Foundations of Computing:Discrete Mathematics

Re-Exam
August 19, 2014

Instructions (Read Carefully)

What to check. In the multiple-choice questions, there is one and only one correct answer. However, to demonstrate partial knowledge, you are allowed to check 2 or more boxes, but this earns you less than full points for that question. If the correct answer is not among your checked boxes, the score will even be negative. If you don't check anything (or check *all* boxes) your score is 0. Also, if you check boxes at random, your expected score is 0. For more details, read:

[G. S. Frandsen, M. I. Schwartzbach: A singular choice for multiple choice. SIGCSE Bulletin 38(4): 34–38 (2006)].

Useful Definitions. At the end of this document, you can find some definitions that could be useful for answering some questions.

Info about you. Write your full name and CPR no. on every page (top-right).

- IMPORTANT -
Only information written on these 8 pages will be evaluated.
Anything else that you hand-in will NOT be considered for the final evaluation!

1. Answer the following multiple choice questions:

PLS (a) (*1 pt.*) Let $P(x)$ and $Q(x)$ be two predicates. Which of the following formulas is always true?

- [A] $(\forall x)\Big((\neg P(x) \vee \neg Q(x)) \wedge \neg P(x) \wedge \neg Q(x)\Big)$
- [B] ✔ $(\forall x)\Big(\neg(\neg P(x) \vee \neg Q(x)) \vee \neg P(x) \vee \neg Q(x)\Big)$
- [C] $(\forall x)\Big((\neg P(x) \vee \neg Q(x)) \vee \neg P(x) \vee \neg Q(x)\Big)$
- [D] $(\forall x)\Big(\neg(\neg P(x) \wedge \neg Q(x)) \wedge \neg P(x) \wedge \neg Q(x)\Big)$

GT (b) (*1 pt.*) The degree sequence of a graph is the sequence of the degrees of the vertices of the graph in non-increasing order. Let G be a graph that has the following degree sequence: $5, 4, 3, 2, 2, 0$. Which of the following statements is true?

- [A] ✔ G cannot exist: no graph can have such a degree sequence.
- [B] G exists, and it is a graph with six nodes.
- [C] G exists and can be divided into two graphs, one of which with a single node and no edges.
- [D] By the hand-shake theorem, G exists and has 8 edges.

PLS (c) (*1 pt.*) Let A, B and C be three sets. What is the set $A \cap (A \cup B \cup C) \setminus A$ equal to?

- [A] A
- [B] $A \cup B \cup C$
- [C] $B \cup C$
- [D] ✔ $\emptyset$ (the empty set)

FS (d) (*1 pt.*) Which of the following functions from $\mathbb{Z}$ to $\mathbb{Z}$ is one-to-one:

- [ ] A $f_A(x) = x^6$
- [ ] B $f_B(x) = x^4$
- [ ] C $f_C(x) = x^2$
- [x] D $f_D(x) = x$

PLS (e) (*1 pt.*) Let S be the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Which of the following sequences has value 55?

- [x] A $\sum_{j \in S} j$
- [ ] B $\sum_{j \in S} 1$
- [ ] C $\sum_{j \in S} j^3$
- [ ] D $\sum_{j \in S} (j-1)^j$

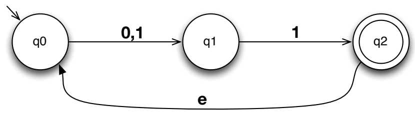
R (f) (*1 pt.*) Consider the relation $\mathcal{R} = \{(1,1),\ (3,3),\ (1,3)\}$ over $\{1,3,4\} \times \{1,3,4\}$. What is its reflexive closure?

- [ ] A $\mathcal{R}_1 = \{(1,1),\ (3,3),\ (4,4)\}$
- [ ] B $\mathcal{R}_2 = \{(1,1),\ (3,3),\ (4,4),\ (1,3)\}$
- [ ] C $\mathcal{R}_4 = \{(1,1),\ (3,3),\ (1,3),\ (3,1)\}$
- [x] D $\mathcal{R}_4 = \{(1,1),\ (3,3),\ (1,3)\}$

(there is an error in the solution for this exam: the correct answer to 1f is B)

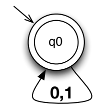
ARS (g) (*1 pt.*) Which of the following automata recognises the language defined by the regular expression $\left((0 \cup 1)1^*\right)^*$

A

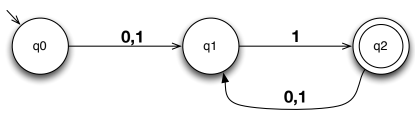


e is the empty string

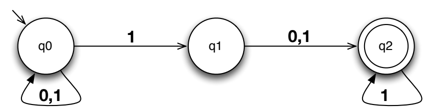
B



C



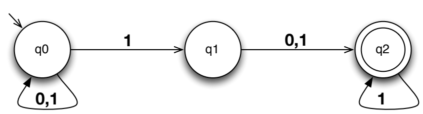
D



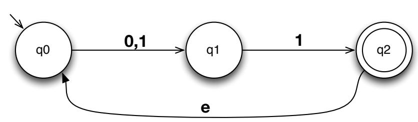
ARS (h) (*1 pt.*) Consider the regular grammar G whose productions are:

$$
\begin{array}{lcl}
S & \rightarrow & 0S \mid 1S \mid 1A \\
A & \rightarrow & 0B \mid 1B \\
B & \rightarrow & 1B \mid \epsilon
\end{array}
$$

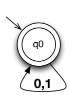
What is the automaton recognising the language generated by G?



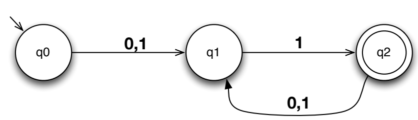
A



B **e** is the empty string



C



D

PL (i) (*1 pt.*) Which of the following statements is true:

- [A] If a language is recognised by a Turing Machine, then it can be recognised by an FSA.
- [B] Every language is either recognised by a Turing Machine or an FSA.
- [C] Every language is recognised by a Turing Machine.
- [D] ✔ If a language is recognised by an FSA, then it can be recognised by a Turing Machine.

CP (j) (*1 pt.*) On Mars, each Martian alien has an ID card with a unique string associated to it. Such strings consist of 5 or 6 alphabetic letters followed by an hyphen "-" and 2 *different* digits. For example, USXJKF-20 or LAAXQ-13 are allowed ID card strings. On Mars, the alphabet has only the letters W A Z Å C Æ Ø, and 4 digits $\{0, 1, 2, 3\}$. How many different ID cards can there be on Mars:

- [A] $\binom{7}{5} \cdot 4^2 + \binom{7}{6} \cdot 4^2$
- [B] $(7^5 + 7^6) \cdot 4^2$
- [C] ✔ $(7^5 + 7^6) \cdot 4 \cdot 3$
- [D] $(8^5 + 8^6) \cdot 4 \cdot 3$

CP (k) (*1 pt.*) The city of Berlin is hosting the YR-Compose Festival (YRCF), a competition for young music composers. Among the participants, 50% plays piano, 30% violin and the remaining 20% plays guitar. Only 10% of the pianists and 10% of the guitarists take part to YRCF for the first time. On the other hand, 30% of the violinists are at their first experience with YRCF. Knowing that the first person who plays at the competition is someone who never participated to YRCF before, what is the probability that such a person is a guitarist?

- [x] A 0.125
- [ ] B 0.16
- [ ] C There is not enough information for computing it
- [ ] D 0.10

2. (*4 pt.*) Answer the following question. Be brief but precise, your correct use of mathematical notation is an important aspect.

Prove by mathematical induction that for any positive integer greater than or equal to 3 ($n \in \{3, 4, 5, \ldots\}$) we have that

$$n^2 > 2n + 1$$

Write Solution Below.

Some useful information for the exam

Functions. A function f from A to B is a subset of the Cartesian product $A \times B$ such that:

$$\forall a \in A.\ \exists_1 b \text{ such that } (a, b) \in f$$

Recall that $\exists_1 b$ means there exists a unique b. We write $f(a) = b$ for $(a, b) \in f$.

A function f from A to B is one-to-one (or injective) whenever

$$\forall a, a' \in A.\ f(a) = f(a') \text{ implies } a = a'$$

Relations. A relation $\mathcal{R}$ on a set A is a subset of the cartesian product $A \times A$.

A relation $\mathcal{R}$ on A is reflexive whenever

$$\forall a \in A.\ (a, a) \in \mathcal{R}$$

The reflexive closure of a binary relation R on A is the smallest reflexive relation on A that contains R.

Handshake Theorem. Let G be an indirected graph. Then,

$$\sum_{v \in V} deg(v) = 2m$$

where m is the number of edges of G and V is the set of vertices.

Bayes'Theorem. Bayes' Theorem allows to manipulate conditional probabilities:

$$p(A_i|B) \ = \ \frac{p(B|A_i)p(A_i)}{p(B)}$$

such that $p(B) = p(B|A_1)p(A_1) + p(B|A_2)p(A_2) + p(B|A_3)p(A_3)$.