A NEW SIMPLE AUTO-TUNING METHOD FOR PID CONTROLLERS

Damir Vrančić, Youbin Peng*, and Janko Petrovčič

J. Stefan Institute
Jamova 39, SI-1000 Ljubljana, Slovenia
e-mail: damir.vrancic@ijs.si
home page: http://www-e2.ijs.si/Damir.Vrancic

*Free University of Brussels
F. D. Roosevelt 50, CP. 165, B-1050 Brussels, Belgium
e-mail: peng@labauto.ulb.ac.be

Abstract: A simple procedure of tuning the PID controllers, based on the so-called magnitude optimum technique, is given. The tuning procedure requires only the process open-loop step response in order to calculate the PID controller parameters.

Keywords: PID controllers, amplitude optimum, moment method, step response, self-tuning regulators

1. INTRODUCTION

During the past 50 years, many different approaches for tuning the PID types of controllers have been developed. Each tuning method has its advantages and drawbacks. In general, the more information that can be derived from the process, the better are the tuning results that can be obtained. However, getting more process information requires more extensive process identification and extensive computations, with longer and more complicated experiments on the testing plant. Moreover, it has been recognised (see Peterka and Åström, 1980) that developing *accurate models* for the process industry, and identifying the parameters within them, is often not worthwhile; the problem is, rather, how to identify the *controller* of the process.

On the other hand, tuning rules which gather less process information (e.g. process gain, lag and rise times, or the process ultimate point) generally do not give the optimal response, since the available information is based only on few identified process parameters. For instance, there are some quite different processes with the same pair of lag and rise times (or ultimate points); clearly, these processes require different controller parameters. Several problems, which arise when using the Ziegler-Nichols tuning rules, are reported in Hang and Cao, (1993); Hang and Sin, (1991); Voda and Landau, (1995). Some authors have suggested the introduction of so-called set-point weighting approach (Hang and Cao, 1993) and of the additional change of the tuning rules (Åström and Hägglund, 1995). The improved closed-loop response has shown that they were successful for some types of processes, but the rules were still based on measurements of the process lag and rise time (or ultimate point), so their applicability to a wide spectrum of processes still remains an open problem.

Our approach was to find a simple tuning procedure which could easily be executed by the real-time auto-tuning algorithm, and which would give the optimal controller settings for a large class of process models. It has been decided that the tuning algorithm should be based on the process step response, because it can be simply obtained on the real plant. Instead of detecting the lag and rise times (which can be difficult in a noisy environment), we tried to extract as much information as possible from the process step response by using the function which is quite inert to the high-frequency process noise and can be simply performed by the real-time digital algorithm: an integration (Nishikawa et al., 1984; Vrančić et al., 1996). The multiple integration approach is applied, originally used as one of the identification techniques (see e.g. Rake, 1987; Strejc, 1959).

It was also decided to use the "magnitude" ("modulus", "Betrags") optimum (MO) tuning approach (Åström and Hägglund, 1995; Hanus, 1975; Kessler, 1955; Umland and Safiuddin, 1990). This technique guarantees fast, stable and non-oscillatory closed-loop behaviour for a large class of industrial processes.

In our research we found the relation between the five integrals (areas), obtained directly from the process step response (using the multiple integration approach), and the MO technique. This allows for the optimal PID controller parameters to be calculated merely from the integrals of the process open-loop step response, regardless of the process order.

2. THEORETICAL EVALUATION

The tuning procedure for the PID controller is given for processes which can be approximated by the following transfer function:

$$G_P(s) = K_{PR}\frac{1+b_1s+b_2s^2+\cdots+b_ms^m}{1+a_1s+a_2s^2+\cdots+a_ns^n}e^{-sT_{del}}, \qquad (1)$$

where K_{PR} denotes the process steady-state gain and a_1 to a_n and b_1 to b_m are the corresponding parameters ($m \leq n$) of the process transfer function. A parameter T_{del} represents the process pure time delay.

The PID controller is given by the following transfer function:

$$G_C(s) = \frac{U(s)}{E(s)} = K\left(1+\frac{1}{sT_i}+\frac{sT_d}{1+sT_f}\right), \qquad (2)$$

where U and E denote the Laplace transforms of the controller output and the control error ($e=w-y$), respectively. The controller parameters K, T_i, T_d and T_f denote proportional gain, integral time constant, differential time constant and filter time constant, respectively.

The PID controller in a closed-loop configuration with a process is shown in Fig. 1, where d denotes a load disturbance.

A tuning goal was to find such a controller that makes the closed-loop amplitude (magnitude) frequency response from the set-point to the plant output as flat and as close to unity as possible for a large bandwidth for a given plant and controller structure. This technique is called the magnitude optimum (Umland and Safiuddin, 1990) or modulus optimum technique (Åström and Hägglund, 1995). Such design objective results in a fast and non-oscillatory closed-loop time response for a large class of process models.

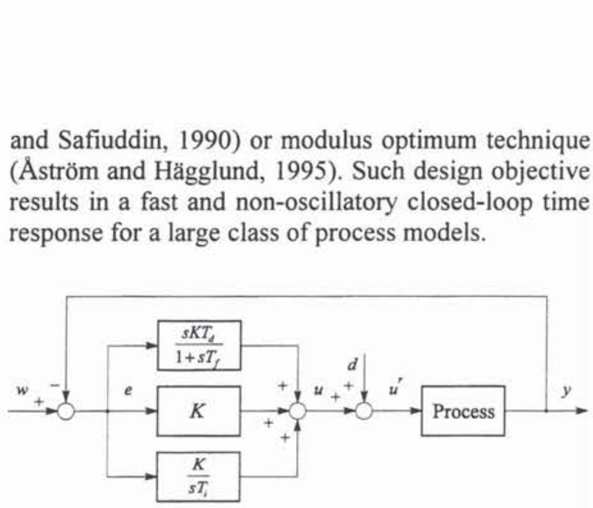

Fig. 1. The closed loop system with PID controller.

Following the procedure given by Hanus, (1975), such tuning goal can be achieved by moving the zeros of the function $\mathbf{Re}\{G_P(j\omega)G_C(j\omega)\}+1/2$ toward $\omega=0$.

When developing the pure time delay in (1), into the Taylor series:

$$e^{-sT_{del}} = 1 - sT_{del} + \frac{(sT_{del})^2}{2!} - \frac{(sT_{del})^3}{3!} + \cdots, \qquad (3)$$

the open-loop system transfer function can be expressed in the following way:

$$G_P(s)G_C(s) = \frac{d_0+d_1s+d_2s^2+d_3s^3+\cdots}{c_0s+c_1s^2+c_2s^3+c_3s^4+\cdots}. \qquad (4)$$

Parameters c_i and d_i can be calculated by inserting (1) to (3) into (4):

$$c_0 = T_i,\ c_1 = a_1T_i,\ c_2 = a_2T_i,\ c_3 = a_3T_i, \cdots \qquad (5a)$$

$$d_0 = K_{PR}K,\ d_1 = K_{PR}K\left[T_i + b_1 - T_{del}\right],$$
$$d_2 = K_{PR}K\left[T_iT_d + b_1T_i + b_2 - T_{del}(T_i + b_1) + \frac{T_{del}^2}{2}\right], \qquad (5b)$$

$$d_3 = K_{PR}K\begin{bmatrix} b_1T_iT_d + b_2T_i + b_3 - \\ -T_{del}(T_iT_d + b_1T_i + b_2) + \\ +\frac{T_{del}^2}{2}(T_i + b_1) - \frac{T_{del}^3}{6} \end{bmatrix}, \qquad (5c)$$

$$d_4 = K_{PR}K\begin{bmatrix} b_2T_iT_d + b_3T_i + b_4 - \\ -T_{del}(b_1T_iT_d + b_2T_i + b_3) + \\ +\frac{T_{del}^2}{2}(T_iT_d + b_1T_i + b_2) - \\ -\frac{T_{del}^3}{6}(T_i + b_1) + \frac{T_{del}^4}{24} \end{bmatrix}, \qquad (5d)$$

$$d_5 = K_{PR}K\begin{bmatrix} b_3T_iT_d + b_4T_i + b_5 - \\ -T_{del}\left(b_2T_iT_d + b_3T_i + b_4\right) + \\ +\frac{T_{del}^{2}}{2}\left(b_1T_iT_d + b_2T_i + b_3\right) - \\ -\frac{T_{del}^{3}}{6}\left(T_iT_d + b_1T_i + b_2\right) + \\ +\frac{T_{del}^{4}}{24}\left(T_i + b_1\right) - \frac{T_{del}^{5}}{120} \end{bmatrix} \tag{5e}$$

Note that T_f is fixed to zero so as to simplify a derivation of the PID controller parameters.

In order for the PID controller parameters to be found, as required by the presented magnitude optimum criterion, the first three (n=0..2) equations from the following set of equations are to hold (Hanus, 1975):

$$\sum_{i=0}^{2n+1}(-1)^i d_i c_{2n+1-i} = \frac{1}{2}\sum_{i=0}^{2n}(-1)^i c_i c_{2n-i} \tag{6}$$

When inserting (5) into (6), by setting n=0..2, the following PID controller parameters can be expressed by the unknown process parameters:

$$K = \frac{\begin{matrix} a_1^3 - a_1^2b_1 + a_1b_2 - 2a_1a_2 + a_2b_1 + a_3 - b_3 + \\ +T_{del}\left(a_1^2 - a_1b_1 - a_2 + b_2\right) + \frac{T_{del}^2}{2}\left(a_1 - b_1\right) + \frac{T_{del}^3}{6} \end{matrix}}{2K_{PR}\begin{bmatrix} -a_1^2b_1 + a_1a_2 + a_1b_1^2 - a_3 - b_1b_2 + b_3 + \\ +T_{del}\left(a_1 - b_1\right)^2 + T_{del}^2\left(a_1 - b_1\right) + \frac{T_{del}^3}{3} - \\ -T_d\left(a_1 - b_1 + T_{del}\right)^2 \end{bmatrix}} \tag{7}$$

$$T_i = \frac{\begin{matrix} a_1^3 - a_1^2b_1 + a_1b_2 - 2a_1a_2 + a_2b_1 + a_3 - b_3 + \\ +T_{del}\left(a_1^2 - a_1b_1 - a_2 + b_2\right) + \frac{T_{del}^2}{2}\left(a_1 - b_1\right) + \frac{T_{del}^3}{6} \end{matrix}}{\begin{matrix} a_1^2 - a_1b_1 - a_2 + b_2 + T_{del}\left(a_1 - b_1\right) + \frac{T_{del}^2}{2} - \\ -T_d\left(a_1 - b_1 + T_{del}\right) \end{matrix}} \tag{8}$$

$$T_d = f\left(a_1 \cdots a_5, b_1 \cdots b_5, T_{del}\right) \tag{9}$$

Note that the explicit result for the derivative time constant is not given due to the limitation of the space.

In order for the method to be applied, the real process must be approximated by the transfer function (1), which requires an explicit identification of the parameters K_{PR}, a_1 ... a_5, b_1 ... b_5, and T_{del}. Note that the identified model of the process must be of the same or a higher order than the real process so as to avoid the modelling error which could appear due to the insufficient order of the model. Therefore, calculation of (7) to (9), by using the process identification, is frequently impossible in practice.

However, it will be shown that the explicit identification of the process model can be avoided by using the concept of multiple integration (Strejc, 1959).

When applying the step-change ΔU at the process input, the following five areas can be expressed by integrating the process step response (y):

$$A_1 = y_1(\infty) = K_{PR}\left(a_1 - b_1 + T_{delay}\right), \tag{10}$$

$$A_2 = y_2(\infty) = K_{PR}\left[b_2 - a_2 + \frac{A_1a_1}{K_{PR}} - T_{delay}b_1 + \frac{T_{delay}^2}{2}\right] \tag{11}$$

$$A_3 = y_3(\infty) = K_{PR}\begin{bmatrix} a_3 - b_3 + \frac{A_2a_1}{K_{PR}} - \frac{A_1a_2}{K_{PR}} + \\ +T_{delay}b_2 - \frac{T_{delay}^2b_1}{2} + \frac{T_{delay}^3}{6} \end{bmatrix}, \tag{12}$$

$$A_4 = y_4(\infty) = K_{PR}\begin{bmatrix} b_4 - a_4 + \frac{A_3a_1}{K_{PR}} - \frac{A_2a_2}{K_{PR}} + \\ +\frac{A_1a_3}{K_{PR}} - T_{delay}b_3 + \frac{T_{delay}^2b_2}{2} - \\ -\frac{T_{delay}^3b_1}{6} + \frac{T_{delay}^4}{24} \end{bmatrix}, \tag{13}$$

$$A_5 = y_5(\infty) = K_{PR}\begin{bmatrix} a_5 - b_5 + \frac{A_4a_1}{K_{PR}} - \frac{A_3a_2}{K_{PR}} + \\ +\frac{A_2a_3}{K_{PR}} - \frac{A_1a_4}{K_{PR}} + T_{delay}b_4 - \\ -\frac{T_{delay}^2b_3}{2} + \frac{T_{delay}^3b_2}{6} - \\ -\frac{T_{delay}^4b_1}{24} + \frac{T_{delay}^5}{120} \end{bmatrix}, \tag{14}$$

where

$$\begin{aligned} y_1(t) &= \int_0^t\left(K_{PR} - \frac{y(\tau)}{\Delta U}\right)d\tau, \\ y_2(t) &= \int_0^t\left(A_1 - y_1(\tau)\right)d\tau, \\ y_3(t) &= \int_0^t\left(A_2 - y_2(\tau)\right)d\tau, \\ y_4(t) &= \int_0^t\left(A_3 - y_3(\tau)\right)d\tau, \\ y_5(t) &= \int_0^t\left(A_4 - y_4(\tau)\right)d\tau \end{aligned} \tag{15}$$

In order to clarify the mathematical derivation, the graphical representations of the first two areas (A_1 and A_2) are shown in Figures 2 and 3.

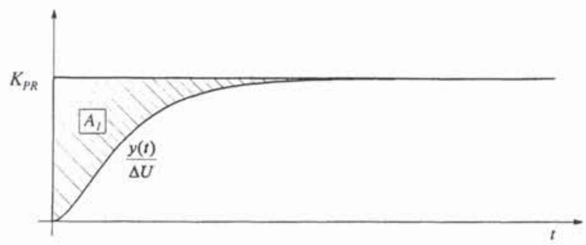

Fig. 2. The graphical representation of area A_1.

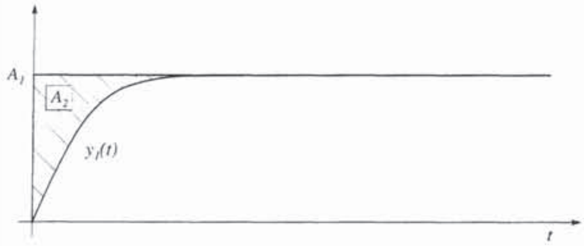

Fig. 3. The graphical representation of area A_2.

When inserting the calculated areas (10) to (14), obtained from the process step response, into equations (7) to (9), the following result is obtained:

$$K = \frac{1}{2K_{PR}\alpha} \tag{16}$$

$$T_i = \frac{A_1}{K_{PR}(1+\alpha)} \tag{17}$$

$$T_d = \frac{A_3 A_4 - A_2 A_5}{A_3 A_3 - A_1 A_5} \tag{18}$$

where

$$\alpha = \frac{A_1 A_2 - T_d A_1^2}{K_{PR} A_3} - 1 \tag{19}$$

Now obviously only areas A_1 to A_5, and the process gain K_{PR} are needed to calculate the unknown controller parameters.

As can be seen from equations (10) to (15), the areas can be calculated from the process open-loop step response by simple numerical integration while the gain K_{PR} can be determined from the steady-state value of the step response in the usual way.

The PID controller tuning procedure can therefore go on as follows:

- measure a process step response
- find a process steady-state gain K_{PR} and areas A_1, to A_5 (by numerical integration (summation) from the start to the end of the process step response).
- calculate PID controller parameters by using expressions (16) to (19).

The main point is, however, that the PID controller parameters can be tuned exactly, according to the amplitude optimum criterion (6), for a wide spectrum of process models (1), only by measuring the process open-loop step response.

3. REAL-TIME EXPERIMENTS WITH AUTO-TUNING ALGORITHM

The real-time experiments were performed by using the auto-tuning algorithm written in the program language PASCAL (some details are given in Vrančić et al., 1997a) with the Burr-Brown acquisition system PCI-20000.

Two real-time experiments were performed on the laboratory set-ups. The first experiment was made on a pneumatic set-up (process), given by Fig. 4. The input of the process is the current reference i_{in} (4/20 mA) on the servo-driven valve V_1 and the output is the pressure p_1 between valves V_1 and V_2 (transferred to the voltage u_{out} by using the pressure-to-voltage transmitter in the range from 0 to 10V).

Fig. 5 shows the results of the closed-loop experiment on the pneumatic set-up, when using the auto-tuning algorithm. After the process open-loop step response is obtained (at t=1.2sec), the following values of the process gain K_{PR}, and areas A_1 to A_5 are measured: K_{PR}=-0.0782, A_1=-0.0248, A_2=-4.919$\cdot 10^{-3}$, A_3=-7.746$\cdot 10^{-4}$, A_4=-1.039$\cdot 10^{-4}$, A_5=-1.219$\cdot 10^{-5}$.

The following PI and PID controller parameters and factors α are obtained from (16) to (19):

	α	K	T_i	T_d
PI	1.014	-6.31	0.158	
PID	0.313	-20.43	0.241	0.069

The closed-loop process responses (see Fig. 6) are quite good. It is obvious that the closed-loop time response is faster when using the PID controller, without significant increase of the process overshoot. It can be observed that the closed-loop responses are quite different when comparing the process responses on positive and on negative reference changes. The reason is that the process is non-linear.

The second experiment was made on a motor-generator laboratory set-up which is shown in Fig. 7.

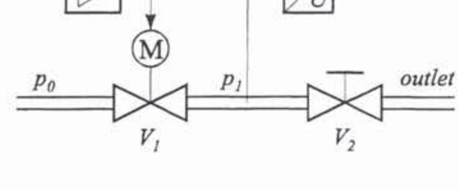

Fig. 4. Pneumatic set-up.

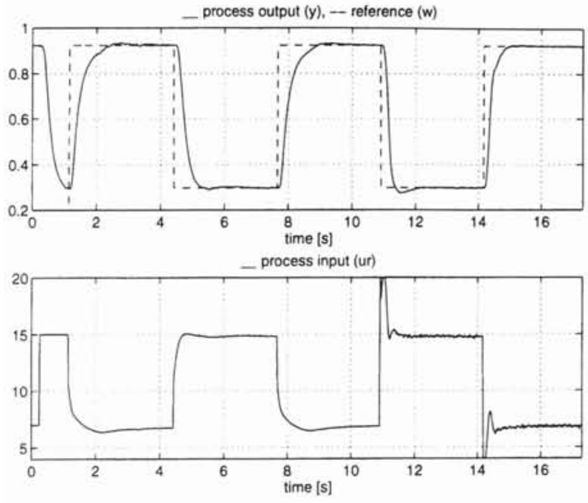

Fig. 5. The system responses under auto-tuning algorithm for the pneumatic set-up.

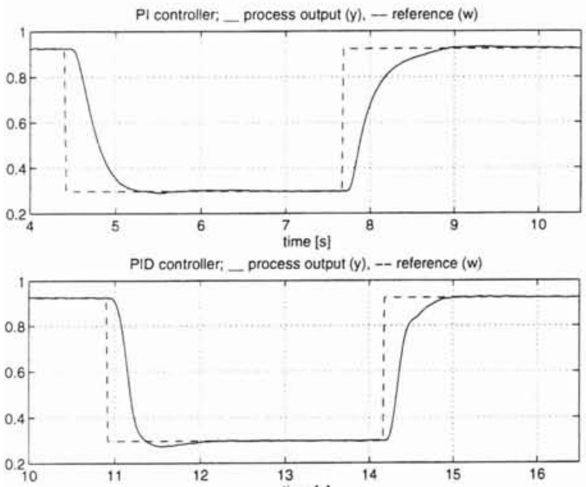

Fig. 6. The closed-loop process responses under PI and PID controller for the pneumatic set-up (the detailed view of the Fig. 5).

The input of the process is the voltage on the input of the amplifier (u_{in}) which drives the motor, and the output is the speed of the motor-generator system, obtained by the speed-to-voltage converter (u_{out}). Both, input and output signals, are in the range from 0 to 10V.

The system time responses, when driven by the auto-tuning algorithm, are shown in Fig. 8.

The following values of the process gain, and areas are obtained: K_{PR}=0.7144, A_1=0.187, A_2=3.198 $\cdot 10^{-2}$, A_3=4.357 $\cdot 10^{-3}$, A_4=4.989 $\cdot 10^{-4}$, A_5=4.881 $\cdot 10^{-5}$.

The following PI and PID controller parameters and factors α are obtained from (16) to (19):

	α	K	T_i	T_d
PI	0.921	0.759	0.136	
PID	0.223	3.14	0.214	0.062

Both closed-loop time responses (when using PI and PID controller) are quite good (see Fig. 9). It is obvious that the closed-loop time response is faster when using the PID controller, without significant increase of the process overshoot. The process non-linear behaviour can be clearly observed from the different closed-loop responses at higher and at lower reference values.

4. CONCLUSIONS

The purpose of this paper was to give a simple tuning method for the PI(D) controller, suitable for a large class of processes. A frequency domain tuning criterion (magnitude optimum) was chosen. It was shown that the parameters of the PID controller can be calculated from the step response by using the multiple integration method in a very simple way.

Tests on the laboratory plants showed that the method is quite robust to the process high-frequency noise and non-linearity. The main point, however, is that in spite of the quite demanding frequency criterion - and the fact that the calculation of the controller parameters is based on a quite complex process model - the implementation, using the time domain approach, is very simple and straightforward.

The advantages of the new tuning approach are:

- There is no need to detect the process inflection point and the slope from the process step response (difficult to do for noisy processes) as required by some other tuning methods.
- The method is based on integrations (summations) and is therefore suitable for use in the auto-tuning algorithms (as shown in examples).
- The controller parameters are calculated exactly, according to the given MO criterion, for a wide spectrum of process models.

The drawback of such an approach is that the method requires a stable open-loop process response in order to determine the appropriate controller parameters. Moreover, the closed-loop stability is not guaranteed.

However, the new tuning method has been tested by Vrančić, (1995) and Vrančić et al., (1995) and the experimental results showed very good closed-loop time responses for nine different process models.

Besides the given two examples, the new tuning approach has shown excellent closed-loop responses also for the high-order processes, the highly non-minimal phase processes and processes with larger time delays (Vrančić, 1995).

The new method can also be successfully used for improving the classical tuning rules (see Vrančić, 1995). The extension of the method (the areas) to other types of controllers so as PID controllers with D or/and P part only partially connected to the reference (Åström and Hägglund, 1995; Hang and

Cao, 1993) or multivariable controllers (Vrančić et al., 1997b) is also straightforward.

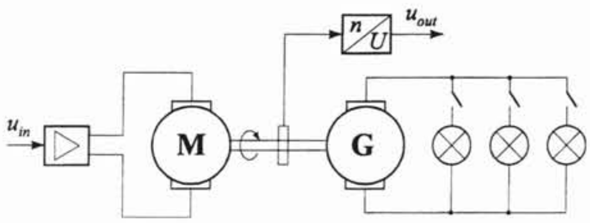

Fig. 7. Motor-generator laboratory set-up.

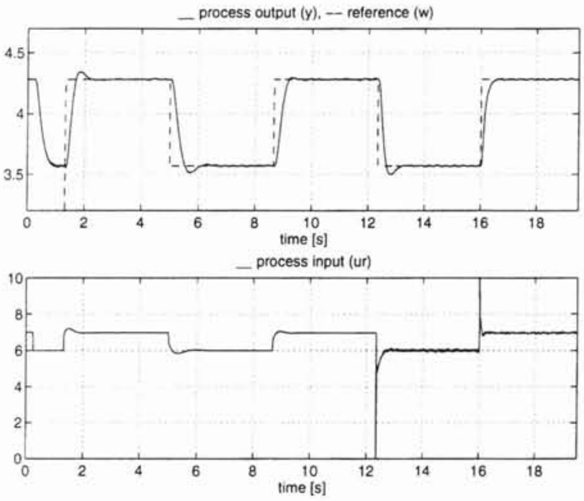

Fig. 8. The system responses under auto-tuning algorithm for the motor-generator set-up.

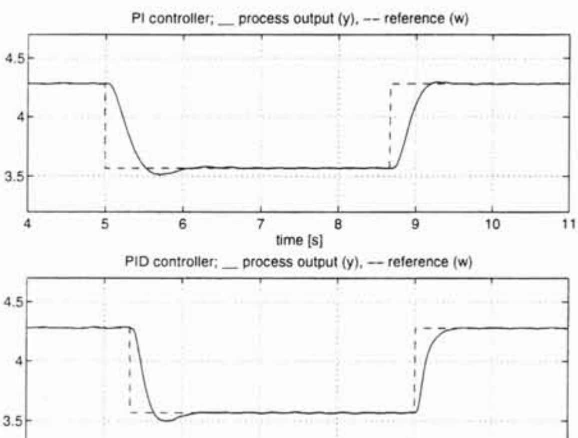

Fig. 9. The closed-loop process responses under PI and PID controller for the motor-generator set-up (the detailed view of the Fig. 8).

ACKNOWLEDGEMENT

The authors would like to thank three anonymous reviewers for providing many useful comments.

REFERENCES

Åström, K. J., and T. Hägglund (1995). PID Controllers: Theory, Design, and Tuning. *Instrument Society of America*, 2nd edition.

Hang, C. C., and L. S. Cao (1993). Improvement of Transient Response by Means of Setpoint Weighting. *12th World Congress IFAC*, Sydney, Proceedings, **3**, pp. 69-72.

Hang, C. C., and K. K. Sin (1991). On-line Auto-tuning of PID Controllers Based on the Cross-correlation Technique. National University of Singapore, Department of Electrical Engineering, *report* **CI-91-4**.

Hanus, R. (1975). Determination of controllers parameters in the frequency domain. Journal A, **XVI** (3).

Kessler, C. (1955). Über die Vorausberechnung optimal abgestimmter Regelkreise Teil III. Die optimale Einstellung des Reglers nach dem Betragsoptimum. Regelungstechnik, Jahrg. **3**, pp. 40-49.

Nishikawa, Y., N. Sannomiya, T. Ohta, and H. Tanaka (1984). A Method for Auto-tuning of PID Control Parameters. Automatica, **20** (3), pp. 321-332.

Peterka, V., and K. J. Åström (1980). Control of multivariable systems with unknown but constrained parameters. Automatica, **16**, pp. 393-398.

Rake, H. (1987). Identification: Transient- and Frequency-Response Methods. Systems & Control Encyclopedia; Theory, Technology, Applications, Madan G Singh, ed., *Pergamon Press.*

Strejc, V. (1959). Näherungsverfrahren für aperiodische Übergangscharakteristiken. *Regelungstechnik* 7, pp. 124-128.

Umland, J. W., and M. Safiuddin (1990). Magnitude and Symmetric Optimum Criterion for the Design of Linear Control Systems: What Is It and How Does It Compare with the Others?. IEEE Transactions on Industry Applications, **26** (3), pp. 489-497.

Voda, A. A., and I. D. Landau (1995). A Method for the Auto-calibration of PID Controllers. Automatica, **31** (1), pp. 41-53.

Vrančić, D. (1995). A new PI(D) tuning method based on the process reaction curve. *J. Stefan Institute, Report DP-7298.*

Vrančić, D., Y. Peng, C. Danz (1995). A comparison between different PI controller tuning methods. *J. Stefan Institute, Report DP-7286.*

Vrančić, D., Y. Peng, S. Strmčnik, R. Hanus (1996). A new tuning method for PI controllers based on a process step response. *Pre-prints of the CESA'96 IMACS Multiconference, Symposium on Control, Optimization and Supervision*, Lille, **2**, pp. 790-794.

Vrančić, D., Y. Peng, J. Petrovčič, and R. Hanus (1997a). A new tuning method for PID controllers. Accepted by the 4th IFAC Conference on System Structure and Control, Bucharest, October 23-25.

Vrančić, D., Y. Peng, J. Lieslehto, S. Strmčnik, and R. Hanus (1997b). Design of MIMO PI Controllers Using the Multiple Integration Approach. Accepted by the IMACS World Congress, Berlin, August 24-29.