NLSC Quadcopter Project

Task 1: Linear control

Due Date: May 5th, 2023 @ 11:59PM

Instructions:

- The materials provided for this task are:
  - The introduction to the project: `0_Introduction.pdf`;
  - This script: `1_LinearControl.pdf`;
  - A template `Simulink` file: `Model.slx`;
  - A initialisation `MATLAB` file: `ModelParameters.m`;
- You should only modify the "Linear Control" sub-block from the `Simulink` template. You can set the reference and initial conditions by setting the variables `X_init` and `X_ref` in the `MATLAB` file `ModelParameters.m`. The provided `Simulink` model will use those. Although we encourage you to experiment with the `Simulink` model, we might not be able to grade your work if you change other blocks in the submitted version.
- Use the Q&A forum of the corresponding task in Moodle to ask questions.
- The submission portal will be available in Moodle. It will be automatically closed at the specified deadline. Submissions will only be considered if submitted before the deadline.
- Use the filenames for deliverables as defined below in Section 1.5.
- If you choose to deliver digitalized handwritten notes, make sure they are clearly visible.

---

1 Linear Control: LQR

1.1 Goal

The goal of this task is to apply design techniques from linear systems control to effectively control the quadcopter around a collection of operating points.

1.2 Definitions

A set of generalised coordinates that fully describe the state of the system is given by:

$$X = \begin{bmatrix} x & y & z & \dot{x} & \dot{y} & \dot{z} & \phi & \theta & \psi & p & q & r \end{bmatrix}^T \tag{1}$$

The control input can be written as:

$$U = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 \end{bmatrix}^T \tag{2}$$

1.3 System Model

Task 1.a: Write the equations of motion of the system in the form $\dot{X} = f(X, U)$. Show that the points $X_{ss}(\psi) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \psi & 0 & 0 & 0 \end{bmatrix}^T, \forall \psi \in [0, 2\pi]$ are equilibrium points of the non-linear system for the input $U_{ss} = \begin{bmatrix} mg & 0 & 0 & 0 \end{bmatrix}^T$.

Task 1.b: Find the linearised system matrices A and B around $X_{ss}(\psi)$ and U_{ss}, as a function of ψ.

1.4 Control Implementation

Task 1.c: Design an LQR controller for the operating point $X_{ss}(\psi)$ and U_{ss} with $\psi = 0$. You can use the `MATLAB` command `lqr`. Take the cost matrices, Q and R to be identity matrices of appropriate sizes.

Task 1.d: Implement the controller in `Simulink`, by modifying the "Linear Control" sub-block in the provided `Simulink` file `Model.slx`.

Task 1.e: Simulate the time evolution of the system from different initial conditions and reference values. In particular show the results for $X_{init} = [1, 1, 1, 0, 0, 0, 0, 0, \psi_0, 0, 0, 0]^T$ with $\psi_0 \in \{\frac{2\pi}{16}, \frac{2\pi}{8}, \frac{2\pi}{4}, \frac{2\pi}{2}\}$, regulating the system to the origin. Plot the position x, y, z and orientation ψ, θ, ϕ in two separate figures for each value of ψ_0. Briefly explain the observed behavior.

Task 1.f: What happens to the system response if you set the reference x, y, z coordinates to a non-zero value? Explain the observed behavior. What happens if the initial condition is X_{init} defined above with $\psi_0 = \frac{\pi}{4}$ and the reference is $X_{ref} = [0, 0, 0, 0, 0, 0, 0, 0, \frac{\pi}{2}, 0, 0, 0]^T$? What goes wrong? How can one fix it?

1.5 Deliverables

- `task1ab.pdf`: PDF file with the answers to tasks 1.a and 1.b. Digitalised handwritten notes are also accepted.
- `task1c.m`: `MATLAB` file with the answer to task 1.c.
- `task1d.slx`: `Simulink` file with the controller implemented in task 1.d.
- `task1ef.pdf`: PDF file with the 8 plots generated in task 1.e and corresponding explanation, as well as the answers to task 1.f.