JAI RF Cavity Design Tutorial

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Assume you are designing the RF system for a modern accelerator. Describe the process and the steps of transferring power from the grid to the beam. Discuss technology options available and justify your choices.
Let's say that I'm designing a really fancy proton linac (like the ESS). I have done a little reserach to understand what on earth I actually need to power this thing. It comes down to a few really important things:

- Modulator. This should be a thing that takes in the regular AC power from the grid, and turns it into pulsed power. If I understand correctly, it is essentially a capacitative energy storage device which "charges up" and then releases a pulse in a short time using a fast switch.
- Amplifier. This thing takes pulse power from the modulator and aplifies it, all at a frequency of a couple of hundred MHz. Usually, this would be a klystron. I found that I can assume the klystron efficiency, the ratio of the power supplied to the klystron (as a DC-power supply) and the high-frequency power output, to be around 60%. The amplification can be somewhere between 40 and 60 dB (10^4 to 10^6 gain).
- Distribution and feedback. The pulses from klystrons need to get distributed to the the RF cavities themselves, and a feedback loop needs to be implemented in order to keep the pulsing at the correct phase and amplitude.

I need to have the klystrons run at a few hundred MW (peak power). There are some alternatives to klystrons available; solid state aplifiers seem to be more expensive and not yet very established, requiring water cooling, etc. They also seem to provide lower peak power than klystrons. There are also magnetrons which can be used instead of klystrons; these tend to be cheaper and appear to be more efficient than (I found up to 90%), however, they also seem to be limited to low peak power due to increased heating issues. These should be good for lower energy accelerators - not the fancy cutting-edge high-energy linac that **I** would be building though, obviously.

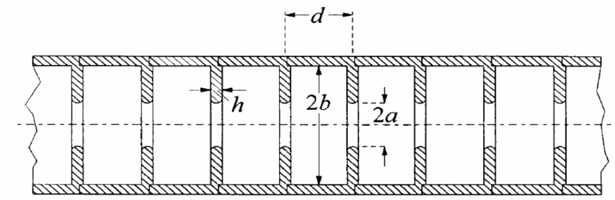


Figure 1

2

The SLAC accelerating linac structure has the design shown in figure 1 with the following dimensions:

- **2b = 82.474 mm**
- **2a = 22.606 mm**
- **h = 5.842 mm**
- **d = 35.001 mm**

Assuming that the cavities are operated in the 2/3 mode with a phase velocity z = 1 and a supplied power of 35 MW, what is the total accelerating voltage and energy gain per metre for a SLAC structure of length l = 3 m.?

To get the total energy gain per particle, I can make use of the equation

$$R_s = K\sqrt{P_{RF} l R_s} \tag{1}$$

where P_{RF} is the supplied RF power, K is a correction factor ($\simeq 0.8$), l is the length of the cavity, and R_s is the Shunt impedance calculated via the empirical formula

$$R_s = 5.12 \cdot 10^8 \frac{\beta_z(1-\eta)^2}{p + 2.61\beta_z(1-\eta)} \left(\frac{\sin D/2}{D/2}\right)^2 \tag{2}$$

Here we know that $\beta_z = \frac{v_\phi}{c} = 1$, $p = 3$ since we are operating in the 2/3 mode (one wavelength within the cavity spans over exactly 3 cells, $\lambda_z = 3d$), $\eta = h/d$ refers to the geometry of the cavity, and $D = \frac{2\pi}{p}(1-\eta)$. I plugged these numbers into a Desmos calculator and got

$$R_s = 52.93 M\Omega/m \tag{3}$$

$$U \simeq 53.64 MV \implies U/l \simeq 19.9 MV/m \tag{4}$$

The energy gan per particle per metre is about 20mV/m in this RF cavity. I did not get to use the iris and cavity widths a, b. I hope that's expected.

3

Using SuperFish, design a 10 MHz normal conducting pill-box cavity. Assuming an accelerating gradient of 3 MV/m, present your findings including the key figures of merit. What is the transit-time factor for protons of 4 GeV (kinetic energy) being accelerated with this cavity?

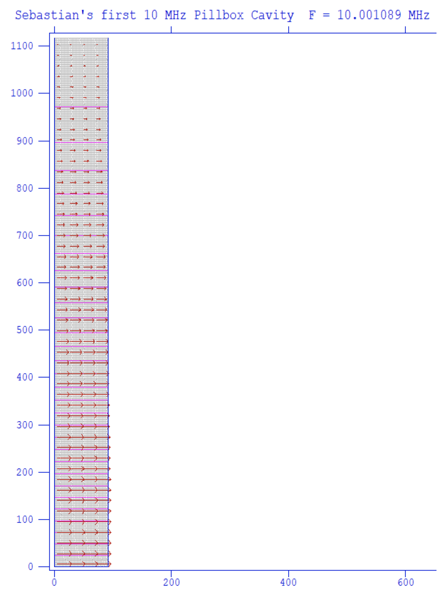
I basically just played around with the geometry of the pillbox cavity example we did in class, and found (quite logically) that if I increase the diameter of the cavity, the resonant frequency decreases. By playing around and halving diameter change intervals, I arrived at the following solution, which is pretty close:

```
-------------------------------------------------------
Sebastian's first 10 MHz Pillbox Cavity

PARTICLE H+,
$reg kprob=1,           ; Superfish problem
dx=5,                  ; X mesh spacing
freq=10.,             ; Starting frequency in MHz
icylin=1

xdri=92.,ydri=1147.3 $      ; Drive point location

$po x=0.0,y=0.0 $       ; Start of the boundary points
$po x=0.0,y=1147.3 $
$po x=92,y=1147.3 $
$po x=92,y=0.0 $
$po x=0.0,y=0.0 $
-------------------------------------------------------
```



with the following SFO output:

```
--------------------------------------------------------------------
Field normalization (NORM = 0):     EZERO =      1.00000 MV/m
Frequency                                 =     10.00109 MHz
Particle rest mass energy                 =   938.272029 MeV
Beta =  0.0613825          Kinetic energy =        1.773 MeV
Normalization factor for E0 =  1.000 MV/m =     5191.202
Transit-time factor                       =    0.0001174
Stored energy                             =  453.8802114 Joules
Using standard room-temperature copper.
Surface resistance                        =      0.82506 milliOhm
Normal-conductor resistivity              =      1.72410 microOhm-cm
Operating temperature                     =      20.0000 C
Power dissipation                         =      51.9486 kW
Q    =   549028.          Shunt impedance =       17.710 MOhm/m
--------------------------------------------------------------------
```

What confuses me is that everything seems to be normalized to 1.0 MV/m on-axis field, and I didn't figure out a way to change the scaling in superfish without moving to DTL and other modules.

4

A new 50 GeV (kinetic energy) proton synchrotron, the PS2 accelerator, has been proposed to replace the CERN PS. The new PS2 will be in a new ring tunnel of mean radius 215 m. and will receive an injected beam at 4 GeV (kinetic energy) from a new linear accelerator - the Superconducting Proton Linac (SPL). The 1.8 T magnetic field of the bending magnets is excited by a sine wave oscillating between injection and top energy at a frequency of 0.3 Hz. Given that the mass of the proton is 0.9383 GeV:

- **What is the revolution frequency at 4 GeV, 20 GeV and 50 GeV?**

The revolution frequency can be simply calculated as

$$f_r = \frac{v}{2\pi R} \tag{5}$$

where the particle velocity can be found using the relativistic γ factor computed from the particle kinetic energy E_k and its rest energy E_0

$$\gamma = \frac{E}{E_0} = 1 + \frac{E_k}{E_0} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \implies v = c\sqrt{\frac{E_k}{E_0 + E_k}} \tag{6}$$

hence

$$f_r = \frac{c}{2\pi R}\sqrt{\frac{E_k}{E_0 + E_k}} \tag{7}$$

Assuming that the energy given are E_k, we have $f_r(4\text{ GeV}) = 0.200$ MHz, $f_r(20\text{ GeV}) = 0.217$ MHz, and $f_r(50\text{ GeV}) = 0.220$ MHz

- **Assuming the revolution frequency at 20 GeV, calculate the voltage per turn necessary to match the maximum rate of the rise of the field.**

Here we need to do a few more calculations. I can use the relationship for the energy gain is provided by the RF voltage I learned in one of the longitudinal dynamics lectures:

$$\Delta E_{turn} = 2\pi Re\rho\dot{B} = eV\sin(\phi_s) \tag{8}$$

where R is the tunnel radius, ρ the bending radius, and ϕ_s the synchronous phase.

First, I need to calculate how the magnetic field changes as a function of time. I assume that $B_{max} = 1.8$ T is the magnetic field at the maximum proton energy $E_k = 50$ GeV. From here, I can use the half-empirical formula $B\rho = 3.3365p[Tm]$ where p is the particle momentum given in GeV/c. That can be calculated using the relativistic momentum-energy relation

$$E^2 = (pc)^2 + (m_0c^2)^2 \implies (E_k + E_0)^2 = (pc)^2 + E_0^2 \implies p = \frac{\sqrt{2E_0E_k + E_k^2}}{c} \tag{9}$$

This means $p_{max}(E_k = 50\text{GeV}) = 50.930$ GeV/c and thus the bending radius $\rho \simeq 94.404$ m. This value needs to remain constant, so the minimum magnetic field can be calculated from the proton momentum at the minimum kinetic energy $E_k(min) = 4$ GeV as $B_0 = \frac{3.3365p_{min}}{\rho} = \frac{3.3365\cdot 4.848}{94.404}$ T $\simeq 0.171$ T since $p_{min}(E_k = 4\text{ GeV}) = 8.848$ GeV/c, using the above equation. This means that the magnetic field changes as

$$B(t) = B_0 + \Delta B\sin(2\pi f_B t) \tag{10}$$

where $\Delta B = B_{max} - B_0 \simeq 1.629$ T and $f_B = 0.3$ Hz. This means that

$$\dot{B} = \Delta B \cdot 2\pi f_B\cos(2\pi f_B) \tag{11}$$

and

$$\dot{B}_{max} = \Delta B \cdot 2\pi f_B = 1.629\text{T} \cdot 2\pi \cdot 0.3\text{Hz} \simeq 3.07\text{ T/s} \tag{12}$$

this means that

$$\Delta E_{turn} = 4\pi^2 Re\rho\Delta B f_B = 4\pi^2 \cdot 215\text{m} \cdot 94.404\text{m} \cdot 1.629\text{T} \cdot 0.3\text{Hz [eV]} \simeq 0.392 \text{ MeV} \tag{13}$$

- **If $\sin\phi_s = \sin 60^o$, what is the peak voltage necessary in the cavity? Note that $\phi_s = 0$ corresponds to the zero crossing of the accelerating voltage and the particle is not accelerated.**

The peak voltage In the cavity can be calculated using $\Delta E_{turn} = 2\pi Re\rho\dot{B} = eV\sin(\phi_s)$ as well:

$$V_{peak} = \frac{2\pi R\rho\dot{B}_{max}}{\sin\phi_s} = \frac{4\pi}{\sqrt{3}}R\rho\dot{B}_{max} = 261 \text{ kV} \tag{14}$$

- **Given a harmonic number of 32, what are the RF frequencies at 4 GeV and 50 GeV for $\phi_s = 0$?**

Acceleration of protons increases their revolution frequency, and thus the RF frequency has to follow this development. We know that $f_{RF} = h \cdot f_r$ where $h = 32$ is the harmonic number. Using the previously calculated revolution frequencies, we get

$$f_{RF}(4 \text{ GeV}) = 32 \cdot f_r(4 \text{ GeV}) = 32 \cdot 0.200 \text{ MHz} = 6.4 \text{ MHz} \tag{15}$$

$$f_{RF}(50 \text{ GeV}) = 32 \cdot f_r(50 \text{ GeV}) = 32 \cdot 0.220 \text{ MHz} = 7.04 \text{ MHz} \tag{16}$$