European Option Pricing

Black–Scholes & Monte Carlo Implementation

Abstract

This project implements a complete **European option pricer** using both the analytical **Black–Scholes model** and a **Monte Carlo simulation**. It also computes the main **Greeks** to analyze risk sensitivities. The project is structured into three main parts, each corresponding to a key concept in option pricing.

1 Black–Scholes Analytical Pricing

The first part implements the **Black–Scholes framework** for European call and put options.

The variables d_1 and d_2 are computed from the model parameters. Using these quantities, closed-form formulas are applied to obtain:

- The **Call** price
- The **Put** price

The model accounts for the following parameters:

- Spot price (S_0)
- Strike price (K)
- Time to maturity (T)
- Volatility (σ)
- Risk-free interest rate (r)
- Continuous dividend yield (q)

This section provides a *theoretical benchmark price* under the standard Black–Scholes assumptions (log-normal dynamics, constant volatility, frictionless markets).

2 Monte Carlo Pricing

The second part prices the same European options using a **Monte Carlo simulation**. The process follows these steps:

1. The underlying asset price at maturity is simulated using a **Geometric Brownian Motion** (GBM).
2. Random shocks are drawn from a **Standard Normal Distribution** ($\mathcal{N}(0, 1)$).
3. A large number of simulated terminal prices is generated.
4. For each simulation, the option payoff is computed.
5. The final option price is obtained by **discounting the average payoff**.

This numerical approach illustrates how option pricing can be performed without closed-form solutions and highlights the flexibility of Monte Carlo methods, which can be extended to *exotic options*.

3 Greeks Computation (Finite Differences)

The third part focuses on the computation of the **Greeks**, which measure the sensitivity of the option price to model parameters. While some Greeks admit analytical expressions in the Black–Scholes framework, numerical methods are widely used in practice.

Here, derivatives are approximated using **Finite Difference Methods**.

Numerical Differentiation Approach

Each Greek corresponds to a partial derivative of the option price with respect to a given parameter (spot, volatility, time, or interest rate). To approximate these derivatives:

- A small perturbation (ε) is applied to the parameter of interest.
- The option price is recomputed for $(x + \varepsilon)$ and $(x - \varepsilon)$.
- The derivative is approximated using a **Central Difference Scheme**, which provides a good trade-off between accuracy and numerical stability:

$$\frac{\partial V}{\partial \theta} \approx \frac{V(\theta + \varepsilon) - V(\theta - \varepsilon)}{2\varepsilon}$$

Implemented Greeks

Delta (Δ) Sensitivity to the spot price (computed analytically).

Gamma (Γ) Second-order sensitivity to the spot price (finite differences).

Vega (ν) Sensitivity to volatility.

Theta (Θ) Sensitivity to time decay.

Rho (ρ) Sensitivity to the interest rate.