Nonlinear Systems and Control

Quadcopter Project - Task 3: Sliding Mode Control

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1 Task 3a

Choose the sliding manifold as a function of the quadcopter states.

Solution: By calling $\eta = \psi$ and $\xi = r$, the sliding manifold can be chosen as:

$$\sigma = \xi - \phi(\eta) = r + a\psi\frac{\cos(\theta)}{\cos(\phi)} + q\tan(\phi) = 0, \quad a > 0 \tag{1}$$

2 Task 3b

Derive the reduced order model inside the sliding manifold, and show that its origin is asymptotically stable.

Solution: The reduced order dynamics can be written as:

$$\dot{\eta} = \dot{\psi} = q\frac{\sin(\phi)}{\cos(\theta)} + r\frac{\cos(\phi)}{\cos(\theta)}$$

By computing r from (1), it follows that:

$$\dot{\eta} = q\frac{\sin(\phi)}{\cos(\theta)} - \frac{\cos(\phi)}{\cos(\theta)}\left[a\psi\frac{\cos(\theta)}{\cos(\phi)} + q\tan(\phi)\right] = q\frac{\sin(\phi)}{\cos(\theta)} - a\psi - q\frac{cos(\phi)sin(\phi)}{cos(\theta)cos(\phi)} = -a\psi$$

and since $a > 0$, the origin is asymptotically stable.

3 Task 3c

Compute the derivative of the sliding manifold as a function of the quadcopter states, inputs, and parameters.

Solution:

$$\dot{\sigma} = \dot{\xi} - \frac{\partial\phi(\eta)}{\partial\eta}\dot{\eta} = \dot{r} + a\frac{\cos(\theta)}{\cos(\psi)}\dot{\psi} = \frac{1}{I_z}\left[(I_x - I_y)pq + u_4 - k_r r\right] + a\frac{\cos(\theta)}{\cos(\phi)}\left[q\frac{\sin(\phi)}{\cos(\theta)} + r\frac{\cos(\phi)}{\cos(\theta)}\right]$$

$$= \frac{(I_x - I_y)}{I_z}pq + \frac{1}{I_z}u_4 + \frac{k_r}{I_z}u_4 + aq\tan(\phi) + ar$$

4 Task 3d

By starting from the Lyapunov function $V(\sigma) = \frac{1}{2}\sigma^2$, design a control law that uses the derivative of the Lyapunov function such that all trajectories that do not start on the chosen manifold converge to the manifold in finite time.

Solution: First of all, u_4 is designed in order to cancel the known terms of $\dot{\sigma}$:

$$u_4 = -(I_x - I_y)pq - aI_z q\tan(\phi) - aI_z r + I_z v$$

$$\rightarrow \dot{\sigma} = v - \frac{k_r}{I_z}r = v + \Delta x$$

where $|\Delta x| \leq \frac{k_r^{max}}{I_z} = \rho(x)$. Then:

$$\dot{V} = \sigma\dot{\sigma} \leq \sigma v + |\sigma|\rho(x) = -\beta_0|\sigma|$$

where $\beta_0 > 0$. Finally, by choosing:

$$v = -(\rho(x) + \beta_0)sgn(\sigma)$$

a control law such that all trajectories converge to the manifold in finite time is obtained.

5 Task 3g

Plot two figure each of which has three subplots (top: ψ, middle: r, bottom: u_4). The first figure shows the trajectories before solving the chattering problem, whereas the second shows the trajectories after solving the chattering problem. All trajectories should be displayed for 10 s starting from an initial condition $[\psi_0, r_0] = [\frac{\pi}{8}, 0]$ with the reference $\psi_r = 0$.

Solution: Control parameters: $\rho(x) = \frac{10^{-3}}{I_z}$, $a = 0.8$, $\beta_0 = 1$.

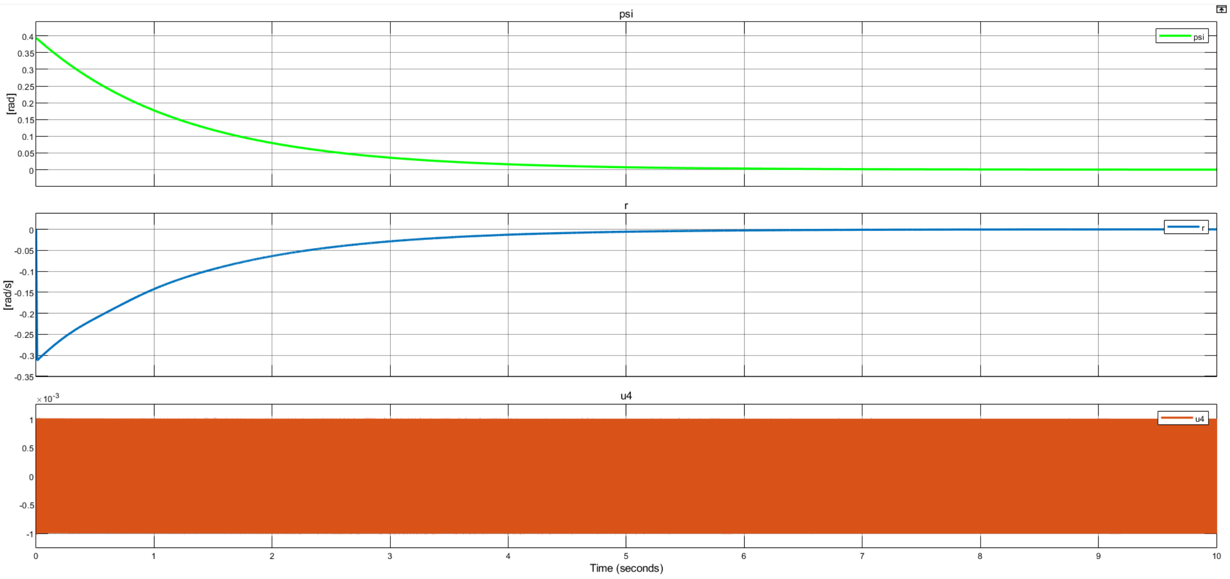

Figure 1: Trajectories before solving the chattering problem.

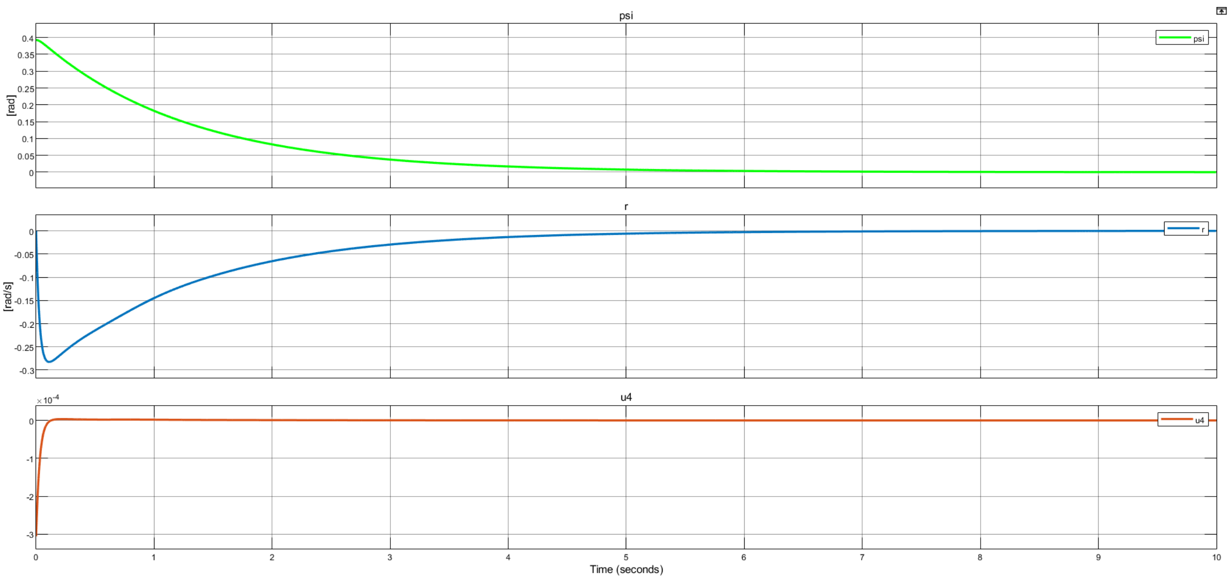

Figure 2: Trajectories before solving the chattering problem.