1.1 Protons, 50 GeV (KE) = E_2 (final) ← this is relativistic mass, I suppose...

tunnel: R = 215 m

E_1 = 4 GeV (KE, injected)

B = 1,8 T

↑ f_B = 0,3 Hz sine wave

m_p = 0,9383 GeV = m_0 (rest mass..?)

↳ $m_0 c^2$ = 0,9383 GeV, actually

a) $p = \gamma m_0 v$

$$E = \sqrt{p^2c^2 + (m_0 c^2)^2} \rightarrow p = \left(\frac{E^2 - (m_0 c^2)^2}{c^2}\right)^{\frac{1}{2}} = \sqrt{\frac{E^2}{c^2} - m_0^2 c^2}$$

@ 4 GeV: $p_1^2 = \frac{4^2 \, GeV^2}{c^2} - \frac{0,9383^2 \, GeV^2}{c^2} \Rightarrow p_1 \simeq 3,888 \frac{GeV}{c}$

@ 50 GeV: $p_2^2 = \frac{50^2 \, GeV^2}{c^2} - \frac{0,9383^2 \, GeV^2}{c^2} \Rightarrow p_2 \simeq 49,99 \frac{GeV}{c}$

b) Rigidity $B\rho = 3,3356 \, p$ Tm   p in $\frac{GeV}{c}$ already

@ 4 GeV: $B\rho = 3,3356 \cdot 3,888$ Tm $\simeq$ 12,97 Tm

@ 50 GeV: $B\rho = 3,3356 \cdot 49,99$ Tm $\simeq$ 166,75 Tm

c)

↳ $\rho = \frac{B\rho}{B} = \frac{12,97}{1,8} \simeq 7,2$ m @ 4 GeV

$\rho = \frac{166,75}{1,8} \frac{Tm}{T} \simeq 92,63$ m @ 50 GeV } ... ? The bending radius can be shorter ↳ dipoles just won't be everywhere, obviously...

d) needs to be true that

$\int ds = 2\pi \int d\rho$

also: $\sigma = \frac{BL}{p/e} \rightarrow L = \frac{\sigma p}{Be} \rightarrow \frac{S}{C} = \frac{2\pi p}{Be 2\pi R} = \frac{p}{eBR} = \frac{\rho}{R}$ ✓

magnet length

↳ $S = 2\pi\rho$, $C = 2\pi R$ } $\frac{S}{C} = \frac{2\pi\rho}{2\pi R} = \frac{\rho}{R} =$

$\frac{7,2}{215} \simeq 0,034$ @ 4 GeV

$\frac{92,63}{215} \simeq 0,43$ @ 50 GeV

↑ Tunnel circumference

1.2 Betatron: $R = 0{,}2\,m$ beam radius

$f = 50\,Hz$

$B_{max} = 0{,}8\,T \rightarrow \bar{B} \simeq 0{,}637$ ← effective magn. field...?

Wideroe condition: field at R exactly half the average field over the magnet area.

Faraday law: $\mathcal{E} = -\dfrac{d\Phi_B}{dt}$

Betatron condition: $\Phi = 2\pi R^2 B$

energy of an electron there: $E = BeRc$

$(E = pc \;,\; eBr = mv = p \nearrow)$

the peak energy will be simply $E = 0{,}8 \cdot 0{,}2 \cdot 3 \cdot 10^8\,eV =$

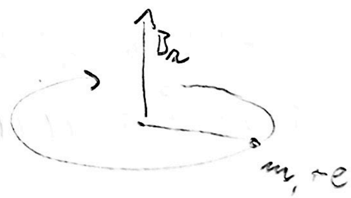
$= 0{,}48 \cdot 10^8\,eV = \underline{48\,MeV}$

1.3 $\dfrac{mv^2}{R} = q\,v B_z \quad q = e$; $v = \dfrac{2\pi R}{T} = 2\pi R f = \omega R$

$\omega = \dfrac{2\pi}{T}$

$\hookrightarrow \dfrac{m}{\cancel{R}}\,\omega \cancel{R} = e B_z$

$\hookrightarrow \boxed{\omega = \dfrac{e B_z}{m}}$



✓ as simple as that, I believe...

now numerically: $B = 1{,}5\,T$

$\left.\begin{array}{l} \\ \dfrac{e}{m} = 9{,}58 \cdot 10^7 \dfrac{C}{kg} \end{array}\right\}\ \omega = 1{,}5 \cdot 9{,}58\;10^7\;s^{-1} = \underline{1{,}437 \cdot 10^8\,s^{-1}}$

a)

1.4| muon lifetime ~ 2.2 μs (rest) = τ_0 → $\tau = \gamma \tau_0$ ← lifetime in lab frame

$l \simeq \frac{p}{mc} c \tau_0$

$\gamma = \frac{E}{E_0}$ — Rest mass of muon $E_0 \simeq 105{,}7\,\text{MeV}$

↳ $\boxed{\tau = \frac{E}{E_0} \tau_0}$ — as simple as that, I believe...

@ 50GeV: $\tau = \frac{50}{0{,}1057} \cdot 2{,}2\,\mu s \simeq \underline{1041\,s}$ (~ 17 min)

@ 4GeV: $\tau = \frac{4}{0{,}1057} \cdot 2{,}2\,\mu s \simeq 83\,s$ (~ 1 min 23 s)

b) muon storage ring: $B\rho = \frac{p}{e} = 33{,}356 \cdot p\ [T/m]$ ← $[p] = \frac{GeV}{c}$

$B = 6T$

$\rho/R = 0{,}7$

1.5| a)

energy loss per turn: $-\frac{C E^4}{\rho} \equiv w$ ← lepton energy

$\rho = 7m$ (magnet bending radius)

$C = \frac{4\pi}{3} \frac{r}{(m_0 c^2)^3}$ ← electron radius $\simeq 2{,}82 \cdot 10^{-15}\,m$; m_0 ← electron rest mass

$E = 3GeV$ (electron beam energy)

numerically: $C = \frac{4\pi}{3} \cdot \frac{2{,}82 \cdot 10^{-15}\,m}{(9{,}11 \cdot 10^{-31} \cdot 3^2 \cdot 10^{16})^3} \frac{m\,s^6}{kg^3 m^6} \simeq 2{,}14 \cdot 10^{25} \frac{s^6}{m^5 kg^3}$

↳ $w = -2{,}14 \cdot 10^{25} \frac{s^6}{m^5 kg^3} \cdot \frac{\left(3 \cdot 10^9 \cdot 1{,}602 \cdot 10^{-19} \frac{kg\,m^2}{s^2}\right)^4}{7\,m} = 1{,}63 \cdot 10^{-25} \frac{s^6\,kg^4\,m^8}{kg^3\,m^6\,s^8} =$

$= 1{,}63 \cdot 10^{-25} \frac{kg\,m^2}{s^2} - J =$

$\doteq \underline{\underline{1{,}02\,\text{MeV}}}$

loses about 1 MeV every turn

↳

b) $1\text{Å} = 10^{-10}$ m $= 0{,}1$ nm

rough scales: crystal structure : NaCl ~ 0,3 nm separation dist.

DNA size ~ 2,5 nm

Benzene ~ 1,4 nm bond size

Bohr radius ~ 0,0529 nm

nucleus size ~ 10^{-15} m (fm)

Roughly, once the light wavelength is shorter than the object studied, it should be able to resolve it

↳ crystals, DNA, Benzene, .. OK

atoms (individual), nuclei ... NOT OK

c) neutron De-Broglie wavelength: $\lambda = \frac{h}{p} = \frac{h}{mv}$

$E = h\nu$

$\lambda = \frac{c}{\nu}$ $\quad p = \left(\frac{E^2}{c^2} - m_0^2 c^2\right)^{\frac{1}{2}}$

$$\hookrightarrow \lambda = h\left(\frac{E^2}{c^2} - m_0^2 c^2\right)^{-\frac{1}{2}}$$

$$\hookrightarrow \lambda^2\left(\frac{E^2}{c^2} - m_0^2 c^2\right) = h^2$$

$$E^2 = \left(\frac{h^2}{\lambda^2} + m_0^2 c^2\right) c^2$$

where $p^2 = \frac{h^2}{\lambda^2}$

$$\Rightarrow E = c\left(\frac{h^2}{\lambda^2} + m_0^2 c^2\right)^{\frac{1}{2}}$$ or just $E = (p^2c^2 + (m_0c^2)^2)^{\frac{1}{2}}$

$$\boxed{E = \left(\frac{h^2 c^2}{\underbrace{\lambda^2}_{10^{-10}\,m}} + (\underbrace{m_0}_{\text{neutron rest mass}} c^2)^2\right)^{\frac{1}{2}}}$$, obviously

$$E = \left(\frac{(6{,}62 \cdot 10^{-34})^2 \cdot 9 \cdot 10^{16}}{10^{-20}} + (1{,}67 \cdot 10^{-27} \cdot 9 \cdot 10^{16})^2\right)^{\frac{1}{2}} J \simeq 1{,}2\,\text{TeV}$$ wow, very big...

1.6

a) $m_p = 0{,}938\,GeV$

collider: $E_{C.M.} \simeq 2E_B$ (sum of two beam energies)

collission E = 7TeV/beam

$$E_{CM} \simeq \sqrt{2\, \underset{\text{target mass}}{m_T}\, \underset{\text{beam energy}}{E_B}} \leftarrow \text{ if stationary target}$$

$$\hookrightarrow 2E_B = \sqrt{2 m_T E_R}$$ required beam energy

$$\hookrightarrow \boxed{\frac{2E_B^2}{m_T} = E_R} \quad \rightarrow E_R \sim \frac{2\cdot(7\cdot 10^{12})^2}{0{,}938\cdot 10^9}\, eV \simeq 10^{17}\, eV \sim 100\,000\ TeV..!$$

↳ The energy of a beam required to provide the same E_{CM} for stationary target case as in collider case is some 5 orders of magnitude higher!

b) $E_K = \frac{1}{2} M v^2$

E_{beam} (total) $= 1{,}15\cdot 10^{11} \cdot 7\ TeV = 8{,}05\cdot 10^{23}\, eV \simeq 12{,}9\cdot 10^4\ J$

Mini Cooper: $E_K \simeq \frac{1}{2}\cdot 1{,}3\cdot 10^3 \cdot v^2$ kg

$$\hookrightarrow v^2 \cdot \frac{1}{2} M = E_{Beam} \rightarrow v \simeq \sqrt{\frac{2E_B}{M}} = \sqrt{\frac{2\cdot 12{,}9\cdot 10^4}{1{,}3\cdot 10^3}}\ \frac{m}{s}$$

$$\simeq 14\ \frac{m}{s} = 50\ \frac{km}{h}$$

The Mini-Cooper would be travelling at ~ 50 km/h.