägerfrequenz

- Basisband-Signal (modulierendes, informationstragendes Signal)



- Bandpass-Signal (moduliertes Signal)



Modulation

- Information wird durch die systematische Veränderung einer oder mehrerer Eigenschaften des Trägersignals übertragen: der Amplitude, der Phase, der Frequenz
- Modulationsverfahren zur Übertragung analoger Signale:
  - Amplitudenmodulation (**AM**, engl. Amplitude Modulation)
  - Frequenzmodulation (**FM**, engl. Frequency Modulation)
- Modulationsverfahren zur Übertragung digitaler Signale:
  - Amplitudenumtastung (ASK, engl. Amplitude Shift Keying)
  - Frequenzumtastung (FSK, engl. Frequency Shift Keying)
  - Phasenumtastung **(PSK**, engl. Phase Shift Keying)

Modulation

- Definition:

  Modulation ist definiert als der Vorgang, bei dem eine Charakteristik einer Trägerwelle einem informations-tragenden Signal entsprechend verändert wird.

- Informations-tragendes Signal=modulierendes Signal
  (engl: information-bearing signal, **modulating signal)**

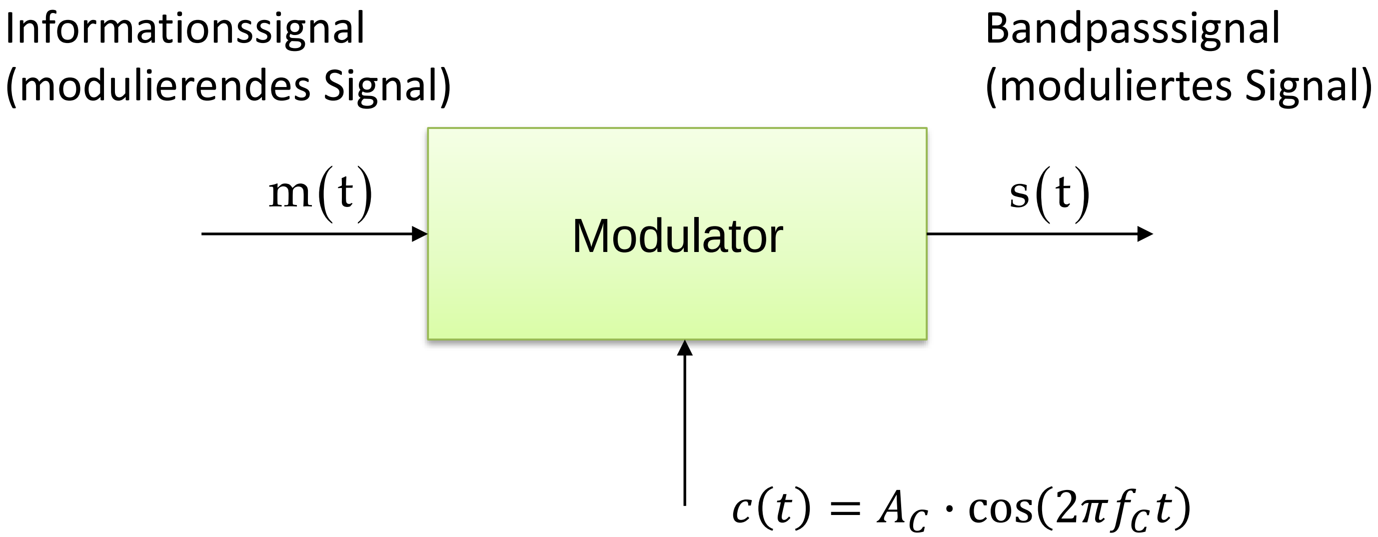
- Verändertes Trägersignal: moduliertes Signal
  (engl: **modulated signal)**

- Trägersignal
  (engl: **carrier** signal):

$$c(t) = \underbrace{A_c}_{\textbf{amplitude}} \cos\left(2\pi \underbrace{f_c}_{\textbf{frequency}} t + \underbrace{\theta}_{\textbf{phase}}\right)$$

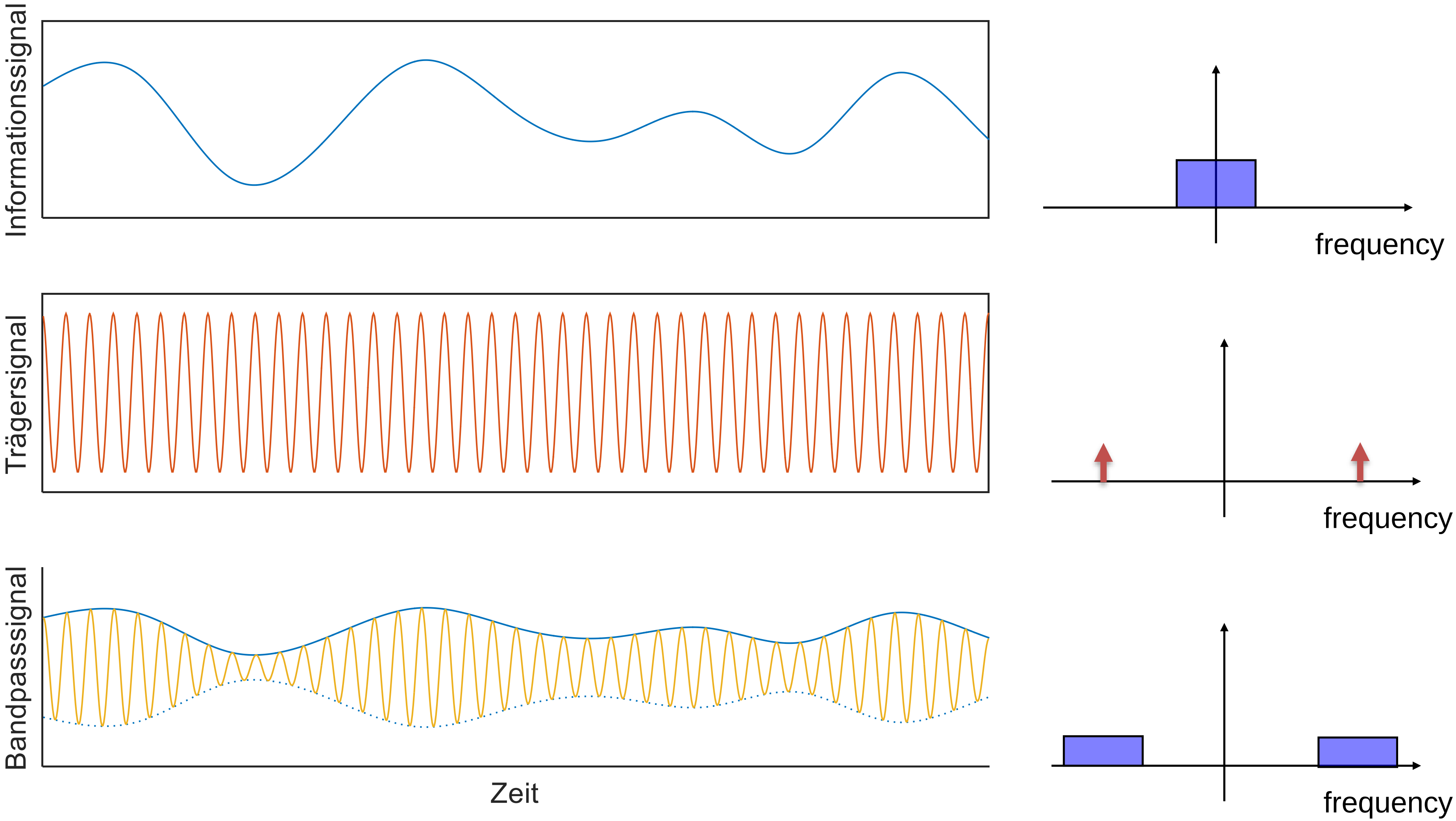
Analoge Amplitudenmodulation

$$s(t) = A_c \cdot m(t) \cdot \cos(2\pi f_c t)$$

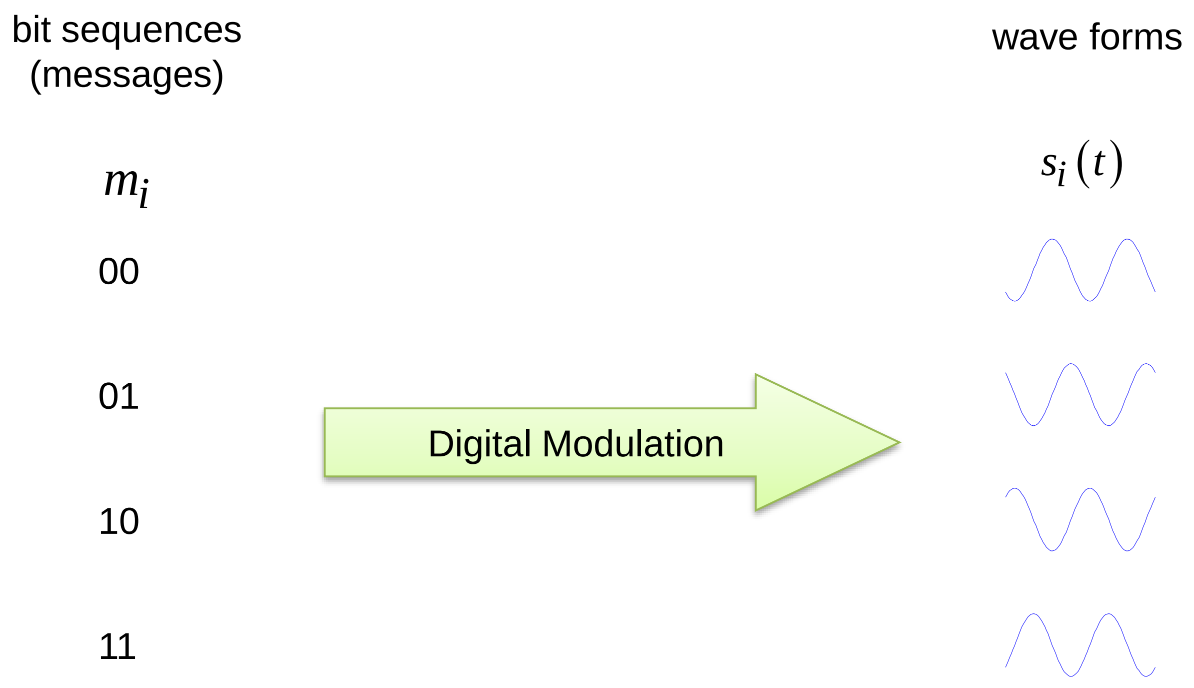


Trägersignal mit Trägerfrequenz f_C und Amplitude A_C

Beispiel: Analoge Amplituden Modulation



General Principle of Digital Modulation

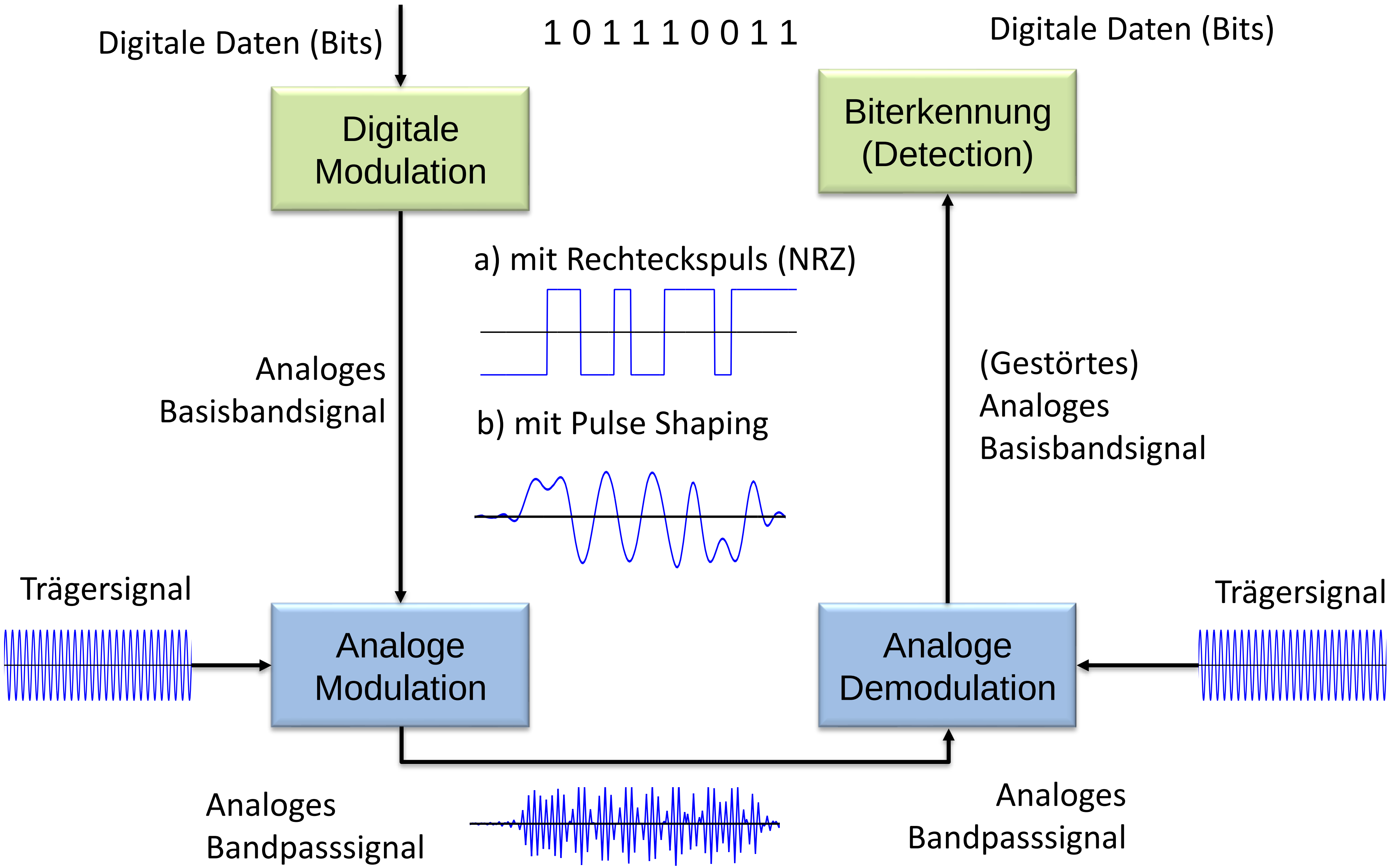


- Mapping of sequences of one or more bits to different wave forms
  - sequence of bits is called a SYMBOL
  - duration of a wave form is called SYMBOL DURATION T_s (Symboldauer)
  - symbol rate $f_s = \frac{1}{T_s}$ (inverse of symbol duration)

Digitale Modulation

- Die digitale Modulation wird in zwei Schritten ausgeführt:
  - ein analoges Basisbandsignal wird generiert, das die Veränderung der Amplitude, Phase oder Frequenz beschreibt
    - dieser Vorgang nennt sich Baseband Processing (Signalverarbeitung im Basisband) und kann sender- und empfängerseitig in Software durchgeführt werden. Dies wird als Software Defined Radio (SDR) bezeichnet.
    - oft wird auch dieser Teilschritt als Digitale Modulation bezeichnet, z.B. in Matlab
  - das analoge Basisbandsignal wird über AM auf die Trägerfrequenz geschoben
    - dieser Schritt wird als RF Processing bezeichnet und ist im RF Front End in Hardware implementiert

Digitale Modulation



Digital Modulation

- Considerations in the design of digital modulation techniques
  - high data rate
  - high spectral efficiency
  - high power efficiency
  - robustness to channel impairments
  - low power/cost implementation

- Categories of digital modulation
  - linear modulation
    - amplitude and phase modulation
    - mPSK (**BPSK**, **QPSK**, 8PSK) e.g. EDGE, UMTS, WiMAX, DVB
    - **mQAM** e.g. HSDPA, WiMAX, WLAN, DVB, DSL
  - non-linear modulation
    - frequency modulation
    - GMSK, Gaussian Minimum Shift Keying (e.g. GSM)

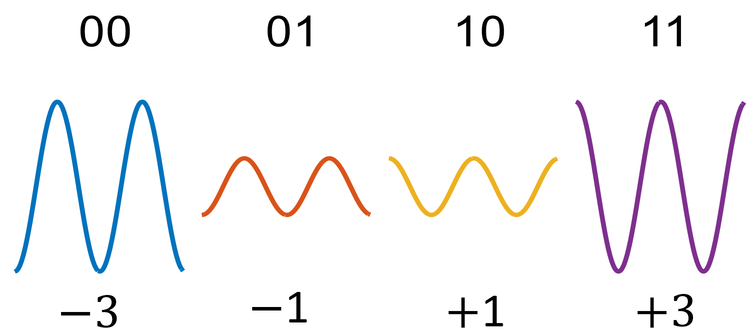
Digitale Modulation

- Wave forms may be distinguished by different characteristics:

Amplitude

ASK, Amplitude Shift Keying (Amplitudenumtastung)

Wave forms with different AMPLITUDES

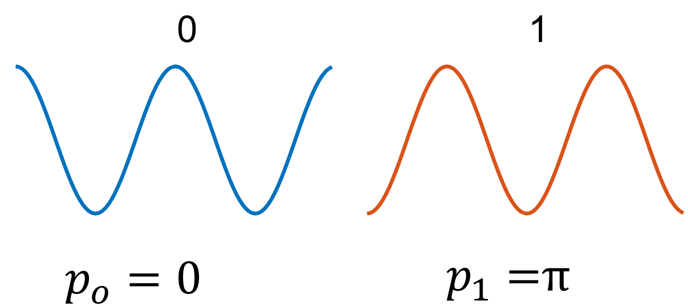


$f_C = 4Hz$

Phase

PSK, Phase Shift Keying (Phasenumtastung)

Wave forms with different PHASE offset

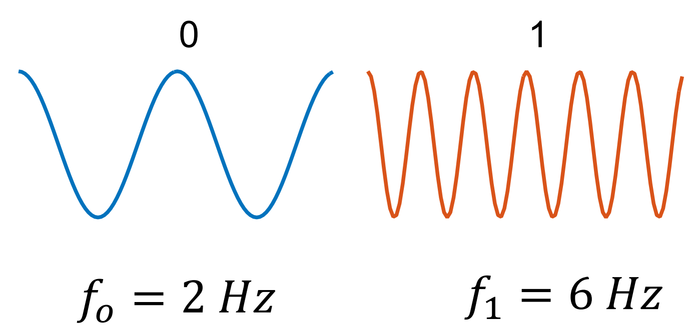


$f_C = 4Hz$

Frequency

FSK, Frequency Shift Keying (Frequenzumtastung)

Wave forms with different Frequencies



$f_C = 4Hz$

Übersicht Kapitel 4: Leitungscodierung und Modulation

Darstellung eines Symbols als komplexe Zahl

- Ein moduliertes Signal wird mathematisch definiert als

$$\begin{aligned} s(t) &= s_I(t) \cdot cos(2\pi f_c t) - s_Q(t) \cdot sin(2\pi f_c t) \\ &= \mathrm{A}(t) \cdot \cos\big(2\pi f_c t + \varphi(t)\big) \end{aligned}$$

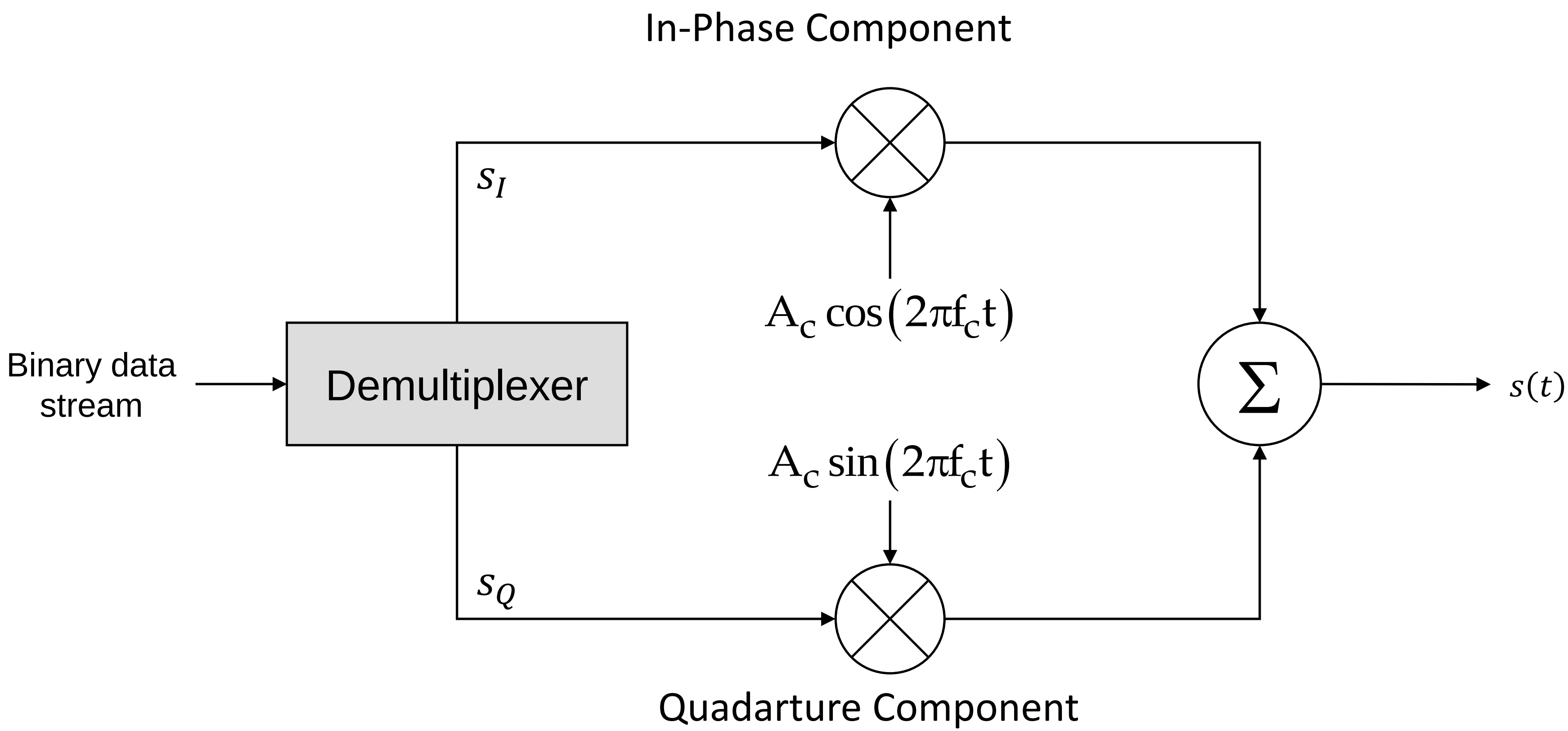
- Das Signal besteht aus einem Cosinusträger, der mit der In-Phase Komponente s_I moduliert wird und einem Sinusträger, der mit der negierten Quadrature-Komponente s_Q moduliert wird.
- Ein Symbol ist folglich vollständig definiert über seine In-Phase-Komponente s_I und seine Quadrature-Komponente s_Q
- Ein Symbol wird daher als komplexe Zahl geschrieben

$$s = s_I + j \cdot s_Q$$

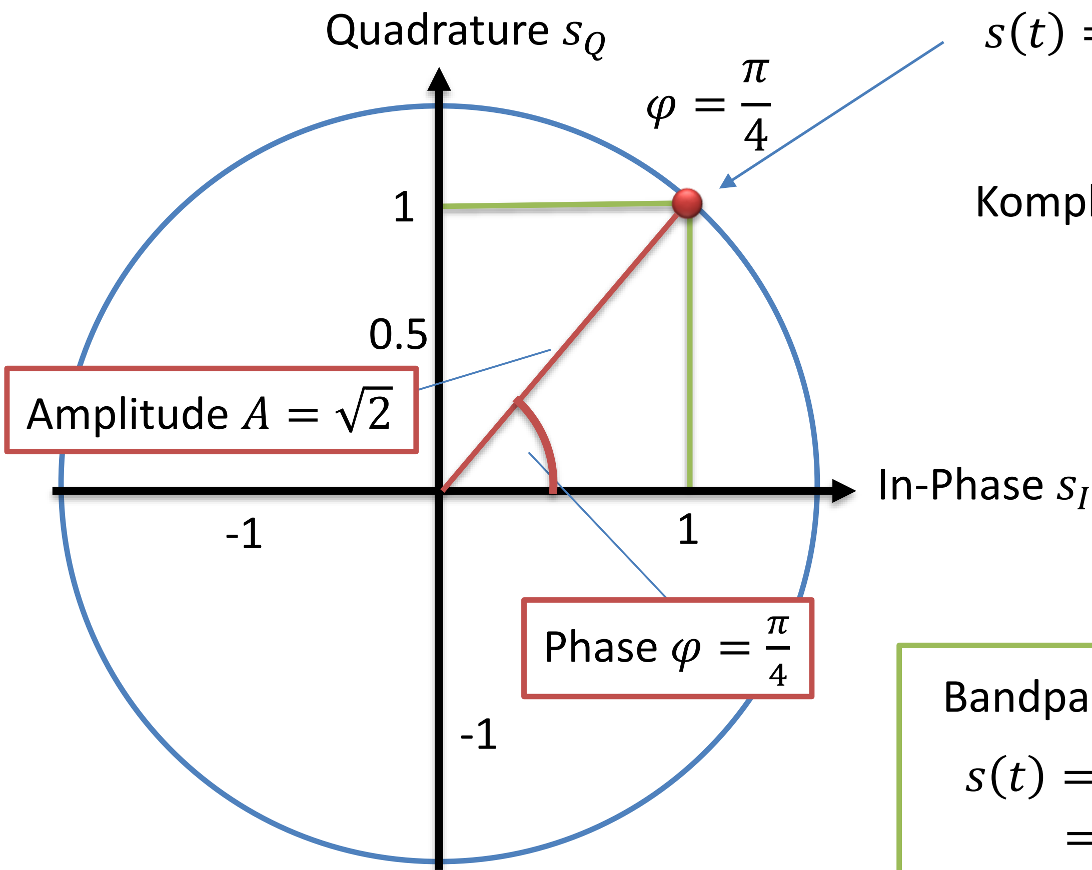
- Ein Symbol ist alternativ auch über seine Amplitude A und seine Phase φ vollständig definiert

$$A = \sqrt{{s_I}^2 + {s_Q}^2} \qquad \varphi = \tan^{-1} \frac{s_Q}{S_I}$$

Modulation


In-Phase Component
s_I
$A_c \cos(2\pi f_c t)$
Binary data stream
Demultiplexer
Σ
$s(t)$
$A_c \sin(2\pi f_c t)$
s_Q
Quadarture Component

Signaldarstellung im Koordinatensystem



Moduliertes Signal

$$s(t) = \cos(2\pi f_c t) - sin(2\pi f_c t) = \cos\left(2\pi f_c t + \frac{\pi}{4}\right)$$

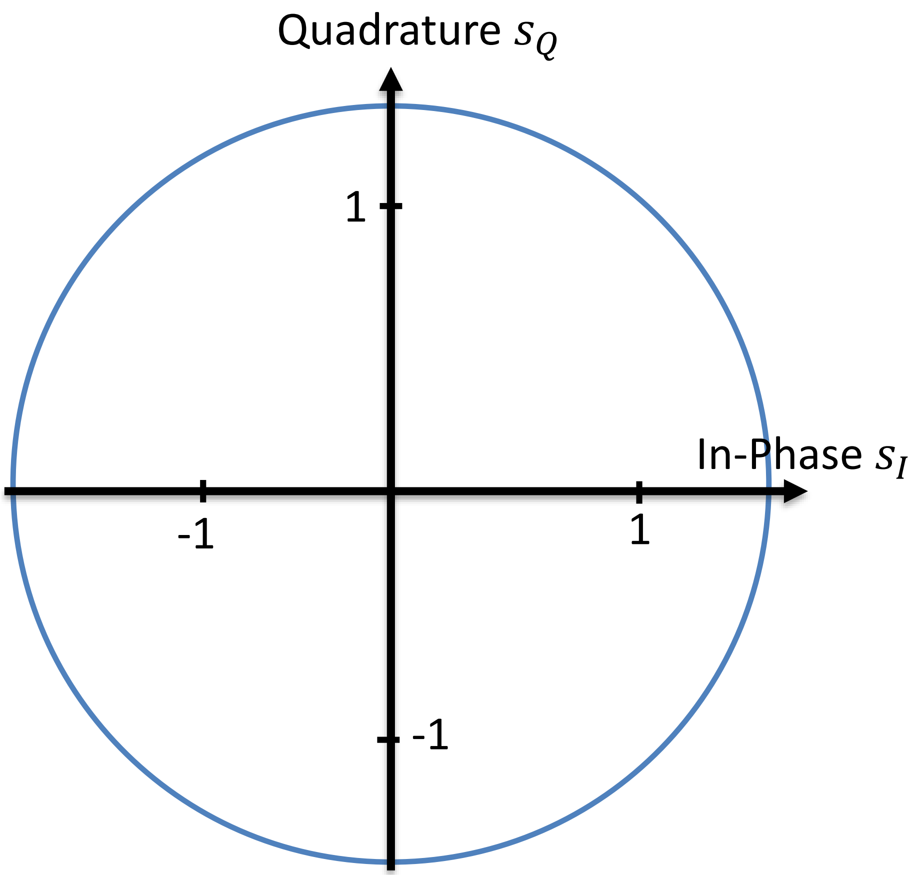
Komplexes Symbol: $s = 1 + j$

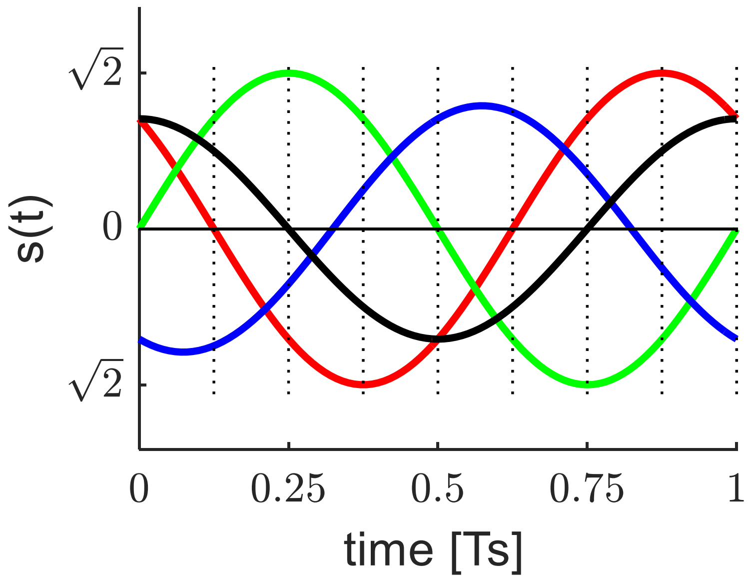
Bandpasssignal:

$$s(t) = s_I \cdot cos(2\pi f_c t) - s_Q \cdot sin(2\pi f_c t)$$
$$= \mathrm{A} \cdot \cos(2\pi f_c t + \varphi)$$

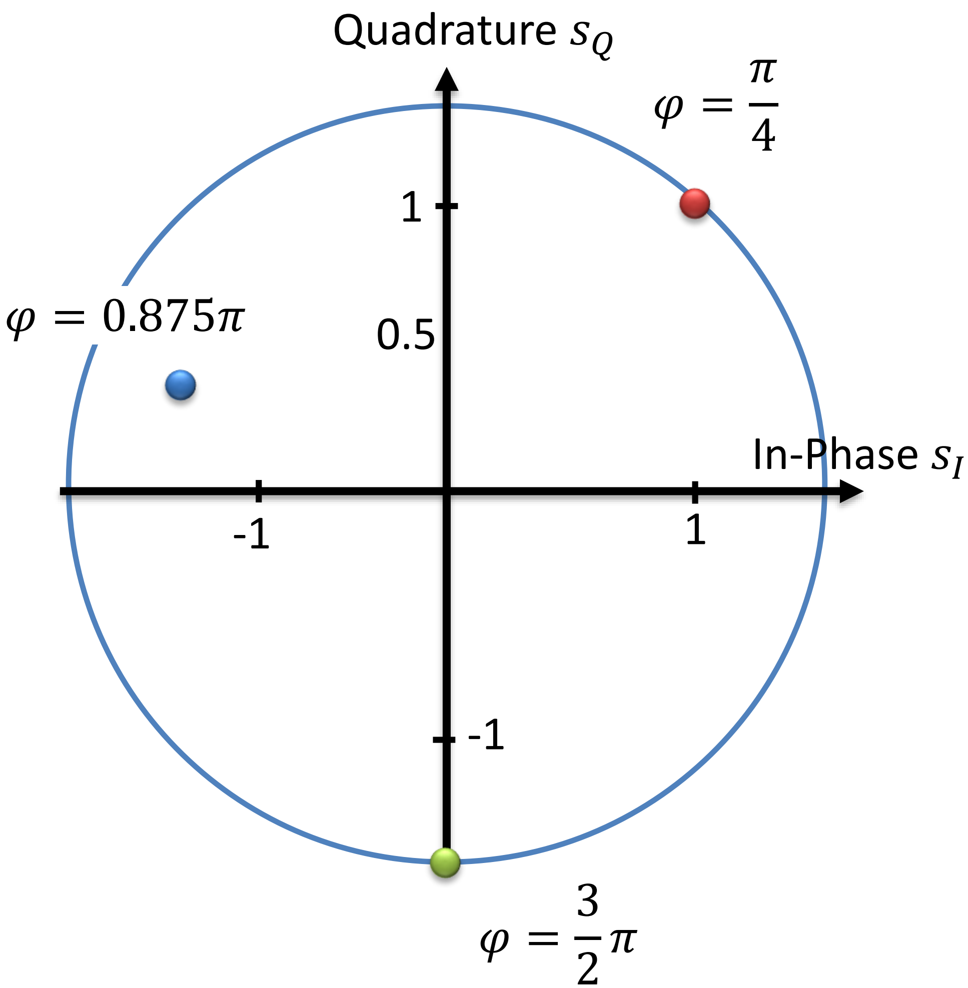
Komplexes Symbol: $s = s_I + j \cdot s_Q$

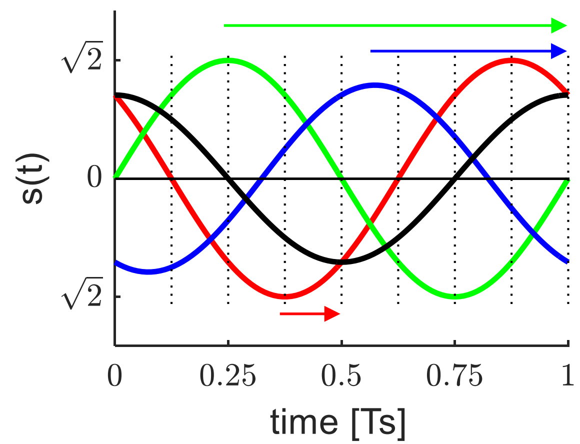
Signaldarstellung im Koordinatensystem





Signaldarstellung im Koordinatensystem





Übersicht Kapitel 4: Leitungscodierung und Modulation

4.1 Prinzip der Leitungscodierung und Modulation

4.2 Übertragung im Basisband

4.3 Übertragung auf einer Trägerfrequenz

- 4.3.1 Analoge und digitale Modulation
- 4.3.2 Mathematische und grafische Signaldarstellung
- **4.3.3 Modulationsarten**
- 4.3.4 Bandbreite von modulierten Signalen
- 4.3.5 Signalverfälschung im Funkkanal
- 4.3.6 Demodulation
- 4.3.7 Pulsformung

4.4 Zusammenfassung

Modulationsarten

- Eine Modulationsart bzw. ein Modulationsschema beschreibt, wie viele verschiedene Symbole übertragen werden können und wie diese Symbole im Konstellationsdiagramm angeordnet sind
- Generell gilt:
  - je größer die Anzahl der Symbole desto größer die Datenrate

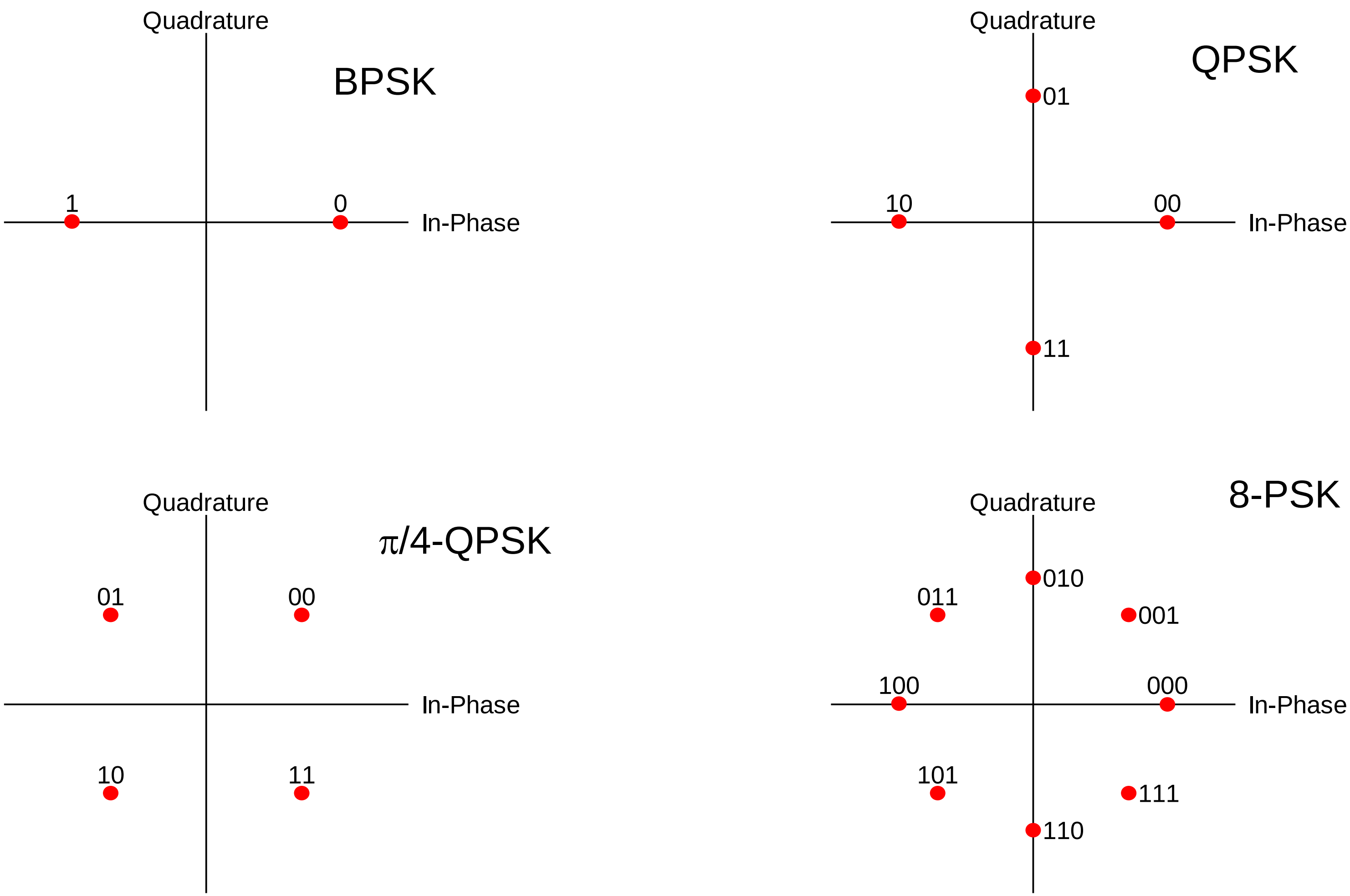
$$\boxed{Bits\ pro\ Symboldauer = \log_2(Anzahl\ Symbole)}$$

    - dies gilt unabhängig von der Bandbreite
    - Beispiel: bei einer Bandbreite von 1 MHz und 16 Symbolen wird eine Datenrate von 4 Mbps erzielt
  - je größer die Anzahl der Symbole desto größer ist die Fehleranfälligkeit
    - der Abstand zwischen zwei Symbolen im Konstellationsdiagramm legt fest, wie stark ein Rauschsignal sein muss, um einen Bitfehler zu verursachen

Phasenumtastung (Phase Shift Keying, PSK)

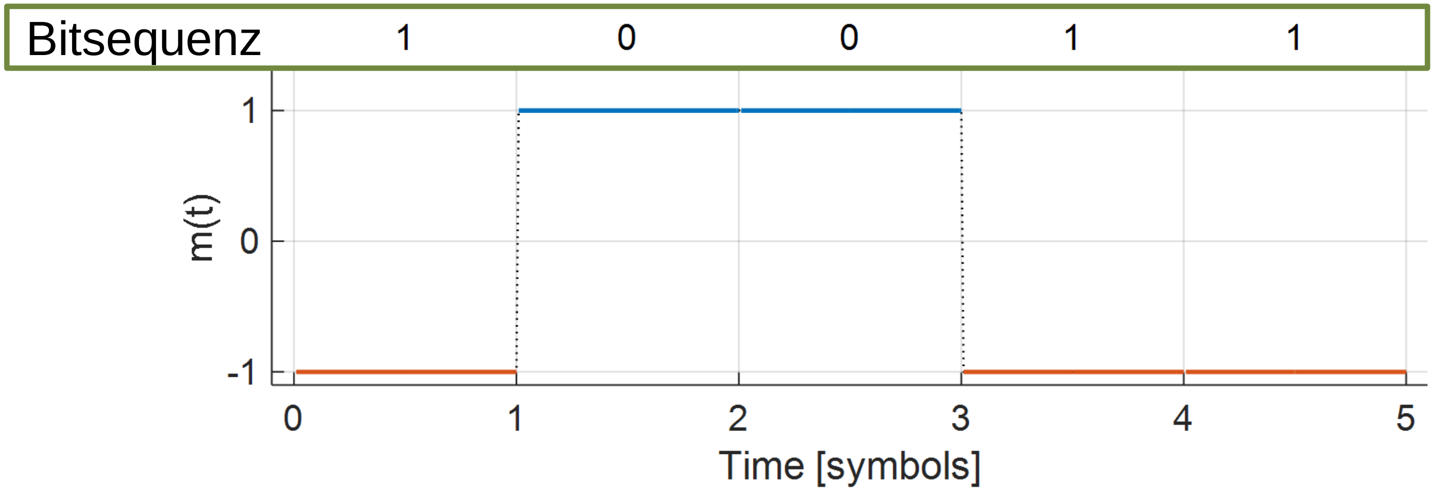
- alle Symbole liegen auf einem Kreis um den Nullpunkt des Konstellationsdiagramm, sie haben gleiche Amplitude und unterscheiden sich nur durch die Phase
  - m-PSK:
    - m Symbole mit $\log_2 m$ Bits pro Symboldauer
  - BPSK (Binary Phase Shift Keying):
    - 2 Symbole, 1 Bits pro Symboldauer
  - QPSK (Quadrature Phase Shift Keying):
    - 4 Symbole, 2 Bits pro Symboldauer
- BPSK und QPSK werden in Systemen mit komplexeren Transceivern für niedrige Datenraten eingesetzt (Mobilfunk, WLAN)
- 8-PSK wird in EDGE für höhere Datenraten eingesetzt

Signal Constellations for Phase Shift Keying

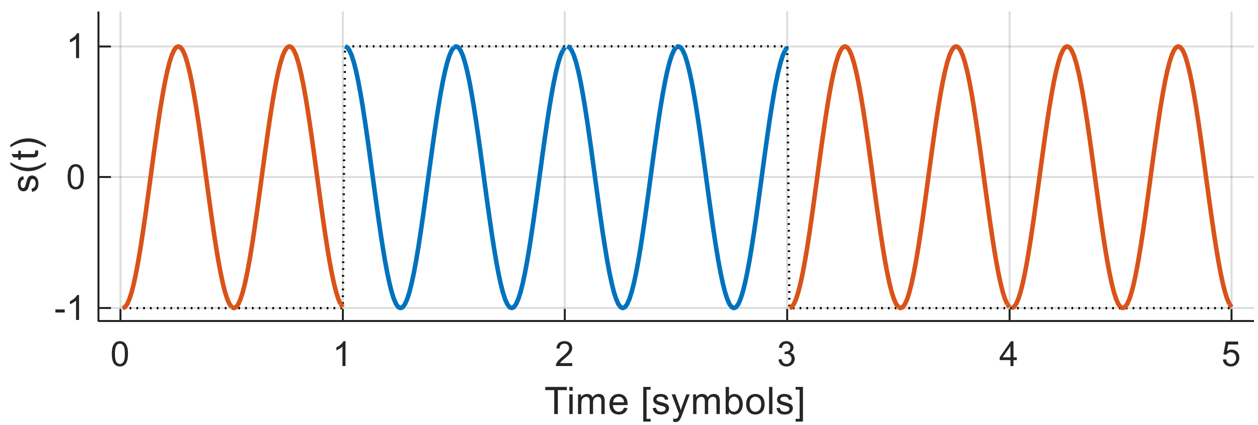


Beispiel: Binary Phase Shift Keying (BPSK)

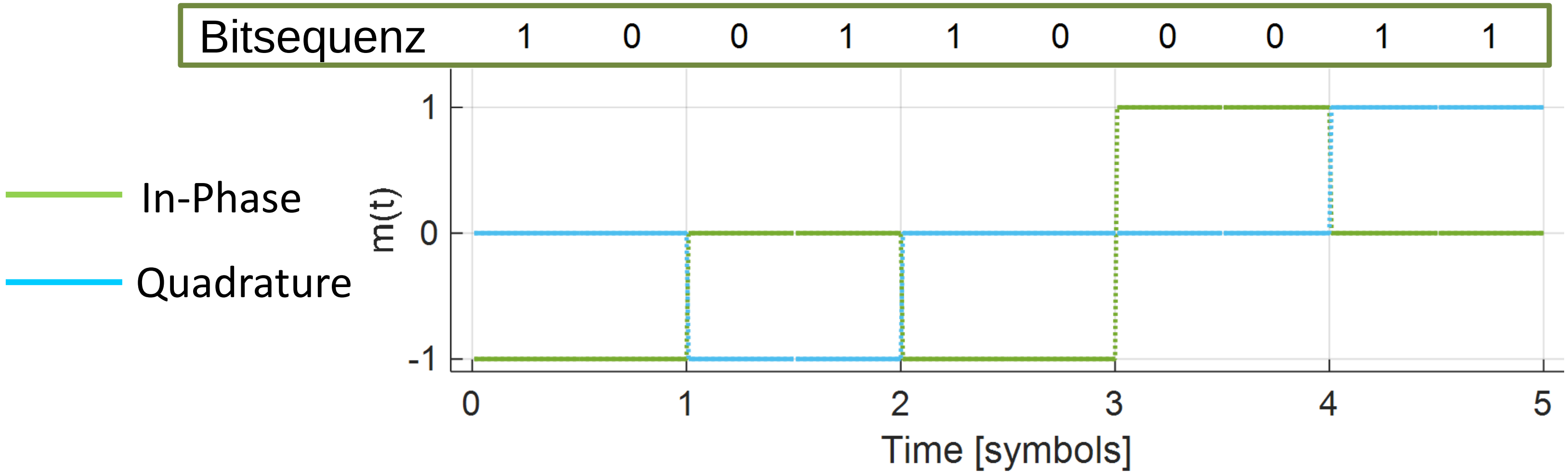
- Basisbandsignal (entspricht NRZ)



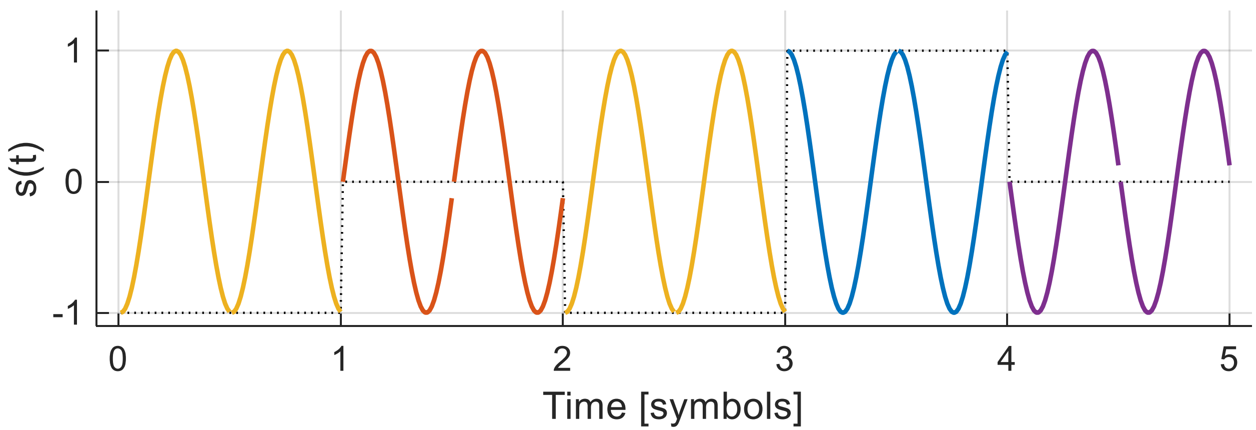
- Auf Trägerfrequenz $f_C = 2\ Hz$ moduliertes Signal



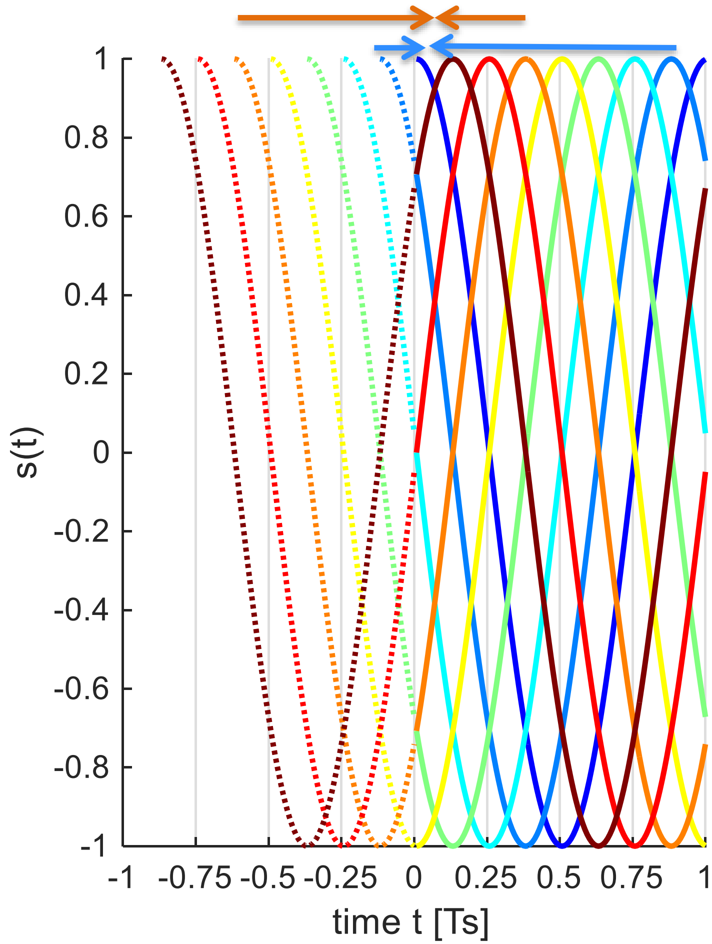
- In-Phase und Quadrature Basisbandsignale (entspricht NRZ)



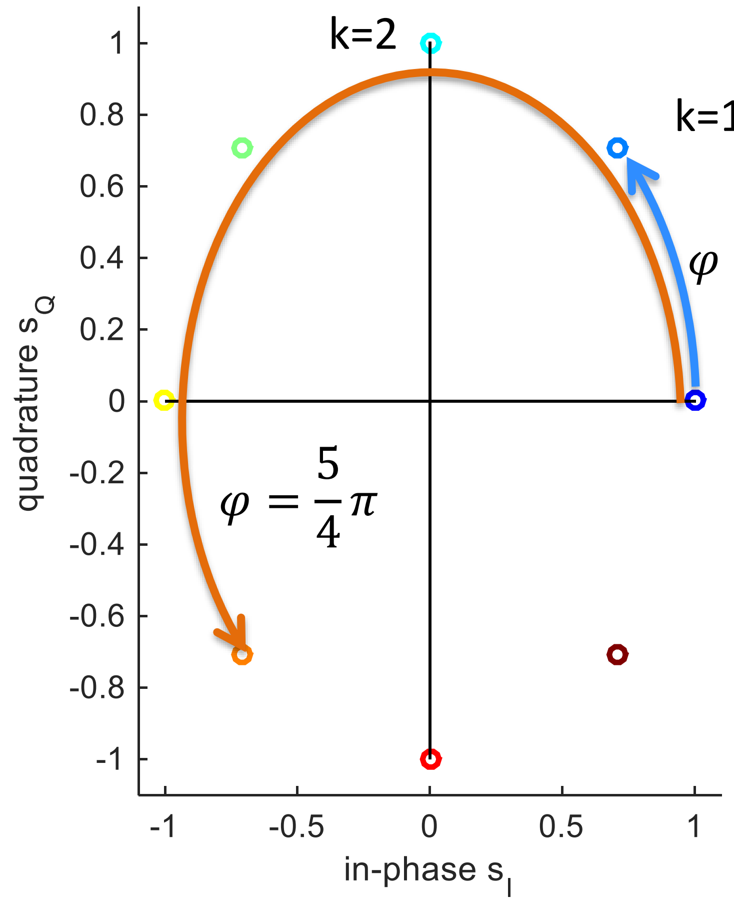
- Auf Trägerfrequenz $f_C = 2\ Hz$ moduliertes Signal



Signal-Konstellation 8-PSK

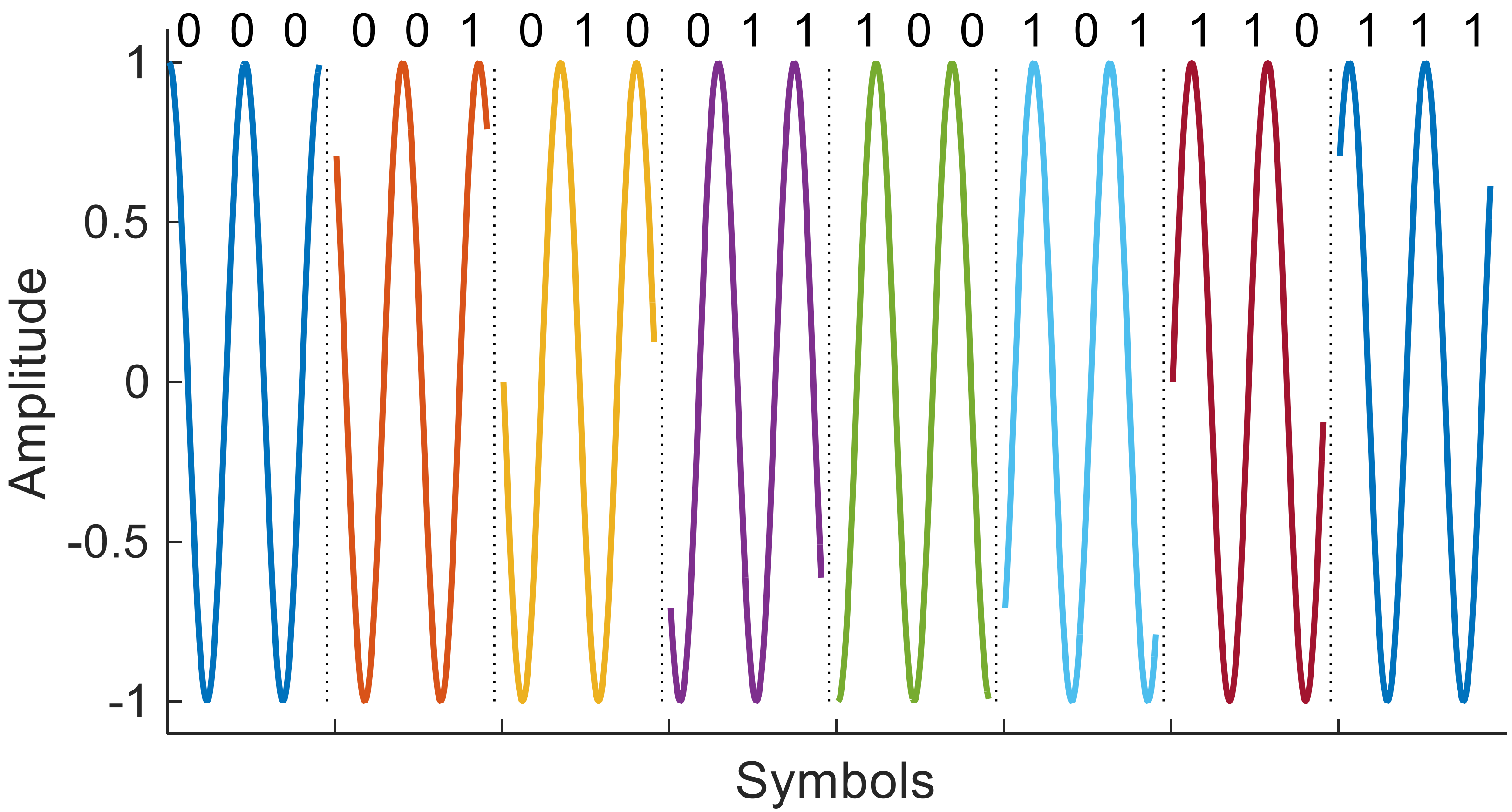


$$s(t) = \cos\left(2\pi f_c t + k \cdot \frac{\pi}{4}\right), k = 0, \dots, 7$$

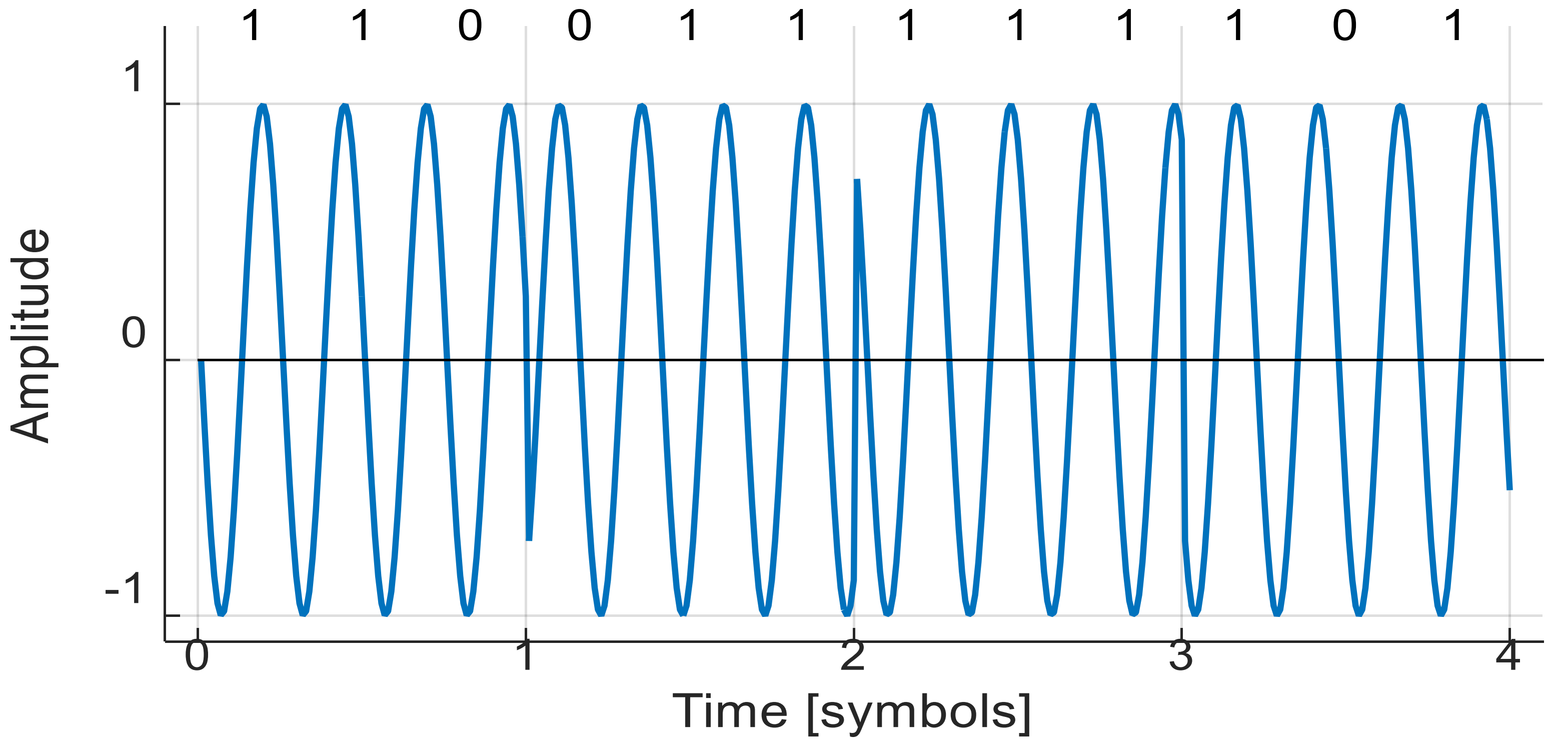


$$\varphi = \tan^{-1}\frac{s_Q}{s_I}$$

8-PSK-Symbole



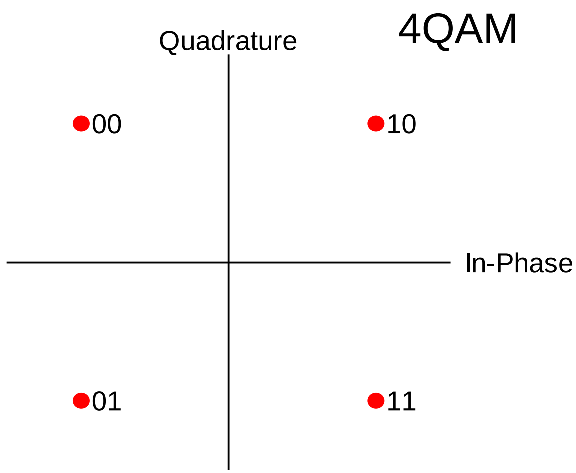
8-PSK-Signal

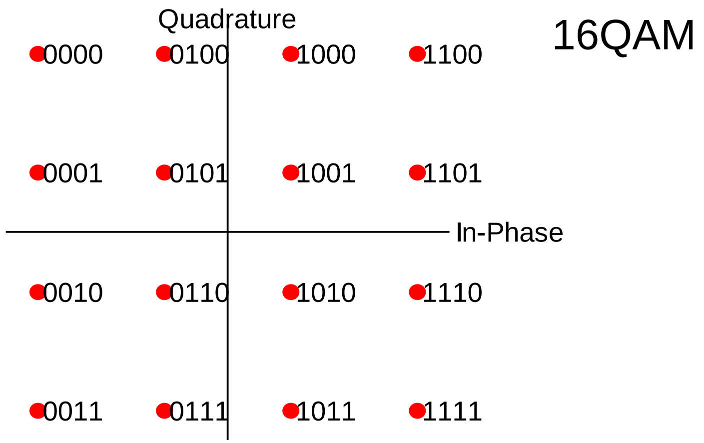


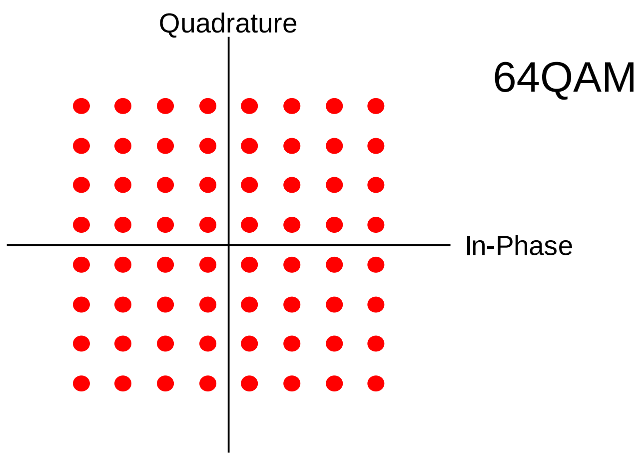
Quadrature-Amplitude Modulation (QAM)

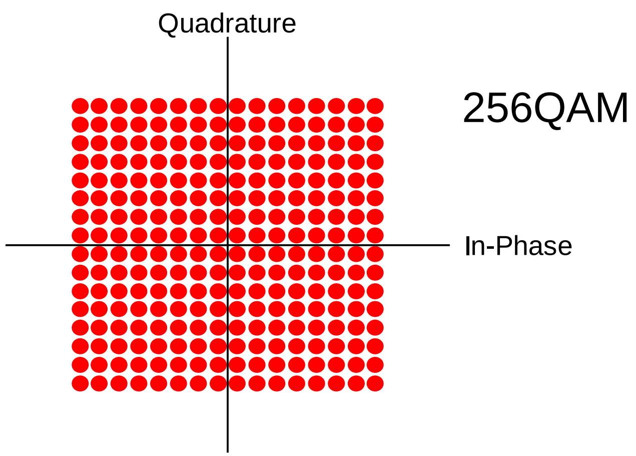
- Alle Symbole liegen im Konstellationsdiagramm auf einer regelmäßigen Gitterstruktur im Konstellationsdiagramm, sie unterscheiden sich in Amplitude und Phase
  - m-QAM:
    - m Symbole mit $\log_2 m$ Bits pro Symboldauer
    - üblich ist $m = 4^k$ mit $k = 1, 2, 3, \ldots$
  - 4-QAM: 4 Symbole, 2 Bits pro Symboldauer
    - identisch mit rotiertem QPSK
  - 16-QAM: 16 Symbole, 4 Bits pro Symboldauer
  - 64-QAM: 64 Symbole, 6 Bits pro Symboldauer
- QAM wird in Systemen mit komplexeren Transceivern für hohe Datenraten eingesetzt (Mobilfunk, WLAN, DSL, Kabelmodem)
  - bis zu 1024 QAM in WLAN und bis zu 32768-QAM in DSL
- Die Codierung über unterschiedliche Amplituden erfordert eine exakte Kenntnis des Funk-Kanals (der Empfangsleistung und der Phasenverschiebung), der über das Versenden von bekannten Symbolen in einer Präambel oder als Pilotsymbol bestimm wird.

Signal Constellations for Quadrature Amplitude Modulation



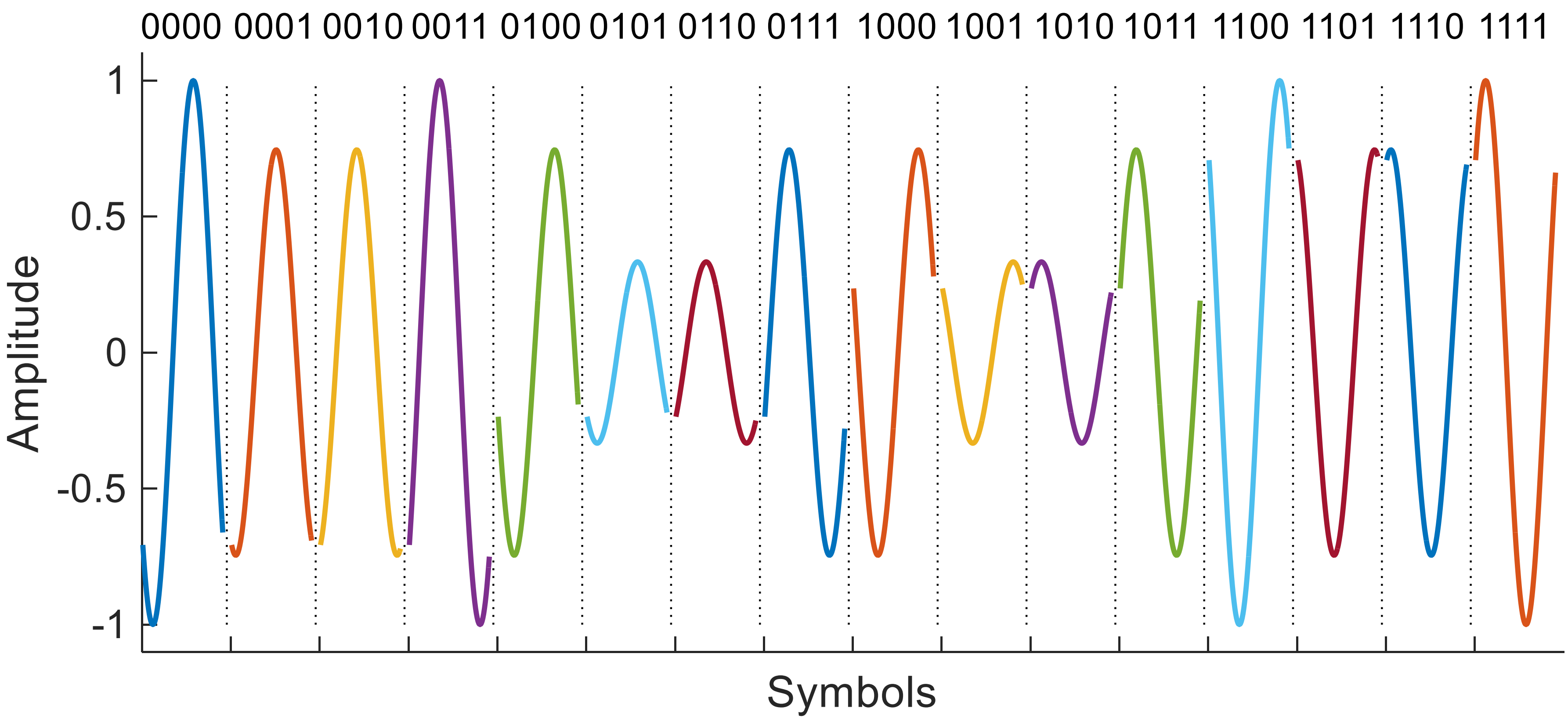






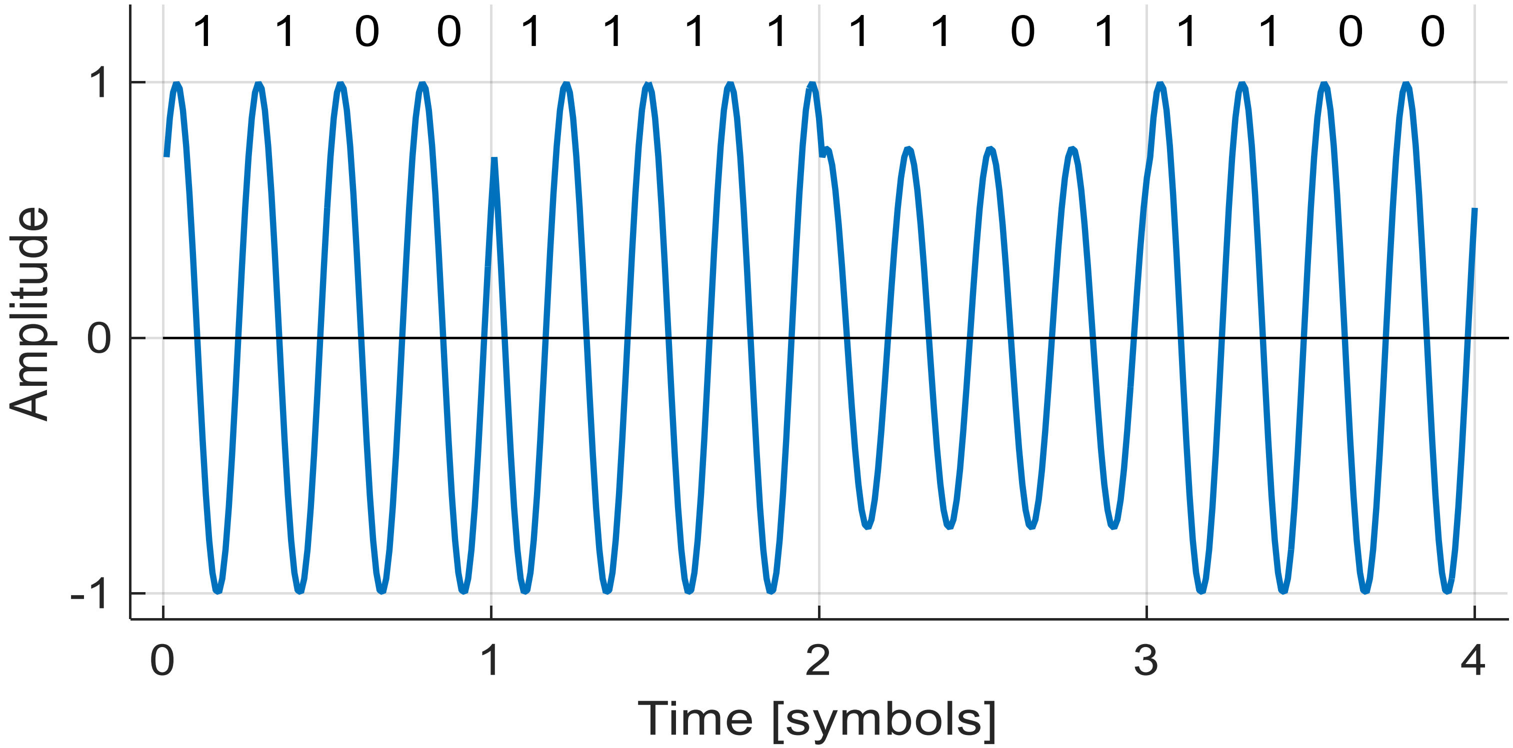
16-QAM-Symbole

- Trägerfrequenz: 1Hz



16-QAM Signal

- Trägerfrequenz: 5 Hz



Modulation Schemes in Practice

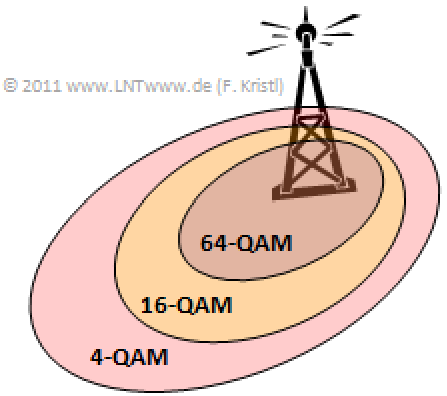
WLAN IEEE 802.11 a/b/g

Data Rate (Mbps)	Modulation	Coding Rate	Data bits per OFDM symbol
6	BPSK	1/2	24
9	BPSK	3/4	36
12	QPSK	1/2	48
18	QPSK	3/4	72
24	16-QAM	1/2	96
36	16-QAM	3/4	144
48	64-QAM	2/3	192
54	64-QAM	5/6	216

Convolutional Codes in/out

LTE

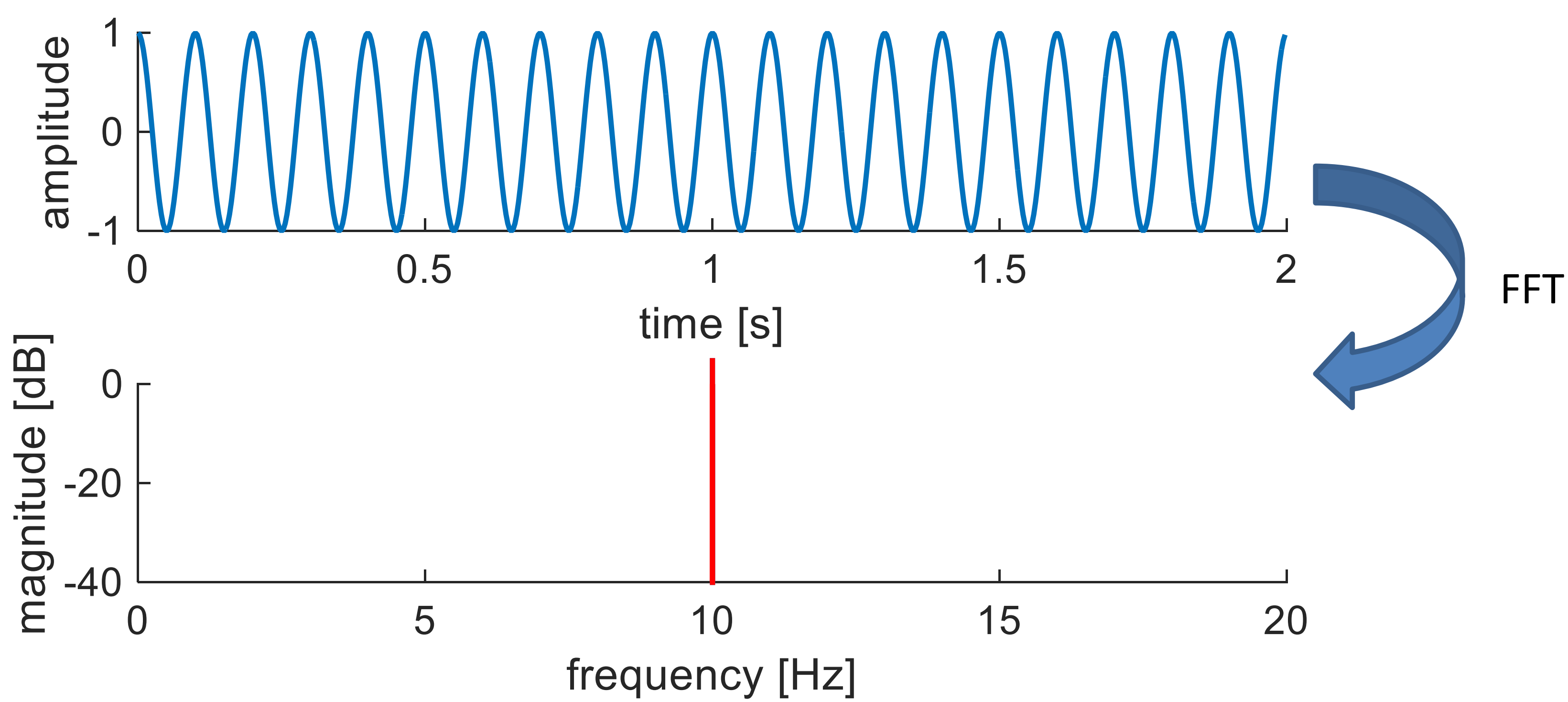
Category		1	2	3	4	5
Peak rate Mbps	DL	10	50	100	150	300
	UL	5	25	50	50	75
Capability for physical functionalities						
RF bandwidth		20MHz				
Modulation	DL	QPSK, 16QAM, 64QAM				
	UL	QPSK, 16QAM				QPSK, 16QAM, 64QAM
Multi-antenna						
2 Rx diversity		Assumed in performance requirements.				
2x2 MIMO		Not supported	Mandatory			
4x4 MIMO		Not supported				Mandatory



Übersicht Kapitel 4: Leitungscodierung und Modulation

Bandwidth and Spectrum of a Modulated Signal

- How much bandwidth does an unmodulated carrier occupy?
  - infinitesimal small spectrum (for an infinite signal)
  - the power spectrum of an unmodulated carrier is a Dirac impulse at the carrier frequency
    - example with $f_c = 10\ Hz$

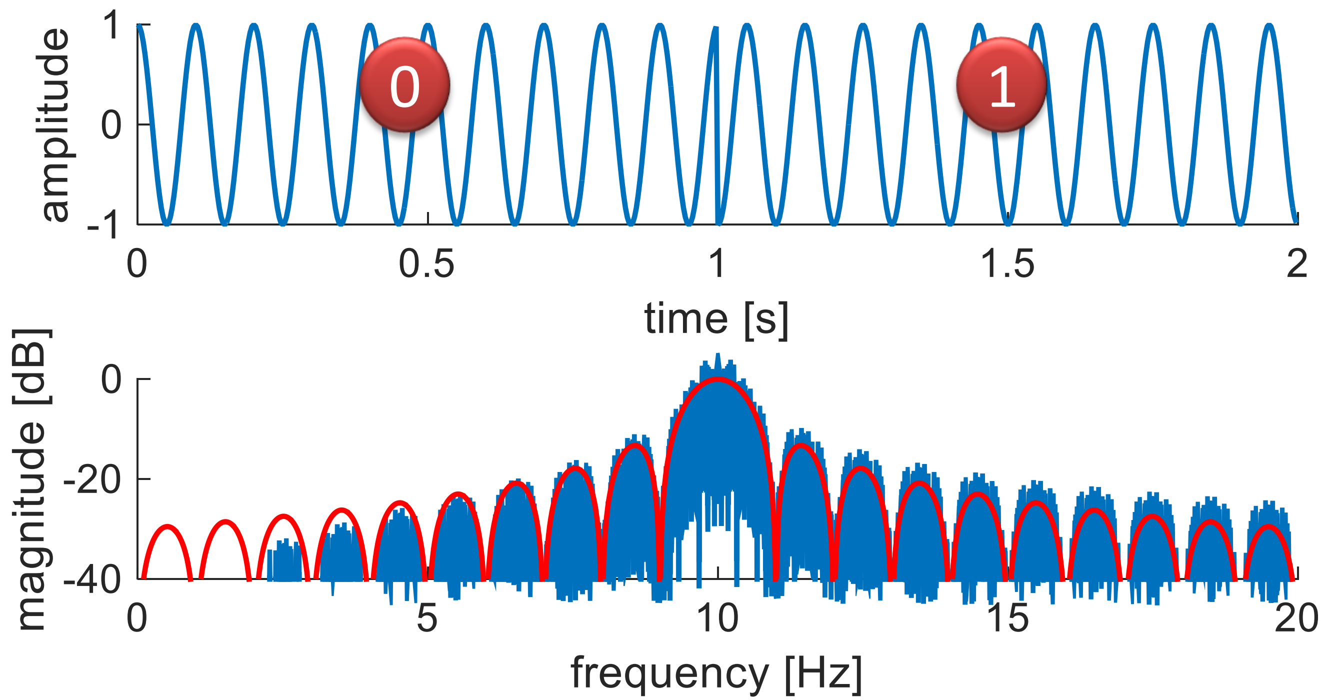


Bandwidth and Spectrum of a Modulated Signal

- What happens to the power spectrum of a BPSK modulated signal with a symbol rate of $1\ Hz$?
  - a symbol rate of $1\ Hz$ means that the duration of a symbol is $1s$ and the signals consists of wave forms with a duration of $1s$ that are either cosine or negative cosine curves

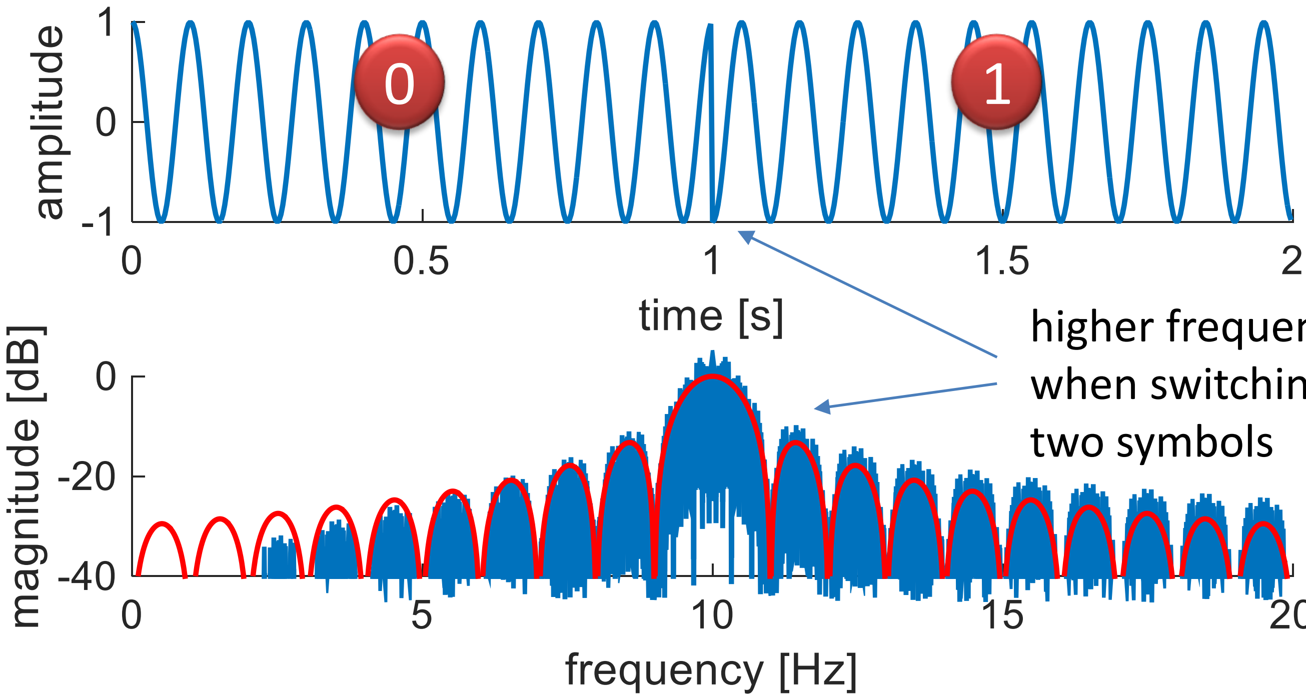
signal in time domain

power spectrum of the signal



Bandwidth and Spectrum of a Modulated Signal

jump in the signal means "infinite frequency"; the signal cannot be "fully" transmitted in a limited band

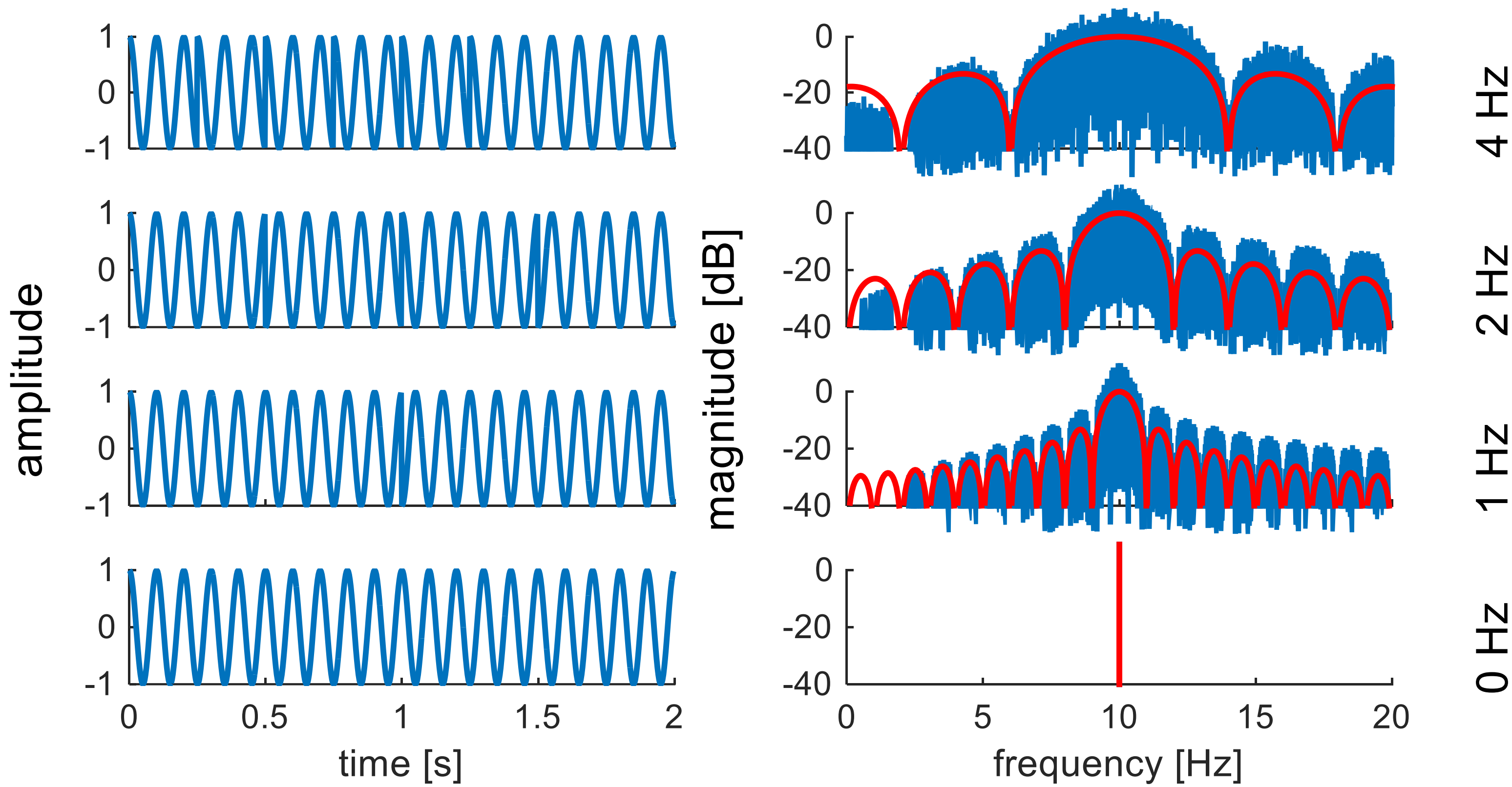


higher frequencies appear when switching between two symbols

- the signal "occupies" infinite spectrum
  - occupy means "energy is transmitted in that spectrum"
- the main energy is between 9 Hz and 11 Hz; a band around the carrier frequency of twice the symbol rate
  - the energy after the first null is clearly weaker and can be filtered away by a bandpass or actually a lowpass on the baseband signal

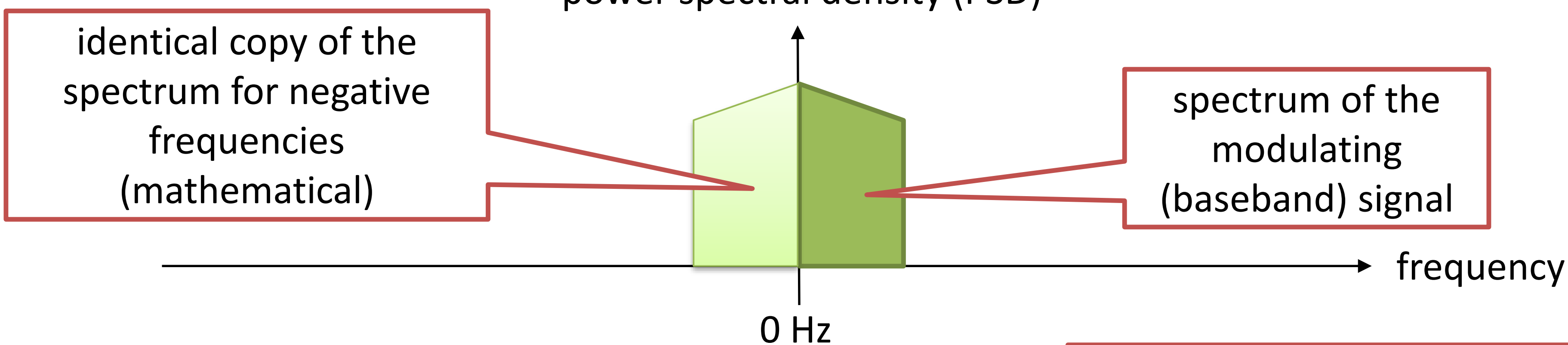
Bandwidth and Spectrum of a Modulated Signal

- The bandwidth of a signal increases proportional to the symbol rate

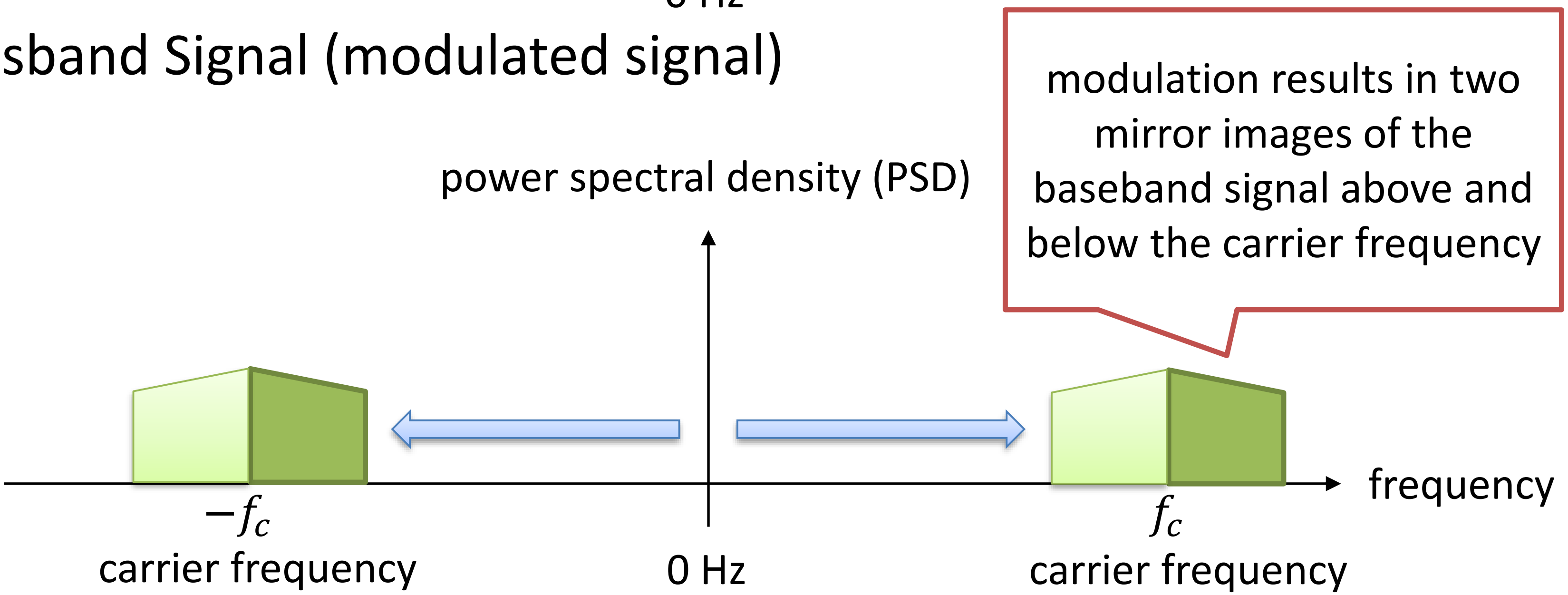


Baseband and Passband Signal in Spectrum

- Real Baseband Signal (modulating, information-bearing signal)

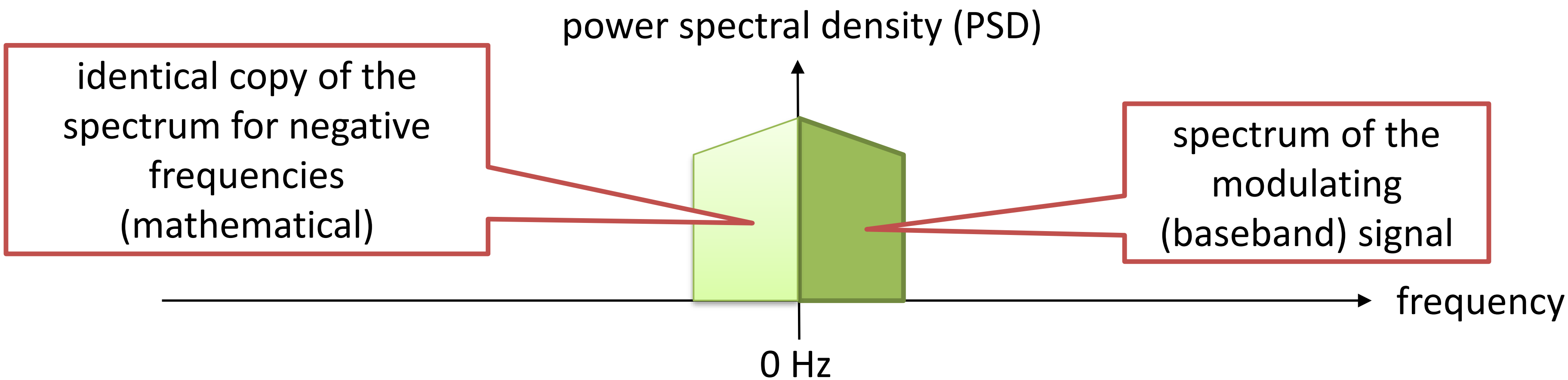


- Passband Signal (modulated signal)

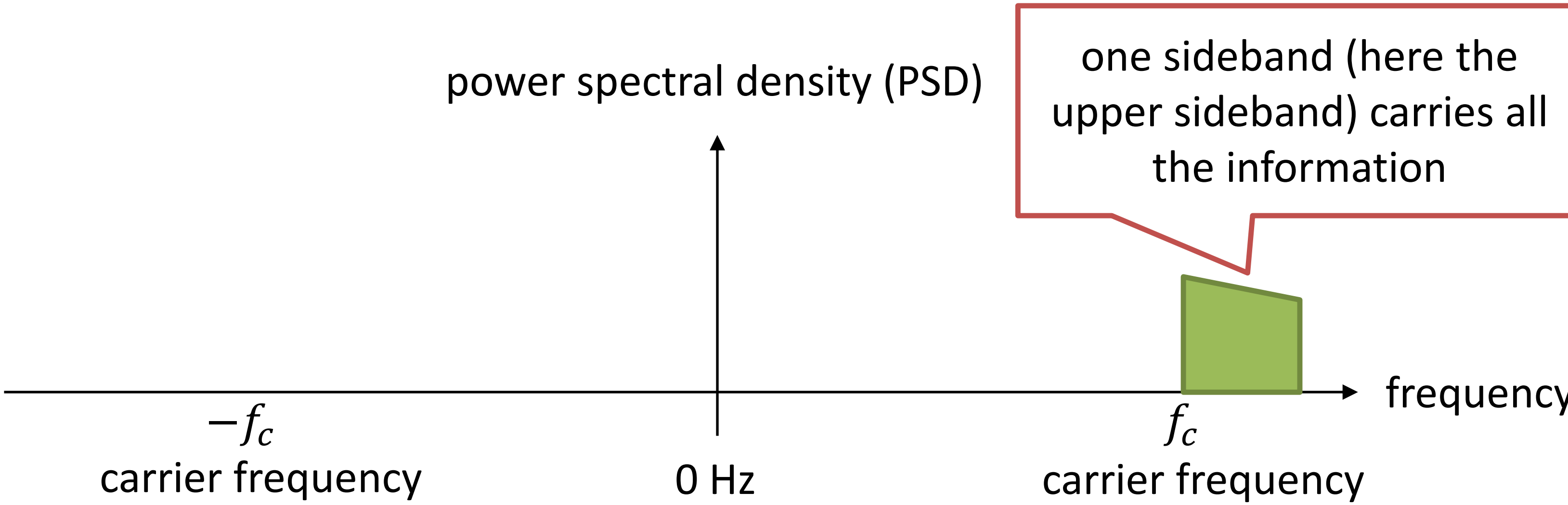


Baseband and Passband Signal in Spectrum

- Real Baseband Signal (modulating, information-bearing signal)



- Passband Signal (modulated signal)

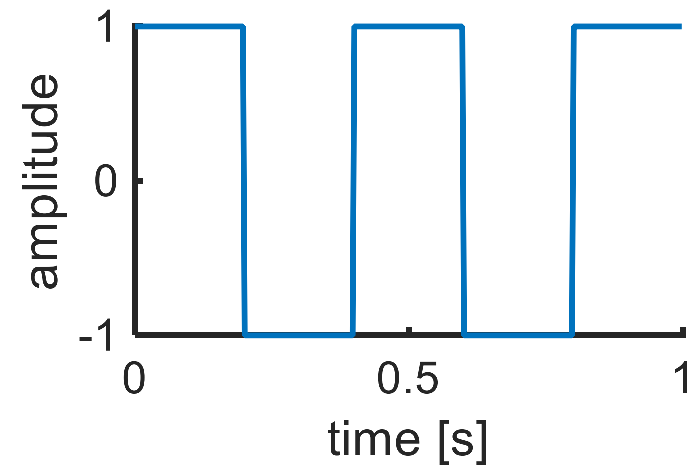


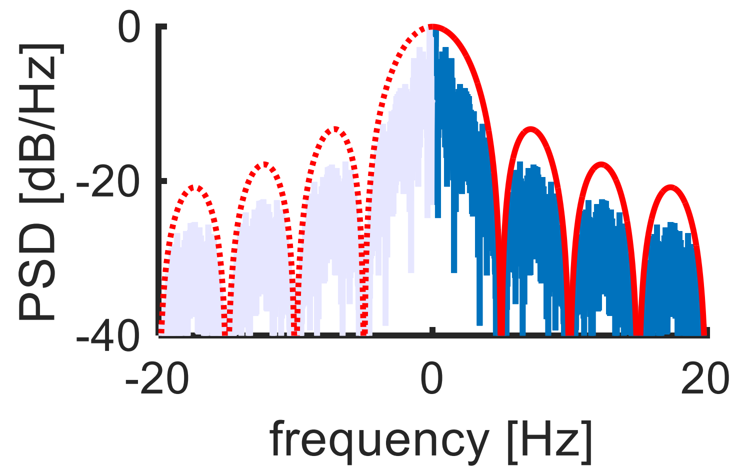
Spectrum of a 5Hz BPSK Signal

real-world signals in the time domain are always real-valued (voltage over time)

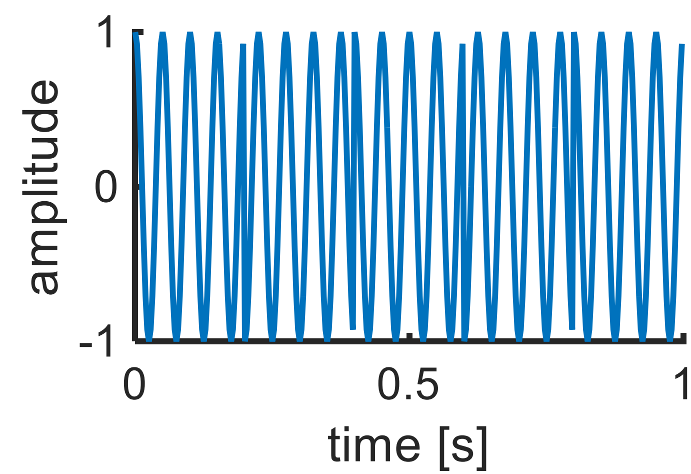
Modulation shifts and doubles the spectrum of a baseband signal though it carries the same information. One half is just a copy of the other half.

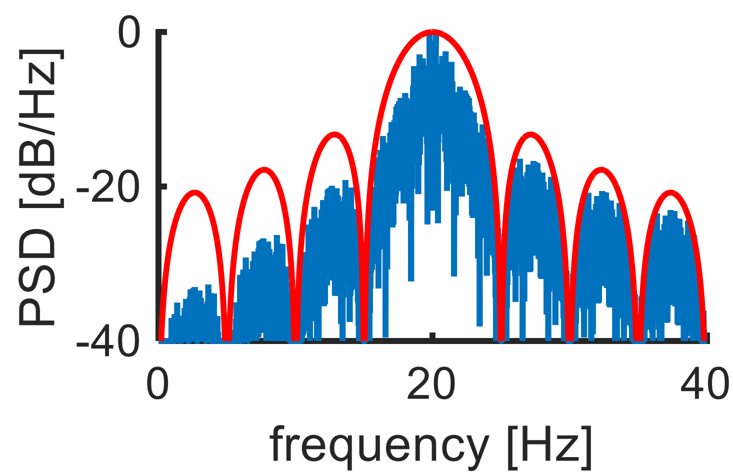
baseband (modulating) signal





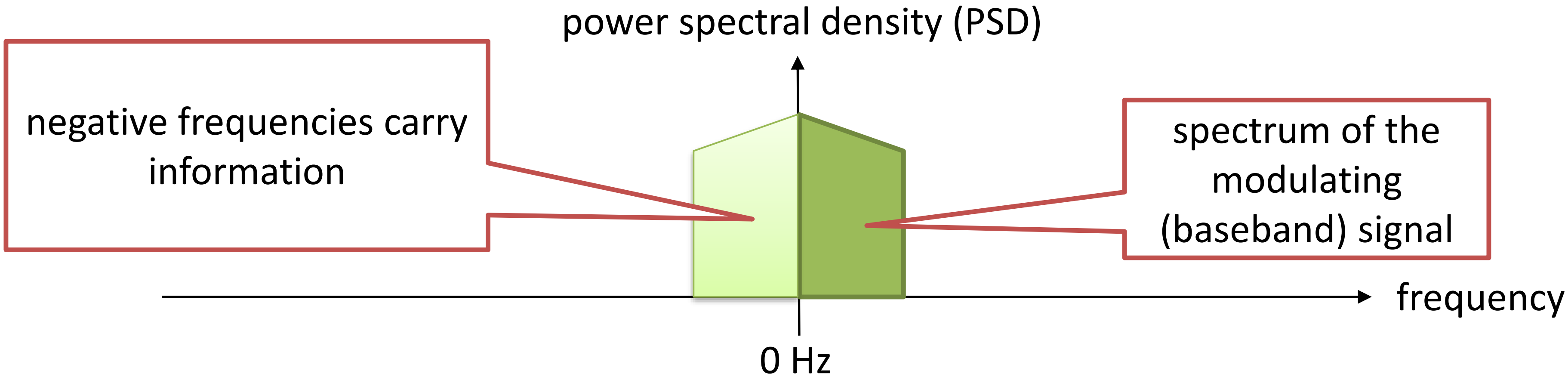
passband (modulated) signal



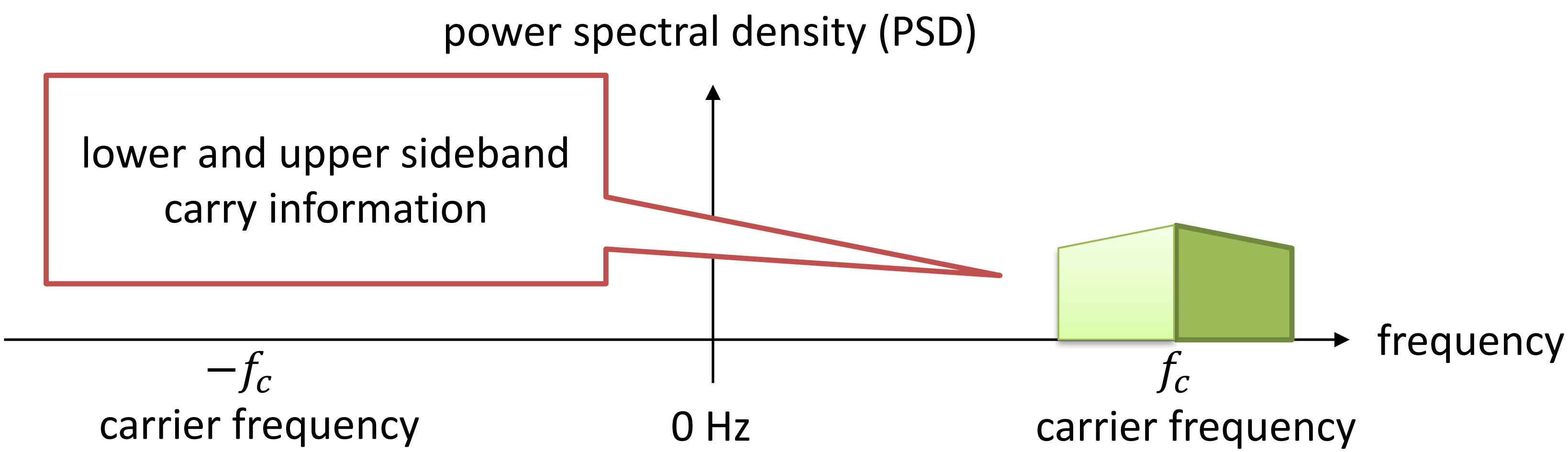


Baseband and Passband Signal in Spectrum

- Complex Baseband Signal (modulating, information-bearing signal)
  - double bits per symbol by using sine and cosine carrier for modulation

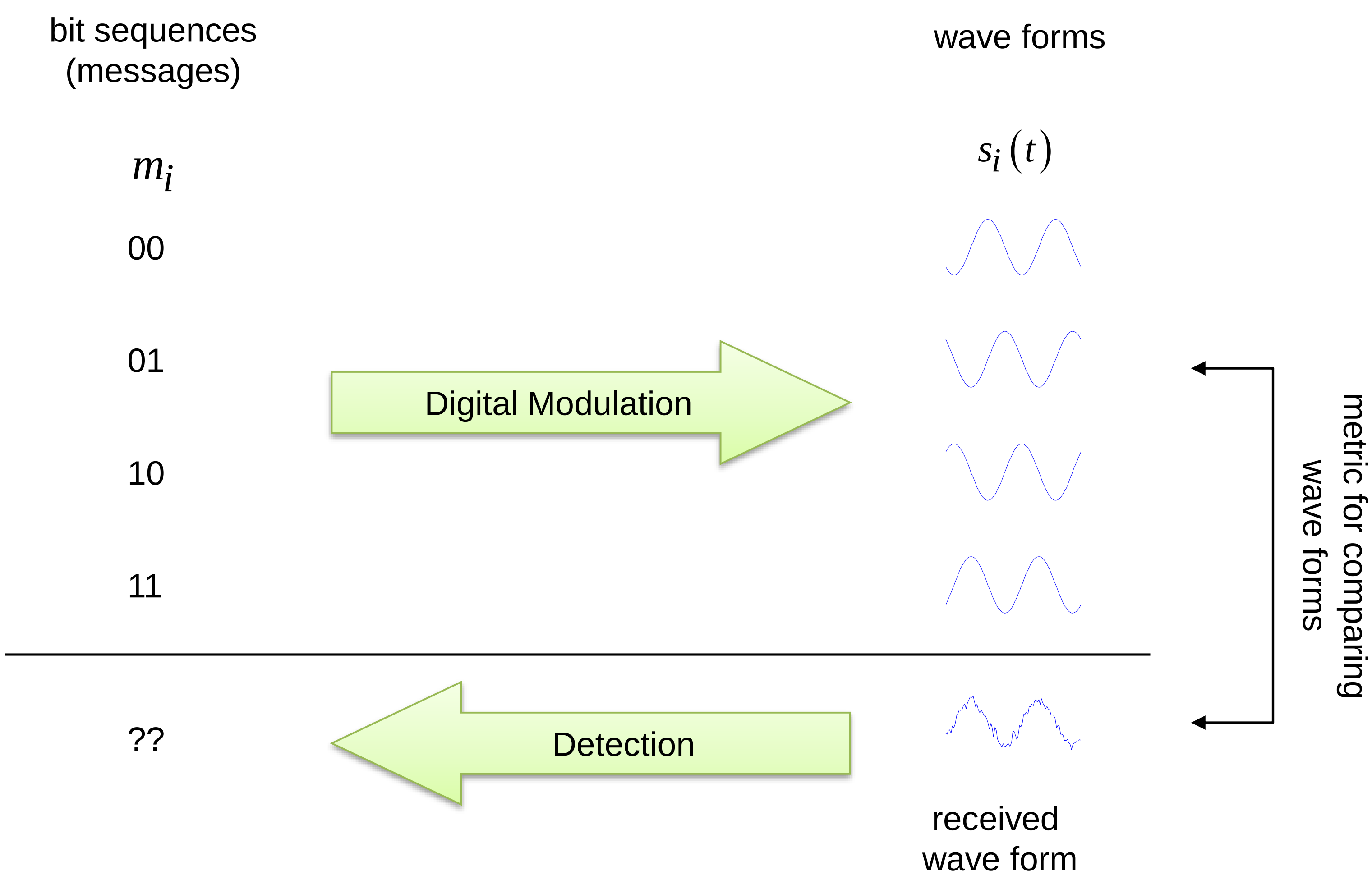


- Passband Signal (modulated signal)



Übersicht Kapitel 4: Leitungscodierung und Modulation

General Principle of Digital Modulation



Shannon Capacity

The Shannon Capacity C is a theoretical upper bound on the data rate that is achievable on a channel with bandwidth B and a signal-to-noise ratio SNR. The Shannon bound is one of the most fundamental laws in communication theory.

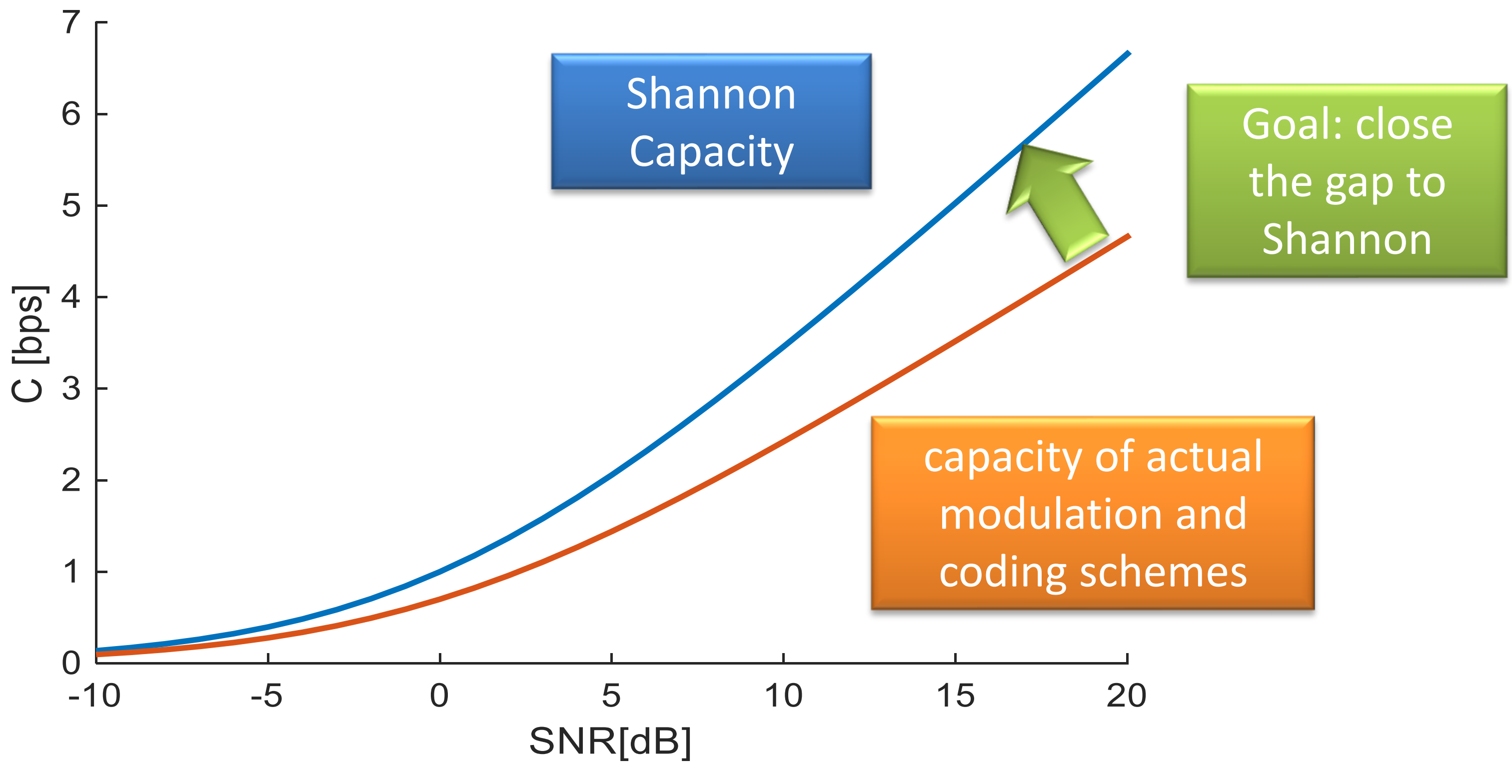
channel bandwidth (Hz)

signal-to-noise ratio (linear)

$$C = B \cdot \log_2(1 + SNR)$$

channel capacity (data rate bps)

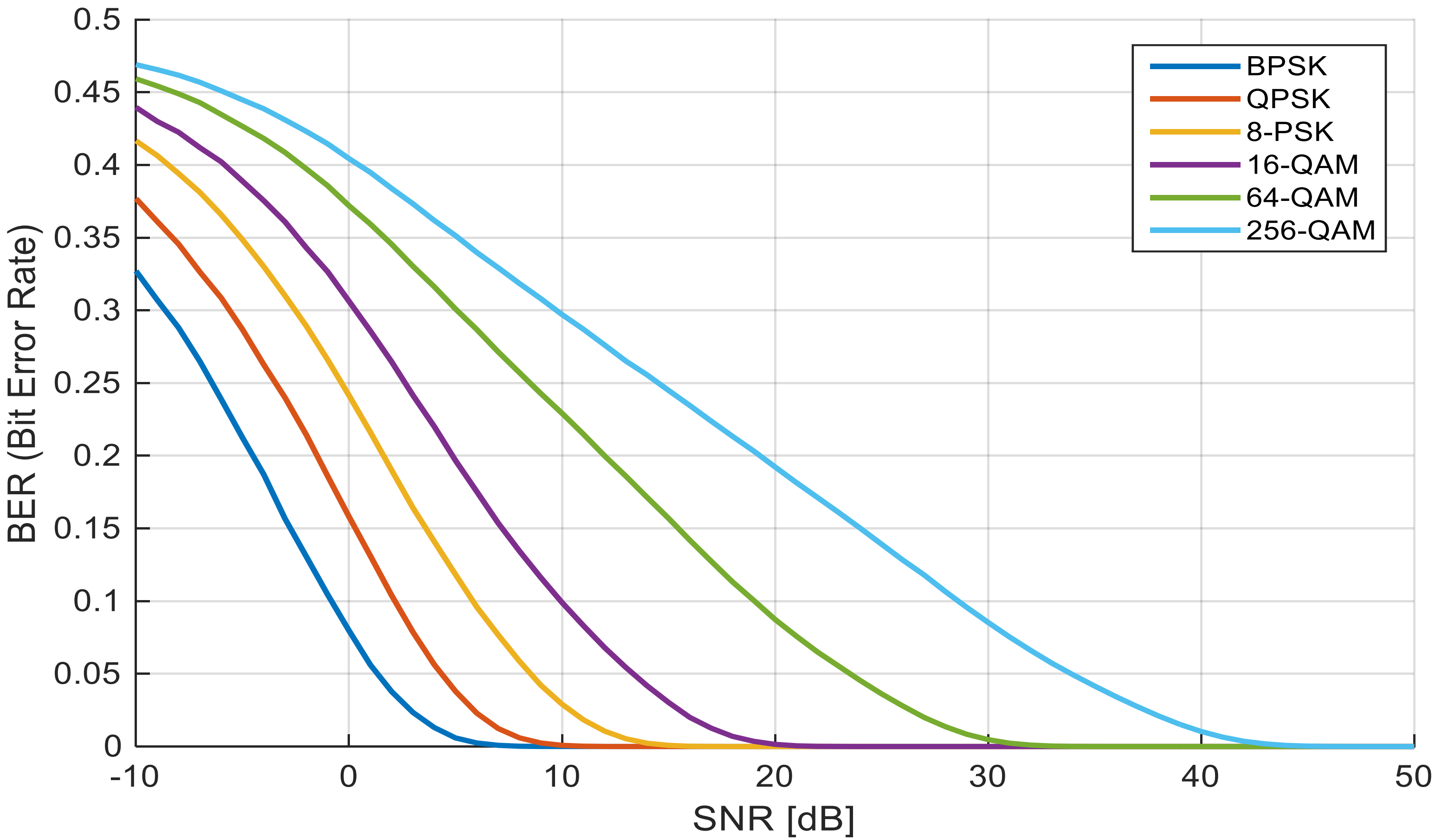
Objective of Modulation and Coding



- the SNR (Signal-to-Noise-Ratio) determines the data rate over a wireless channel
  - different modulation and coding schemes work are used depending on the SNR to provide optimal performance

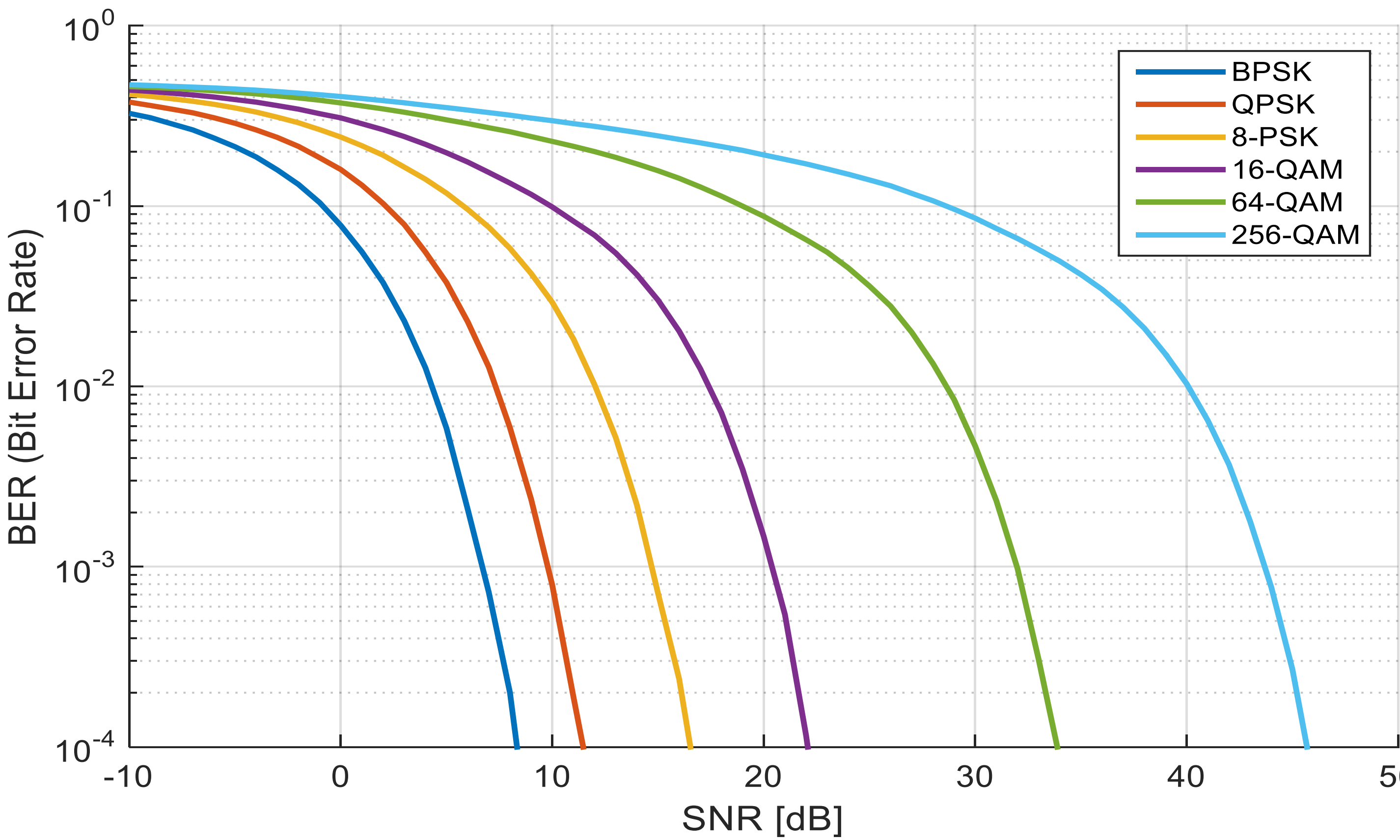
Bit Error Rate of Different Modulation Schemes

The higher the modulation order, the higher the bit error rate



Bit Error Rate of Different Modulation Schemes

In logarithmic scale we see that bit errors can always occur – even on a very good channel with very high SNR an a robust modulation scheme – but then the probability of an error is very low of course



About Decibel

- Wikipedia: The **decibel (dB)** is a logarithmic unit that expresses the ratio of two values of a physical quantity, often power or intensity.

$$z\,[dB] = 10 \cdot \log_{10}\frac{x}{y} \qquad \frac{x}{y} = 10^{z[dB]/10}$$

  – The value x is by $z\ dB$ larger than the value y.
- One of these quantities is often a reference value and in this case the decibel expresses the absolute level of the physical quantity.
  – Example:
    - for dBm the reference value (y) is $1mW$
    - How much is a transmit power of 10 dBm?

$$\frac{x}{y} = 10^{z[dB]/10} \Longrightarrow x = 10^{10dB/10} \cdot 1mW = 10mW$$

  – Other values are dBW related to 1W or dBi related to an isotropic antenna.
- Decibel values are typically used to express transmit power, received power, noise power and in particular power ratios
  – the number are more handy and sums are easier than products

Questions about Decibel

1. My car is by $3dB$ faster than a truck. The truck goes by $80km/h$. How fast is my car?

2. Is it correct to say that the transmit power is $30dB$?

3. The time for a lecture is 90 minutes, a student is sleeping for 9 minutes, the rest of the time the student is awake and very interested in the lecture. How can we express the sleeping time related to the lecturing period?

4. An access points transmits with $100mW$, the smartphone receives a signal that is $60dB$ weaker. How strong is the signal received at the smartphone?

Overview Decibel for Powers

Linear scale (Watts) | **Logarithmic scale (decibel)**

ratio of powers (unitless scalar) $x = \frac{a}{b}$ $\longrightarrow$ $x[dB] = 10 \cdot \log_{10}(x)$

dB relates to a ratio of two powers

power value x

$\nearrow$ $x[dBm] = 10 \cdot \log_{10}\left(\frac{x}{1mW}\right)$

dBm relates to the ratio of a power to 1mW

$\searrow$ $x[dBW] = 10 \cdot \log_{10}\left(\frac{x}{1W}\right)$

dBW relates to the ratio of a power to 1W

$x = 10^{x[dB]/10}$ $\longleftarrow$ $x[dB]$

$x[mW] = 10^{x[dBm]/10}$ $\longleftarrow$ $x[dBm]$

$x[W] = 10^{x[dBW]/10}$ $\longleftarrow$ $x[dBW]$

the formulae:

$$x[dB] = 10 \cdot \log_{10}(x)$$

$$x = 10^{x[dB]/10}$$

What is the Signal-to-Noise Ratio?

- Signal-to-noise Ratio:
  - ratio of received power to noise power
  - ratio of received symbol energy to noise energy per symbol
    - energy=power x time

- Symbol energy:

$$E_s = \int_0^{T_s} |s(t)|^2 dt \quad \text{(signal)}$$

$$E_s = P_{tx} \cdot T_s \qquad \text{(transmitter)}$$

$$E_s = P_{rec} \cdot T_s \qquad \text{(receiver)}$$

- Noise
  - power density:

$$N_0 = -174 dBm/Hz$$

  - power in a band with bandwidth B:

$$N_0 \cdot B = -174\, dBm/Hz + B[dBHz]$$

Signal-to-Noise Ratio

- Signal-to-noise Ratio:
  - ratio of received power to noise power

$$SNR = \frac{P_{rec}[mW]}{N_0[mW/Hz] \cdot B[Hz]}$$

  - ratio of received symbol energy to noise energy per symbol

$$SNR = \frac{E_s[mWs]}{N_0[mW/Hz]} = \frac{P_{rec}[mW] \cdot T_s[s]}{N_0[mW/Hz]}$$

- Symbol energy over noise: E_s/N_0 (speak: ESNO)
  - equal to SNR
- Bit energy over noise: E_b/N_0 (speak: EBNO)
  - equal to SNR divided by number of bits per symbol (modulation order)

Background: Signal-to-Noise-Ratio and Energy per Symbol/Bit

- The signal-to-noise ratio is strictly the ratio of signal power to noise power at an instance of time t

$$SNR(t) = \frac{P_{signal}(t)}{P_{noise}(t)} = \frac{|s(t)|^2}{|n(t)|^2}$$

- In real system and in computer simulations signals only exist at discrete sampling times and the signal and noise power of a sampling time period are present in one sampling point. The more useful definition of SNR is that the SNR is the ratio of mean signal power at a random instance of time to the expectation of the noise power.

$$SNR = \frac{E\left[|s(t)|^2\right]}{E[|n(t)|^2]} = \frac{E\left[|s(t)|^2\right]}{VAR[|n(t)|]}$$

- A power integrated over a period of time yields an energy. In particular, the power integrated of a symbol period yields the symbol energy

$$E_s = \int_{t=0}^{T_s} |s(t)|^2 dt.$$

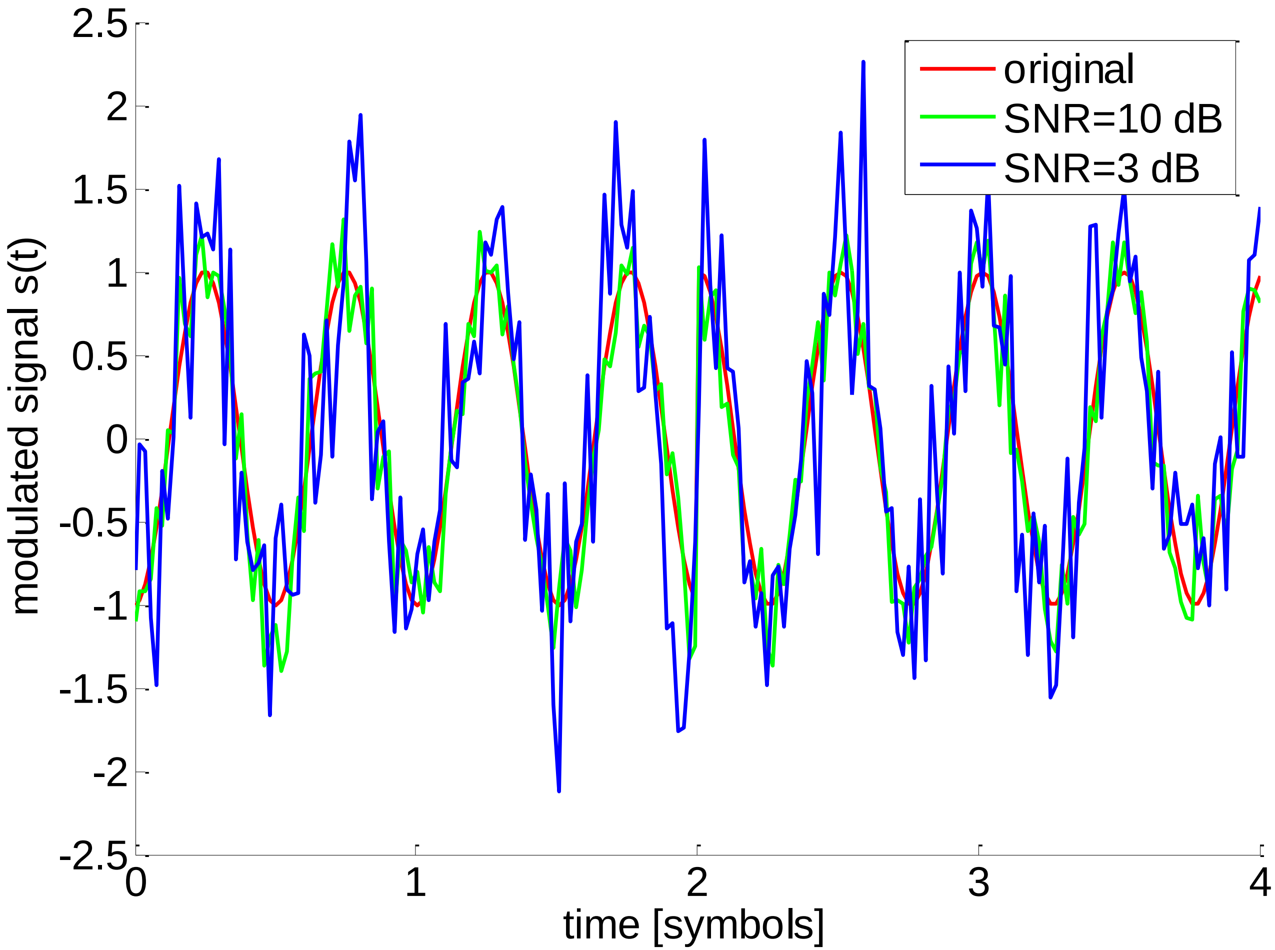
- If a symbol represents M Bits, then the energy per Bit is $E_b = E_s/M$ since the symbol energy is split over M symbols.
- The ratio of signal energy to noise energy over a period of time is the $\boldsymbol{E_s/N_0}$ (spoken Esno) which is equal to the SNR. The energy per bit, over the noise power is called $\boldsymbol{E_b/N_0}$ (spoken Ebno).

Example: E_s/N_0 for WiFi

- WLAN:
  - transmit power: 100 mW (20dBm) over 20 MHz band
  - receive power P_{rec}: 100 nW (-40dBm) over 20 MHz band
    - by 60 dB less than transmit power
- Noise power is specified over a certain band with bandwidth B
  - e.g. for Wi-Fi and a 20 MHz bandwidth the noise power is
    $$E[P_{noise}] = N_0 \cdot B = -174dBm/Hz + 73\text{ dB}Hz = -101\ dBm$$
  - note: 20 MHz = 73 dBHz

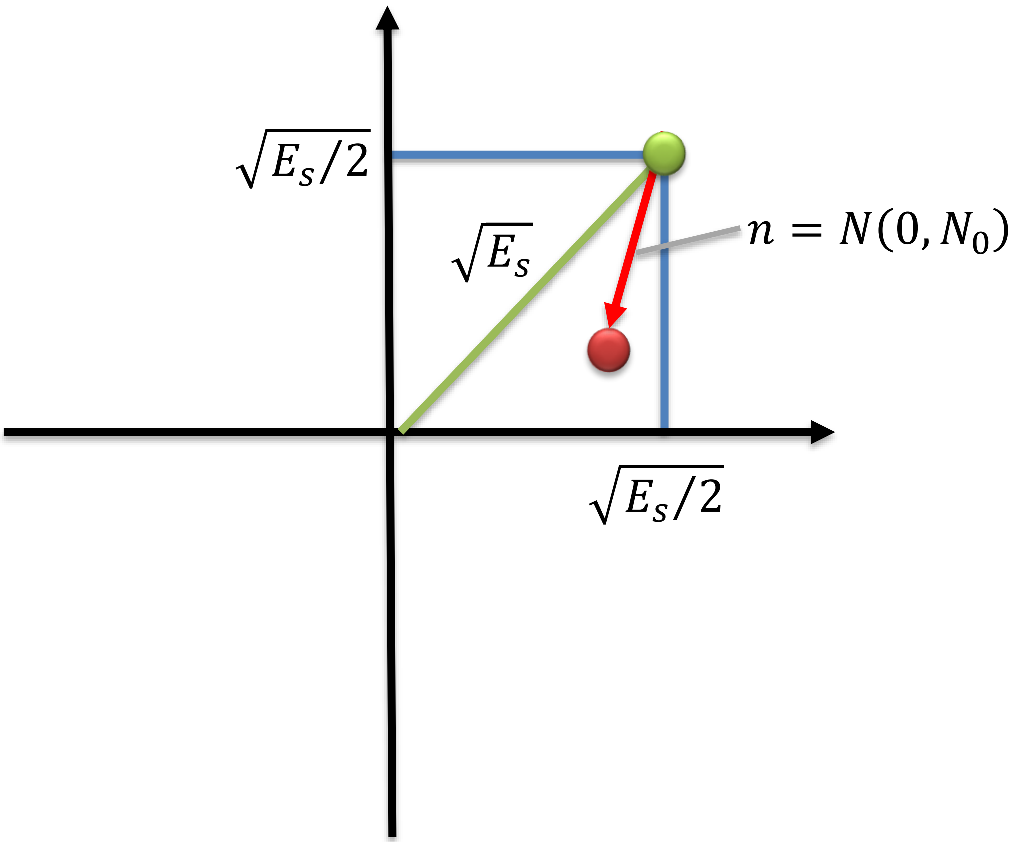
    1kHz=30dBHz,1MHz=60dBHz,10MHz=70dBHz, 20 MHz=73 dBHz
- The (instantaneous) signal to noise ratio (SNR) is the ratio of powers, the instantaneous receive signal power over the expectation of the instantaneous noise power
  $$SNR = \frac{P_{rec}}{E[P_{noise}]} = \frac{P_{rec}}{N_0 \cdot B} = -40\ dBm - (-101\ dBm) = 61\ dB$$
- The symbol energy E_s is the signal power integrated over a symbol period or the expected signal power multiplied by the symbol period
  $$E_s = E[P_{rec}] \cdot T_s = E[P_{rec}]/B = -113\ dBm/Hz$$
- The symbol energy over noise, the E_s/N_0, is the ratio of two energies
  $$E_s/N_0 = \frac{E[P_{rec}] \cdot T_s}{N_0} = -113\ dBm/Hz - \left(-\frac{174dBm}{Hz}\right) = 61\ dB$$

BPSK Signal with AWGN



Noise on Complex Symbols

Diagram scaled by absolute values



received signal:

$$r = \alpha_{PL} \cdot s$$

noisy received signal:

$$r^n = r + n$$

complex Gaussian noise:

$$n \sim N(0, N_0)$$

in-phase of received signal:

$$r_I^n = r_I + n_I$$

quadrature of received signal:

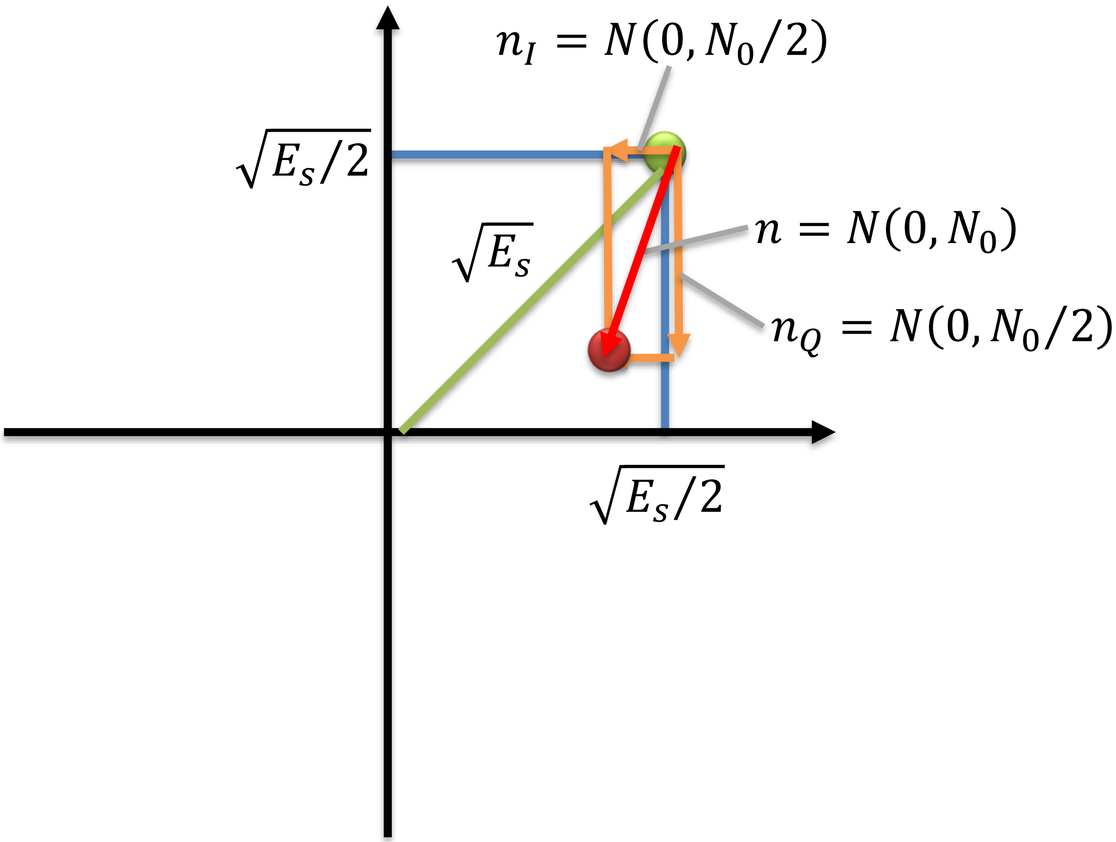
$$r_Q^n = r_Q + n_I$$

in-phase/quadrature noise

$$n_I \sim N(0, N_0/2\,)$$
$$n_Q \sim N(0, N_0/2\,)$$

Noise on Complex Symbols

Diagram scaled by absolute values



received signal:

$$r = \alpha_{PL} \cdot s$$

noisy received signal:

$$r^n = r + n$$

complex Gaussian noise:

$$n \sim N(0, N_0)$$

in-phase of received signal:

$$r_I^n = r_I + n_I$$

quadrature of received signal:

$$r_Q^n = r_Q + n_I$$

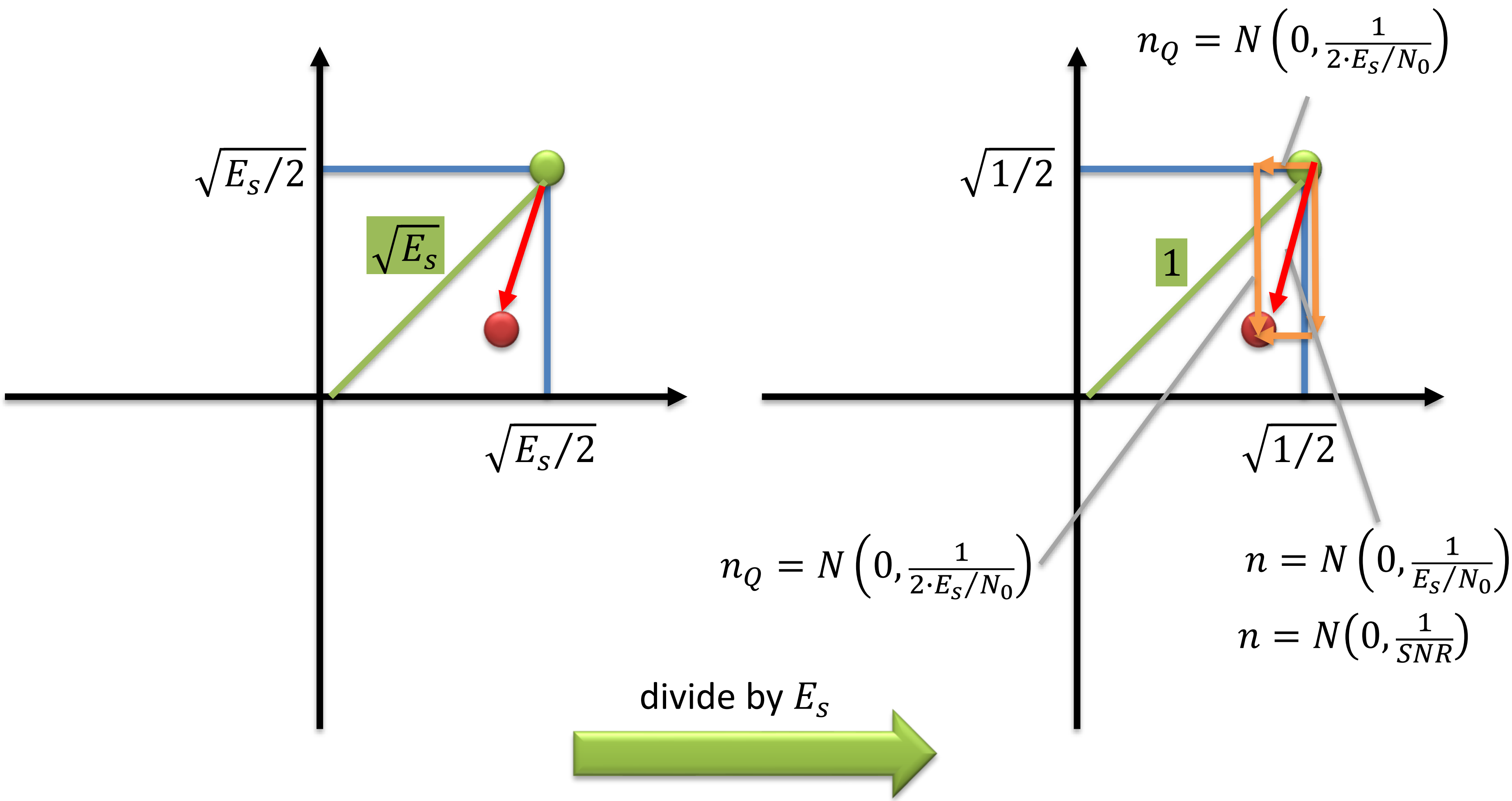
in-phase/quadrature noise

$$n_I \sim N(0, N_0/2\,)$$
$$n_Q \sim N(0, N_0/2\,)$$

Noise on Complex Symbols

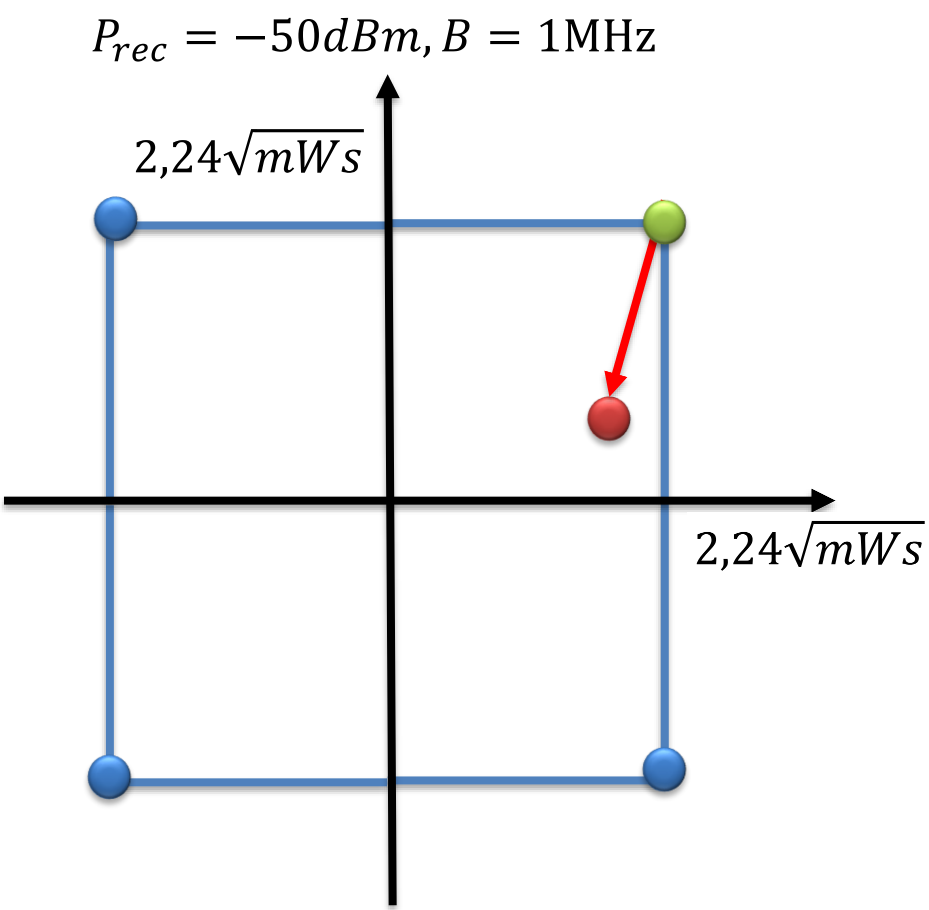
Diagram scaled by absolute values

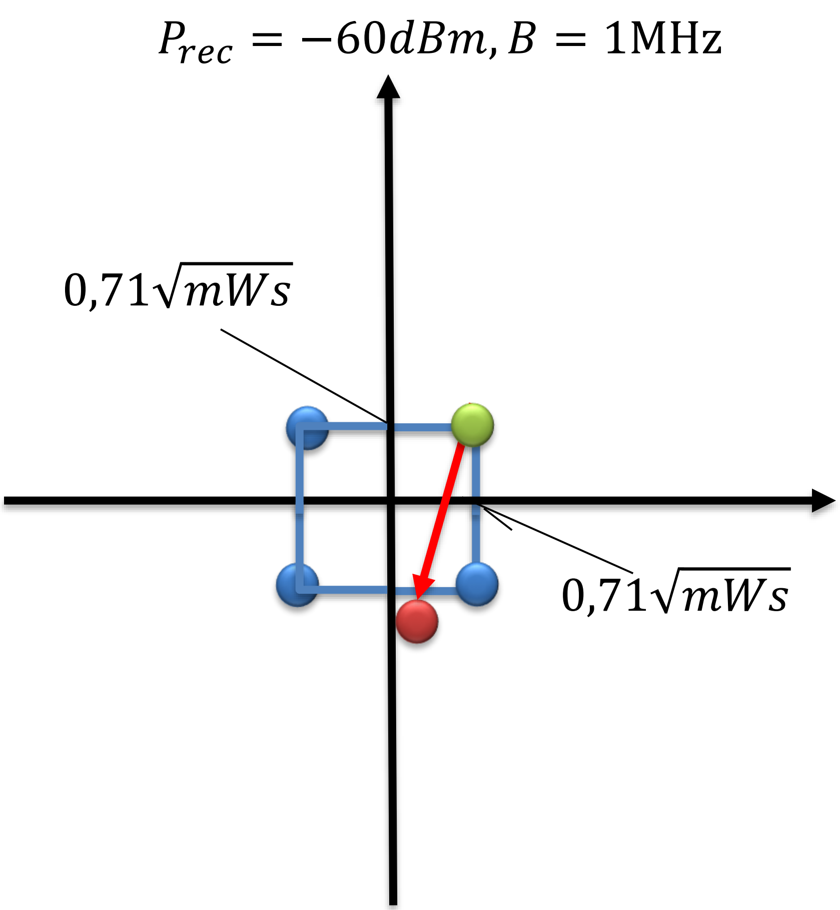
Diagram scaled by relative values



Impact of Received Power

- Energy of received symbol: $E_s = P_{rec} \cdot T_s$



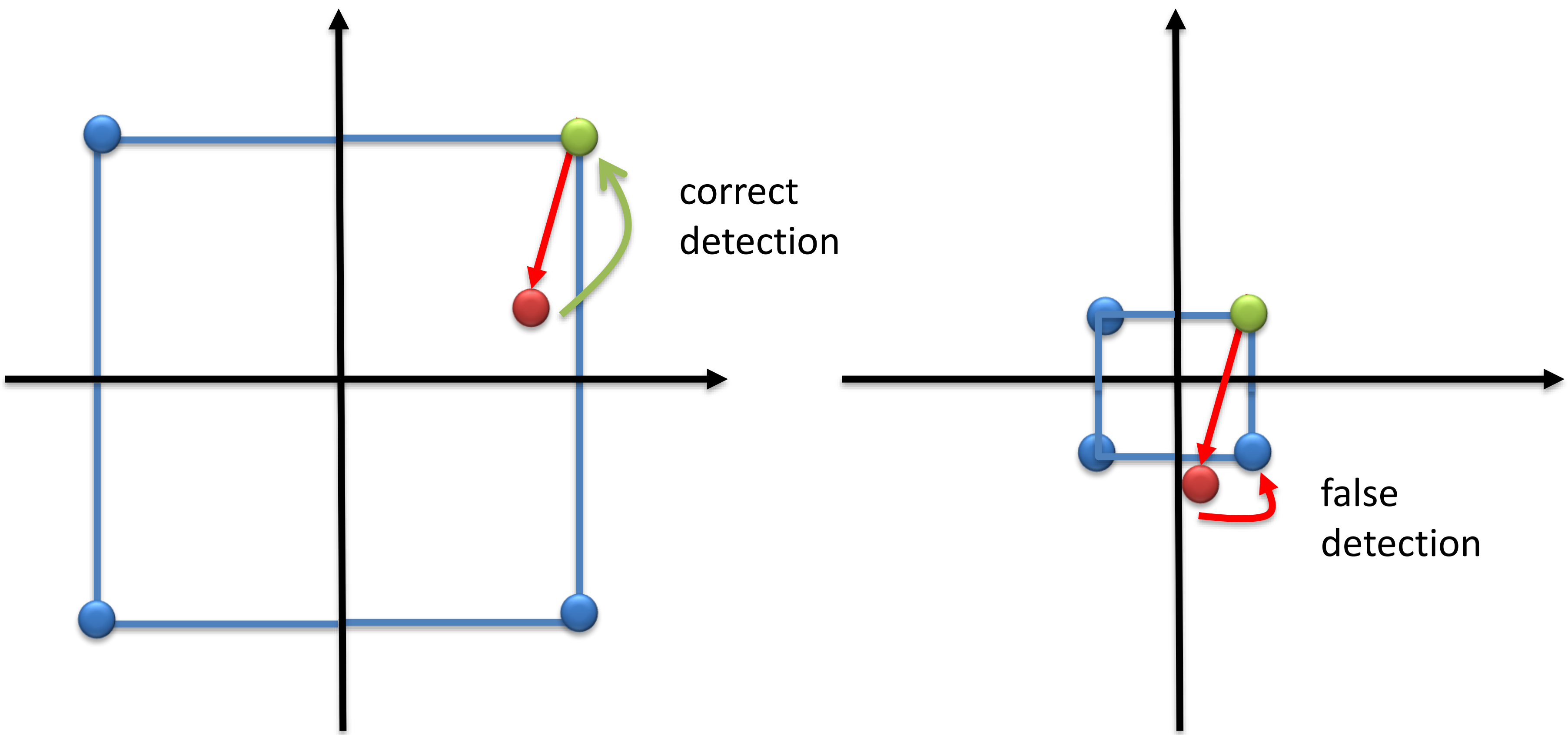


$E_s = P_{rec} \cdot T_s = -110\text{dBm/Hz}$

$E_s = P_{rec} \cdot T_s = -120\text{dBm/Hz}$

Detection at Receiver

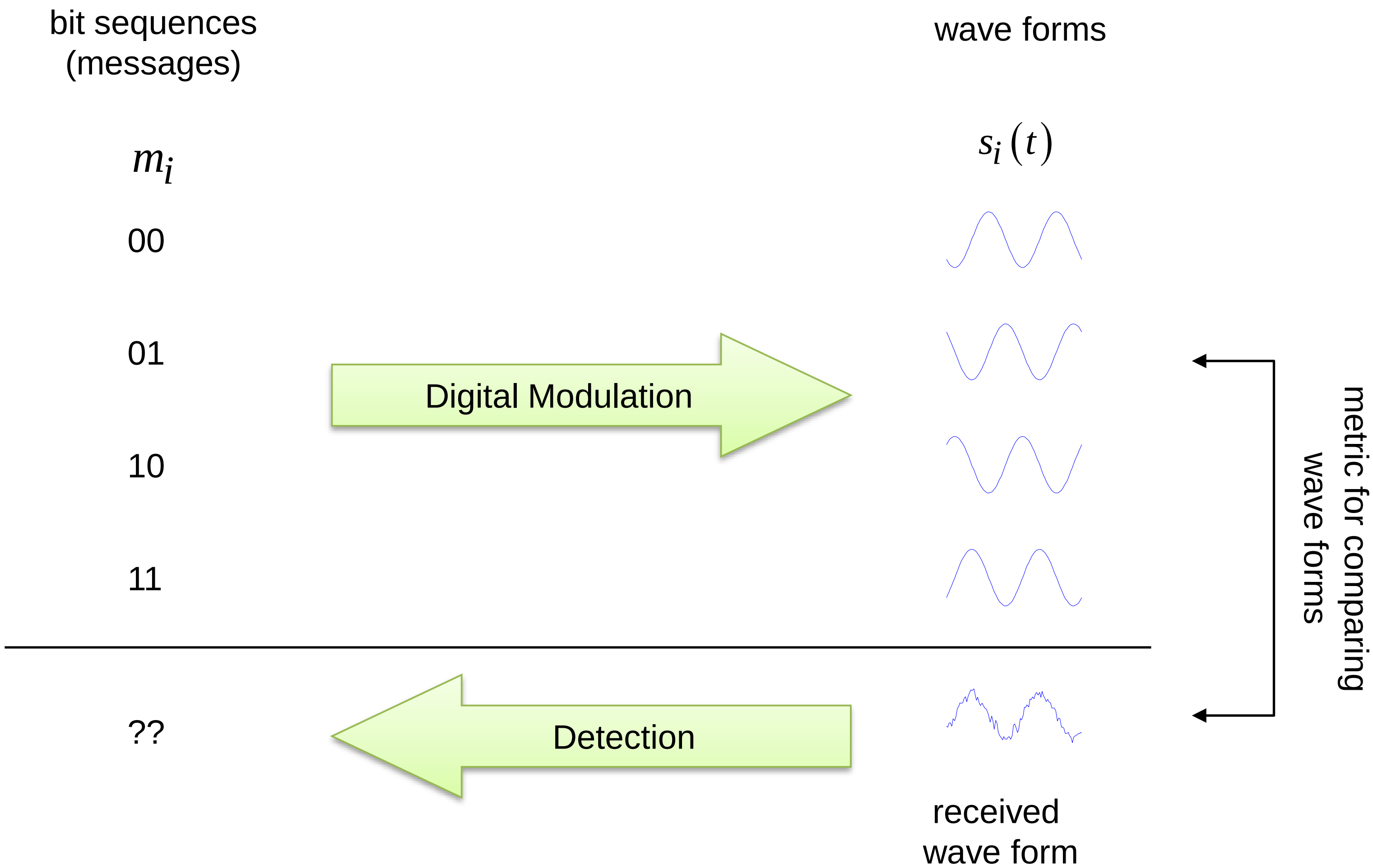
- Detection: identify closest symbol in constellation diagram



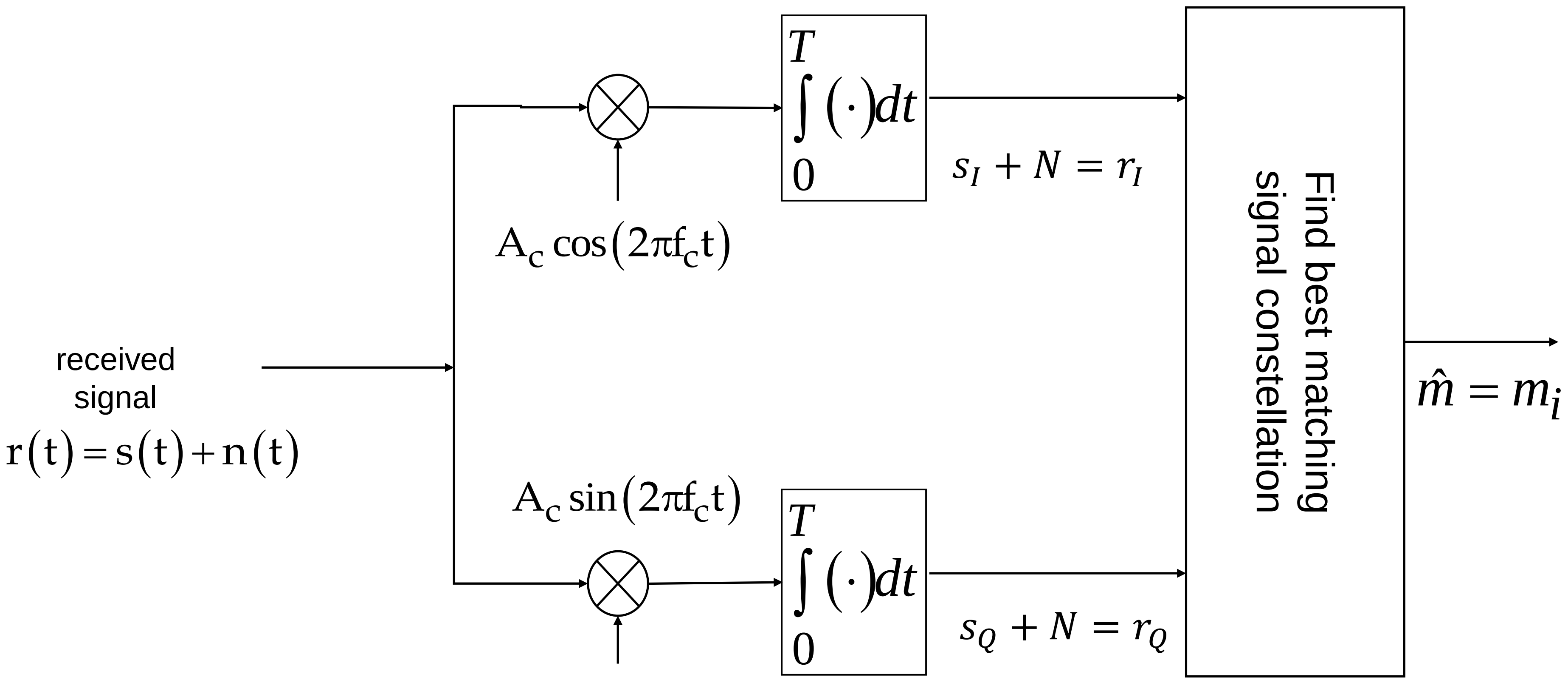
- Bit errors occur if the received power is too low

Übersicht Kapitel 4: Leitungscodierung und Modulation

General Principle of Digital Modulation



Receiver Structure



What happens at the receiver?

- Signal at the receiver:

$$r(t) = s_I \cdot \cos 2\pi f_c t - s_Q \cdot sin\, 2\pi f_c t$$

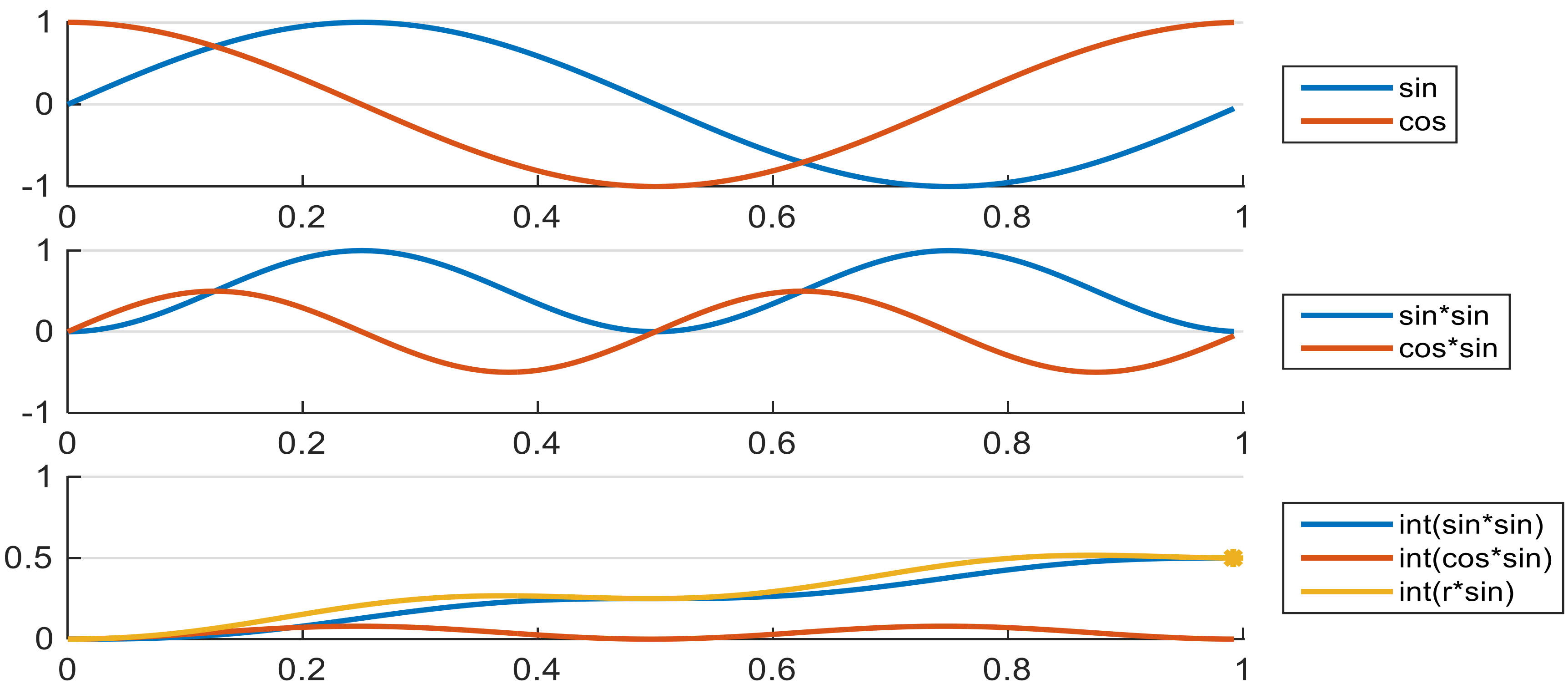
- Target of decoding:
  - extract s_I and s_Q from the received signal
- Demodulate, integrate and dump
  - demodulate:
    - same as modulate (plus low pass filter)
    - multiply with sine/cosine signal
  - integrate:
    - integrate product over symbol period
    - sum over all sampling points
  - dump: sample last value at end of symbol period
- In mathematical terms:

$$r_I = \int_{t=0}^{T_S} r(t) \cdot \cos 2\pi f_c t\, dt = \int_{t=0}^{T_S} \left(s_I \cdot \cos 2\pi f_c t - s_Q\, sin\, 2\pi f_c t\right) \cdot \cos 2\pi f_c t\, dt = \frac{s_I}{2}$$

$$r_Q = \int_{t=0}^{T_S} r(t) \cdot -sin\, 2\pi f_c t\, dt = \int_{t=0}^{T_S} \left(s_I \cdot \cos 2\pi f_c t - s_Q\, sin\, 2\pi f_c t\right) \cdot -sin\, 2\pi f_c t\, dt = \frac{s_Q}{2}$$

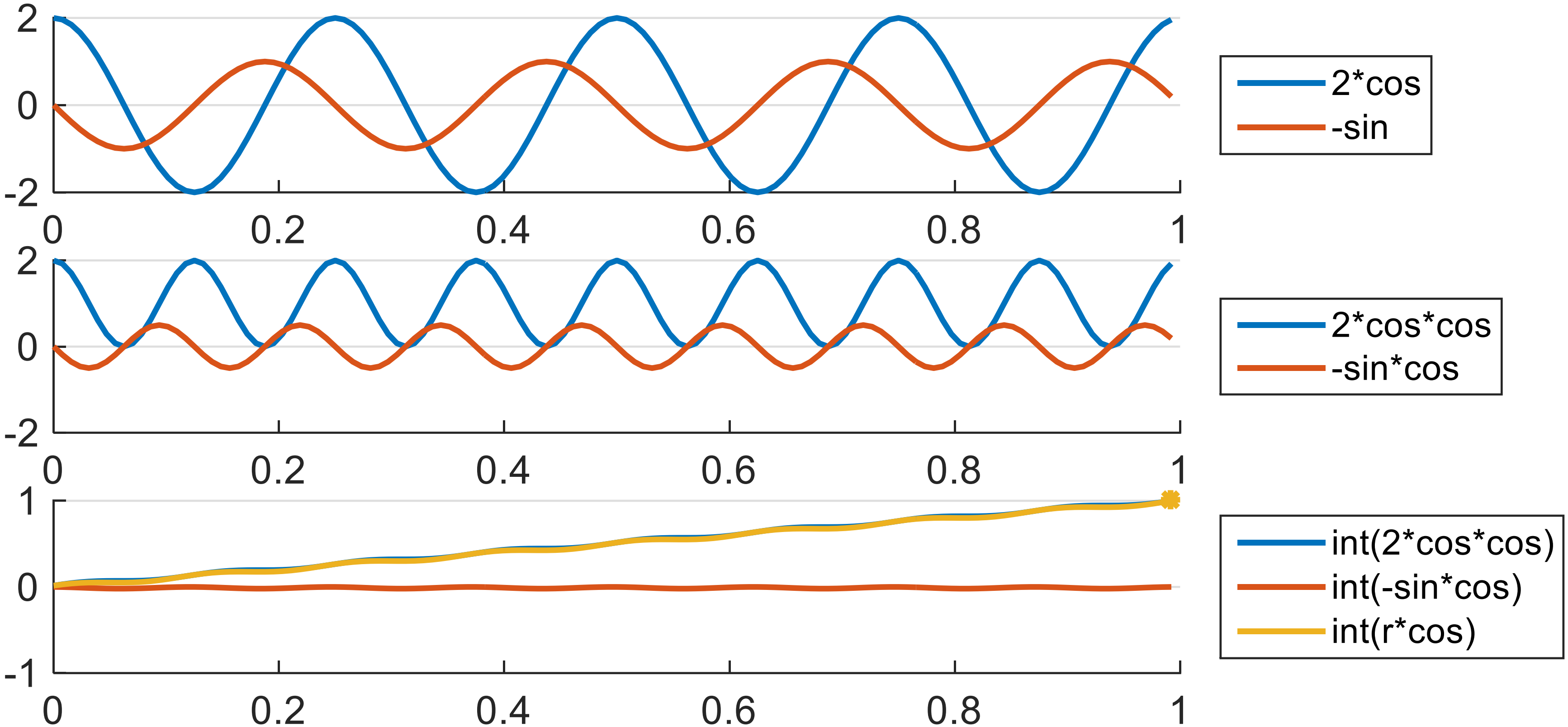
Demodulate, Integrate, and Dump

- Regenerate quadrature by multiplying complex signal with sine wave, integrating over the symbol period and dumping the result
- Note that the integral over the squared sine wave is 0.5 and not 1.
- Example: $s_I = 1, s_Q = -1$



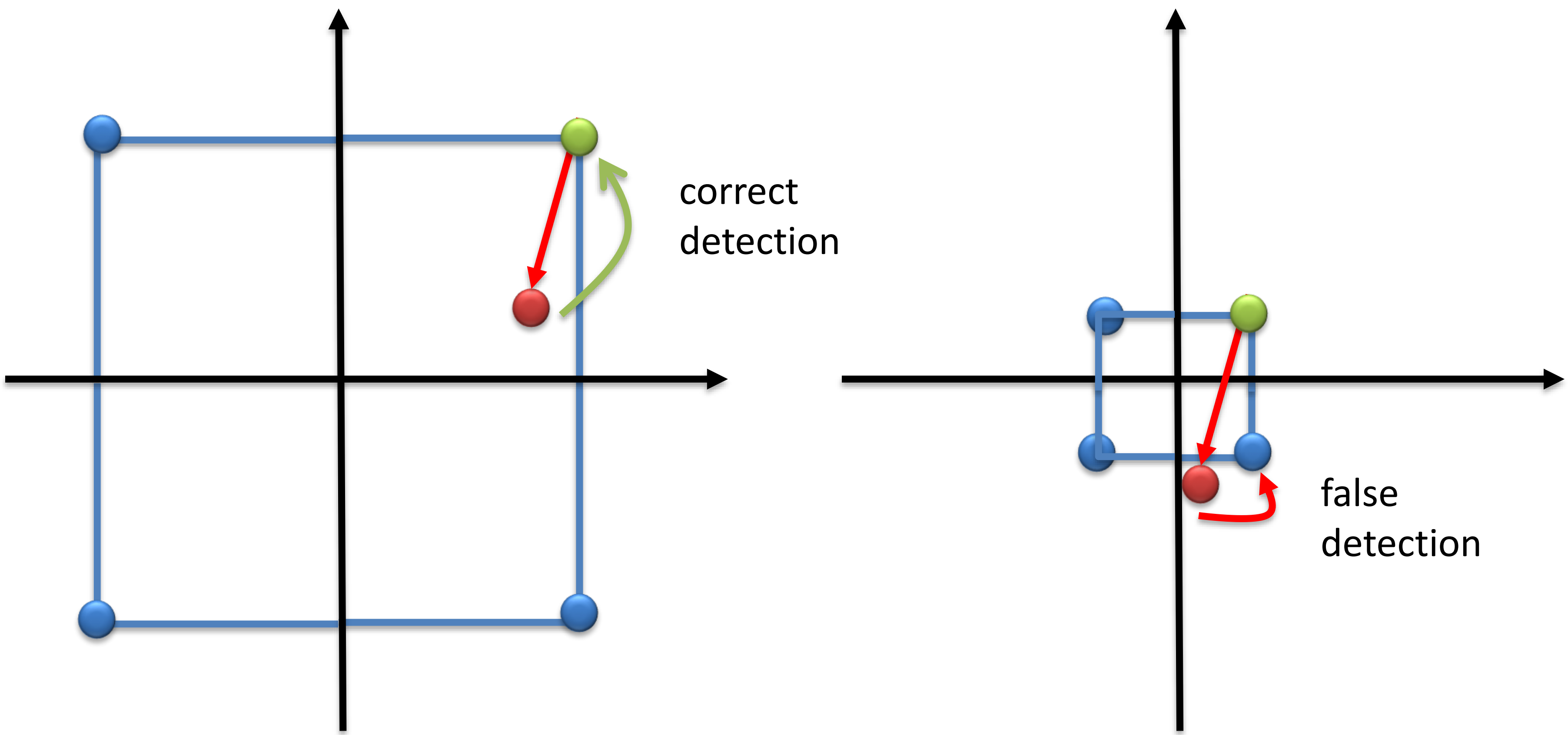
Demodulate, Integrate, and Dump

- Regenerate in-phase by multiplying complex signal with cosine wave, integrating over the symbol period and dumping the result



Detection at Receiver

- Detection: identify closest symbol in constellation diagram

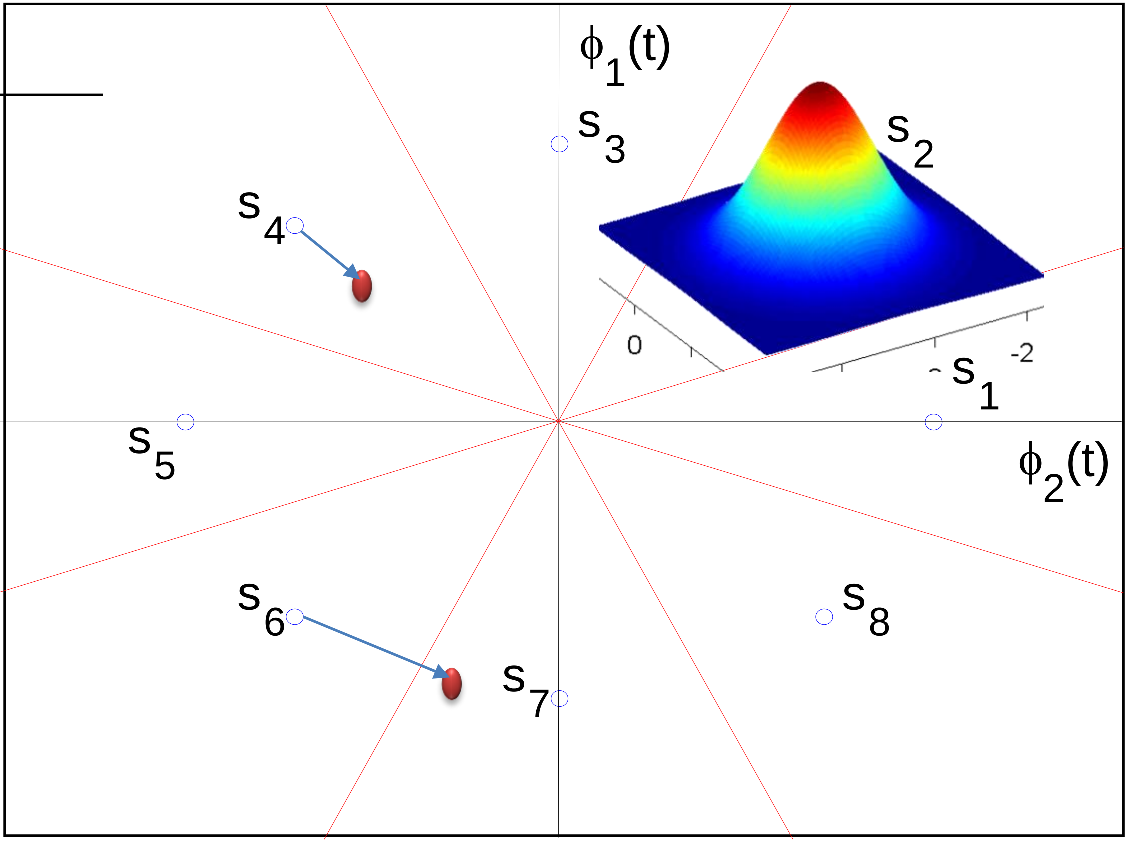


- Bit errors occur if the received power is too low

Decision Regions

Decision region Z_4

$$Z_i = \left\{ r \middle| \forall j : \|r - s_i\| < \|r - s_j\| \right\}$$

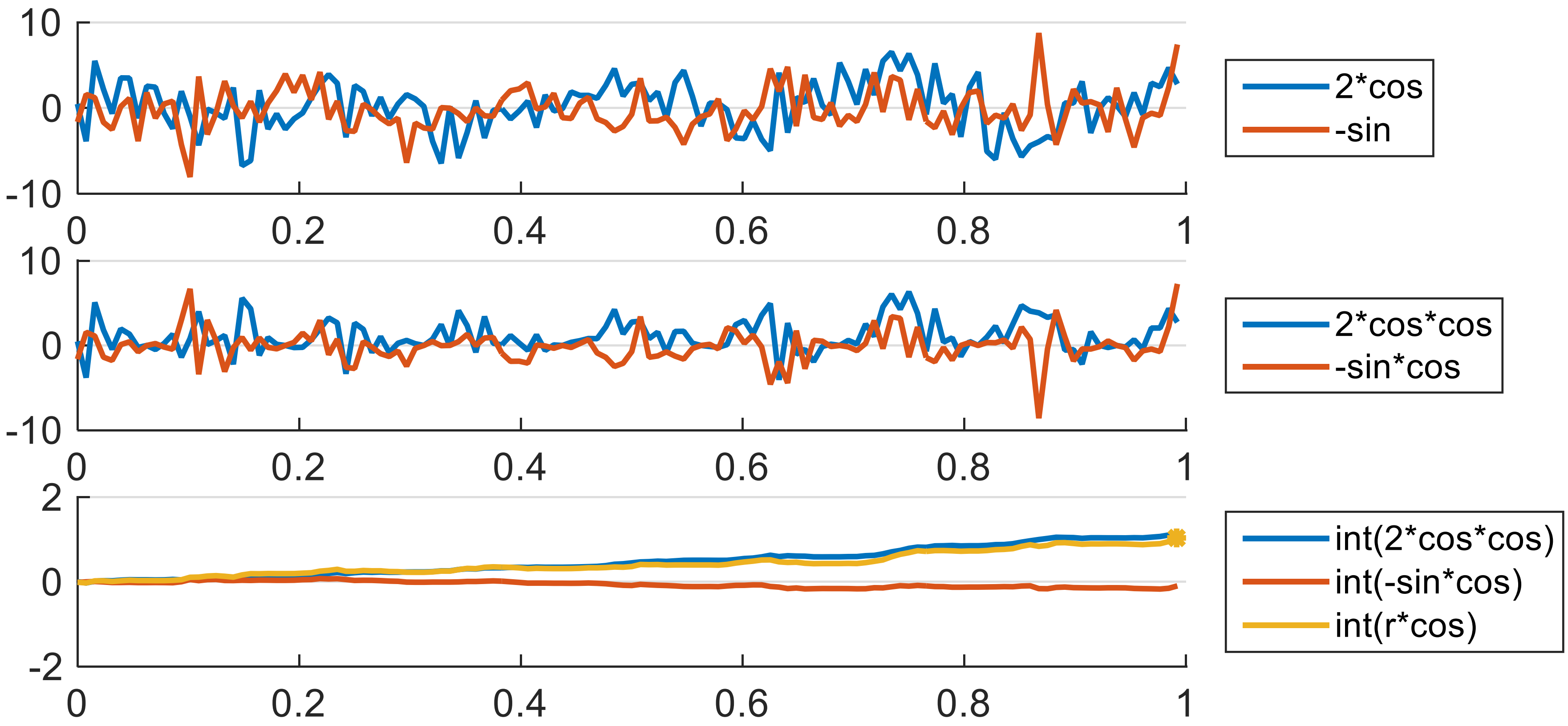


error probability

$$P_e = P\left\{ r \notin Z_i \middle| s_i \text{ sent} \right\}$$

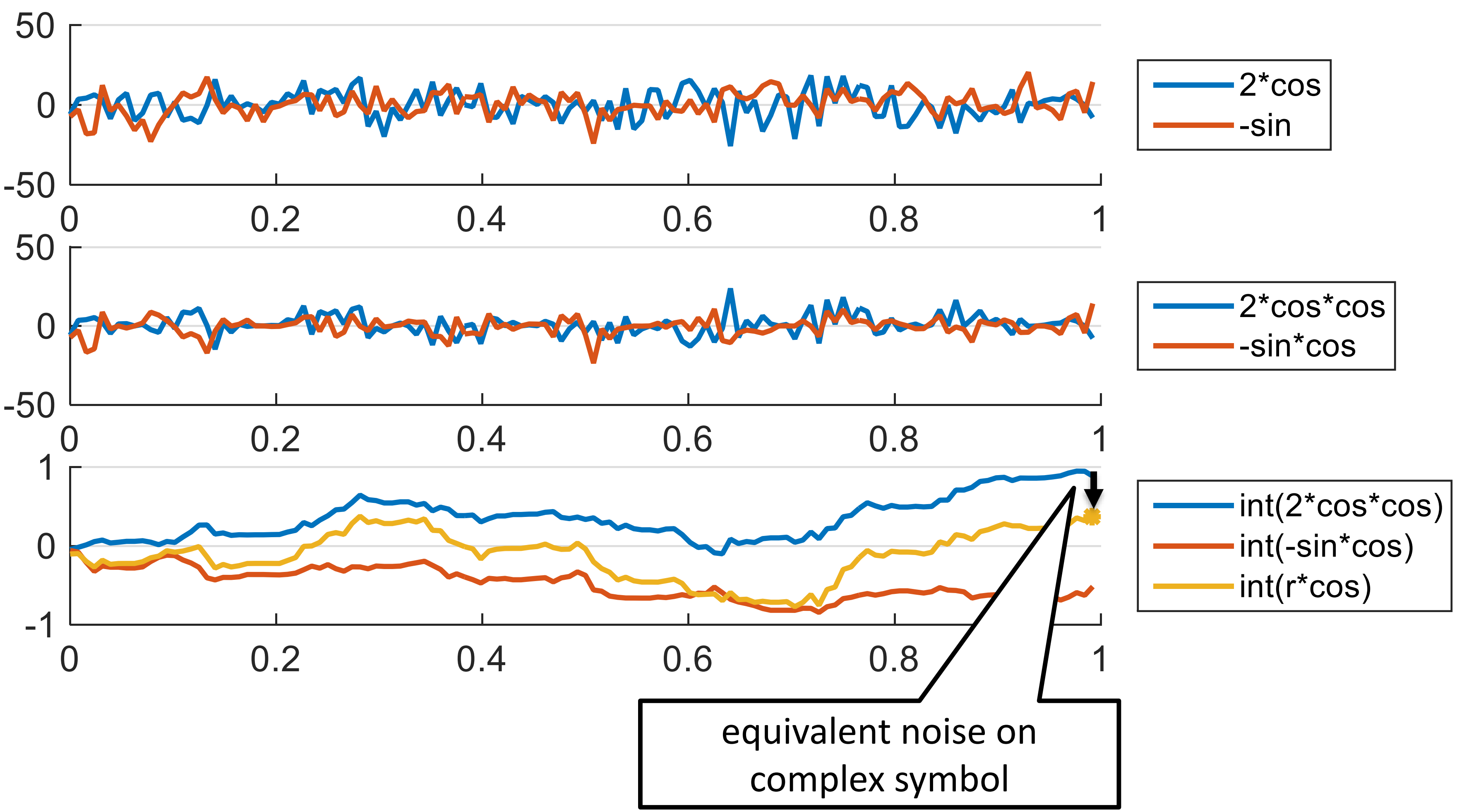
Noisy ($E_s/N_0 = 10dB$)

- Signal is hard to detect but detector yields a clear “1”
- Example: s_I =2, $s_Q = 1$



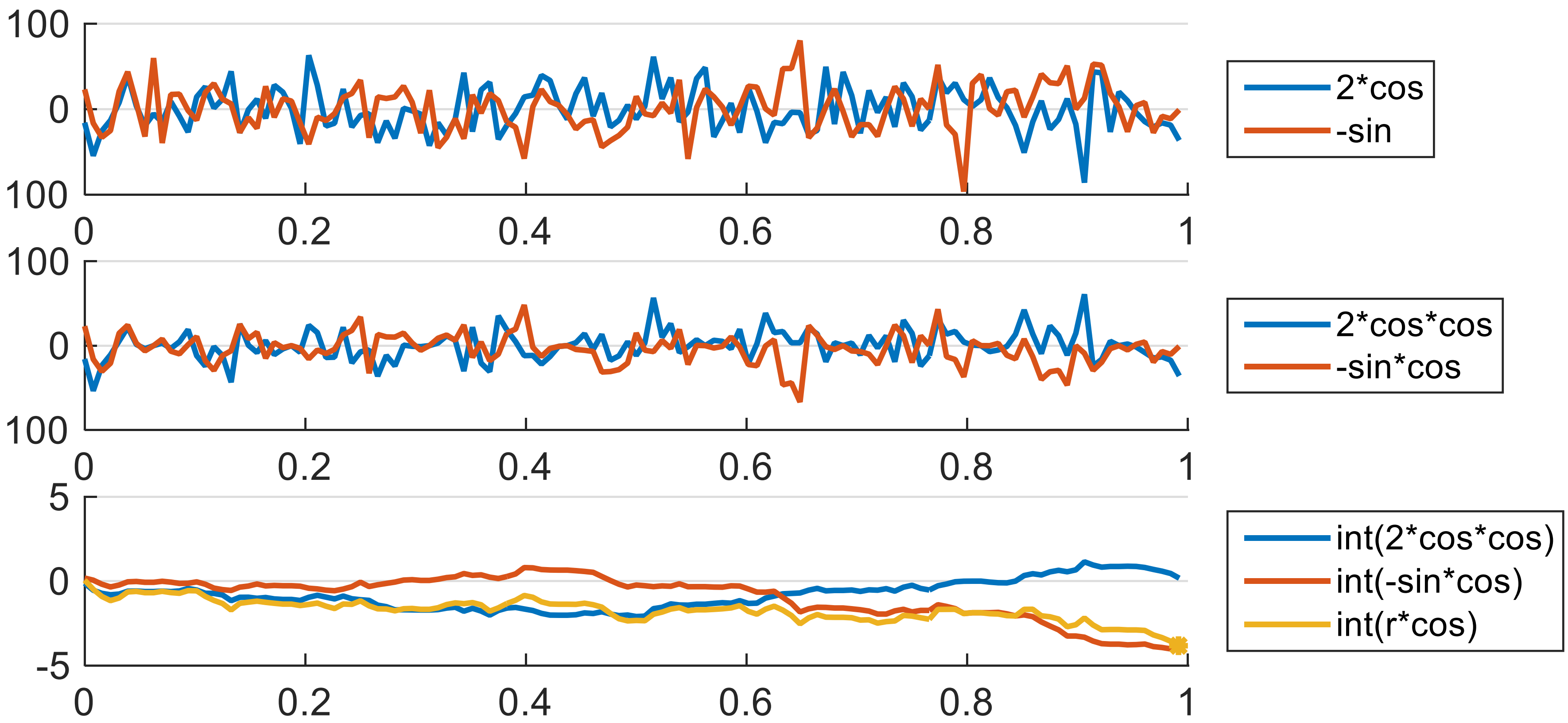
Noisy ($E_s/N_0 = 0\ dB$)

- Detection point is shifted but still positive. In case of QAM we might see an amplitude error.

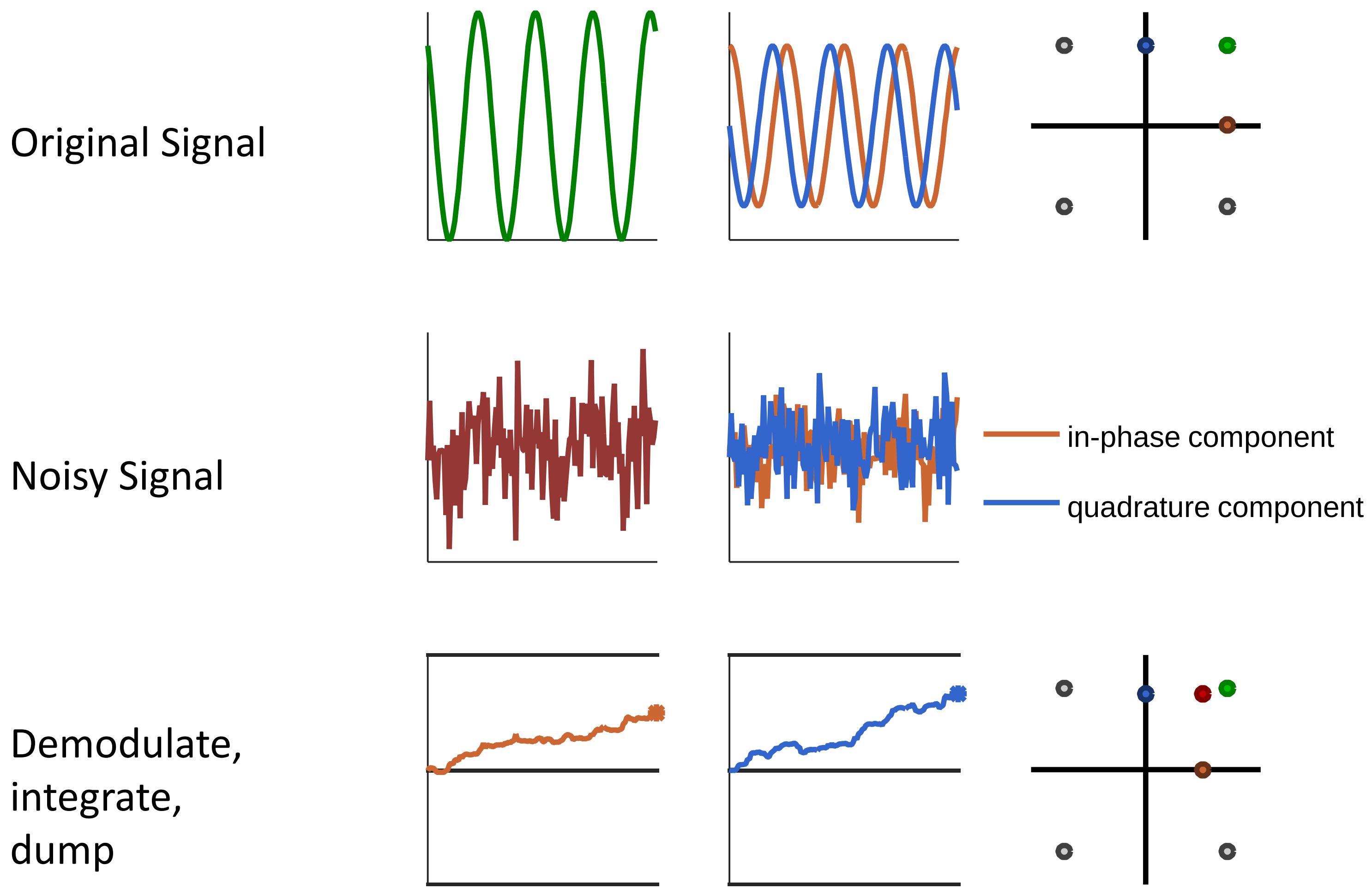


Noisy ($E_s/N_0 = -10\ dB$)

- Noise dominates, signal would be detect as “-1” even with QPSK



Signal in Different States (No Error)

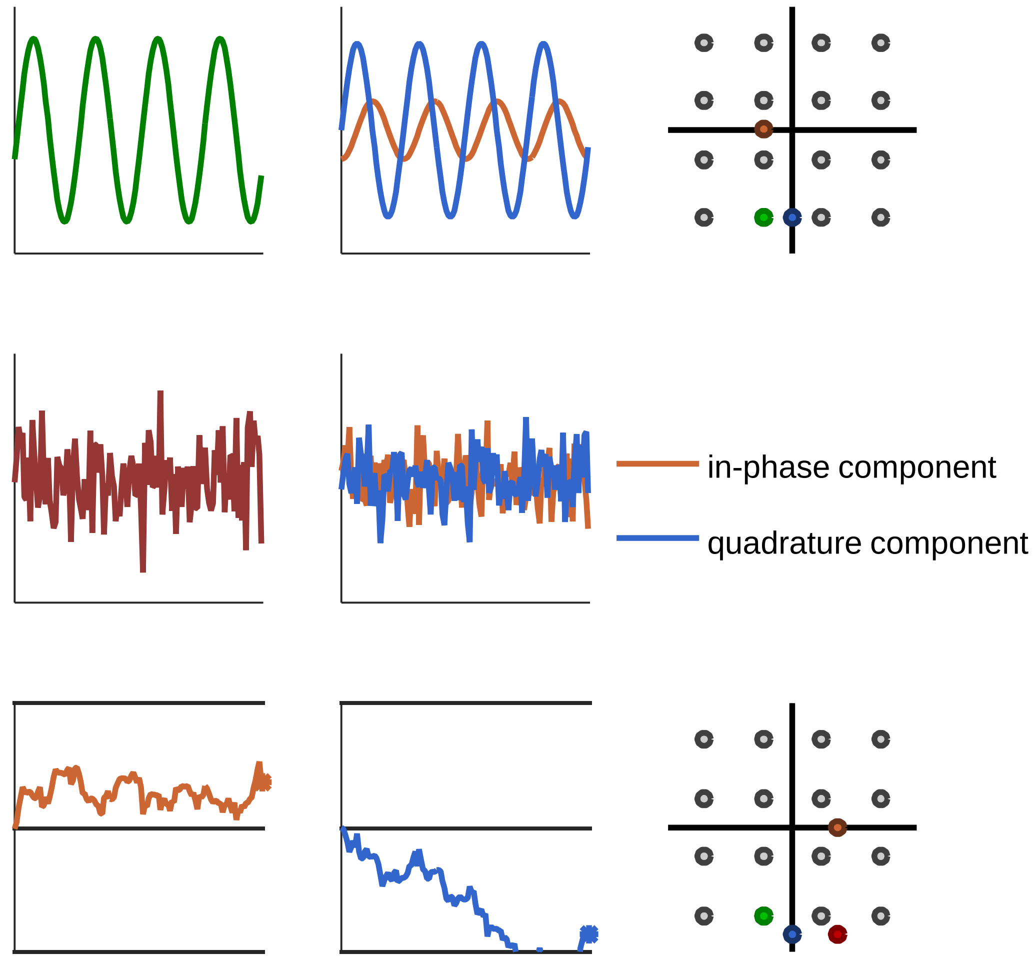


Signal in Different States (Error)

Original Signal

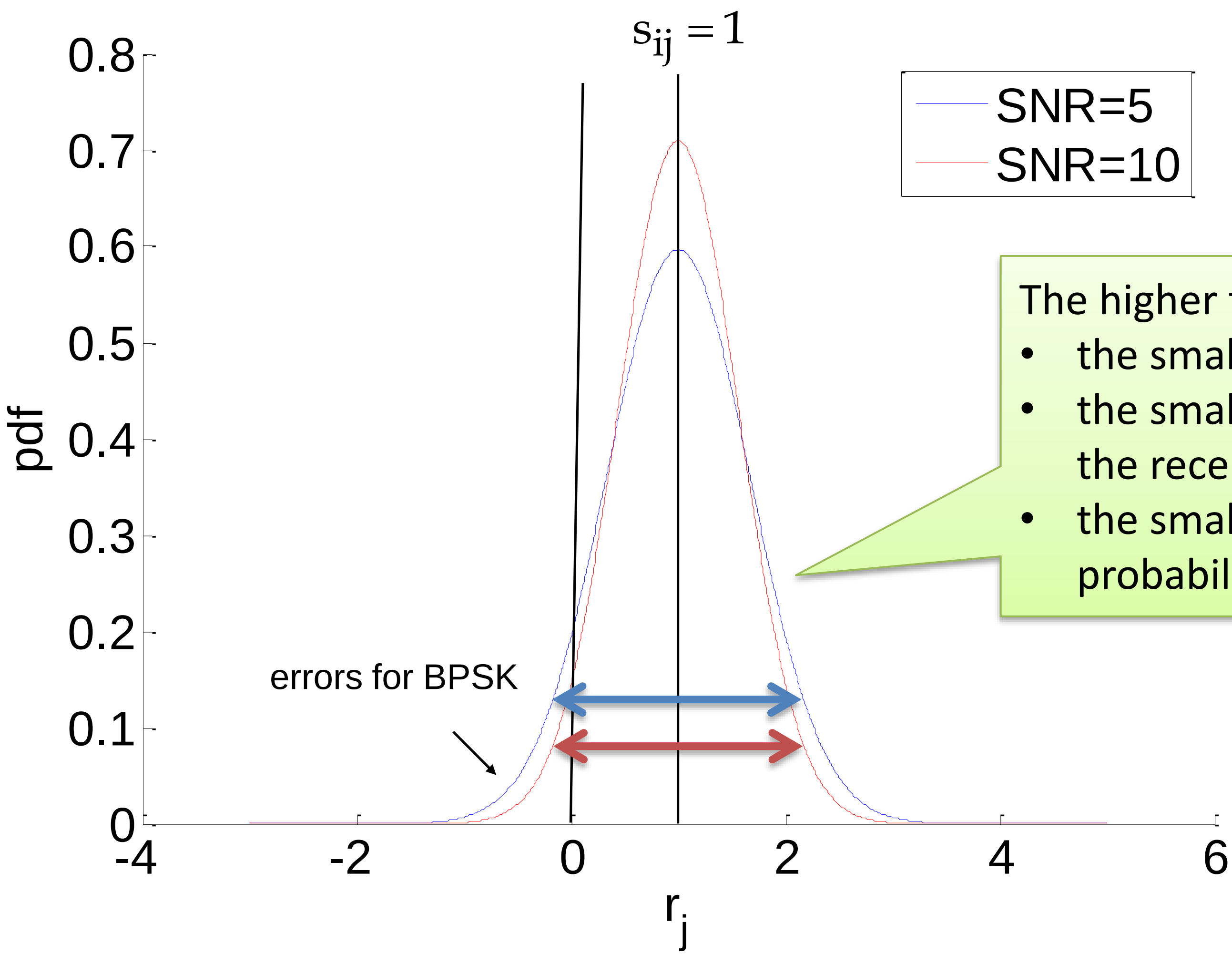
Noisy Signal

Demodulate,
integrate,
dump



Übersicht Kapitel 4: Leitungscodierung und Modulation

Distribution of Received Signal Constellation



The higher the SNR,
- the smaller the noise power
- the smaller the variance of the received signal
- the smaller the bit error probability

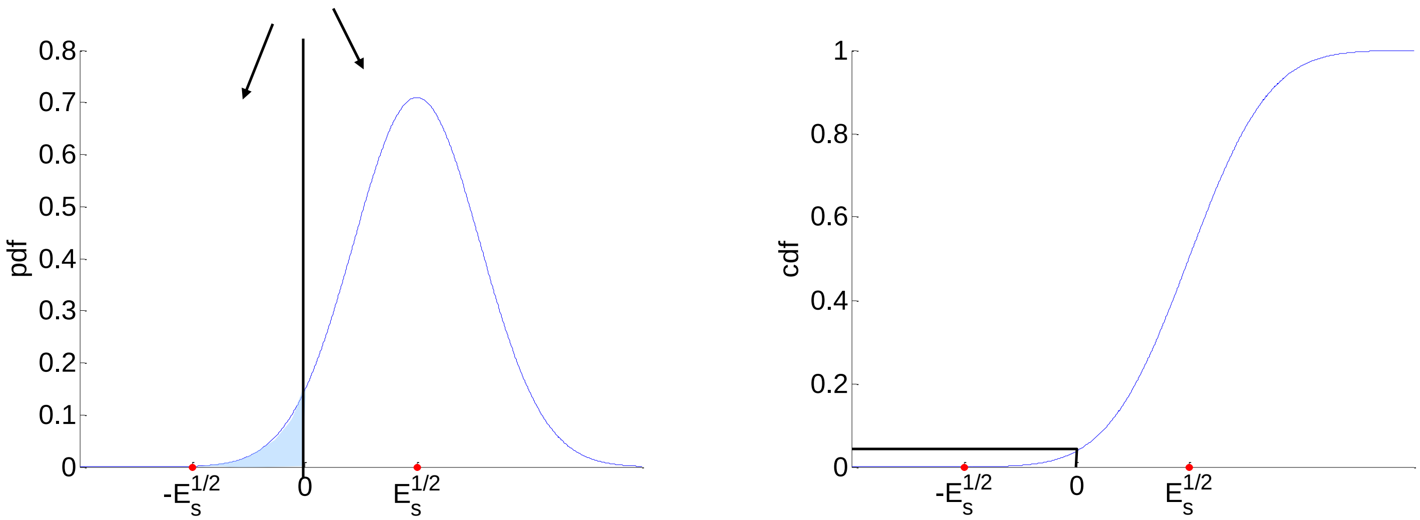
Bit Error Probability for BPSK with AWGN

received signal:

$$r_j = s_{ij} + N\left(0, \sqrt{N_0/2}\right)$$

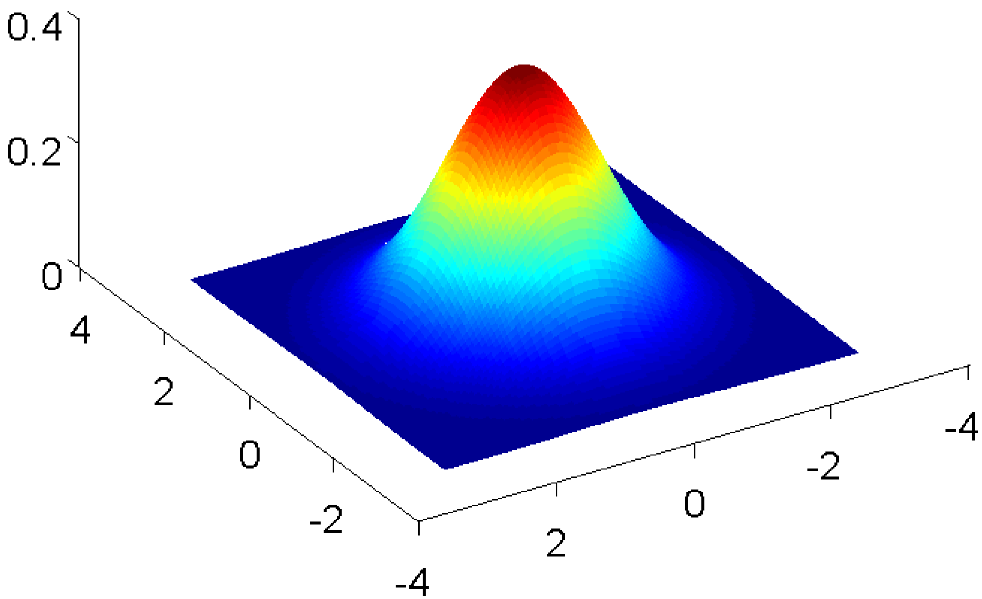
consider the case "1" was sent, i.e. $s_{ij} = \sqrt{E_s}$

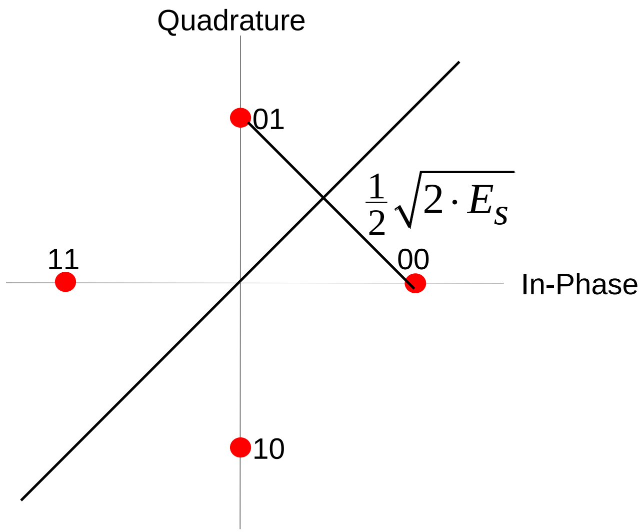
decision regions



$$P_b = P\{r_1 < 0\} = P\left\{\sqrt{E_s} + N\left(0, \sqrt{N_0/2}\right) < 0\right\} = P\left\{N(0,1) > \frac{\sqrt{E_s}}{\sqrt{N_0/2}}\right\} = Q\left(\sqrt{\frac{2E_s}{N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

Bit Error Probability for QPSK with AWGN





$$P\{00 \to 01\} = P\left\{N\left(0, \sqrt{N_0/2}\right) > \tfrac{1}{2}\sqrt{2E_s}\right\} = Q\left(\sqrt{\frac{E_s}{N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

Bit and Symbol Error Probabilities

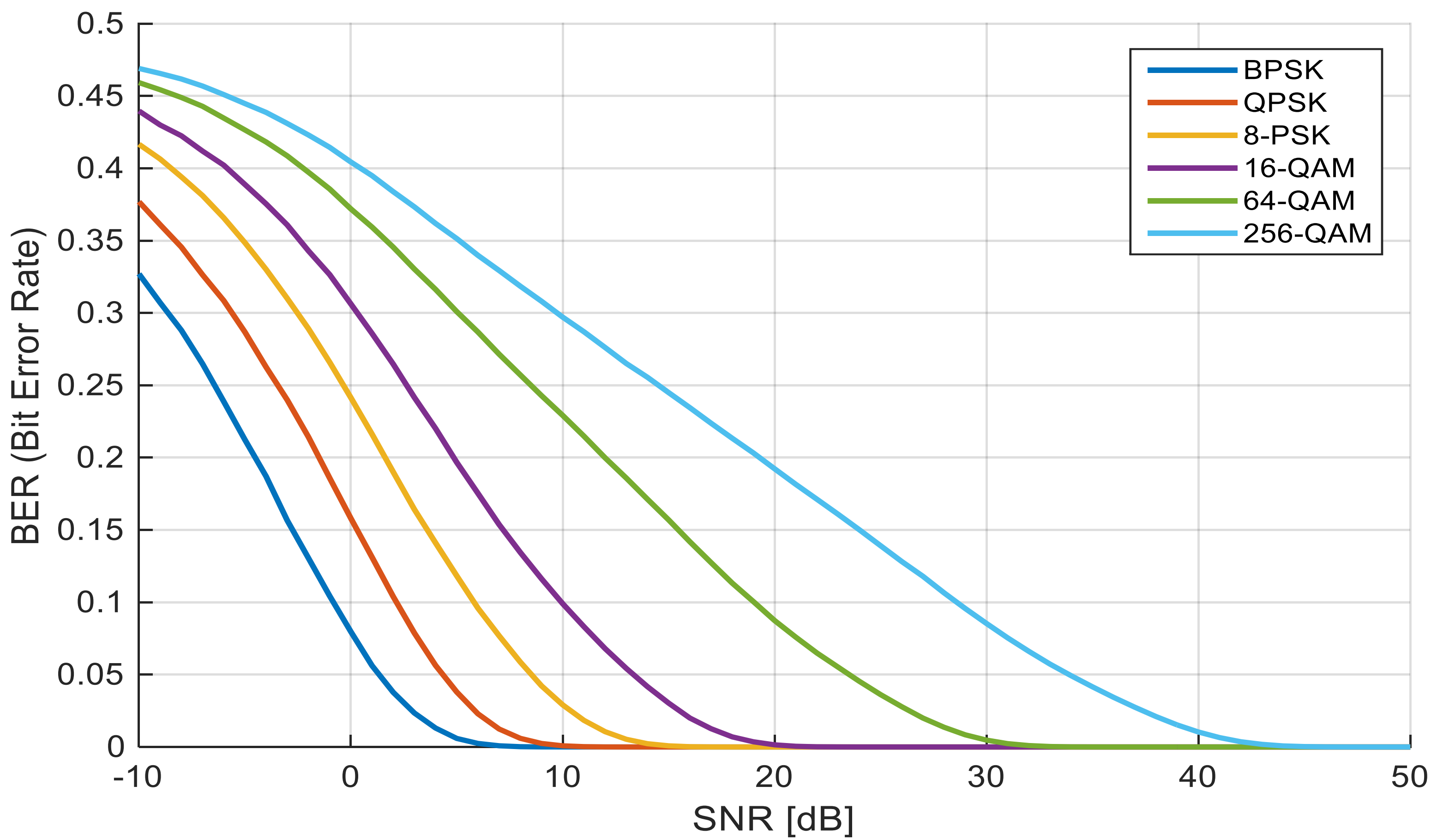
	Symbol	Bit
BPSK	$Q\left(\sqrt{2\frac{E_s}{N_0}}\right)$	$Q\left(\sqrt{2\frac{E_b}{N_0}}\right)$
QPSK	$1-\left(1-Q\left(\sqrt{\frac{E_s}{N_0}}\right)\right)^2$	$Q\left(\sqrt{2\frac{E_b}{N_0}}\right)$
mPSK (approximation)	$2Q\left(\sqrt{2\frac{E_s}{N_0}}\sin\left(\frac{\pi}{M}\right)\right)$	$\frac{2}{\log_2 M}Q\left(\sqrt{2\cdot\log_2 M\cdot\frac{E_b}{N_0}}\sin\left(\frac{\pi}{M}\right)\right)$
mQAM (approximation)	$4Q\left(\sqrt{\frac{3}{M-1}\cdot\frac{\bar{E}_s}{N_0}}\right)$	$\frac{4}{\log_2 M}Q\left(\sqrt{\frac{3}{M-1}\cdot\log_2 M\cdot\frac{\bar{E}_b}{N_0}}\right)$

Approximations:

$$E_b/N_0 \approx \frac{1}{\log_2 M}\cdot E_s/N_0 \qquad p_b \approx \frac{1}{\log_2 M}p_s$$

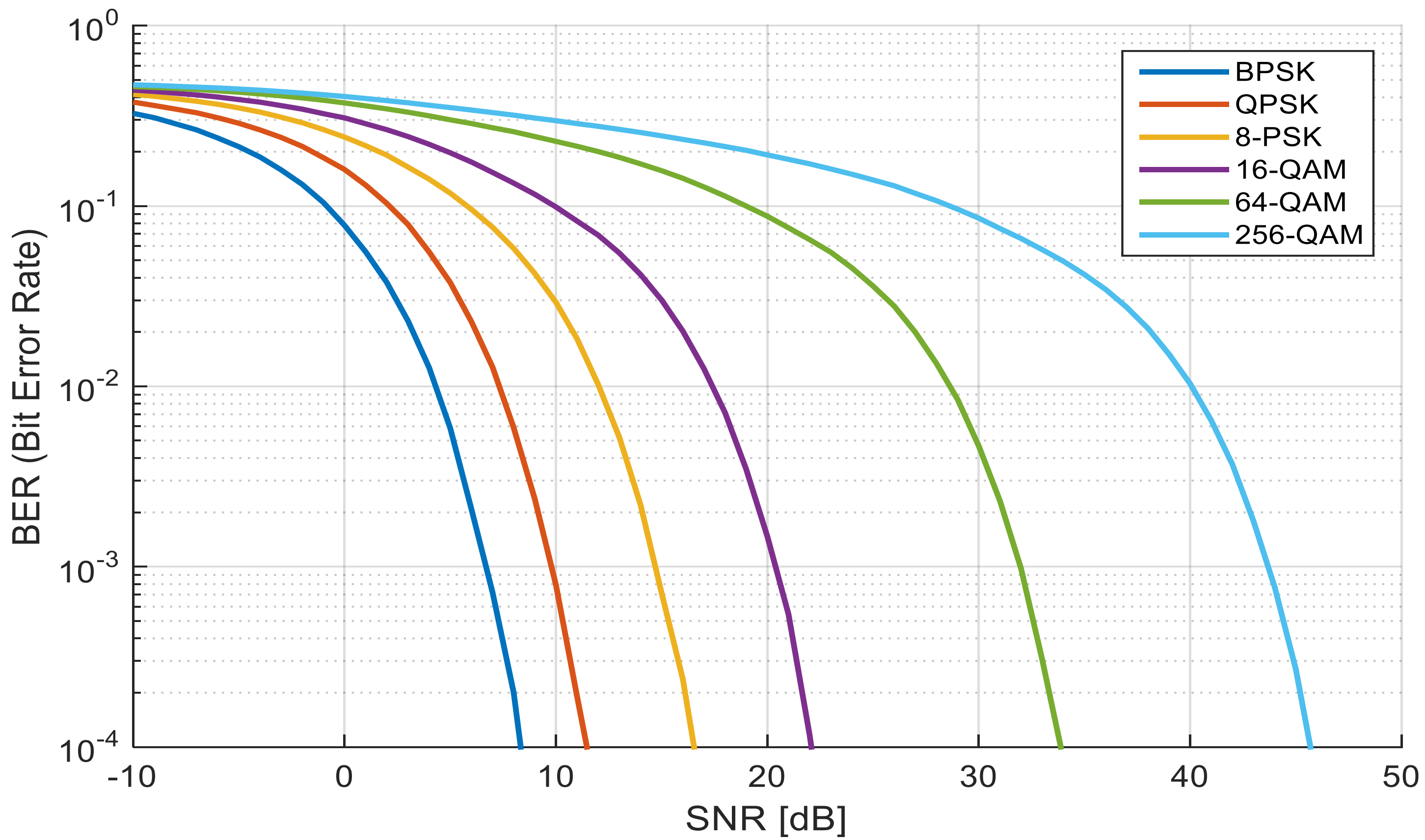
Bit Error Rate of Different Modulation Schemes

The higher the modulation order, the higher the bit error rate

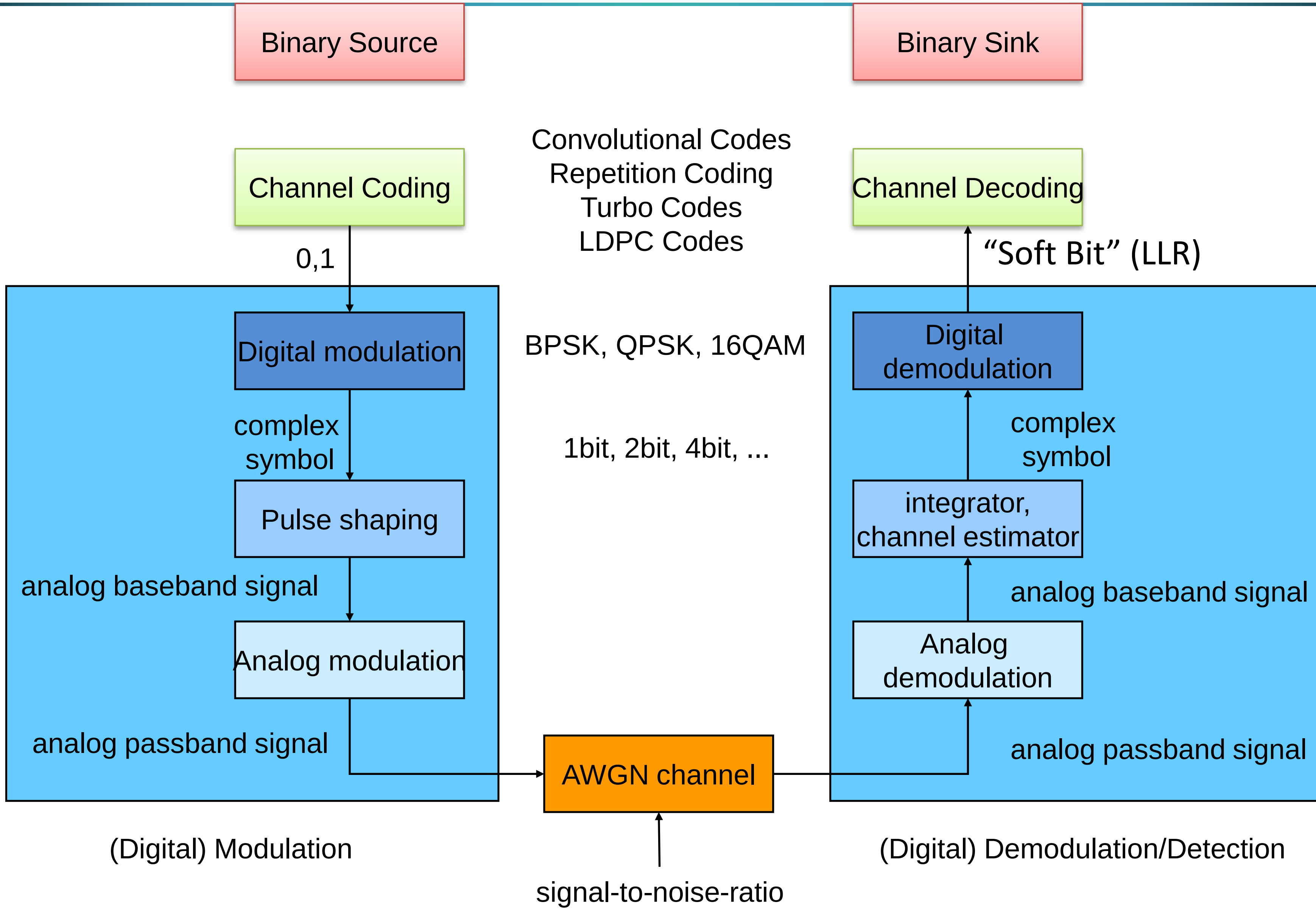


Bit Error Rate of Different Modulation Schemes

In logarithmic scale we see that bit errors can always occur – even on a very good channel with very high SNR an a robust modulation scheme – but then the probability of an error is very low of course

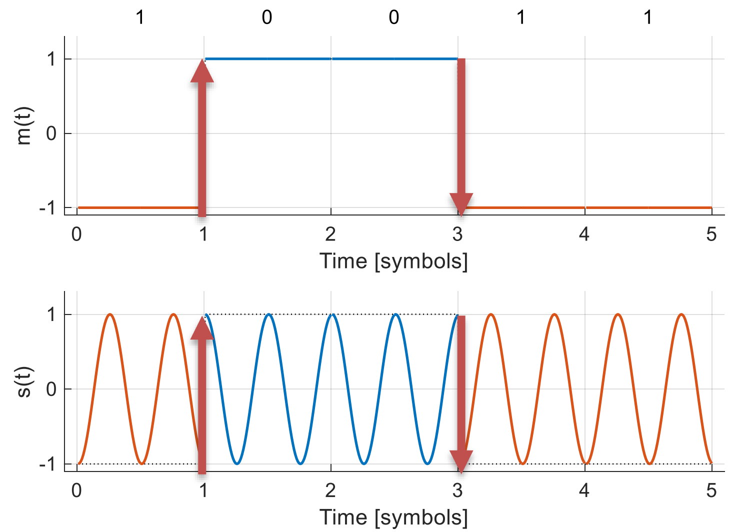


Pulse Shaping



Pulse Shaping to Reduce Bandwidth

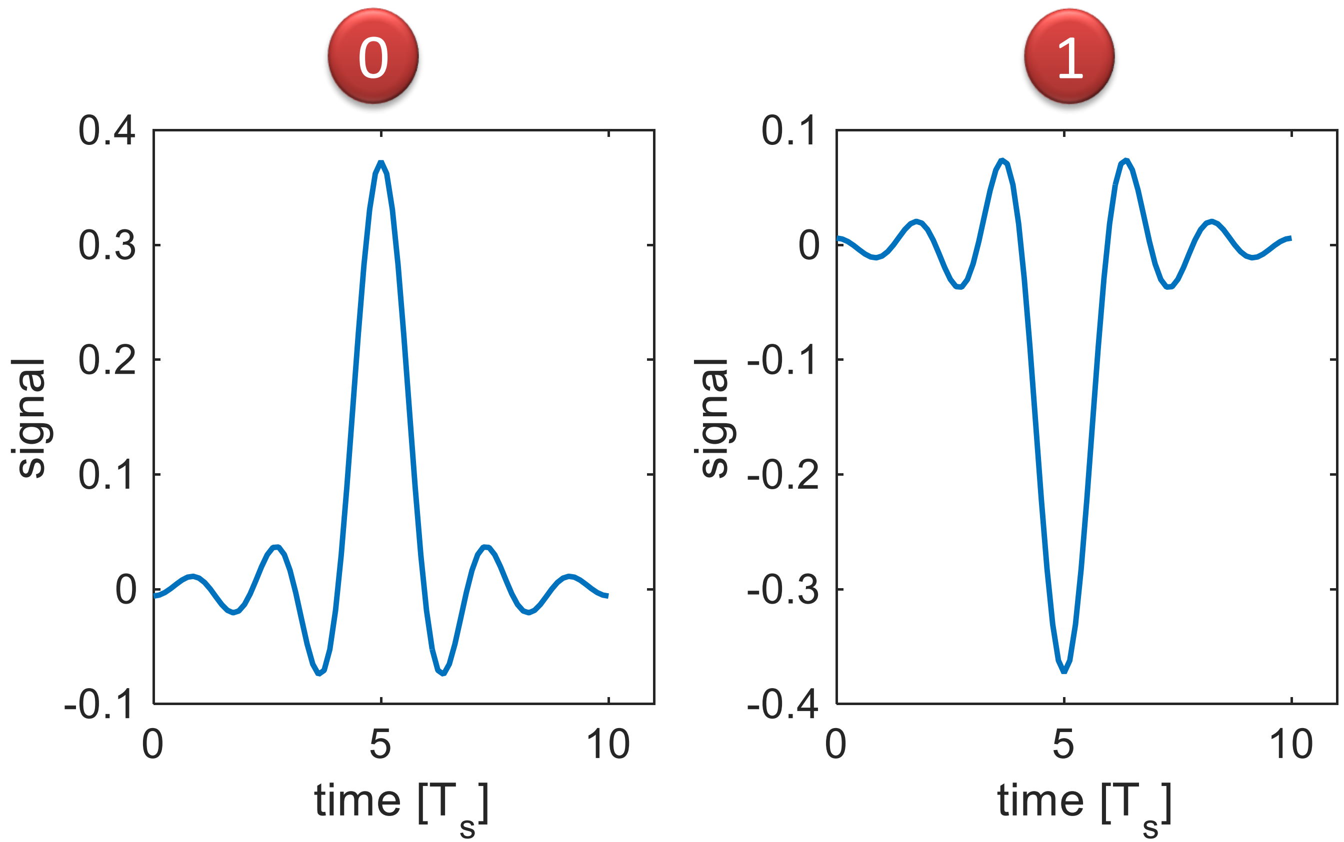
- The wave form or the pulse is the basic baseband signal that is used for digital modulation; simply speaking the wave represents a “+1”. In digital modulation the analog wave form is then multiplied with the digital signal amplitude.
- So far the wave form was a rectangular pulse; with BPSK the wave form was multiplied with the digital signal “+1” and “-1”. The RECTANGULAR PULSE has bad properties
  - abrupt changes in the signal
  - broad spectrum
  - high bandwidth demand
  - any jump in the signal occupies an infinity spectrum with the power falling off rather slowly with higher frequencies



- Longer and smoother wave forms overlapping in time lead to a more smooth transition between different symbols
  - (root) raised cosine pulse (next slide)
  - half-sine pulse (use the half of a sine curve instead of the rectangle

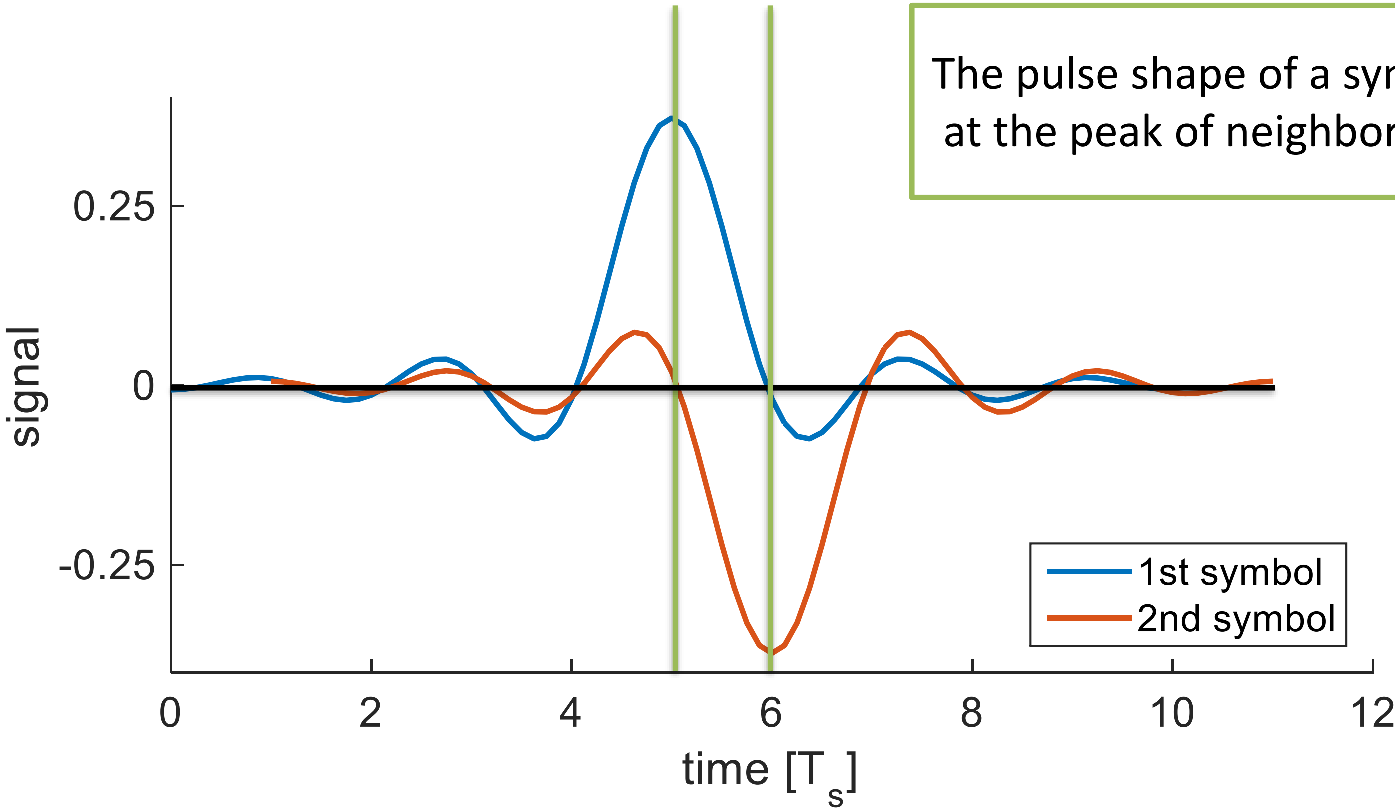
Root Raised Cosine Pulse

- The “Root Raised Cosine Pulse” spans multiple symbol periods
  - in the example the pulse duration is 11 symbol periods
  - the pulse is zero at the peak of neighboring symbols



Root Raised Cosine Pulse

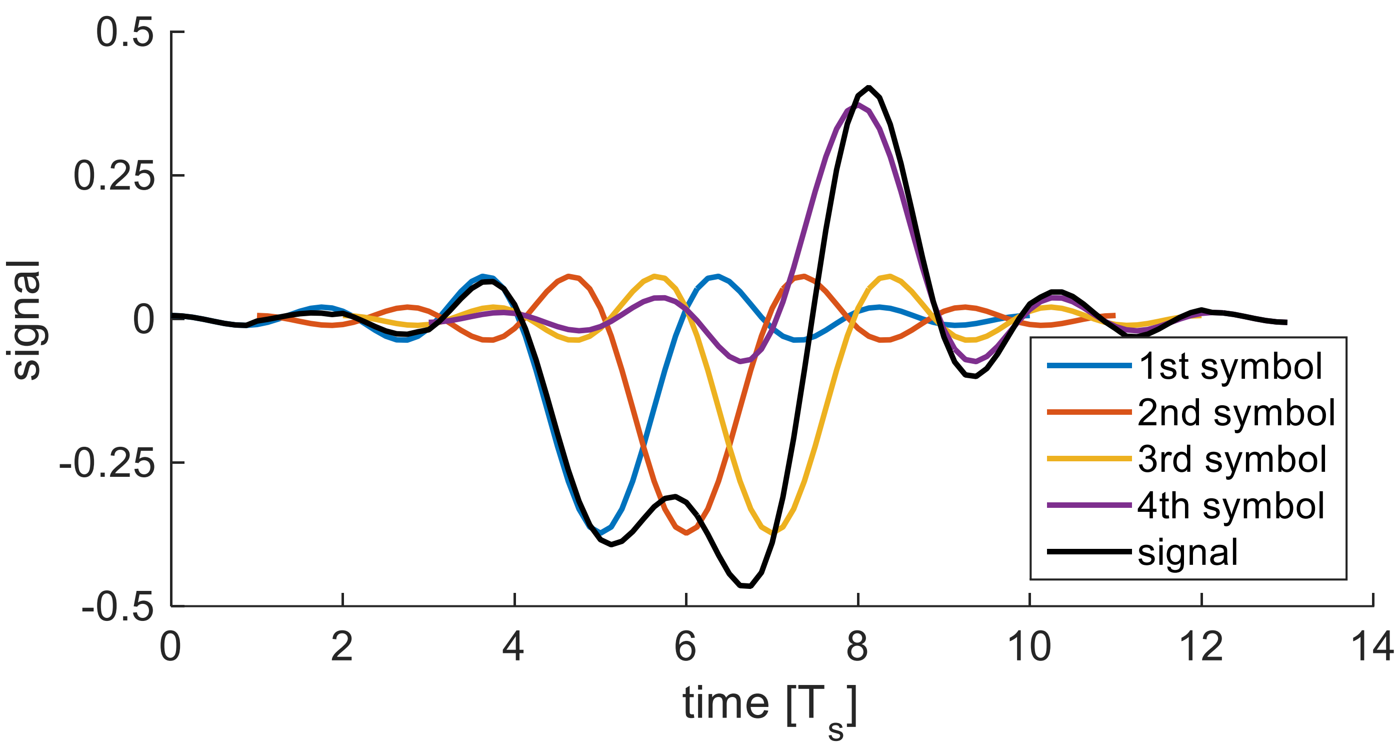
- A new symbol is still transmitted per symbol period; since the pulses are longer than a symbol period they overlap
- All overlapping pulses are summed up to form the transmitted signal



The pulse shape of a symbol is zero at the peak of neighboring pulses.

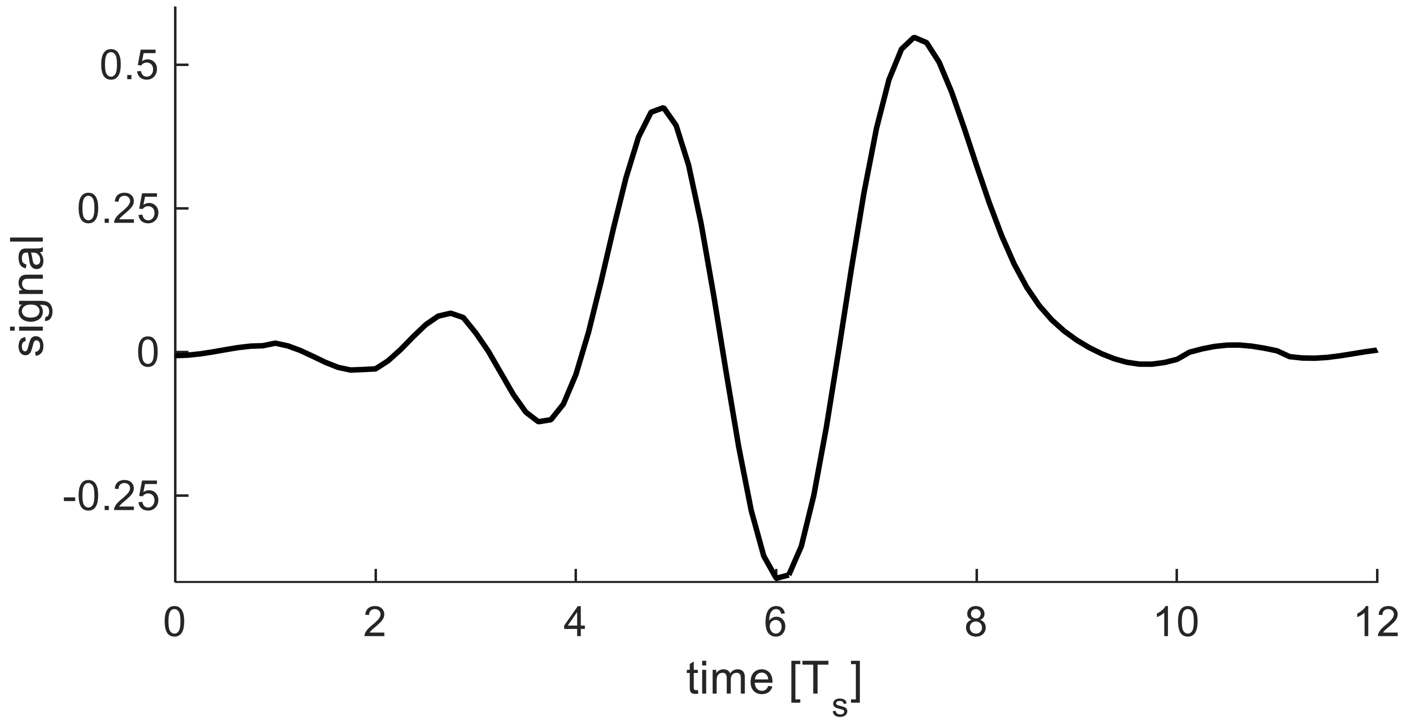
Root Raised Cosine Pulse

- The example shows 4 symbols (“1”,”1”,”1”,”-1”) in different colors
- The resulting signal is the sum of the symbols and shown in black

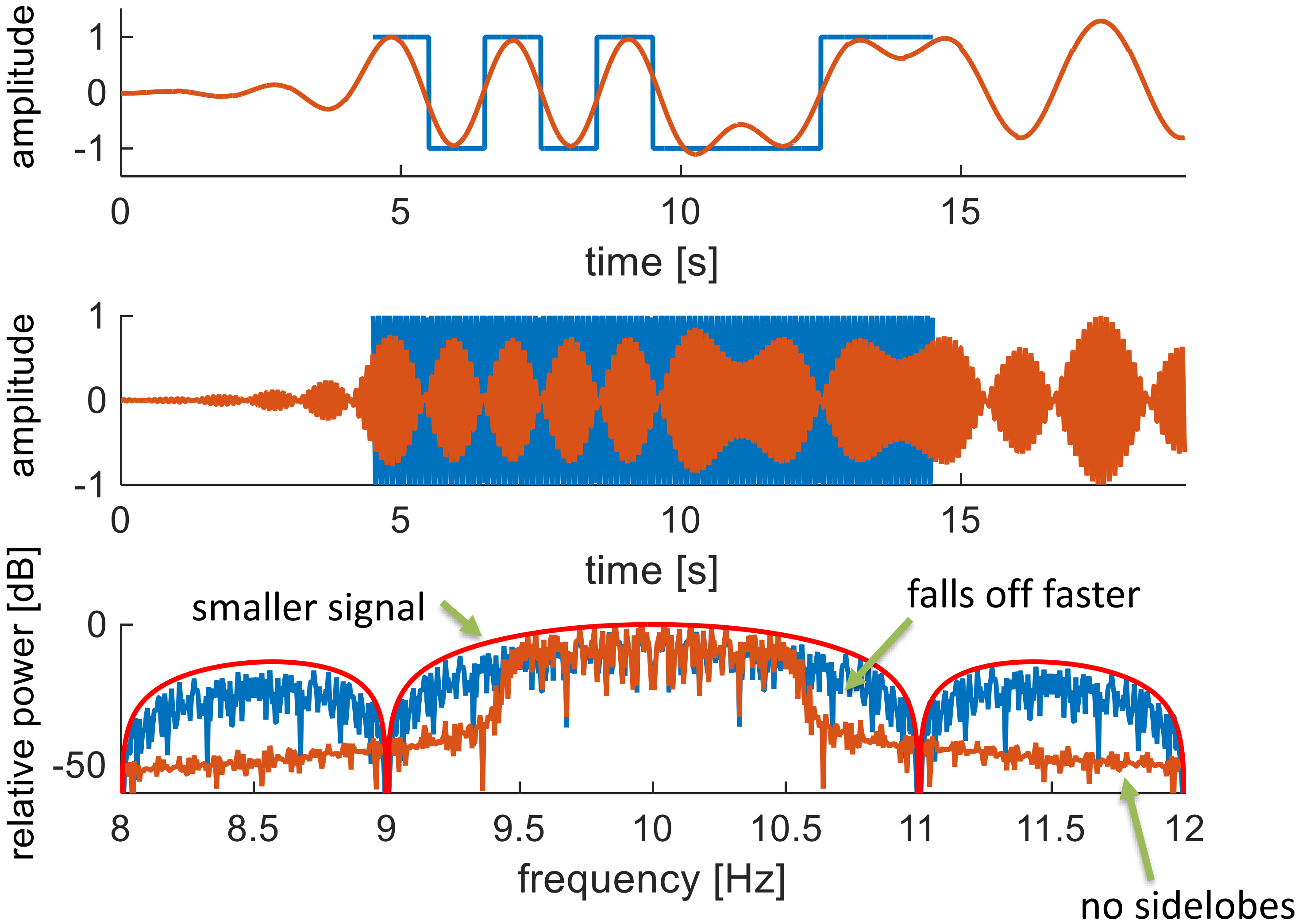


Root Raised Cosine Pulse

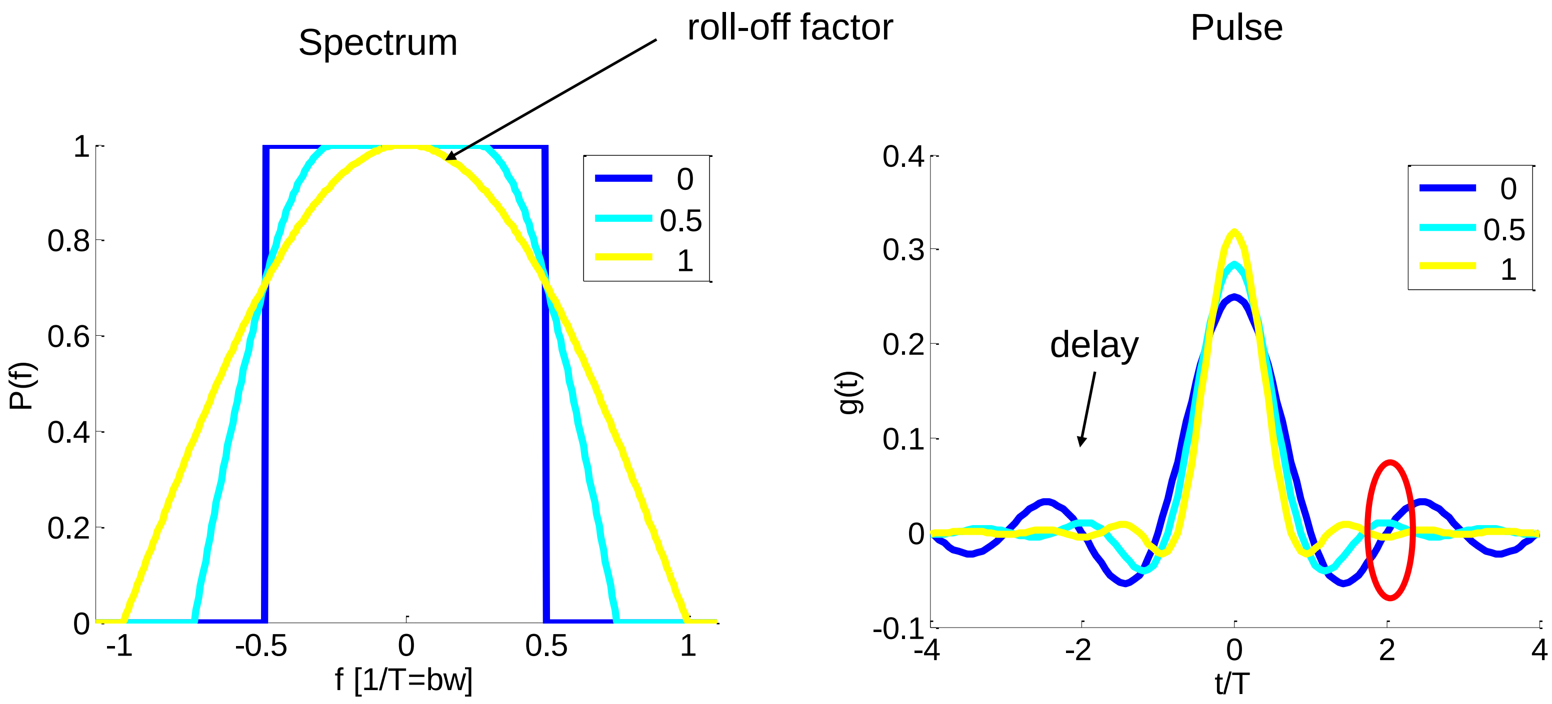
- The resulting signal is much smoother than the signal with rectangular pulses
  - jumps in the signals are avoided
  - the signal is smaller and falls off faster (see next slide)



Root Raised Cosine Pulse



Root Raised Cosine Pulse



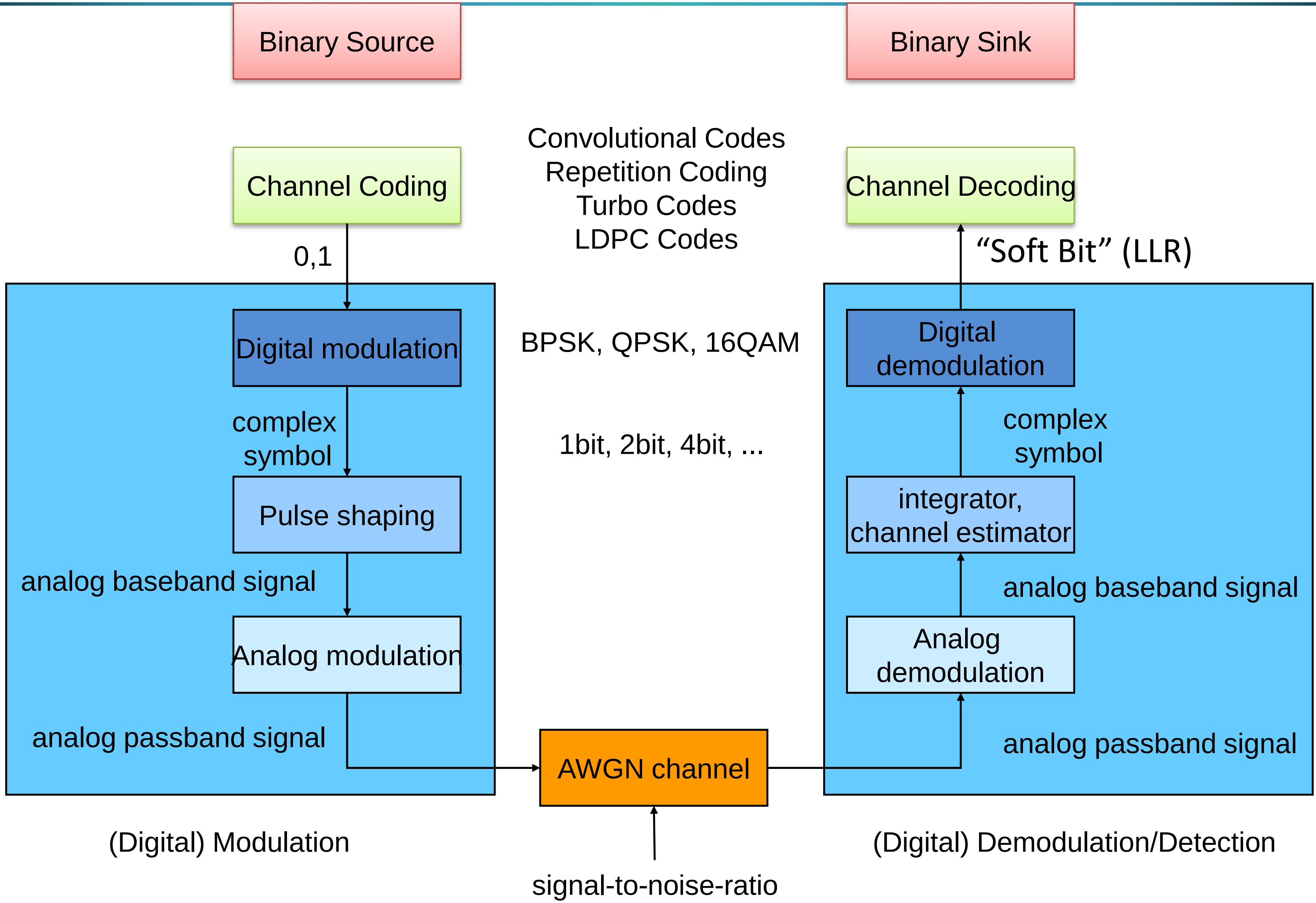
- The **roll-off factor** of the root raised cosine pulse is a trade-off between a rapid decay of signal power in spectrum and the duration of a pulse in time domain
  - a roll-off factor of “0” produces a rectangular shape in frequency domain and an infinite signal in time domain

Übersicht Kapitel 4: Leitungscodierung und Modulation

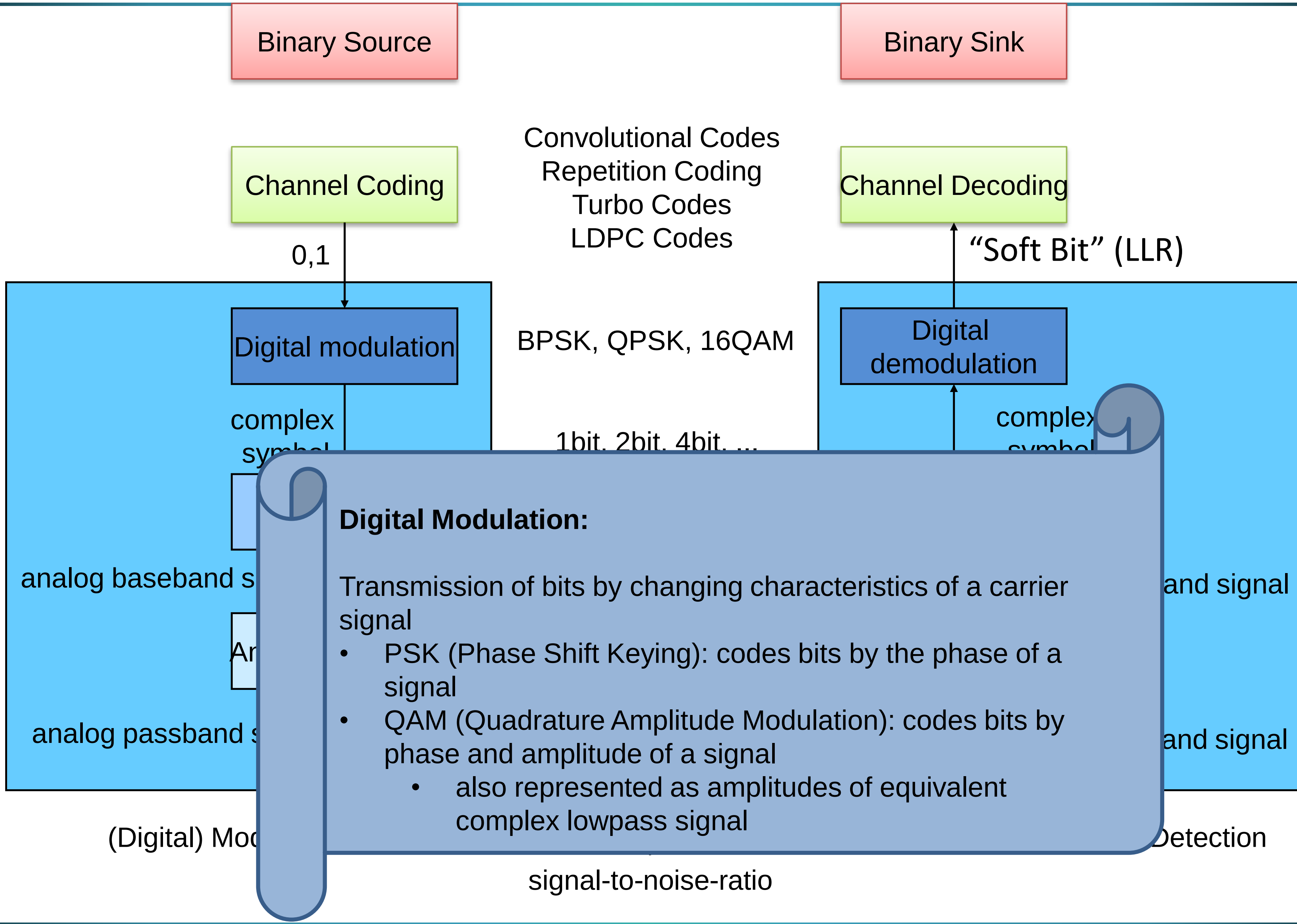
Zusammenfassung

- Bandbasssignal
  - befindet sich im Frequenzbereich um die Trägerfrequenz, der Frequenz des Trägersignals
  - belegt die doppelte Bandbreite des modulierenden Basisbandsignals
  - entspricht der Summe eines modulierten Cosinusträgers und eines modulierten Sinusträgers, also der Summe zweier Basisbandsignale
- Übertragung von Bits durch Digitale Modulation: Veränderungen der Amplitude und Phase des Trägersignals pro Symbol
  - Phase Shift Keying: BPSK, QPSK, 8-PSK
  - Quadrature Amplitude Modulation: 16-QAM, 64-QAM, …
- Darstellung eines Signals als komplexes Symbol über In-Phase- und Quadrature-Komponente

Summary: Digital Modulation



Summary: Digital Modulation



Digital Modulation:

Transmission of bits by changing characteristics of a carrier signal

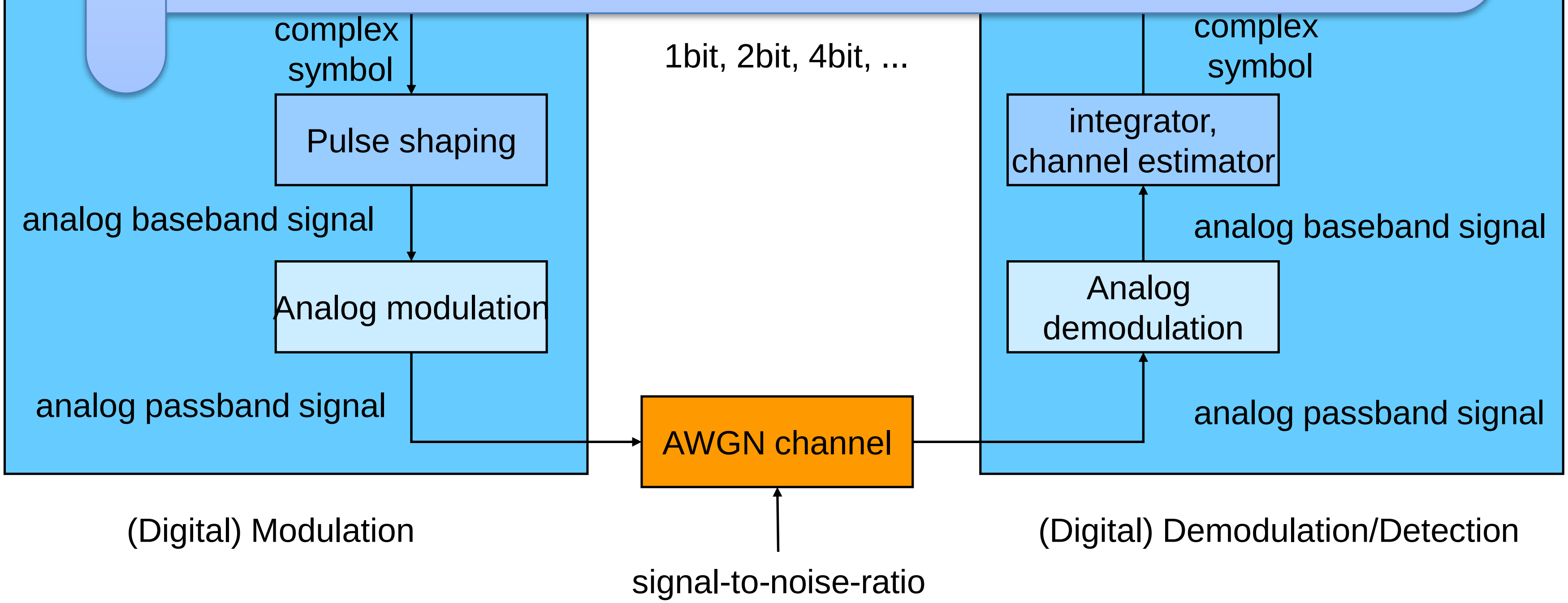
- PSK (Phase Shift Keying): codes bits by the phase of a signal
- QAM (Quadrature Amplitude Modulation): codes bits by phase and amplitude of a signal
  - also represented as amplitudes of equivalent complex lowpass signal

Summary: Digital Modulation

Pulse Shaping:

Pulse shaping converts a complex symbol into an analogue baseband signal with

- the pulse shape is a compromise between signal power decay in frequency domain and delay in time domain
- pulse shaping and recovery is done by applying filter and matched filter at sender and receiver

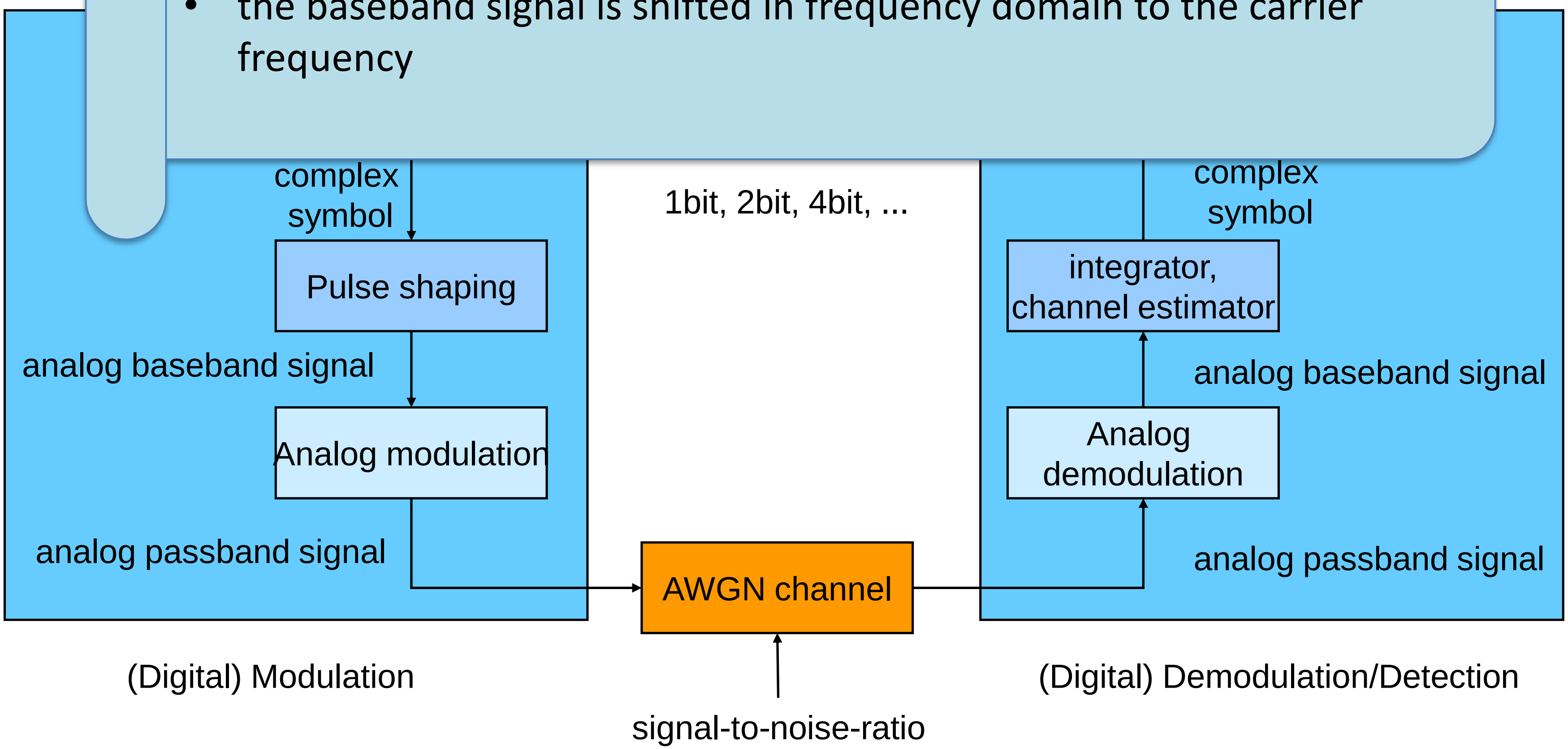


Summary: Digital Modulation

Analog Modulation:

Analog modulation changes the carrier signal according to the baseband signal

- the baseband signal is shifted in frequency domain to the carrier frequency



Summary: Digital Modulation

Binary Source

Binary Sink

Channel Coding

Convolutional Codes
Repetition Coding
Turbo Codes
LDPC Codes

Channel Decoding

0,1

"Soft Bit" (LLR)

Digital modulation

BPSK, QPSK, 16QAM

Digital demodulation

complex symbol

1bit, 2bit, 4bit, ...

complex symbol

analog baseband signal

analog passband signal

(Digital) Modulation

Detection

signal-to-noise-ratio

Digital Demodulation (Detection):

Signal detection determines the best matching constellation of a noisy signal

- the output of soft demodulation is a soft bit, the final decision will be taken by the channel decoder

Summary Digital Modulation

- Transmission of bits by changing characteristics of a carrier signal
  - PSK (Phase Shift Keying): codes bits by the phase of a signal
  - QAM (Quadrature Amplitude Modulation): codes bits by phase and amplitude of a signal
    - also represented as amplitudes of equivalent complex lowpass signal
- Pulse shaping converts a complex symbol into an analogue baseband signal with
  - the pulse shape is a compromise between signal power decay in frequency domain and delay in time domain
  - pulse shaping and recovery is done by applying filter and matched filter at sender and receiver
- Analog modulation changes the carrier signal according to the baseband signal
  - the baseband signal is shifted in frequency domain to the carrier frequency
- Signal detection determines the best matching constellation of a noisy signal
  - the output of soft demodulation is a soft bit, the final decision will be taken by the channel decoder