NLSC Quadcopter Project

Task 3: Sliding Mode Control

Due Date: June 9th, 2023 @ 11:59PM

Instructions:

- Check the Introduction file that specifies the system model.
- Use the Q&A forum of the corresponding task in Moodle to ask questions.
- The submission portal will be available in Moodle. It will automatically be closed at the specified deadline.
- Submissions will only be considered if submitted before the deadline.
- Use the filenames for deliverables as define below.
- In case of delivering digitized handwritten notes, make sure they are clearly visible.

Goal

The goal of this task is to apply sliding mode control to effectively control the nonlinear quadcopter yaw dynamics in the presence of uncertainty.

Problem setup

The equations of motion of a quadrotor are nonlinear (e.g., nonlinear kinematics and dynamics) and uncertain (e.g., parameter uncertainty). Sliding mode control is an advanced control scheme which aims to robustly control a nonlinear system in the presence of uncertainty. Although this scheme is feasible for the full quadcopter model, it might require advanced analysis. In this task, a simpler setup is considered, where the position of the quadcopter along $x - y - z$ axes are controlled by an LQR controller (which is already designed), and the yaw dynamics are controlled using sliding mode control (which is to be designed).

Recall that the quadcopter model mentioned in the `Introduction.pdf` file has four inputs, namely u_1, u_2, u_3 and u_4, which are used to control four outputs, namely the position x, y, z and the heading ψ, respectively. As stated earlier, u_1, u_2 and u_3 are designed using LQR to control the quadcopter position. It is required to design u_4 as a function of the quadcopter state vector $[x,\ y,\ z,\ \dot{x},\ \dot{y},\ \dot{z},\ \phi,\ \theta,\ \psi, p,\ q,\ r]$ and input vector $[u_1,\ u_2,\ u_3]$ to regulate the quadcopter heading to the origin in the presence of uncertainty whose source is assumed to be the drag coefficient k_r. Although the actual value of this parameter is $10^{-5} kg\, m^2\, s^{-1}$, this value is assumed NOT to be known by the controller which assumes that this parameter can lie anywhere between $0\ kg\, m^2\, s^{-1}$ and $10^{-3}\ kg\, m^2\, s^{-1}$ (i.e., $k_r \in [0, 10^{-3}]$). While designing the controller, consider the following bounds: $(\phi, \theta, \psi) \in [-\frac{\pi}{3}, \frac{\pi}{3}] \times [-\frac{\pi}{3}, \frac{\pi}{3}] \times [-\frac{\pi}{3}, \frac{\pi}{3}]$ and $(p, q, r) \in [-1, 1] \times [-1, 1] \times [-1, 1]$.

The following files are attached with this task:

1. `Introduction.pdf` file which gives a brief explanation of the quadcopter model.

2. `ModelParameters.m` file where all model and simulation parameters can be found and should NOT be modified.

3. `Model.slx` file where the controller can be designed.

In the `Model.slx` file, the quadcopter model is found in the `Model` block which should NOT be modified. The `Sliding Mode Control` block consists of two main blocks; the `MATLAB` function `xyzControl` where LQR is used for position control and which should NOT be modified and the `MATLAB` function `yawControl` which should be modified to design the sliding mode controller for the yaw dynamics. The inputs to this function are the quadcopter state vector `X`, the quadcopter reference vector `Ref` and the input vector `u123` which comprises the inputs u_1, u_2 and u_3, respectively. The output of this block is the input u_4. The inputs and outputs of this function should NOT be modified.

Control Design and Implementation

Answer the following questions to design and implement a sliding mode controller for the yaw dynamics

Task 3.a Choose the sliding manifold as a function of the quadcopter states.

Task 3.b Derive the reduced order model inside the sliding manifold, and show that its origin is asymptotically stable.

Task 3.c Compute the derivative of the sliding manifold as a function of the quadcopter states, inputs and parameters.

Task 3.d By starting from the Lyapunov function $V(s) = \frac{1}{2}s^2$, design a control law that uses the derivative of the Lyapunov function such that all trajectories that do not start on the chosen manifold converge to the manifold in finite time.

Task 3.e Implement the designed control law in simulations.

Task 3.f Solve the chattering problem in simulations.

Task 3.g Plot two figures each of which has three subplots (top: the yaw angle ψ, middle: the angular velocity r, bottom: the control input u_4). The first figure shows the trajectories before solving the chattering problem, whereas the second shows the trajectories after solving the chattering problem. All trajectories should be displayed for ten seconds starting from an initial condition $[\psi_0, r_0] = [\pi/8, 0]$ with the reference $\psi_r = 0$.

Deliverables

- `Task3.pdf` file with the answers to Tasks 3.a, 3.b, 3.c, 3.d, 3.e, 3.f, and 3.g. The file should NOT exceed two pages.

- `yawControl.m` file with the `MATLAB` function `yawControl` where the controllers in Tasks 3.e and 3.f are implemented. Mention the values of the control parameters explicitly in the beginning of this file.

Hints

- It is recommended to change the solver type from "Variable-step" to "Fixed-step" with a relatively small fixed-step size (e.g., $1e-5$ or lower) if the sign function is used within the controller. To do so, go to "Simulation/Model Configuration Parameters" in the menu-bar and change the "Solver selection - Type and Fixed-step size" in the "Solver" tab.