Deadline: Fr. July 5 (online) or Mo. July 8, 10:00 AM (hand-in)
Drop your printed or legible handwritten submissions into the boxes at Samelsonplatz, or upload a `.pdf` file via LearnWeb.

Remark 1. This is the last hand in tutorial for this semester. For each 20 points you earned, you will get 0.5 bonus points in the exam. (capped at 4)

1 Proximal Gradients (10 points)

A. [10p] Perform 2 iterations of ISTA and FISTA, using $s = 0.5$ and $\lambda = 0.1$ on the dataset below

x_1	x_2	x_3	y
0	1	0	1
0.5	0.5	1	0.75
1	0	0	0.5

2 Laplace Priors (10 points)

Hint: Let X be a Random Variable (RV) with density function $f(x)$. Then $Y = X^{-1}$ has the density function $\frac{1}{y^2} f(\frac{1}{y})$.

A. [2p] Explain the role of the prior $p(\beta)$ in Bayesian Linear Regression

B. [4p] Given $p(\beta_i \mid \tau_i^2) = \mathcal{N}(\beta_i \mid 0, \tau_i^2)$ and $p(\tau_i^2) = \operatorname{Exp}(\tau_i^2 \mid \frac{1}{2}\lambda^2)$, show that $z_i = \frac{1}{\tau_i^2}$ has the conditional distribution

$$p(z_i \mid \beta_i) = \operatorname{InvGauss}\left(z \,\middle|\, \sqrt{\frac{\lambda^2}{\beta_i^2}}, \lambda^2\right) \tag{1}$$

C. [2p] Given that $p(y \mid X, \beta, \sigma^2) = \mathcal{N}(y \mid X\beta, \sigma^2 I)$ and $p(\sigma^2) = \operatorname{IG}(\sigma^2 \mid a, b)$, show that σ^2 has the conditional distribution

$$p(\sigma^2 \mid X, y, \beta, \tau^2) = \operatorname{IG}(\sigma^2, a', b')$$

with $a'_\sigma = a_\sigma + \frac{1}{2}N$ and $b'_\sigma = b_\sigma + \frac{1}{2}\|y - X\beta\|_2^2$.

D. [2p] Show that in general $\mathbb{E}[X] \neq \mathbb{E}[X^{-1}]^{-1}$ by providing an example.