

# COM1029 Data Structures and Algorithms

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## 1 Algorithms

### 1.1 History of Algorithms

My boy Muhammad ibn Musa al-Khwarizmi was the first to write a book on the systematic solution of linear and quadratic equations. He was a Persian mathematician, astronomer, and geographer during the Abbasid Caliphate, a scholar in the House of Wisdom in Baghdad. He was the first to introduce the concept of algorithm to the Western world. The word algorithm comes from the Latin word *algorismus*, which is a Latinization of his name. He is also known for his work on algebra, which is derived from the Arabic word *al-jabr*.

### 1.2 Algorithm analysis

#### 1.2.1 Time complexity

Example (find the average of an array of  $n$  integers)

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```
% find the average of an array of n integers
int sum = 0;                // 1
for (int i = 0; i < n; i++) { // n
    sum += i;                // n
}
return sum / n;              // 1
```

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Quadratic example:

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```
% find the sum of an array of n integers
int[] a = someArray;        // 1
int sum = 0;                  // 1
for (int i = 0; i < len(a); i++) { // n
    for (int j = 0; j < len(a[i]); j++) { // n^2
        sum += a[i][j];           // n^2
    }
}
```

---

```
}  
return sum;           // 1
```

---

### 1.2.2 Dominant term

The dominant term is the one with highest power (degree) in a function

Example: the cubic function  $f(N) = 10N^3 + N^2 + 40N + 80$

- $10N^3$  is the dominant term (among  $10N^3$ ,  $N^2$ ,  $40N$  and  $80$ )
- When we look at  $f(1000) = 10,001,00,080$ , we see that  $10,000,000,000$  is due to the  $10N^3$  term
- If we use the approximation  $f(N) = 10N^3$ , then we would only be 0.01% out

The value of  $f(N)$  is largely determined by the dominant term, for sufficiently large  $N$

- The meaning of 'sufficiently large' varies according to the function

### 1.2.3 Big O notation

Compared to evaluating the dominant term, Big O notation is a more general way of expressing the time complexity of an algorithm. It is a way of expressing the upper bound of a function.

Example:  $f(N) = 10N^3 + N^2 + 40N + 80$

- $f(N)$  is  $O(N^3)$
- $f(N)$  is  $O(N^2)$
- $f(N)$  is  $O(N)$
- $f(N)$  is  $O(1)$