Graphs & Trees (G)

	Chapter	Lecture	Assignment
Graphs & Trees	1.3, 10	11	6

10. (6 pts) Each of the four descriptions below specify a graph. For one of those four descriptions it is impossible to construct a graph that matches the description. Which description is the impossible one?

☐ A simple graph with two connected components and a simple circuit.

☐ A full binary tree with 9 vertices.

☐ A graph with 6 vertices: one vertex of degree 0, one of degree 1, one of degree 2, one of degree 3, and two of degree 4.

☐ A tree with 6 vertices and total degree 12.

9. (6 pts) Consider the following three descriptions of a graph:

(A) A simple graph with 6 vertices: one vertex of degree 0, one of degree 1, one of degree 2, one of degree 3, one of degree 4 and one of degree 5.

(B) A tree with 5 vertices and an Euler trail.

(C) A tree with 5 vertices and a total degree of 10.

Which statement is **true**?

☐ (B) and (C) exist, but (A) does not.

☐ (B) exists, but neither (A) nor (C) do.

☐ (A) and (B) exist, but (C) does not.

☐ (C) exists, but neither (A) nor (B) do.

10. (6 pts) Each of the four descriptions below specify a graph. For one of those four descriptions it is impossible to construct a graph that matches the description. Which description is the impossible one?

[A] A simple graph with two connected components and a simple circuit.

[B] A full binary tree with 9 vertices.

[C] A graph with 6 vertices: one vertex of degree 0, one of degree 1, one of degree 2, one of degree 3, and two of degree 4.

☒ A tree with 6 vertices and total degree 12.

Solution: A tree with 6 vertices must have 5 edges according to Theorem 14. According to the Handshake Theorem, a graph with 5 edges has total degree 10 – not 12.

9. (6 pts) Consider the following three descriptions of a graph:

(A) A simple graph with 6 vertices: one vertex of degree 0, one of degree 1, one of degree 2, one of degree 3, one of degree 4 and one of degree 5.

(B) A tree with 5 vertices and an Euler trail.

(C) A tree with 5 vertices and a total degree of 10.

Which statement is **true**?

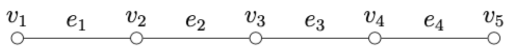
[A] (B) and (C) exist, but (A) does not.

☒ (B) exists, but neither (A) nor (C) do.

[C] (A) and (B) exist, but (C) does not.

[D] (C) exists, but neither (A) nor (B) do.

Solution: (A) not exist. Such a graph would have total degree of $0+1+2+3+4+5 = 15$. However, according to the Handshake Theorem, every graph has an even total degree. (B) does exist:

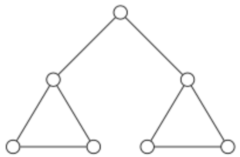


(C) does not exist. According to the Handshake Theorem, a graph with total degree 10 must have exactly 5 edges. However, according to Theorem 14 on page 14, a tree with 5 vertices must have exactly 4 edges.

9. (6 pts) Let n be a positive integer. What is the total degree of a tree with n vertices?

- ☐ $2n$.
- ☐ $2n-1$.
- ☐ $2n-2$.
- ☐ $n-1$.

8. (6 pts) Let G be the following graph:



Which of the following statements is **true**?

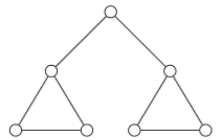
- ☐ G is a tree.
- ☐ G has an Euler circuit.
- ☐ G has an Euler trail.
- ☐ G has two connected components.

9. (6 pts) Let n be a positive integer. What is the total degree of a tree with n vertices?

- A $2n$.
- B $2n-1$.
- ✔ $2n-2$.
- D $n-1$.

Solution: A tree with n vertices must have $n-1$ edges according to Theorem 14. According to the Handshake Theorem, a graph with $n-1$ edges has total degree $2(n-1)=2n-2$.

8. (6 pts) Let G be the following graph:

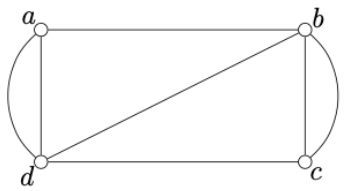


Which of the following statements is **true**?

- A G is a tree.
- B G has an Euler circuit.
- ✔ G has an Euler trail.
- D G has two connected components.

Solution: The graph is connected, two vertices have degree 3, while all remaining vertices have degree 2. Hence, the graph has an Euler trail.

10. (6 pts) Let G be the following graph:



Which of the following statements is **true**?

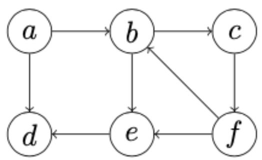
☐ G is not connected.

☐ G has an Euler circuit.

☐ G has an Euler trail.

☐ G is a simple graph.

9. (2 pts) Consider the following directed graph $G = (V, E)$.



Which of the following statements is *false*?

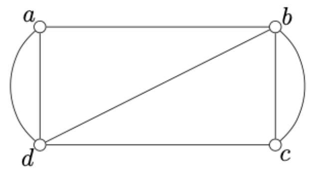
☐ G is a simple directed graph

☐ $H = (\{f\}, \{\})$ is a strongly connected component of G

☐ G is weakly connected

☐ The path b, c, f, b is a simple circuit of length 3

10. (6 pts) Let G be the following graph:



Which of the following statements is **true**?

$\boxed{\text{A}}$ G is not connected.

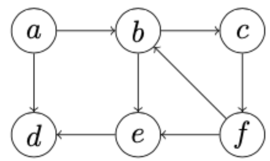
$\boxed{\text{B}}$ G has an Euler circuit.

☑ G has an Euler trail.

$\boxed{\text{D}}$ G is a simple graph.

Solution: Both a and c have an odd degree (namely 3) all other vertices have an even degree. Hence, there is an Euler trail from a to c.

9. Consider the following directed graph $G = (V, E)$.



Which of the following statements is *false*?

$\boxed{\text{A}}$ G is a simple directed graph

☑ $H = (\{f\}, \{\})$ is a strongly connected component of G

$\boxed{\text{C}}$ G is weakly connected

$\boxed{\text{D}}$ The path b, c, f, b is a simple circuit of length 3

3. (15 pts) There are three conditions for a graph listed below. For each of these conditions give *either* an example of a graph that satisfies the condition, *or* a reason why no such graph exists. In order to give an example, either draw the corresponding graph or give the triple (V, E, f) of vertices, edges and edge-endpoint function.
Hint: Use the definitions and theorems about graphs and trees on pages 15–16.

(a) A tree with 5 vertices and a total degree of 10.

(b) A tree with 5 vertices and an Euler trail.

(c) A simple graph with 6 vertices: one vertex of degree 0, one of degree 1, one of degree 2, one of degree 3, one of degree 4 and one of degree 5.

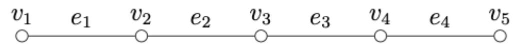
3. (15 pts) There are three conditions for a graph listed below. For each of these conditions give *either* an example of a graph that satisfies the condition, *or* a reason why no such graph exists. In order to give an example, either draw the corresponding graph or give the triple (V, E, f) of vertices, edges and edge-endpoint function.
Hint: Use the definitions and theorems about graphs and trees on pages 16–17.

(a) A tree with 5 vertices and a total degree of 10.

Solution: Such a graph does not exist. According to the Handshake Theorem, a graph with total degree 10 must have exactly 5 edges. However, according to Theorem 13 on page 17, a tree with 5 vertices must have exactly 4 edges.

(b) A tree with 5 vertices and an Euler trail.

Solution: Such a graph does exist:



It has the Euler trail $v_1e_1v_2e_2v_3e_3v_4e_4v_4$.

(c) A simple graph with 6 vertices: one vertex of degree 0, one of degree 1, one of degree 2, one of degree 3, one of degree 4 and one of degree 5.

Solution: Such a graph does not exist. Such a graph would have total degree of $0+1+2+3+4+5 = 15$. However, according to the Handshake Theorem, every graph has an even total degree.
Alternative Solution: The vertex with degree 5 must have an edge to each other vertex, because a simple graph may not have loops or parallel edges. But that means that there cannot be a vertex with degree 0.

13. (12 pts) There are three conditions for a graph listed below. For each of these conditions give *either* an example of a graph that satisfies the condition, *or* a justification why no such graph exists. In order to give an example, either draw the corresponding graph or give the triple (V, E, f) of vertices, edges and edge-endpoint function. In order to give a justification that a graph of the given description cannot exist, use the definitions and theorems about graphs and trees on pages 16–18.

(a) A graph with 4 vertices 7 edges and an Euler circuit.

(b) A simple connected graph with a circuit.

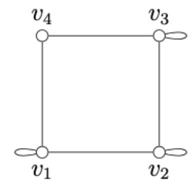
(c) A full binary tree with 8 vertices.

Above question appears multiple times in previous exams

13. (12 pts) There are three conditions for a graph listed below. For each of these conditions give *either* an example of a graph that satisfies the condition, *or* a justification why no such graph exists. In order to give an example, either draw the corresponding graph or give the triple (V, E, f) of vertices, edges and edge-endpoint function. In order to give a justification that a graph of the given description cannot exist, use the definitions and theorems about graphs and trees on pages 15–17.

(a) A graph with 4 vertices 7 edges and an Euler circuit.

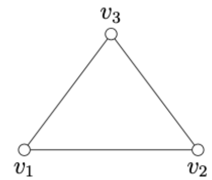
Solution: Such a graph does exist:



The graph has an Euler circuit since each vertex has an even degree.

(b) A simple connected graph with a circuit.

Solution: Such a graph exists:



(c) A full binary tree with 8 vertices.

Solution: Such a graph does not exist. According to Theorem 15 on page 17, a full binary tree with n internal vertices has exactly $n+1$ leaves. That means, every full binary tree has $2n+1$ vertices, i.e. an odd number of vertices. Hence, there is no full binary tree with an even number of vertices.