Learning in Job Search: Insights on Belief Updating

Sebastian Brown[*], Kenneth Chan[†]

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Abstract

We use a nationally representative survey (the labor supplement of the Survey of Consumer Expectations) to study individuals' behavioral biases in learning about their wage distributions. Using a recently developed test for Bayesianism (Augenblick and Rabin, 2021), we find strong evidence of non-Bayesian learning. Specifically, among respondents who receive wage offers, we find an average movement in beliefs that is roughly 543% of the reduction in their beliefs' uncertainty, 443% more than the Bayesian benchmark, a result consistent with base-rate neglect and/or overreaction to signals. This estimate remains large and significant when examining subsets of the data separated by different labor market and demographic variables. We further examine the heterogeneity in agents' learning by identifying specific non-Bayesian patterns that can be explained by multiple priors models and updating rules that accommodate motivated beliefs.

1 Introduction

Canonical job search models typically assume that the agent knows the underlying wage distribution (Mortensen, 1970; McCall, 1970). This class of model has an optimal

[*]Department of Economics, UC Santa Barbara. sebastian_brown@ucsb.edu.
[†]Department of Economics, UC Santa Barbara. kenneth_chan@ucsb.edu.

strategy of setting a reservation value and accepting the first offer that is better than the reservation value (Weitzman, 1979). While these models give us valuable insights into job search behavior, it is often unrealistic to assume that people know the underlying wage distribution.

Another class of search models assumes an unknown underlying distribution (Rothschild, 1978; Rosenfield and Shapiro, 1981; Talmain, 1992; Li and Yu, 2018), where the agent has to learn about the underlying distribution over time by observing the offers they receive.[1] While these models are more realistic, the empirical application of these models has been limited due to insufficient high-quality data on job searchers' beliefs. Consequently, there has been a lack of research on how job seekers learn and update their beliefs in light of new information. Examining the process of learning and belief updating can provide valuable insights into the effectiveness of information provision policy in a job search setting. Our paper hopes to fill this gap and provide some insights into how job searchers incorporate new information into existing beliefs.

Moreover, providing information has been shown to be a potent policy tool for changing people's beliefs and behavior. In a series of recent field experiments, providing information has been shown to help students make more informed decisions about their college majors (Wiswall and Zafar, 2015), improve people's knowledge about COVID-19 (Sadish et al., 2021), eliminate statistical racial discrimination in a patient's choice of medical professional (Chan, 2022), and reduce disagreement about the extent of racial discrimination (Haaland and Roth, 2023). In a job search context, Arni (2016) found that a coaching program was successful in increasing job-finding rates among the treated job searchers by 9 percent.[2] The author argues that coaching helped workers to have more realistic expectations and search more effectively. In another experiment, Gee (2018) randomly displayed information to LinkedIn users about the number of workers applying for a specific job. She showed that

[1] These models are search models where an agent is looking for a good with the lowest price or a product with the highest quality. These models can be extended easily to a job search context.

[2] We interpret this coaching program as a form of providing information to help the job searcher.

this additional labor market information increased the probability of a worker completing a job application by 2.5 percent. Overall, these studies highlighted have shown the effectiveness of information treatments in affecting people's behavior in various settings.

Unlike most of the above papers which use a treatment intervention to manipulate people's beliefs, in this paper, we utilize a government dataset (the Survey of Consumer Expectations) to study an individual's capacity to learn about the wage distribution without any experimental manipulation. As our data are representative and free of the cognitive influences of an experimental setting, it is also a more externally relevant measure of how individuals process information in a true labor market context.

Our main analysis employs a recently developed test for Bayesian updating in beliefs from Augenblick and Rabin (2021). There are two statistics required for this test. Belief movement which is the squared difference of changes in belief, and uncertainty reduction which is the amount of reduction in the belief's variance from updating. If the agent is Bayesian, the average movement in beliefs will be equal to the average uncertainty reduction. Using this test, we find an average movement in beliefs that is over five times the average reduction in uncertainty. We can reject that people are updating their beliefs in a Bayesian manner, and our results suggest that on average people overupdate relative to the Bayesian benchmark. This is consistent with biases such as overreaction to signals and base rate neglect.

Our remaining analysis attempts to examine specific behavioral patterns that are inconsistent with the Bayesian prediction. With these results, we are able to test against various non-Bayesian updating rules (Grether, 1980; Epstein, Noor and Sandroni, 2010; Ortoleva, 2012; Hagmann and Loewenstein, 2017; Ba, 2022) to determine which model can best describe how job searchers update their beliefs.

The paper that is closest to ours is Conlon, Pilossoph, Wiswall and Zafar (2018). Despite having a similar research question and using the same dataset, our papers differ in a few ways. Firstly, our paper is primarily interested in using the data to test against various updating rules, while their paper focuses on how learning can affect people's behavior like their reservation wage. Secondly, our approach uses the Augenblick and Rabin (2021) test which circumvents the need to compute the Bayesian benchmark to compare if people are Bayesian. This test also allows us to test for Bayesianism with fewer and more conservative assumptions. Their analysis assumes that the agent is updating in a Gaussian framework where the priors and signals (with a correctly perceived variance) are normally distributed. Our analysis does not rely on this parametric assumption, we only assumed log-normality in the wage distribution to compare our data across time as described in Section 5. Thirdly, our approach allows for updating without a job offer, as people may change their beliefs with news about the labor market. This allows us to make statements about respondents without a job offer which is more representative of the population. Lastly, we build on this paper by conducting additional *individual-level* analyses that allow us to uncover additional details about labor market learning. Specifically, we measure and discuss the prevalence of various learning patterns in the data.

The remainder of the paper proceeds as follows. Section 2 reviews relevant literature. Section 3 describes the data and our process of fitting distributions to the data. Section 3 outlines a simple theoretical model of labor market updating and briefly describes the general theory of the Augenblick-Rabin test for Bayesianism. Section 5 explains our empirical strategy, detailing the steps that we took to apply the Augenblick-Rabin test to this dataset. Section 6 then lists our main results. Section 7, shows that our result is robust to measurement errors. In section 8, we look at other belief updating patterns at the individual level. Finally, section 9 concludes and provides suggestions for future work.

2 Literature Review

Our work is related to two strains of literature: search theoretic models with unknown distributions and non-Bayesian updating. In these models, the underlying distribution from which offers are drawn, while fixed, is also unknown to the agent.[3] A key characteristic of this class of model is that the agent has to update his beliefs about the underlying distribution as he searches.

Rothschild (1978) wrote the first search model with unknown distribution, and he showed that the static reservation value no longer applies in most scenarios as the agent will continuously update his expectations about the future. He also pointed out that having the ability to recall a previously rejected offer will affect the optimal search strategy when the distribution is unknown. When recall is not possible, the optimal strategy can be characterized by reservation value that is a function of the agent's beliefs (Rothschild, 1978; Rosenfield and Shapiro, 1981). When recall is possible, the strategy is to set a decreasing sequence of reservation values (Talmain, 1992).

For tractability of the model, the agent is typically assumed to have a Dirichlet prior on the wage distribution (Rothschild, 1978; Rosenfield and Shapiro, 1981; Talmain, 1992) or a Gamma distribution on the arrival rate of offers Potter (2021). This is because the Dirichlet and Gamma distribution has a conjugacy property[4] which provides a closed-form expression for the posterior. Recently, Li and Yu (2018) deviated from the conjugate prior assumption and they assume the agent believes that there is a set of possible distributions and all of the distributions in this set satisfy the monotone likelihood ratio property (MLRP). The MLRP allowed them to establish first-order stochastic dominance between distributions and posterior beliefs, which allows them to perform monotone comparative statics (Milgrom and Shannon, 1994) on the reservation value.

While these search models typically assume people are updating their beliefs in a Bayesian manner, behavioral economists have shown that people are not Bayesian. Behavioral theorists have come up with belief updating models to accommodate these non-Bayesian behavior (Grether, 1980; Epstein, Noor and Sandroni, 2010; Ortoleva, 2012; Hagmann and

[3] An alternative way to model this is to let the arrival rate of offers be unknown, see Potter (2021). However, assuming the underlying distribution is unknown is the main way of introducing uncertainty into the model.

[4] Under Bayesian updating, the posterior belief and the prior beliefs are from the same parametric family of distribution.

Loewenstein, 2017; Ba, 2022). Some of the more well-known biases include base rate neglect (Kahneman and Tversky, 1972), a phenomenon where people underweight their priors, and conservatism bias (Phillips and Edwards, 1966), a situation where people are insensitive to new information. While most of the empirical evidence is from lab experiments (see Benjamin (2019) for a survey of the experimental literature on belief updating), some recent papers have used field data to show that people update their beliefs in a non-Bayesian manner (Conlon, Pilossoph, Wiswall and Zafar, 2018; Bordalo, Gennaioli, Porta and Shleifer, 2019; Bordalo, Gennaioli, Ma and Shleifer, 2020; Augenblick and Rabin, 2021).

There is a burgeoning literature that empirically studies learning in job search and how people's behavior deviates from the Bayesian benchmark. Firstly, Kudlyak, Lkhagvasuren and Sysuyev (2014) finds that job seekers first apply to jobs that match their education levels. But with prolonged unemployment, they apply to jobs that require a lower education level. They argue that this is evidence that searching workers learn to adjust their expectations downward over the unemployment spell. Similarly, Mueller, Spinnewijn and Topa (2021) found that unemployed workers adjust their beliefs downwards but not sufficiently. This results in the long-term unemployed displaying an optimistic bias in job finding.

Contrary to Mueller, Spinnewijn and Topa (2021) findings, Conlon, Pilossoph, Wiswall and Zafar (2018) use the SCE data and find that people overupdate their beliefs relative to the Bayesian benchmark. They estimated that on average, wage expectations increase by \$0.47 for every one-dollar increase in observed wage offer, while the Bayesian benchmark is estimated to be \$0.16.

Instead of focusing on wage expectation, Potter (2021) examines how the expectation of offer arrival rate changes over time using data from the Great Recession. Using a calibrated model, he showed that learning can explain the job search dynamics during the Great Recession.

Aside from these papers, our work also relates more generally to a broader, yet still relatively new, literature on behavioral job search (e.g. DellaVigna et al. (2017); see Cooper and Kuhn (2020) for a recent review), as well as more general work studying employed job search (e.g. Faberman et al. (2022), Ahn and Shao (2021)).

3 Data

3.1 Data Source Description

Our primary data source is the Survey of Consumer Expectations (SCE), which is a nationally representative survey administered monthly by the New York Federal Reserve. The survey is divided into two parts, a core set of questions that remain the same every month and a supplementary set of questions that rotates between several different economic topics. Our analysis focuses on the labor market supplement, which is administered every March, July, and November. Subjects can remain in the survey for up to a year, and are replaced on a rolling basis. Therefore, we are able to observe the same subject for up to a maximum of three times for the labor force questions and twelve times for the core questions. For a more in-depth description of the structure and administration of the SCE, see Armantier et al. (2017).

The main advantage of this dataset is that it explicitly elicits expectations from survey respondents, meaning there is no need to indirectly infer beliefs as in many previous papers (see, for example, Potter (2021), Spinnewijn (2015)). Another benefit is its panel structure, which, while short, still does allow for within-subject observations of the evolution in beliefs over time. A third benefit is that it features mostly employed individuals, allowing study of a previously-understudied class of workers (as highlighted in Faberman et al. (2022)).

This last benefit, however, comes at the cost of including fewer unemployed individuals. While the SCE has been included in analyses of unemployed workers (e.g. Mueller et al. (2021); Conlon et al. (2018); Faberman et al. (2022)), the fraction of the sample that is unemployed is very small, chiefly because the SCE is a representative sampling of the entire population, not of the unemployed. As noted in table 1 in section 3.3, only 3.9% of our final

Think again about the job offers that you may receive within the coming four months. What do you think is the percent chance that the job with the best offer will have an annual salary for the first year of...

The best offer is the offer you would be most likely to accept.

Less than [0.8* OO2a2] dollars (1)	______	% (1)
Between [0.8* OO2a2] dollars and [0.9* OO2a2] dollars (2)	______	% (2)
Between [0.9* OO2a2] dollars and [1.0* OO2a2] dollars (3)	______	% (3)
Between [1.0* OO2a2] dollars and [1.1* OO2a2] dollars (4)	______	% (4)
Between [1.1* OO2a2] dollars and [1.2* OO2a2] dollars (5)	______	% (5)
More than [1.2* OO2a2] dollars (6)	______	% (6)

Figure 1: Survey Question OO2b from SCE

sample is unemployed.[5]

3.2 Survey Question Description

Our analysis uses a few survey questions which merit more detailed explanations. First, we use the distributional question labeled "OO2b" as shown in Figure 1 to measure the mean and variance of an individual's prior.

Here the label "OO2a2" refers to the following question: "*Think about the job offers that you may receive within the coming four months. Roughly speaking, what do you think the annual salary for the best offer will be for the first year?*" Therefore, the categories of this question may be taken to mean, "What is the probability your maximum offer will be 80% or less of your guess of the average maximum offer?", "What is the probability your maximum offer will be 80% to 90% of your guess of the average maximum offer?", and so on. Since the question is about the maximum wage offer, not the average, we must take steps to recover the single wage offer distribution from this question. This process is outlined in section 5.1.

Hereafter we will refer to wage "bins," by which we mean the ranges of wages implied by the survey questions. These range from bin 1, the lowest bin, consisting of all wages 80% times the expected best offer and below, to bin 6, the highest bin, consisting of all wages 120% times the expected best offer and above. A challenge working with this question is

[5]It is therefore not surprising that Mueller et al. (2021) augments the SCE by including a second dataset, the Survey of Unemployed Workers in New Jersey, which as its name suggests, focuses entirely on unemployed individuals.

that the bins of wages are redefined with each new survey taken, including within subjects. The size of the wage bins is determined by their response to "OO2a2," which differs across every response to the survey. To make the responses comparable between an individual's consecutive surveys, we fit a log-normal distribution to each individual's responses, after making the adjustments discussed in section 5.1. This fitting also gives us the mean and variance used in our principal analysis for each individual distribution. For an explanation of this fitting process, see section 5.2.

While our main analysis uses the mean from the fitted log-normal distribution, we also use the question "OO2a" to measure an individual's expectation of the average wage for a given period to graph the sample distribution of expected offers in figure 1 in the next section. This question reads "*Think about the job offers that you may receive within the coming four months. Roughly speaking, what do you think the average annual salary for these offers will be for the first year*?" We do not use this question for our main analysis since unlike OO2a2, it does not have a corresponding question (OO2b) to elicit a probability distribution. However, its responses do allow us to see how closely our fitted means follow the directly-elicited means, as highlighted in the next section.

Finally, we also use questions about the wages and number of offers received, as well as a question on the number of offers expected. The survey asks individuals to report their three best offers, but it also collects information on the number of offers expected and received. Hence, we can tell if an individual received more offers than they have space to report. Further explanation of how we incorporate the expected number of offers to recover the individual wage distribution is given in section 5.1.

3.3 Data Structure and Summary Statistics

Our final dataset is organized such that each observation contains information from two consecutive labor market surveys for a single individual. While most individuals in the dataset appear only twice, for those that appear three times, there will therefore be two separate observations: one for the update from period 1 to period 2, and one for the update from period 2 to period 3.

The composition of our final dataset by employment status (defined as the employment status of the individual in each before-after pair's first period) and whether they received at least one wage offer is given Table 1:

Count	All Individuals	Got Offer?		Searched?		
		Yes	No	Yes	No	Unknown
Total observations	4,859	978	3,881	1,212	3,252	395
Unique individuals	3,430	847	2,879	981	2,457	290
Number unemployed	189	59	130	99	81	9
Number employed	3,948	804	3,144	1,012	2,568	368
Number not in labor force	665	104	561	74	578	13
Missing employment status	57	11	46	27	25	5
Data Range	3/2015-3/2020					

Table 1: Dataset composition. Observations consist of information from two consecutive surveys for a given individual. Unique individual counts for subcategories do not add up to total since individuals could have changed search status and whether they received an offer between periods if appearing for three periods. Unique individual counts for subcategories counts an individual for that category if appearing in it at least once.

In the following graphs, we report on the expected and realized wage offers, both in the wage amount and number of offers. One concern is that the beliefs from the survey are not incentivized and that this may affect the quality of the responses. In figure 3 below, we see that most of the responses are close to the average offers all the survey respondent has reported receiving. This suggests that at the aggregate level, people are not randomly responding to the survey questions. We can also see that the aggregate log distributions for expected offers and reported offers seem to follow a bell curve, consistent with our assumptions of log normality for individual distributions.

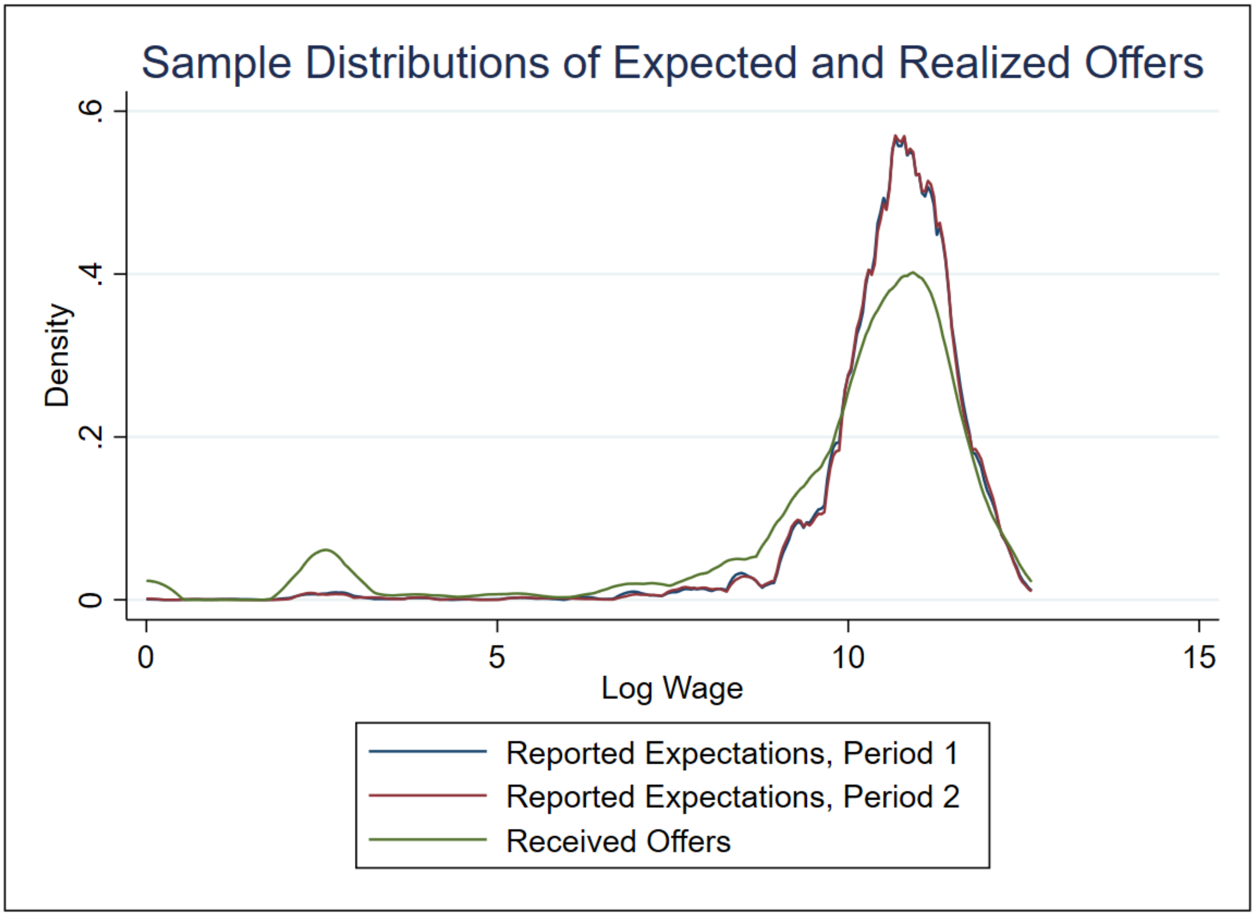

Figure 2: Distribution of reported expected wages, fitted expected wages, and received wages. Reported expected wages and fitted expected wages include all individuals, including those who did not receive an offer.

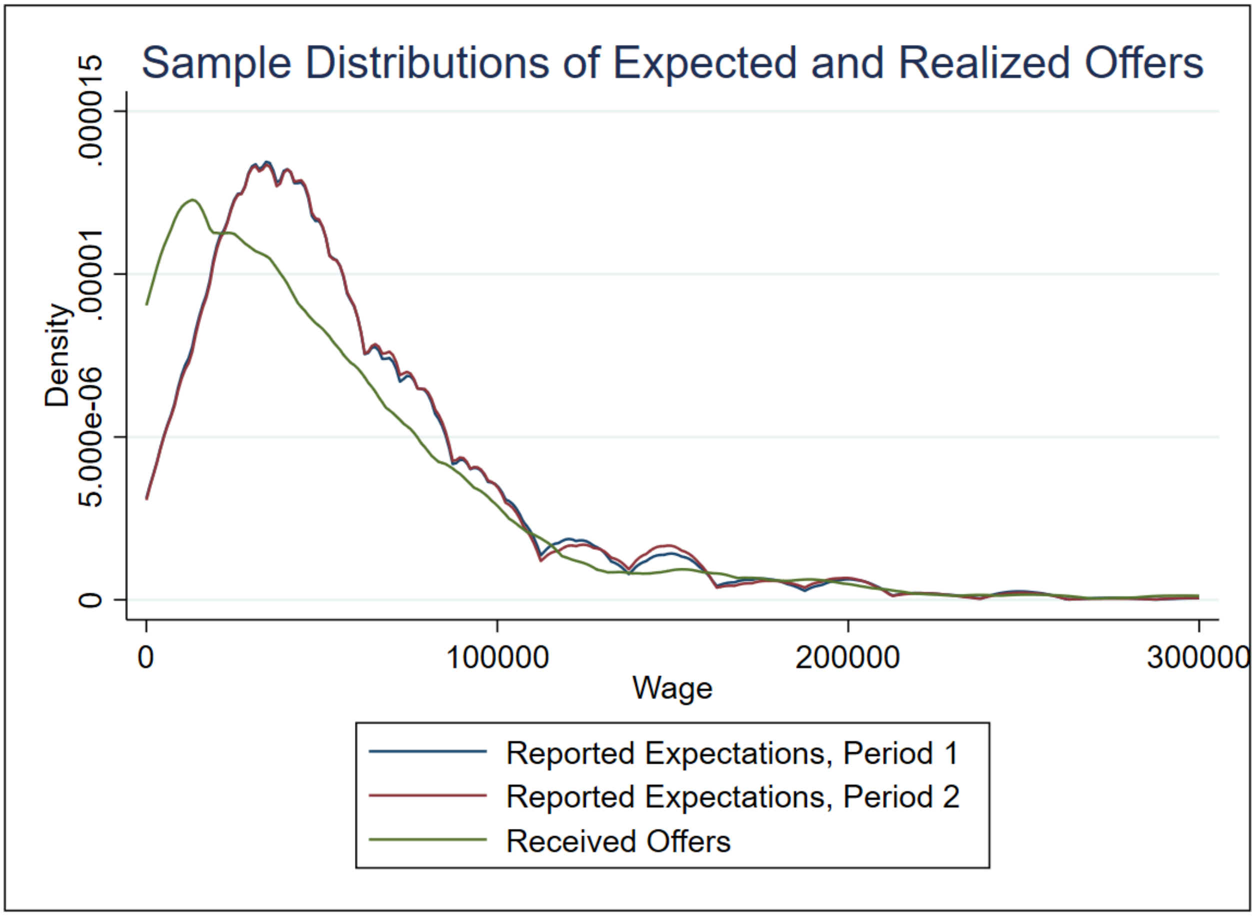

Figure 3: Distribution of reported expected wages, fitted expected wages, and received wages. Reported expected wages and fitted expected wages include all individuals, including those who did not receive an offer. Fitted expected wages are the means of the fitted distributions using the process discussed in section 5.2.

4 Theoretical Model

We will now provide a model of belief dynamics related to the job search setting we are studying. We will follow Li and Yu's (2018) method of modeling uncertainty. Unlike their model which is interested in obtaining comparative statics across different signal realizations, we are primarily interested in the belief dynamics, hence we will not need to assume MLRP in the set of distributions. This model also differs from the Augenblick and Rabin (2021) setting where the state of the world is a probability distribution and not an outcome. We assume that the agent's reported beliefs in the survey are a mixture distribution. We will need to first verify that for Bayesian agents, the reported beliefs from the mixture distribution have the martingale property to apply their test.

Suppose there is an agent who is searching for a job and believes there is a set of possible wage distributions, $\mathcal{F}$, that he is drawing his wages from. We will index the density functions in $\mathcal{F}$, by a parameter θ taking values in an ordered set Θ. The agent has a non-degenerate belief g_t over θ at time t. We will also assume that the agent places a non-zero probability weight on the true wage distribution, F, he is drawing from. The agent's belief about the wage he will receive is a mixture distribution of the possible wage distributions in $\mathcal{F}$.

In each period, the agent can observe a signal that reveals some information about the wage distribution he is drawing from. We let the set of possible signal realizations be $X \subseteq \mathbb{R}^n$; we can think of the signal realization $x \in X$ as a vector of wages of the job offer the agent received or news related to the labor market. The conditional density function is denoted as $p(x_t|\theta)$, which reflects the likelihood of observing signal x at time t conditioned on the wage distribution being f_θ.

In the survey questionnaire "OO2b," the respondents provided a probability distribution about the best wage offer they will receive. We can recover the single wage offer following the procedure in section 5.1, which gives us data about the respondent's probability distribution about the wage offer they will receive as probabilities over several wage bins. We denote the probability of the wages being in bin $i \in \{1, 2, \dots, n\}$ at time t as π_t^i.[6]

Under the model we have constructed, we will partition the wages into n wage bins $\{[a_0, a_1), [a_1, a_2), \dots [a_n, a_{n+1})\}$. In the survey, $a_0 = 0$ and $a_{n+1} = \infty$. This partition covers the entire support of the wage distribution. We will assume that the agent's reported belief can be represented by the expression below

$$\pi_t^i = \int_{\theta' \in \Theta} g_t(\theta') \int_{a_{i-1}}^{a_i} f(w|\theta') dw d\theta' \tag{1}$$

where w is the wage drawn from the distribution. The inner integral is the probability

[6] In the survey, there are 6 bins, but we are keeping the model general.

of receiving a wage offer within the wage bin from the distribution indexed by θ'. The outer integral integrates over the agent's beliefs about the distribution he is drawing from.

Our primary test tests if people's belief follows the martingale property. Formally, the martingale property states that the expected posterior beliefs have to be equal to the prior belief, $\mathbb{E}(g_{t+1}(\theta|x)|g_t(\theta)) = g_t(\theta)$. Intuitively, this property states that the agent should not expect himself to change his belief before seeing the signal. If the agent expects himself to change his belief, he should have already done so which leads to inconsistency with the current prior.

If the agent's updating rule on g has the martingale property, the probabilities π^i will have the martingale property as well.[7]

$$\mathbb{E}(\pi^i_{t+1}|g_t) = \int_{\theta'\in\Theta} \mathbb{E}(g_{t+1}(\theta'|x_{t+1})|g_t) \int_{a_{i-1}}^{a_i} f(w|\theta')dw = \int_{\theta'\in\Theta} g_t(\theta') \int_{a_{i-1}}^{a_i} f(w|\theta')dw = \pi^i_t$$

This allows us to perform a martingale test on the statistics reported to determine if people are updating their beliefs in a Bayesian manner.

4.1 Updating Rules

We will be testing the data against some updating rules to determine which of these updating rules can best explain people's behavior.

4.1.1 Updating Rules with Martingale Property

1. Bayesian updating

$$g^{bayes}_{t+1}(\theta|x_{t+1}) = \frac{g_t(\theta)p(x_{t+1}|\theta)}{\int_{\theta'\in\Theta} g_t(\theta')p(x_{t+1}|\theta')} \tag{2}$$

Bayesian updating is the standard updating rule in microeconomic theory. This updating rule has many desired properties such as the martingale property, which makes models

[7] We can easily verify that $\mathbb{E}(\pi^i_{t+1}|g_t)$ converges absolutely since it has to be less than 1 and the integrands are non-negative. We can apply Fubini's theorem and interchange the order of expectation and the integral.

tractable.[8]

2. Affine Transformation of Bayesian Belief and prior (Epstein, Noor and Sandroni, 2010)[9]

$$g_{t+1}^{bias}(\theta|x_{t+1}) = (1-\lambda)g_t(\theta) + \lambda g_{t+1}^{bayes}(\theta|x) \tag{3}$$

The Epstein, Noor and Sandroni (2010) updating rule is attractive as it preserves the martingale property, and yet it accomodates over and under-update relative to the Bayesian benchmark. $\lambda \geq 0$ is the parameter that determines the degree of over and underreaction. When $\lambda < 1$ we have underreaction and $\lambda > 1$ we have overreaction.[10] When $\lambda = 1$ we have the standard Bayesian updating.

4.1.2 Updating Rules without Martingale Property

3. Exponential distortion to prior and conditional probabilities (Grether, 1980)

$$g_{t+1}^{bias}(\theta|x_{t+1}) = \frac{g_t(\theta)^a p(x_{t+1}|\theta)^b}{\int_{\theta' \in \Theta} g_t(\theta')^a p(x_{t+1}|\theta')^b} \tag{4}$$

This updating rule can accommodate several updating biases but it violates the martingale property. $a \geq 0$ is the weight the agent places on the prior; when $a < 1$, we have base rate neglect and when $a > 1$, we have confirmation bias. $b \geq 0$ is the weight the agent places on the signal; when $b < 1$ we have underreaction to signals, and when $b > 1$ we have overreaction to signals. When $a = 1$ and $b = 1$ we have the standard Bayes' rule.

This model is widely used in the analysis of experimental data because the odds ratio can be log-linearized and estimated with a linear regression.

[8] Another useful property is divisibility (Cripps, 2018), where updating the signals sequentially or simultaneously does not affect the posterior belief. In the ambiguity literature, dynamic consistency (Epstein and Le Breton, 1993), receiving more information about the state of the world does not change the agent's optimal contingent plan, leads to Bayesian updating under expected utility theory.

[9] This model nests cursed belief (Eyster and Rabin, 2005), where the updated belief is just a convex combination between the Bayesian posterior and the prior.

[10] There is an upper bound on λ to ensure that it is a valid probability measure.

4.1.3 Non-Falsifiable Updating Rules without Martingale Property

We now mention two more updating rules that appears to be consistent with the data, but we have no test to falsify the model with this data. These models have too many free-parameters which allows it to rationalize most updating patterns.

4. Affine Transformation of Bayesian Belief and reference belief (Hagmann and Loewenstein, 2017)

$$g_{t+1}^{bias}(\theta|x_{t+1}) = (1-\lambda)\mu(\theta) + \lambda g_{t+1}^{bayes}(\theta|x) \tag{5}$$

In this model, μ is a reference belief which is a belief that the agent wants to have (for utility reason), and $\lambda \geq 0$ is parameter that draws the updated belief towards the reference belief. This model has been use to explain motivated beliefs (Bénabou and Tirole, 2002; Eil and Rao, 2011). A key prediction from this model is the agent will update asymmetrically, the agent over-updates when the signal moves the prior towards the reference belief and under-updates when the signal moves the prior away from the reference belief.

For instance, if an individual is optimistic about the wage offers he can receive. We set μ to be an optimistic belief (more likely for high wages to be drawn), and when $\lambda \neq 1$, the agent's belief will update towards this reference belief.

5. Multiple Prior models (e.g. (Ortoleva, 2012; Ba, 2022))

In this class of model, the agent typically entertains two or more possible prior beliefs and there is a switching rule the agent adopts to switch between prior. For example, in Ortoleva (2012), the agent has a belief over which prior is most likely to be correct and he updates this distribution and all the priors with Bayes' rule. When he sees a surprising signal, the probability of seeing this signal with the current prior is below a threshold, he reevaluates which prior is most likely to be correct. This class of models violates the martingale property when a switch in prior occurs.

These models are typically used in misspecified learning models, where an agent incorrectly believe that the true states are impossible. Unlike the earlier models, when the agent assigns zero probability to an event in the prior, the updated belief for the event will always remain zero. This helps us to distinguish this class of updating rules from the other updating rules that were mentioned earlier.

While we do not have a test to falsify this class model, most of the behavioral patterns appear to be consistent with this model's prediction. This model is also intuitively appealing as it allows agents to be non-dogmatic about zero-probability events.

5 Empirical Strategy

There are two main challenges in working with this data. Firstly, the survey measured the probability distribution of the best wage offer, not the individual wage offer. The distribution of the best wage offer is a function of the arrival rate of job offers which may change over time as well. As such, when the respondent's belief changes, we can't attribute it completely to the respondent updating his belief about the wage distribution. To address this, we recover the single wage offer distribution from the reported maximum.

Secondly, the bins of the wage offer differ across every individual and time period. The bins of the wage offer are constructed using percentages of the expected best wage offers. If subjects report different best wage expectations in different time periods, the wage bins will have different intervals across each response. For example, when an individual changes his wage expectation, the data is as follows. The wage bins are not comparable across time.

	$w < 52,000$	$52,000 \leq w < 58,500$	$58,500 \leq w < 65,000$	$65,000 \leq w < 71,500$	$71,500 \leq w < 78,000$	$w \geq 78,000$
$p(\cdot)$	0	0.2	0.3	0.4	0.1	0

Table 2: Period 1 Beliefs

	$w <$ 56,800	$56,800 \leq w <$ 63,900	$63,900 \leq w <$ 71,000	$71,000 \leq w <$ 78,100	$78,100 \leq w <$ 85,200	$w \geq$ 85,200
$p(\cdot)$	0	0	0.7	0.2	0.1	0

Table 3: Period 2 Beliefs

To make the bins comparable across time, we fit a log-normal distribution to period 2's belief. This allows us to estimate what the updated beliefs are in the period 1's wage bins.

5.1 Recovering the Single Wage Offer from the Maximum

There are two reasons why we want to work with a single-wage offer distribution instead of the best-wage offer distribution directly. Firstly, the best wage offer distribution is a function of the individual wage distribution and the number of offers the respondent expects. The latter is likely to be a function of the respondent's search effort. For instance, if the respondent is actively searching for a job, the worker may expect to receive more job offers, and the respondent will report a "better" best-wage distribution. This makes it challenging to deduce if the respondents are Bayesian if the search effort differs across time. To alleviate this concern, we estimate their single-wage distribution using the data.

Secondly, given the structure of the data, we have to make some parametric assumptions to fit a distribution of the data to the responses and make them comparable across time. While it is well known that the single wage distribution can be modeled well by a log-normal distribution (e.g. Clementi and Gallegati (2005)), it is challenging to justify using a log-normal distribution to fit the best (maximum) wage offer distribution.

We recover the single-wage offer using the following procedure. Assuming that all wages are drawn from the same single wage offer distribution independently, we let the CDF from a single wage offer be $F(w)$, the CDF of the maximum wage distribution from n offer is $F^n(w)$. From the maximum wage distribution given in "OO2b," we take the nth root to obtain the CDF of the individual wage offer. For individuals who expect to receive zero wage offers, we assume the distribution they report in this question is the distribution of the single wage offer distribution.

5.2 Distribution Fitting

We use the Simulated Method of Moments (SMM) to fit log-normal distributions to the five points on individuals' posterior cumulative density functions recorded in question OO2b. The main reason we fit the posterior distribution is that unbiased measurement error in the posterior distribution does not affect the Augenblick and Rabin (2021) test statistics.[11] The horizontal values of these five points are the inverse CDF values located at the wage bin thresholds (e.g. 80% of OO2a2, 90% of OO2a2, and so on), and the vertical values are survey responses of OO2b with the adjustments described in 5.1. We find the mean and standard deviation which minimize the horizontal distance in log wages (using the L2 norm) between points on the simulated CDFs with the same CDF values as the five points and the five points themselves. An example is given in figure 3, where the blue points are points on the CDF implied by survey responses, and the orange points are points that minimize the horizontal distance between the orange curve (a log normal CDF) and the blue points.[12]

[11] We discuss this in greater detail in Section 7.

[12] Note here than we use the CDF to do the fitting rather than the PDF. This is because there is no bijection between a log-normal PDF and its inverse and because the survey responses are binned rather than for individual wages.

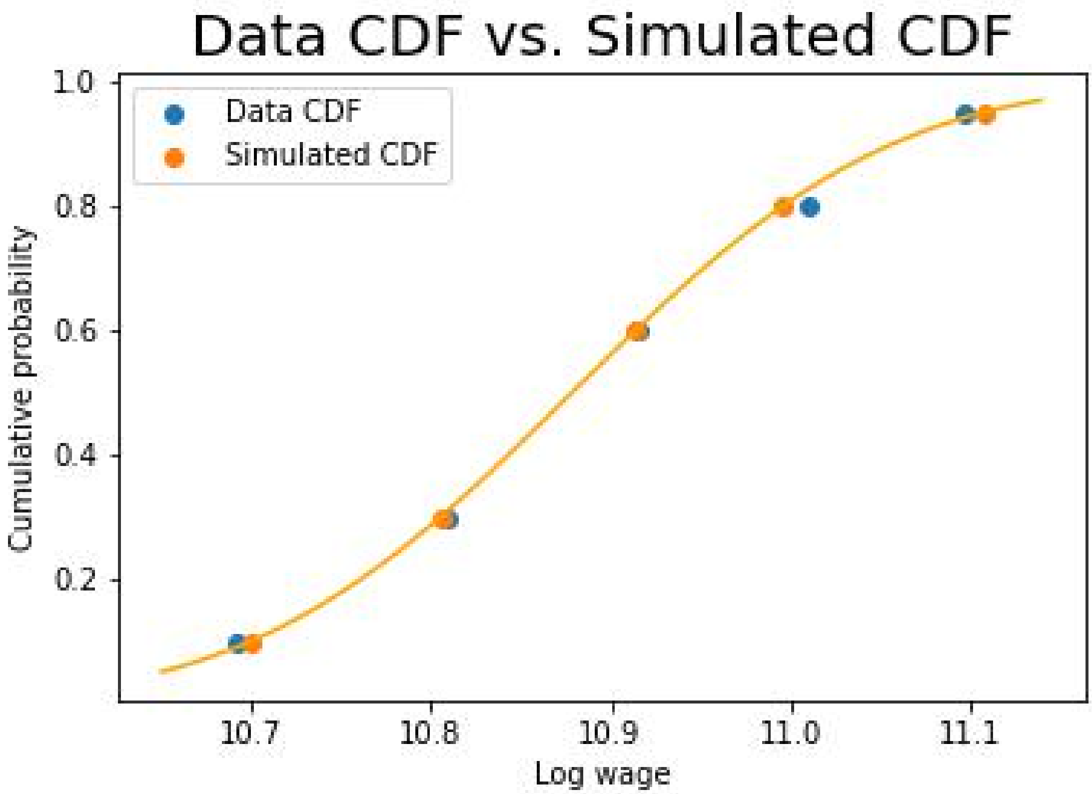

Figure 4: Example fitted distribution.

Since this process gives us an entire distribution for the posterior, we can directly estimate the probability weight placed over the ranges of the posterior distribution that align with the prior wage bins defined by the survey questions by directly estimating the weight over these bins from the simulated distribution.

5.3 Martingale Tests

5.3.1 Test Description

The martingale property can be tested directly, and we will be using a test proposed by Augenblick and Rabin (2021) to do so. There are alternative tests, such as testing if the updated beliefs average out to be the prior belief. The main benefit of the Augenblick and Rabin (2021) test is that it is closely related to some of the more prominent belief-updating biases, such as base-rate neglect and underreaction to signals. This will give us insights into the type of belief updating bias the agent is exhibiting. Moreover, circumvents the need to compute the Bayesian beliefs, by estimating the conditional probabilities. This allows us to make fewer assumptions for the analysis.

The test involves computing two statistics: (1) belief movement, $m_{1,2}$, and (2) uncertainty reduction, $r_{1,2}$, as shown in equations 6 and 7 respectively. Let π_t^i denote the probability assigned to bin i at time t. With h bins, the two statistics are defined as

$$m_{1,2} \equiv \sum_{i=1}^{6} (\pi_2^i - \pi_1^i)^2 \tag{6}$$

$$r_{1,2} \equiv \sum_{i=1}^{6} \pi_1^i(1 - \pi_1^i) - \pi_2^i(1 - \pi_2^i) \tag{7}$$

Both statistics have an intuitive interpretation. The belief movement is the total squared difference between period 2's belief and period 1's belief. This captures how much belief is changing regardless of the direction of change. For uncertainty reduction, the statistics can be interpreted as a measurement of the "variance" of the belief. If we treat each bin like a Bernoulli distribution, the expression in the summation is the variance of the Bernoulli distribution in period 1 minus the variance of the Bernoulli distribution in period 2. This is summed across all the bins. This gives us a proxy of the amount of uncertainty in the belief distribution.

If the belief updating rule satisfies the martingale property, the expected belief movement will be equal to the expected uncertainty reduction. This means that if the agent's belief is expected to move greatly, we will expect the agent to become more certain about the state.[13]

The ideal test will require us to elicit the respondents' beliefs at every possible signal realization to compute the expected belief movement and uncertainty reduction. This will allow us to determine if an individual is Bayesian. Since we only observe single updated

[13]To show this, we have to expand the terms and apply the law of iterated expectations. With the martingale property, the terms will cancel out nicely.

belief profile based on the signals the respondent observed in the last four months, we can only determine if the population is Bayesian at the aggregate level. Given that there are n observations, we compute the average belief movement as $\overline{m}_{1,2} \equiv \frac{1}{n}\sum_{j=1}^{n} m_{1,2}^{j}$ and the average uncertainty reduction as $\overline{r}_{1,2} \equiv \frac{1}{n}\sum_{j=1}^{n} r_{1,2}^{j}$. We then compute the average excess belief movement statistic:

$$X = \overline{m}_{1,2} - \overline{r}_{1,2} \tag{8}$$

where "excess" refers to the amount of movement exceeding reduction.

The standard error for this test is

$$s_{1,2} = \sqrt{\frac{1}{n}\sum\left(m_{1,2}^{j} - r_{1,2}^{j} - (\overline{m}_{1,2} - \overline{r}_{1,2})\right)^{2}} \tag{9}$$

Under the null hypothesis that the population is Bayesian, we expect $X = 0$. With the central limit theorem, we can compute the Z-statistic that converges to a standard normal distribution:

$$Z \equiv \frac{\sqrt{n}}{s_{1,2}}(\overline{m}_{1,2} - \overline{r}_{1,2}) \xrightarrow{n\to\infty} N(0,1) \tag{10}$$

An excess belief movement has a different interpretation depending on the amount of uncertainty reduction. For instance, with an excess belief movement of 0.01, the agent is closer to Bayesian if the uncertainty reduction is large compared to the uncertainty reduction when it is small. To address this concern, we also compute the normalized excess movement, which can be interpreted as the percentage of excess belief movement relative to the amount of uncertainty reduction. Under the null hypothesis that people are Bayesian, we expect $X_{norm} = 1$.

$$X_{norm} = \frac{\overline{m}_{1,2}}{\overline{r}_{1,2}} = \frac{X}{\overline{r}_{1,2}} + 1 \tag{11}$$

As proposed by Augenblick and Rabin (2021), we will report the excess belief movement, the Z statistics and the normalized belief movement in our results. Overall, this test will help us to reject the hypothesis that people's updating rule has the martingale property, which includes Bayesian updating.

5.3.2 Test Assumptions

The validity of this excess belief movement test hinges on two critical assumptions. Firstly, the agent's prior has to be correct. If the agent reported an incorrect prior and updated this incorrect prior in a Bayesian manner, the equality relationship between belief movement and uncertainty reduction will not hold. Consider a simple two-state model with a fully revealing signal. Suppose the correct prior that state 1 occurred is 0.5 but the agent holds an incorrect prior of 0.7. If this trial is repeated many times the agent will expect that 70% of the time, the posterior belief is 1 and 0 in the remaining 30% of the time. However, when the data is collected, half the time the posterior belief will be 1 and 0 otherwise. We may incorrectly conclude that the agent is non-Bayesian. Unfortunately, we are unable to validate this assumption, however, Augenblick and Rabin (2021) showed that their test has low power in detecting incorrect prior beliefs in a calibration exercise. This means that even when respondents begin with a wrong prior but are updating in a Bayesian manner, it is unlikely for the test to reject the hypothesis that people are non-Bayesian when they have wrong priors.

Secondly, the individual wage distribution has to remain stable over time. If the individual wage distribution is changing over time, the equality of the expected belief movement and uncertainty reduction will not hold. Intuitively, when the state is ever-changing, we expect beliefs to change constantly and the uncertainty will not be resolved. To test this assumption we perform a KL divergence test to determine if the offers received by the survey respondents are the same across time.

We have about 500 subjects who reported wage offers they received in the first and second surveys. We test whether the (log) average offer distribution differs between the pre-period 1 response and the between period 1 and 2 responses for these individuals to get a sense of wage offer distribution stability in our sample. The graph below plots both offer distributions.

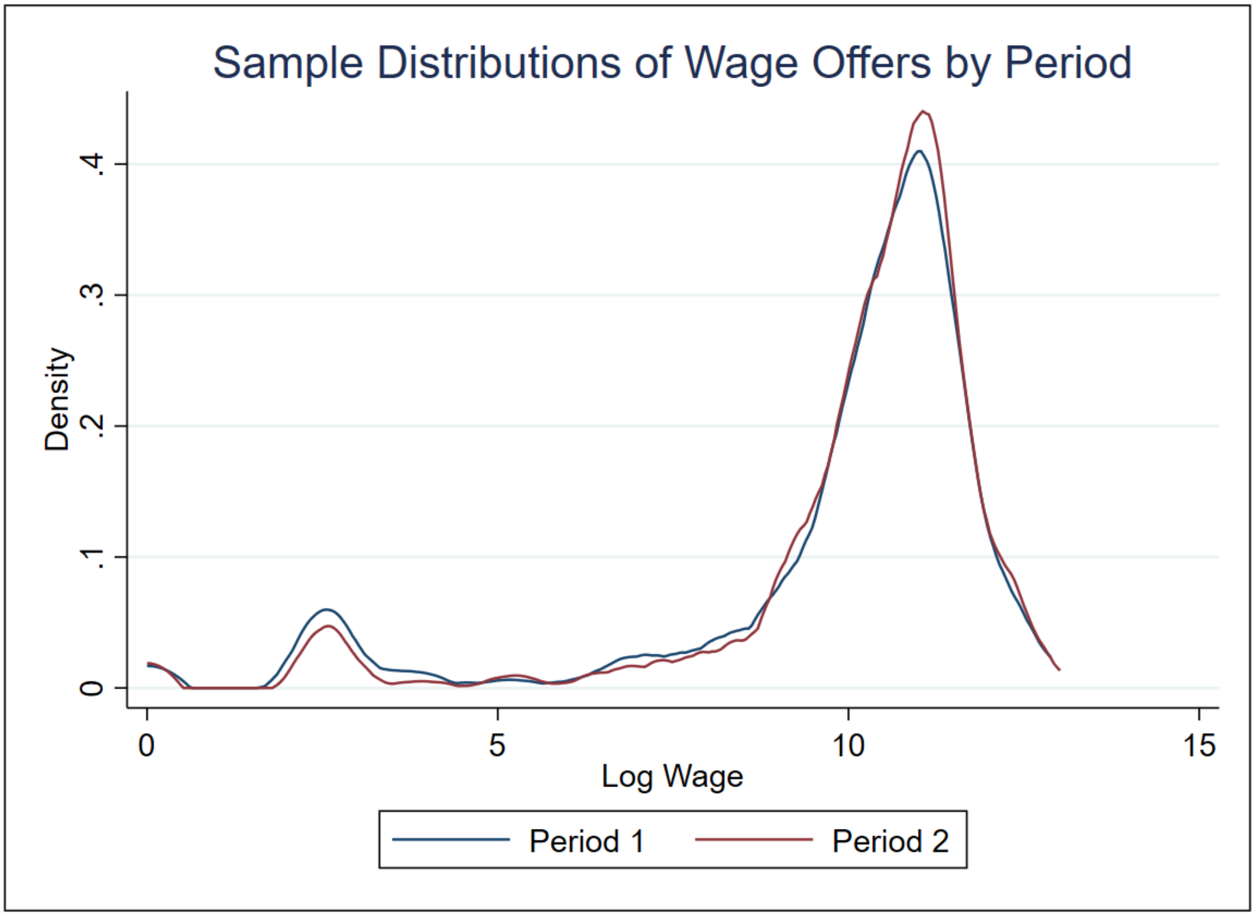

Figure 5: Period Wage Distributions

We formally test whether the two distributions differ using the Kullback–Leibler divergence.

Statistic	Value
Entropy	0.0179318
Standard Error	0.0091245
Histogram Bins	20
Comparison Observations	854
Reference Observations	854

Table 4: Kullback–Leibler Divergence Statistics for Period Log Wage Offer Distributions.

We note that the lower bound of the 95% confidence interval is 0.0000354, so that the difference between the distributions by this measure may be very close to 0. Therefore, we do not find strong evidence that the distribution undergoes major changes between periods on average.

6 Bayesian Updating (Martingale) Test Results

6.1 Martingale Test Statistics

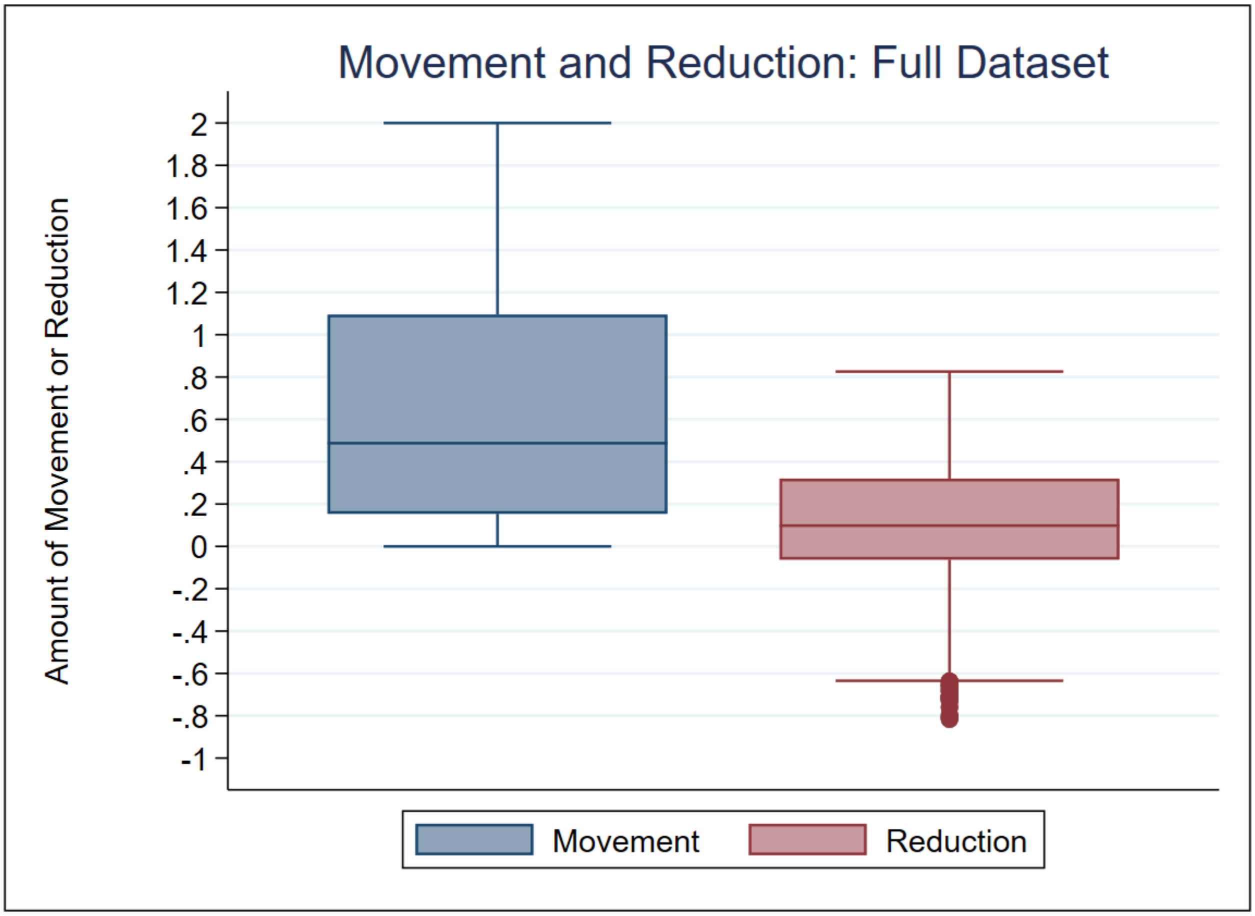

Figure 6: Movement and reduction summary statistics.

We first present the result from the excess belief movement. Overall we found excess belief movement relative to the amount of uncertainty reduction as shown in column 1 of table 5 and the box plot in figure 6. We found an excess belief movement of X =

0.5314 and this is statistically different from 0. The normalized excess belief movement of $X_{norm} = 5.2186$ which means that belief is moving 421% more than the amount of uncertainty reduction. Excess belief movement suggests that people are over-updating relative to the Bayesian benchmark, and this is consistent with updating biases such as base rate neglect and overreaction to signals.

Statistic	All Individuals	Got Offer?		Searched?	
		Yes	No	Yes	No
X	.5314	.5924	.5161	.5321	.5290
Z	58.4407	27.8964	51.4247	28.9208	48.0481
X_{norm}	5.2186	5.4344	5.1600	5.1430	5.4421
Observations	4,859	978	3,881	1,212	3,252

Table 5: Excess movement statistics: Main results.

We hypothesized that people who received a job offer would update in a more Bayesian manner because job offers are individualized feedback about one's wage distribution, while other information like labor market news are at the population level. But surprisingly, we found the opposite effect. Respondents who received an offer updates in a more non-Bayesian manner compared to people who do not have a job offer.

We also examine the heterogeneous effect between respondents who are searching for a job and not searching. Intuitively, people who are actively searching for a job are likely to put in more thought in forming their wage expectation. This could mean job searchers are more Bayesian than non-job searchers. However, we found that both groups are equally non-Bayesian; this could be suggestive that increased mental focus in job seeking does not help with becoming more Bayesian.

Figure 7 below provides an alternative look into the martingale test results. The provided box plot compares belief movement and uncertainty reduction measures of four different groups of individuals. The graph shows the surprising result that movement and reduction remain fairly similar across both categories, contrary to our prediction that those with personal wage and those who are searching would behave in a more Bayesian manner.

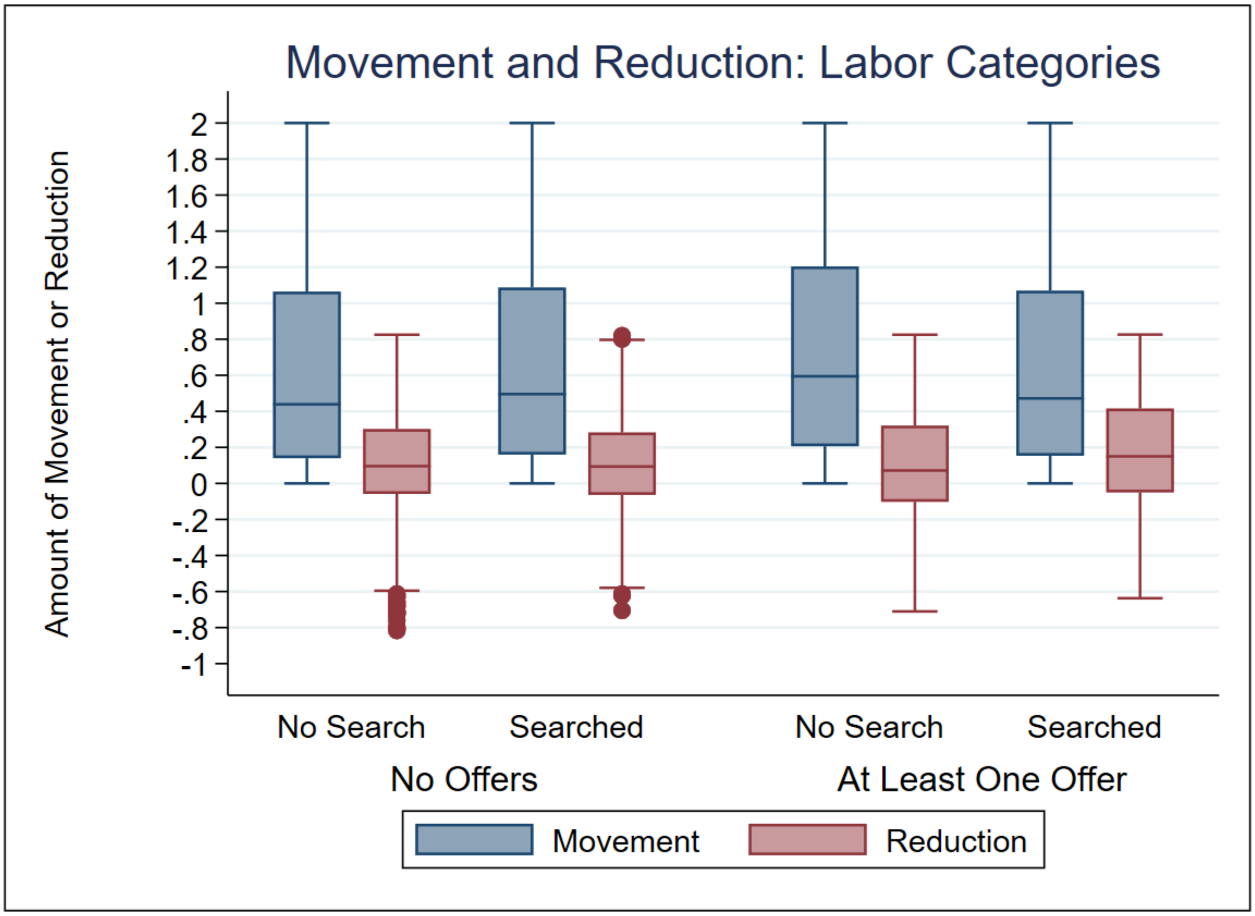

Figure 7: Movement and reduction summary statistics.

6.2 Heterogeneous Effect by Demographics

We also examine heterogeneous effects across different demographic groups. These differences, while not causal, might suggest labor market updating phenomena for further study.

First, when we divide the population by age, we find that the oldest group (individuals 60 and older) had the lowest normalized excess movement of any demographic group (not just compared to other age groups). While the oldest group and the middle group (individuals 40 to 59) had similar excess movement values, the normalized value for the middle group is nearly twice that of the older group. This is because the oldest group had the highest level of uncertainty reduction among all demographic groups (0.14, while most other groups had about 0.10).

When we divide the sample by education level, individuals with at least some college education are closer to the Bayesian benchmark compared to individuals with only a high school diploma or lower. We found a normalized excess belief movement that is 1.36 points higher for those with a lower level of education, while both groups had similar uncertainty reduction statistics of about 0.1. Unlike in the previous comparison, here the difference in the normalized statistic between the demographic groups is because those with a higher level of education are less reactive in changing where they think the mean is, not because they become more sure about their guess than the lower education group as the older group did compared to the other age groups.

The difference in normalized movement statistics between the gender groups was the smallest of the group differences, with the female group having a normalized statistic 0.4243 higher than the male group. For the demographic race variable, the difference between the minority and majority group statistics was 1.4109.

Statistic	Age			Some College		Gender		Race	
	18 to 39	40 to 59	60 and Above	Yes	No	Female	Male	White	Non-white
X	.4614	.5707	.5666	.5194	.6756	.5618	.5070	.5204	.5885
Z	33.0062	39.9160	27.5887	55.8095	17.7766	40.0788	42.5914	53.0069	24.7031
X_{norm}	5.2187	6.3975	3.9800	5.1160	6.4728	5.4509	5.0266	5.0216	6.4325
Observations	1,704	2,063	1,092	4,484	375	2,177	2,680	4,070	789

Table 6: Excess movement statistics: Demographic groups.

6.3 Test Implications for Updating Rules

Given the result that people are non-Bayesian, what are some updating rules that can generate this data? Firstly, we can reject all updating rules with the martingale property. This includes Bayesian updating and the Epstein, Noor and Sandroni (2010) model.

The Grether (1980) model as described in equation 4 can be consistent with the results obtained from this test. If we set $a < 1$ and/or $b > 1$, which suggests the agent exhibits base-rate neglect and overreaction to signals, we can get excess belief movement. The intuition is that if people overweight the signal they observe or underweight their prior beliefs, this will result in over-updating in the beliefs compared to the Bayesian benchmark which results in excess belief movement. This finding supports the robust finding from lab experiments where people neglect their priors (Kahneman and Tversky, 1972; Benjamin, 2019; Esponda, Vespa and Yuksel, 2020).

The Hagmann and Loewenstein (2017) updating rule and the multiple prior model (Ortoleva, 2012; Ba, 2022) can produce excess belief movement in the beliefs with an appropriate combination of model parameters.

7 Robustness Check: Measurement Error

In surveys, it is possible to potentially misreport their beliefs or round off some of the probabilities. In this section, we address the concern that our primary result might be driven by this measurement error.

7.1 Theoretical Analysis

The normalized excess belief movement statistics is significantly larger than the estimates Augenblick and Rabin (2021) got in their empirical application in their paper. To further understand why we have such a large estimate we perform a Monte Carlo Simulation where we allow for measurement error.

Consider that the agent has a true belief of π_t but reports a distorted $\hat{\pi}_t = \pi_t + \epsilon_t$, where ϵ_t is the measurement error. In a two-state model, assuming that the measurement error term is mean zero with variance σ^2_ϵ and uncorrelated with recent belief and error realizations ($\mathbb{E}(\epsilon_t \pi_t) = \mathbb{E}(\epsilon_t \pi_{t-1}) = \mathbb{E}(\epsilon_t \epsilon_{t-1})$), Augenblick and Rabin (2021) showed that the excess belief movement will be equal to $2\sigma^2_{\epsilon_t}$.

Generalizing this to n states, we show that the excess belief movement will be equal to $\sum_{i=1}^{n} 2\sigma^2_{\epsilon^i_t}$.[14] With measurement error, the excess belief movement to be equal to the total variance of the measurement error in the prior belief multiplied by 2. One interesting feature of this test is only measurement error in the prior affects the test statistics. This motivates our method of fitting a log-normal distribution to the posterior to compare it to the prior beliefs, which minimizes the error term in the prior.

This result tells us that only the measurement error in the first period will affect the excess belief movement statistics. In the calibration exercise, we only have to introduce the measurement error to the prior belief. Moreover, given that we have an excess belief movement of more than 0.5, this suggests that the total variance of the measurement error has to be at least 0.25 to explain the result we are observing.

7.2 Monte Carlo Simulation

We perform a Monte Carlo simulation to determine how much measurement error is required to produce the result we obtained. The Monte Carlo simulation is designed to mirror the setting as closely as possible. Since there are 6 wage bins and most subjects responded to the survey twice, we have 6 states and with only 2 periods we will have only 1 signal realization and 1 update. With 4866 respondents in our dataset, we generated 5000 pairs of priors and posteriors in each simulation.

Since our statistics of interest only depend on measurement error in the prior, we only introduce measurement errors to the prior beliefs. We draw our simulated prior data from

[14] Refer to appendix A.1 for the poof.

a Dirichlet distribution that is centered around a uniform prior distribution.[15] Since we are working with uniform priors, the parameters of the Dirichlet distribution are equal. We then scale the parameters to adjust the variance of the distribution to match the normalized excess belief movement that we obtained.

With our assumptions on the measurement error as discussed in the earlier subsection, we can show that the belief movement will be larger by $\sum_{i=1}^{n} \sigma^2_{\epsilon^i_t}$ and the uncertainty reduction is smaller by $\sum_{i=1}^{n} \sigma^2_{\epsilon^i_t}$ compared to the scenario without measurement.[16] Assuming that only measurement error is driving the results we observe we can compute the supposed Bayesian belief movement and uncertainty reduction statistics. Based on the result we obtained the supposed Bayesian belief movement and uncertainty reduction is 0.392, and we try to match this statistics in our calibration exercise.

It is important to note that, there is infinitely many possible combinations of prior beliefs and distribution of posterior beliefs that can produce the same desired belief movement statistics. For this calibration exercise, we assume a uniform prior. The main reason for this assumption is we can construct a uniform distribution of the posterior beliefs with symmetric posteriors which is easier to work with.

The posterior belief is then selected to match the supposed Bayesian belief movement of 0.392. One of the possible posterior beliefs that is selected in this calibration exercise is $(0.75, 0.2, 0.05, 0, 0, 0)$. To obtain the full distribution of posterior beliefs, we can permutate the order of the probabilities and each of these permutations will be realized with equal probabilities.

Finally, the variance of the error term is not an intuitive measure of the amount of measurement error. To quantify measurement error in a more intuitive manner, we define $\hat{\pi}^i_1$ as the prior beliefs drawn from the Dirichlet distribution. The measurement error, $\Delta \equiv \sum_{i=1}^{6} |\hat{\pi}^i_1 - \pi^i_{prior}|$, is the total "distance" between the beliefs drawn from the Dirichlet

[15]The draws from the Dirichlet distributions are a valid probability distribution that has to sum to 1. This approach is better than adding an error that is normally distributed which may cause the probability measure to be negative or exceed 1.

[16]This is only true if there is only measurement error in the prior.

distribution and true prior.

We are interested in the normalized excess movement and Δ, this process is simulated 10,000 times. In Table 7, we report the average statistics computed from the 10,000 Monte Carlo simulations and the interval that contains 95% of our simulation result.

We see that in order to obtain an excess belief movement from our dataset assuming the respondents are Bayesian, we need the survey respondents to misreport their prior beliefs by about 98 percentage points. The measurement error required to rationalize the behavior is very large and is unlikely to explain the result we have obtained.

Statistic	Uniform Prior with Matched Belief Movement
X	0.5275
	[0.5158, 0.5388]
X_{norm}	5.2349
	[5.0219, 5.4579]
Δ	0.9831
	[0.9765, 0.9899]

Table 7: Measurement error calibration statistics.

8 Individual Level Analysis

We now present a few more analyses designed to provide additional insights on the updating rules without the martingale property.

8.1 Zero Probability Analysis

The Grether (1980) model makes a stark prediction where once an event is thought to be impossible, no signal can change the agent's mind about this. However, in a the Hagmann and Loewenstein (2017) model and the multiple prior model (Ortoleva, 2012; Ba, 2022), it is possible for the agent to update to a positive probability. In this data, we observe many instances where individuals who thought it was impossible to get a certain range of wage offers changed their minds when they are surveyed again.

One important caveat is in the Grether (1980) model, there is a special case of $a = 0$ (perfect base rate neglect). In this situation, it may be possible for depending on how 0^0 is defined in the model. When we find behavioral patterns that rejects the Grether (1980) model, we are rejecting the model when $a > 0$.

The main idea of this analysis is to find beliefs which were initiall reported to be zero, but which become positive in the second survey. In general, this cannot be done directly, because the bin definitions change for each distribution, as described previously in the empirical strategy section. We use two different approaches for detecting this behavior.

8.1.1 Subset Method

The first method (hereafter referred to as the "subset method") relies on identifying wage bins from the posterior distribution that are a subsets of wage bins on the prior distribution. We attempt to identify behavioral patterns as shown in the tables below which shows a survey response.

	$w < 52,000$	$52,000 \leq w < 58,500$	$58,500 \leq w < 65,000$	$65,000 \leq w < 71,500$	$71,500 \leq w < 78,000$	$w \geq 78,000$
$p(\cdot)$	0	0	0.7	0.2	0.1	0

Table 8: Period 1 Beliefs

	$w < 44,000$	$44,000 \leq w < 49,500$	$49,500 \leq w < 55,000$	$55,000 \leq w < 60,500$	$60,500 \leq w < 66,000$	$w \geq 66,000$
$p(\cdot)$	0	0.3	0.4	0.3	0	0

Table 9: Period 2 Beliefs

Note that the individual places zero probability on the wage bin $w < \$52,000$ in period 1's belief. When the individual updates their beliefs, the second bin, $\$44,000 < w < \$49,500$ is a subset of the bin from the first period distribution, and there is a probability weight of 30% in this bin. We can reject the this individual is updating his belief in a inconsistent with the Grether (1980) model.

The strength of this approach is that it is nonparametric and can be used directly with the data without any additional assumptions. However, this method is restrictive as it requires zero probability weight and a posterior bin to be a subset of the prior bin to identify such behavior. As such, we should interpret the results as a lower bound of the number of the number of people who are inconsistent with the Grether (1980) model.

8.1.2 Simulated Cutoff Method

The second method (hereafter referred to as the "simulated cutoff method") relies on the belief fitting process described in section 5. The following table shows the fitted period 2 beliefs for the same individual from the earlier table.

	$w < 52,000$	$52,000 \leq w < 58,500$	$58,500 \leq w < 65,000$	$65,000 \leq w < 71,500$	$71,500 \leq w < 78,000$	$w \geq 78,000$
$p(\cdot)$	0.4095	0.5880	0.0026	0	0	0

Table 10: Period 2 Fitted Beliefs

Note that since the fitting the beliefs gives us a full distribution, we are able to directly compare the fitted posterior beliefs to the raw prior beliefs using the wage bins from the prior. However, since the fitted distribution is a log normal distribution, it will technically have positive weight over any positive wage values.[17] We label the individual as being inconsistent with the Grether (1980) model if the estimated posterior distribution has a probability weight that exceeds a certain threshold in wage bins that were initially given a

[17] The last three bins shown are exactly 0, but this is only because the table values are rounded to four decimal places.

zero probability weight.

The lowest cutoff is for a fitted bin to have at least 0.5% probability to be considered non-zero, while the highest cutoff is for a fitted bin to have at least 10% probability. We choose 10% probability to provide some leeway for errors from estimating the posterior belief.

8.1.3 Results

The results of the two methods for finding people who are inconsistent with the Grether (1980) model is shown in the chart below. Overall, we estimate that about 20-50% of the observations in our data that updates in a way that is inconsistent with the Grether (1980) model.

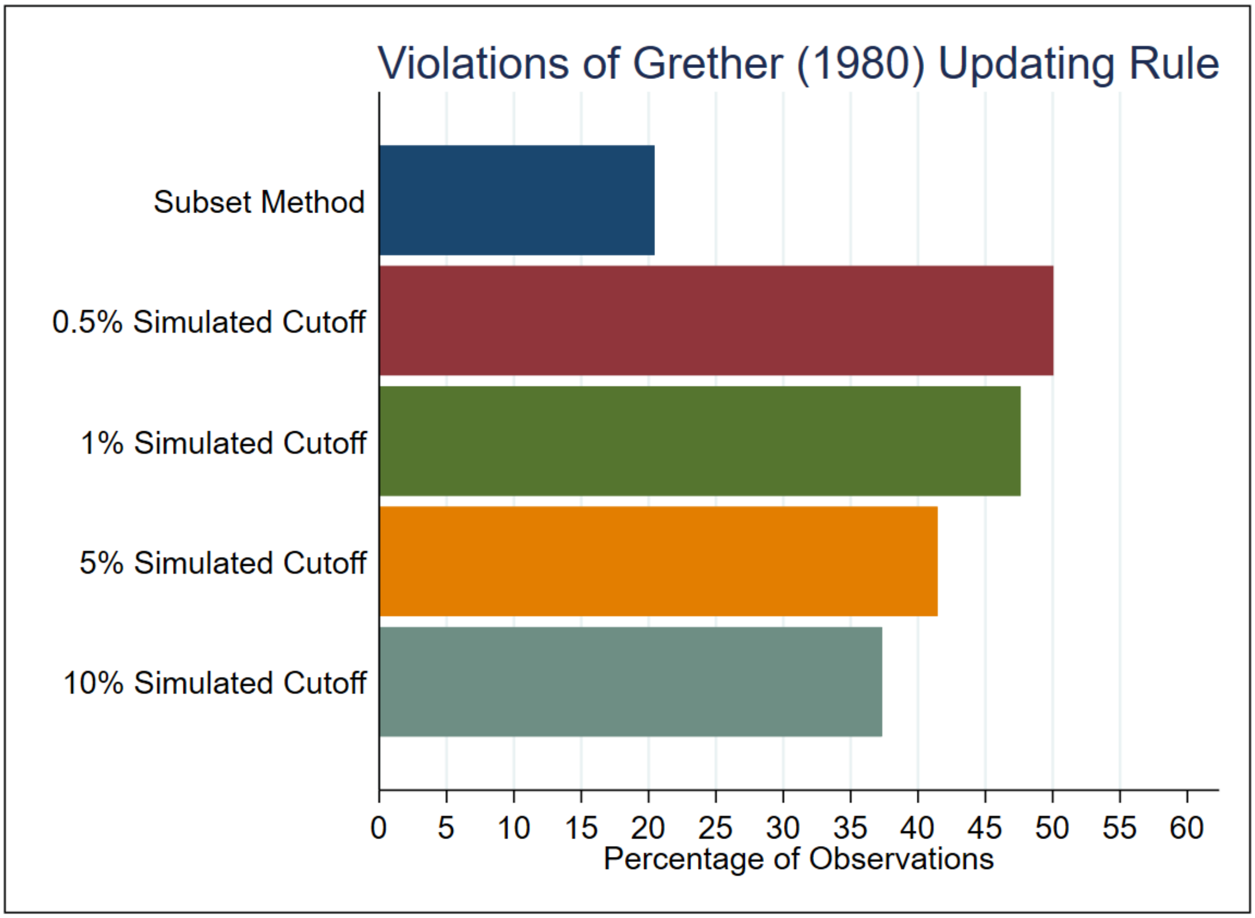

Figure 8: Fraction of observations which are inconsistent with Grether (1980).

Using the most conservative subset method, about 20% of the individuals update a zero probability event into a positive probability event. When we make some parametric assumption about the updated belief distribution, we find that about 37-50% of the subjects are in consistent with the Grether (1980) model.

8.2 Other non-Bayesian Updating Patterns

For individuals who reported receiving a job offer, we have some information about the signals they have received during the job search. This allows us to examine other updating patterns as well.

Under a standard Bayesian updating job search model, when an agent receives a wage offer that is lower than the prior mean, the agent would adjust his wage expectation downwards. Conversely, when the agent receives a wage offer that is higher than his prior mean, he will adjust his expectation upwards. In a Gaussian updating framework, the mean of the posterior belief should always be a convex combination between the prior mean and the wage offer.

Motivated by this framework, we look at individuals whose mean updated beyond the wage from the reported job offers ("overshooting") and those whose mean moved in the opposite direction of any wage offer ("wrong direction"). We found individuals who exhibit these updating patterns, but we also see that there is an asymmetry in the direction of the update which is consistent with motivated beliefs (Bénabou and Tirole, 2002; Eil and Rao, 2011) and Hagmann and Loewenstein (2017) model. The fraction of these updating pattern is summarized in the figure 9.

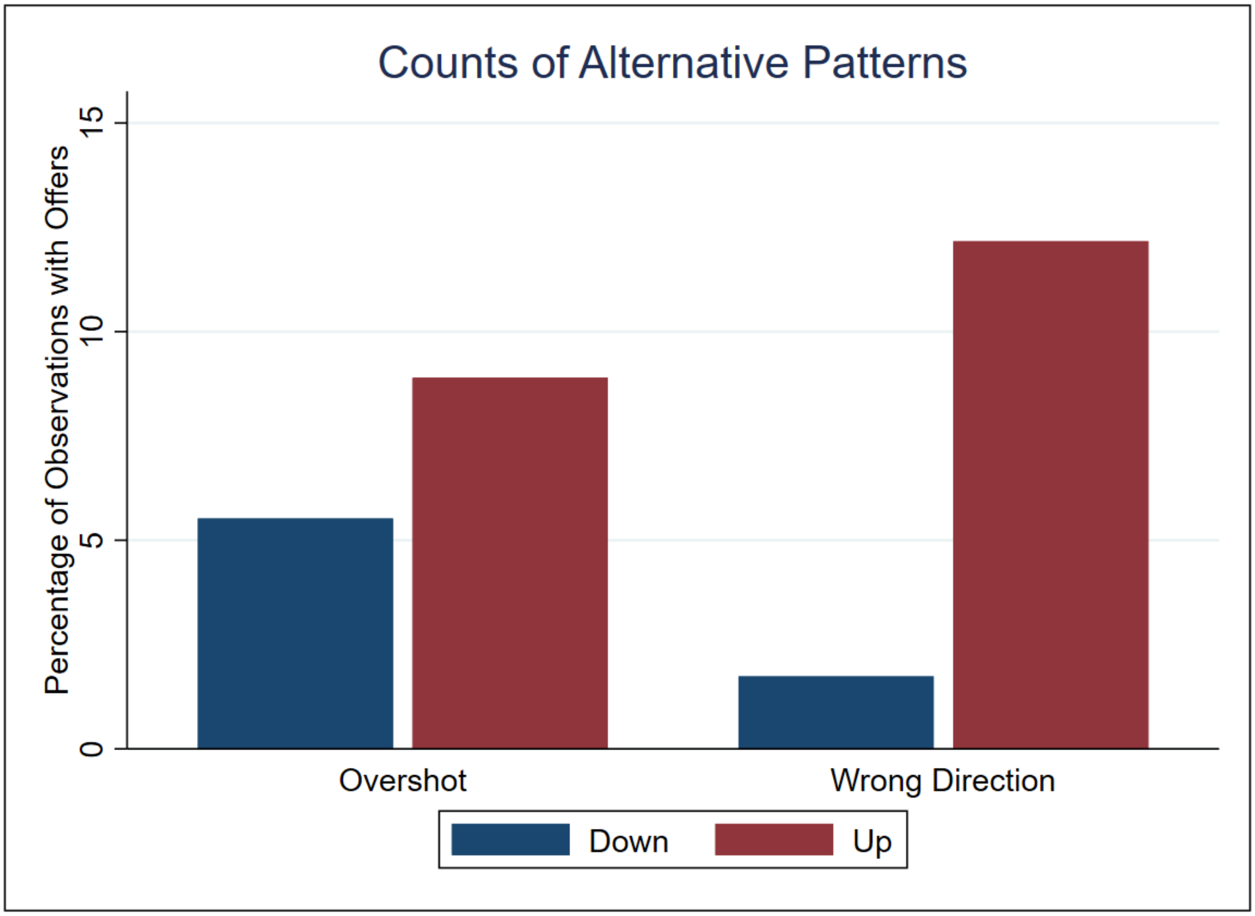

Figure 9: Counts of specific patterns in data as percentages.

We note that for both of these measures, the number of individuals making this mistake by updating upward exceeds the number of individuals making this mistake by updating downward. Specifically, we see that 87 individuals (8.9% of all observations with offers) overshoot upward, while 54 individuals (5.52% of all observations with offers) overshoot downward. The difference is more stark for those updating in a direction opposite to all reported wage offers, with only 17 (1.74 % of all observations with offers) updating downward and 119 (12.17% of all observations with offers) updating upward.

Because the number of people receiving a signal to adjust belief upward is different from the number of people receiving a signal to adjust the belief downwards, we cannot just compare the occurrence of these mistakes. In figure 10, we see that among subjects who received an upward signal about 24% of the subject overshot compared to 11% of subjects who overshot when received a downward signal. When subjects receive a downward signal 22% of the subject updated upward instead compared to 5% of subjects who updated downwards when a signal to update upwards is received.

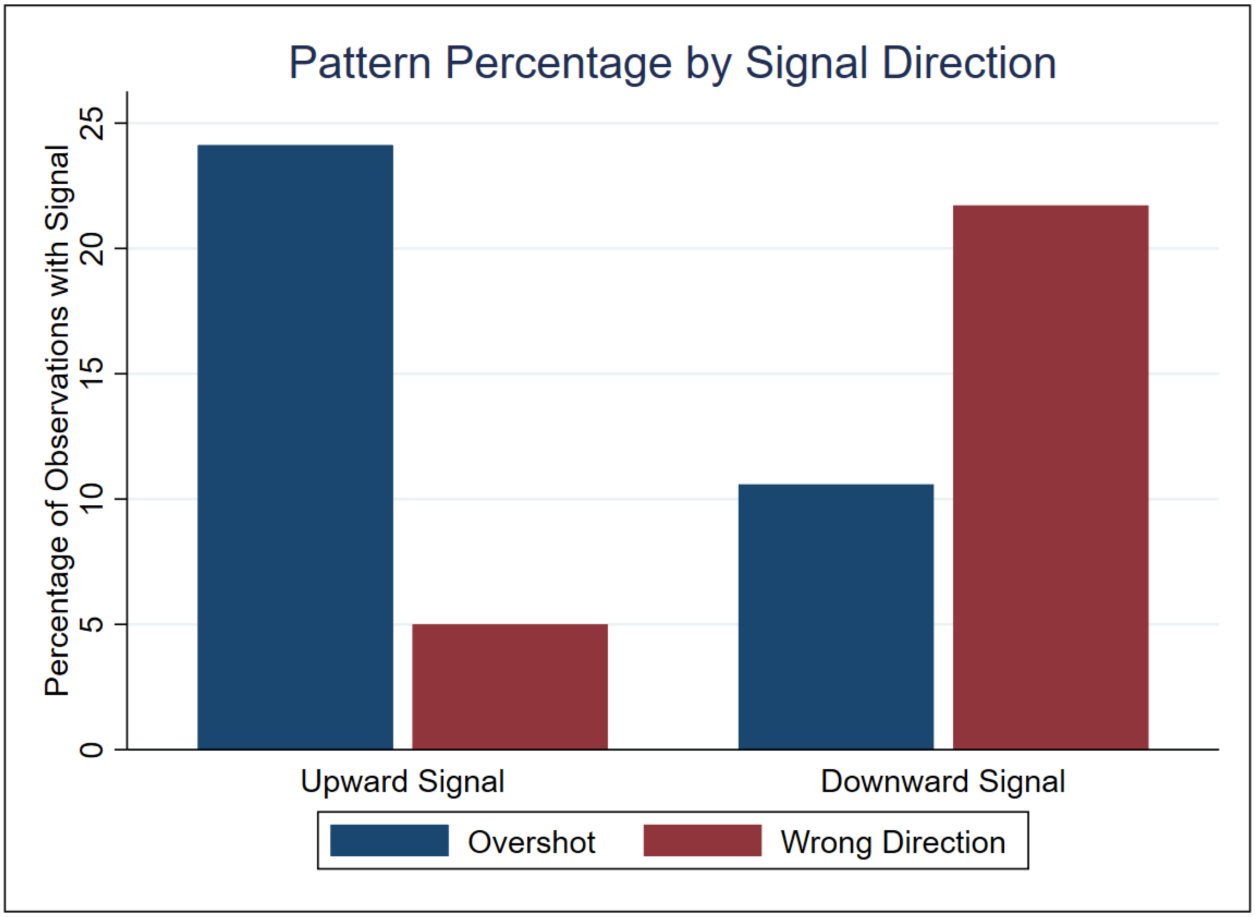

Figure 10: Signal direction defined as relative to reported first period average offer expectation.

This behavior is consistent with motivated beliefs (Bénabou and Tirole, 2002; Eil and Rao, 2011) where individuals want to have an optimistic outlook on their job prospect. In a theoretical model like Hagmann and Loewenstein (2017), we set the reference belief to an "optimistic" belief and we can obtain this asymmetric updating pattern where beliefs would move upwards. If we set λ to be large it is possible for people to update in the wrong direction as well.

Another possible explanation for people updating in the wrong direction is the gambler's fallacy (Rabin, 2002).[18] When an individual sees a low offer, he thinks that the next offer is more likely to be high. In this dataset, we see that people exhibit this bias in a self-serving way as the gambler's fallacy is more likely to manifest when they are supposed to update their belief downwards.

9 Discussion

We found significantly more movement than uncertainty reduction in our data. This is consistent with base-rate neglect and overreaction to signals. We find updating patterns that can reject Bayesian updating and Epstein, Noor and Sandroni (2010) updating rule. A significant proportion of individual exhibit updating pattern that is inconsistent with Grether (1980) model.

Moreover, we detected evidence of asymmetric updating. When providing information that can cause individuals to adjust their wage expectation downwards, we may need a stronger or informative signal. This behavioral pattern can be explained by the Hagmann and Loewenstein (2017) updating model.

Our findings suggest that information intervention is a powerful tool for influencing people's beliefs in a job search setting, due to over-updating relative to the Bayesian benchmark. However, in our information-rich world, where individuals are continuously exposed to a stream of new information, it is essential to recognize that the effect of providing a single piece of information may be short-lived as people may overreact to the new information they receive or neglect the older information. Further, since we did not find that the overreaction bias was reduced for individuals receiving a signal directly from the labor market personalized to them, it could be that not all sources of labor information may have the same level of impact on affecting beliefs. To ensure a more lasting impact on beliefs, it might be necessary to adopt a strategy of consistently providing new and relevant information over time, as well as experimenting with different types of labor market information to see which

[18] This is conjecture posited by Mueller, Spinnewijn and Topa (2021) as well who analyzed the SCE data.

kind individuals use the most effectively.

Our results are limited in a few important ways that leave scope for future work. First, our sample chiefly includes employed individuals, so it is difficult for us to comment on the differences between employed and unemployed learning, with our analysis mostly applying to learning among employed individuals. Second, we find a large effect among individuals with no wage offers, implying that our main result may be the product of unobservable factors or survey design. Future analyses could test the sensitivity of question phrasing and propose possibly more effective survey techniques. Another limitation is that we observe most individuals for only two periods, which limits our ability to study learning over extended time periods. Future analyses could collect higher-frequency data, allowing for a better study of the change in learning biases over time and across different time horizons.

Another potential direction for future work would be to more closely examine the mechanisms behind the differences in updating for the demographic variables. For example, the result that the group of individuals 60 and over have higher uncertainty reduction than the rest of the demographic groups might have implications for how those in or close to retirement learn about the labor market differently than others.[19] Differences in learning might have important implications for studying how individuals choose to retire or stay retired. A field experiment on this topic might also be able to comment further on the role of motivated self-image concerns, employment opportunities, and control over one's own employment status in learning about the labor market, as each of these factors is likely to be different for those approaching or in retirement compared to the rest of the population.

[19] While only roughly 6% of individuals in the 40 to 59 group reported being out of the labor force, roughly 42% of the 60 and older group reported being out of the labor force.

Appendix A Proofs

A.1 Measurement Error

Suppose the survey respondent report a distorted prior of $\hat{\pi}_t = \pi_t + \epsilon_t$, where ϵ_t is the measurement error. In a two-state model, assuming that the measurement error term is mean zero with variance σ_ϵ^2 and uncorrelated with recent belief and error realizations ($\mathbb{E}(\epsilon_t \pi_t) = \mathbb{E}(\epsilon_t \pi_{t-1}) = \mathbb{E}(\epsilon_t \epsilon_{t-1})$).

The belief movement will be larger by σ_ϵ^2

$$\begin{aligned}
\mathbb{E}(M_{t,t+1}) &= \mathbb{E}(\pi_{t+1} + \epsilon_{t+1} - \pi_t - \epsilon_t)^2 \\
&= \mathbb{E}[(\pi_{t+1} - \pi_t)^2 + 2(\pi_{t+1} + \pi_t)(\epsilon_{t+1} - \epsilon_t) + (\epsilon_{t+1} - \epsilon_t)^2] \\
&= \mathbb{E}[(\pi_{t+1} - \pi_t)^2 + \epsilon_{t+1}^2 - 2\epsilon_{t+1}\epsilon_t + \epsilon_{t+1}^2] \\
&= \mathbb{E}[(\pi_{t+1} - \pi_t)^2] + \sigma_{\epsilon_t}^2 + \sigma_{\epsilon_{t+1}}^2
\end{aligned}$$

The uncertainty reduction will be smaller by σ_ϵ^2

$$\begin{aligned}
\mathbb{E}(R_{t,t+1}) &= \mathbb{E}[(\pi_t + \epsilon_t)(1 - \pi_t - \epsilon_t) - (\pi_{t+1} + \epsilon_{t+1})(1 - \pi_{t+1} - \epsilon_{t+1})] \\
&= \mathbb{E}[(\pi_t)(1 - \pi_t - \epsilon_t) + \epsilon_t(1 - \pi_t - \epsilon_t) - (\pi_{t+1})(1 - \pi_{t+1} - \epsilon_{t+1}) + \epsilon_{t+1}(1 - \pi_{t+1} - \epsilon_{t+1})] \\
&= \mathbb{E}[(\pi_t)(1 - \pi_t) - \pi_{t+1}(1 - \pi_{t+1})] - \sigma_{\epsilon_t}^2 + \sigma_{\epsilon_{t+1}}^2
\end{aligned}$$

The expected excess belief movement statistics for a Bayesian agent with measurement error is

$$\mathbb{E}(M_{t,t+1}) - \mathbb{E}(R_{t,t+1}) = 2\sigma_{\epsilon_t}^2$$

If we generalize this to n states,

$$
\begin{aligned}
\mathbb{E}(M_{t,t+1}) &= \mathbb{E}\left[\sum_{i=1}^{n}(\pi_{t+1}^{i}+\epsilon_{t+1}^{i}-\pi_{t}^{t}-\epsilon_{t}^{i})^{2}\right] \\
&= \mathbb{E}\left[\sum_{i=1}^{n}(\pi_{t+1}^{i}-\pi_{t}^{i})^{2}+2(\pi_{t+1}^{i}-\pi_{t}^{i})(\epsilon_{t+1}^{i}-\epsilon_{t}^{i})+(\epsilon_{t+1}^{i}-\epsilon_{t}^{i})^{2}\right] \\
&= \mathbb{E}\left[\sum_{i=1}^{n}(\pi_{t+1}^{i}-\pi_{t}^{i})^{2}\right]+\sum_{i=1}^{n}(\sigma_{\epsilon_{t}}^{i})^{2}+(\sigma_{\epsilon_{t+1}}^{i})^{2}
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{E}(R_{t,t+1}) &= \mathbb{E}\left[\sum_{i=1}^{n}(\pi_{t}^{i}+\epsilon_{t}^{i})(1-\pi_{t}^{i}-\epsilon_{t}^{i})+(\pi_{t+1}^{i}+\epsilon_{t+1}^{i})(1-\pi_{t}^{i}-\epsilon_{t+1}^{i})\right] \\
&= \mathbb{E}\left[\sum_{i=1}^{n}(\pi_{t}^{i})(1-\pi_{t}^{i}-\epsilon_{t}^{i})+\epsilon_{t}^{i}(1-\pi_{t}^{i}-\epsilon_{t}^{i})-(\pi_{t+1}^{i})(1-\pi_{t+1}^{i}-\epsilon_{t+1}^{i})+\epsilon_{t+1}^{i}(1-\pi_{t+1}^{i}-\epsilon_{t+1}^{i})\right] \\
&= \mathbb{E}\left[\sum_{i=1}^{n}(\pi_{t}^{i})(1-\pi_{t}^{i})-\pi_{t+1}^{i}(1-\pi_{t+1}^{i})\right]+\sum_{i=1}^{n}(\sigma_{\epsilon_{t+1}}^{i})^{2}-(\sigma_{\epsilon_{t}}^{i})^{2}
\end{aligned}
$$

The excess belief movement is

$$
\mathbb{E}(M_{t,t+1})-\mathbb{E}(R_{t,t+1})=\sum_{i=1}^{n}2(\sigma_{\epsilon_{t}}^{i})^{2}
$$

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