Exercise sheet 4

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Machine Learning Lab - Exercise Sheet 4
Author: Sebastian Pineda Arango ID: 246098 Universität Hildesheim - Data Analytics Master

0.1 Exercise Sheet 4

We donwload the following datasets:

- Airfare and demand (http://www.stat.ulf.edu/winter/data/airq402.dat)
- Wine Quality (http://archive.ics.uci.edu/ml/datasets/Wine+Quality)

In this notebook, we aim to preprocess two datasets and then create a prediction model (logistc regression) training using different techniques. In this case, the first dataset is about Bank Marketing (from now on, it is ogint be called *Dataset1*). The second dataset corresponds to Occupancy detection (from now on, it is going to be called *Dataset2*).

For modelling, we am to implement four different apporaches to find the parameters of the logistic function:

- Stochastic gradient descent
- Stochastic gradient descent with momentum
- Stochastic gradient descent with bold driver
- Stochastic gradient descent with adagrad.

0.2 Preprocessing

The first part of this notebook focuses in reading the data and preprocessing it through following transformations:

- Converting non numeric values to numeric values
- Drop rows
- Splitting train and test set

```
In [1]: import pandas as pd
        import numpy as np
        import matplotlib.pyplot as plt

        %matplotlib inline

        dataset1 = pd.read_csv("bank-full.csv", sep=";")
        print("Size of dataset1 before dropping NA:", dataset1.shape)
```

```
dataset1 = dataset1.dropna()
print("Size of dataset1 after dropping NA:", dataset1.shape)
dataset1.head()
```

```
Size of dataset1 before dropping NA: (45211, 17)
Size of dataset1 after dropping NA: (45211, 17)
```

```
Out[1]:    age           job  marital  education default  balance housing loan  \
        0   58    management  married   tertiary      no     2143     yes   no
        1   44    technician   single  secondary      no       29     yes   no
        2   33  entrepreneur  married  secondary      no        2     yes  yes
        3   47   blue-collar  married    unknown      no     1506     yes   no
        4   33       unknown   single    unknown      no        1      no   no

           contact  day month  duration  campaign  pdays  previous poutcome   y
        0  unknown    5   may       261         1     -1         0  unknown  no
        1  unknown    5   may       151         1     -1         0  unknown  no
        2  unknown    5   may        76         1     -1         0  unknown  no
        3  unknown    5   may        92         1     -1         0  unknown  no
        4  unknown    5   may       198         1     -1         0  unknown  no
```

```
In [2]: dataset2 = pd.read_csv("occupancy_data/datatraining.txt", sep=",")
        print("Size of dataset2 before dropping NA:", dataset2.shape)
        dataset2 = dataset2.dropna()
        print("Size of dataset2 after dropping NA:", dataset2.shape)
        dataset2.head()
```

```
Size of dataset2 before dropping NA: (8143, 7)
Size of dataset2 after dropping NA: (8143, 7)
```

```
Out[2]:                   date  Temperature  Humidity  Light     CO2  HumidityRatio  \
        1  2015-02-04 17:51:00        23.18   27.2720  426.0  721.25       0.004793
        2  2015-02-04 17:51:59        23.15   27.2675  429.5  714.00       0.004783
        3  2015-02-04 17:53:00        23.15   27.2450  426.0  713.50       0.004779
        4  2015-02-04 17:54:00        23.15   27.2000  426.0  708.25       0.004772
        5  2015-02-04 17:55:00        23.10   27.2000  426.0  704.50       0.004757

           Occupancy
        1          1
        2          1
        3          1
        4          1
        5          1
```

0.3 Exploratory analysis

We perform a quick exploratory analysis of the data. First, we describe the numerical data using a python function. Then, we make hisotgram plots for all the categorical variables, so that we can see how their values are distributed.

In [3]: dataset1.describe()

Out[3]:

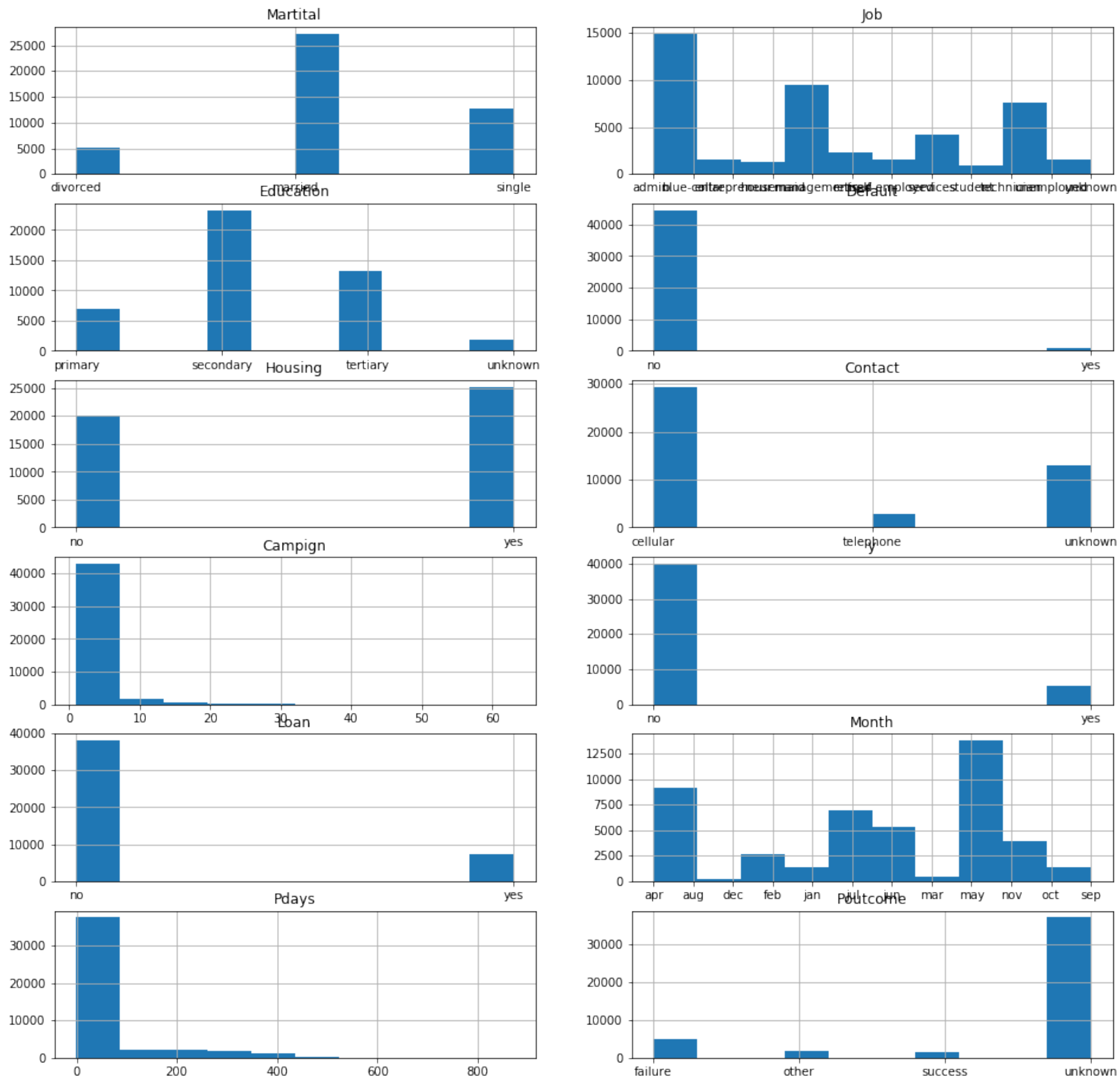
```
                age        balance           day      duration      campaign  \
count  45211.000000   45211.000000  45211.000000  45211.000000  45211.000000
mean      40.936210    1362.272058     15.806419    258.163080      2.763841
std       10.618762    3044.765829      8.322476    257.527812      3.098021
min       18.000000   -8019.000000      1.000000      0.000000      1.000000
25%       33.000000      72.000000      8.000000    103.000000      1.000000
50%       39.000000     448.000000     16.000000    180.000000      2.000000
75%       48.000000    1428.000000     21.000000    319.000000      3.000000
max       95.000000  102127.000000     31.000000   4918.000000     63.000000

              pdays      previous
count  45211.000000  45211.000000
mean      40.197828      0.580323
std      100.128746      2.303441
min       -1.000000      0.000000
25%       -1.000000      0.000000
50%       -1.000000      0.000000
75%       -1.000000      0.000000
max      871.000000    275.000000
```

In [4]:

```
fig, ax = plt.subplots(6,2, figsize=(16,16))
ax[0,0].set_title('Martital')
dataset1.marital.hist(ax=ax[0,0])
ax[0,1].set_title('Job')
dataset1.job.hist(ax=ax[0,1])
ax[1,0].set_title('Education')
dataset1.education.hist(ax=ax[1,0])
ax[1,1].set_title('Default')
dataset1.default.hist(ax=ax[1,1])
ax[2,0].set_title('Housing')
dataset1.housing.hist(ax=ax[2,0])
ax[2,1].set_title('Contact')
dataset1.contact.hist(ax=ax[2,1])
ax[3,0].set_title('Campign')
dataset1.campaign.hist(ax=ax[3,0])
ax[3,1].set_title('y')
dataset1.y.hist(ax=ax[3,1])
ax[4,0].set_title('Loan')
dataset1.loan.hist(ax=ax[4,0])
ax[4,1].set_title('Month')
dataset1.month.hist(ax=ax[4,1])
ax[5,0].set_title('Pdays')
dataset1.pdays.hist(ax=ax[5,0])
ax[5,1].set_title('Poutcome')
dataset1.poutcome.hist(ax=ax[5,1])
```

Out[4]:

```
<matplotlib.axes._subplots.AxesSubplot at 0x268eac18f60>
```



From the previous analysis for the dataset1, we can have some insights the following:

- There are 8 categorical variables (marital, job, education, contact, poutcome). This are converted to numerical using one hot encoding.
- There are 3 binary variables (default, housing, loan), which has two possible values (yes/no). They are converted to 1 (if y is equal to 'yes') or 0 (if y is equal to 'no').
- There are 2 ordinal variables (month and campaign). They are going to be converted to numbers directly.
- Some categorical variables have 'unknown' class. They are handled as a new category, instead of replacing or dropping these values.
- pdays have a lot of values equals to -1

Now we describe the second dataset.

```
In [5]: dataset2.describe()
```

```
Out[5]:        Temperature     Humidity        Light          CO2  HumidityRatio  \
        count  8143.000000  8143.000000  8143.000000  8143.000000    8143.000000
        mean     20.619084    25.731507   119.519375   606.546243       0.003863
        std       1.016916     5.531211   194.755805   314.320877       0.000852
        min      19.000000    16.745000     0.000000   412.750000       0.002674
        25%      19.700000    20.200000     0.000000   439.000000       0.003078
        50%      20.390000    26.222500     0.000000   453.500000       0.003801
        75%      21.390000    30.533333   256.375000   638.833333       0.004352
        max      23.180000    39.117500  1546.333333  2028.500000       0.006476

                 Occupancy
        count  8143.000000
        mean      0.212330
        std       0.408982
        min       0.000000
        25%       0.000000
        50%       0.000000
        75%       0.000000
        max       1.000000
```

0.4 Encoding of features

We want now to convert all the categorical features of dataset1 to numbers using one-hot encoding. The ordinal features like 'month' and 'campaing' are converted to number.

```
In [6]: categorical_features = ['marital', 'job', 'education', 'contact', 'poutcome']
        print("Size before encoding:", dataset1.shape)
        dataset1 = pd.get_dummies( dataset1, columns = categorical_features)
        print("Size after encoding:",dataset1.shape)
        dataset1.head()
```

```
Size before encoding: (45211, 17)
Size after encoding: (45211, 38)
```

```
Out[6]:    age default  balance housing loan  day month  duration  campaign  pdays  \
        0   58      no     2143     yes   no    5   may       261         1     -1
        1   44      no       29     yes   no    5   may       151         1     -1
        2   33      no        2     yes  yes    5   may        76         1     -1
        3   47      no     1506     yes   no    5   may        92         1     -1
        4   33      no        1      no   no    5   may       198         1     -1

                    ...          education_secondary education_tertiary  \
        0           ...                            0                  1
        1           ...                            1                  0
        2           ...                            1                  0
        3           ...                            0                  0
```

```
4        ...                          0                   0

   education_unknown  contact_cellular  contact_telephone  contact_unknown  \
0                  0                 0                  0                1
1                  0                 0                  0                1
2                  0                 0                  0                1
3                  1                 0                  0                1
4                  1                 0                  0                1

   poutcome_failure  poutcome_other  poutcome_success  poutcome_unknown
0                 0               0                 0                 1
1                 0               0                 0                 1
2                 0               0                 0                 1
3                 0               0                 0                 1
4                 0               0                 0                 1

[5 rows x 38 columns]
```

In [7]:
```
#converting the month to number
months = ['jan', 'feb', 'mar', 'apr', 'may', 'jun', 'jul', 'aug', 'sep', 'oct', 'nov',
month_num = [months.index(i) for i in dataset1.month]

dataset1.month = month_num
dataset1.head()
```

Out[7]:
```
   age default  balance housing loan  day  month  duration  campaign  pdays  \
0   58      no     2143     yes   no    5      4       261         1     -1
1   44      no       29     yes   no    5      4       151         1     -1
2   33      no        2     yes  yes    5      4        76         1     -1
3   47      no     1506     yes   no    5      4        92         1     -1
4   33      no        1      no   no    5      4       198         1     -1

         ...          education_secondary education_tertiary  \
0        ...                            0                  1
1        ...                            1                  0
2        ...                            1                  0
3        ...                            0                  0
4        ...                            0                  0

   education_unknown  contact_cellular  contact_telephone  contact_unknown  \
0                  0                 0                  0                1
1                  0                 0                  0                1
2                  0                 0                  0                1
3                  1                 0                  0                1
4                  1                 0                  0                1

   poutcome_failure  poutcome_other  poutcome_success  poutcome_unknown
0                 0               0                 0                 1
```

```
1                   0                0                 0                 1
2                   0                0                 0                 1
3                   0                0                 0                 1
4                   0                0                 0                 1

[5 rows x 38 columns]
```

The column *month* has been now converted to number. We also want to convert the binary variables (i.e. housing, loan,...) to numbers. This varables are originally (yes/no) strings, therefore we have to perform a further step to get the binary value as follows.

In [8]:
```python
binary_tf = lambda x: int(x=="yes")

dataset1.housing = dataset1.housing.apply(binary_tf)
dataset1.loan = dataset1.loan.apply(binary_tf)
dataset1.default = dataset1.default.apply(binary_tf)
dataset1.y = dataset1.y.apply(binary_tf)

dataset1.head()
```

Out[8]:
```
   age  default  balance  housing  loan  day  month  duration  campaign  \
0   58        0     2143        1     0    5      4       261         1
1   44        0       29        1     0    5      4       151         1
2   33        0        2        1     1    5      4        76         1
3   47        0     1506        1     0    5      4        92         1
4   33        0        1        0     0    5      4       198         1

   pdays         ...          education_secondary  education_tertiary  \
0     -1         ...                            0                   1
1     -1         ...                            1                   0
2     -1         ...                            1                   0
3     -1         ...                            0                   0
4     -1         ...                            0                   0

   education_unknown  contact_cellular  contact_telephone  contact_unknown  \
0                  0                 0                  0                1
1                  0                 0                  0                1
2                  0                 0                  0                1
3                  1                 0                  0                1
4                  1                 0                  0                1

   poutcome_failure  poutcome_other  poutcome_success  poutcome_unknown
0                 0               0                 0                 1
1                 0               0                 0                 1
2                 0               0                 0                 1
3                 0               0                 0                 1
4                 0               0                 0                 1

[5 rows x 38 columns]
```

0.5 Splitting train-test data

We want th split both datasets in train and test data. For that, we create a split_train_test function. Then, the function is used with a train/test split corresponding to 80%/20%.

```
In [9]: def split_train_test(data, train_pct, features, target):

            '''This functions divides "data"  in train and test set.
            The percentage give to the train data is determined by "train_pct".
            The "features" argument determine a list of features to consider.
            The "target" arugment indicates the the variable to predict.'''

            #getting the total number of training samples
            data_size = data.shape[0]
            train_size = int(train_pct*data_size)

            #shuffling indexes to separate train and test randoming
            idx = np.arange(0,data_size)
            np.random.shuffle(idx)

            #creating test indexes
            train_idx = idx[:train_size]

            #creating test indexes
            test_idx = idx[train_size:]

            #selecting train data (features)
            X_train = data[features].iloc[train_idx,]

            #selecting train data (target)
            y_train = data[target].iloc[train_idx,]

            #selecting test data (features)
            X_test = data[features].iloc[test_idx,]

            #selecting test data (target)
            y_test = data[target].iloc[test_idx,]

            #stacking a column of ones to the training and test set
            X_train = np.hstack((X_train, np.ones((X_train.shape[0],1))))
            X_test = np.hstack((X_test, np.ones((X_test.shape[0],1))))

            return X_train, y_train, X_test, y_test

In [10]: #using split_train_test function to split the dataset1
         features1 = list(dataset1.columns) #list of features
         target1 = 'y'
         features1.remove('y')
         X_train1, y_train1, X_test1, y_test1 = split_train_test(dataset1, 0.8, features1, targ
```

```
print("Verifying dataset sizes ...")
print("Size train set for dataset1:", X_train1.shape)
print("Size test set for dataset1:", X_test1.shape)


#using split_train_test function to split the dataset2
features2 = ['Temperature', 'Humidity', 'Light', 'CO2', 'HumidityRatio'] #list of fea
target2 = 'Occupancy'
X_train2, y_train2, X_test2, y_test2 = split_train_test(dataset2, 0.8, features2, targ
print("Size train set for dataset2:", X_train2.shape)
print("Size test set for dataset2:", X_test2.shape)
```

```
Verifying dataset sizes ...
Size train set for dataset1: (36168, 38)
Size test set for dataset1: (9043, 38)
Size train set for dataset2: (6514, 6)
Size test set for dataset2: (1629, 6)
```

0.6 Linear classification with gradient ascent

0.6.1 Exercise 1: SGA

First we want to perform stochastic gradient ascent and stochastic gradient ascent with momentum. In stochastic gradient ascent,we calculate the gradient using only one sample at a time. If we use the momentum addition, we calcualte the current gradient update as a weighted sum of the gradient and the last gradient. It means, with momentum, the update would be:

$\triangle\beta_i = \alpha \triangle \beta_{i-1} + \mu \bigtriangledown_\beta l$

As follows, we implement different functions to performe the stochastic gradient descent (ascent).

- Prediction function:

$\hat{y} = \frac{1}{1+e^{-x\beta}}$

- Loss function:

$l = \sum_i^n ylog(\hat{y}) + (1-y)log(1-\hat{y})$
Where $y$is the ground truth.

- Gradient of loss function respect to β:

$\bigtriangledown_\beta l = X(y-\hat{y})$

Loss in this case is refered programamtically to the log-likelihood that we want to optimize. However we must take into account that $loss = -(log\_likelihood)$.

```
In [11]: def loss_function(y, y_pred):

             '''This function implements the loss function (logitic loss9)'''
```

```
    loss=  -(np.sum( np.log(y_pred[y==1,]))  + np.sum(np.log(1-y_pred[y==0,])))
    return loss

def grad(X, y, beta):

    '''This function implements the gradient of the logistic loss'''

    grad=X.T*(y-sigmoid(X, beta))
    return grad


def sigmoid (X, beta):

    '''This function implements the sigmoid function (prediction function for
    logistic regression)'''

    z = X*beta
    y = 1.0/(1.0+np.exp(-z))
    return y


    return out

def train (X_train, y_train, X_test, y_test, grad_func, max_iter, learning_rate=0.000

    '''This function trains a logistic function using Stochastic gradient descent'''

    #casting to type numpy.matrix
    X_train = np.matrix(X_train)
    y_train = np.matrix(y_train).T
    X_test =np.matrix(X_test)
    y_test = np.matrix(y_test).T

    #initializing beta
    beta =  np.matrix(np.zeros((X_train.shape[1],1)))

    #training set size
    n_samples_train = X_train.shape[0]

    #index list
    idx = np.arange(0,n_samples_train)

    #initilaizing lists
    loss_train_list = []
    loss_test_list = []
    diff_list = []
    last_loss_train = 0
```

```
    #iterating over max_iter
    for i in range(max_iter):

        if(i %10==0):
            print("epoch:", i)

        #shuffling dataset indexes
        np.random.shuffle(idx)

        #calculating stochastic gradient using one sample per time
        for j in idx:
            #updating parameters
            g=grad_func(X_train[j,:], y_train[j], beta)
            beta = beta + learning_rate*g

        #saving values to track performance
        loss_train = loss_function(y_train, sigmoid(X_train, beta))
        loss_test = loss_function(y_test, sigmoid(X_test, beta))
        diff = np.abs(last_loss_train-loss_train)
        last_loss_train = loss_train

        loss_train_list.append(loss_train)
        loss_test_list.append(loss_test)
        diff_list.append(diff)

    return beta, loss_train_list, loss_test_list, diff_list


def train_with_momentum (X_train, y_train, X_test, y_test, grad_func, max_iter, learn

    '''This function trains a logistic regression model using stochastic gradient des
    and momentum to boost the convergence.'''

    #casting to type numpy.matrix
    X_train = np.matrix(X_train)
    y_train = np.matrix(y_train).T
    X_test =np.matrix(X_test)
    y_test = np.matrix(y_test).T

    #initializing beta
    beta =  np.matrix(np.zeros((X_train.shape[1],1)))

    #training set size
    n_samples_train = X_train.shape[0]

    #index list
    idx = np.arange(0,n_samples_train)
```

```
        #initilaizing lists
        loss_train_list = []
        loss_test_list = []
        diff_list = []

        last_loss_train =0


        #iterating over max_iter
        for i in range(max_iter):

            if(i %25==0):
                print("epoch:", i)

            #shuffling dataset indexes
            np.random.shuffle(idx)

            #to save last value for momentum
            last_grad=0

            #calculating stochastic gradient using one sample per time
            for j in idx:

                #updating parameters with momentum
                g=grad_func(X_train[j,:], y_train[j], beta) + 0.9*last_grad
                last_grad=g
                beta = beta + learning_rate*g

            #saving values to track performance
            loss_train = loss_function(y_train, sigmoid(X_train, beta))
            loss_test = loss_function(y_test, sigmoid(X_test, beta))
            diff = np.abs(last_loss_train - loss_train)
            last_loss_train = loss_train

            loss_train_list.append(loss_train)
            loss_test_list.append(loss_test)
            diff_list.append(diff)


        return beta, loss_train_list, loss_test_list, diff_list
```

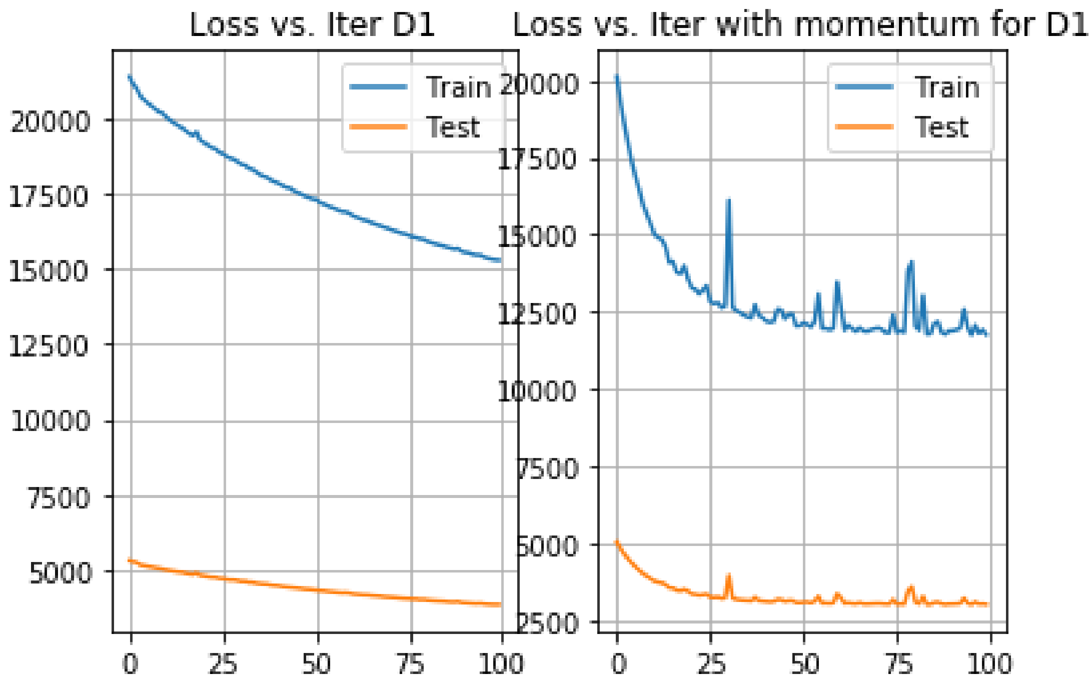
In [12]:
```
beta1, loss_train1, loss_test1, diff_list1 = train(X_train1, y_train1, X_test1, y_tes
beta1_m, loss_train1_m, loss_test1_m, diff_list1_m = train_with_momentum(X_train1, y_
```

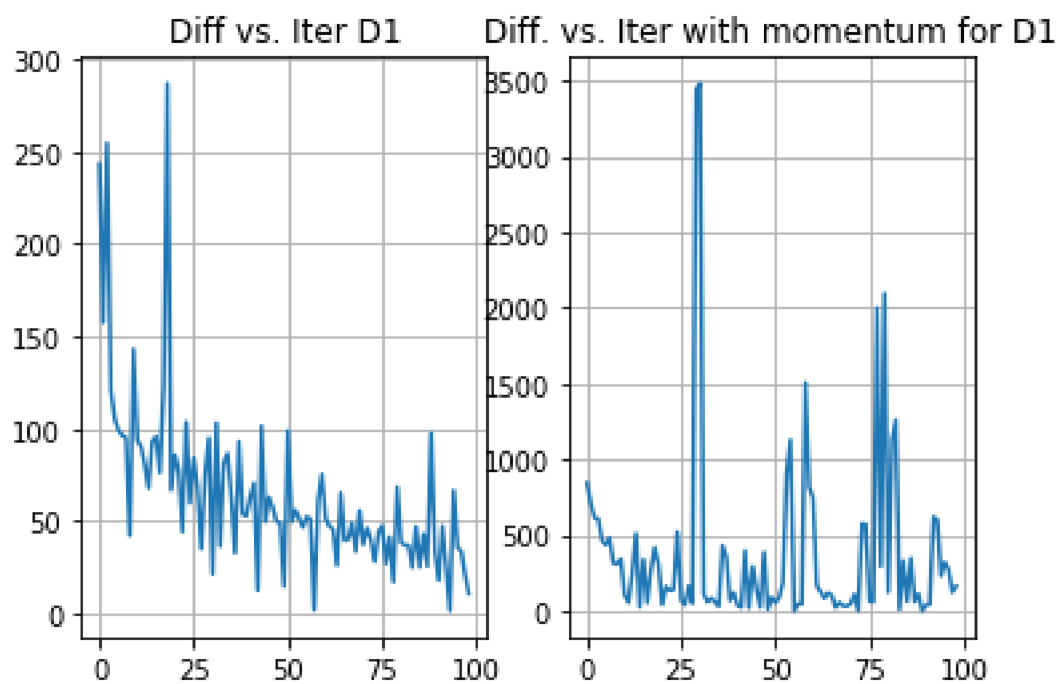
```
epoch: 0
epoch: 10
```

```
epoch: 20
epoch: 30
epoch: 40
epoch: 50
epoch: 60
epoch: 70
epoch: 80
epoch: 90
epoch: 0
epoch: 25
epoch: 50
epoch: 75
```

In [13]:
```python
fix, ax = plt.subplots(1,2)
ax[0].plot(loss_train1)
ax[0].plot(loss_test1)
ax[0].legend(("Train", "Test"))
ax[0].grid()
ax[0].set_title("Loss vs. Iter D1")

ax[1].plot(loss_train1_m)
ax[1].plot(loss_test1_m)
ax[1].legend(("Train", "Test"))
ax[1].grid()
ax[1].set_title("Loss vs. Iter with momentum for D1")

fix, ax = plt.subplots(1,2)
ax[0].plot(diff_list1[1:])
ax[0].grid()
ax[0].set_title("Diff vs. Iter D1")

ax[1].plot(diff_list1_m[1:])
ax[1].grid()
ax[1].set_title("Diff. vs. Iter with momentum for D1")
```

Out[13]:
```
Text(0.5,1,'Diff. vs. Iter with momentum for D1')
```



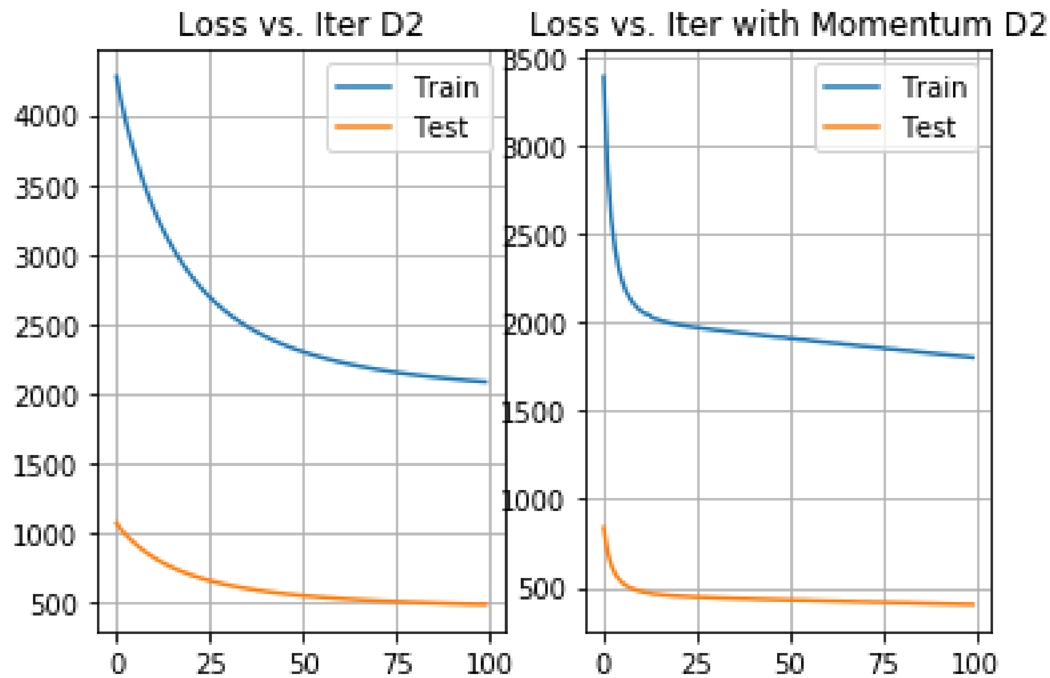


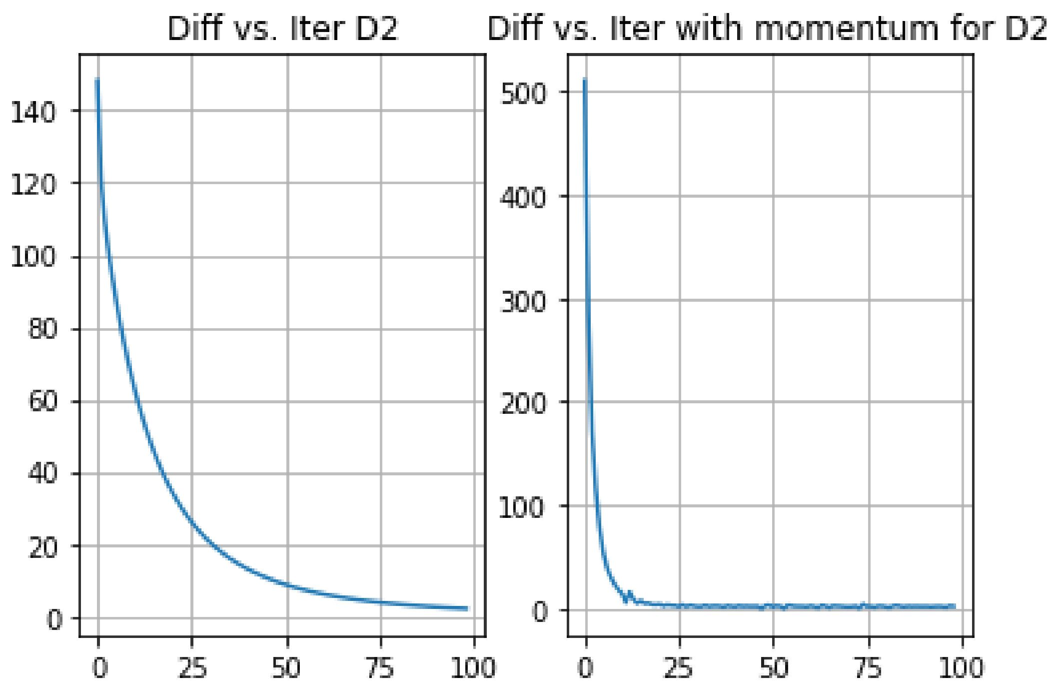
In [14]:
```
beta2, loss_train2, loss_test2, diff_list2= train(X_train2, y_train2, X_test2, y_test2
beta2_m, loss_train2_m, loss_test2_m, diff_list2_m = train_with_momentum(X_train2, y_t
```

```
fix, ax = plt.subplots(1,2)
ax[0].plot(loss_train2)
ax[0].plot(loss_test2)
ax[0].legend(("Train", "Test"))
ax[0].grid()
ax[0].set_title("Loss vs. Iter D2")

ax[1].plot(loss_train2_m)
ax[1].plot(loss_test2_m)
ax[1].legend(("Train", "Test"))
ax[1].grid()
ax[1].set_title("Loss vs. Iter with Momentum D2")


fix, ax = plt.subplots(1,2)
ax[0].plot(diff_list2[1:])
ax[0].grid()
ax[0].set_title("Diff vs. Iter D2")

ax[1].plot(diff_list2_m[1:])
ax[1].grid()
ax[1].set_title("Diff vs. Iter with momentum for D2")
```

```
epoch: 0
epoch: 10
epoch: 20
epoch: 30
epoch: 40
epoch: 50
epoch: 60
epoch: 70
epoch: 80
epoch: 90
epoch: 0
epoch: 25
epoch: 50
epoch: 75
```

```
Out[14]: Text(0.5,1,'Diff vs. Iter with momentum for D2')
```





Observation: both datasets have different losses, however we can see that we momentum the convergence speed increase in both datasets. This show that moments permits to calcualte better gradients in each update of the stochastic gradient descent, since it somehow has a memory of the last iteration. Also, as the train log-likelihood increases, the test log-likehilhood also does. Without momentum, the intial difference between succesive steps were short, how ever with momentum the difference between them increased.

0.6.2 Bold-driver

Now, we implement bold driver, which is also a method to choose a better step-length at each iteration.

```
In [20]: def loss_function_stochastic(y, y_pred):

             '''This fucntion implements loss function for only one sample.
             It is intended to be used for stochastic gradient descent'''

             if(y[0]==1):
                 loss=  -np.log(y_pred)
             elif (y[0]==0):
                 loss = -np.log(1-y_pred)
             return loss


         def step_bold_driver1(X_train, y_train, beta, grad, learning_rate=0.000001):

             '''This function uses bold driver to find a suitable step-length.'''


             a1=1.000001
             a2=0.5
             l1 = loss_function_stochastic(y_train, sigmoid(X_train,beta))
             l2=loss_function_stochastic(y_train, sigmoid(X_train,beta+learning_rate*grad))

             learning_rate = learning_rate*a1

             while(l1<l2 ):
                 learning_rate=learning_rate*a2
                 l2= loss_function_stochastic(y_train, sigmoid(X_train,beta+learning_rate*grad

             return learning_rate

         def step_bold_driver(learning_rate, f_new, f_old):

             a1=1.1
             a2 = 0.5

             if(f_new<f_old):
                 learning_rate = learning_rate*a1
             else:
```

```
        learning_rate = learning_rate*a2

    return learning_rate


def train_with_bold_driver(X_train, y_train, X_test, y_test, grad_function, max_iter,

    '''This function traind a logistic regression model using bold driver to find a s

    X_train = np.matrix(X_train)
    y_train = np.matrix(y_train).T
    X_test =np.matrix(X_test)
    y_test = np.matrix(y_test).T

    #initializing beta
    beta =  np.matrix(np.zeros((X_train.shape[1],1)))

    #training set size
    n_samples_train = X_train.shape[0]

    #index list
    idx = np.arange(0,n_samples_train)

    #initilaizing lists
    loss_train_list = []
    loss_test_list = []
    diff_list = []
    last_loss_train = 1e10
    print(max_iter)
    #iterating over max_iter
    for i in range(max_iter):
        if((i%25)==0):
            print("epoch:", i)

        #shuffling dataset indexes
        np.random.shuffle(idx)

        #calculating stochastic gradient using one sample per time
        for j in idx:

            #updating parameters
            g=grad_function(X_train[j,:], y_train[j], beta)
            beta = beta + learning_rate*g


        #saving values to track performance
        loss_train = loss_function(y_train, sigmoid(X_train, beta))
        loss_test = loss_function(y_test, sigmoid(X_test, beta))
```

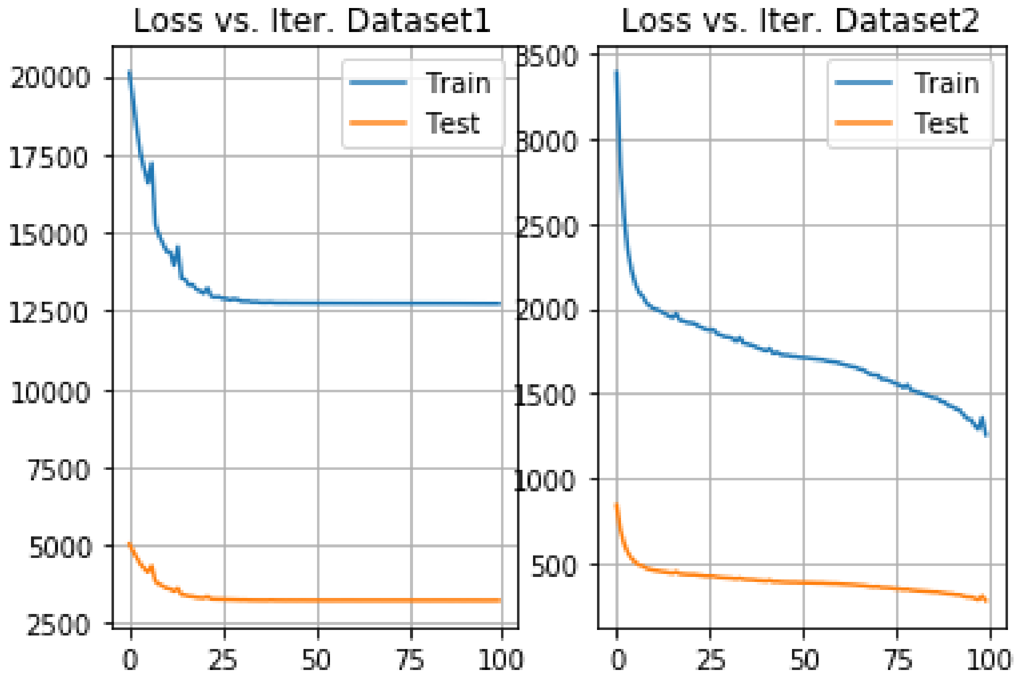
```
            diff = np.abs(loss_train-last_loss_train)

            learning_rate = step_bold_driver(learning_rate, loss_train, last_loss_train)
            last_loss_train = loss_train

            loss_train_list.append(loss_train)
            loss_test_list.append(loss_test)
            diff_list.append(diff)

            if(diff==0): break

        return beta, loss_train_list, loss_test_list, diff_list
```

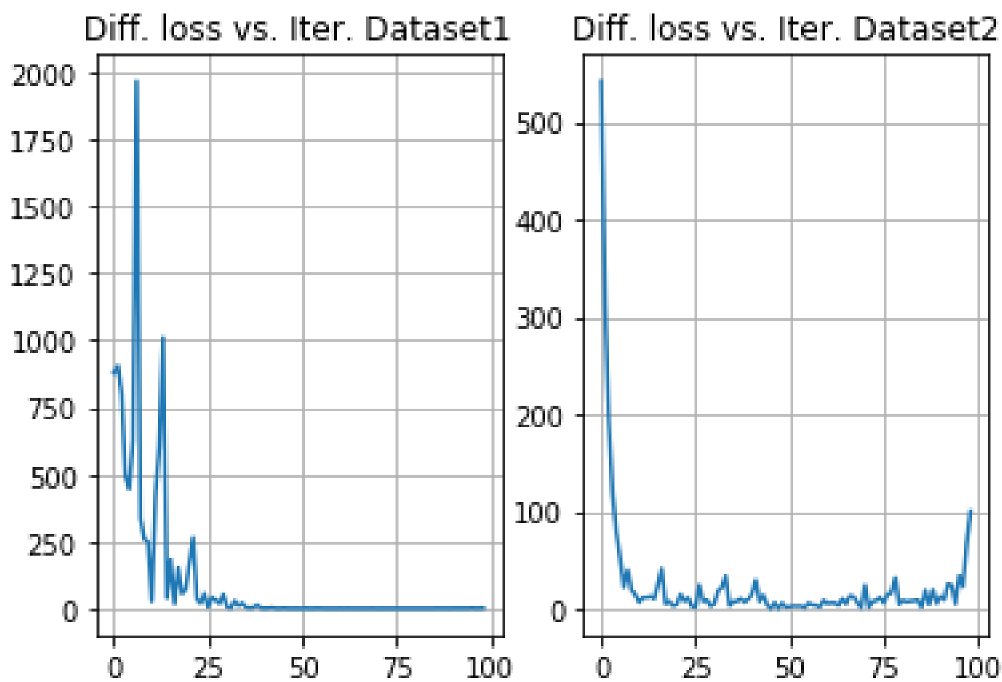
In [21]:
```
beta1, loss_train1, loss_test1, diff_list1= train_with_bold_driver(X_train1, y_train1
beta2, loss_train2, loss_test2, diff_list2 = train_with_bold_driver(X_train2, y_train2
```

```
100
epoch: 0
epoch: 25
epoch: 50
epoch: 75
100
epoch: 0
epoch: 25
epoch: 50
epoch: 75
```

In [22]:
```
fix, ax = plt.subplots(1,2)
ax[0].plot(loss_train1)
ax[0].plot(loss_test1)
ax[0].legend(("Train", "Test"))
ax[0].grid()
ax[0].set_title("Loss vs. Iter. Dataset1")

ax[1].plot(loss_train2)
ax[1].plot(loss_test2)
ax[1].legend(("Train", "Test"))
ax[1].grid()
ax[1].set_title("Loss vs. Iter. Dataset2")

fix, ax = plt.subplots(1,2)
ax[0].plot(diff_list1[1:])
ax[0].grid()
ax[0].set_title("Diff. loss vs. Iter. Dataset1")

ax[1].plot(diff_list2[1:])
ax[1].grid()
ax[1].set_title("Diff. loss vs. Iter. Dataset2")
```

```
Out[22]: Text(0.5,1,'Diff. loss vs. Iter. Dataset2')
```





0.6.3 Adagrad

We now implement Adragad, we uses an adaptative step length that invovles calculating the Hessian of the loss function. This produces a decrease in the step length at each iteration.

```
In [23]: def train_with_adagrad(X_train, y_train, X_test, y_test, grad_function, max_iter, lea

             '''This function trains a logistic regression model using adagrad to find a suita

             #transforming data typ
             X_train = np.matrix(X_train)
             y_train = np.matrix(y_train).T
             X_test =np.matrix(X_test)
             y_test = np.matrix(y_test).T

             #initializing beta
             beta =  np.matrix(np.zeros((X_train.shape[1],1)))
             h = np.matrix(np.zeros((X_train.shape[1],1)))
             #training set size
             n_samples_train = X_train.shape[0]

             #index list
             idx = np.arange(0,n_samples_train)

             #initilaizing lists
             loss_train_list = []
             loss_test_list = []
             diff_list = [] #list of difference between last loss and current loss for train s

             last_loss_train = 0


             #iterating over max_iter
             for i in range(max_iter):

                 if((i%25)==0):
                     print("epoch:", i)

                 #shuffling dataset indexes
                 np.random.shuffle(idx)

                 #calculating stochastic gradient using one sample per time
                 for j in idx:

                     #updating parameters
                     g=grad_function(X_train[j,:], y_train[j], beta)
                     h = h + np.multiply(g, g)
                     adagrad= np.multiply(1/np.sqrt(h), g)
                     beta = beta + learning_rate*g
```

```
            #saving values to track performance
            loss_train = loss_function(y_train, sigmoid(X_train, beta))
            loss_test = loss_function(y_test, sigmoid(X_test, beta))
            diff = np.abs(loss_train-last_loss_train)
            last_loss_train = loss_train

            loss_train_list.append(loss_train)
            loss_test_list.append(loss_test)
            diff_list.append(diff)

            if(diff==0): break

        return beta, loss_train_list, loss_test_list, diff_list
```

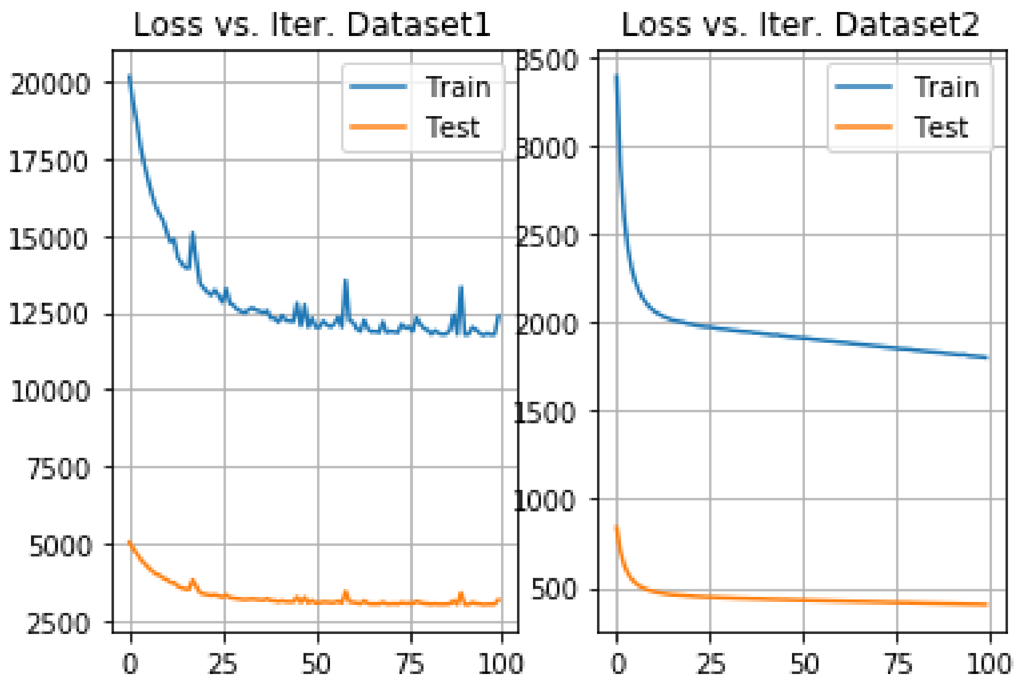
In [24]:
```
beta1, loss_train1, loss_test1, diff_list1 = train_with_adagrad(X_train1, y_train1, X_
beta2, loss_train2, loss_test2, diff_list2 = train_with_adagrad(X_train2, y_train2, X_
```

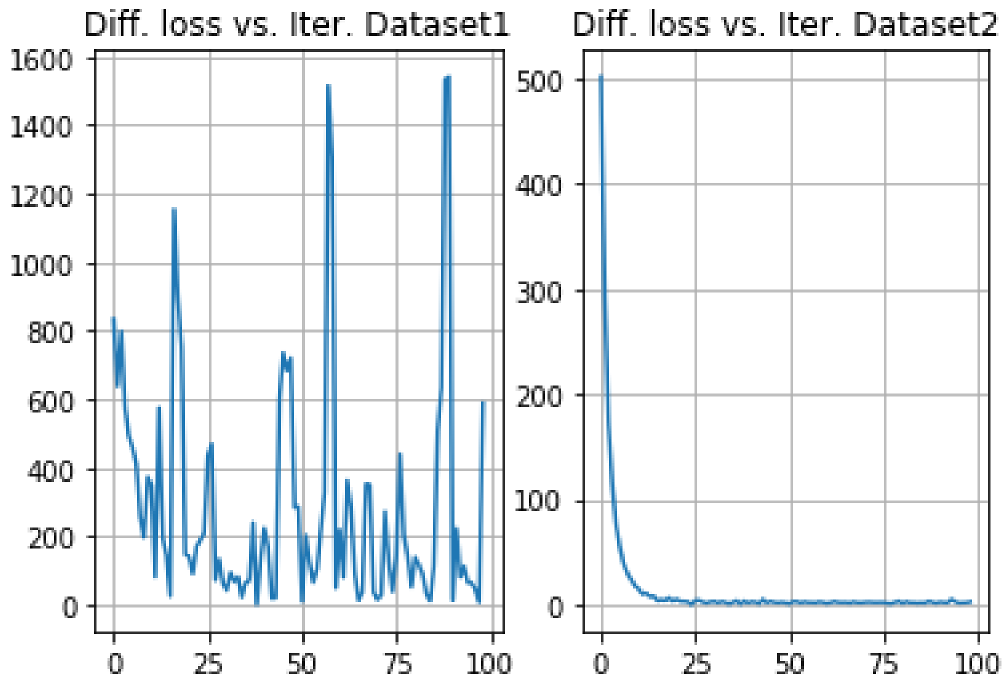
```
epoch: 0


C:\Users\User\Anaconda3\lib\site-packages\ipykernel_launcher.py:43: RuntimeWarning: divide by z
C:\Users\User\Anaconda3\lib\site-packages\ipykernel_launcher.py:43: RuntimeWarning: invalid val


epoch: 25
epoch: 50
epoch: 75
epoch: 0
epoch: 25
epoch: 50
epoch: 75
```

In [25]:
```
fix, ax = plt.subplots(1,2)
ax[0].plot(loss_train1)
ax[0].plot(loss_test1)
ax[0].legend(("Train", "Test"))
ax[0].grid()
ax[0].set_title("Loss vs. Iter. Dataset1")

ax[1].plot(loss_train2)
ax[1].plot(loss_test2)
ax[1].legend(("Train", "Test"))
ax[1].grid()
ax[1].set_title("Loss vs. Iter. Dataset2")

fix, ax = plt.subplots(1,2)
ax[0].plot(diff_list1[1:])
```

```
ax[0].grid()
ax[0].set_title("Diff. loss vs. Iter. Dataset1")

ax[1].plot(diff_list2[1:])
ax[1].grid()
ax[1].set_title("Diff. loss vs. Iter. Dataset2")
```

Out[25]: Text(0.5,1,'Diff. loss vs. Iter. Dataset2')





With Adagrad, in each iteration the step-length is reduced, so it is difficult to get better improvements in lot of iterations. Bold dirver, in the other hand, was able to modify dinamically the step-length so that it was still decrease in the losslog in a high number of iterations. As we can see comparing the plots for dataset2, Bold Driver was able to make a better improvement at high number of iterations. In Adagrad, we must update the step length constantly, which is also computationally expensive, while we only modify the step-length at the end of the iteration using Bold Driver.

0.7 Conclusions

- Stochastic gradient descent permits optimize a function, aking step in the gradient direction of only a sample. However, it moves overall in the direction of the real gradient (accounting the whole dataset). SGD is slower than gradient descent since it makes computations over each sample. Nevertheless, it is a good optimion when the number of data samples is large and therefore calculating the whole gradient turns out to be expensive.
- Momentum improves convergence speed, since it tooks a direction of update improved by the direction of the last update. It even performs close as good as step length selection heuristics like bold driver and adagrad.
- Momentum, bold driver and adagrad have improved the speed convergence for both dataset, so they are a good compliment in the moment of using stochastic gradient descent. The difference between sucesive iterations is bigger using bold driver, momentum and adagrad, than without using it.
- For all the approaches, the higher the logloss in train set was, the higher the log-loss in test set was. This show a good generalization. In general, int he plots we can see that the test set log-l was higher than train-set but this happens because test set is shorter and therefore involves calculating log-loss with a summatory having less terms.