# Earthquake Shaking of Multistory Buildings

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**Abstract**

Hooke's law, Newton's second law of motion, matrices, vectors, differential equations, eigenvalues, and eigenvectors all contribute to modeling the effects of an earthquake on multistory buildings. In this paper, we aim to determine and graph resonance frequencies that will put a multistory building in danger of destruction.

## 1 Introduction

An earthquake is one of the most devastating natural disasters that result from two pieces of Earth's crust slipping past one another. An earthquake is "a sudden release of strain energy in the Earth's crust, resulting in waves shaking that radiate outwards from the earthquake source," according to the British Geological Survey [1]. The surface where they slip is called the fault or fault plane. When the edges of the faults are stuck together, the rest of the block is still in motion. Potential energy is stored up, and once the force overcomes the friction of the jagged edges, stored-up potential energy is released. The energy radiates outwards in all directions through seismic waves, which a seismometer can measure. These waves disperse through Earth's layers, eventually reaching the surface and forcing buildings and other structures to shake. Unpredictable by nature, these sudden strains have the ability to level buildings with a high enough magnitude [4]. With an estimated 350 building-damaging earthquakes occurring annually across the globe, knowing when it's necessary to evacuate is critical [3]. Measuring and analyzing frequencies of earthquakes and multistory buildings can help determine when a building needs to be evacuated. By modeling, solving, and interpreting the mathematics behind earthquake effects on multi-story buildings, we will determine when floors of multi-story buildings are expected to experience the highest oscillations.

# 2 Background

## 2.1 Resonance Frequency

Frequency can be described as the number of waves that pass a fixed point in unit time. Higher frequencies have many waves in a short amount of time, meaning they have a short period wavelength. Lower frequencies are the opposite and have waves with a long-period wavelength. The resonance frequency describes the natural frequency where an object vibrates at a higher peak amplitude.

The Incorporated Research Institutions for Seismology (IRIS) found that tall and large buildings tend to have low resonant frequencies, whereas short and small buildings tend to have high resonant frequencies. Specifically, they found that every ten stories correspond to an average resonant frequency period of one-second [2]. Matching resonance frequencies maximizes vibration. Thus, if the period of ground motion matches the natural resonance of a building, the building will undergo the largest oscillations possible and suffer the most damage.

While our model didn't account for ground material, an improved future model might. It was determined that soft sediment has a low resonant frequency, whereas hard bedrock has a high resonant frequency. Therefore, structures should be placed on grounds with opposing resonance frequencies to minimize earthquake damage. In other words, tall or large buildings should be built on hard bedrock, and short or small buildings should be built on soft sediment. In our model, a building is deemed safe if its resonance frequency has a dissimilar period to the earthquake's frequency [2].

## 2.2 Hooke's Law

Herein we adapt a model as presented in Zill and Wright [5], which is described for completeness. First, we define $x_i$ as the horizontal displacement of the $i$th floor from equilibrium. Since the equilibrium position is a fixed point on the ground, $x_0 = 0$. Second, we will assume that the $i$th floor has mass $m_i$ with each successive floor connected by an elastic connector with effects resembling a spring. Hooke's law, which states $F_s = -kx$, where $F_s$ is the spring force, $k$ is the spring constant, and $x$ is the spring stretch or compression, defines our spring system.

Applying Hooke's Law to our model, we get the following where $i$ is the restoring forces between two floors, and $k_i$ is our proportionality spring constant between the $i$th and $(i+1)$th floors.

$$F = k_i(x_{i+1} - x_i)$$

Here, $x_{i+1} - x_i$ is the displacement or shift of the $(i+1)$th floor relative to the $i$th floor. We assume the proportionality constant between the first floor and the ground is $k_0$. See Figures 1 and 2 below.

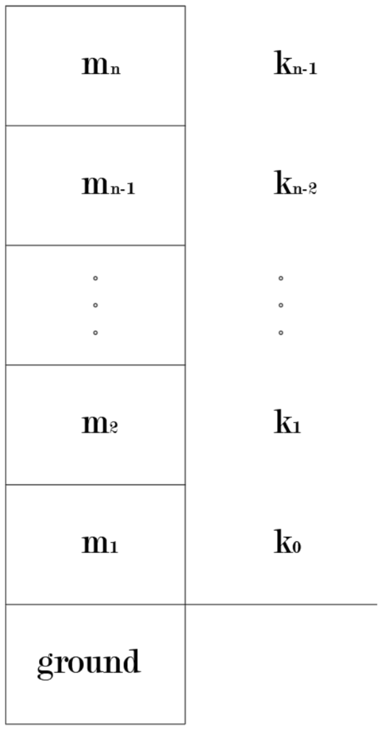


Figure 1: Floors of building

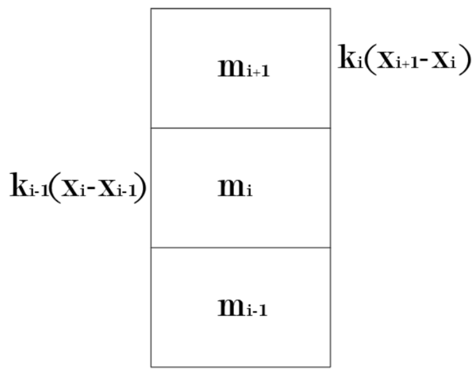


Figure 2: Forces of ith floor

## 2.3 Newton's Second Law

Newton's second law of motion states $F = ma$, where $F$ is force, $m$ is mass, and $a$ is acceleration. Applying this law to each floor of our model, the following linear differential equations can be derived.

$$
\begin{aligned}
m_1 \frac{d^2 x_1}{dt^2} &= -k_0 x_1 + k_1(x_2 - x_1) \\
m_2 \frac{d^2 x_2}{dt^2} &= -k_1(x_2 - x_1) + k_2(x_3 - x_2) \\
. \qquad & \qquad . \\
m_i \frac{d^2 x_i}{dt^2} &= -k_{i-1}(x_i - x_{i-1}) + k_i(x_{i+1} - x_i) \\
. \qquad & \qquad . \\
. \qquad & \qquad . \\
. \qquad & \qquad . \\
m_n \frac{d^2 x_n}{dt^2} &= -k_{n-1}(x_n - x_{n-1})
\end{aligned}
$$

Above, $m_i$ represents each successive floor after the first, and $m_n$ represents the top floor. Solving and graphing our system will reveal the effect of each floor's displacement on the next.

# 3 Assumptions

## 3.1 Matrices and Vector

The following matrices and vector define all floors for a multistory building with $n$ floors.

$$\mathbf{M} = \begin{pmatrix} m_1 & 0 & 0 & \cdots & 0 \\ 0 & m_2 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & m_n \end{pmatrix}$$

$$\mathbf{K} = \begin{pmatrix} -(k_0+k_1) & k_1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ k_1 & -(k_1+k_2) & k_2 & 0 & \cdots & 0 & 0 & 0 \\ 0 & k_2 & -(k_2+k_3) & k_3 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & k_{n-2} & -(k_{n-2}+k_{n-1}) & k_{n-1} \\ 0 & 0 & 0 & 0 & \cdots & 0 & k_{n-1} & -k_{n-1} \end{pmatrix}$$

$$\mathbf{X}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{pmatrix}$$

$\mathbf{M}$ is the mass matrix, $\mathbf{K}$ is the stiffness matrix, and $\mathbf{X}(t)$ is a vector with $x_n(t)$ values. Thus, we can write our system of differential equations defined earlier in matrix form

$$\mathbf{M}\frac{d^2\mathbf{X}}{dt^2} = \mathbf{KX} \quad \text{or} \quad \mathbf{MX}'' = \mathbf{KX}$$

Observe that the mass matrix is a diagonal matrix with the mass of the $i$th floor being the $i$th diagonal element. Therefore, the mass matrix's inverse is easily calculated below.

$$\text{Inverse Mass Matrix}$$
$$\mathbf{M}^{-1} = \begin{pmatrix} m_1^{-1} & 0 & 0 & \cdots & 0 \\ 0 & m_2^{-1} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & m_n^{-1} \end{pmatrix}$$

The following can represent the second-order matrix differential equation

$$\begin{aligned} \mathbf{X}'' &= (\mathbf{M}^{-1}\mathbf{K})\mathbf{X} \\ \mathbf{A} &= \mathbf{M}^{-1}\mathbf{K} \\ \therefore \mathbf{X}'' &= \mathbf{AX} \end{aligned}$$

## 3.2 Eigenvalues and Eigenvectors

The eigenvalues of $\mathbf{A}$ reveal the stability of the building during an earthquake. The eigenvalues of $\mathbf{A}$ are negative and distinct. The natural resonance frequencies of a building are the square roots of the negatives of the eigenvalues. If $\lambda_i$ is the $i$th eigenvalue, $\omega_i = \sqrt{-\lambda_i}$ is the $i$th frequency, for $i = 1, 2, \ldots, n$.

# 4 Calculations

## 4.1 Solving Differential Equations

The first model will be a notional three-story building. Assuming each floor has mass $m = 5,000kg$ and restoring force constant $k = 10000\frac{kg}{s^2}$ [5], we conduct the following calculations to reveal a differential equation for each floor.

$$
\begin{aligned}
m_1\frac{d^2x_1}{dt^2} &= -k_0x_1 + k_1(x_2 - x_1) \\
m_2\frac{d^2x_2}{dt^2} &= -k_1(x_2 - x_1) + k_2(x_3 - x_2) \\
m_3\frac{d^2x_3}{dt^2} &= -k_2(x_3 - x_2)
\end{aligned}
$$

Plugging in $m$ and $k$ we get:

$$
\begin{aligned}
5000\frac{d^2x_1}{dt^2} &= -10000x_1 + 10000(x_2 - x_1) \\
5000\frac{d^2x_2}{dt^2} &= -10000(x_2 - x_1) + 10000(x_3 - x_2) \\
5000\frac{d^2x_3}{dt^2} &= -10000(x_3 - x_2)
\end{aligned}
$$

Solving the first differential equation, we get:

$$
\begin{aligned}
5000\frac{d^2x_1}{dt^2} &= -10000x_1 + 10000(x_2 - x_1) \\
\frac{d^2x_1}{dt^2} &= \frac{-10000x_1 + 10000(x_2 - x_1)}{5000} \\
\frac{d^2x_1}{dt^2} &= \frac{-10000x_1 + 10000x_2 - 10000x_1}{5000} \\
\frac{d^2x_1}{dt^2} &= \frac{-20000x_1 + 10000x_2}{5000} \\
\frac{d^2x_1}{dt^2} &= -4x_1 + 2x_2
\end{aligned}
$$

Solving the second differential equation, we get:

$$\begin{aligned}
5000\frac{d^2x_2}{dt^2} &= -10000(x_2 - x_1) + 10000(x_3 - x_2) \\
\frac{d^2x_2}{dt^2} &= \frac{-10000x_2 + 10000x_1 + 10000x_3 - 10000x_2}{5000} \\
\frac{d^2x_2}{dt^2} &= \frac{-20000x_2 + 10000x_1 + 10000x_3}{5000} \\
\frac{d^2x_2}{dt^2} &= -4x_2 + 2x_1 + 2x_3
\end{aligned}$$

Solving the third differential equation, we get:

$$\begin{aligned}
5000\frac{d^2x_3}{dt^2} &= -10000(x_3 - x_2) \\
\frac{d^2x_3}{dt^2} &= \frac{-10000(x_3 - x_2)}{5000} \\
\frac{d^2x_3}{dt^2} &= \frac{-10000x_3 + 10000x_2}{5000} \\
\frac{d^2x_3}{dt^2} &= -2x_3 + 2x_2
\end{aligned}$$

Therefore, the final three differential equations are

$$\begin{aligned}
\frac{d^2x_1}{dt^2} &= -4x_1 + 2x_2 \\
\frac{d^2x_2}{dt^2} &= -4x_2 + 2x_1 + 2x_3 \\
\frac{d^2x_3}{dt^2} &= -2x_3 + 2x_2
\end{aligned}$$

## 4.2 Solving Matrices

Referencing previously defined assumptions, the following matrices are obtained for a three-story building.

$$\mathbf{M} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \qquad \mathbf{K} = \begin{pmatrix} -(k_0 + k_1) & k_1 & 0 \\ k_1 & -(k_1 + k_2) & k_2 \\ 0 & k_2 & -k_2 \end{pmatrix}$$

$$\mathbf{M}^{-1} = \begin{pmatrix} \frac{1}{m_1} & 0 & 0 \\ 0 & \frac{1}{m_2} & 0 \\ 0 & 0 & \frac{1}{m_3} \end{pmatrix}$$

Substituting our $m$ and $k$ values into the three-story building matrices, we get

$$\mathbf{M} = \begin{pmatrix} 5000 & 0 & 0 \\ 0 & 5000 & 0 \\ 0 & 0 & 5000 \end{pmatrix}$$

$$\mathbf{M}^{-1} = \begin{pmatrix} \frac{1}{5000} & 0 & 0 \\ 0 & \frac{1}{5000} & 0 \\ 0 & 0 & \frac{1}{5000} \end{pmatrix}$$

$$\mathbf{K} = \begin{pmatrix} -(10000 + 10000) & 10000 & 0 \\ 10000 & -(10000 + 10000) & 10000 \\ 0 & 10000 & 10000 \end{pmatrix}$$

$$= \begin{pmatrix} -20000 & 10000 & 0 \\ 10000 & -20000 & 10000 \\ 0 & 10000 & 10000 \end{pmatrix}$$

Next, we must calculate the **A** matrix, defined earlier as $\mathbf{A} = M^{-1}K$.

$$\mathbf{A} = \begin{pmatrix} -\dfrac{k_0 + k_1}{m_1} & \dfrac{k_1}{m_1} & 0 \\ \dfrac{k_1}{m_2} & -\dfrac{k_1 + k_2}{m_2} & \dfrac{k_2}{m_2} \\ 0 & \dfrac{k_2}{m_3} & -\dfrac{k_2}{m_3} \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} -\dfrac{20000}{5000} & \dfrac{10000}{5000} & 0 \\ \dfrac{10000}{5000} & -\dfrac{20000}{5000} & \dfrac{10000}{5000} \\ 0 & \dfrac{10000}{5000} & -\dfrac{10000}{5000} \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} -4 & 2 & 0 \\ 2 & -4 & 2 \\ 0 & 2 & -2 \end{pmatrix}$$

It should be observed that **A** has the lead coefficients of our differential equation. Using **A**, we can now begin calculating our general solution for our system of differential equations.

## 4.3 Solving Eigenvalues and Eigenvectors

To find the eigenvalues, we take **A** and subtract $\lambda * \mathbf{I}$, where **I** is the identity matrix. Next, we take the determinate and set it equal to zero, then solve for $\lambda$. i.e

$$\det[(\mathbf{A} - \lambda\mathbf{I})] = 0$$

Each $\lambda_1, \lambda_2, \lambda_3, \ldots \lambda_n$ represents an eigenvalue. After finding all $\lambda$ values, we will substitute the values in the equation $\mathbf{AV}_i = \lambda_i$ or $(\mathbf{A} - \lambda_i * \mathbf{I})\mathbf{V}_i = 0$. Then, we calculate each eigenvector **V** using each corresponding eigenvalue

$\lambda$. Verifying our calculations with Mathematica, we get the following values.

$$\lambda_1 \approx -6.49396$$
$$\lambda_2 \approx -3.10992$$
$$\lambda_3 \approx -0.396125$$
$$\mathbf{V}_1 \approx \begin{pmatrix} 1.80194 \\ -2.24698 \\ 1 \end{pmatrix}$$
$$\mathbf{V}_2 \approx \begin{pmatrix} -1.24698 \\ -0.554958 \\ 1 \end{pmatrix}$$
$$\mathbf{V}_3 \approx \begin{pmatrix} 0.445042 \\ 0.801938 \\ 1 \end{pmatrix}$$

When an earthquake occurs, a large horizontal force is applied to the first floor. Our model implies that each floor is oscillatory in nature and can therefore be modeled with sine and cosine functions. Thus, the following equation can be used to model shaking

$$\mathbf{X} = \mathbf{V}\cos(\omega t) \quad \text{and} \quad \mathbf{X} = \mathbf{V}\sin(\omega t).$$

$\mathbf{V}$ represents a column matrix of constants. Substituting one of the functions above into $\mathbf{X}'' = \mathbf{A}\mathbf{X}$ results in $(\mathbf{A} + \omega^2\mathbf{I})\mathbf{V} = 0$. Earlier, we defined $\omega_i = \sqrt{-\lambda_i}$, meaning $\lambda = -\omega^2$. $\lambda$ represents an eigenvalue of $\mathbf{A}$ with corresponding eigenvector $\mathbf{V}$. The eigenvalues $\lambda_i = -\omega_i^2$ where $i = 1, 2$ of $\mathbf{A}$ are negative. Therefore, $\omega_i = \sqrt{-\lambda_i}$ is a real number representing a circular vibration frequency. By superposition of solutions, the general solution of $\mathbf{X}''$ is

$$\begin{aligned}\mathbf{X} &= c_1\mathbf{V}_1\cos(\omega_1 t) + c_2\mathbf{V}_1\sin(\omega_1 t) + c_3\mathbf{V}_2\cos(\omega_2 t) + c_4\mathbf{V}_2\sin(\omega_2 t) + c_5\mathbf{V}_3\cos(\omega_3 t) \\ &\quad + c_6\mathbf{V}_3\sin(\omega_3 t) \\ &= \Big(c_1\cos(\omega_1 t) + c_2\sin(\omega_1 t)\Big)\mathbf{V}_1 \\ &\quad + \Big(c_3\cos(\omega_2 t) + c_4\sin(\omega_2 t)\Big)\mathbf{V}_2 + \Big(c_5\cos(\omega_3 t) + c_6\sin(\omega_3 t)\Big)\mathbf{V}_3\end{aligned}$$

where $\mathbf{V}_i$ is a real eigenvector corresponding to $\lambda_i$, since there are three of them after all. The result given above generalizes. If $\omega_1^2, \omega_2^2, \omega_3^3, \ldots \omega_n^2$

are distinct negative eigenvalues and $\mathbf{V}_1, \mathbf{V}_2, \dots, \mathbf{V}_n$, are corresponding real eigenvectors of the $n\times n$ coefficient matrix $\mathbf{A}$. Thus, the homogeneous second-order system $\mathbf{X}'' = \mathbf{A}\mathbf{X}$ has general solution

$$\mathbf{X} = \sum_{i=1}^{n} (c_{2i-1}\cos(\omega_i t) + c_{2i}\sin(\omega_i t))\mathbf{V}_i,$$

where the $c_i$ represent arbitrary constants.

### 4.3.1 Calculating Omega

As defined above, the $\omega$ values are calculated by taking the square root of the negative eigenvalue. Hence, our numerical $\omega$ values are

$$\begin{aligned} \omega_1 &\approx 2.54832 \\ \omega_2 &\approx 1.7635 \\ \omega_3 &\approx 0.629384 \end{aligned}$$

With these $\omega$ values, we now have all the information to write a general solution and calculate the arbitrary constants as shown below.

$$\begin{aligned} \mathbf{X} = &\Big(c_1\cos(\omega_1 t) + c_2\sin(\omega_1 t)\Big)\mathbf{V}_1 \\ &+ \Big(c_3\cos(\omega_2 t) + c_4\sin(\omega_2 t)\Big)\mathbf{V}_2 \\ &+ \Big(c_5\cos(\omega_3 t) + c_6\sin(\omega_3 t)\Big)\mathbf{V}_3 \\ \mathbf{X} = &\Big(c_1\cos(2.54832t) + c_2\sin(2.54832t)\Big)\begin{pmatrix} 1.80194 \\ -2.24698 \\ 1 \end{pmatrix} \\ &+ \Big(c_3\cos(1.7635t) + c_4\sin(1.7635t)\Big)\begin{pmatrix} -1.24698 \\ -0.554958 \\ 1 \end{pmatrix} \\ &+ \Big(c_5\cos(0.629384t) + c_6\sin(0.629384t)\Big)\begin{pmatrix} 0.445042 \\ 0.801938 \\ 1 \end{pmatrix} \end{aligned}$$

### 4.3.2 Calculating C Values

In order to calculate $c_1$ through $c_6$, we set up initial conditions for $\mathbf{X}(0)$ and $\mathbf{X}'(0)$ choosing the following.

$$\mathbf{X}(0) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{X}'(0) = \begin{pmatrix} 1 \\ 3 \\ 7 \end{pmatrix}$$

In this case, the $\mathbf{X}'(0)$ will have the first-floor displacement of one unit, where the second floor will have a displacement three times the first floor, and the third floor will have a displacement of seven times the first floor. Inputting our initial conditions into Mathematica resulted in the following $c$ values

$$\begin{aligned} c_1 &= 0 \\ c_2 &= 0.0870025 \\ c_3 &= 0 \\ c_4 &= 0.80973 \\ c_5 &= 0 \\ c_6 &= 8.5009 \end{aligned}$$

Inputting our calculated eigenvectors, c-values, and omega values, the solution for our function is calculated below.

$$\begin{aligned} \mathbf{X} = 0.0870025 \sin(2.54832t) \begin{pmatrix} 1.80194 \\ -2.24698 \\ 1 \end{pmatrix} + 0.80973 \sin(1.7635t) \begin{pmatrix} -1.24698 \\ -0.554958 \\ 1 \end{pmatrix} \\ + 8.5009 \sin(0.629384t) \begin{pmatrix} 0.445042 \\ 0.801938 \\ 1 \end{pmatrix} \end{aligned}$$

# 5 Results and Analysis

Using various initial conditions, we used Mathematica to construct the following graphs. All models plot time t, measured in seconds, and floor displacement measured in arbitrary units. Both units can be expressed as needed based on initial condition units. See Section 8 for the Mathematica code for constructing all models.

## 5.1 Three-Story Building

### 5.1.1 Example 1

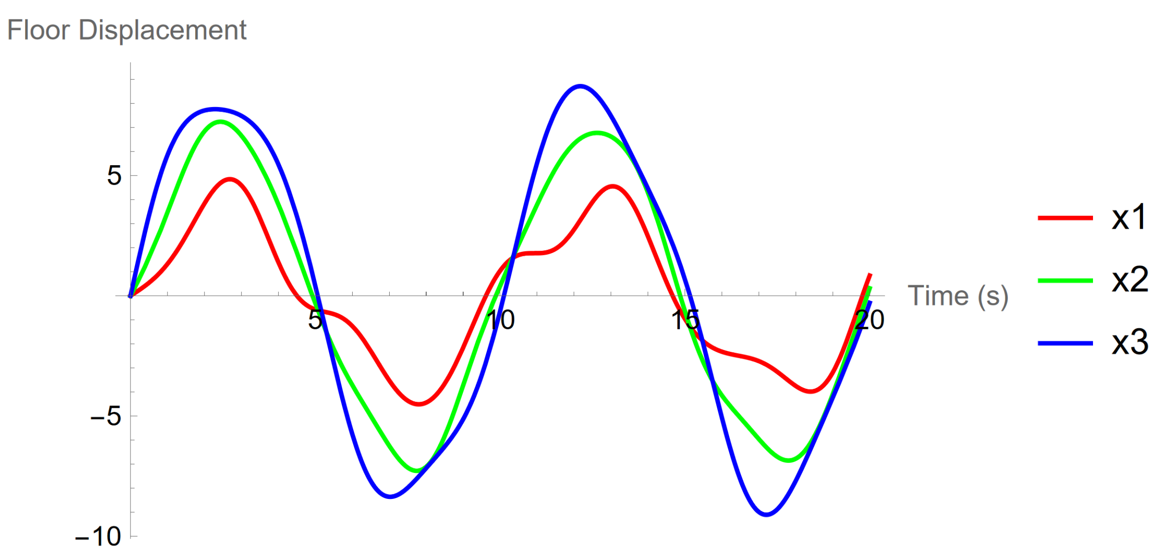


Figure 3: 3 Story Building

With initial conditions $\mathbf{X}' = (1, 3, 7)$ and $0 \leq t \leq 20$, Figure 3 plots all floors for our three-story model. As expected, the top floor experienced the most displacement, the bottom floor experienced the least displacement, and the middle floor had displacement between the two. Because our building was only three stories, the total overall displacement was minimal. Shorter buildings are more rigid and therefore have lower $\mathbf{K}$ values. As $\mathbf{K}$ increases, a building becomes less stiff, its resonance frequency period increases (becomes a lower frequency), and it tolerates larger oscillations or floor displacement before collapsing. In other words, as a building's number of stories increases, it can withstand higher frequencies but is more vulnerable to lower frequencies with longer periods, as we'll see in the 33-story building. This agrees with our initial resonance research from IRIS.

Visually, we can see that our wavelength period is roughly 10 seconds, which is verified by dividing $2\pi$ by our smallest omega value, $\omega_3$. $\frac{2\pi}{0.629384} \approx 9.983$. Therefore, if an earthquake's frequency matches a wavelength of 9.983 seconds, the building will undergo the largest oscillations possible and suffer the most damage. The more the wavelengths differ, the less damage a building will incur, if any. In Figure 4, we graphed the same model over a longer time span $0 \leq t \leq 100$.

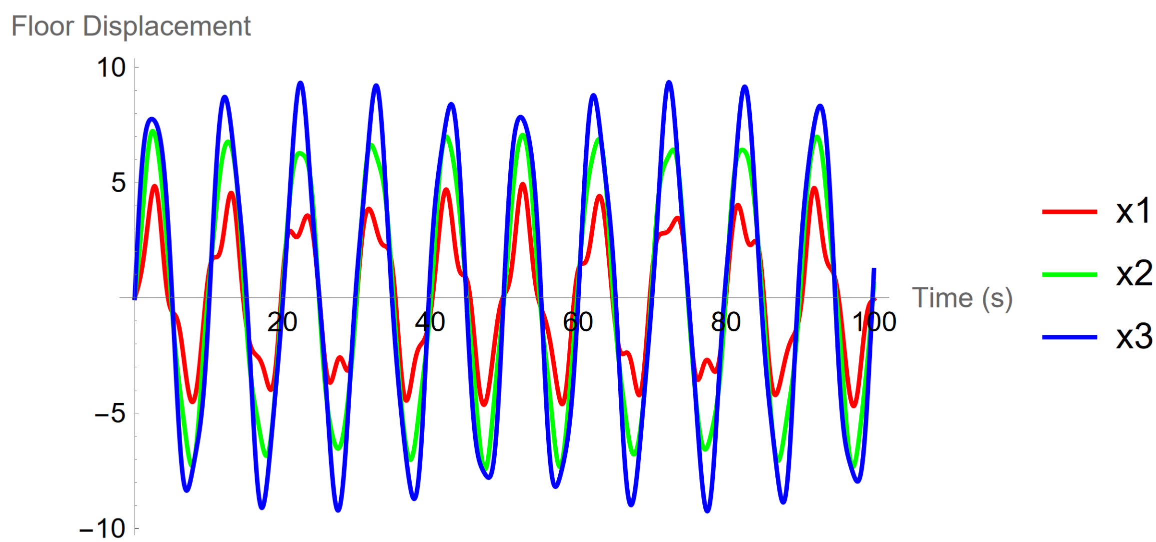


Figure 4: 3 Story Building

As seen in Figure 4, assuming no dampening occurs, the oscillation pattern would repeat every 50 seconds.

### 5.1.2 Example 2

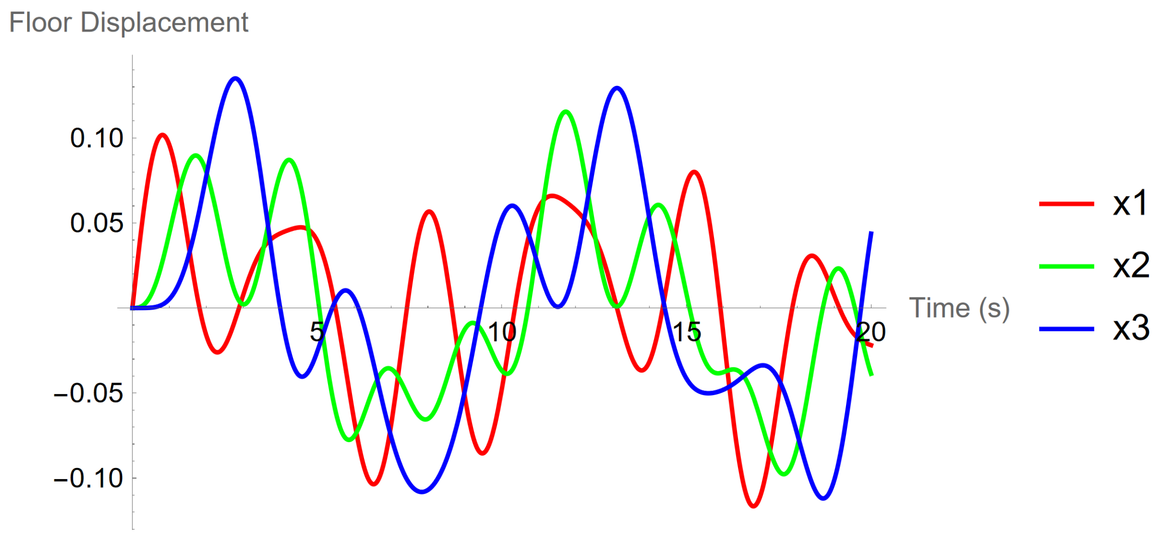


Figure 5: 3 Story Building

Figure 5 is another three-story model with different initial conditions. Here $\mathbf{X}'(0) = (0.2, 0, 0)$ from $0 \leq t \leq 20$. The same trend continues where generally, the top floor experiences the most peak displacement. However, on this small interval, the bottom and middle floors seem to have similar displacement, which is unusual since the middle floor should have more. If we look over a longer time interval from $0 \leq t \leq 100$, as shown in Figure 6, the expected trend occurs, suggesting Figure 5's time interval was too small to analyze.

Since our omega value is the same, the wavelength period is still 9.983 seconds, meaning this building would undergo the largest oscillations at similar frequencies as Example 1.

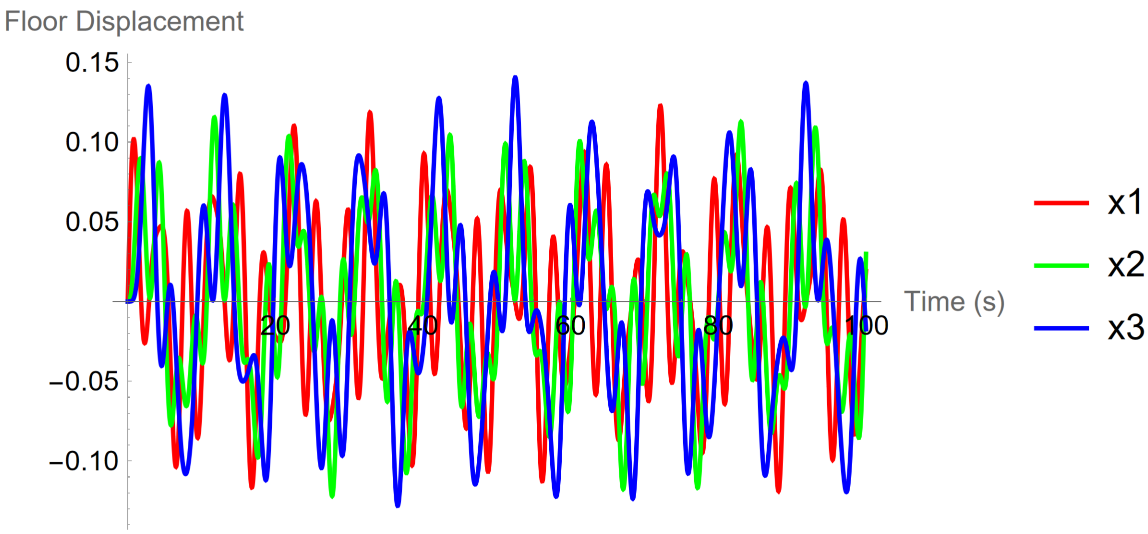


Figure 6: 3 Story Building

## 5.2 Thirty-Three Story Building

### 5.2.1 Example 3

Using similar methods, we modeled a thirty-three-story building with $m$ and $k$ values $m = 2,700,000kg$ and $k = 1,350,000kg/s^2$. Our initial conditions were $\mathbf{X}'(0) = 2, 8, 10, 12, 16, 18, 15, 2, 1, 9, 8, 4, 5, 4, 3, 2, 1, 9, 4, 6, 2, 3, 2, 26, 2, 1, 3, 6, 9, 7, 8, 9, 10)$. For readability, we only plotted the bottom floor, the sixteenth (middle) floor, and the top floor. The trend continues as expected. The displacement and wavelength period here is noticeably much larger than the three-story building, meaning our thirty-three-story building oscillates more at a lower frequency. See Figure 7.

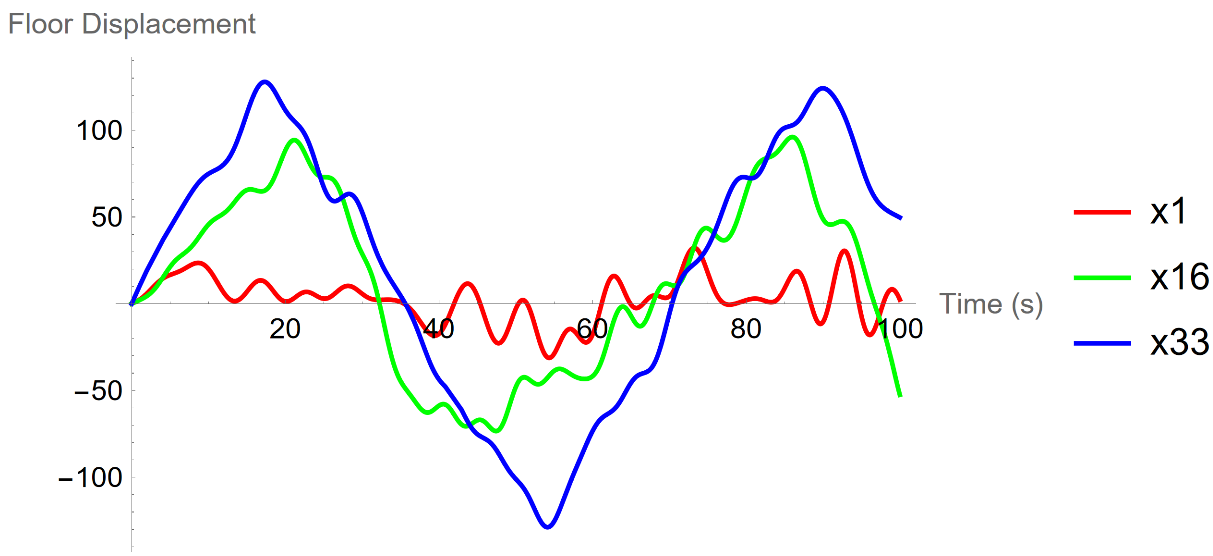


Figure 7: 33 Story Building

Once again, with the same initial conditions over a 500-second time span, we get Figure 8.

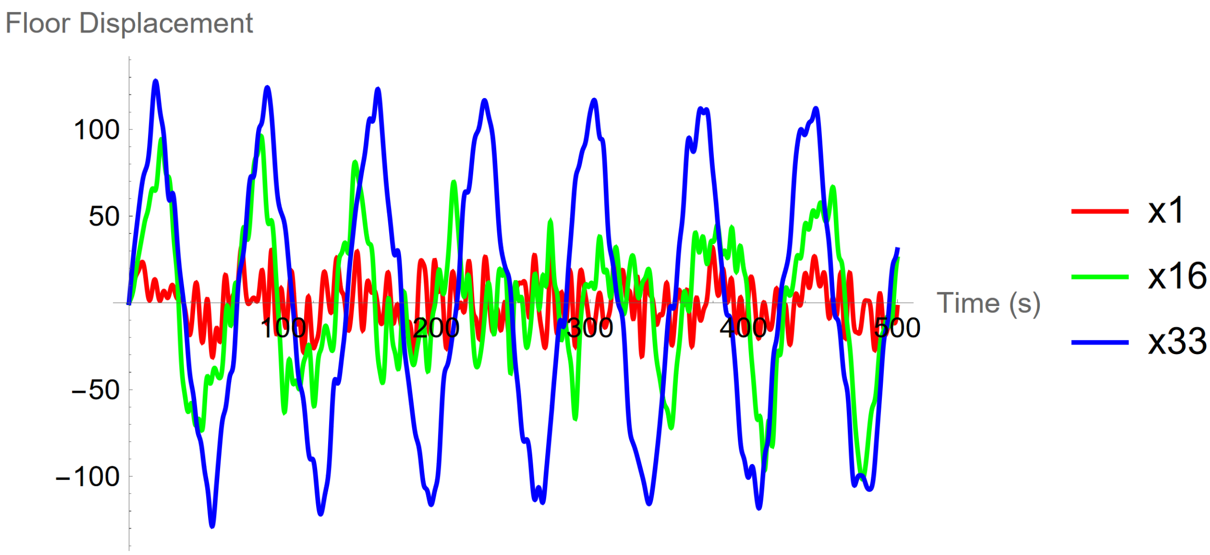


Figure 8: 33 Story Building

#### 5.2.2 Example 4

For our final example, initial conditions $\mathbf{X}'(0) = 0.3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$ caused us to run into a numerical error since $x_{22} - x_{33}$ were so small that Mathematica rounded them to zero. Hence, our graph incorrectly shows zero displacements occurred on those upper floors. See Figure 9. To fix this, we changed the initial conditions to $\mathbf{X}'(0) = (0.3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.02, 0.02, 0.02, 0.02, 0.02, 0.02, 0.02, 0.02, 0.02, 0.02, 0.02, 0.02)$ and graphed $t$ over longer time interval $0 \leq t \leq 500$, which gave us a more accurate Figure 10.

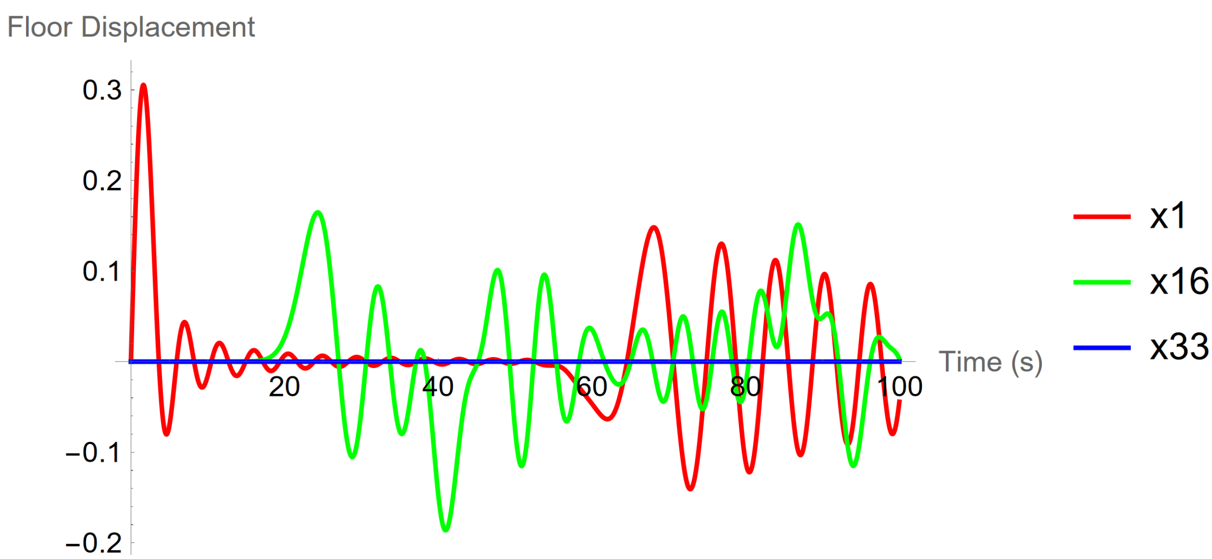


Figure 9: 33 Story Building

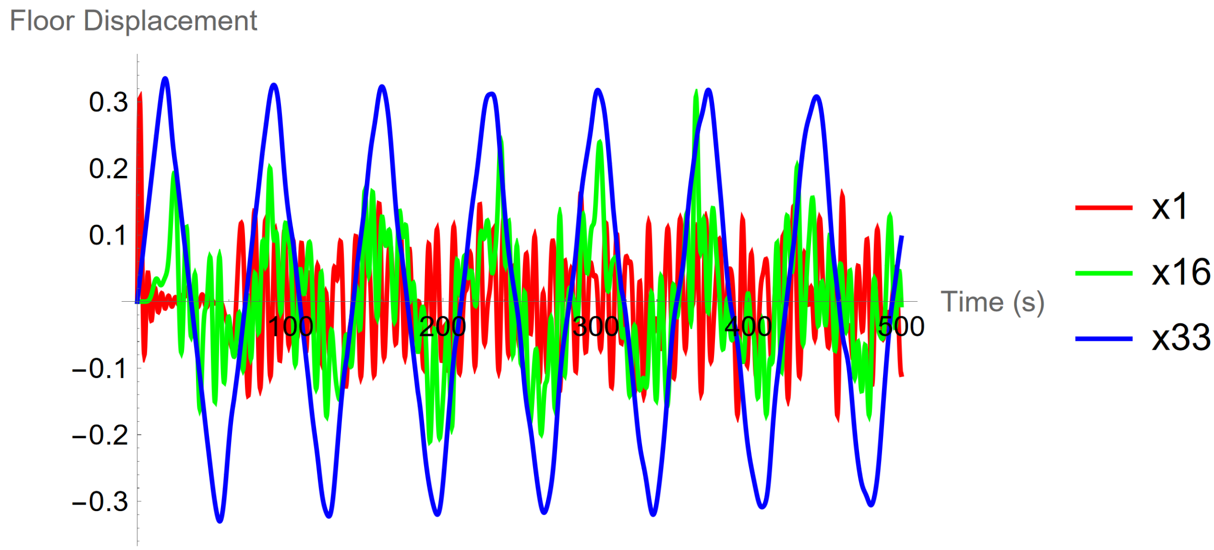


Figure 10: 33 Story Building

# 6 Conclusion

The closer an earthquake's frequency response to a given building's natural resonance frequency, the more oscillations and, ultimately, damage a building will undergo. Typically, the higher a story is on a building, the more it will be displaced during an earthquake. Overall, buildings with many stories sway more and have lower resonant frequencies with longer wavelengths. On the other hand, buildings with few stories sway less and have higher resonant frequencies with shorter wavelengths. Future models could be improved by accounting for dampening, torque, and foundation type. Research and testing to find more accurate initial conditions, $K$-values, and masses could also improve accuracy.

# 7 Acknowledgment

We want to thank and acknowledge Dr. Nicholas Kirby for being our patient mentor and working with us after switching topics last minute. We would also like to thank anyone who took the time to read through our project.

# 8 Links To Mathematica Files

Below are the links to the google drive doc, which includes the files we created to model our buildings.

Three-Story Building File:

`https://drive.google.com/file/d/1PyVMWFbAczpd8duzdMDaGPHlZq_Xx7M5`

Thirty-Three-Story Building File:

`https://drive.google.com/file/d/1Rdur53EJIlpRZMM_gM39Ya50gg8mvWYq`

# References


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