Exercise8SebastianPineda

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Machine Learning Lab - Exercise Sheet 8

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In this notebook we want to create a model that predicts the rating of a user on a product, i.e. a recommender system. To do that, matriz factorization approach is used. The recommeder system uses MovieLens data [1]. This dataset has been wiedly used to model different kinds of recommender systems. The notebook's outline is as follows:

1. Statistical analysis of the dataset.
2. Implementation of a basic matrix factorization (MF) technique for recommender systems.
3. Recommendation system using scikit-learn

0.0.1 Exercise 1-A:Statistical analysis of Wine dataset

```
In [13]: import numpy as np
         import pandas as pd
         import matplotlib.pyplot as plt

         %matplotlib inline
```

```
In [14]: #reading red wine dataset
         dataset = pd.read_csv("winequality-red.csv", sep=";")

         print("Size of dataset2 before dropping NA:", dataset.shape)
         dataset = dataset.dropna()
         dataset.head()
```

```
Size of dataset2 before dropping NA: (1599, 12)
```

```
Out[14]:    fixed acidity  volatile acidity  citric acid  residual sugar  chlorides  \
        0            7.4              0.70         0.00             1.9      0.076
        1            7.8              0.88         0.00             2.6      0.098
        2            7.8              0.76         0.04             2.3      0.092
        3           11.2              0.28         0.56             1.9      0.075
        4            7.4              0.70         0.00             1.9      0.076

           free sulfur dioxide  total sulfur dioxide  density    pH  sulphates  \
        0                 11.0                  34.0   0.9978  3.51       0.56
```

```
1                     25.0                  67.0   0.9968  3.20        0.68
2                     15.0                  54.0   0.9970  3.26        0.65
3                     17.0                  60.0   0.9980  3.16        0.58
4                     11.0                  34.0   0.9978  3.51        0.56

   alcohol  quality
0      9.4        5
1      9.8        5
2      9.8        5
3      9.8        6
4      9.4        5
```

For the preprocessing, we will make some exploratory analysis and also see correlations between the variables.

```
In [15]: dataset.describe()
```

```
Out[15]:        fixed acidity  volatile acidity  citric acid  residual sugar  \
         count    1599.000000       1599.000000  1599.000000     1599.000000
         mean        8.319637          0.527821     0.270976        2.538806
         std         1.741096          0.179060     0.194801        1.409928
         min         4.600000          0.120000     0.000000        0.900000
         25%         7.100000          0.390000     0.090000        1.900000
         50%         7.900000          0.520000     0.260000        2.200000
         75%         9.200000          0.640000     0.420000        2.600000
         max        15.900000          1.580000     1.000000       15.500000

                  chlorides  free sulfur dioxide  total sulfur dioxide      density  \
         count  1599.000000          1599.000000           1599.000000  1599.000000
         mean      0.087467            15.874922             46.467792     0.996747
         std       0.047065            10.460157             32.895324     0.001887
         min       0.012000             1.000000              6.000000     0.990070
         25%       0.070000             7.000000             22.000000     0.995600
         50%       0.079000            14.000000             38.000000     0.996750
         75%       0.090000            21.000000             62.000000     0.997835
         max       0.611000            72.000000            289.000000     1.003690

                         pH    sulphates      alcohol      quality
         count  1599.000000  1599.000000  1599.000000  1599.000000
         mean      3.311113     0.658149    10.422983     5.636023
         std       0.154386     0.169507     1.065668     0.807569
         min       2.740000     0.330000     8.400000     3.000000
         25%       3.210000     0.550000     9.500000     5.000000
         50%       3.310000     0.620000    10.200000     6.000000
         75%       3.400000     0.730000    11.100000     6.000000
         max       4.010000     2.000000    14.900000     8.000000
```

The describe method gives us important insight from data:

- There are aboyt 1600 samples
- Mean and median of variables are relative close, except for total sulfur dioxide (this variable may have outlier values).
- All variables are positive.
- Free sulfur dioxide also have a very high maximum value. This could indicate also the existence of outliers.

In [16]: dataset.corr()

```
Out[16]:                       fixed acidity  volatile acidity  citric acid  \
         fixed acidity              1.000000         -0.256131     0.671703
         volatile acidity          -0.256131          1.000000    -0.552496
         citric acid                0.671703         -0.552496     1.000000
         residual sugar             0.114777          0.001918     0.143577
         chlorides                  0.093705          0.061298     0.203823
         free sulfur dioxide       -0.153794         -0.010504    -0.060978
         total sulfur dioxide      -0.113181          0.076470     0.035533
         density                    0.668047          0.022026     0.364947
         pH                        -0.682978          0.234937    -0.541904
         sulphates                  0.183006         -0.260987     0.312770
         alcohol                   -0.061668         -0.202288     0.109903
         quality                    0.124052         -0.390558     0.226373

                              residual sugar  chlorides  free sulfur dioxide  \
         fixed acidity              0.114777   0.093705            -0.153794
         volatile acidity           0.001918   0.061298            -0.010504
         citric acid                0.143577   0.203823            -0.060978
         residual sugar             1.000000   0.055610             0.187049
         chlorides                  0.055610   1.000000             0.005562
         free sulfur dioxide        0.187049   0.005562             1.000000
         total sulfur dioxide       0.203028   0.047400             0.667666
         density                    0.355283   0.200632            -0.021946
         pH                        -0.085652  -0.265026             0.070377
         sulphates                  0.005527   0.371260             0.051658
         alcohol                    0.042075  -0.221141            -0.069408
         quality                    0.013732  -0.128907            -0.050656

                              total sulfur dioxide   density        pH  sulphates  \
         fixed acidity                   -0.113181  0.668047 -0.682978   0.183006
         volatile acidity                 0.076470  0.022026  0.234937  -0.260987
         citric acid                      0.035533  0.364947 -0.541904   0.312770
         residual sugar                   0.203028  0.355283 -0.085652   0.005527
         chlorides                        0.047400  0.200632 -0.265026   0.371260
         free sulfur dioxide              0.667666 -0.021946  0.070377   0.051658
         total sulfur dioxide             1.000000  0.071269 -0.066495   0.042947
         density                          0.071269  1.000000 -0.341699   0.148506
         pH                              -0.066495 -0.341699  1.000000  -0.196648
         sulphates                        0.042947  0.148506 -0.196648   1.000000
```

```
alcohol                              -0.205654 -0.496180  0.205633   0.093595
quality                              -0.185100 -0.174919 -0.057731   0.251397

                       alcohol   quality
fixed acidity        -0.061668  0.124052
volatile acidity     -0.202288 -0.390558
citric acid           0.109903  0.226373
residual sugar        0.042075  0.013732
chlorides            -0.221141 -0.128907
free sulfur dioxide  -0.069408 -0.050656
total sulfur dioxide -0.205654 -0.185100
density              -0.496180 -0.174919
pH                    0.205633 -0.057731
sulphates             0.093595  0.251397
alcohol               1.000000  0.476166
quality               0.476166  1.000000
```

From the correlation diagrams, it is seen that the variables *alcohol* has a very important correlation (about 0.476). So this predictor could turn to be also very important at prediction time.

0.0.2 Exercise 1-B:Statistical analysis of MovieLens dataset

For the statistical analysis, we import the used libraries and take a look to the dataset.

In [17]:
```
u_cols = ['user_id', 'age', 'sex', 'occupation', 'zip_code']
users = pd.read_csv('data/u.user', sep='|', names=u_cols,
                    encoding='latin-1')
print("Users dataset size:", users.shape)

users.head()
```

```
Users dataset size: (943, 5)
```

Out[17]:
```
   user_id  age sex  occupation zip_code
0        1   24   M  technician    85711
1        2   53   F       other    94043
2        3   23   M      writer    32067
3        4   24   M  technician    43537
4        5   33   F       other    15213
```

In [18]:
```
r_cols = ['user_id', 'movie_id', 'rating', 'unix_timestamp']
ratings = pd.read_csv('data/u.data', sep='\t', names=r_cols,
                      encoding='latin-1')
print("Ratings dataset size:", ratings.shape)
ratings.head()
```

```
Ratings dataset size: (100000, 4)
```

```
Out[18]:    user_id  movie_id  rating  unix_timestamp
         0      196       242       3       881250949
         1      186       302       3       891717742
         2       22       377       1       878887116
         3      244        51       2       880606923
         4      166       346       1       886397596
```

```
In [19]: m_cols = ['movie_id', 'title', 'release_date', 'video_release_date', 'imdb_url']
         m_cols = [ 'movie_id', 'title', 'release_date' , 'video_release_date', 'IMDb URL',
                    'unknown', 'Action', 'Adventure', 'Animation', 'Children', 'Comedy', 'Crime
                    'Documentary', 'Drama','Fantasy','Film-Noir','Horror','Musical',
                    'Mystery','Romance','Sci-Fi','Thriller','War','Western' ]

         movies = pd.read_csv('data/u.item', sep='|', names=m_cols, usecols=range(23),
                              encoding='latin-1')

         print ("Movies dataset size:", movies.shape)

         movies.head()
```

```
Movies dataset size: (1682, 23)
```

```
Out[19]:    movie_id              title release_date  video_release_date  \
         0         1   Toy Story (1995)  01-Jan-1995                 NaN
         1         2   GoldenEye (1995)  01-Jan-1995                 NaN
         2         3  Four Rooms (1995)  01-Jan-1995                 NaN
         3         4  Get Shorty (1995)  01-Jan-1995                 NaN
         4         5     Copycat (1995)  01-Jan-1995                 NaN

                                                     IMDb URL  unknown  Action  \
         0  http://us.imdb.com/M/title-exact?Toy%20Story%2...        0       0
         1  http://us.imdb.com/M/title-exact?GoldenEye%20(...        0       1
         2  http://us.imdb.com/M/title-exact?Four%20Rooms%...        0       0
         3  http://us.imdb.com/M/title-exact?Get%20Shorty%...        0       1
         4  http://us.imdb.com/M/title-exact?Copycat%20(1995)        0       0

            Adventure  Animation  Children    ...     Fantasy  Film-Noir  Horror  \
         0          0          1         1    ...           0          0       0
         1          1          0         0    ...           0          0       0
         2          0          0         0    ...           0          0       0
         3          0          0         0    ...           1          0       0
         4          0          0         0    ...           1          0       0

            Musical  Mystery  Romance  Sci-Fi  Thriller  War  Western
         0        0        0        0       0         0    0        0
         1        0        0        0       0         0    1        0
         2        0        0        0       0         0    1        0
```

```
3         0         0         0        0          0     0          0
4         0         0         0        0          0     1          0

[5 rows x 23 columns]
```

After importing the data, we got the following insights:

- Users dataset has 6 features and 943 different users.
- Movies dataset has 23 features and 1682 different movies.
- Ratings dataset has 4 columns (the three first columns are the most important ones). They encode information about ratings in CSR format.

We want to answer the following questions: - How many users have rated? - How many movies have being rated? - Which are the most (5) rated movies? - Which are the best (5) average rated movies? - Who are the most (5) active users? - Which users tend to give the best ratings? - Which are the range of the valid ratings?

```
In [22]: r_cols = ['user_id', 'movie_id', 'rating', 'unix_timestamp']
         ratings = pd.read_csv('data/u.data', sep='\t', names=r_cols,
                               encoding='latin-1')

         data = ratings.join (movies, on='movie_id', lsuffix="2")
         data = data.join(users, on='user_id', lsuffix="2")
         data.head()
```

```
Out[22]:    user_id2  movie_id2  rating  unix_timestamp  movie_id  \
         0       196        242       3       881250949     243.0
         1       186        302       3       891717742     303.0
         2        22        377       1       878887116     378.0
         3       244         51       2       880606923      52.0
         4       166        346       1       886397596     347.0

                                    title release_date  video_release_date  \
         0                 Jungle2Jungle (1997)  07-Mar-1997                 NaN
         1                   Ulee's Gold (1997)  01-Jan-1997                 NaN
         2         Miracle on 34th Street (1994)  01-Jan-1994                 NaN
         3   Madness of King George, The (1994)  01-Jan-1994                 NaN
         4                   Wag the Dog (1997)  09-Jan-1998                 NaN

                                                 IMDb URL  unknown    ...    \
         0  http://us.imdb.com/M/title-exact?Jungle2Jungle...      0.0    ...
         1  http://us.imdb.com/M/title-exact?Ulee%27s+Gold...      0.0    ...
         2  http://us.imdb.com/M/title-exact?Miracle%20on%...      0.0    ...
         3  http://us.imdb.com/M/title-exact?Madness%20of%...      0.0    ...
         4  http://us.imdb.com/M/title-exact?imdb-title-12...      0.0    ...

           Romance  Sci-Fi  Thriller  War  Western  user_id   age  sex  occupation  \
         0     0.0     0.0       0.0  0.0      0.0    197.0  55.0    M  technician
         1     0.0     0.0       0.0  0.0      0.0    187.0  26.0    M    educator
```

```
2       0.0      0.0        0.0  0.0       0.0     23.0  30.0     F      artist
3       0.0      0.0        0.0  0.0       0.0    245.0  22.0     M     student
4       0.0      0.0        0.0  0.0       0.0    167.0  37.0     M       other

   zip_code
0     75094
1     16801
2     48197
3     55109
4     L9G2B

[5 rows x 32 columns]
```

In [23]:
```
#Basic information
counts_user = data[['user_id','movie_id']].groupby(['user_id']).count()
counts_movies = data[['user_id','title']].groupby(['title']).count()

counts_user.columns = ['rating_count']
counts_movies.columns = ['rating_count']

print("Number of different users:", counts_user.shape[0])
print("Number of different movies:", counts_movies.shape[0])
print("Max. rating:", max(ratings['rating']))
print("Min. rating:", min(ratings['rating']))

print("Max. user id.:", max(ratings['user_id']))
print("Min. user id:", min(ratings['user_id']))

print("Max. movie id:", max(ratings['movie_id']))
print("Min. movie id:", min(ratings['movie_id']))
```

```
Number of different users: 942
Number of different movies: 1663
Max. rating: 5
Min. rating: 1
Max. user id.: 943
Min. user id: 1
Max. movie id: 1682
Min. movie id: 1
```

In [24]:
```
counts_user.sort_values(['rating_count'], ascending=False).head(5)
```

Out[24]:
```
         rating_count
user_id
406.0             737
656.0             685
14.0              636
```

```
451.0                540
277.0                518
```

In [25]:
```
counts_movies.sort_values(['rating_count'], ascending=False).head(5)
```

Out[25]:
```
                              rating_count
title
Legends of the Fall (1994)             582
George of the Jungle (1997)            509
Heavy Metal (1981)                     507
GoodFellas (1990)                      506
Breakdown (1997)                       485
```

In [26]:
```
avg_rating_user = data[['user_id','rating']].groupby(['user_id']).mean()
avg_rating_movie = data[['title', 'rating']].groupby(['title']).mean()

avg_rating_user.columns = ['avg_rating']
avg_rating_movie.columns = ['avg_rating']

std_rating_user = data[['user_id', 'rating']].groupby(['user_id']).std()
std_rating_movie = data[['title', 'rating']].groupby(['title']).std()

std_rating_user.columns = ['std_rating']
std_rating_movie.columns = ['std_rating']
```

In [27]:
```
avg_rating_user.sort_values(['avg_rating'], ascending=False).head(5)
```

Out[27]:
```
         avg_rating
user_id
850.0      4.869565
689.0      4.833333
508.0      4.724138
629.0      4.703704
929.0      4.687500
```

In [28]:
```
avg_rating_user.sort_values(['avg_rating'], ascending=True).head(5)
```

Out[28]:
```
         avg_rating
user_id
182.0      1.491954
406.0      1.834464
446.0      1.985185
686.0      2.050000
775.0      2.058036
```

In [29]:
```
avg_rating_movie.sort_values(['avg_rating'], ascending=False).head(5)
```

Out[29]:
```
                                             avg_rating
title
```

```
Guantanamera (1994)                                   5.0
Maybe, Maybe Not (Bewegte Mann, Der) (1994)           5.0
Last Time I Saw Paris, The (1954)                     5.0
One Fine Day (1996)                                   5.0
That Old Feeling (1997)                               5.0
```

We can now answer the above-mentioned questions:

- *How many users have rated?*

On the dataset of users, we see that there are 942 different users that have rated.

- *How many movies have being rated?*

On the dataset of movies, we see that there are 1682 different rated movies.

- *Which are the most (5) rated movies?*

The most rated movies are:

(1) Legends of the fall
(2) George of the jungle
(3) Heavy metal
(4) GoodFellas
(5) Breakdown

- Which are the best (5) average rated movies?

The best rated movies are:

(1) Guantanamera
(2) Maybe, Maybe Not (Bewegte Mann, der)
(3) Last Time Saw Paris
(4) One Fine Day
(5) That Old Feeling.

- Who are the most (5) active users?

The most active users are the ones with identification:

(1) 406
(2) 656
(3) 14
(4) 451
(5) 277

- Which users tend to give the best ratings?

The users which tend to give the best ratings are:

(1) 850

(2) 689
(3) 508
(4) 629
(5) 929

- Which are the range of the valid ratings?

The range valids for ratings are between 1 and 5.
A data frame is created to analysis the relationship between rating couns and rating mean.

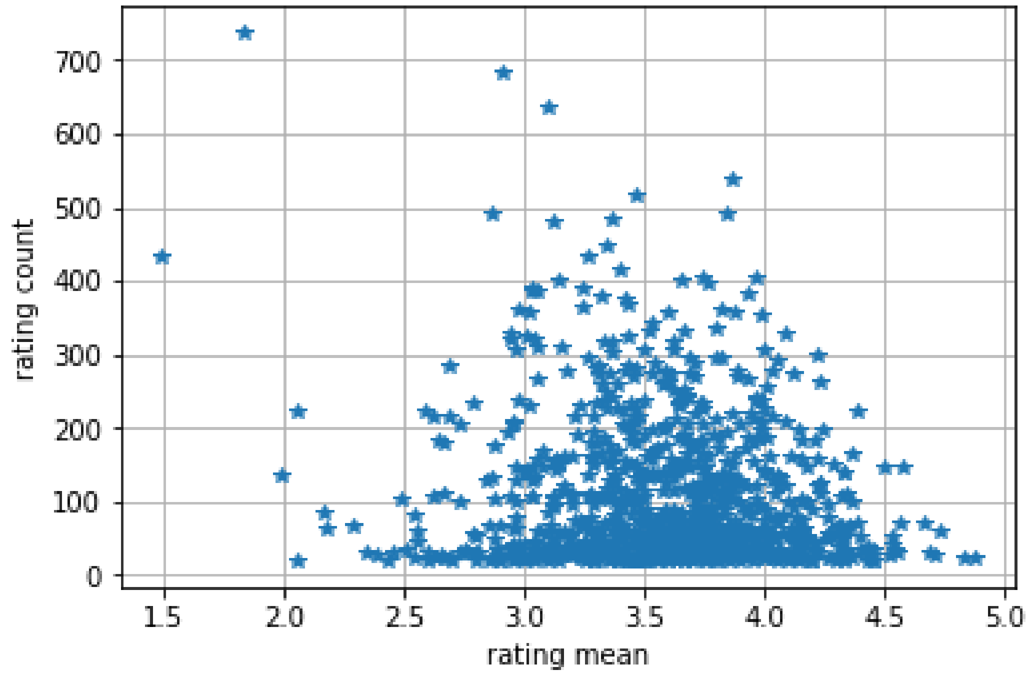
In [30]:
```
user_info = counts_user.join(avg_rating_user, rsuffix='_avg').join(std_rating_user, r
user_info.columns = ['rating_counts', 'rating_mean', 'rating_std']
user_info.head()
```

Out[30]:

user_id	rating_counts	rating_mean	rating_std
2.0	272	3.610294	1.263585
3.0	62	3.709677	1.030472
4.0	54	2.796296	1.219026
5.0	24	4.333333	0.916831
6.0	175	2.874286	1.362963

In [31]:
```
plt.plot( user_info.rating_mean, user_info.rating_counts,'*')
plt.grid()
plt.xlabel("rating mean")
plt.ylabel('rating count')
```

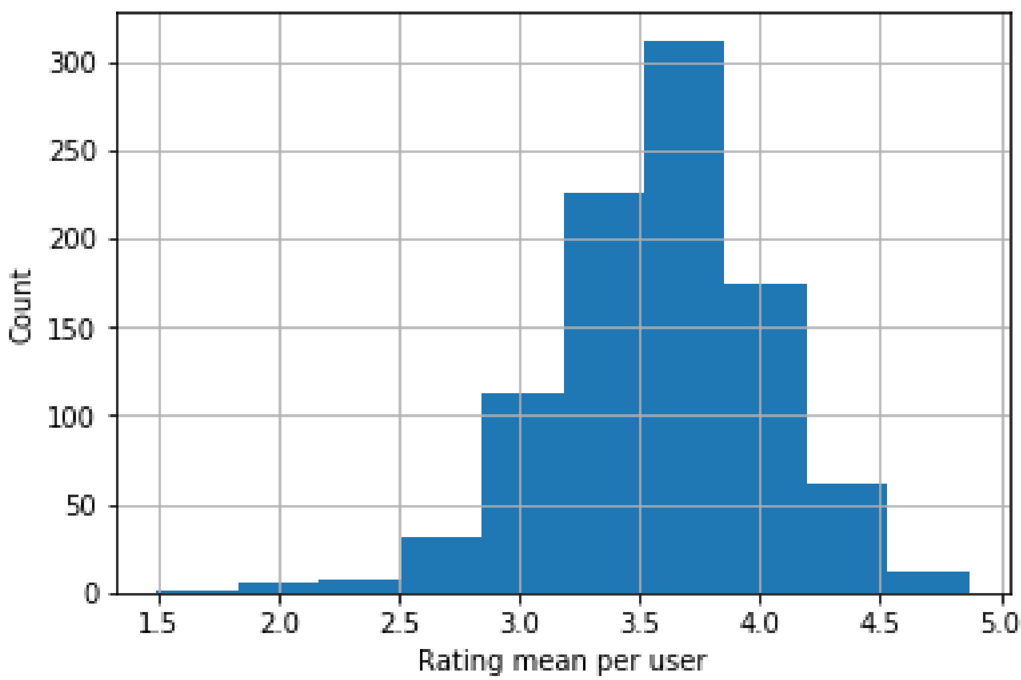
Out[31]: Text(0,0.5,'rating count')



We can see that if the user has to many ratins, the mean value tends to be around 3.5. On the other side, if the mean rating is high or low, the rating count is low. There are, however, some users that have a strange behaviour, since they have too many ratings and low rating mean: this can be considered outliers, and could be *taken out of the data*.

```
In [32]: plt.hist(user_info.rating_mean)
         plt.grid()
         plt.xlabel("Rating mean per user")
         plt.ylabel("Count")
```

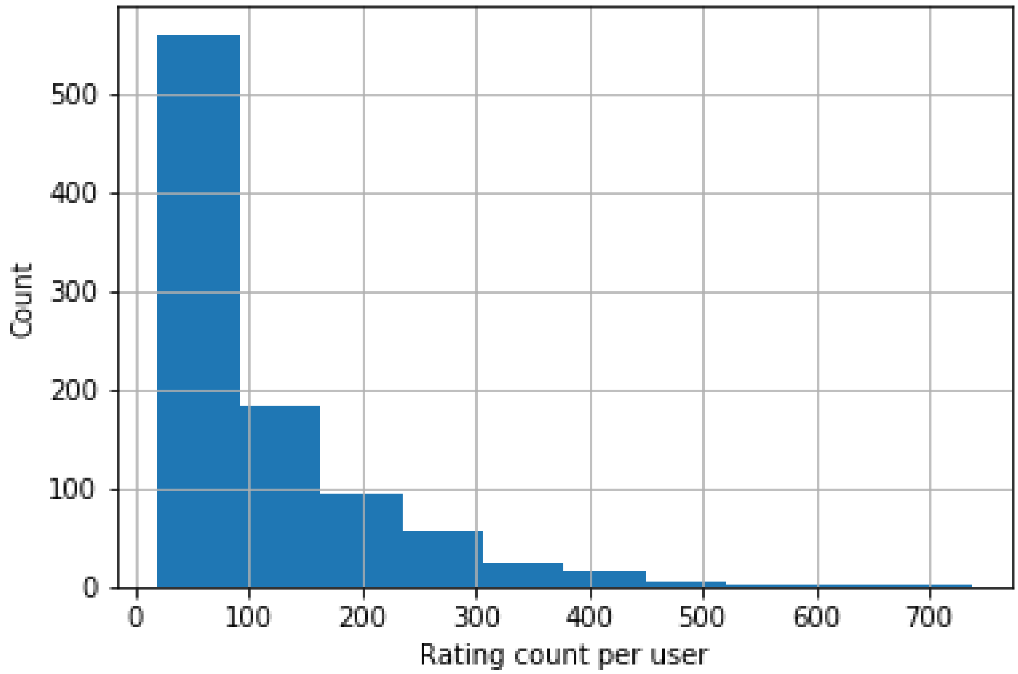
```
Out[32]: Text(0,0.5,'Count')
```



As we can see, the mean rating among the users is around 3.5.

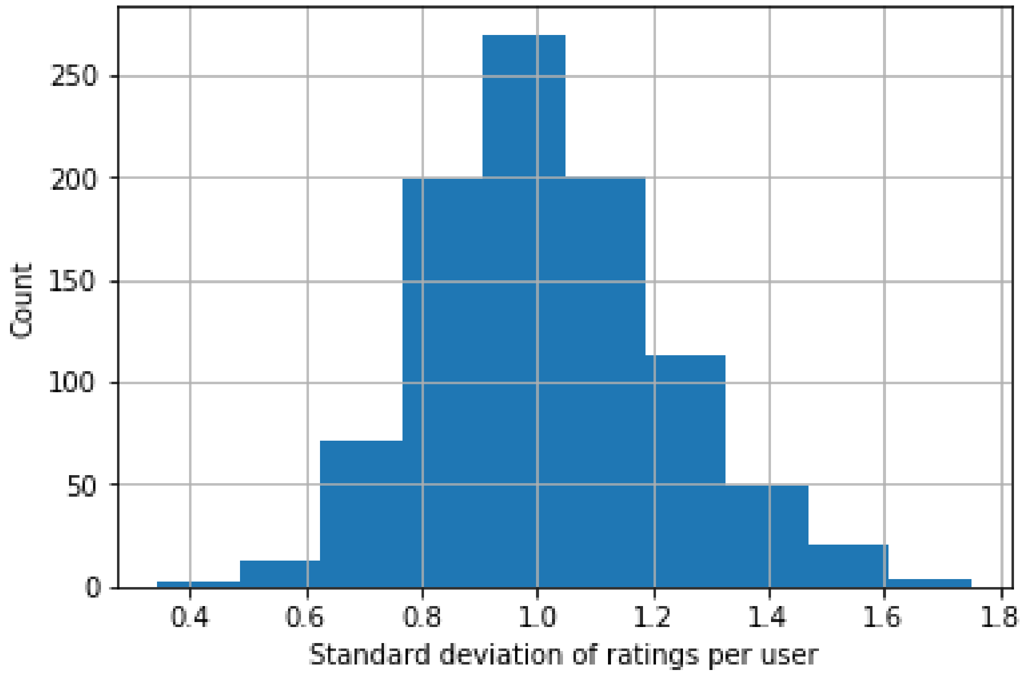
```
In [33]: plt.hist(user_info.rating_counts)
         plt.grid()
         plt.xlabel("Rating count per user")
         plt.ylabel("Count")
```

```
Out[33]: Text(0,0.5,'Count')
```



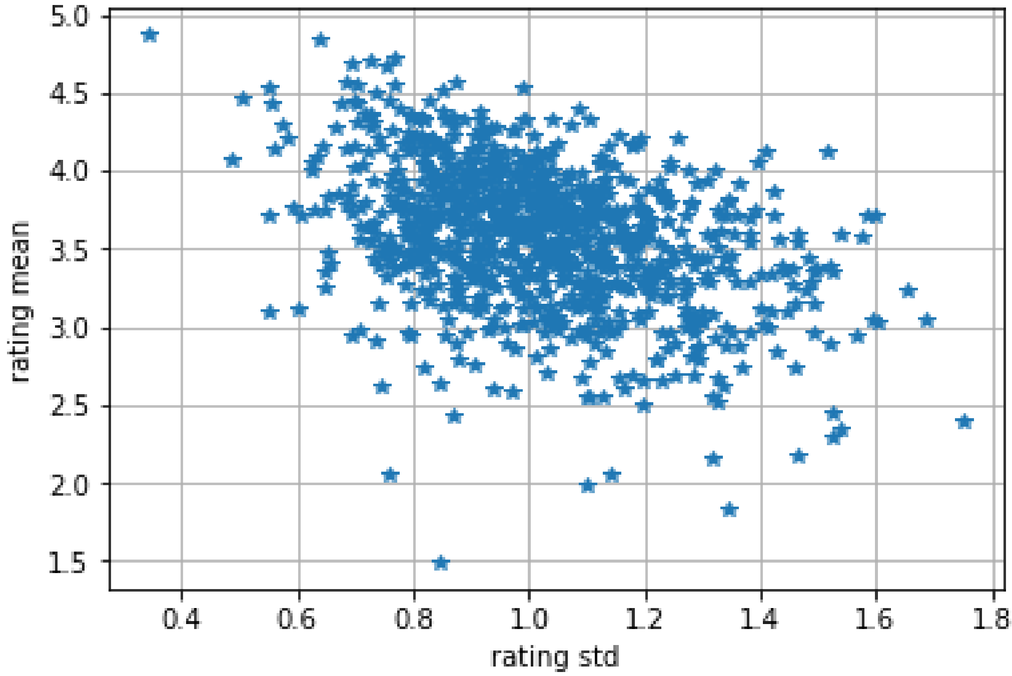
On the other hand, the rating counts among users decresases exponentially: it is more probable to find users with low rating count, while users with high rating count appears rarely.

```
In [34]: plt.hist(user_info.rating_std)
         plt.grid()
         plt.xlabel("Standard deviation of ratings per user")
         plt.ylabel("Count")

Out[34]: Text(0,0.5,'Count')
```



From the above graph, we can also conclude one thing: the mean standard deviation of the ratings is 1. It means that the rating for each user does not deviates too much from the mean rating the user has. It could therefore indicates that the mean rating ro each user could be a good predictor for future ratings.

```
In [35]: plt.plot(user_info.rating_std, user_info.rating_mean, '*')
         plt.grid()
         plt.xlabel('rating std')
         plt.ylabel('rating mean')

Out[35]: Text(0,0.5,'rating mean')
```



According to the above graph, there is a little tendence the rating mean to decrease as the standard deviation increases. It shows that the lower the mean ratings are, the more deviated are the rating from the mean.

To analize better how the mean rating pro user is related with the activity of the user (rating counts), we discretize the mean of rating (using ceil function) and then we calculate the mean of the rating counts. By this method, we find out that the users with a lower discrete mean, have higher rating counts. Of course, one of the causes could be some outliers (some users with many ratings and low mean rating as showed before).

```
In [36]: user_info['discrete_mean'] = np.ceil(user_info.rating_mean)
         user_info[['discrete_mean', 'rating_counts']].groupby(['discrete_mean']).mean()
```

```
Out[36]:                 rating_counts
         discrete_mean
         2.0                435.666667
         3.0                101.306818
         4.0                110.745739
         5.0                 79.210884
```

Now we import user related information and see how it relates to ratings.

```
In [37]: users.index = users.user_id
         users_augmented = users.join(user_info)
         users_augmented.head()
```

```
Out[37]:          user_id  age sex  occupation zip_code  rating_counts  rating_mean  \
         user_id
```

```
1                 1   24   M  technician    85711            NaN          NaN
2                 2   53   F       other    94043          272.0     3.610294
3                 3   23   M      writer    32067           62.0     3.709677
4                 4   24   M  technician    43537           54.0     2.796296
5                 5   33   F       other    15213           24.0     4.333333

          rating_std  discrete_mean
user_id
1                NaN            NaN
2           1.263585            4.0
3           1.030472            4.0
4           1.219026            3.0
5           0.916831            5.0
```

```
In [38]: users_augmented.groupby(['sex'])['rating_mean'].mean()
```

```
Out[38]: sex
         F    3.630362
         M    3.571248
         Name: rating_mean, dtype: float64
```

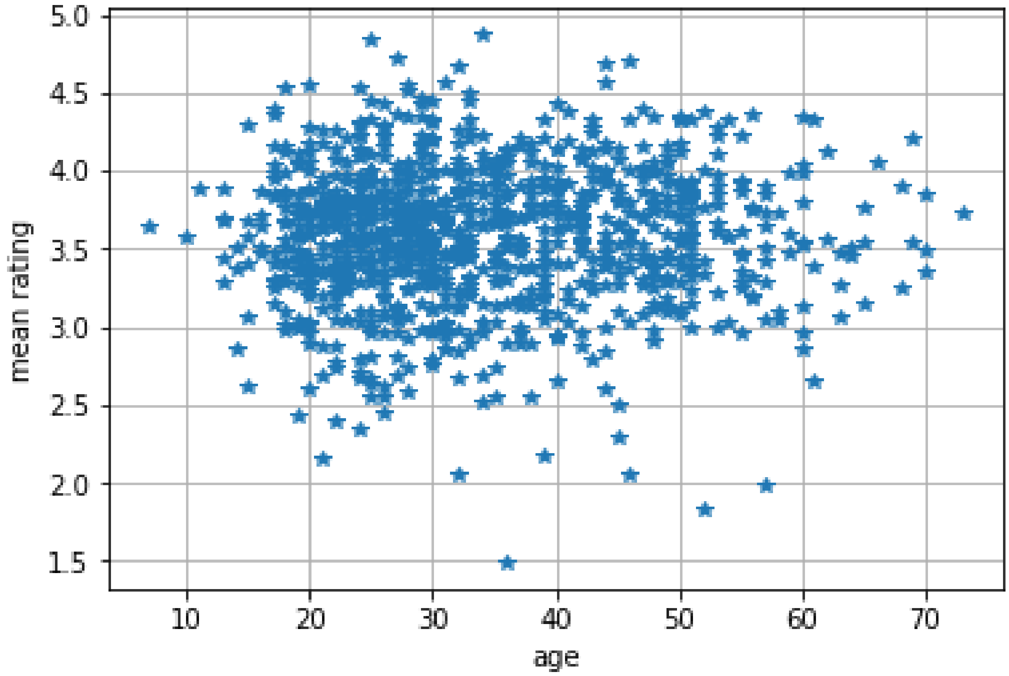
```
In [39]: cr1 = pd.crosstab(users_augmented.discrete_mean, users_augmented.sex)
         cr1 = cr1/np.sum(cr1, 0)
         cr1
```

```
Out[39]: sex                   F         M
         discrete_mean
         2.0            0.000000  0.004484
         3.0            0.076923  0.100149
         4.0            0.754579  0.744395
         5.0            0.168498  0.150972
```

According to the above results, the mean rating among men and women does not differ alot. However, given that someone is woman, could increase a littl the probabilidty that the rating is above 4, where as the probability that the socre is between 3 and 4 could be higher if the person is man.

```
In [40]: plt.plot(users_augmented.age, users_augmented.rating_mean, '*')
         plt.grid()
         plt.xlabel("age")
         plt.ylabel("mean rating")
```

```
Out[40]: Text(0,0.5,'mean rating')
```



The age may have also an impact predicting the rating. We can see, for example, if the people are older, the possibility that they give a rating lower than 3 is lower.

```
In [41]: cr2 = pd.crosstab(users_augmented.discrete_mean, users_augmented.occupation)
         cr2 = cr2/np.sum(cr2, 0)
         cr2

Out[41]: occupation     administrator    artist    doctor  educator  engineer  \
         discrete_mean
         2.0                 0.000000  0.000000  0.000000  0.021053  0.000000
         3.0                 0.101266  0.107143  0.000000  0.094737  0.164179
         4.0                 0.759494  0.821429  0.571429  0.726316  0.716418
         5.0                 0.139241  0.071429  0.428571  0.157895  0.119403

         occupation     entertainment  executive  healthcare  homemaker    lawyer  \
         discrete_mean
         2.0                 0.000000    0.00000        0.00   0.000000  0.000000
         3.0                 0.055556    0.09375        0.00   0.000000  0.083333
         4.0                 0.833333    0.71875        0.75   0.714286  0.833333
         5.0                 0.111111    0.18750        0.25   0.285714  0.083333

         occupation       ...     marketing      none     other  programmer   retired  \
         discrete_mean    ...
         2.0              ...      0.000000  0.000000  0.000000    0.015152  0.000000
         3.0              ...      0.076923  0.333333  0.057143    0.060606  0.000000
         4.0              ...      0.653846  0.666667  0.733333    0.742424  0.928571
```

```
5.0             ...      0.269231  0.000000  0.209524   0.181818  0.071429

occupation     salesman  scientist  student  technician   writer
discrete_mean
2.0            0.000000   0.000000 0.000000    0.000000 0.000000
3.0            0.000000   0.193548 0.102041    0.076923 0.133333
4.0            0.916667   0.741935 0.760204    0.692308 0.688889
5.0            0.083333   0.064516 0.137755    0.230769 0.177778

[4 rows x 21 columns]
```

In the last table, we can see that the occupation can, in fact, be somewhat important to predict the rating. In fact, the probability that the discrete mean rating falls in a given interval varies a lot depending on the occupation.

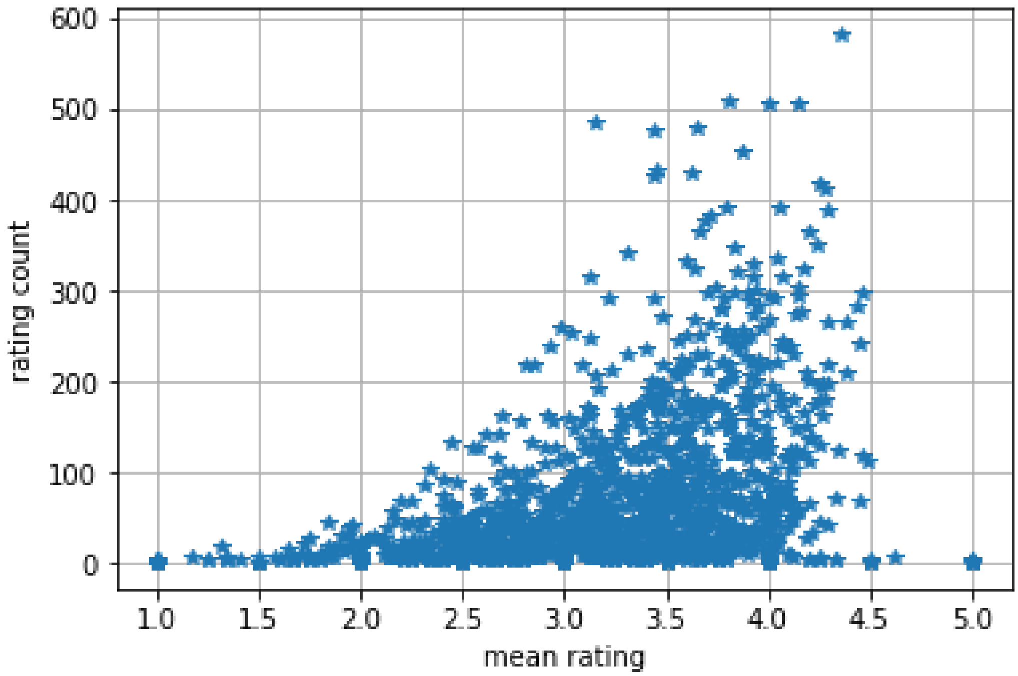
Now, movies data is used to get some insights from them.

In [42]:
```python
movies_rating_info = counts_movies.join(avg_rating_movie).join(std_rating_movie, rsuf
movies_rating_info.head()
```

Out[42]:
```
                           rating_count  avg_rating  std_rating
title
'Til There Was You (1997)             8    3.000000    0.925820
1-900 (1994)                          1    1.000000         NaN
101 Dalmatians (1996)                44    3.704545    1.069222
12 Angry Men (1957)                 137    3.861314    0.908963
187 (1997)                           45    3.333333    1.000000
```

In [43]:
```python
plt.plot(movies_rating_info.avg_rating, movies_rating_info.rating_count, '*')
plt.grid()
plt.xlabel('mean rating')
plt.ylabel('rating count')
```

Out[43]:
```
Text(0,0.5,'rating count')
```



In the last picture, we confirm something that can be obvious: the most a movie is seen, the higher is its mean rating. If a movie is seen a lot, in fact, it could indicates that it is a good movie and therefore, the rating could be higher than normal.

After permoring the previous analyzis, we conclude generally the following:

- There are some users that have many rating with low ratings. These could be considered outlier since their behavior does not match the other users behavior. Therefore, these users are extracted from the dataset. We eliminiate user 406 (since he/she has many ratings and a low rating mean) and user 182 (since his/her mean rating is too low).
- The mean rating of the movie, the mean rating of the user, the popularity of the movie, the activity, the age and occupation of the user could be very potential predictors for the rating. A predictor of the rating which uses these features could be trained to face the cold start problem, however *this predictive algorithm will not be coded in this notebook and remains as a proposal for improvement.*

0.0.3 Exercise 2: Implementation of a basic matrix factorization (MF) technique for recommender systems

We create a recommender system for the MovieLens dataset using matrix factorization. With matrix factorizations, we start from a user-item matrix (R) and find two matrices Q and P. This concept can be summarized in the following equation and the following image [2].

$R = PQ^T$

The matrix R contains the rating given by a user U to an Item I. All ratings of a given user are placed in a given row. On the other hand, all ratings of a given item are placed in a given column. Therefore, the element R_{ij} of the matrix R is the rating of the user i to the item j. The matrix $R \in R^{n \times m}$, while the matrices $P \in R^{n \times k}$ and $Q \in R^{m \times k}$ have a new common dimension K, which corresponds to latent features.

The implemented algorithm for the matrix factorization is the alternating least squares [3] which is essentially applying stochastic gradient ascent (SGD) algorithm. More information about this algorithm can be found in different sources [4][5].

To find the est hyperaparameters we apply K-Fold cross validation. We also create a test set for the final evaluation.

In [45]:
```
def split_train_test(data, train_pct, features, target):

    '''This functions divides "data"  in train and test set.
    The percentage give to the train data is determined by "train_pct".
    The "features" argument determine a list of features to consider.
    The "target" arugment indicates the the variable to predict.'''

    #getting the total number of training samples
    data_size = data.shape[0]
    train_size = int(train_pct*data_size)

    #shuffling indexes to separate train and test randoming
    idx = np.arange(0,data_size)
    np.random.shuffle(idx)

    #creating test indexes
    train_idx = idx[:train_size]

    #creating test indexes
    test_idx = idx[train_size:]

    #selecting train data (features)
    X_train = data[features].iloc[train_idx,]

    #selecting test data (features)
    X_test = data[features].iloc[test_idx,]

    return np.array(X_train), np.array(X_test)

#filtering outlier users
ratings = ratings[ratings.user_id !=406][ratings.user_id!=182]

features = ['user_id', 'movie_id', 'rating'] #list of features
target = []
X_train, X_test= split_train_test(ratings, 0.8, features, target)

print("Training set size:", X_train.shape[0])
print("Test set size:", X_test.shape[0])
```

```
Training set size: 79704
```

```
Test set size: 19926
```

```
In [53]: def threshold(x):

             '''This function truncate the value of x, so that
             it falls inside the valid range of the ratings.'''

             if(x>5): x=5
             elif(x<1): x=1

             return x

         def RMSE(X,p,q):

             '''Returns RMSE of the factorized matrix.
             Inputs are:
             - X: matrix with three columns (above exlained format)
             - p: first factor matrix
             - q: second factor matrix'''

             e = 0
             n = X.shape[0]
             for u, i, r in zip(X[:,0], X[:,1], X[:,2]):
                 e = e + (r - np.dot(p[u,:],q[i,:].T))**2

             rmse = np.sqrt(e/n)

             return rmse

         def RMSE_thr(X,p,q):

             '''Returns RMSE of the factorized matrix. This function threshold
                the predicted values to valid values (within the valid range)
                to measure the RMSE.
             Inputs are:
             - X: matrix with three columns (above exlained format)
             - p: first factor matrix
             - q: second factor matrix'''

             e = 0
             n = X.shape[0]
             for u, i, r in zip(X[:,0], X[:,1], X[:,2]):
                 e = e + (r - threshold(np.dot(p[u,:],q[i,:].T)))**2

             rmse = np.sqrt(e/n)

             return rmse
```

```python
def SGA_MF(X_train, X_test, K, n_p, n_q, alpha, lamb, max_iter=10):

    '''Factorizes a matrix given training data (X_train) and measure RMSE
    on test set (X_test). Other hyperparameters must be given:
    - n_p: number of total users (number of rows for the p matrix)
    - n_q: number of total items (numer of items for the q matrix)
    - K: number of latent features
    - alpha: learning rate
    - lamb: regularization factor'''

    #initializing values
    p = np.random.random((n_p,K))
    q = np.random.random((n_q,K))

    rmse_train_list = []
    rmse_test_list = []

    error = RMSE(X_test, p,q)
    print("Initial RMSE:",error)

    last_rmse = error

    for j in range(max_iter):

        np.random.shuffle(X_train)

        for u, i, r in zip(X_train[:,0], X_train[:,1], X_train[:,2]):
            #error
            e = r - np.dot(p[u,:],q[i,:].T)

            #updating parameters
            for k in range(K):
                p[u,k] = p[u,k] + alpha*(e*q[i,k] - lamb*p[u,k])
                q[i,k] = q[i,k] + alpha*(e*p[u,k] - lamb*q[i,k])

        #measuring rmse
        rmse_train = RMSE(X_train, p,q)
        rmse_test = RMSE(X_test, p, q)

        rmse_train_list.append(rmse_train)
        rmse_test_list.append(rmse_test)

        if (j%10)==0:
            print("Epoch: "+str(j))
            print("RMSE test:", rmse_test)
```

```
            if(np.abs(rmse_train-last_rmse)<0.001):
                break

            last_rmse = rmse_train

        fix, ax = plt.subplots(figsize=(5,5))

        ax.plot(rmse_test_list, "b")
        ax.plot(rmse_train_list, "r")
        ax.legend(("test", "train"))
        ax.set_title("RMSE vs. Iteration lambda="+str(lamb)+" alpha="+ str(alpha)+ " k=" +

        return rmse_train, rmse_test, p, q
```

Now we use the above defined functions to factorize the matrix of ratings.

```
In [56]: n_q = max(ratings["movie_id"])+1
         n_p = max(ratings["user_id"])+1

         #number of samples of training set
         n_train = X_train.shape[0]

         #number of folds
         n_folds = 3

         #initializing folds
         folds = []
         samples_fold = int(n_train/n_folds)

         #creating the k-fold subsets
         for i in range(n_folds):
             folds.append((X_train[(i*samples_fold):((i+1)*samples_fold),:]))

         folds_list = list(range(n_folds+1))

         #initialize list to store the man of each hyperparameter setting
         mean_val_folds = []

         #list of hyperparameters
         max_iter= 10
         k_list = [5,10,20]
         alpha_list = [0.01, 0.001, 0.0001]
         lamb_list = [0.1, 0.01, 0.001]

         hyper_list = [(i,j,k) for i in k_list for j in alpha_list for k in lamb_list]

         for k,alpha,lamb in hyper_list:
```

```
print("Trying hyperparameters set: k=",k,"alpha=", alpha, "lambda=",lamb)

val_rmse_folds = []

for f in range(n_folds):

        #list of folds
        folds_list = list(range(n_folds))
        folds_list.pop(f)

        #selecting test dataset
        X_val_fold = folds[f]

        #merging the folds to create the training dataset
        X_train_fold = folds[folds_list[1]]

        for j in folds_list[1:]:
            X_train_fold = np.vstack((X_train_fold, folds[j]))

        #training the model
        rmse_train, rmse_val, p, q = SGA_MF(X_train_fold, X_val_fold, k, n_p, n_q

        #finding accuracy over the fold
        val_rmse_folds.append(rmse_val)

#findning the mean across all the folds
mean_val_folds.append(np.mean(val_rmse_folds))
```

```
Trying hyperparameters set: k= 5 alpha= 0.01 lambda= 0.1
Initial RMSE: 2.58439235998
Epoch: 0
RMSE test: 1.11802885213
Initial RMSE: 2.55457807063
Epoch: 0
RMSE test: 1.10890083776
Initial RMSE: 2.55675881677
Epoch: 0
RMSE test: 1.11134611213
Trying hyperparameters set: k= 5 alpha= 0.01 lambda= 0.01
Initial RMSE: 2.59571367847
Epoch: 0
RMSE test: 1.10351606194
Initial RMSE: 2.60370595591
Epoch: 0
RMSE test: 1.11086292485
Initial RMSE: 2.61398047898
Epoch: 0
RMSE test: 1.09999987964
```

```
Trying hyperparameters set: k= 5 alpha= 0.01 lambda= 0.001
Initial RMSE: 2.58708658204
Epoch: 0
RMSE test: 1.10497977088
Initial RMSE: 2.58144013238
Epoch: 0
RMSE test: 1.10084442512
Initial RMSE: 2.56887023203
Epoch: 0
RMSE test: 1.10376882527
Trying hyperparameters set: k= 5 alpha= 0.001 lambda= 0.1
Initial RMSE: 2.62220326022
Epoch: 0
RMSE test: 2.15621278969
Initial RMSE: 2.59247221073
Epoch: 0
RMSE test: 2.13072886842
Initial RMSE: 2.59362870921
Epoch: 0
RMSE test: 2.12286244026
Trying hyperparameters set: k= 5 alpha= 0.001 lambda= 0.01
Initial RMSE: 2.59445995359
Epoch: 0
RMSE test: 2.1122794732
Initial RMSE: 2.5796583415
Epoch: 0
RMSE test: 2.09018978487
Initial RMSE: 2.57611757643
Epoch: 0
RMSE test: 2.08760012277
Trying hyperparameters set: k= 5 alpha= 0.001 lambda= 0.001
Initial RMSE: 2.57129108092
Epoch: 0
RMSE test: 2.08045517305
Initial RMSE: 2.5789238714
Epoch: 0
RMSE test: 2.0873448571
Initial RMSE: 2.59474731436
Epoch: 0
RMSE test: 2.10215761556
Trying hyperparameters set: k= 5 alpha= 0.0001 lambda= 0.1
Initial RMSE: 2.56798849388
Epoch: 0
RMSE test: 2.51923891576
Initial RMSE: 2.58723123319
Epoch: 0
RMSE test: 2.53864259951
Initial RMSE: 2.59522287741
```

```
Epoch: 0
RMSE test: 2.54715712432
```

```
C:\Users\User\Anaconda3\lib\site-packages\matplotlib\pyplot.py:523: RuntimeWarning: More than 2
  max_open_warning, RuntimeWarning)
```

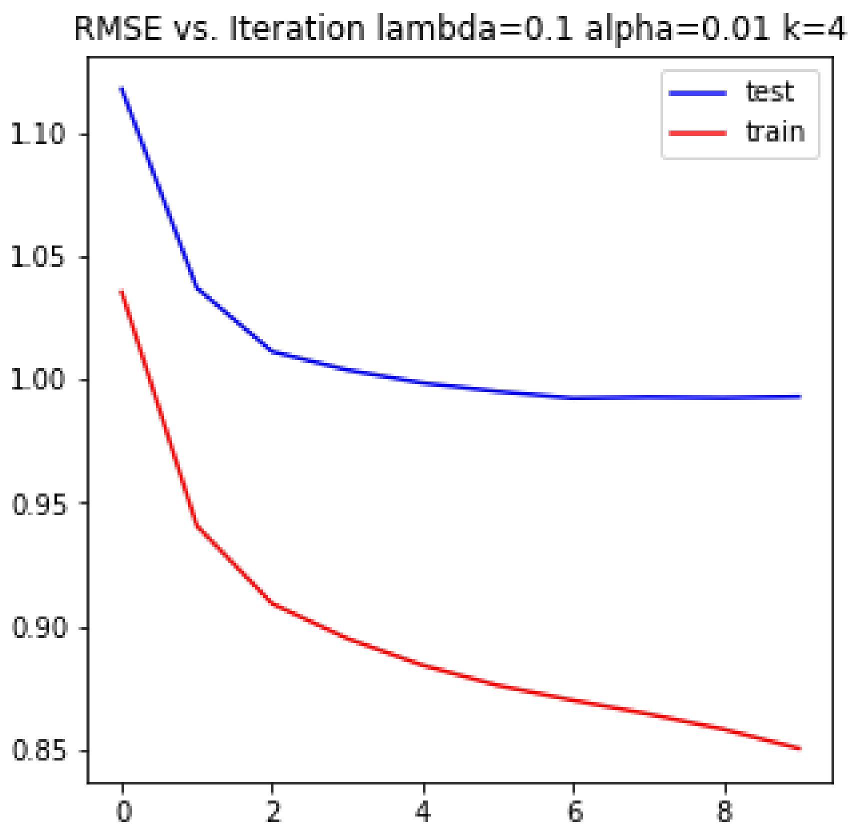
```
Trying hyperparameters set: k= 5 alpha= 0.0001 lambda= 0.01
Initial RMSE: 2.59179241885
Epoch: 0
RMSE test: 2.54192774448
Initial RMSE: 2.6003470739
Epoch: 0
RMSE test: 2.54985773821
Initial RMSE: 2.59842690473
Epoch: 0
RMSE test: 2.54853827413
Trying hyperparameters set: k= 5 alpha= 0.0001 lambda= 0.001
Initial RMSE: 2.56963829377
Epoch: 0
RMSE test: 2.51901920962
Initial RMSE: 2.60858966732
Epoch: 0
RMSE test: 2.55741678621
Initial RMSE: 2.56889893617
Epoch: 0
RMSE test: 2.51884778689
Trying hyperparameters set: k= 10 alpha= 0.01 lambda= 0.1
Initial RMSE: 1.67530133394
Epoch: 0
RMSE test: 1.03493324685
Initial RMSE: 1.68435593554
Epoch: 0
RMSE test: 1.0305674549
Initial RMSE: 1.65738370335
Epoch: 0
RMSE test: 1.02320068796
Trying hyperparameters set: k= 10 alpha= 0.01 lambda= 0.01
Initial RMSE: 1.66353754578
Epoch: 0
RMSE test: 1.033047882
Initial RMSE: 1.69760302812
Epoch: 0
RMSE test: 1.02790513972
Initial RMSE: 1.69914655759
Epoch: 0
RMSE test: 1.02446661563
```

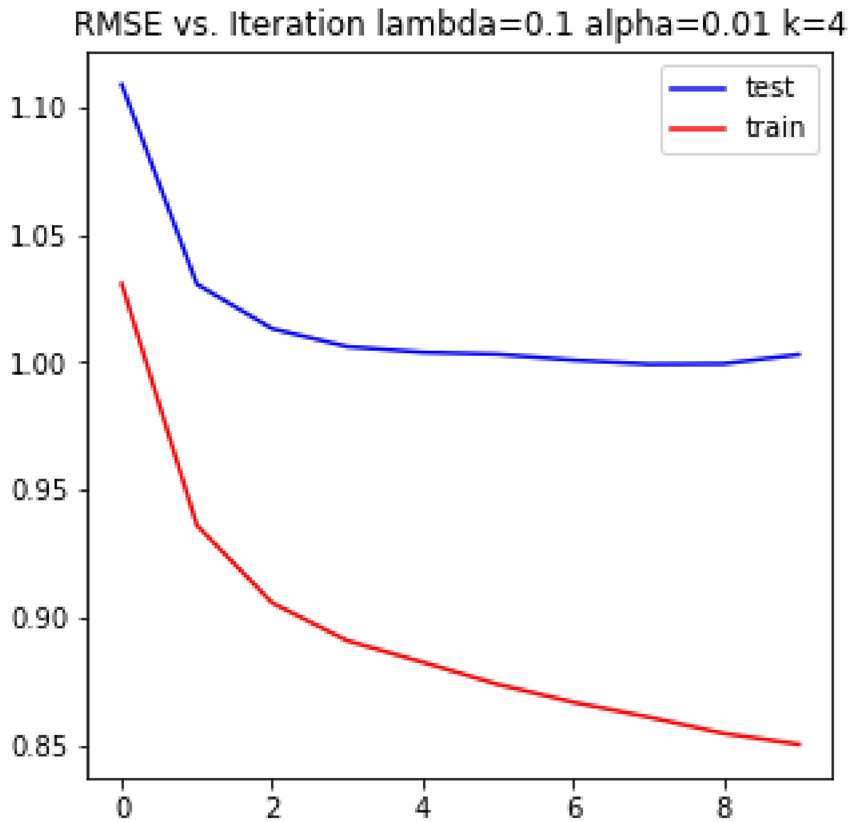
```
Trying hyperparameters set: k= 10 alpha= 0.01 lambda= 0.001
Initial RMSE: 1.70927251377
Epoch: 0
RMSE test: 1.0337270255
Initial RMSE: 1.68569095933
Epoch: 0
RMSE test: 1.03267852794
Initial RMSE: 1.6695082298
Epoch: 0
RMSE test: 1.02591539041
Trying hyperparameters set: k= 10 alpha= 0.001 lambda= 0.1
Initial RMSE: 1.65866856597
Epoch: 0
RMSE test: 1.36865075167
Initial RMSE: 1.71669491761
Epoch: 0
RMSE test: 1.40531366439
Initial RMSE: 1.67619518968
Epoch: 0
RMSE test: 1.38629882696
Trying hyperparameters set: k= 10 alpha= 0.001 lambda= 0.01
Initial RMSE: 1.68447399531
Epoch: 0
RMSE test: 1.37973822917
Initial RMSE: 1.69221577359
Epoch: 0
RMSE test: 1.37718278557
Initial RMSE: 1.70922233603
Epoch: 0
RMSE test: 1.388030158
Trying hyperparameters set: k= 10 alpha= 0.001 lambda= 0.001
Initial RMSE: 1.69905491759
Epoch: 0
RMSE test: 1.38957819721
Initial RMSE: 1.66254331349
Epoch: 0
RMSE test: 1.36616403999
Initial RMSE: 1.6747419636
Epoch: 0
RMSE test: 1.35784760531
Trying hyperparameters set: k= 10 alpha= 0.0001 lambda= 0.1
Initial RMSE: 1.68028130408
Epoch: 0
RMSE test: 1.64282788668
Initial RMSE: 1.68850111038
Epoch: 0
RMSE test: 1.65055307084
Initial RMSE: 1.6768920852
```

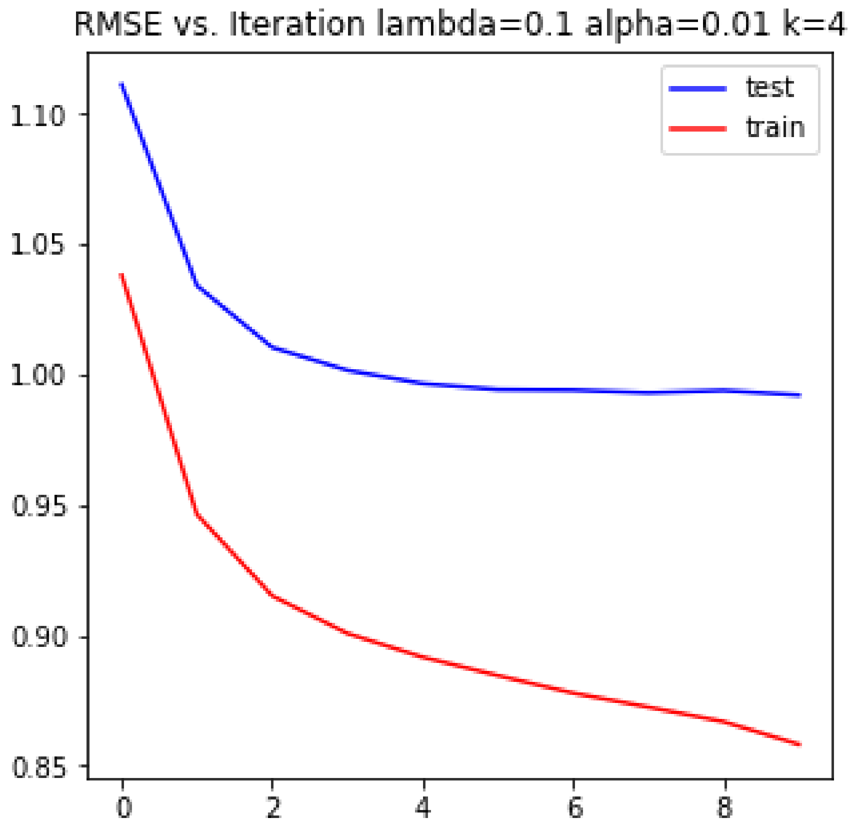
```
Epoch: 0
RMSE test: 1.63907270329
Trying hyperparameters set: k= 10 alpha= 0.0001 lambda= 0.01
Initial RMSE: 1.65183372113
Epoch: 0
RMSE test: 1.61455941187
Initial RMSE: 1.72292282418
Epoch: 0
RMSE test: 1.68083222314
Initial RMSE: 1.67832125479
Epoch: 0
RMSE test: 1.63918529652
Trying hyperparameters set: k= 10 alpha= 0.0001 lambda= 0.001
Initial RMSE: 1.69407497907
Epoch: 0
RMSE test: 1.65313198583
Initial RMSE: 1.68980327056
Epoch: 0
RMSE test: 1.64957384894
Initial RMSE: 1.69319806375
Epoch: 0
RMSE test: 1.65094265367
Trying hyperparameters set: k= 20 alpha= 0.01 lambda= 0.1
Initial RMSE: 2.06404602489
Epoch: 0
RMSE test: 1.0759867601
Initial RMSE: 2.05803109051
Epoch: 0
RMSE test: 1.07933981603
Initial RMSE: 2.09336873226
Epoch: 0
RMSE test: 1.0703691809
Trying hyperparameters set: k= 20 alpha= 0.01 lambda= 0.01
Initial RMSE: 2.09616040081
Epoch: 0
RMSE test: 1.09145513794
Initial RMSE: 2.12916078949
Epoch: 0
RMSE test: 1.09609349393
Initial RMSE: 2.06619775832
Epoch: 0
RMSE test: 1.08286596436
Trying hyperparameters set: k= 20 alpha= 0.01 lambda= 0.001
Initial RMSE: 2.09223612551
Epoch: 0
RMSE test: 1.0902307099
Initial RMSE: 2.07519479237
Epoch: 0
```

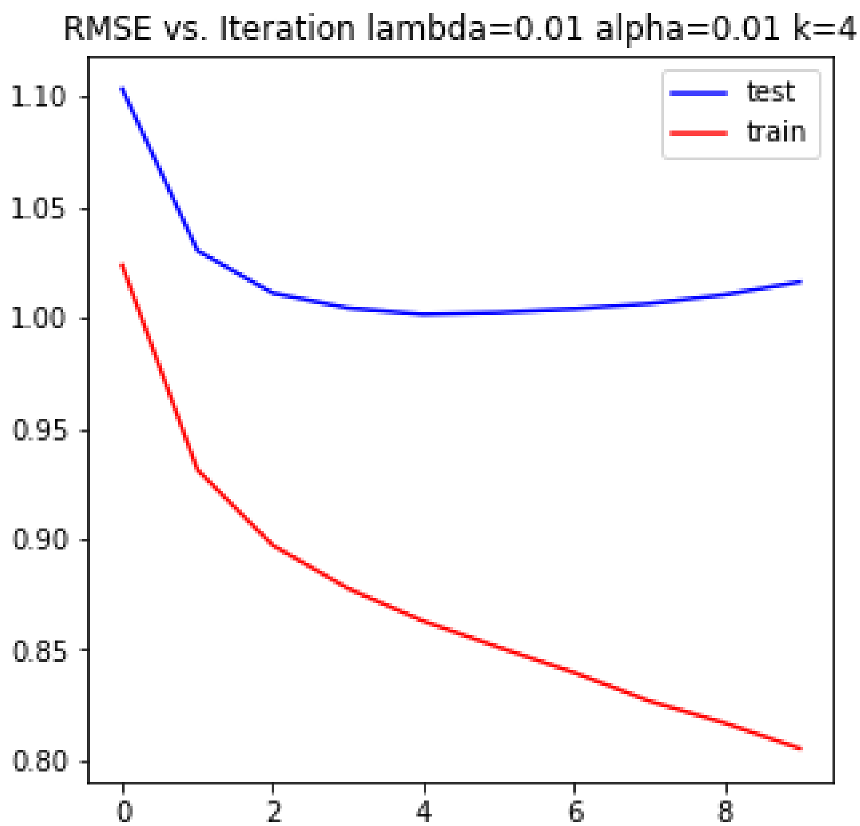
```
RMSE test: 1.09329930257
Initial RMSE: 2.02460492659
Epoch: 0
RMSE test: 1.08584875135
Trying hyperparameters set: k= 20 alpha= 0.001 lambda= 0.1
Initial RMSE: 2.1199195689
Epoch: 0
RMSE test: 1.42780614448
Initial RMSE: 2.15584627896
Epoch: 0
RMSE test: 1.43621807985
Initial RMSE: 2.058471082
Epoch: 0
RMSE test: 1.40943698495
Trying hyperparameters set: k= 20 alpha= 0.001 lambda= 0.01
Initial RMSE: 2.11545869782
Epoch: 0
RMSE test: 1.45272744372
Initial RMSE: 2.09539862723
Epoch: 0
RMSE test: 1.44413167905
Initial RMSE: 2.1215502394
Epoch: 0
RMSE test: 1.43837269053
Trying hyperparameters set: k= 20 alpha= 0.001 lambda= 0.001
Initial RMSE: 2.08348158679
Epoch: 0
RMSE test: 1.44161529904
Initial RMSE: 2.11202252171
Epoch: 0
RMSE test: 1.4450658656
Initial RMSE: 2.08469550346
Epoch: 0
RMSE test: 1.43979139743
Trying hyperparameters set: k= 20 alpha= 0.0001 lambda= 0.1
Initial RMSE: 2.12581454092
Epoch: 0
RMSE test: 1.98951400028
Initial RMSE: 2.04951820019
Epoch: 0
RMSE test: 1.92191109708
Initial RMSE: 2.09226727149
Epoch: 0
RMSE test: 1.95747311821
Trying hyperparameters set: k= 20 alpha= 0.0001 lambda= 0.01
Initial RMSE: 2.12329336146
Epoch: 0
RMSE test: 1.99619619748
```

```
Initial RMSE: 2.10333674654
Epoch: 0
RMSE test: 1.97354770234
Initial RMSE: 2.07647964414
Epoch: 0
RMSE test: 1.95330892976
Trying hyperparameters set: k= 20 alpha= 0.0001 lambda= 0.001
Initial RMSE: 2.10938775611
Epoch: 0
RMSE test: 1.97913371861
Initial RMSE: 2.08908603368
Epoch: 0
RMSE test: 1.96353713447
Initial RMSE: 2.06834659079
Epoch: 0
RMSE test: 1.94418015253
```




RMSE vs. Iteration lambda=0.1 alpha=0.01 k=4
test
train
1.10
1.05
1.00
0.95
0.90
0.85
0
2
4
6
8


RMSE vs. Iteration lambda=0.1 alpha=0.01 k=4
test
train
1.10
1.05
1.00
0.95
0.90
0.85
0
2
4
6
8


RMSE vs. Iteration lambda=0.01 alpha=0.01 k=4
test
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1.00
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0.85
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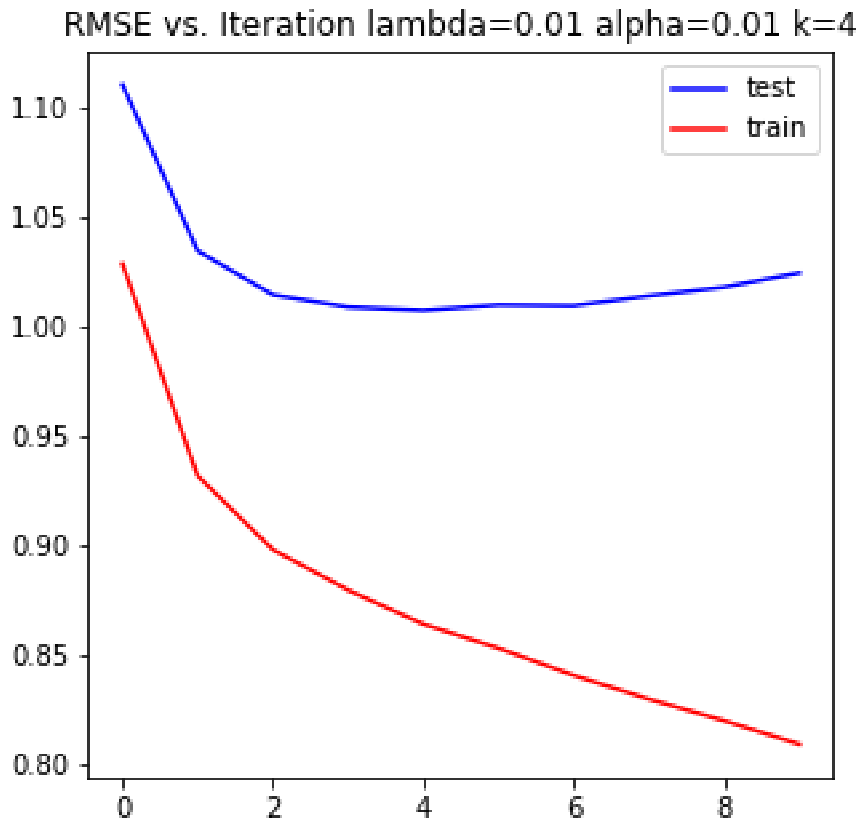
RMSE vs. Iteration lambda=0.01 alpha=0.01 k=4
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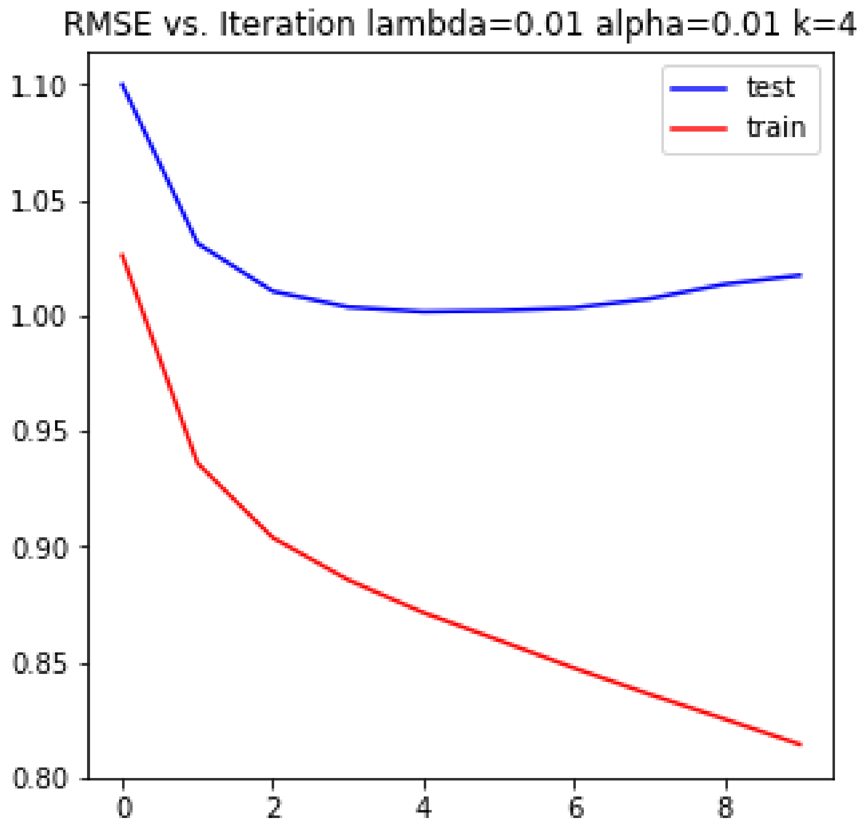
RMSE vs. Iteration lambda=0.01 alpha=0.01 k=4
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1.00
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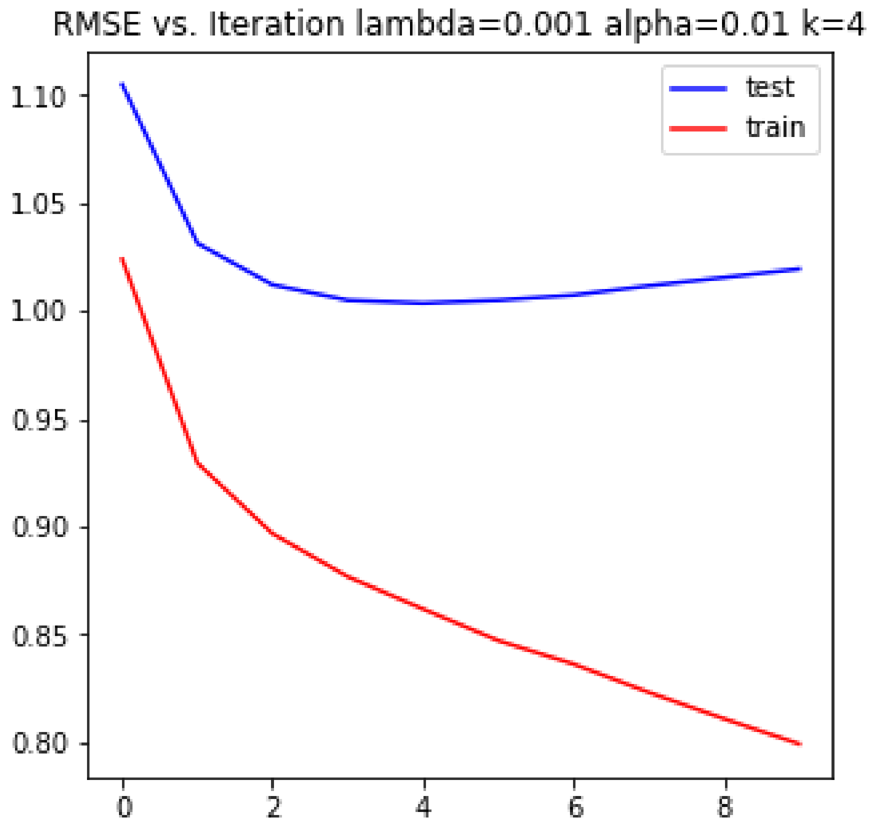

RMSE vs. Iteration lambda=0.001 alpha=0.01 k=4
test
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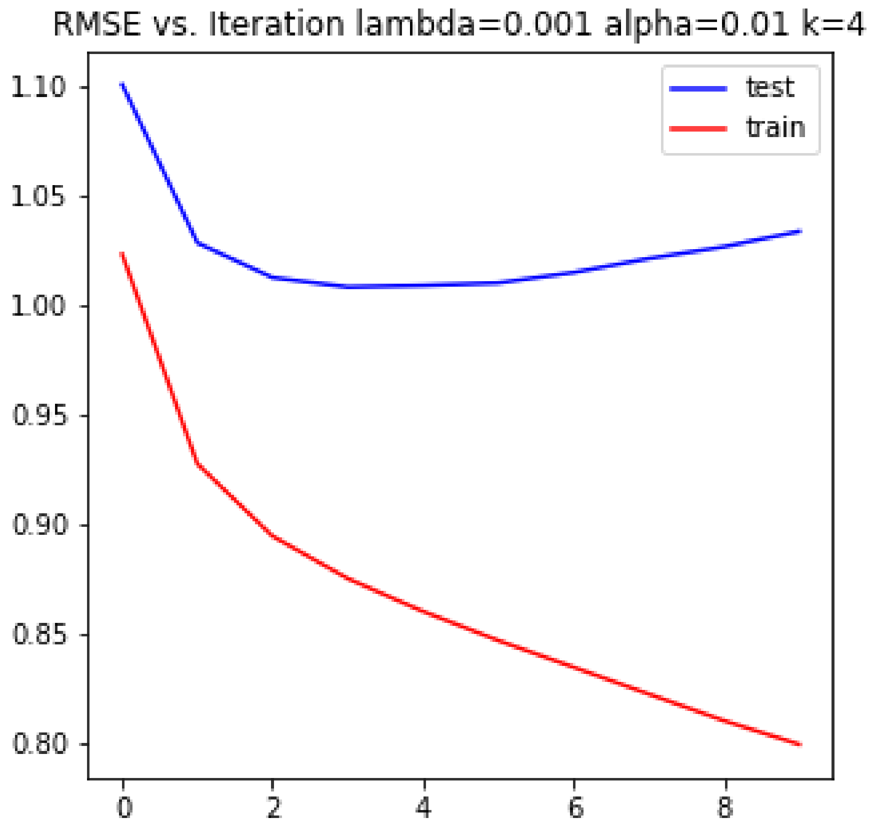
RMSE vs. Iteration lambda=0.001 alpha=0.01 k=4
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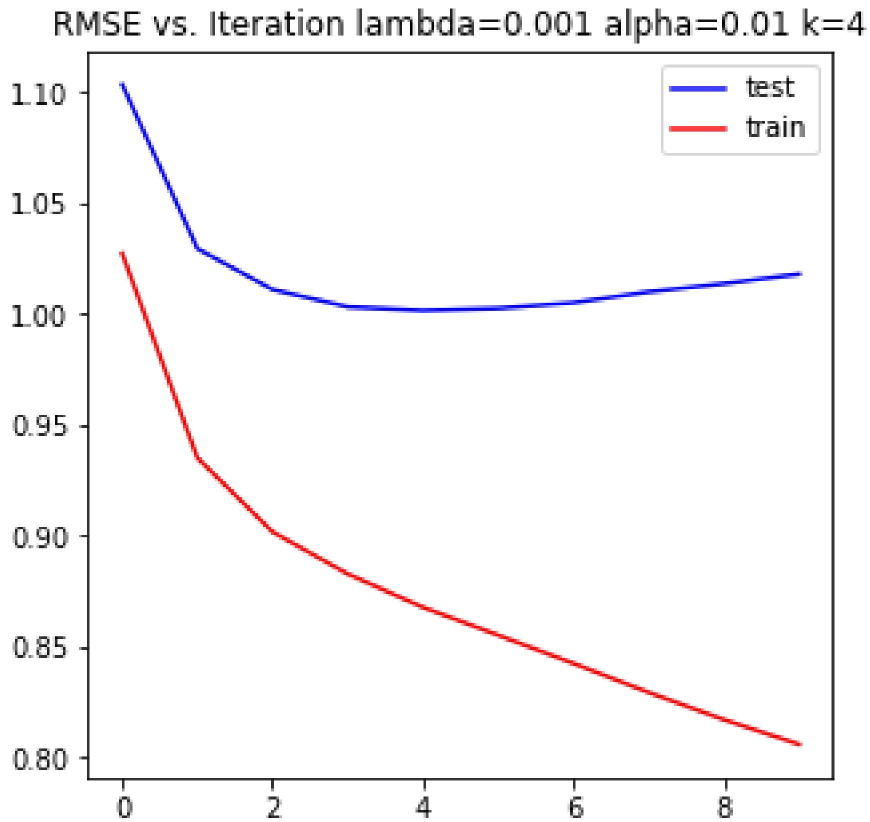
6
8
test
train


RMSE vs. Iteration lambda=0.001 alpha=0.01 k=4
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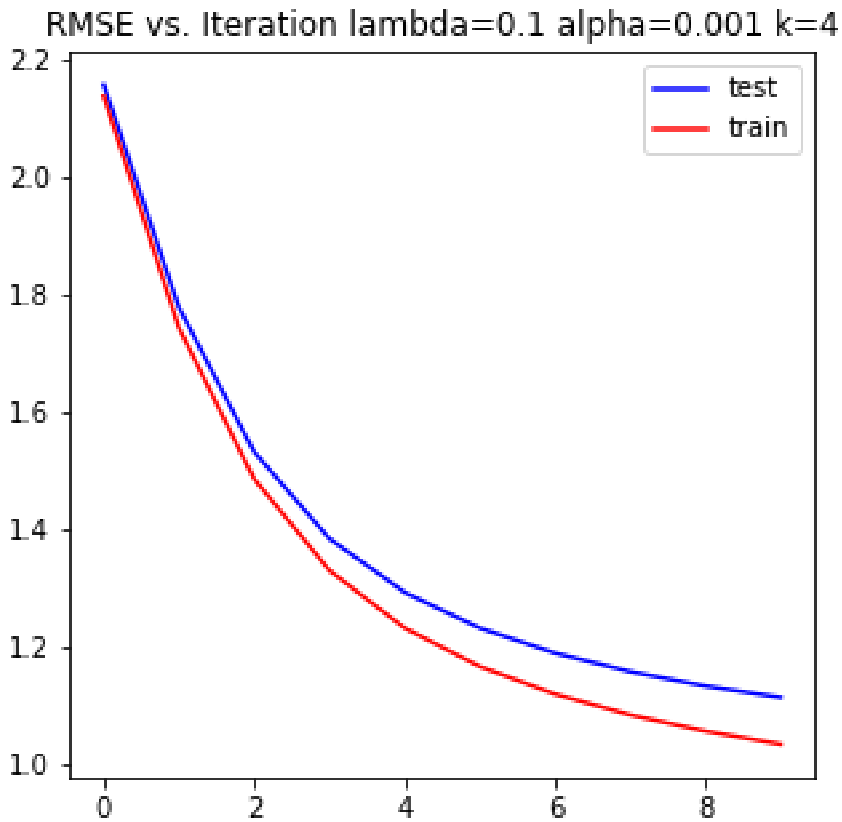
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2.0
1.8
1.6
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1.2
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2
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6
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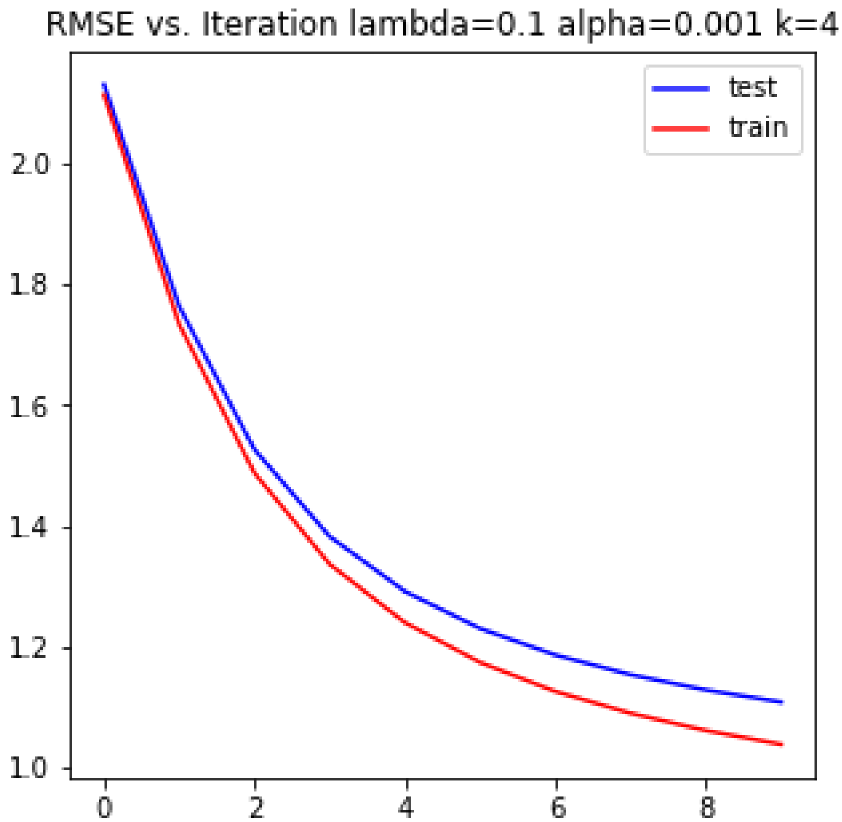
RMSE vs. Iteration lambda=0.1 alpha=0.001 k=4
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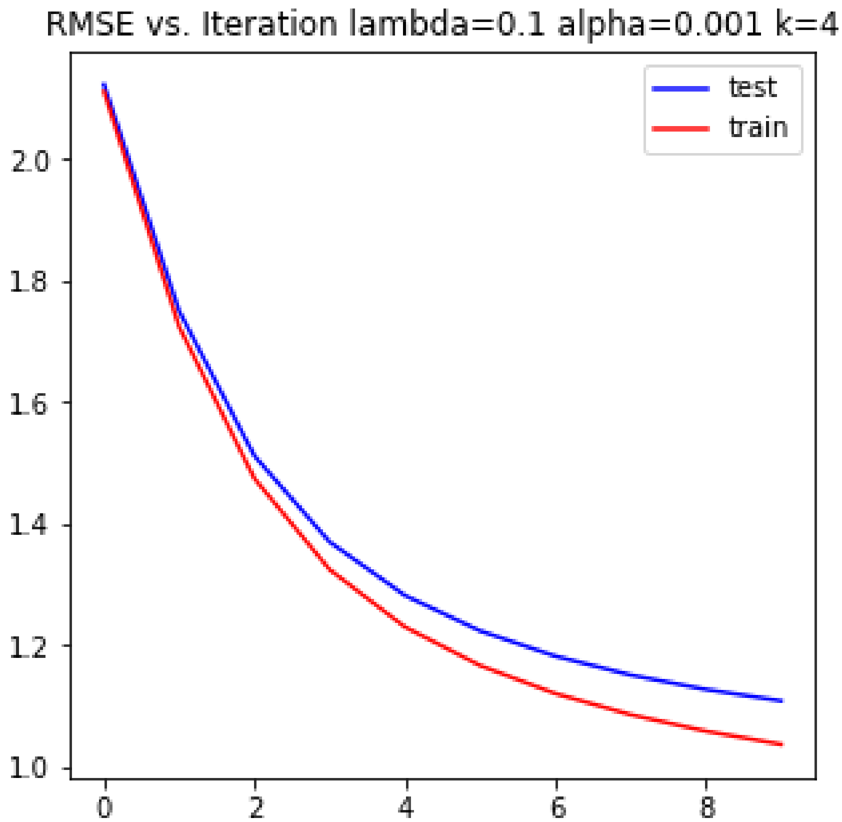
RMSE vs. Iteration lambda=0.1 alpha=0.001 k=4
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train
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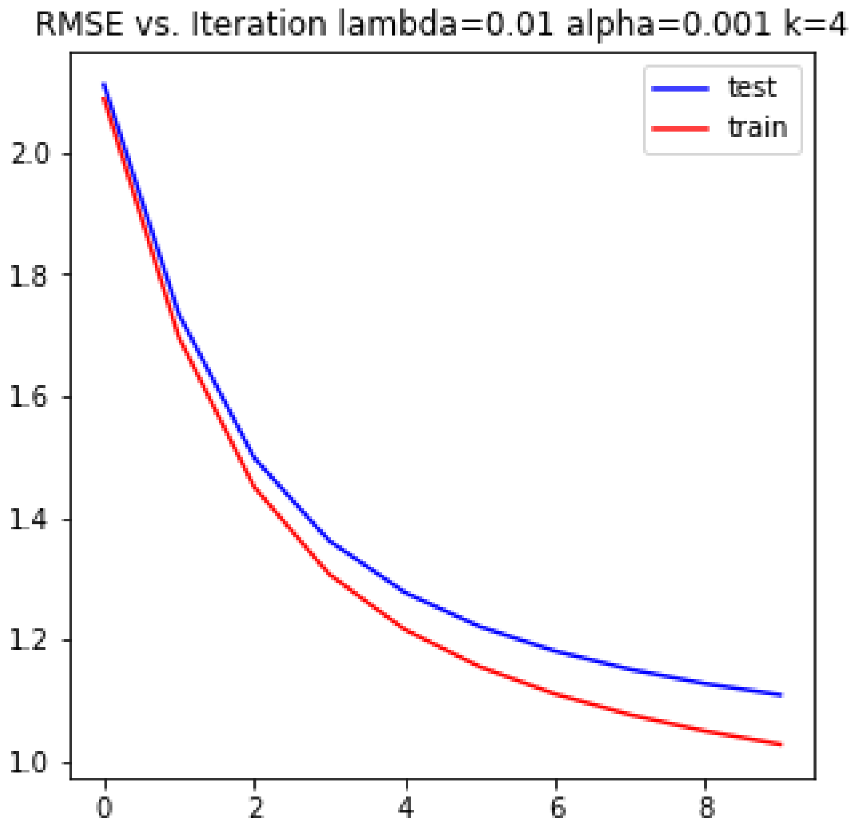

RMSE vs. Iteration lambda=0.01 alpha=0.001 k=4
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train
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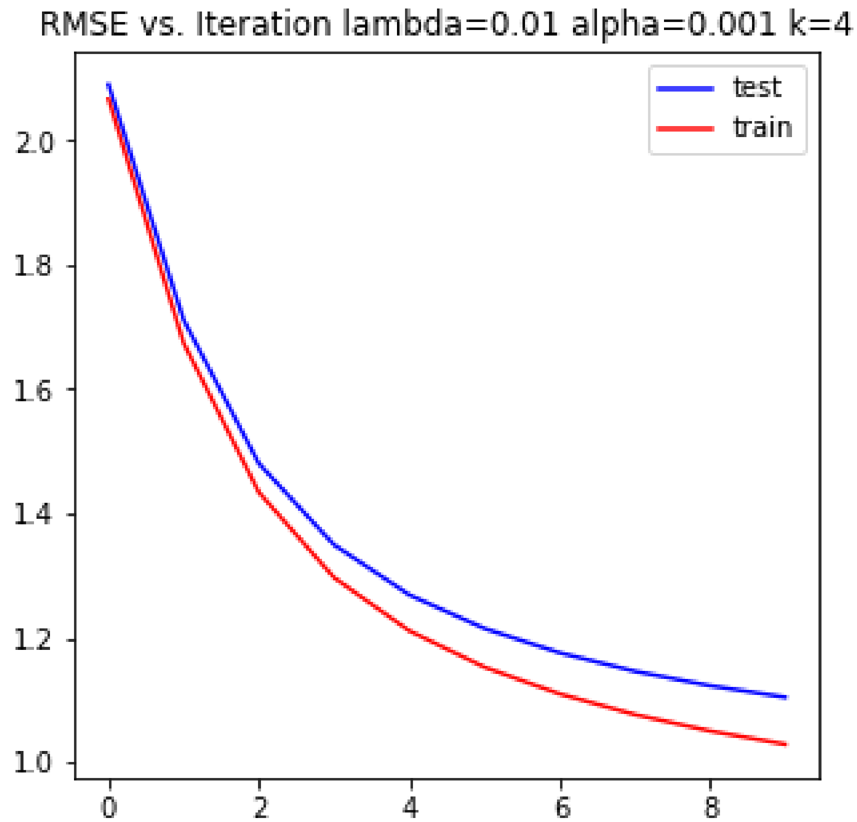
RMSE vs. Iteration lambda=0.01 alpha=0.001 k=4
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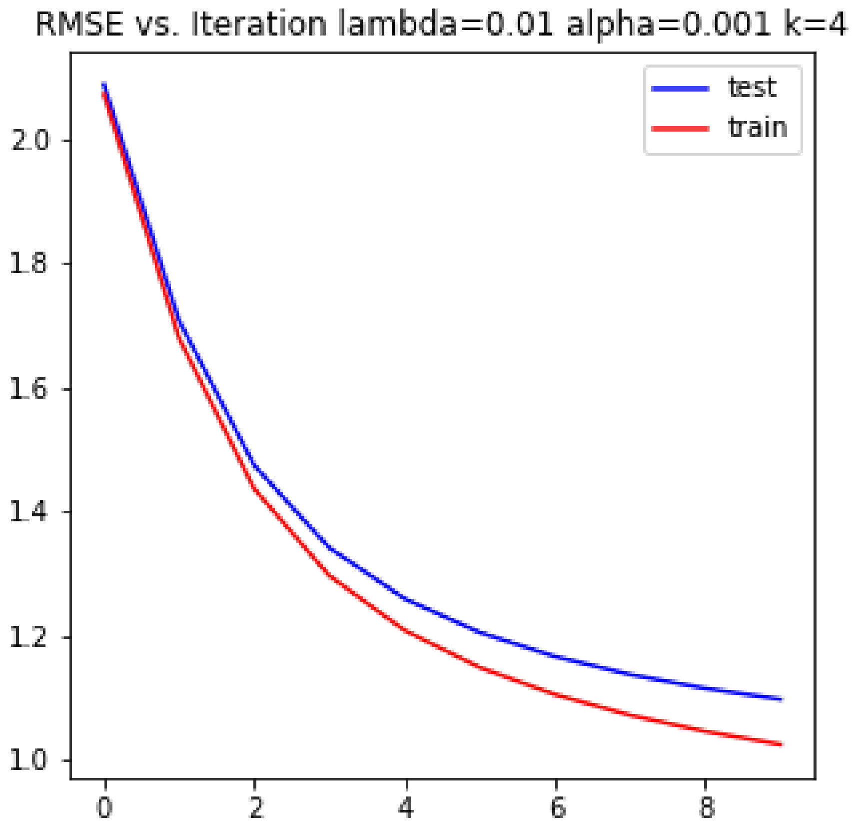
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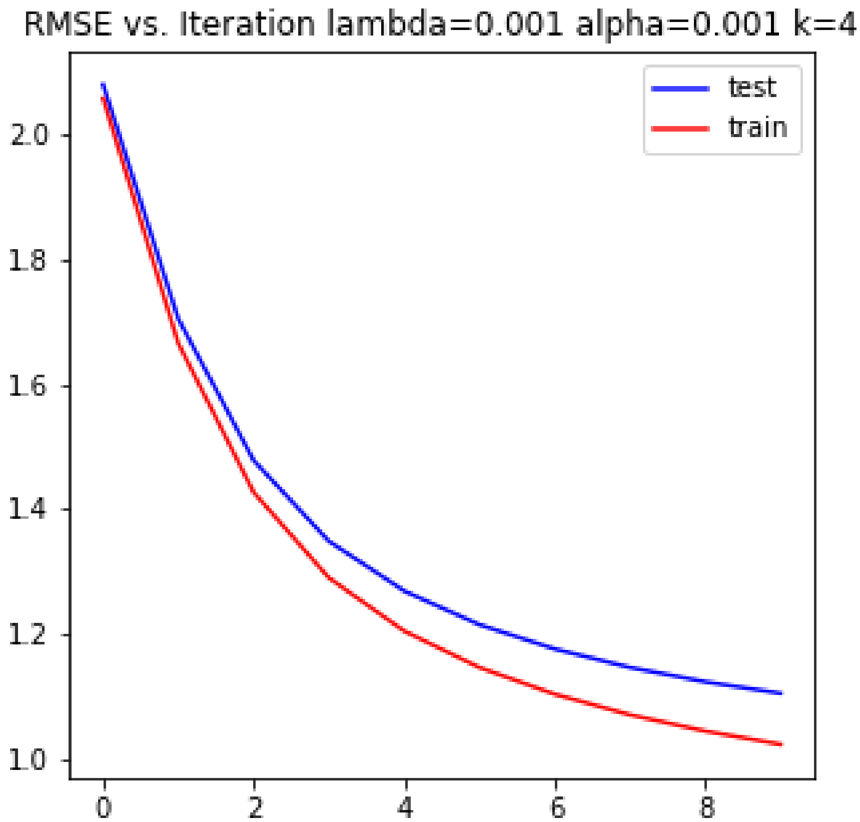
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train
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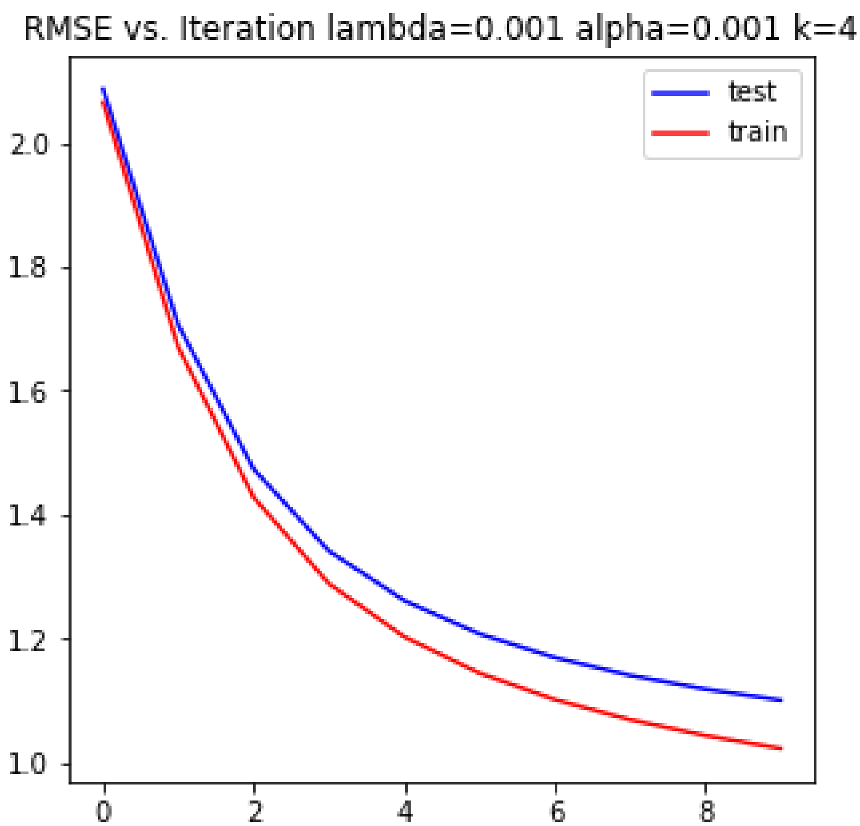
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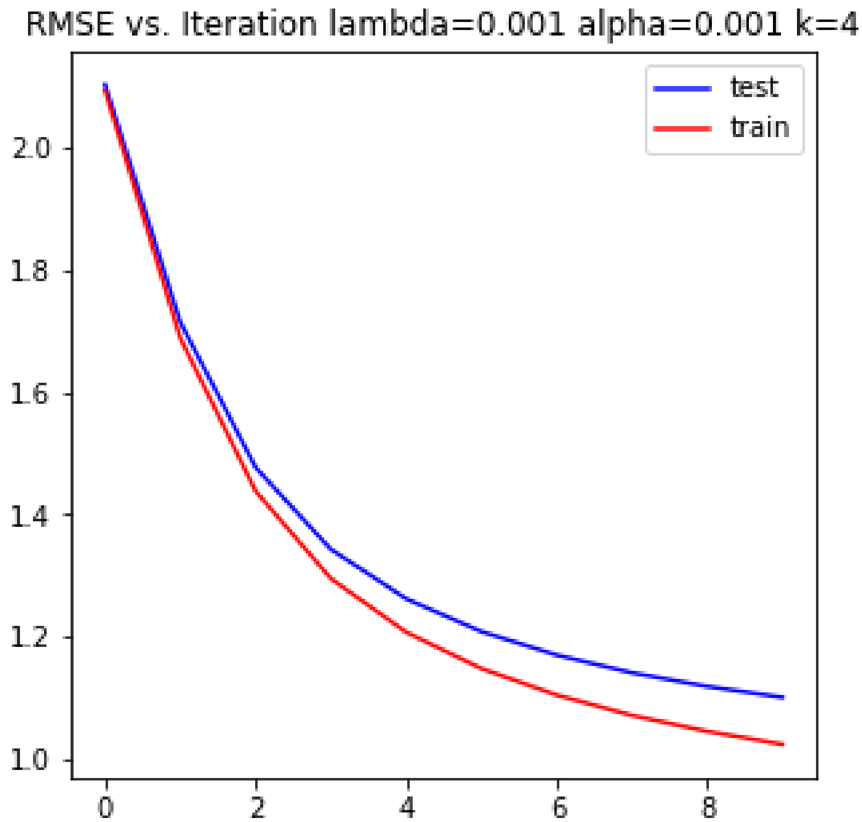
RMSE vs. Iteration lambda=0.001 alpha=0.001 k=4
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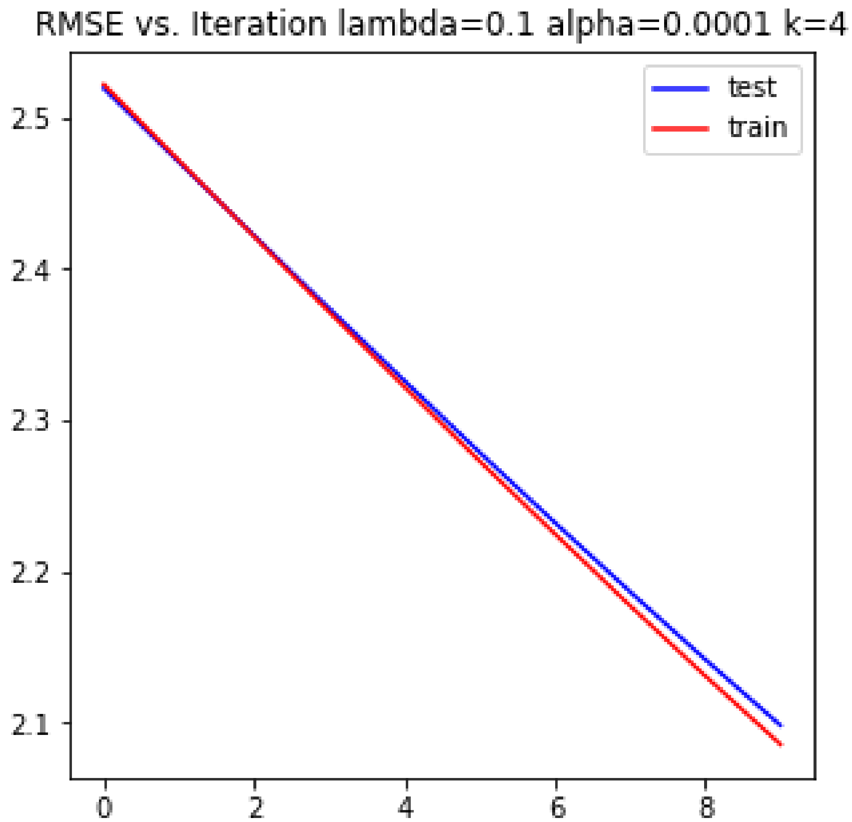
RMSE vs. Iteration lambda=0.1 alpha=0.0001 k=4
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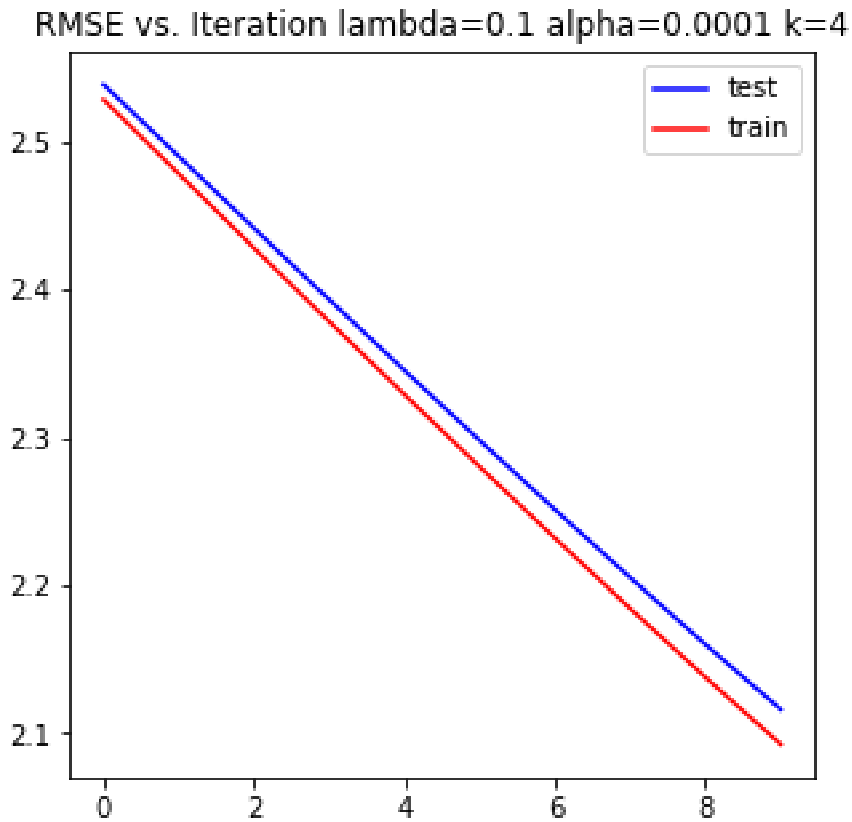

RMSE vs. Iteration lambda=0.1 alpha=0.0001 k=4
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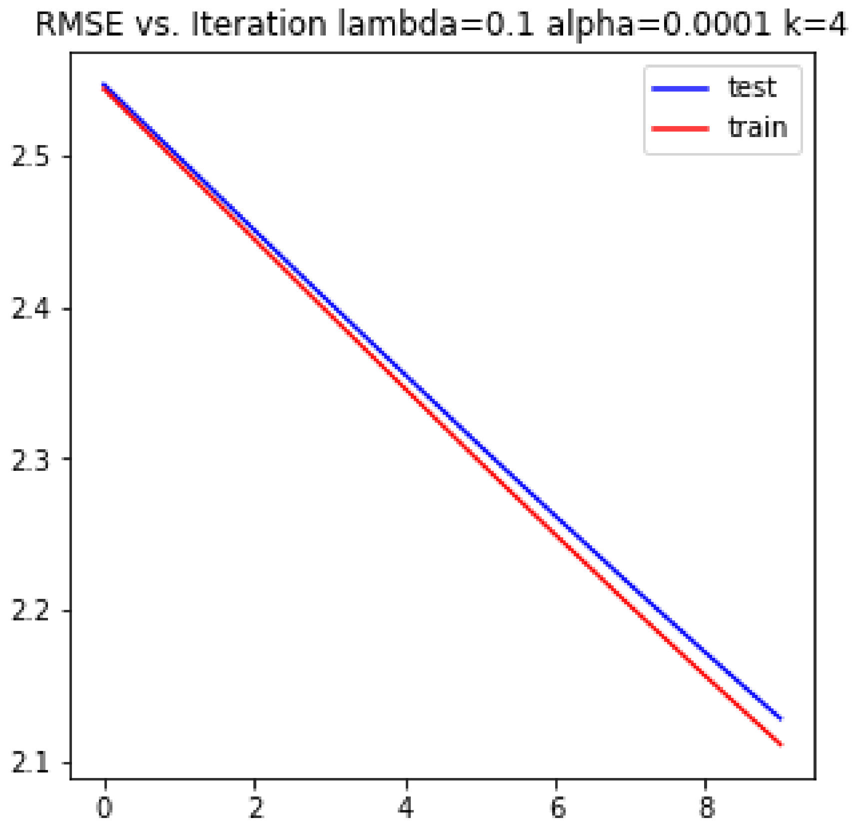
RMSE vs. Iteration lambda=0.1 alpha=0.0001 k=4
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train
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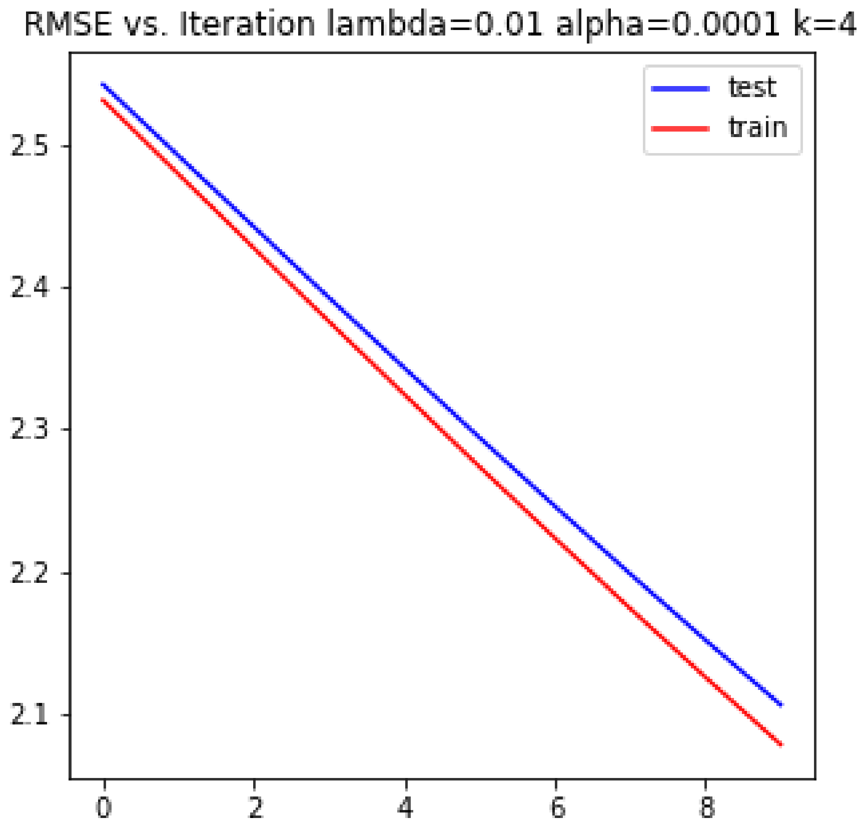
RMSE vs. Iteration lambda=0.01 alpha=0.0001 k=4
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train
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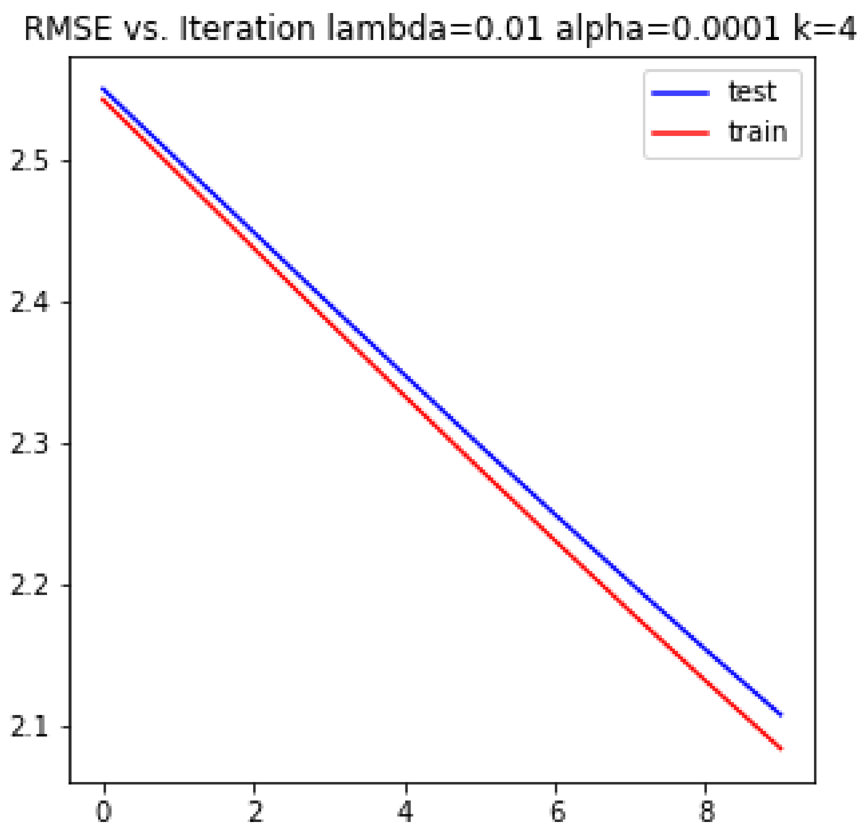

RMSE vs. Iteration lambda=0.01 alpha=0.0001 k=4
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train
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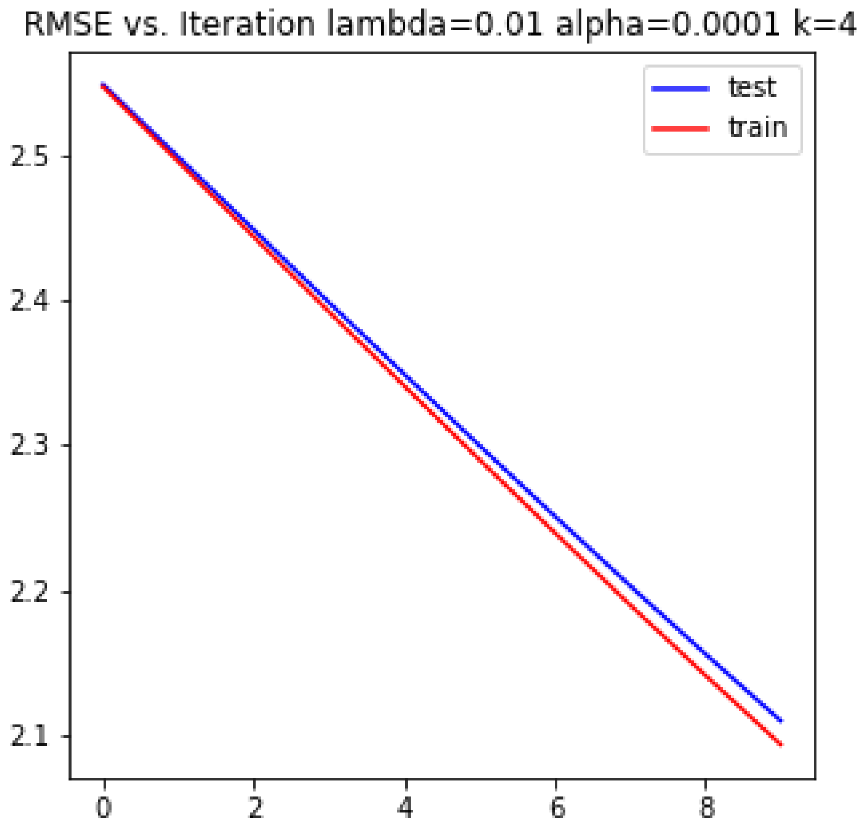
RMSE vs. Iteration lambda=0.01 alpha=0.0001 k=4
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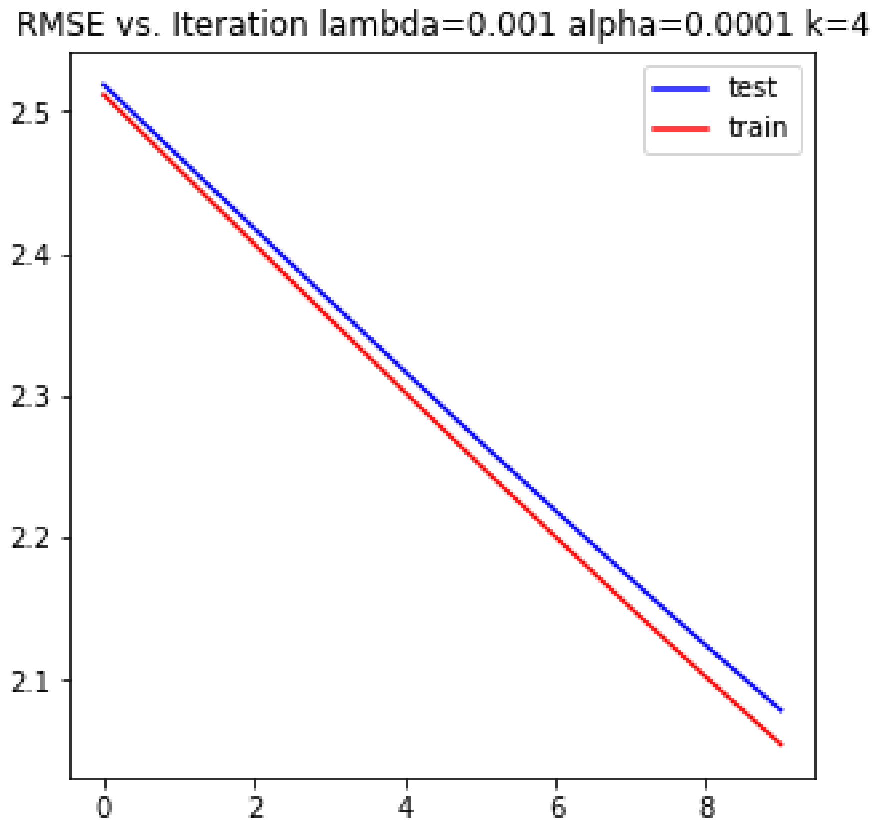
RMSE vs. Iteration lambda=0.001 alpha=0.0001 k=4
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train
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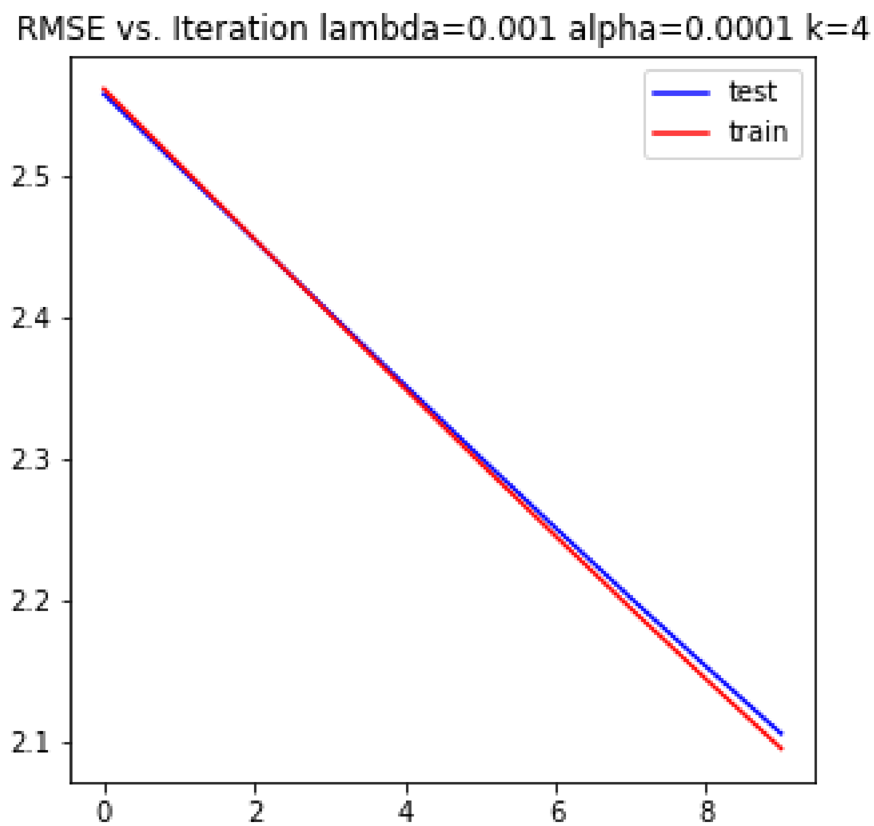
RMSE vs. Iteration lambda=0.001 alpha=0.0001 k=4
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train
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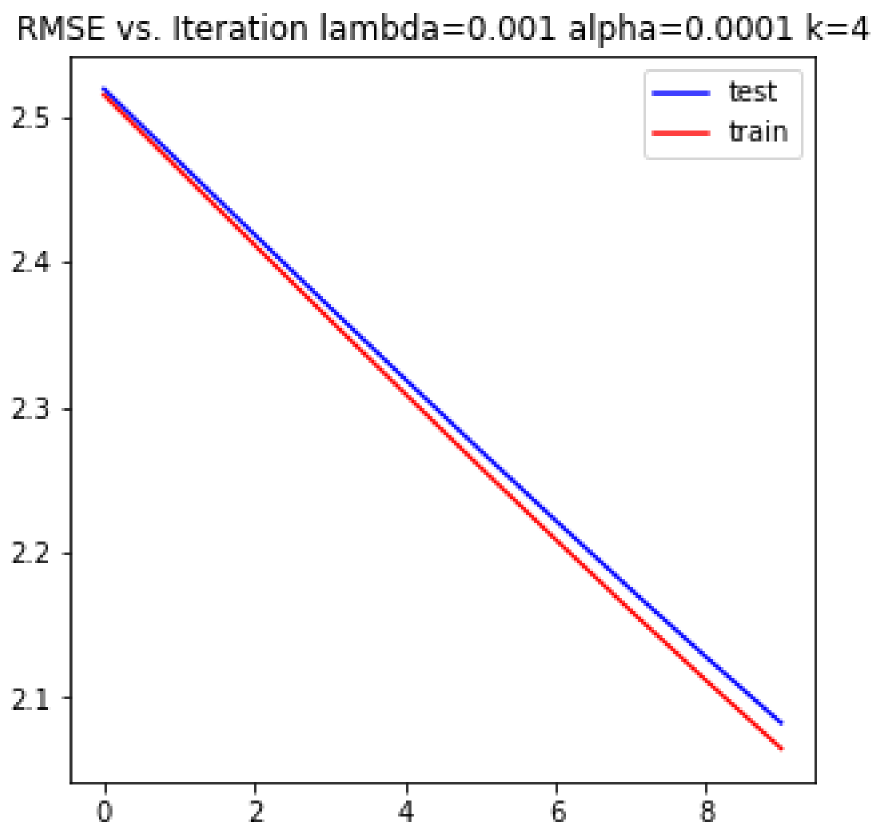
RMSE vs. Iteration lambda=0.001 alpha=0.0001 k=4
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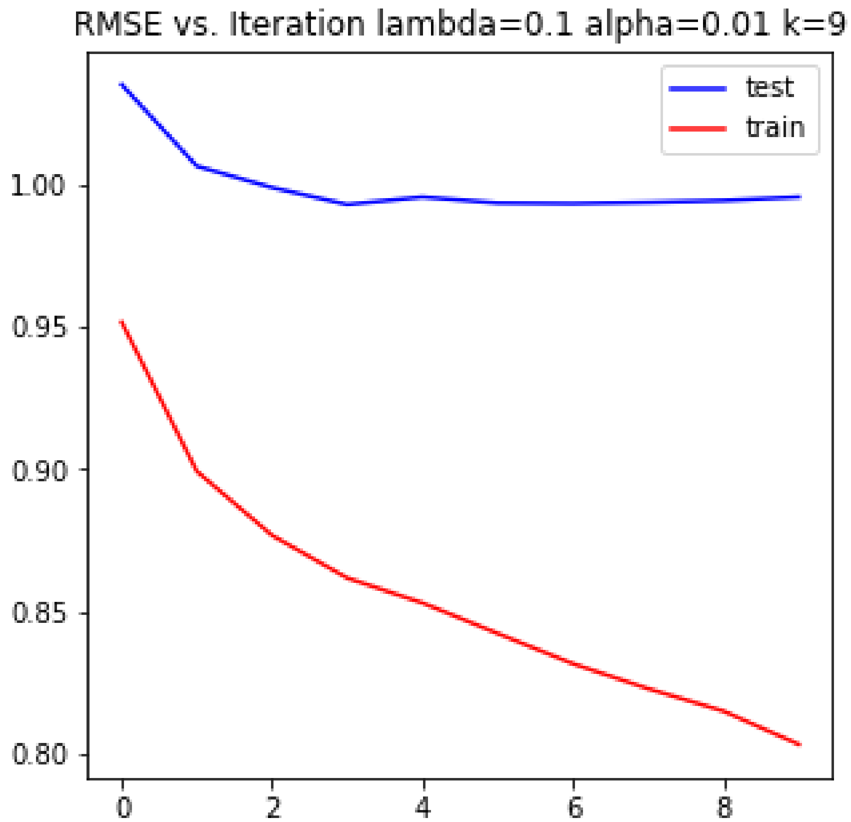
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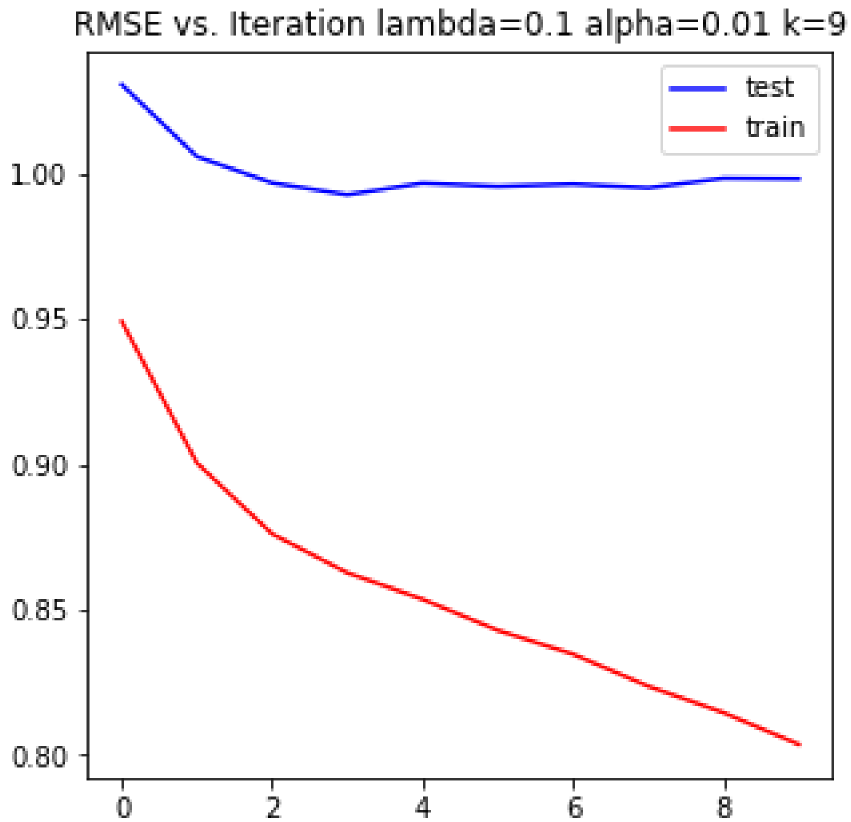
RMSE vs. Iteration lambda=0.1 alpha=0.01 k=9
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train
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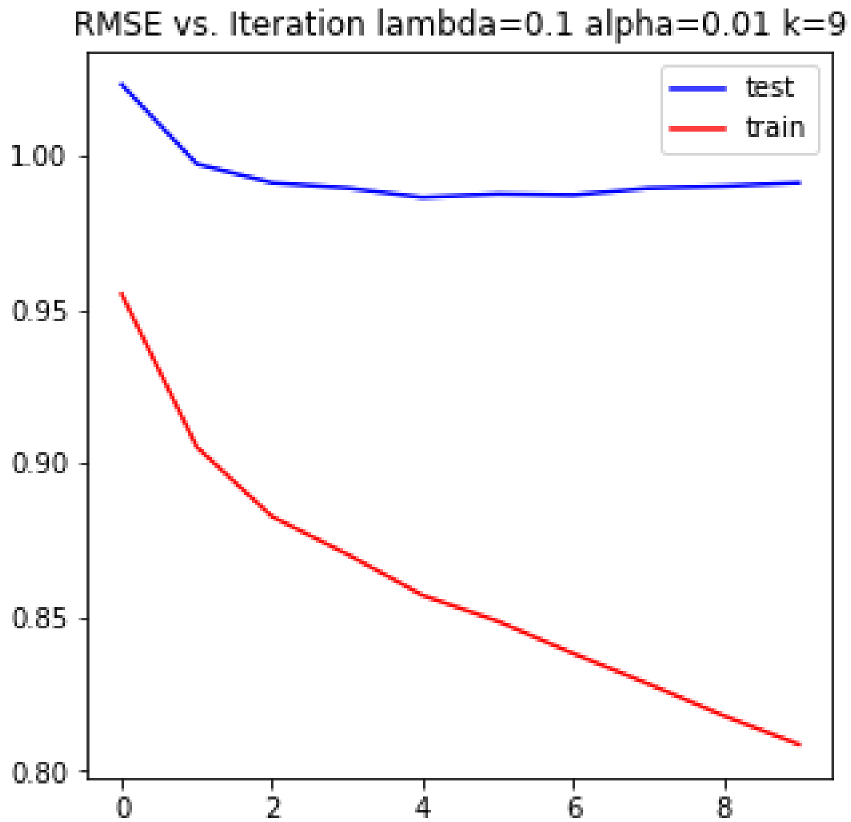
RMSE vs. Iteration lambda=0.1 alpha=0.01 k=9
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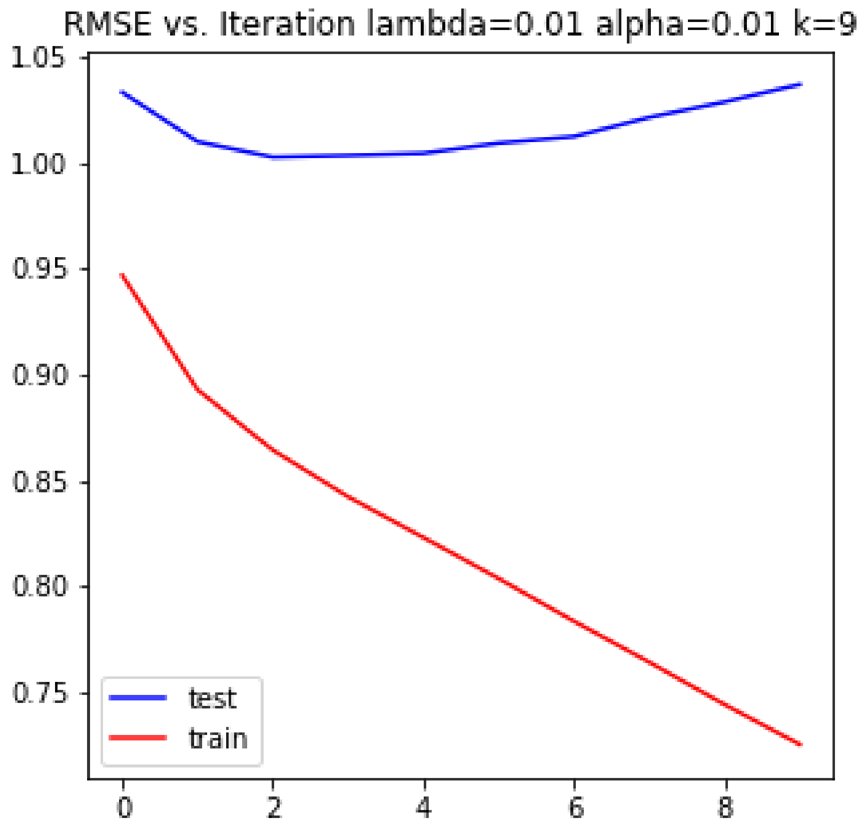
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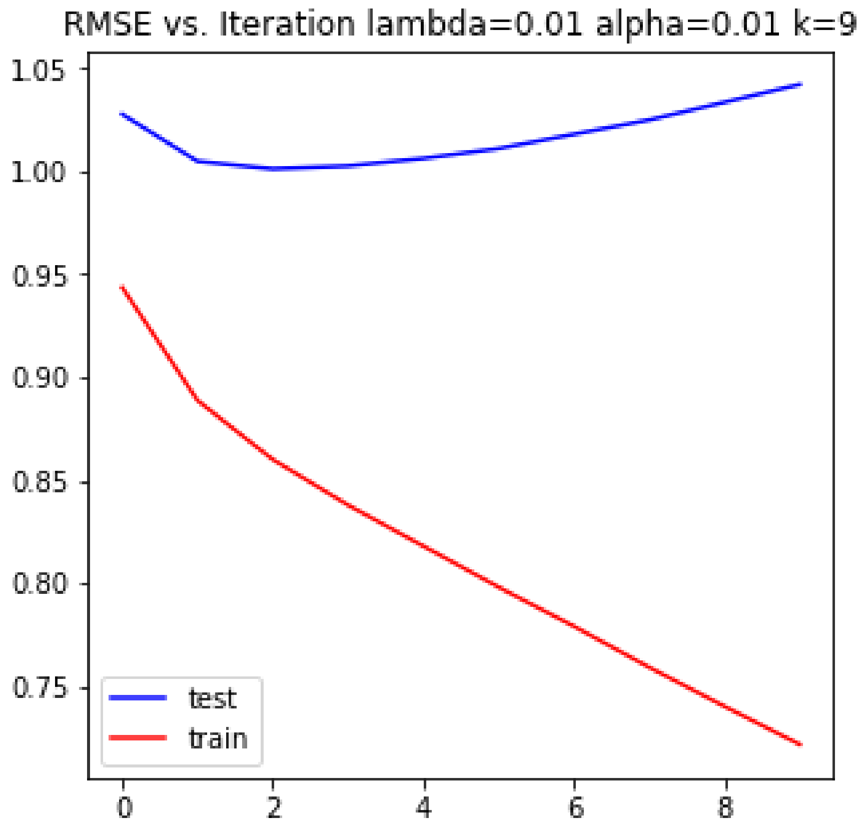
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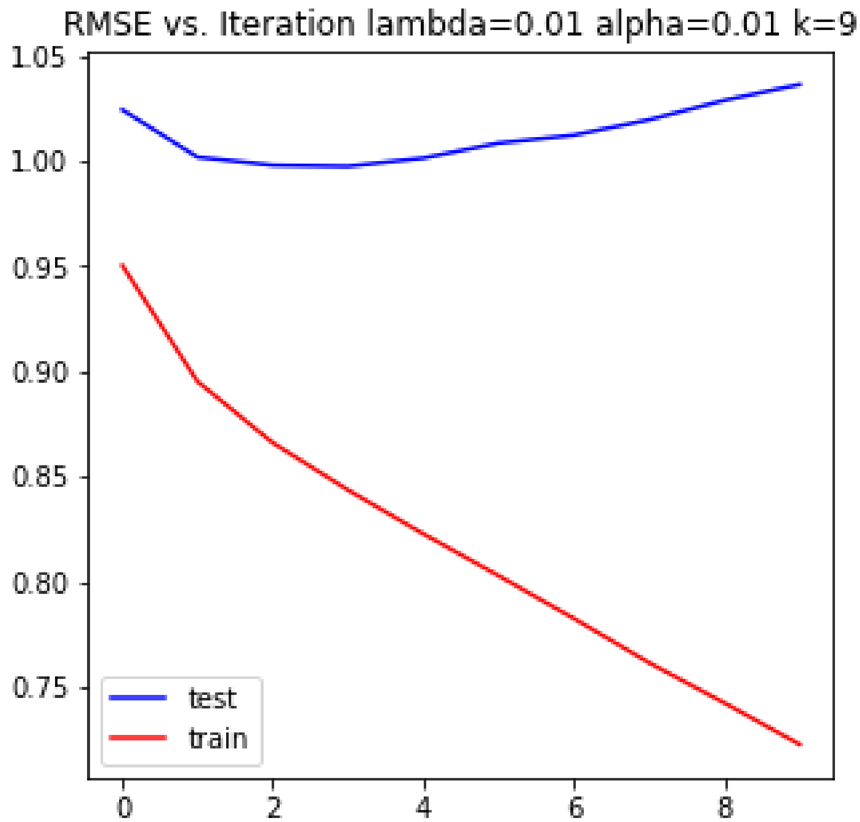
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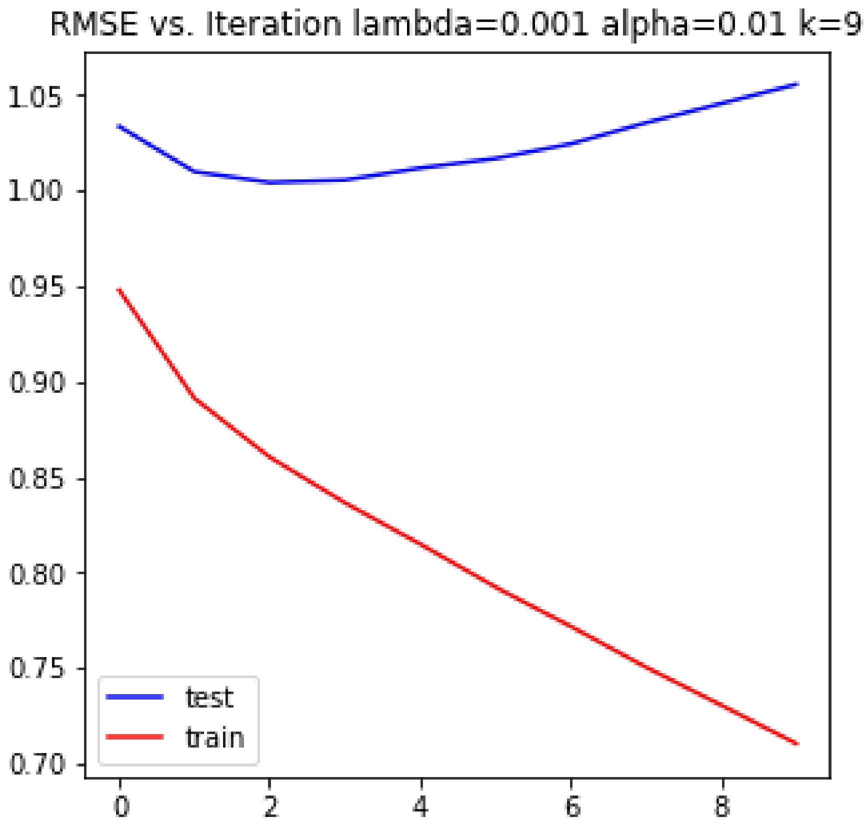
test
train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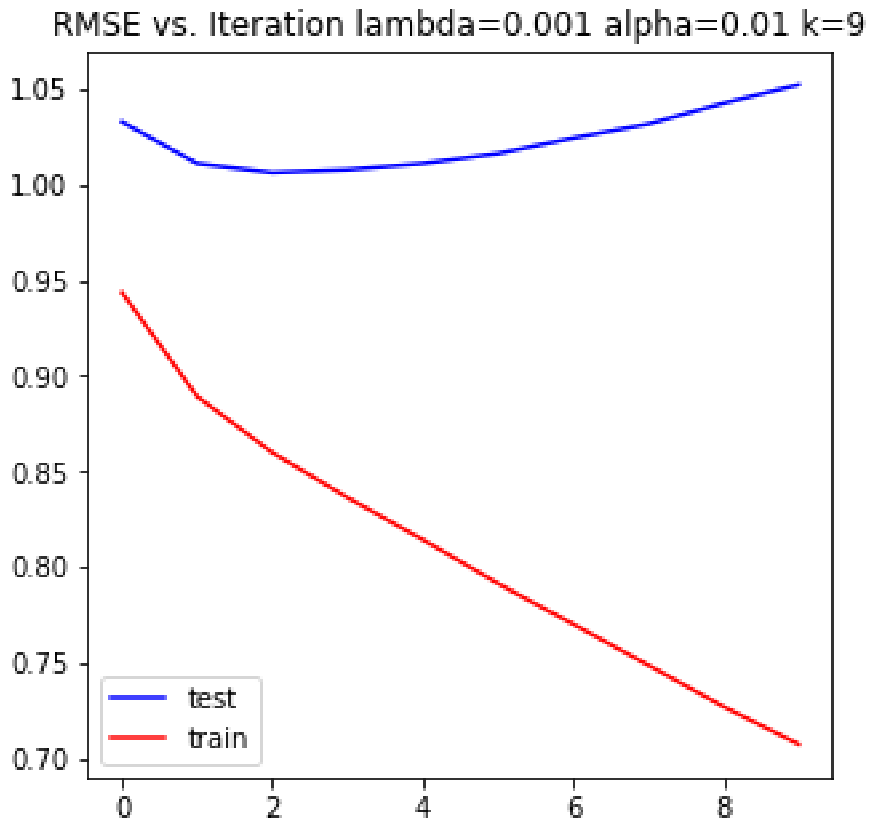
test
train



RMSE vs. Iteration lambda=0.001 alpha=0.01 k=9
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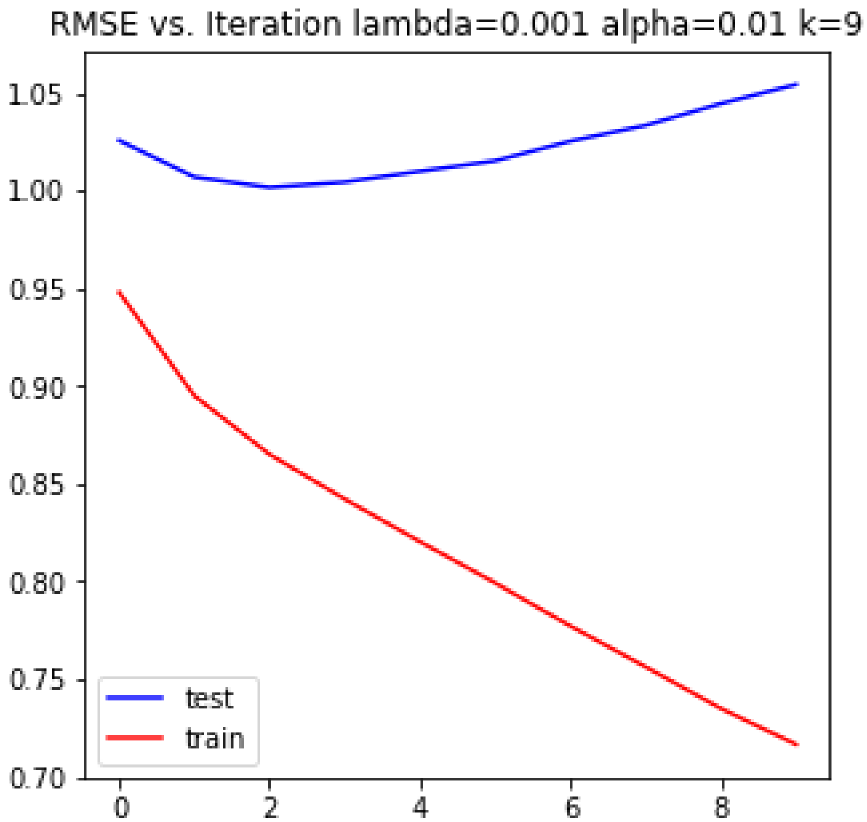
test
train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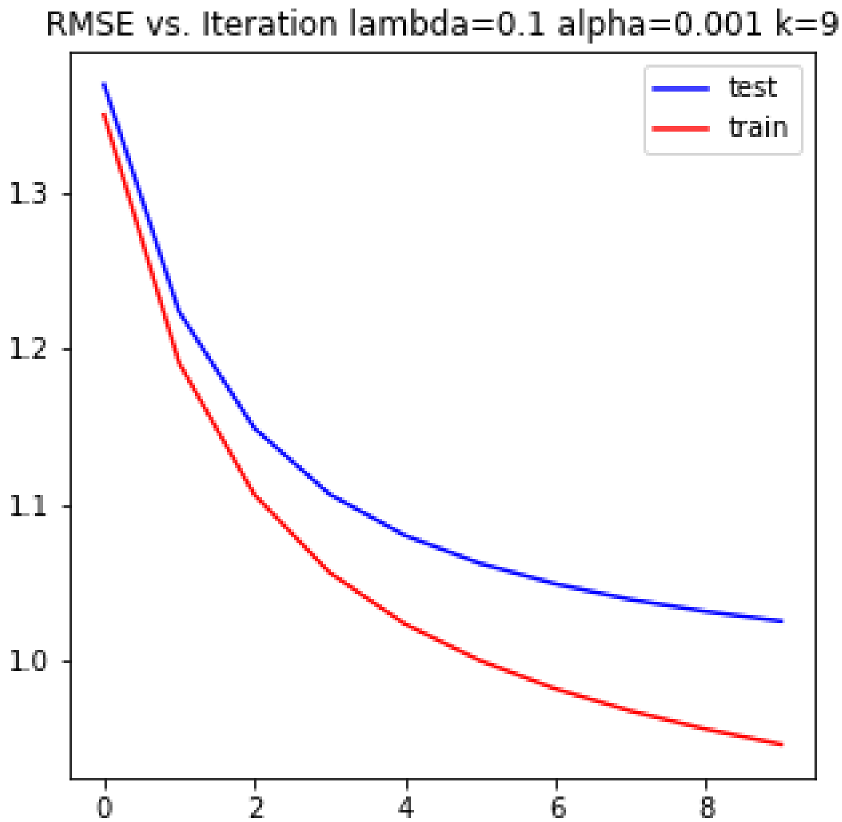
test
train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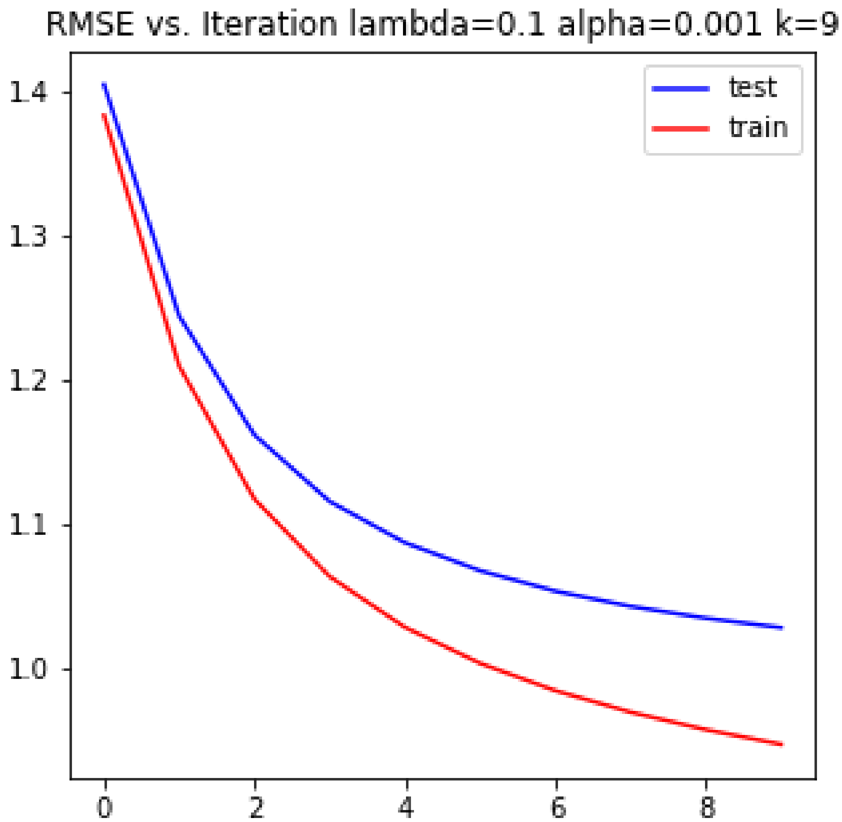
RMSE vs. Iteration lambda=0.1 alpha=0.001 k=9
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train
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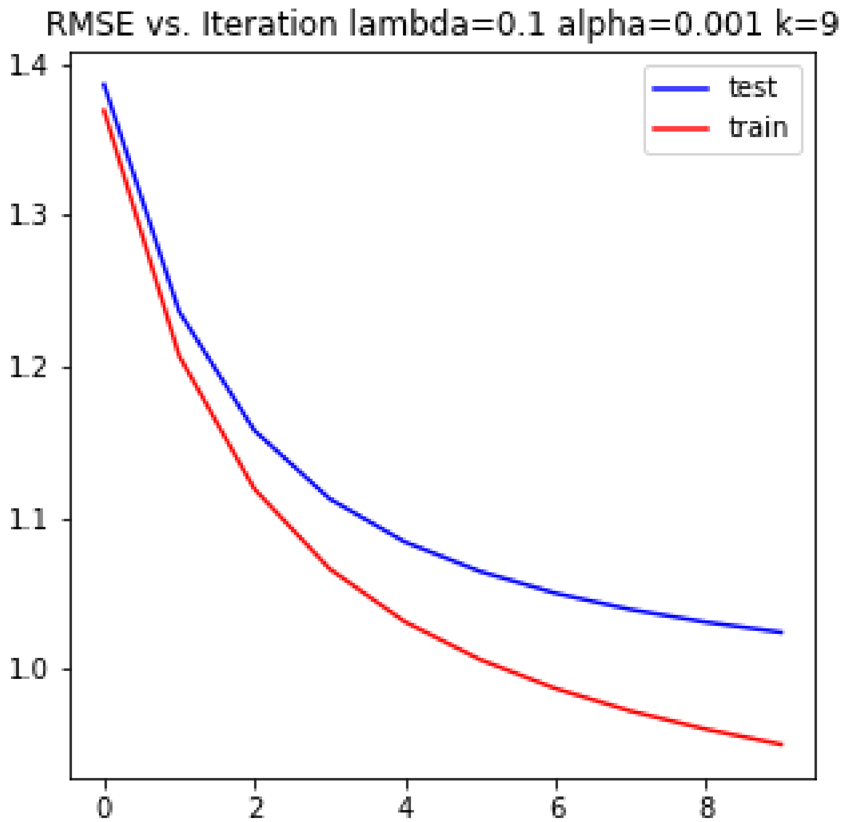
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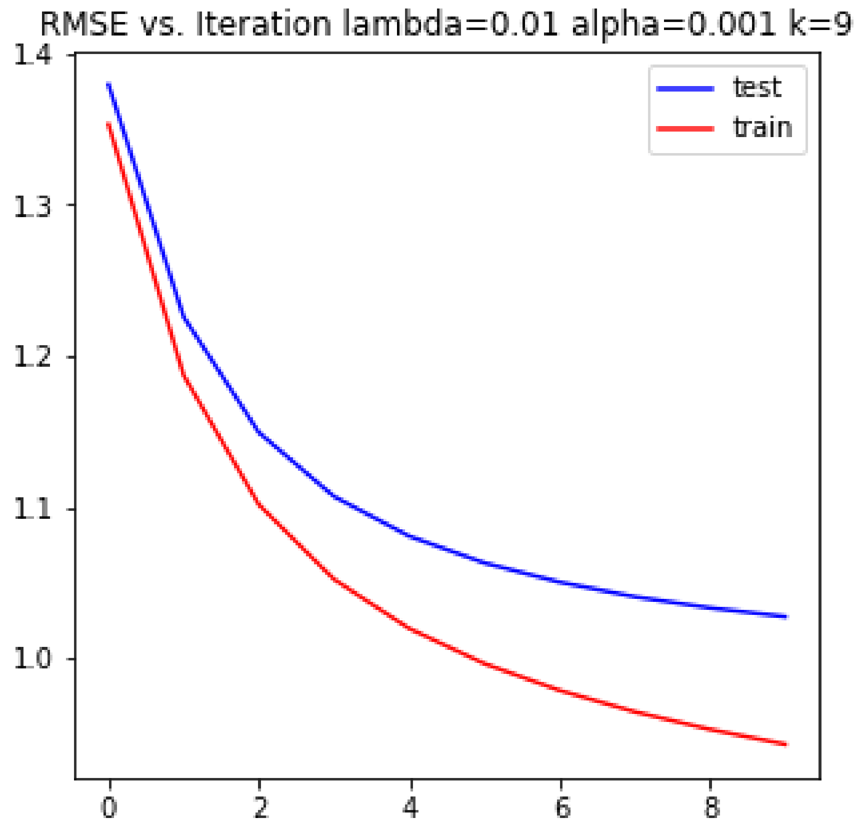
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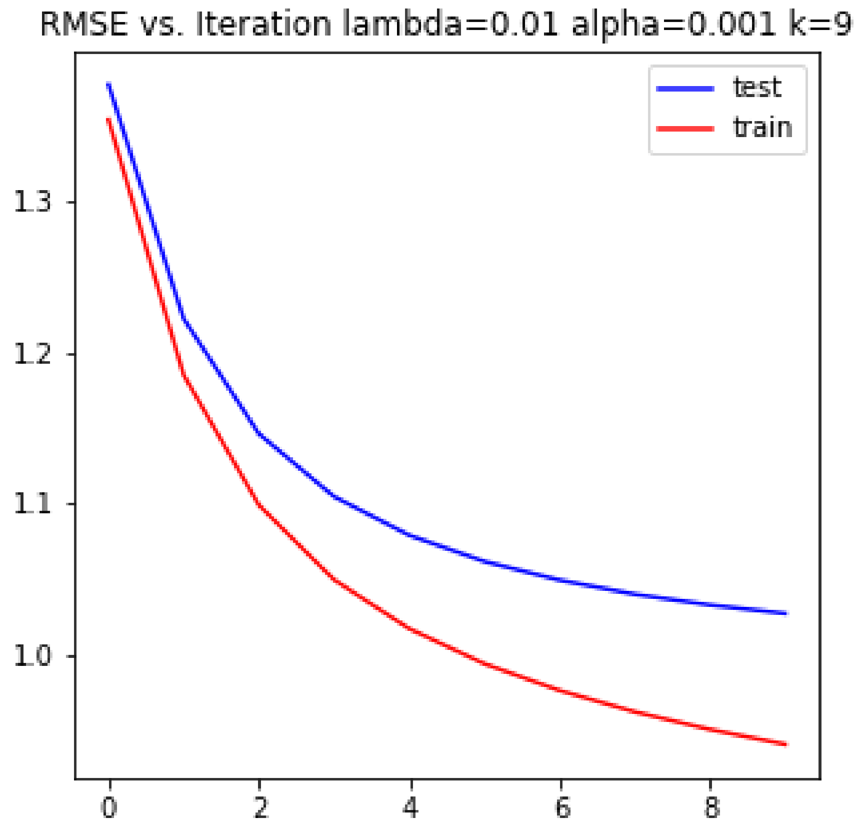
RMSE vs. Iteration lambda=0.01 alpha=0.001 k=9
test
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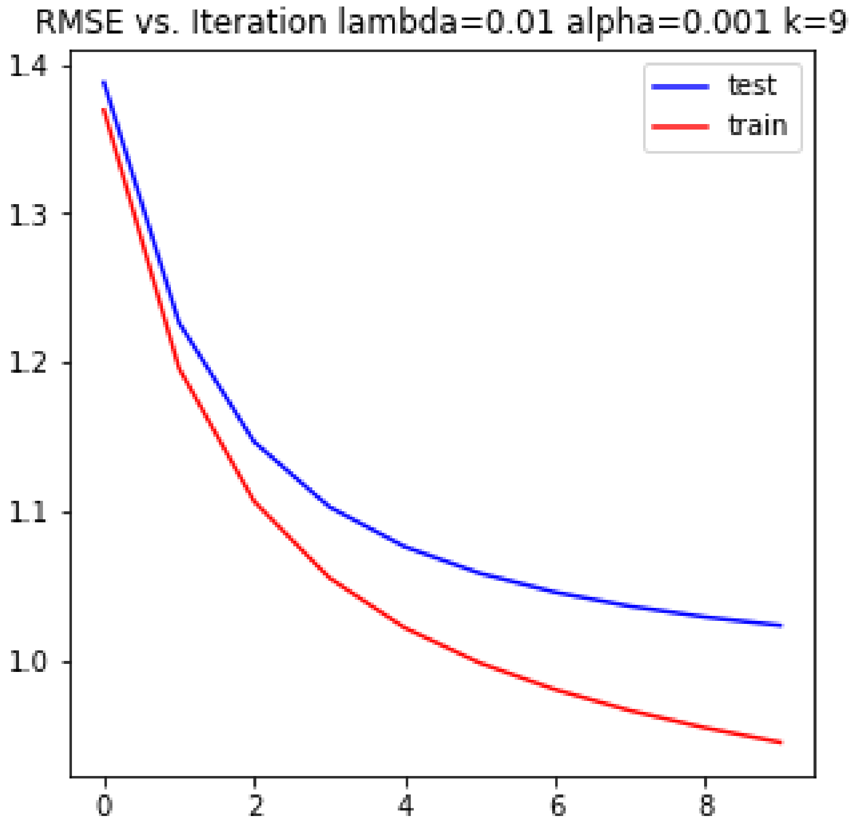

RMSE vs. Iteration lambda=0.01 alpha=0.001 k=9
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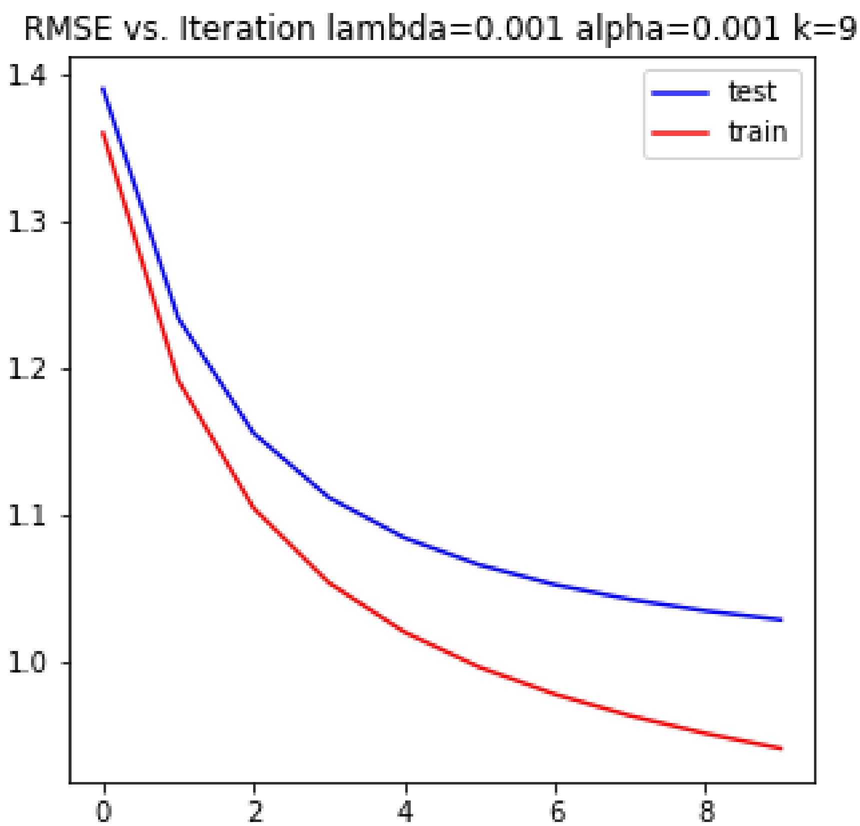
RMSE vs. Iteration lambda=0.001 alpha=0.001 k=9
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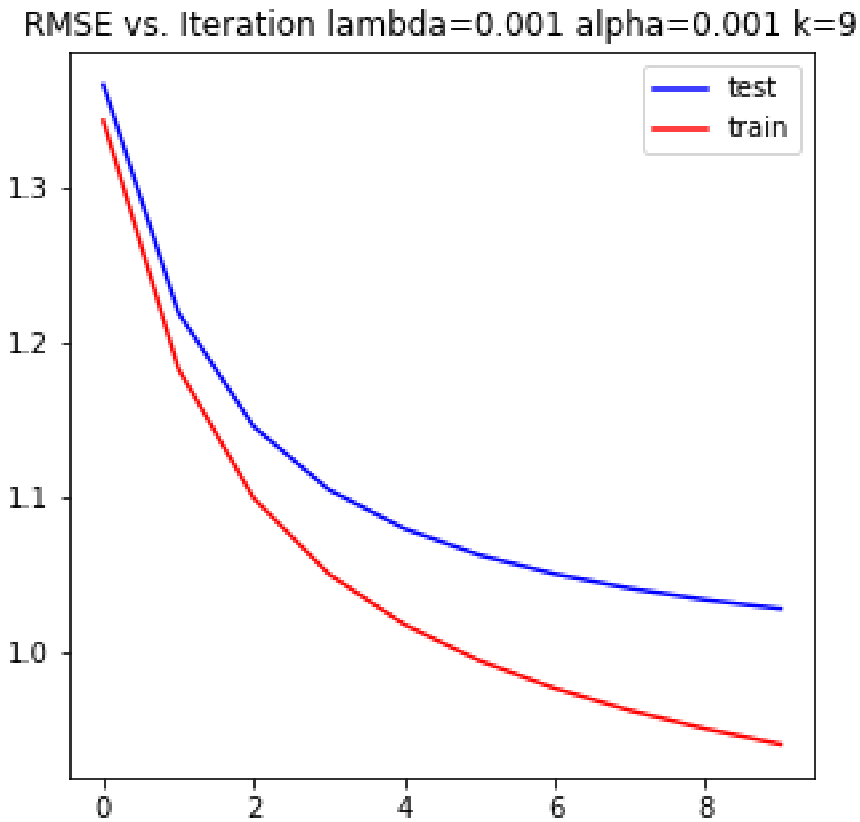
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test
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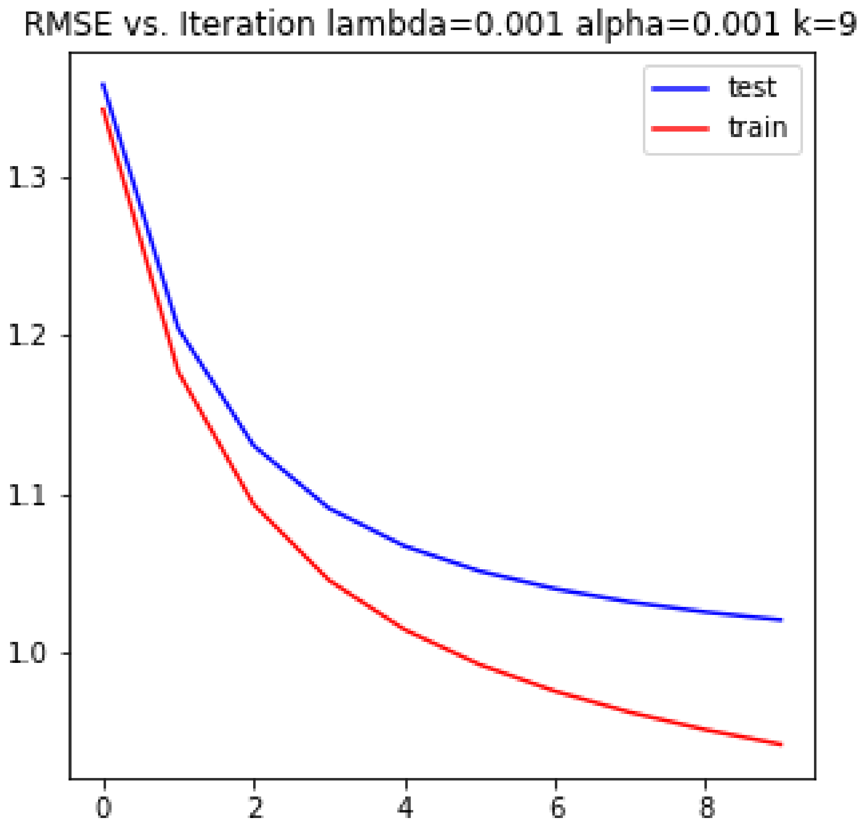
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test
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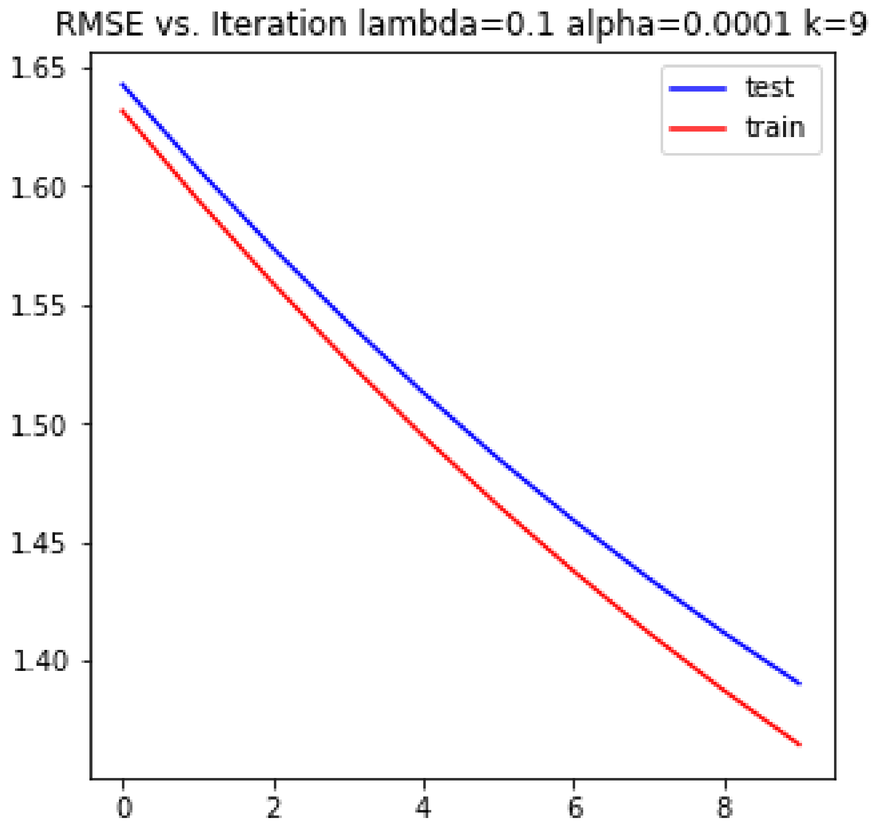
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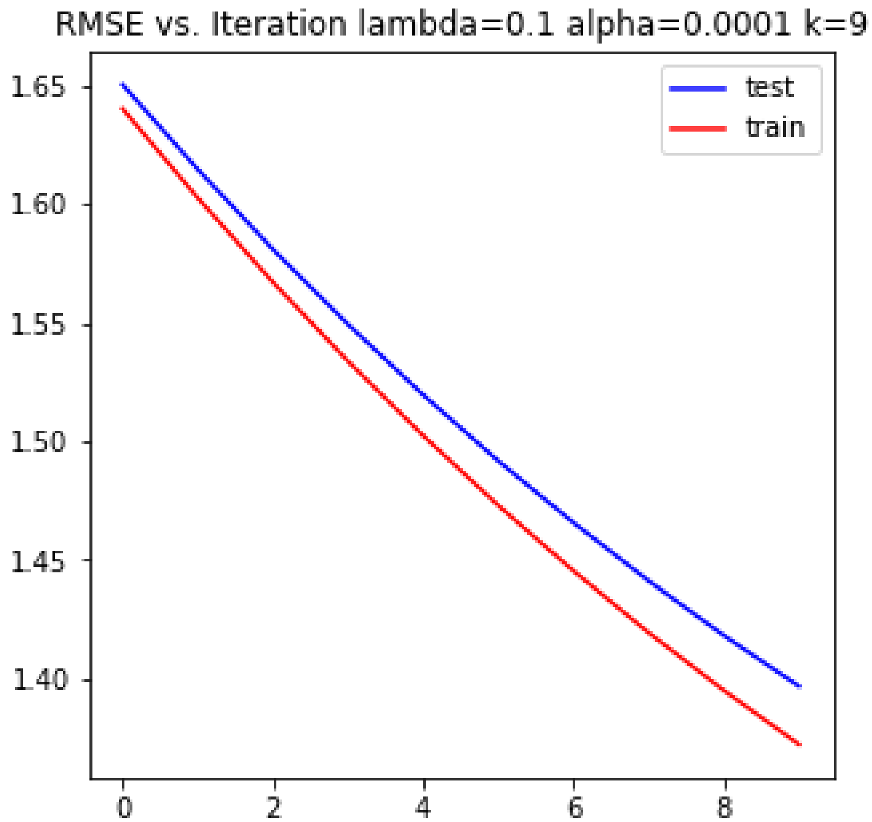

RMSE vs. Iteration lambda=0.1 alpha=0.0001 k=9
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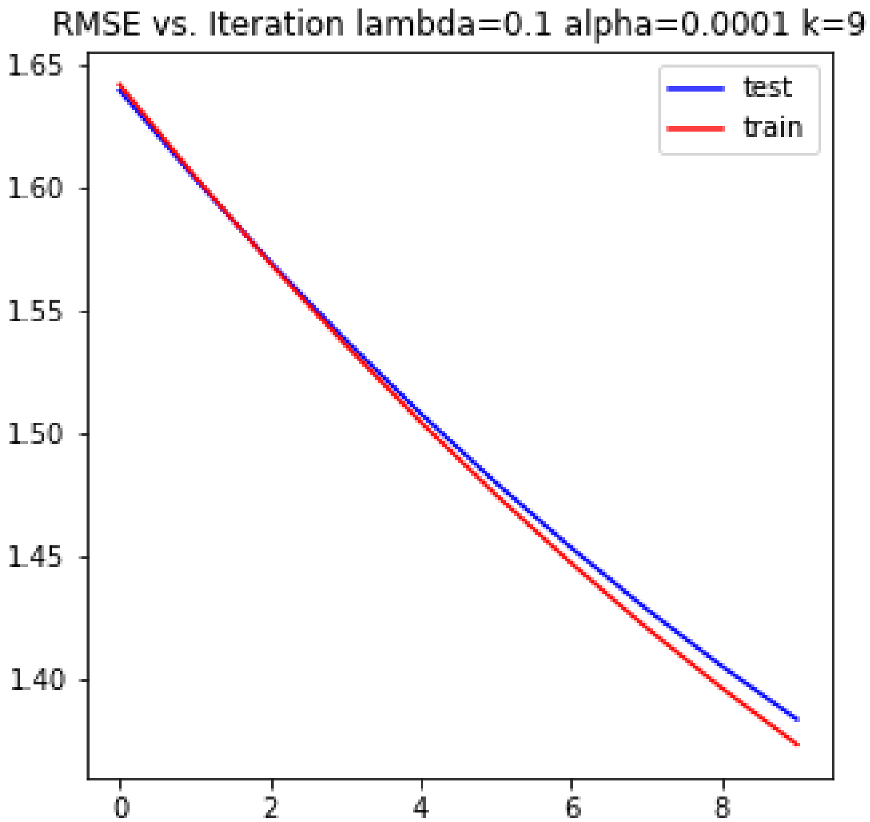
RMSE vs. Iteration lambda=0.1 alpha=0.0001 k=9
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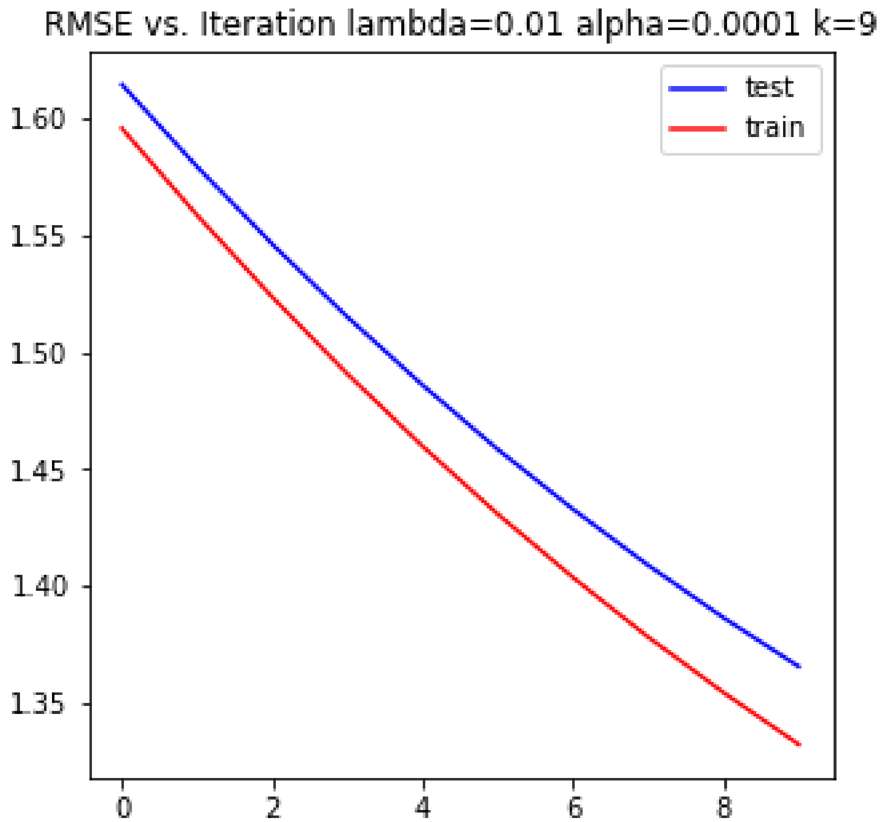
RMSE vs. Iteration lambda=0.01 alpha=0.0001 k=9
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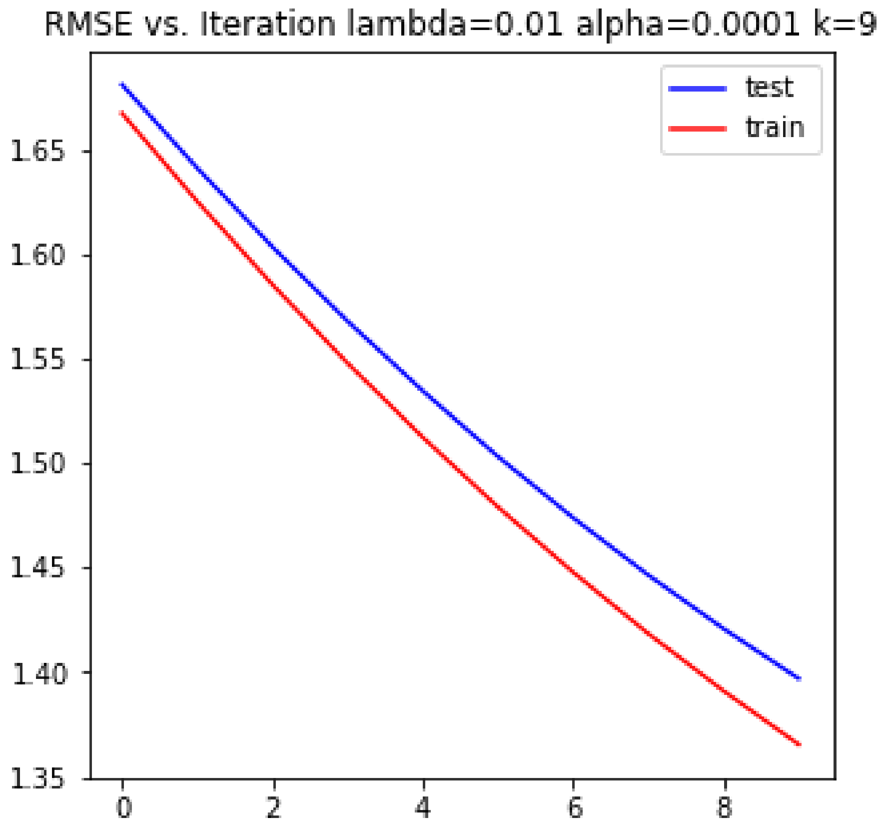

RMSE vs. Iteration lambda=0.01 alpha=0.0001 k=9
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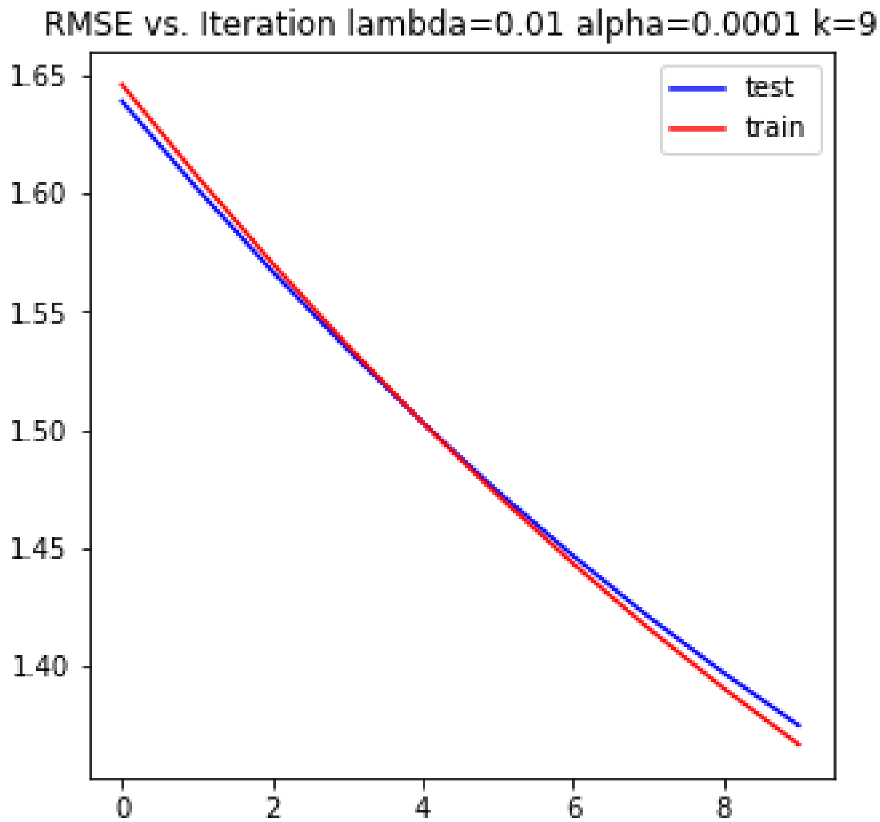
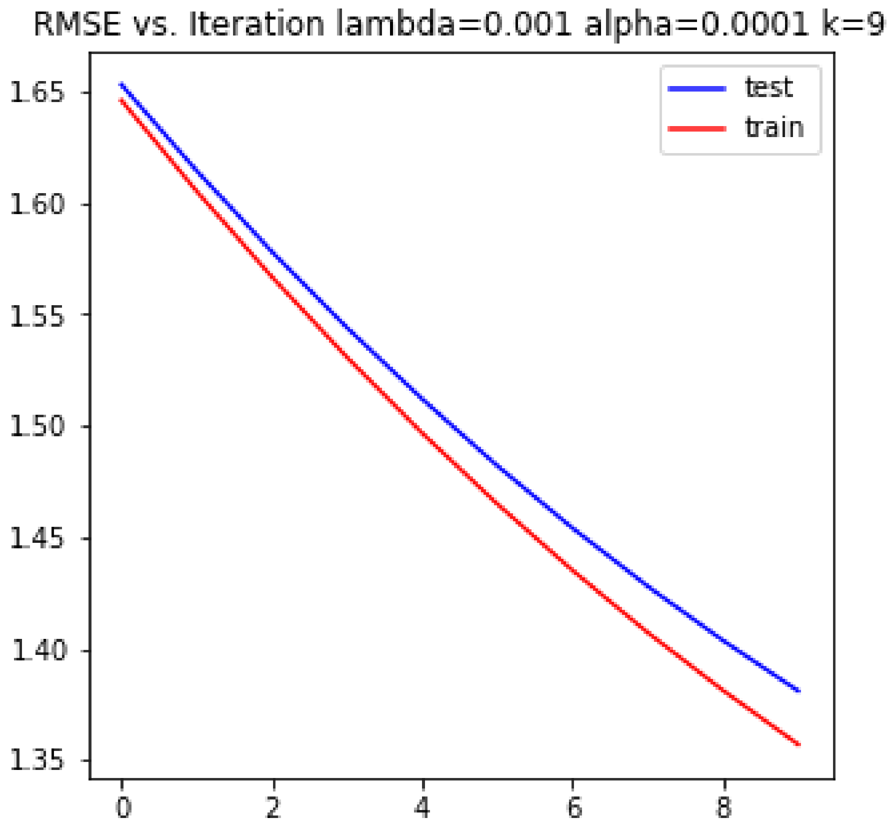
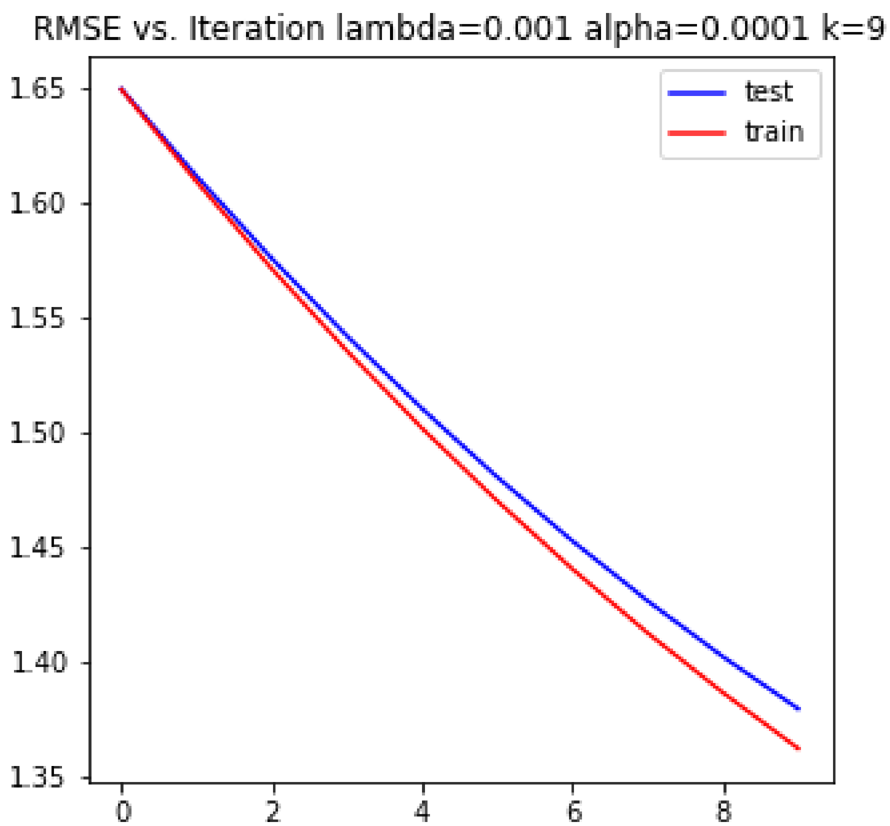
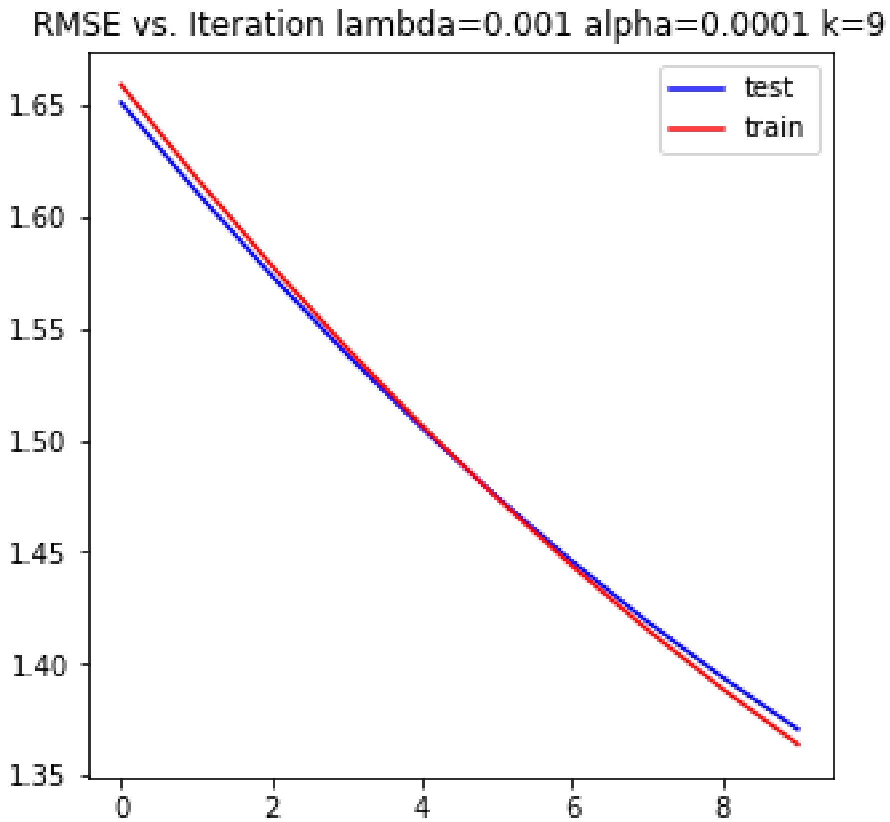
1.65
1.60
1.55
1.50
1.45
1.40
1.35
0
2
4
6
8

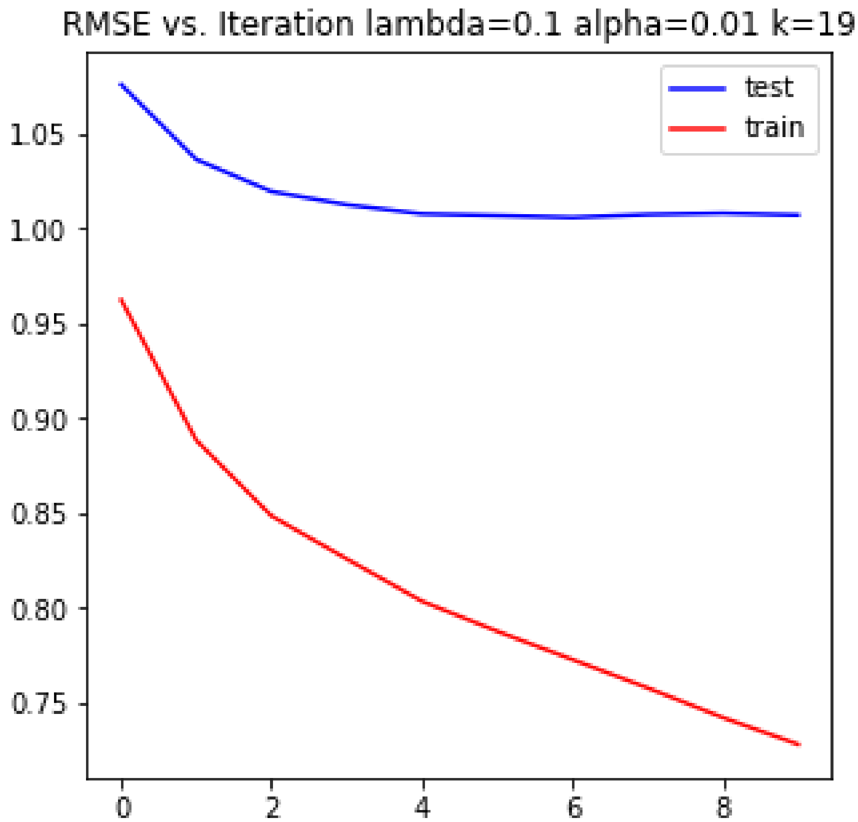

RMSE vs. Iteration lambda=0.01 alpha=0.0001 k=9
test
train
1.65
1.60
1.55
1.50
1.45
1.40
0
2
4
6
8

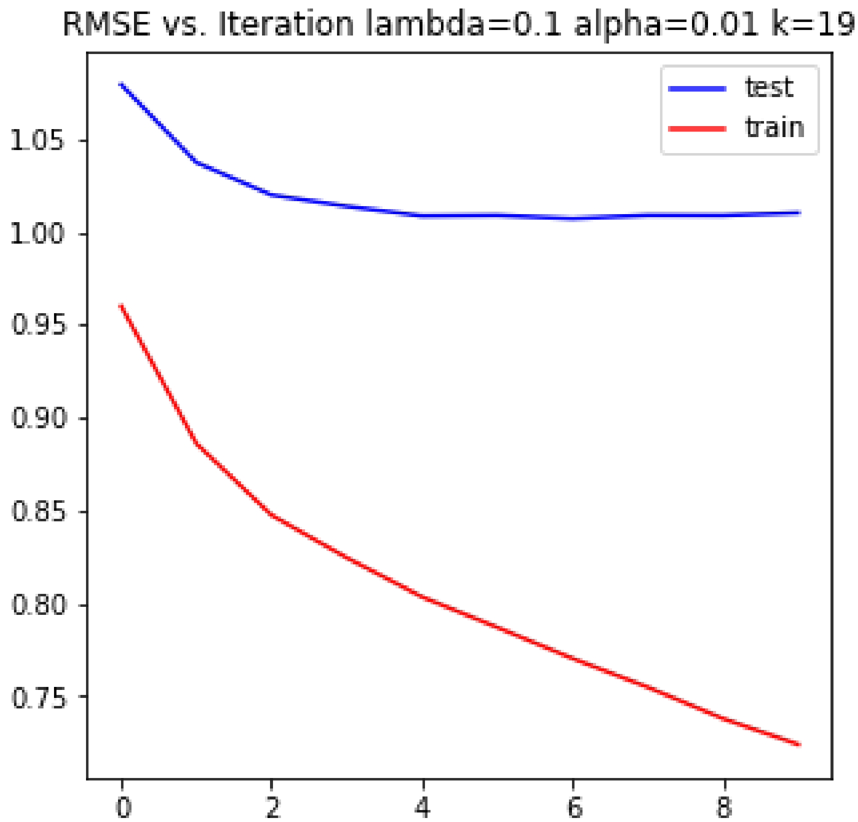


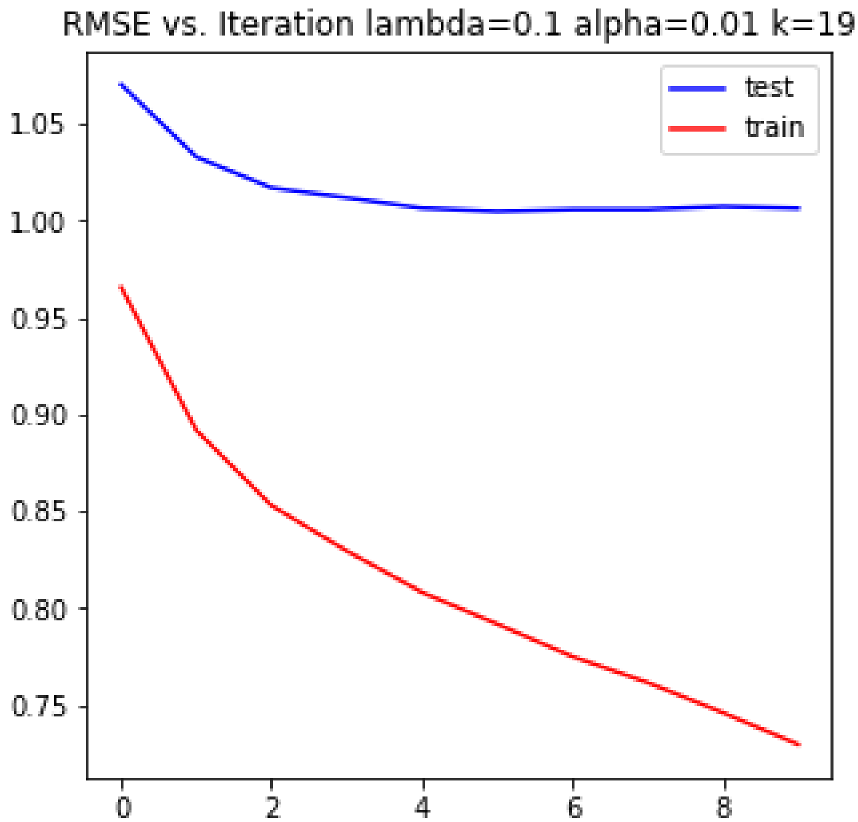
RMSE vs. Iteration lambda=0.001 alpha=0.0001 k=9
test
train
1.65
1.60
1.55
1.50
1.45
1.40
1.35
0
2
4
6
8

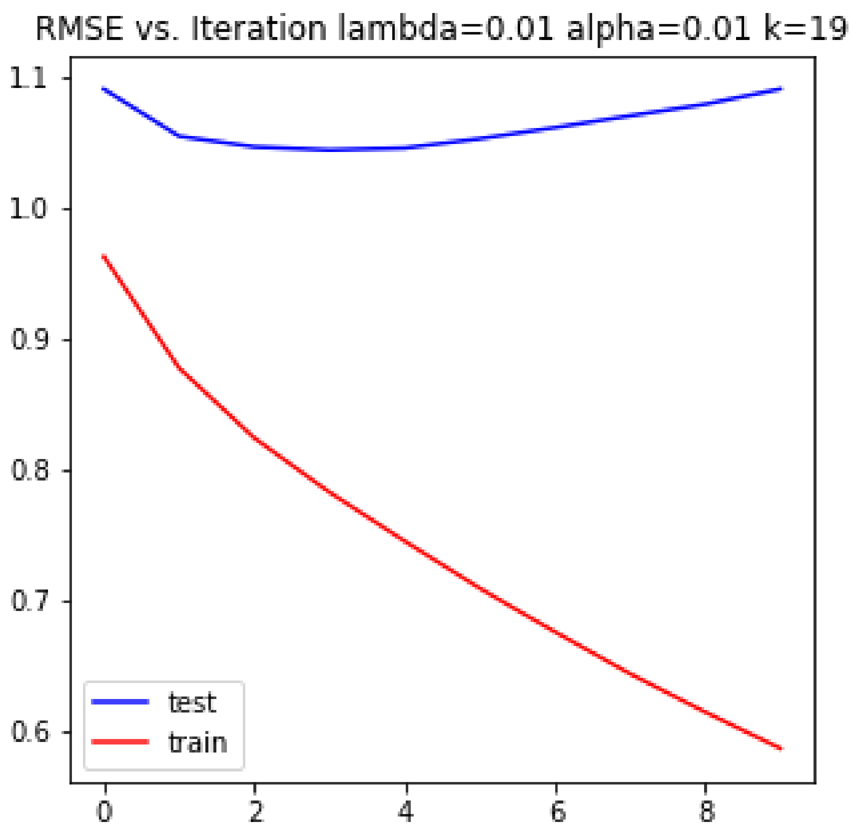

RMSE vs. Iteration lambda=0.001 alpha=0.0001 k=9
test
train
1.65
1.60
1.55
1.50
1.45
1.40
1.35
0
2
4
6
8

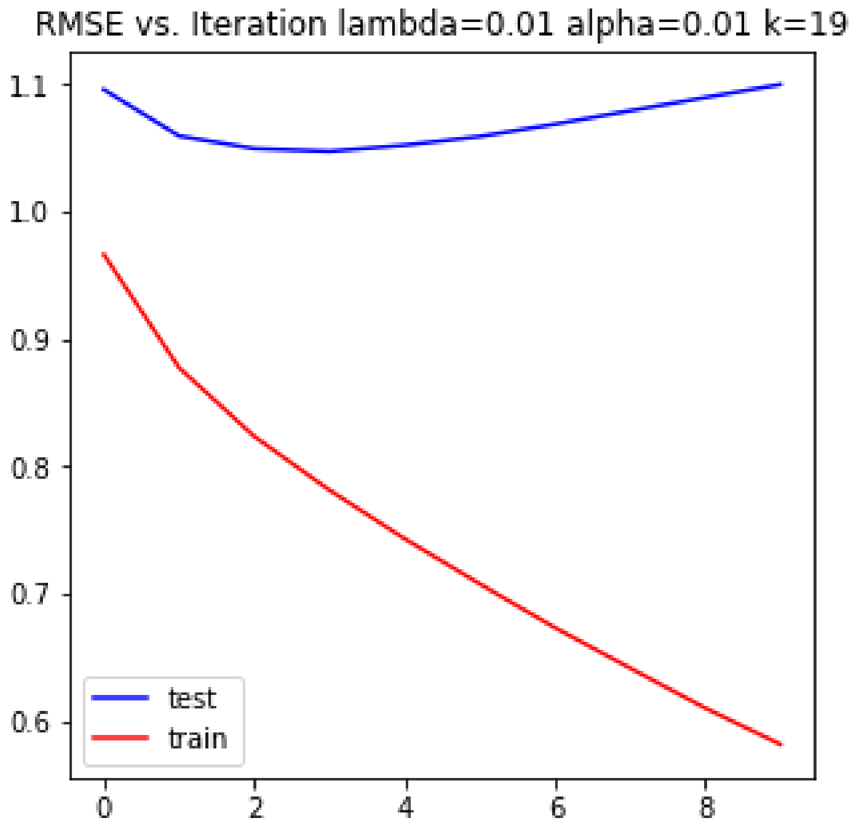

RMSE vs. Iteration lambda=0.001 alpha=0.0001 k=9
test
train
1.65
1.60
1.55
1.50
1.45
1.40
1.35
0
2
4
6
8

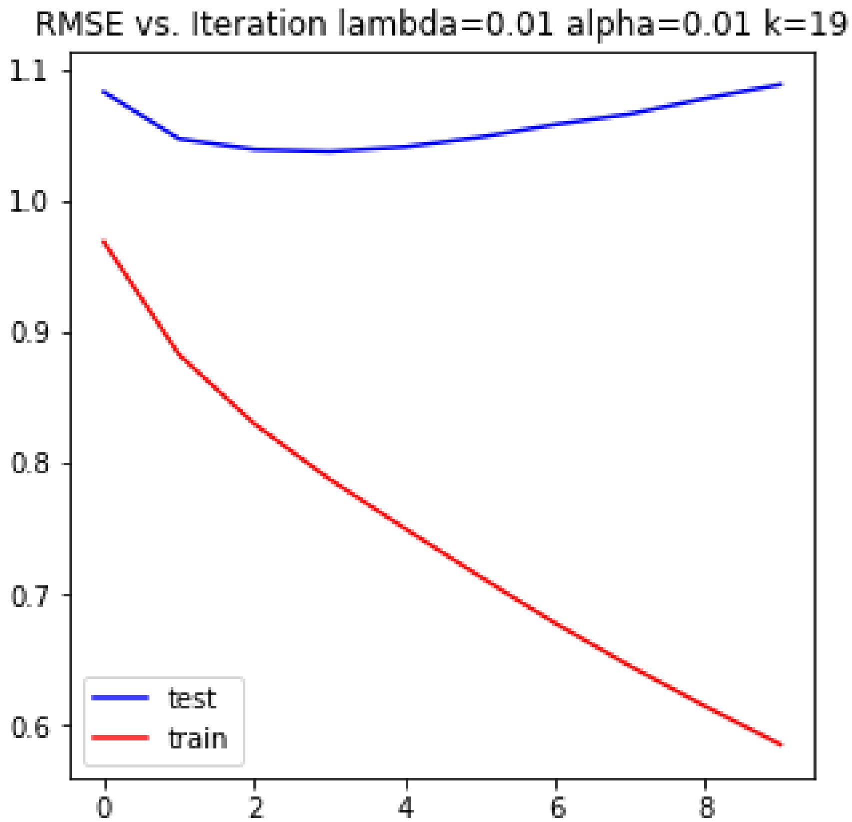

RMSE vs. Iteration lambda=0.1 alpha=0.01 k=19
test
train
1.05
1.00
0.95
0.90
0.85
0.80
0.75
0
2
4
6
8

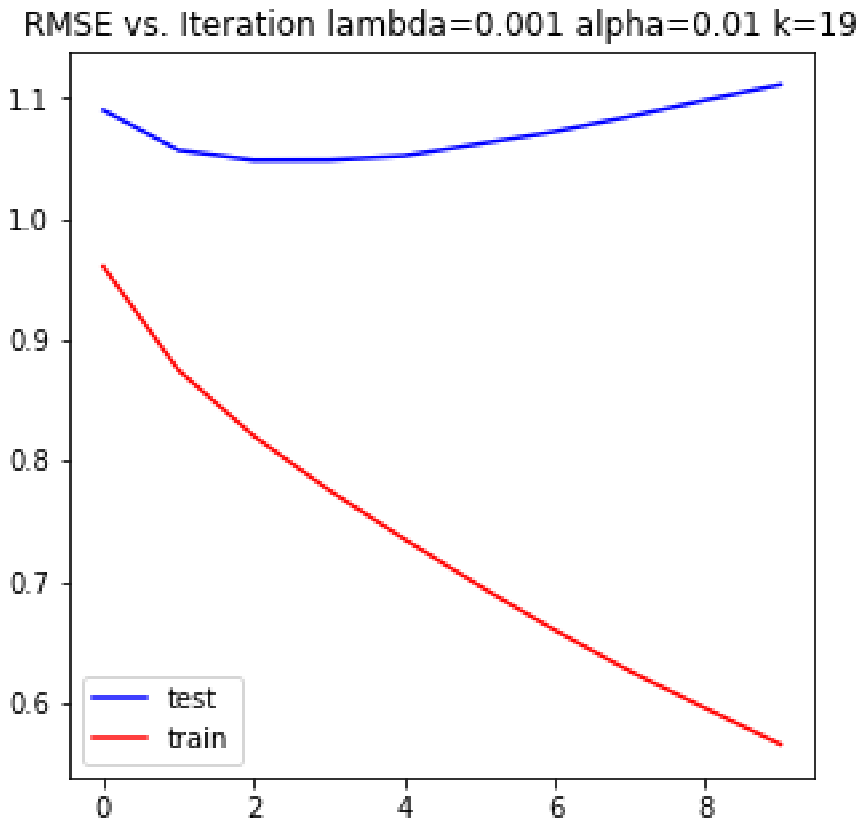

RMSE vs. Iteration lambda=0.1 alpha=0.01 k=19
test
train
1.05
1.00
0.95
0.90
0.85
0.80
0.75
0
2
4
6
8

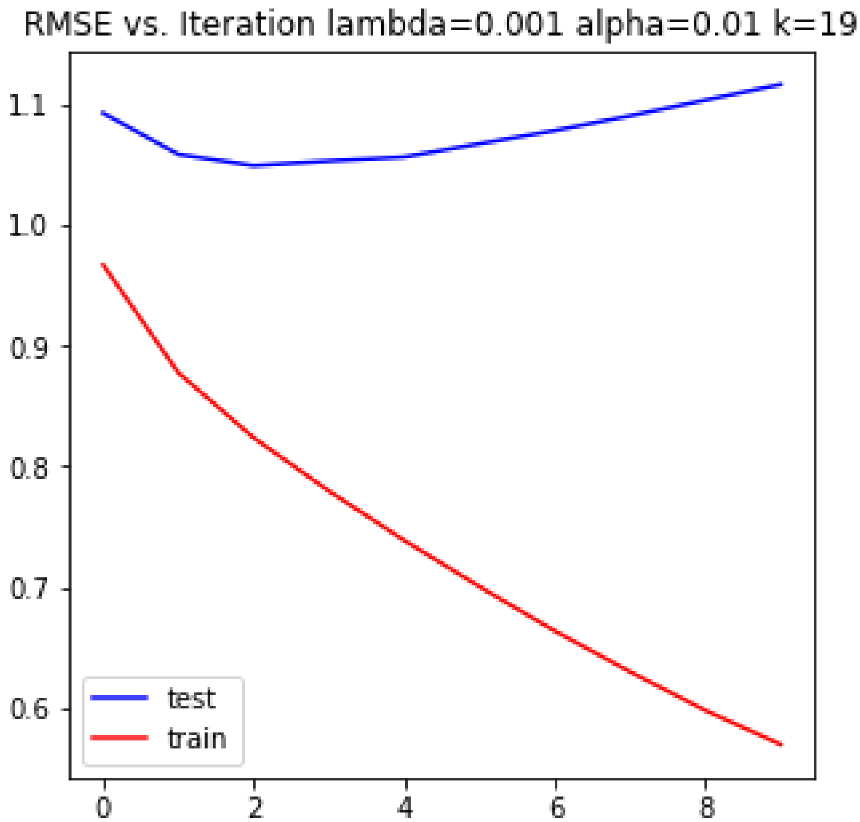

RMSE vs. Iteration lambda=0.1 alpha=0.01 k=19
test
train
1.05
1.00
0.95
0.90
0.85
0.80
0.75
0
2
4
6
8

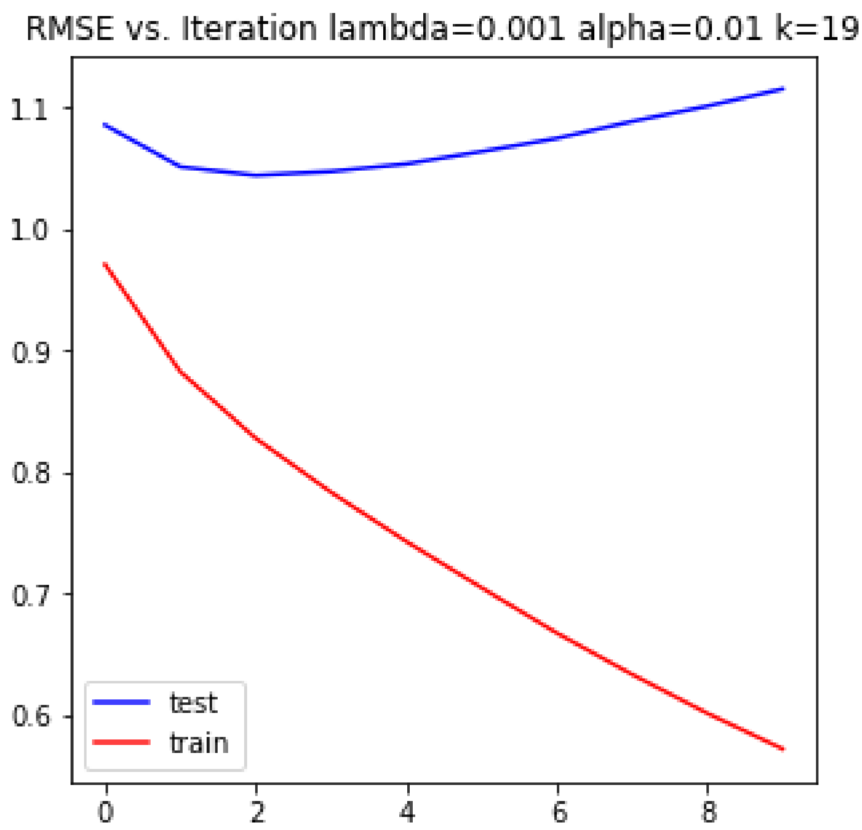

RMSE vs. Iteration lambda=0.01 alpha=0.01 k=19
1.1
1.0
0.9
0.8
0.7
0.6
0
2
4
6
8
test
train


RMSE vs. Iteration lambda=0.01 alpha=0.01 k=19
1.1
1.0
0.9
0.8
0.7
0.6
0
2
4
6
8
test
train

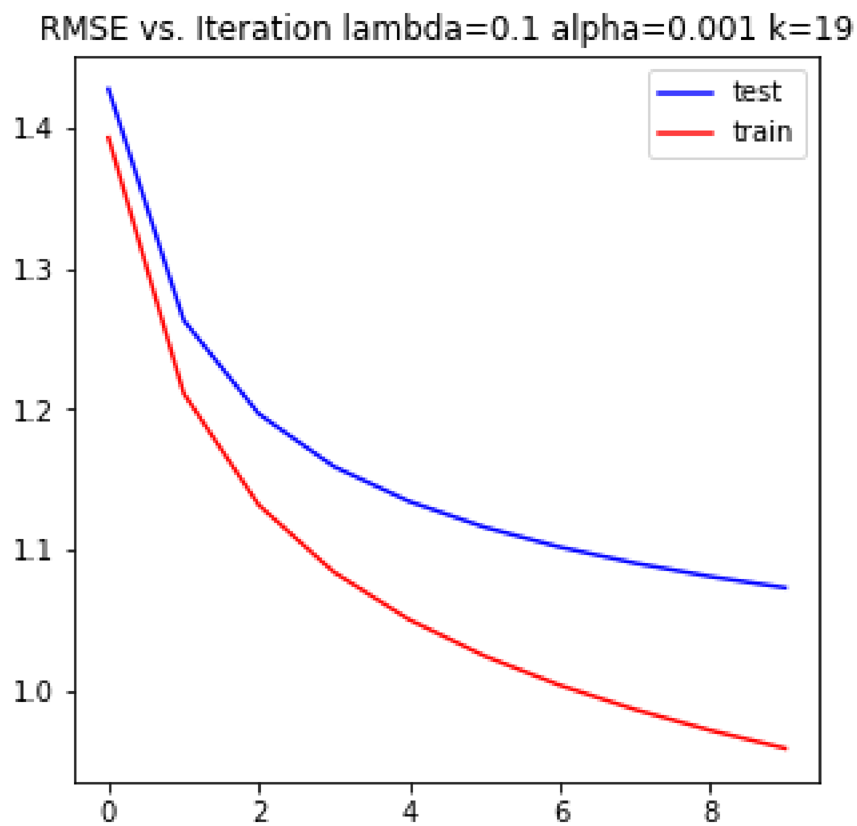

RMSE vs. Iteration lambda=0.01 alpha=0.01 k=19
1.1
1.0
0.9
0.8
0.7
0.6
test
train
0
2
4
6
8

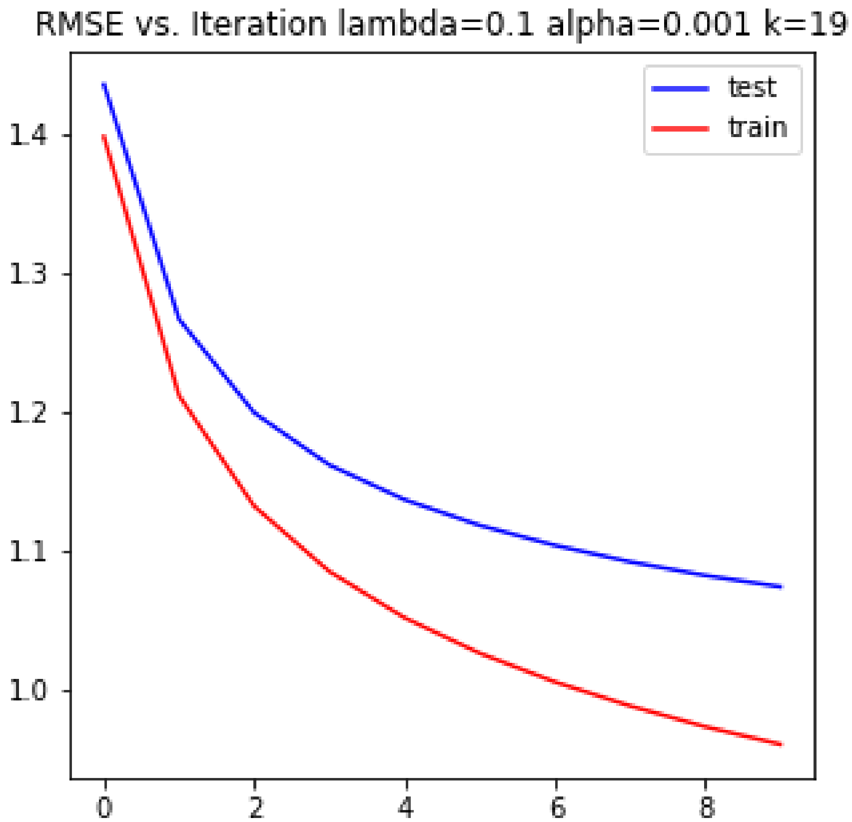

RMSE vs. Iteration lambda=0.001 alpha=0.01 k=19
1.1
1.0
0.9
0.8
0.7
0.6
0
2
4
6
8
test
train

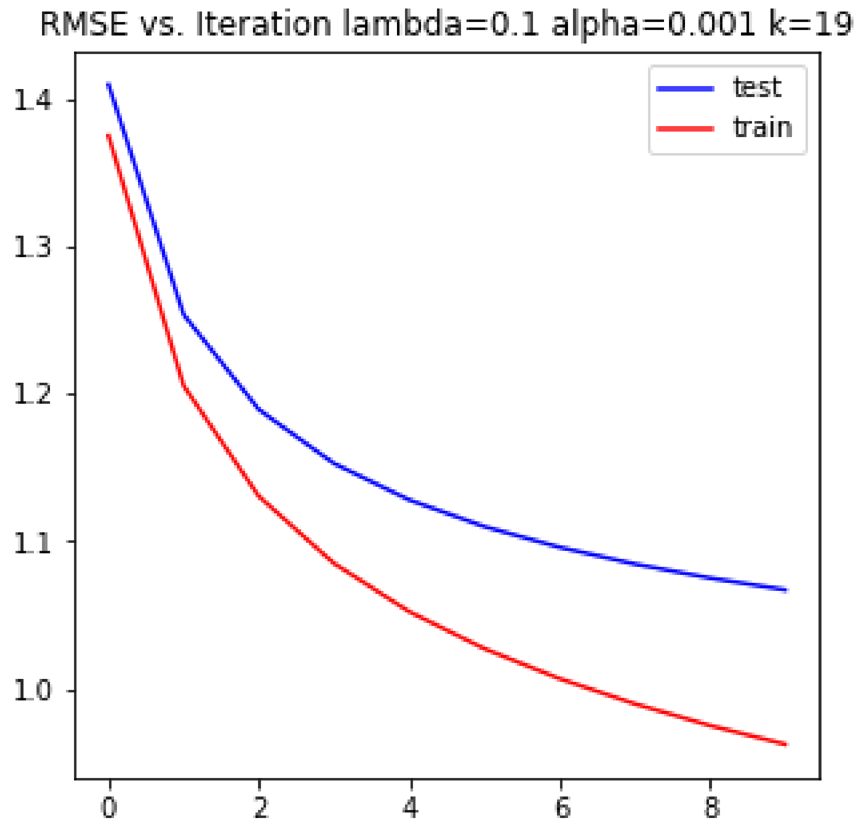

RMSE vs. Iteration lambda=0.001 alpha=0.01 k=19
1.1
1.0
0.9
0.8
0.7
0.6
0
2
4
6
8
test
train

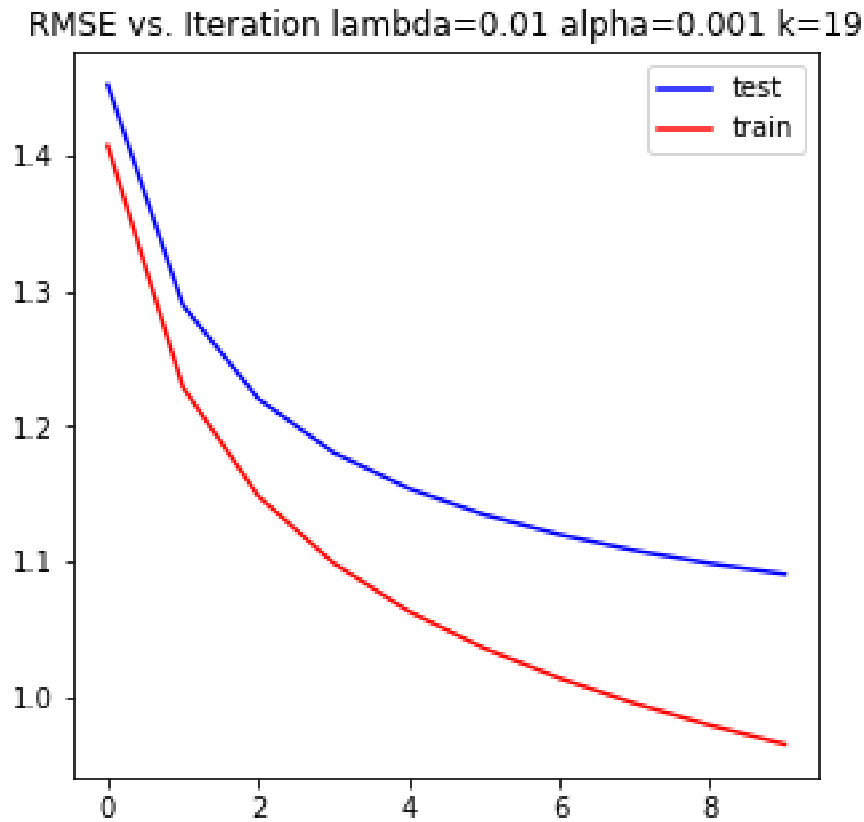


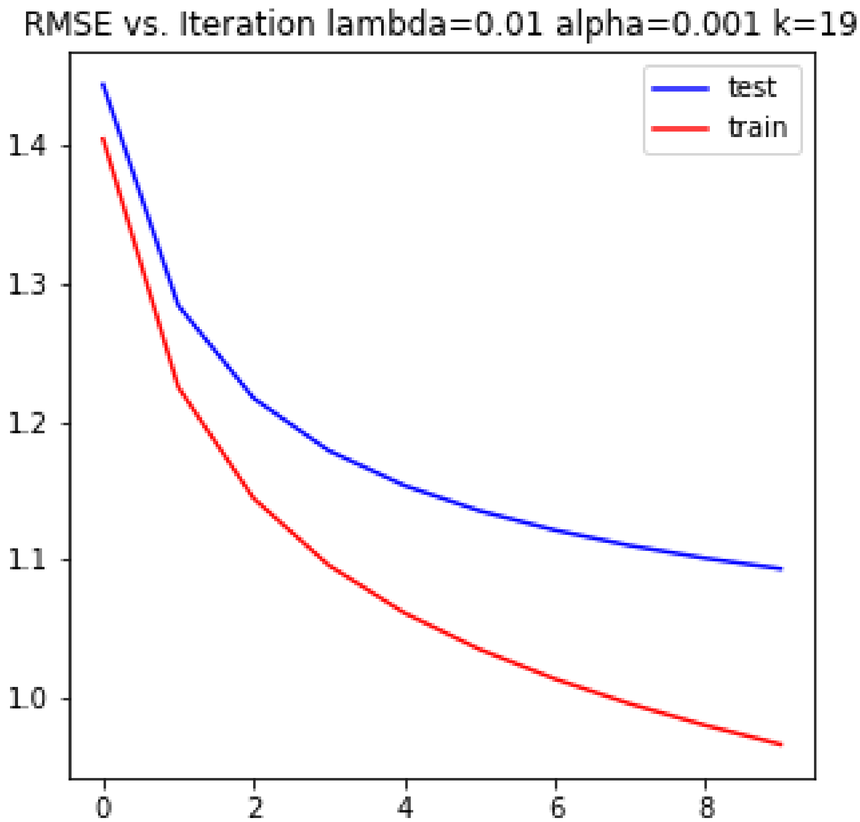
RMSE vs. Iteration lambda=0.001 alpha=0.01 k=19
1.1
1.0
0.9
0.8
0.7
0.6
0
2
4
6
8
test
train

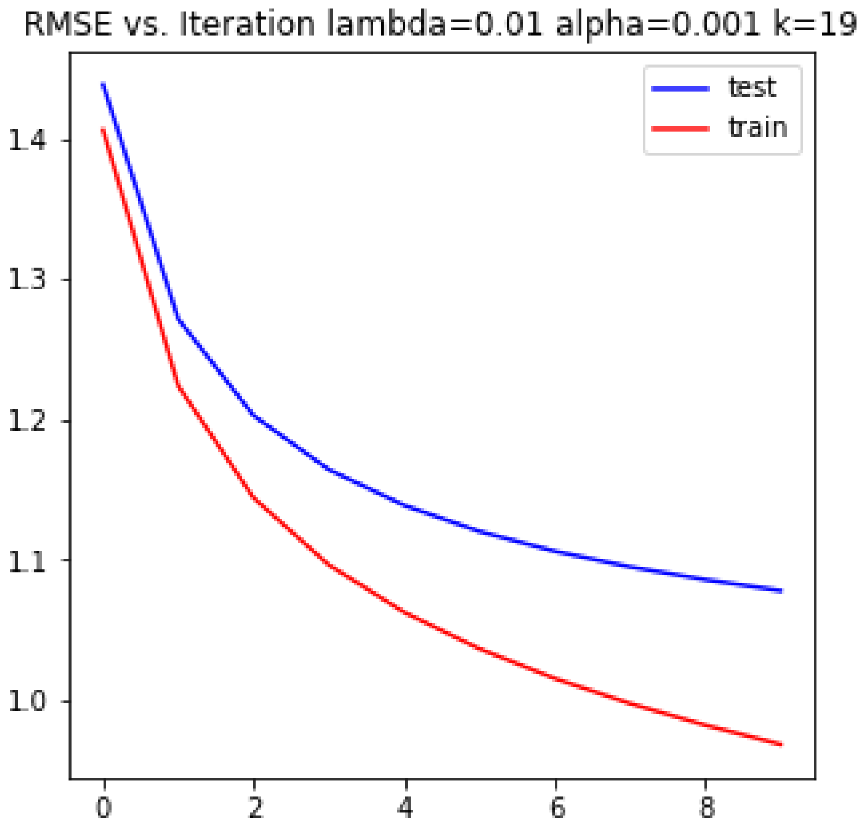

RMSE vs. Iteration lambda=0.1 alpha=0.001 k=19
test
train
1.4
1.3
1.2
1.1
1.0
0
2
4
6
8

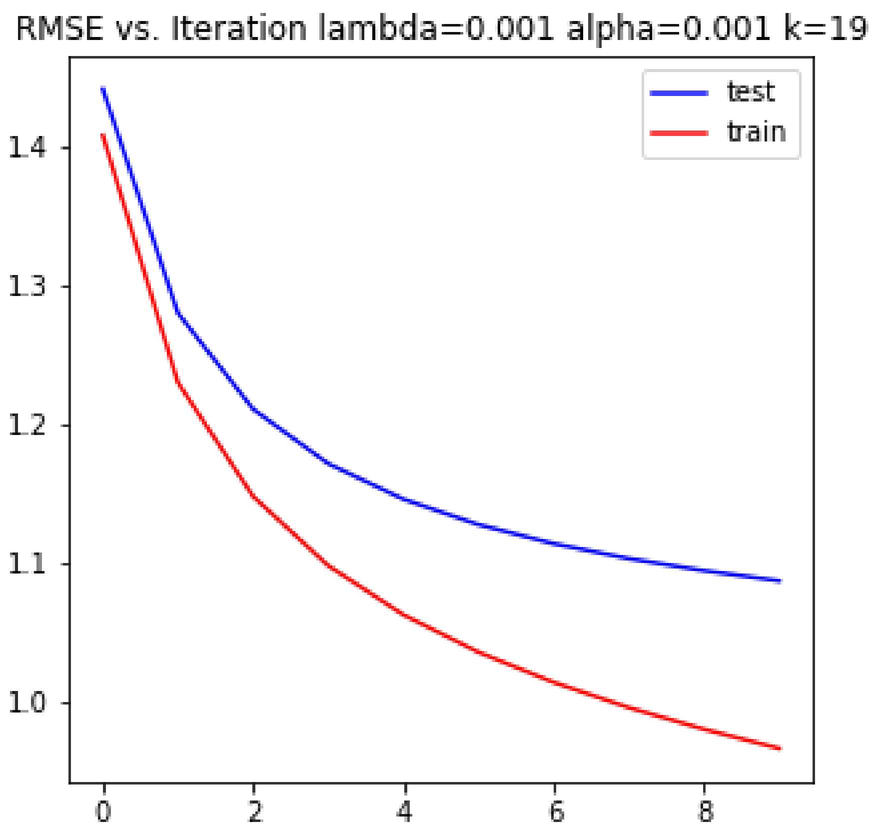

RMSE vs. Iteration lambda=0.1 alpha=0.001 k=19
test
train
1.4
1.3
1.2
1.1
1.0
0
2
4
6
8

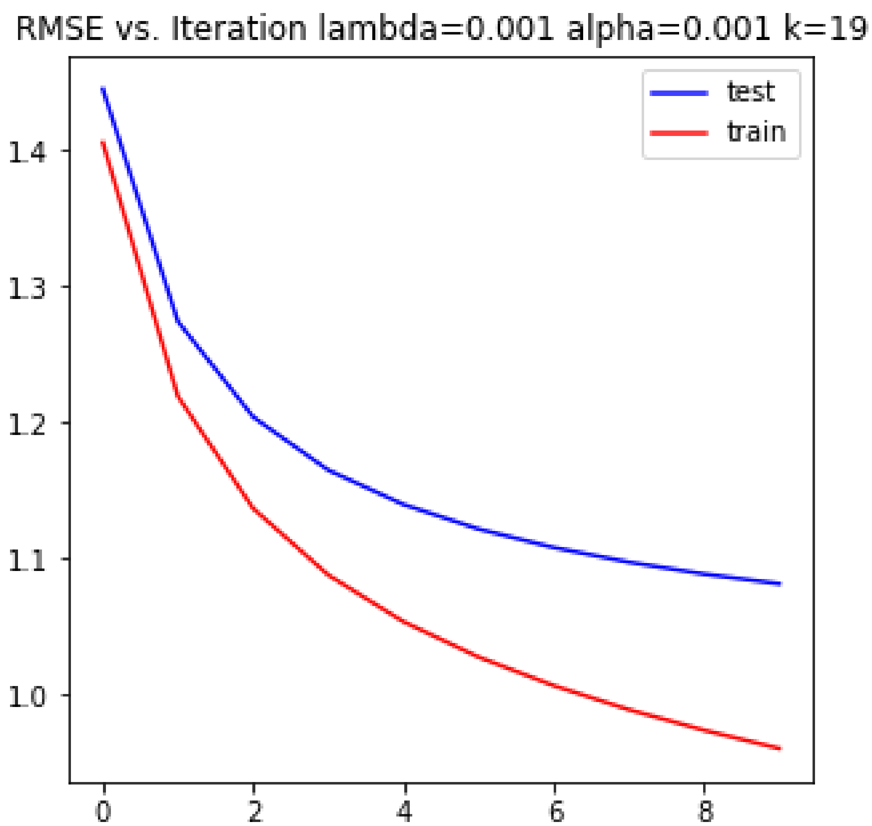

RMSE vs. Iteration lambda=0.1 alpha=0.001 k=19
test
train
1.4
1.3
1.2
1.1
1.0
0
2
4
6
8


RMSE vs. Iteration lambda=0.01 alpha=0.001 k=19
test
train
1.4
1.3
1.2
1.1
1.0
0
2
4
6
8

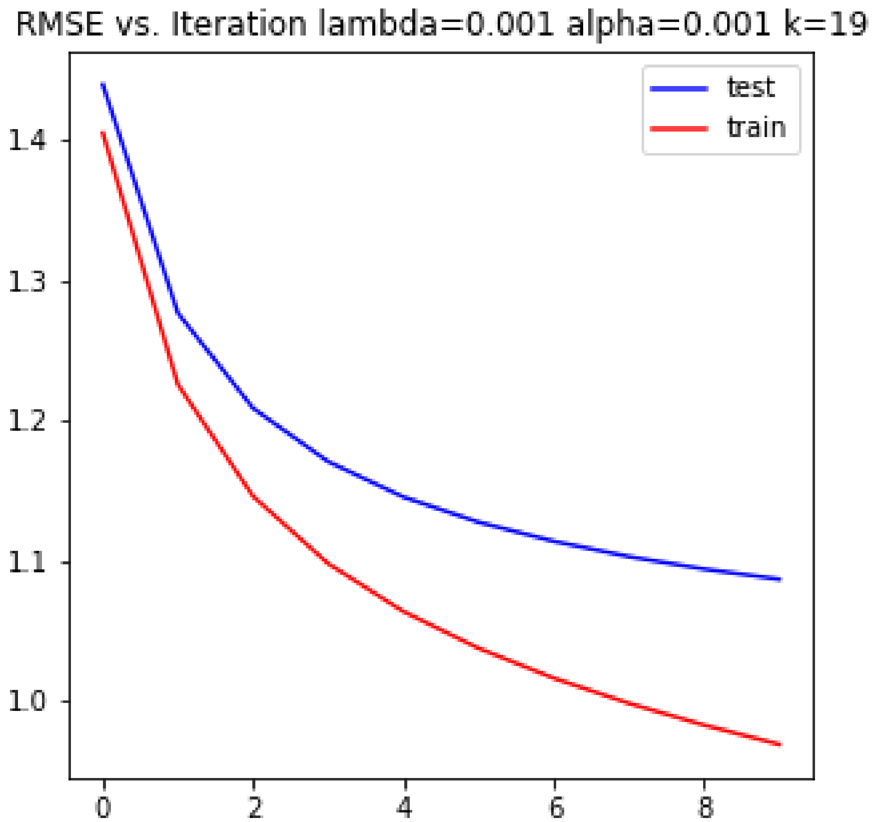

RMSE vs. Iteration lambda=0.01 alpha=0.001 k=19
test
train
1.4
1.3
1.2
1.1
1.0
0
2
4
6
8

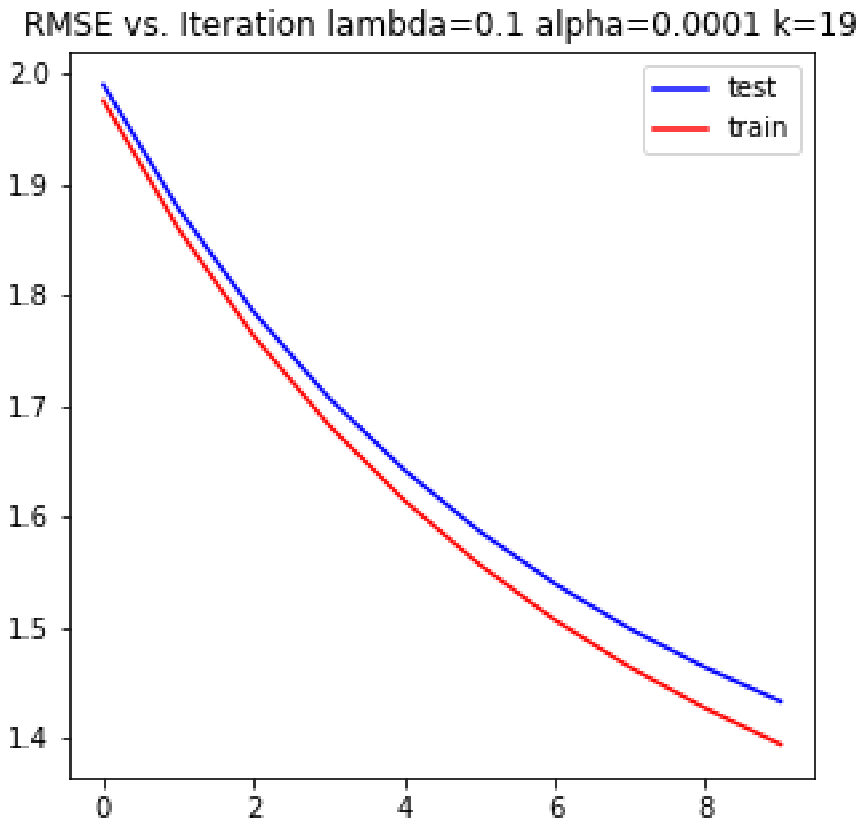

RMSE vs. Iteration lambda=0.01 alpha=0.001 k=19
test
train
1.4
1.3
1.2
1.1
1.0
0
2
4
6
8

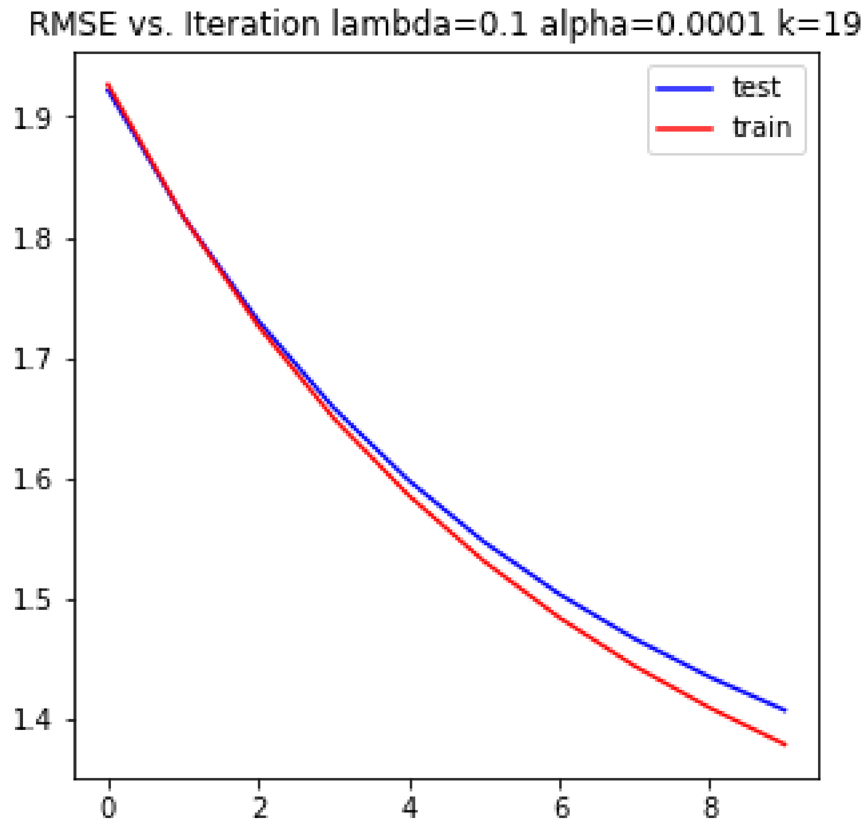

RMSE vs. Iteration lambda=0.001 alpha=0.001 k=19
test
train
1.4
1.3
1.2
1.1
1.0
0
2
4
6
8



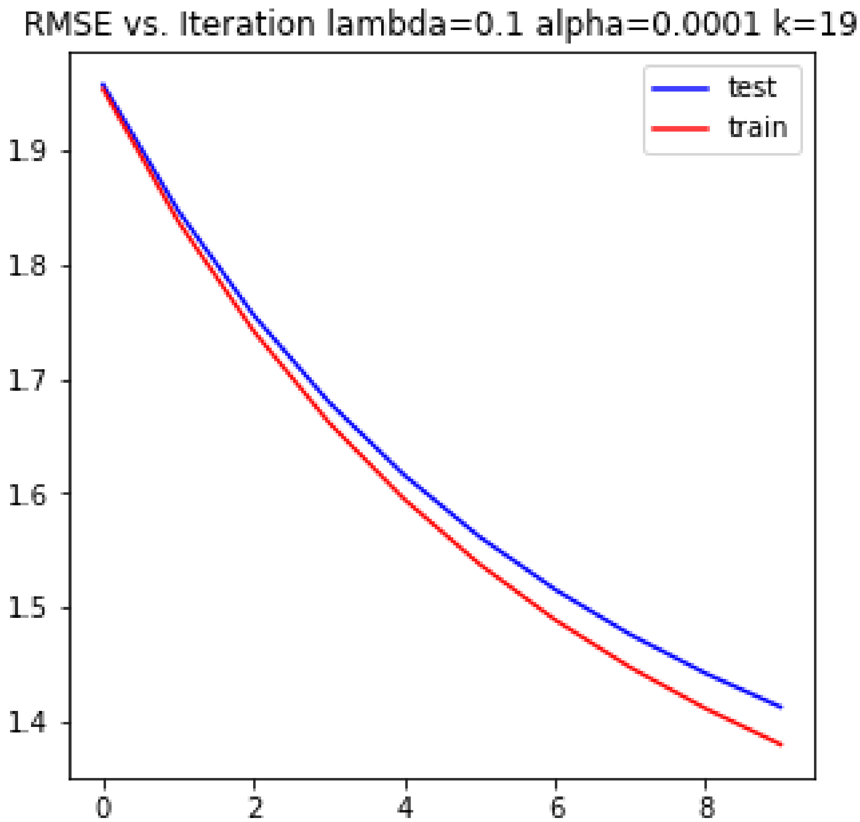
RMSE vs. Iteration lambda=0.001 alpha=0.001 k=19
test
train
1.4
1.3
1.2
1.1
1.0
0
2
4
6
8

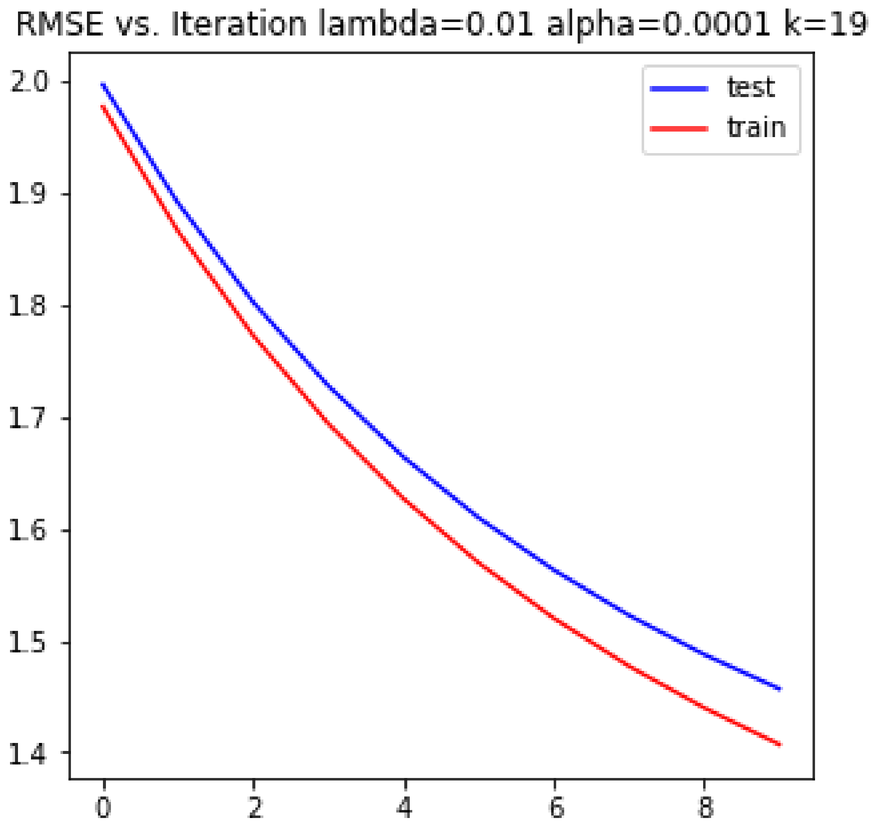

RMSE vs. Iteration lambda=0.001 alpha=0.001 k=19
test
train
1.4
1.3
1.2
1.1
1.0
0
2
4
6
8

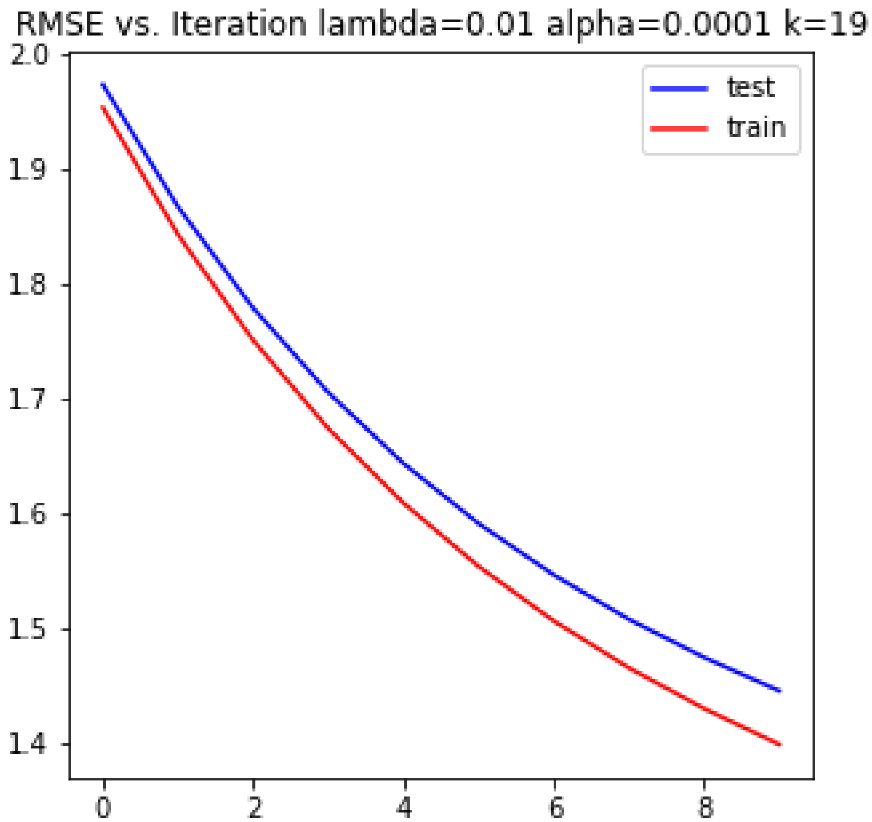

RMSE vs. Iteration lambda=0.1 alpha=0.0001 k=19
test
train
2.0
1.9
1.8
1.7
1.6
1.5
1.4
0
2
4
6
8

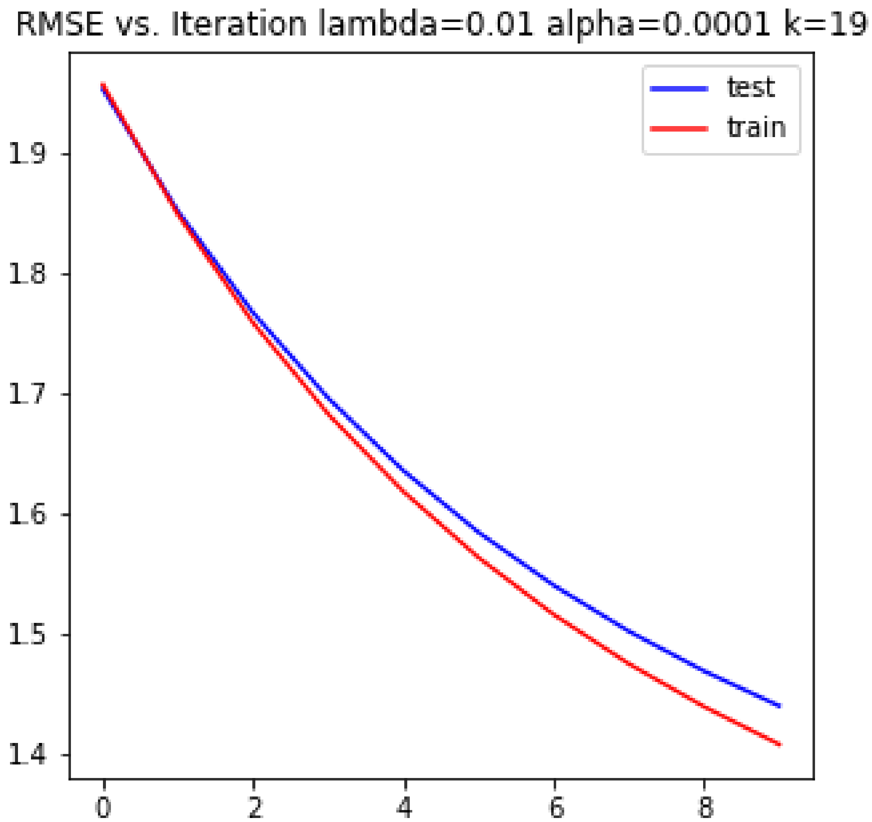


RMSE vs. Iteration lambda=0.1 alpha=0.0001 k=19
test
train
1.9
1.8
1.7
1.6
1.5
1.4
0
2
4
6
8

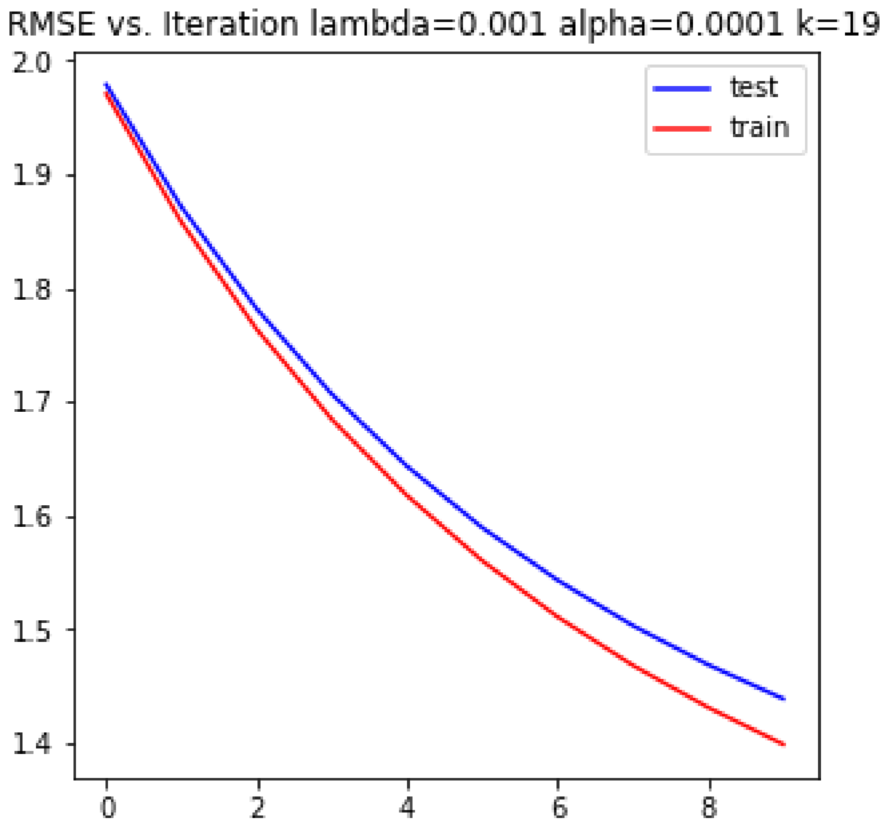

RMSE vs. Iteration lambda=0.1 alpha=0.0001 k=19
test
train
1.9
1.8
1.7
1.6
1.5
1.4
0
2
4
6
8

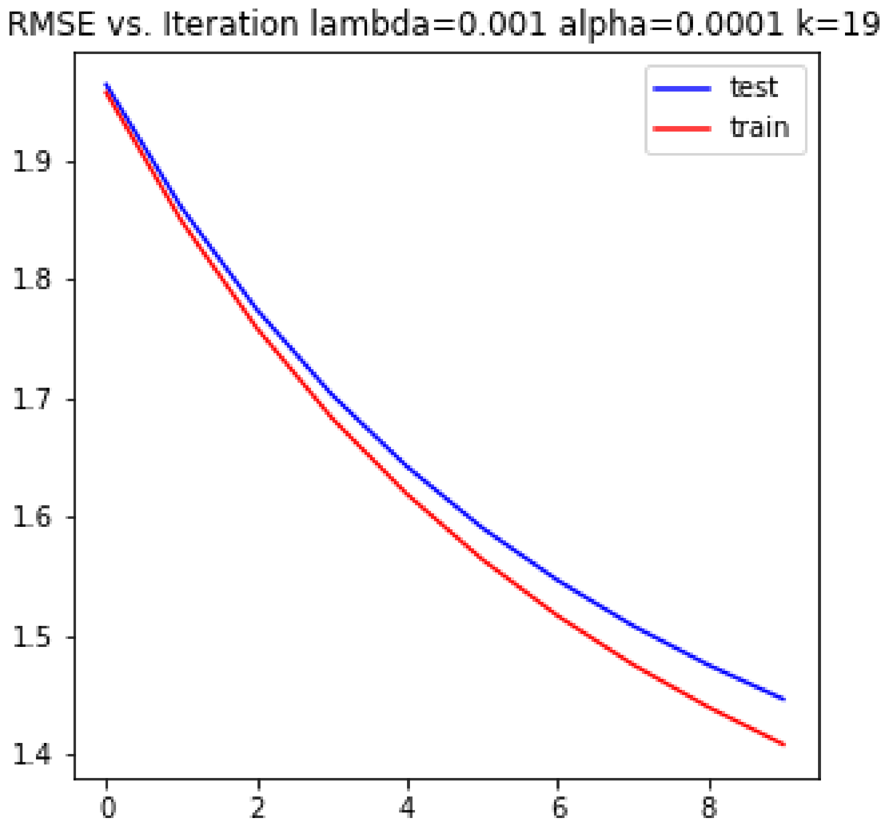

RMSE vs. Iteration lambda=0.01 alpha=0.0001 k=19
test
train
2.0
1.9
1.8
1.7
1.6
1.5
1.4
0
2
4
6
8

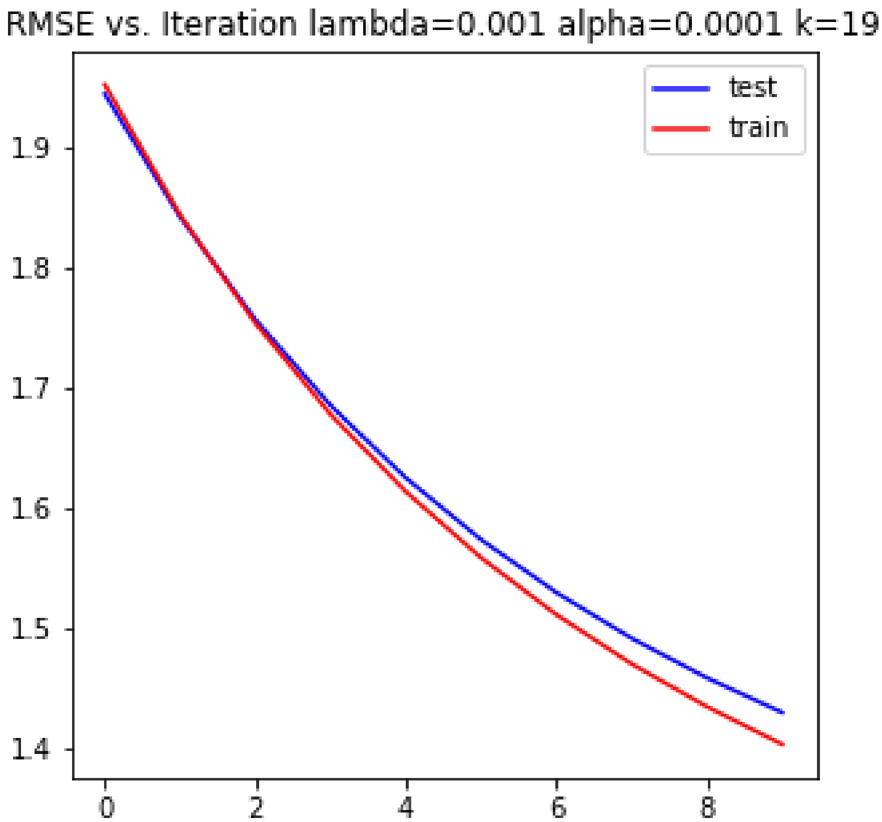

RMSE vs. Iteration lambda=0.01 alpha=0.0001 k=19
test
train
2.0
1.9
1.8
1.7
1.6
1.5
1.4
0
2
4
6
8



RMSE vs. Iteration lambda=0.01 alpha=0.0001 k=19
test
train
1.9
1.8
1.7
1.6
1.5
1.4
0
2
4
6
8


RMSE vs. Iteration lambda=0.001 alpha=0.0001 k=19
test
train
2.0
1.9
1.8
1.7
1.6
1.5
1.4
0
2
4
6
8


RMSE vs. Iteration lambda=0.001 alpha=0.0001 k=19
test
train
1.9
1.8
1.7
1.6
1.5
1.4
0
2
4
6
8




```
In [57]: pd.DataFrame({'Hyperparameter': hyper_list,
                       'RMSE in test':mean_val_folds})
```

Out[57]:

	Hyperparameter	RMSE in test
0	(5, 0.01, 0.1)	0.996116
1	(5, 0.01, 0.01)	1.019427
2	(5, 0.01, 0.001)	1.023773
3	(5, 0.001, 0.1)	1.110568
4	(5, 0.001, 0.01)	1.103436
5	(5, 0.001, 0.001)	1.102232
6	(5, 0.0001, 0.1)	2.114188
7	(5, 0.0001, 0.01)	2.107812
8	(5, 0.0001, 0.001)	2.089272
9	(10, 0.01, 0.1)	0.994921
10	(10, 0.01, 0.01)	1.038508
11	(10, 0.01, 0.001)	1.054160
12	(10, 0.001, 0.1)	1.026043
13	(10, 0.001, 0.01)	1.025887
14	(10, 0.001, 0.001)	1.025858
15	(10, 0.0001, 0.1)	1.390177
16	(10, 0.0001, 0.01)	1.379144

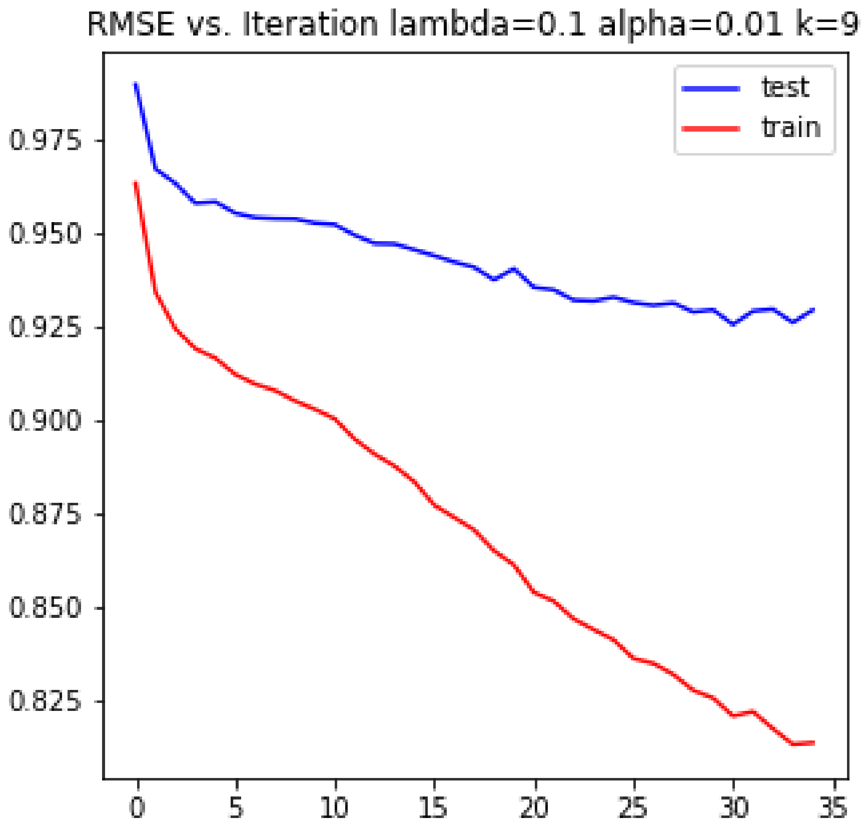
```
17  (10, 0.0001, 0.001)      1.377007
18       (20, 0.01, 0.1)      1.007884
19      (20, 0.01, 0.01)      1.093427
20     (20, 0.01, 0.001)      1.114639
21      (20, 0.001, 0.1)      1.071502
22     (20, 0.001, 0.01)      1.087437
23    (20, 0.001, 0.001)      1.085012
24     (20, 0.0001, 0.1)      1.418129
25    (20, 0.0001, 0.01)      1.447510
26   (20, 0.0001, 0.001)      1.438313
```

We now train the model with the best hyperparrameter set (k=10, alpha=0.01, lambda=0.1) using the whole train dataset and evaluate the model on the test set.

```
In [59]: #defining the best hyperparameter set
         k = 10
         alpha = 0.01
         lamb = 0.1

         #training the final model with the best hyperparameter set
         rmse_train, rmse_val, p, q = SGA_MF(X_train, X_test, k, n_p, n_q, alpha, lamb, max_ite
```

```
Initial RMSE: 1.69325668312
Epoch: 0
RMSE test: 0.989834546594
Epoch: 10
RMSE test: 0.952370912452
Epoch: 20
RMSE test: 0.935617488618
Epoch: 30
RMSE test: 0.925662935787
```




```
In [63]: print("RMSE train error:", rmse_train)
         print("RMSE test error:", rmse_val)
```

```
RMSE train error: 0.813868001303
RMSE test error: 0.929541089533
```

The algorithm converges in about 35 iterations and we obtain an RMSE on test data set equals to 0.929.

0.0.4 Exercise 3: Recommender systems using matrix factorization of scikit-learn

For this part, we would like to compare the previous results with the results of a given library. In this case, we select scikit learn for two reasons:

- Scikit-learn is a very well and wide used library.
- There are some extra dependencies for the "libmf" that could not be addressed.

The scikit -learn library has a module called "decomposition" which has an object called NMF. This object factorizes a matriz in two non-ngative matriz (all components are positive). This, of courses, imposes an aditional constraint in comparision to the previous implemented matriz factorization. The previous implemented matriz factorization, therefore, permits to have negative values.

The object NMF in scikit-learn factorizes a matrix R in the following way:

$R = PQ^T$

Where P and Q are positive. To obtain P matrix, we should apply the method transform on the factorized matrix. To obtain Q, we have to use the variable *components* of the NMF object.

```
In [269]: from scipy.sparse import *
          from scipy import *
          #compressed sparse row matrix

          row = X_train[:,0]
          col = X_train[:,1]
          data = X_train[:,2]

          X_tr_dense = np.array(csr_matrix((data, (row, col)), shape= (n_p, n_q)).todense())
          print("Shape of new matrix:", X_tr_dense.shape)
```

```
Shape of new matrix: (944, 1683)
```

```
In [324]: from sklearn.decomposition import NMF

          model = NMF(n_components=250,init='nndsvd', random_state=10, alpha=0.01, max_iter=20
          P = model.fit_transform(X_tr_dense.astype(float))
          Q_t = model.components_

          print("Shape of P:", P.shape)
          print("Shape of Q:", Q_t.shape)

          RMSE_thr(X_train, P,Q_t.transpose())

          X_=model.inverse_transform(P)
```

```
Shape of P: (944, 250)
Shape of Q: (250, 1683)
```

```
In [326]: RMSE_thr(X_train, P,Q_t.transpose())

Out[326]: 1.4217762680456019
```

```
In [337]: n_q = max(ratings["movie_id"])+1
          n_p = max(ratings["user_id"])+1

          #number of samples of training set
          n_train = X_train.shape[0]

          #number of folds
```

```
n_folds = 3

#initializing folds
folds = []
samples_fold = int(n_train/n_folds)

#creating the k-fold subsets
for i in range(n_folds):
    folds.append((X_train[(i*samples_fold):((i+1)*samples_fold),:]))

folds_list = list(range(n_folds+1))

#initialize list to store the man of each hyperparameter setting
mean_test_folds = []

#list of hyperparameters
max_iter= 1000
k_list = [5,10,20]
lamb_list = [0.1, 0.01, 0.001]

hyper_list = [(i,j) for i in k_list for j in lamb_list]

for k, lamb in hyper_list:

    print("Trying hyperparameters set: k=",k,"lambda=",lamb)
    test_rmse_folds = []

    for f in range(n_folds):

            #list of folds
            folds_list = list(range(n_folds))
            folds_list.pop(f)

            #selecting test dataset
            X_test_fold = folds[f]

            #merging the folds to create the training dataset
            X_train_fold = folds[folds_list[1]]

            for j in folds_list[1:]:
                X_train_fold = np.vstack((X_train_fold, folds[j]))

            #training the model
            row = X_train_fold[:,0]
            col = X_train_fold[:,1]
            data = X_train_fold[:,2]

            X_tr_dense = np.array(csr_matrix((data, (row, col)), shape= (n_p, n_q)).
```

```
                model = NMF(n_components=k,init='random', random_state=10, alpha=lamb, ma
                P = model.fit_transform(X_tr_dense)
                Q_t = model.components_

                print(P.shape)

                #validating model
                row = X_test_fold[:,0]
                col = X_test_fold[:,1]
                data = X_test_fold[:,2]

                X_test_dense = np.array(csr_matrix((data, (row, col)), shape= (n_p, n_q)

                rmse_test = RMSE(X_test_fold, P,Q_t.transpose())

                print("RMSE in test:", rmse_test)

                #finding accuracy over the fold
                test_rmse_folds.append(rmse_test)

        #findning the mean across all the folds
        mean_test_folds.append(np.mean(test_rmse_folds))
```

```
Trying hyperparameters set: k= 5 lambda= 0.1
(944, 5)
RMSE in test: 3.06771959264
(944, 5)
RMSE in test: 3.07199447009
(944, 5)
RMSE in test: 3.07604976549
Trying hyperparameters set: k= 5 lambda= 0.01
(944, 5)
RMSE in test: 3.06755084179
(944, 5)
RMSE in test: 3.07183272794
(944, 5)
RMSE in test: 3.07590103056
Trying hyperparameters set: k= 5 lambda= 0.001
(944, 5)
RMSE in test: 3.06751418195
(944, 5)
RMSE in test: 3.07180630095
(944, 5)
RMSE in test: 3.07589423779
Trying hyperparameters set: k= 10 lambda= 0.1
(944, 10)
RMSE in test: 3.07497547446
```

```
(944, 10)
RMSE in test: 3.07750179996
(944, 10)
RMSE in test: 3.07966267986
Trying hyperparameters set: k= 10 lambda= 0.01
(944, 10)
RMSE in test: 3.07484346262
(944, 10)
RMSE in test: 3.07736686937
(944, 10)
RMSE in test: 3.07953434471
Trying hyperparameters set: k= 10 lambda= 0.001
(944, 10)
RMSE in test: 3.07458231392
(944, 10)
RMSE in test: 3.07719181789
(944, 10)
RMSE in test: 3.07954164209
Trying hyperparameters set: k= 20 lambda= 0.1
(944, 20)
RMSE in test: 3.11859058313
(944, 20)
RMSE in test: 3.11964779573
(944, 20)
RMSE in test: 3.1199079141
Trying hyperparameters set: k= 20 lambda= 0.01
(944, 20)
RMSE in test: 3.11845365387
(944, 20)
RMSE in test: 3.11951521547
(944, 20)
RMSE in test: 3.11981277421
Trying hyperparameters set: k= 20 lambda= 0.001
(944, 20)
RMSE in test: 3.11780279
(944, 20)
RMSE in test: 3.11885198561
(944, 20)
RMSE in test: 3.11980383228
```

In [338]:
```
pd.DataFrame({'Hyperparameter': hyper_list,
              'RMSE in test':mean_test_folds})
```

Out[338]:

	Hyperparameter	RMSE in test
0	(5, 0.1)	3.071921
1	(5, 0.01)	3.071762
2	(5, 0.001)	3.071738

```
3      (10, 0.1)      3.077380
4     (10, 0.01)      3.077248
5    (10, 0.001)      3.077105
6      (20, 0.1)      3.119382
7     (20, 0.01)      3.119261
8    (20, 0.001)      3.118820
```

```
In [343]: row = X_train[:,0]
          col = X_train[:,1]
          data = X_train[:,2]

          X_tr_dense = np.array(csr_matrix((data, (row, col)), shape= (n_p, n_q)).todense())

          model = NMF(n_components=5,init='random', random_state=10, alpha=0.001, max_iter=max_
          P = model.fit_transform(X_tr_dense)
          Q_t = model.components_

          #validating model
          row = X_test[:,0]
          col = X_test[:,1]
          data = X_test[:,2]

          X_test_dense = np.array(csr_matrix((data, (row, col)), shape= (n_p, n_q)).todense())

          rmse_test = RMSE(X_test, P,Q_t.transpose())
          rmse_test_modified = RMSE_thr(X_test, P, Q_t.transpose())
          print("RMSE test:", rmse_test)
          print("RMSE test modified:", rmse_test_modified)
```

```
RMSE test: 2.70768420175
RMSE test modified: 2.4293571516
```

Once we have chosen the best hyperparameter set using cross-validation, we retrained the model and find that the test error using the threshold to the predictions decreased. However, the RMSE applying the scikit-learn function is still higher than the implemented. The main reason for that is that we use a *non-negative matrix factorization* method, it means that the matrix the factor matrix are constraint to be positive, which decreases the capacity of the movdel to learn suitable matrices.

0.1 References

[1] Movie Lens data: https://grouplens.org/datasets/movielens/

[2] Source: data-artisans.com

[3] Alternating least squares: https://datasciencemadesimpler.wordpress.com/tag/alternating-least-squares/

[4] Trevor Hastie, Rahul Mazumder, Jason D. Lee, Reza Zadeh *Matrix Completion and LowRank SVD via Fast Alternating Least Squares* Statistics Department and ICME, Stanford University, 2014.

[5] Recommender systems (University Hildesheim): https://www.ismll.uni-hildesheim.de/lehre/ba-18w/script/6_recommender-systems.pdf