Math Modeling 2021 Project Proposal
Heart Rate Modeling at Critical Running Pace

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1 Introduction & goals of the project

As a runner, I've been watching my own heart rate data on almost daily basis. It has been a key indicator of the state of my physique during both races and practice runs. I have noticed that the maximum heart rate that I can bear while running, without feeling the need to stop or slow down, is somewhere around 190 bpm. Whenever I reach that limit, I experience what runners call "hitting a wall", which essentially means the shutting down of my ability to control the pace with morale - the body says no, and the mind cannot go over it. A great example of this comes from a 10k race I did in March 2021 in Abu Dhabi. My threshold pace at that time (threshold meaning the highest pace that one can sustain for over 30 minutes) was exactly 4:00 min/km (15.0km/h) - and the goal was thus to run an under-40min 10k race. Figure 1 shows the heart rate, speed, and cadence data recorded on my Garmin watch during that race. With the speed being constant (which is fairly hard to achieve, to be honest), the heart rate follows some gradually increasing function, ultimately hitting the 190 bpm limit a couple of hundred meters before the finish line. There, I had to stop for a couple of seconds (hence the drop in running cadence), let the heart rate drop a bit, and then continue. Having a rich library of my own heart rate data on hand, I'd like to delve deeper into the analysis of how the heart rate evolves during such an exercise, possibly looking for some parameter that would help me plan a bit better for the next race.

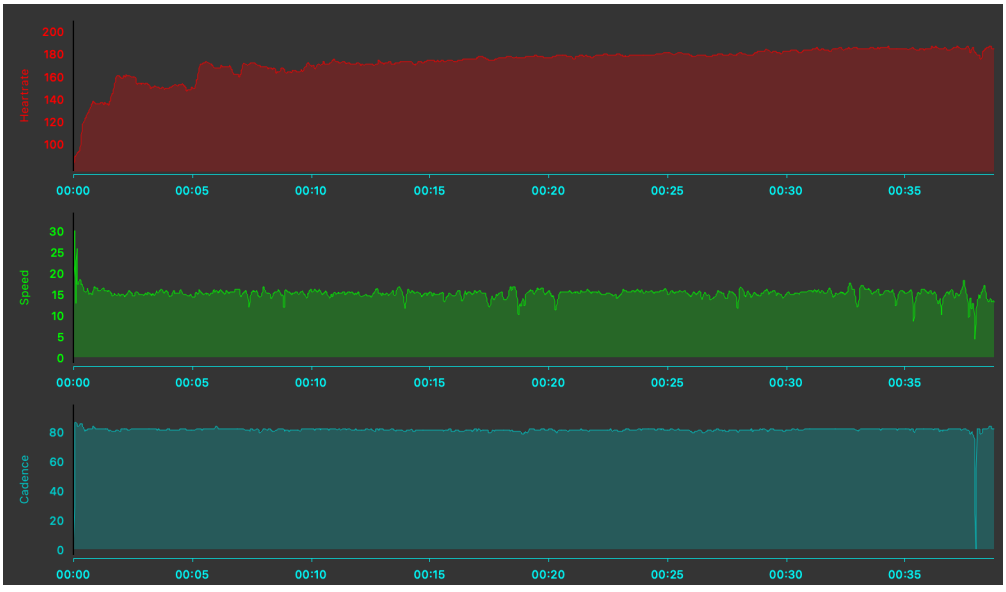

Figure 1:

2 Methods

I have found several papers that deal with heart rate modeling, and two particularly interesting ones [1, 2] propose a system of non-linear ODEs applicable to running exercises:

$$\begin{aligned} \dot{x}_1 &= -a_1 x_1(t) + a_2 x_2(t) + a_2 u^2(t) \\ \dot{x}_2 &= -a_3 x_2(t) + \frac{a_4 x_1(t)}{1 + e^{-(x1(t) - a_5)}} \end{aligned} \tag{1}$$

where $u(t)$ is the running speed at time t, a_1,a_2, etc. are parameters that need to be estimated (fitted with real data, perhaps) and interpreted, x_1 being the deviation of heart rate from the resting heart rate, and x_2 being a variable that reflects the wearing-out of the bodily physique during a prolonged exercise. I believe that i will be able to explain the x_2 variable in greater detail upon delving deeper into the model.

The initial phase of the project includes analyzing the model in greater detail, fitting my own personal data to it, and finding and interpreting of the entering parameters. It would be great to have a clear interpretation of each parameter at the end, knowing which alters the heart rate evolution in the most effective manner. In other words, I believe that each parameter will bear some connection to either some bodily function (such as metabolic rate, rate of oxygen intake, etc.) or the level of some kind of fitness level a running person is at. I'd like to identify which parameter to tweak (should the tweaking be done through some specific training or other preparation) so as not to "hit a wall" next time I'm racing the 40-minute mark. Acknowledging that this is a very broad goal, I believe it is a good starting point, and that the level of achievable analysis will reveal itself naturally.

References

[1] Teddy M Cheng et al. "Nonlinear modeling and control of human heart rate response during exercise with various work load intensities". In: *IEEE Transactions on biomedical engineering* 55.11 (2008), pp. 2499–2508.

[2] Xiaoli Liu et al. "Predicting the heart rate response to outdoor running exercise". In: *2019 IEEE 32nd International Symposium on Computer-Based Medical Systems (CBMS)*. IEEE. 2019, pp. 217–220.