Number Theory (N)

	Chapter	Lecture	Assignment
Number Theory	2.5, 4.5, 4.10, 8,4	4	2

3. (6 pts) Which of the following statements is **true**?

☐ $\gcd(11, 7n) = 1$ for all positive integers n.

☐ $5 \equiv -19 \pmod{12}$.

☐ $33 \mid 11$.

☐ $-5n \bmod 3 = 1$ for all positive integers n.

2. (6 pts) Which of the following statements is **true**?

☐ $2 \mid (2 + c)$ for all positive integers c.

☐ $34 \equiv -14 \pmod{12}$.

☐ $24 = \operatorname{lcm}(6, 4)$.

☐ $\gcd(a, b) = \gcd(a, ab)$ for all positive integers a and b.

3. (6 pts) Which of the following statements is **true**:

☐ $3 \mid (5 \cdot c)$ for all positive integers c.

☐ $\gcd(9, 15) = \gcd(15, 21)$.

☐ $\operatorname{lcm}(9, 15) = \operatorname{lcm}(15, 21)$.

☐ $6 \equiv 12 \pmod{12}$.

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[A] $\gcd(11, 7n) = 1$ for all positive integers n.

☑ $5 \equiv -19 \pmod{12}$.

[C] $33 \mid 11$.

[D] $-5n \bmod 3 = 1$ for all positive integers n.

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☑ $\gcd(9, 15) = \gcd(15, 21)$.

[C] $\operatorname{lcm}(9, 15) = \operatorname{lcm}(15, 21)$.

[D] $6 \equiv 12 \pmod{12}$.

Solution: $\gcd(9, 15) = 3 = \gcd(15, 21)$

3. (6 pts) Which of the following statements is **true**?

☐ $\operatorname{lcm}(18,4) = \operatorname{lcm}(12,9)$.

☐ $8 \mid (8+c)$ for all positive integers c.

☐ $35 \equiv 11 \pmod{9}$.

☐ $\gcd(10,16) = \gcd(16,21)$.

3. (6 pts) Which of the following statements is **true**?

☒ $\operatorname{lcm}(18,4) = \operatorname{lcm}(12,9)$.

$\boxed{\text{B}}$ $8 \mid (8+c)$ for all positive integers c.

$\boxed{\text{C}}$ $35 \equiv 11 \pmod{9}$.

$\boxed{\text{D}}$ $\gcd(10,16) = \gcd(16,21)$.

Solution: $\operatorname{lcm}(18,4) = 36 = \operatorname{lcm}(12,9)$

3. (6 pts) Which of the following is **true** for all positive integers a, b and q?

☐ If $ab \equiv 0 \pmod{q}$ then either $a \equiv 0 \pmod{q}$ or $b \equiv 0 \pmod{q}$.

☐ If $a^2 \equiv a \pmod{q}$ then $a \equiv 1 \pmod{q}$.

☐ If $a \equiv a+b \pmod{q}$ then $b \equiv 0 \pmod{q}$.

☐ If $a^2 \equiv 0 \pmod{q}$ then $a \equiv 0 \pmod{q}$.

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$\boxed{\text{B}}$ If $a^2 \equiv a \pmod{q}$ then $a \equiv 1 \pmod{q}$.

☒ If $a \equiv a+b \pmod{q}$ then $b \equiv 0 \pmod{q}$.

$\boxed{\text{D}}$ If $a^2 \equiv 0 \pmod{q}$ then $a \equiv 0 \pmod{q}$.

Solution: $a \equiv a+b \pmod{q}$ implies by Theorem 3 that $q \mid (a-(a+b))$ and thus $q \mid -b$. Consequently, $q \mid b$, which means that $b \equiv 0 \pmod{q}$.

2. (6 pts) Which of the following is **true** for all integers n?

☐ $(2 \cdot n) \bmod 2 = n$

☐ $(n^2 + 1) \bmod 2 = (n + 1)^2 \bmod 2$

☐ $(2 \cdot n) \bmod 2 = n \bmod 2$

☐ $(n + 2) \bmod 2 = n$

2. (6 pts) Which of the following is **true** for all integers n?

[A] $(2 \cdot n) \bmod 2 = n$

[B] ✔ $(n^2 + 1) \bmod 2 = (n + 1)^2 \bmod 2$

[C] $(2 \cdot n) \bmod 2 = n \bmod 2$

[D] $(n + 2) \bmod 2 = n$

Solution: By definition

$$(n^2 + 1) \bmod 2 = (n + 1)^2 \bmod 2$$

is equivalent to

$$(n^2 + 1) \equiv (n + 1)^2 \pmod 2$$

Using Theorem 3, we need to show that

$$2 \mid ((n^2 + 1) - (n + 1)^2)$$

This follows from the fact that

$$((n^2 + 1) - (n + 1)^2) = (n^2 + 1) - (n^2 + 2n + 1) = -2n$$

because $2 \mid (-2n)$.

2. (1 pt) What is the sum of the binary numbers $(11\,0110)_2$ and $(1111)_2$ as a binary expression?

☐ $(100\,0101)_2$

☐ $(100\,1001)_2$

☐ $(100\,0001)_2$

☐ $(100\,1101)_2$

3. (1 pt) What is the hexadecimal expansion of $(100\,1111\,1100\,0111\,0110)_2$?

☐ $(9F8E6)_{16}$

☐ $(9F8EC)_{16}$

☐ $(4161476)_{16}$

☐ $(4FC76)_{16}$

13. (4 pts) Use the Euclidean algorithm to compute the greatest common divisor of 4 260 and 432.

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13. Use the Euclidean algorithm to compute the greatest common divisor of 4 260 and 432.

Solution: If $a = q \cdot b + r$ then $\gcd(a, b) = \gcd(b, r)$. It then follows that

$$\begin{aligned} 4260 &= 9 \cdot 432 + 372 \\ 432 &= 1 \cdot 372 + 60 \\ 372 &= 6 \cdot 60 + 12 \\ 60 &= 5 \cdot 12 + 0 \end{aligned}$$

Thus

$$\gcd(4260, 432) = \gcd(432, 372) = \gcd(372, 60) = \gcd(60, 12) = \gcd(12, 0) = 12.$$