Exercise 1

Let $L_1 = \{aa, bb, bbb\}$ and $L_2 = \{abba, aab, bb\}$ be two languages over the alphabet $\Sigma = \{a, b\}$. What are the strings of the following languages?

1. $L_1 \circ L_2$
2. $L_1 \cup L_2$
3. $L_1 \cap L_2$
4. Provide a few strings of L_2^*

1: {aaabba, aaaab, aabb, bbabba, bbaab, bbbb, bbbabba, bbbaab, bbbbb}

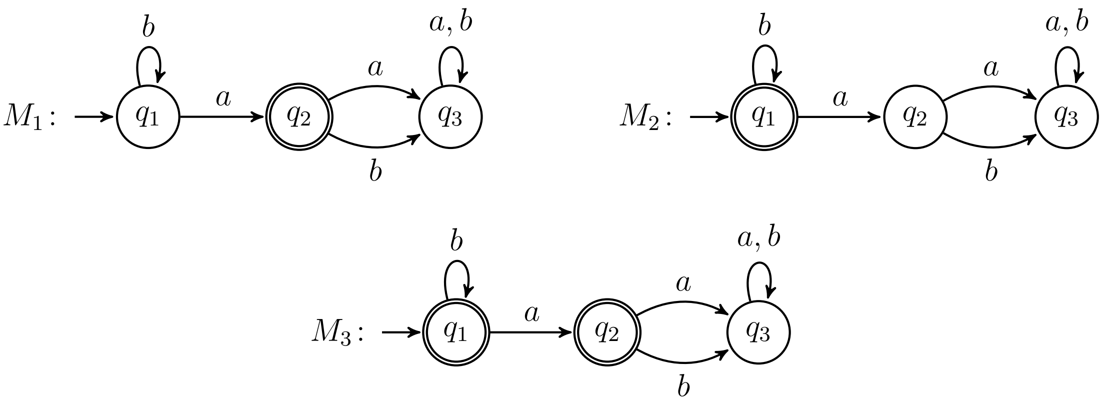
2: {aa, bb, bbb, abba, aab, bb}

3: Ø eller empty string?

4: {ε, abba, aab, bb, bbbb, aabbb, abbabb, bbaab, bbabba ...}

Exercise 2

Given the following three automata M_1, M_2, and M_3



1. Describe the sequence of states that M_1 goes through while reading the inputs:
   (a) $abbbab$
   (b) $ababaab$
   (c) $aaaaa$
   (d) ε
2. Which of the previous sequences are accepting in M_1, M_2 and M_3?
3. Describe the languages recognised by each of the three automata.

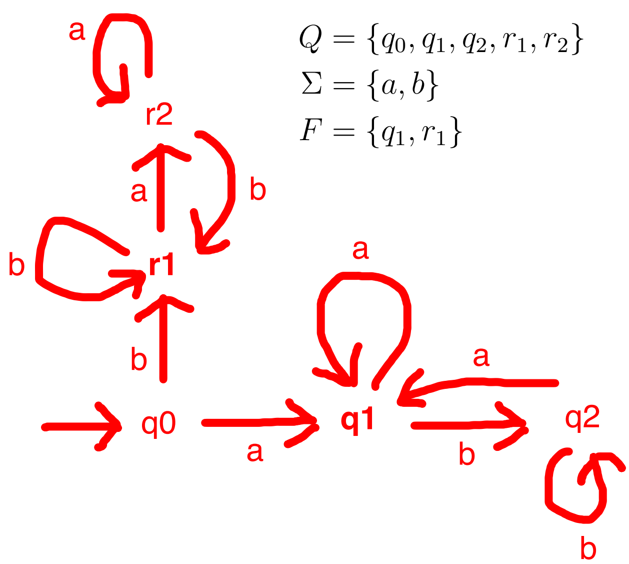
1:
a: q1 -> q3 -> q3 -> q3 -> q3 -> q3 -> q3
b: q1 -> q2 -> q3 -> q3 -> q3 -> q3 -> q3 -> q3
c: a1 -> q2 -> q3 -> q3 -> q3 -> q3
d: q1

2:
M1: d
M2: d
M3: d

3:
L(M1)={w I only b's and ending with a or only 1 a}
L(M2)={w I only symbols b}
L(M3)={w I only b's and ending with a or only 1 a or only symbols b}

Exercise 3*

Provide the state diagram of the automaton $M_4 = (Q, \Sigma, \delta, q_0, F)$ given below and describe its language.



$Q = \{q_0, q_1, q_2, r_1, r_2\}$
$\Sigma = \{a, b\}$
$F = \{q_1, r_1\}$

δ	a	b
q_0	q_1	r_1
q_1	q_1	q_2
q_2	q_1	q_2
r_1	r_2	r_1
r_2	r_2	r_1

a, a, a, a, a, a

b, b, b, b, b, b

b, a, a, a, a, b

a, b, b, b, b, a

Starting with b it must end with b

Starting with a it must end with a

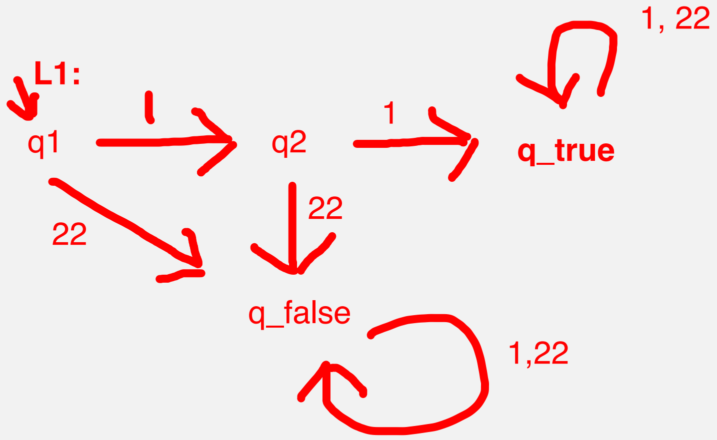
accept states are marked in bold (q1, r1)

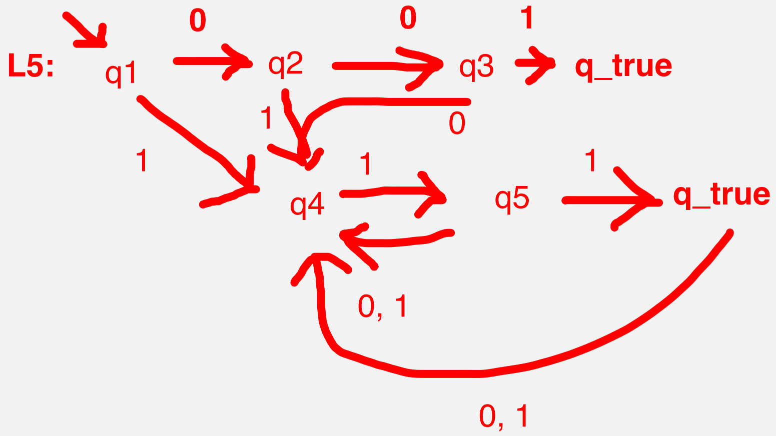
L(M4)={w I w starts and ends with same letter}

Exercise 4*

For each of the following languages, draw the state diagram of an automaton recognizing it.

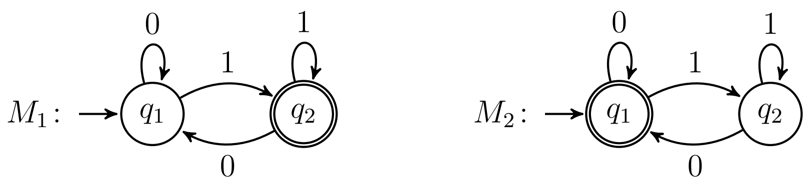
1. $L_1 = \{w \in \{1, 22\}^* \mid w \text{ has } 11 \text{ as prefix}\}$ **$\{1, 22\}^* = \{\varepsilon, 11122, 11221111, ...\}$**
2. $L_2 = \emptyset \subseteq \{0, 1, 2\}^*$
3. $L_3 = \{\varepsilon\} \subseteq \{0, 1, 2\}^*$
4. $L_4 = \{w \in \{\texttt{go}, \texttt{stop}\}^* \mid w = \varepsilon \text{ or ends with } \texttt{stop}\}$
5. $L_5 = \{w \in \{0, 1\}^* \mid w \text{ has } 001 \text{ as prefix or } 11 \text{ as suffix}\}$


L1:
q1
1
q2
1
q_true
1, 22
22
22
q_false
1,22


L5:
q1
0
q2
0
q3
1
q_true
1
1
0
q4
1
q5
1
q_true
0, 1
0, 1

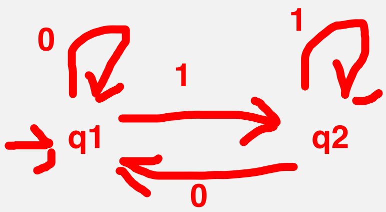
Exercise 5*

Consider the automata M_1 and M_2 given below.



1. Construct an automaton that recognizes the language $L(M_1) \cap L(M_2)$;
2. Construct an automaton for each of the following languages:
   (a) $\overline{L(M_1)}$
   (b) $\overline{L(M_2)}$
   (c) $\overline{L(M_1) \cap L(M_2)}$

1:



m1: ending with 1
m2: ending with 0

q1 and q2 are both acceptable!

2:

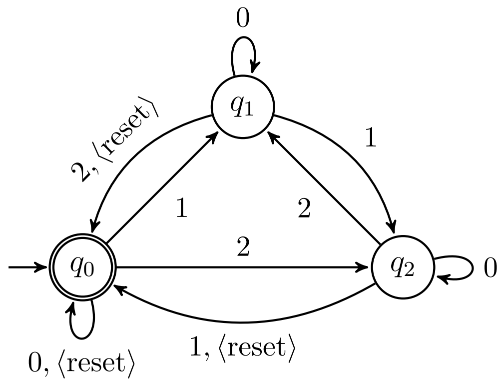
a): L(M2)
b): L(M1)
c): ε

Exercise 6

1. Prove that regular languages are closed under intersection.
   (Hint: Similar construction to the one for the union, only that $F = F_1 \times F_2$);
2. Prove that regular languages are closed under complement.
   (Hint: Transitions don't chance. But, what should be now the accepting states?).

Exercise 7*

Let M be the following automaton over the alphabet $\Sigma = \{0, 1, 2, \langle\text{reset}\rangle\}$.



1. Give examples of accepted and non-accepted words (at least five for each).

2. Define $count(w)$ to be the sum of the numerical symbols in a string $w \in \Sigma^*$. Prove that $L(M) = \{w \mid count(w) = 0 (\text{mod } 3)\}$.
(Hint: Note that M keeps a running count of the sum of the numerical input symbols it reads, modulo 3. Every time it receives the $\langle\text{reset}\rangle$ symbol it resets the count to 0).

1:
Accepted: {111, 12, 222, 2121, 2211 }
Non-accepted: {11111, 1111, 221, 22221, 11121211}

It only accepts if sum mod 3 is 0

2:
Hader proofs

Exercise 8 (optional)

Generalize the automaton M from Exercise 7 so that $L(M) = \{w \mid \mathit{count}(w) = 0 (\mathrm{mod}\ 4)\}$.