Automata, Regex, Grammars (A)

	Chapter	Lecture	Assignment
Automata	12	12	6
Regular expressions	12	12	
Grammars & Models of computation	Rosen	13	

9. (6 pts) Let G be the grammar with vocabulary $V = \{0, 1, S, A\}$, terminal symbols $T = \{0, 1\}$, starting symbol S, and productions

$$\begin{aligned} S &\to 0A1 \\ S &\to 01 \\ A &\to 1S0 \\ A &\to \lambda \end{aligned}$$

Which of the following strings can be generated by G?

☐ 001110

☐ 010101

☐ 101010

☐ 0101

8. (6 pts) Consider the following grammar:

$$\begin{aligned} S &\to AB \\ A &\to aAb \\ A &\to a \\ B &\to Bab \\ B &\to \lambda \end{aligned}$$

Which string belongs to the language generated by this grammar?

☐ $aabbab$

☐ ab

☐ $aaabb$

☐ aba

9. (6 pts) Let G be the grammar with vocabulary $V = \{0, 1, S, A\}$, terminal symbols $T = \{0, 1\}$, starting symbol S, and productions

$$\begin{aligned} S &\to 0A1 \\ S &\to 01 \\ A &\to 1S0 \\ A &\to \lambda \end{aligned}$$

Which of the following strings can be generated by G?

[A] 001110

☑ 010101

[C] 101010

[D] 0101

Solution: 010101 can be generated as follows :

$$S \Rightarrow 0A1 \Rightarrow 01S01 \Rightarrow 010101$$

8. (6 pts) Consider the following grammar:

$$\begin{aligned} S &\to AB \\ A &\to aAb \\ A &\to a \\ B &\to Bab \\ B &\to \lambda \end{aligned}$$

Which string belongs to the language generated by this grammar?

[A] $aabbab$

[B] ab

☑ $aaabb$

[D] aba

Solution: $S \Rightarrow AB \Rightarrow aAbB \Rightarrow aaAbbB \Rightarrow aaabbB \Rightarrow aaabb$

9. (6 pts) Let G be the grammar with vocabulary $V = \{a, b, S, A, B\}$, terminal symbols $T = \{a, b\}$, starting symbol S, and productions

$$\begin{aligned} S &\to AB \\ A &\to aBb \\ A &\to ab \\ B &\to bAa \\ B &\to \varepsilon \end{aligned}$$

Which of the following strings can be generated by G?

☐ $aabbba$

☐ $abbaba$

☐ ba

☐ $bababa$

9. (6 pts) Consider the following grammar:

$$\begin{aligned} S &\to AB \\ A &\to aAb \\ A &\to a \\ B &\to Bab \\ B &\to \epsilon \end{aligned}$$

Which one of the strings below **does not** belong to the language generated by this grammar?

☐ $aaabbab$

☐ $aabab$

☐ $aaabb$

☐ $aabbab$

9. (6 pts) Let G be the grammar with vocabulary $V = \{a, b, S, A, B\}$, terminal symbols $T = \{a, b\}$, starting symbol S, and productions

$$\begin{aligned} S &\to AB \\ A &\to aBb \\ A &\to ab \\ B &\to bAa \\ B &\to \varepsilon \end{aligned}$$

Which of the following strings can be generated by G?

[A] $aabbba$

☑ $abbaba$

[C] ba

[D] $bababa$

Solution: $bababa$ can be generated as follows :

$$S \Rightarrow AB \Rightarrow abB \Rightarrow abbAa \Rightarrow abbaba$$

9. (6 pts) Consider the following grammar:

$$\begin{aligned} S &\to AB \\ A &\to aAb \\ A &\to a \\ B &\to Bab \\ B &\to \epsilon \end{aligned}$$

Which one of the strings below **does not** belong to the language generated by this grammar?

[A] $aaabbab$

[B] $aabab$

[C] $aaabb$

☑ $aabbab$

Solution: The three other strings can be generated as follows:

$$\begin{aligned} S &\Rightarrow AB \Rightarrow aAbB \Rightarrow aaAbbB \Rightarrow aaabbB \Rightarrow aaabbBab \Rightarrow aaabbab \\ S &\Rightarrow AB \Rightarrow aAbB \Rightarrow aabB \Rightarrow aabBab \Rightarrow aabab \\ S &\Rightarrow AB \Rightarrow aAbB \Rightarrow aaAbbB \Rightarrow aaabbB \Rightarrow aaabb \end{aligned}$$

9. (6 pts) Consider the following grammar:

$$
\begin{aligned}
S &\to AB \\
S &\to \varepsilon \\
A &\to Sa \\
B &\to bS
\end{aligned}
$$

Which one of the strings below belongs to the language generated by this grammar?

☐ *baba*

☐ *abab*

☐ *ababa*

☐ *aabb*

9. (6 pts) Let G be the grammar with vocabulary $V = \{a, b, S, A, B\}$, terminal symbols $T = \{a, b\}$, starting symbol S, and productions

$$
\begin{aligned}
S &\to AB \\
A &\to aBb \\
A &\to \varepsilon \\
B &\to bAa \\
B &\to ba
\end{aligned}
$$

Which one of the following strings can be generated by G?

☐ *aabbba*

☐ *bababa*

☐ *ab*

☐ *ababab*

9. (6 pts) Consider the following grammar:

$$
\begin{aligned}
S &\to AB \\
S &\to \varepsilon \\
A &\to Sa \\
B &\to bS
\end{aligned}
$$

Which one of the strings below belongs to the language generated by this grammar?

A *baba*

☑ *abab*

C *ababa*

D *aabb*

Solution: *abab* can be generated as follows :

$$S \Rightarrow AB \Rightarrow SaB \Rightarrow SabS \Rightarrow Sab \Rightarrow ABab \Rightarrow SaBab \Rightarrow SabSab \Rightarrow Sabab \Rightarrow abab$$

(6 pts) Let G be the grammar with vocabulary $V = \{a, b, S, A, B\}$, terminal symbols $T = \{a, b\}$, starting symbol S, and productions

$$
\begin{aligned}
S &\to AB \\
A &\to aBb \\
A &\to \varepsilon \\
B &\to bAa \\
B &\to ba
\end{aligned}
$$

Which one of the following strings can be generated by G?

A *aabbba*

☑ *bababa*

C *ab*

D *ababab*

Solution: *bababa* can be generated as follows :

$$S \Rightarrow AB \Rightarrow B \Rightarrow bAa \Rightarrow baBba \Rightarrow bababa$$

7. (6 pts) Which of the following regular expressions defines a language that contains the string *abab*, but does not contain the string *baba*?

☐ $(a \mid b)^*$

☐ $(a \mid b)^*(b \mid \lambda)$

☐ $(a \mid b)(ba)^*$

☐ $(b^*a)^*(b\lambda)$

8. (6 pts) Which of the regular expressions given below defines a language that **does not** contain the string *baaba* ?

☐ $(a|b)^*$

☐ $(aa|b)^*$

☐ $(ab|ba)^*a^*$

☐ $(a|b)(a|ab)^*(ab|\varepsilon)$

8. (6 pts) Which one of the regular expressions given below defines a language that **does not** contain the string *abbab*?

☐ $(a|b)^*$

☐ $(a|b)(b|ba)^*(ab|\varepsilon)$

☐ $(a|b)^*(a|bb)$

☐ $b^*(ab|ba)^*b^*$

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☐ $(a \mid b)^*(b \mid \lambda)$

☐ $(a \mid b)(ba)^*$

☑ $(b^*a)^*(b\lambda)$

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A $(a|b)^*$

☑ $(aa|b)^*$

C $(ab|ba)^*a^*$

D $(a|b)(a|ab)^*(ab|\varepsilon)$

8. (6 pts) Which one of the regular expressions given below defines a language that **does not** contain the string *abbab*?

A $(a|b)^*$

B $(a|b)(b|ba)^*(ab|\varepsilon)$

☑ $(a|b)^*(a|bb)$

D $b^*(ab|ba)^*b^*$

7. (6 pts) Consider the following six strings:

$w_1 = \lambda$ $\quad$ $w_4 = baba$
$w_2 = aaa$ $\quad$ $w_5 = baaba$
$w_3 = bbb$ $\quad$ $w_6 = bbababaa$

Which of those strings belong to the language defined by the regular expression $(a^*b)^*(\lambda \mid aa)$?

☐ w_2, w_3, w_6

☐ w_1, w_3, w_6

☐ w_1, w_4, w_5

☐ w_1, w_5, w_6

8. (6 pts) Consider the following six strings:

$w_1 = 0101$ $\quad$ $w_4 = \lambda$
$w_2 = 000$ $\quad$ $w_5 = 01001$
$w_3 = 111$ $\quad$ $w_6 = 00101011$

Which of those strings belong to the language defined by the regular expression $(\lambda \mid 00)(10^*)^*$?

☐ w_1, w_2, w_5

☐ w_1, w_3, w_5

☐ w_2, w_4, w_6

☐ w_3, w_4, w_6

8. (6 pts) Consider the following six

$w_1 = \varepsilon$
$w_2 = baba$
$w_3 = aaa$

Which of those strings belong t
$(a^*b)^*(\varepsilon \mid aa)$?

☐ w_2, w_4, w_6

☐ w_1, w_5, w_6

☐ w_2, w_3, w_4

☐ w_1, w_3, w_5

8. (6 pts) Consider the following six strings:

$w_1 = \varepsilon$
$w_2 = a$
$w_3 = ba$

Which of those strings belong to the language
$(ba \mid \epsilon)(a^*b)^*$?

[A] w_2, w_4, w_6

[B] w_1, w_5, w_6

[C] w_2, w_3, w_4

☑ w_1, w_3, w_5

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$w_2 = aaa$ $\quad$ $w_5 = baaba$
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Which of those strings belong to the language defined by the regular expression $(a^*b)^*(\lambda \mid aa)$?

[A] w_2, w_3, w_6

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[C] w_1, w_4, w_5

[D] w_1, w_5, w_6

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[B] w_1, w_3, w_5

[C] w_2, w_4, w_6

8. (6 pts) Consider the following six strings:

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$w_2 = baba$ $\quad$ $w_5 = bbababaa$
$w_3 = aaa$ $\quad$ $w_6 = bbb$

Which of those strings belong to the language defined by the regular expression $(a^*b)^*(\varepsilon \mid aa)$?

[A] w_2, w_4, w_6

☑ w_1, w_5, w_6

[C] w_2, w_3, w_4

[D] w_1, w_3, w_5

12. (10 pts) Construct a finite-state automaton A with input alphabet $\{a, b\}$ that recognises the set of strings starting with an even number of 'a's followed by a single 'b'. That is, A must satisfy

$$L(A) = \{a^{2n}b \mid n \geq 0\}$$

Describe the automaton A using a next-state table *or* a transition diagram.

$w_2 = a$ $\qquad$ $w_5 = bbab$

$w_3 = ba$ $\qquad$ $w_6 = baba$

Which of those strings belong to the language defined by the regular expression $(ba \mid \epsilon)(a^*b)^*$?

☐ w_2, w_4, w_6

☐ w_1, w_5, w_6

12. (10 pts) Construct a finite-state automaton A with input alphabet $\{0, 1\}$ that recognises the set of all strings with an even number of '1's and exactly one '0'. That is, A must satisfy

$$L(A) = \{w \in \{0,1\}^* \mid |w|_0 = 1 \text{ and } |w|_1 \text{ even}\}$$

Describe the automaton A using a next-state table *or* a transition diagram. Make sure your automaton is deterministic, i.e. each state has exactly one transition for each symbol of the input alphabet.

13. (7 pts) Construct a finite-state automaton A with input alphabet $\{a, b\}$ that recognises the set of all strings with at most 2 occurrences of b. That is, A must satisfy

$$L(A) = \{w \in \{a,b\}^* \mid |w|_b \leq 2\}$$

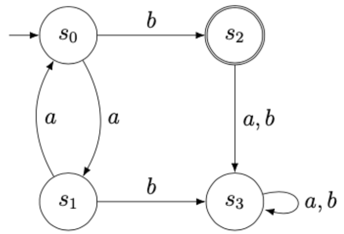
Describe the automaton A using a next-state table *or* a transition diagram.

12. (10 pts) Construct a finite-state automaton A with input alphabet $\{a, b\}$ that recognises the set of strings starting with an even number of 'a's followed by a single 'b'. That is, A must satisfy

$$L(A) = \{a^{2n}b \mid n \geq 0\}$$

Describe the automaton A using a next-state table *or* a transition diagram.

Solution:



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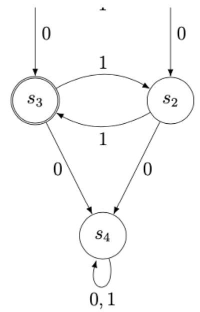
Solution:



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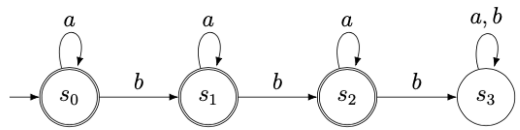

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Describe the automaton A using a next-state table *or* a transition diagram. Make sure your automaton is deterministic, i.e. each state has exactly one transition for each symbol of the input alphabet.

13. (10 pts) Construct a finite-state automaton A with input alphabet $\{a, b\}$ that recognises the set of all strings that have an even number of occurrences of a and at least one occurrence of b. That is, A must satisfy

$$L(A) = \{w \mid |w|_a \text{ even} \wedge |w|_b \geq 1\}$$

Describe the automaton A using a next-state table *or* a transition diagram.

12. (10 pts) Construct a finite-state automaton A with input alphabet $\{0, 1\}$ that recognises the set of all strings with an even number of '1's and exactly one '0'. That is, A must satisfy

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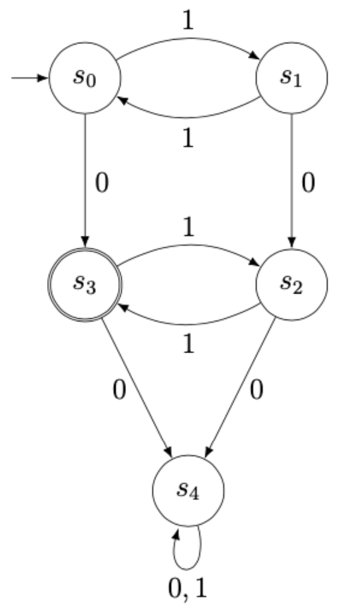

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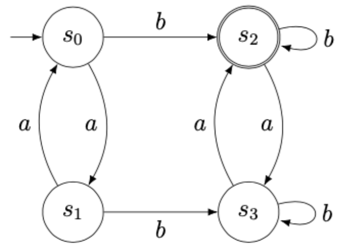

13. (10 pts) Construct a finite-state automaton A with input alphabet $\{a, b\}$ that recognises the set of all strings that start with a and end with ab. That is, A must satisfy

$$L(A) = \{aw \mid w \in \{a, b\}^*\} \cap \{wab \mid w \in \{a, b\}^*\}$$

For example, A should accept the strings ab, aab, and $abab$.
Describe the automaton A using a next-state table *or* a transition diagram.

4. (7 pts) Construct a finite-state automaton A with input alphabet $\{a, b\}$ that accepts the set of all strings that end with ab. That is, A must satisfy

$$L(A) = \{wab \mid w \in \{a, b\}^*\}$$

Describe the automaton A using a next-state table *or* a transition diagram.

14. (8 pts) Construct a finite-state automaton A with input alphabet $\{a, b\}$ that accepts the set of all strings that **do not** end with aa. That is, A must satisfy

$$L(A) = \{a, b\}^* - \{waa \mid w \in \{a, b\}^*\}$$

Describe the automaton A using a next-state table *or* a transition diagram.

13. (10 pts) Construct a finite-state automaton A with input alphabet $\{a, b\}$ that recognises the set of all strings that start with a and end with ab. That is, A must satisfy

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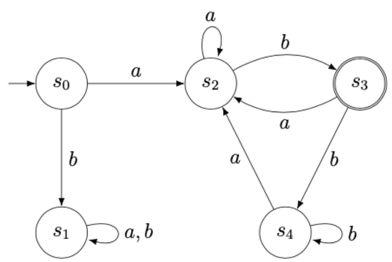
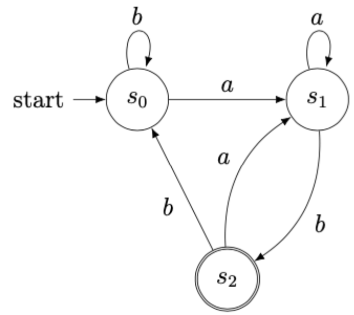


4. (7 pts) Construct a finite-state automaton A with input alphabet $\{a, b\}$ that accepts the set of all strings that end with ab. That is, A must satisfy

$$L(A) = \{wab \mid w \in \{a, b\}^*\}$$

Describe the automaton A using a next-state table *or* a transition diagram.

Solution:

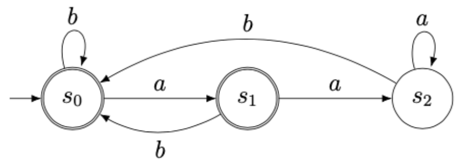


14. (8 pts) Construct a finite-state automaton A with input alphabet $\{a, b\}$ that accepts the set of all strings that **do not** end with aa. That is, A must satisfy

$$L(A) = \{a, b\}^* - \{waa \mid w \in \{a, b\}^*\}$$

Describe the automaton A using a next-state table *or* a transition diagram.

Solution:



14. (8 pts) Construct a finite-state automaton A with input alphabet $\{a, b\}$ that accepts the set of all strings that start with a and end with b. That is, A must satisfy

$$L(A) = \{awb \mid w \in \{a, b\}^*\}$$

Describe the automaton A using a next-state table *or* a transition diagram.

14. (4 pts) Make a deterministic finite state automaton to recognise the regular set denoted by the regular expression:

$$(0 \cup 1)^* 1 (0 \cup 1)^* 1 (0 \cup 1)^*$$

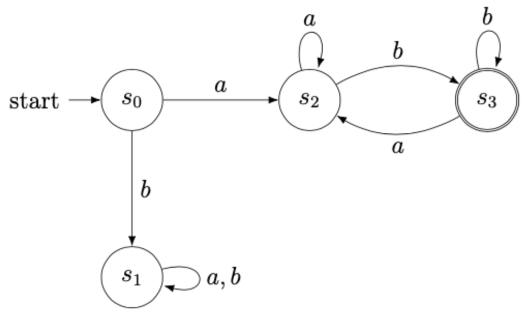
Be sure to specify the entire machine formally.

14. (8 pts) Construct a finite-state automaton A with input alphabet $\{a, b\}$ that accepts the set of all strings that start with a and end with b. That is, A must satisfy

$$L(A) = \{awb \mid w \in \{a, b\}^*\}$$

Describe the automaton A using a next-state table *or* a transition diagram.

Solution:



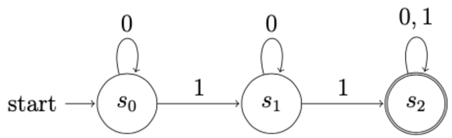
14. Make a deterministic finite state automaton to recognise the regular set denoted by the regular expression:

$$(0 \cup 1)^* 1 (0 \cup 1)^* 1 (0 \cup 1)^*$$

Be sure to specify the entire machine formally.

Solution: A picture of an automaton or a table and set specification are two possible solutions. Please note that these are not the only possible solutions.

Possibility 1:



Possibility 2: $M = (S, I, f, s_0, F)$, where

- $S = (s_0, s_1, s_2)$,
- $I = \{0, 1\}$,
- $f : S \times I \to S$ is given by

	Input	
state	0	1
s_0	s_0	s_1
s_1	s_1	s_2
s_2	s_2	s_2

- $F = \{s_2\}$

13. (12 pts) In the following you are asked to construct grammars that generate languages over the alphabet $\{a, b\}$. Remember to give all components of the grammar: its terminal symbols, non-terminal symbols, the starting symbol and the productions of the grammar.

(a) Construct a grammar that generates the language $\{a^{2n}b^n \mid n \geq 0\}$.

(b) Construct a grammar that generates the language $\{a^n b^m \mid m \geq n \text{ and } n \geq 0\}$.

10. (2 pts) Consider the grammar $G = (V, T, S, P)$, where P is the set of productions

$$P = \{S \to AB, B \to cBd, A \to aAb, A \to e, B \to e\}.$$

Which of the following statements is *true*?

☐ The vocabulary is $V = \{a, b, c, d, e\}$

☐ The language generated by G is $\{a^n e b^n c^m e d^m \mid n \geq 0, m \geq 0\}$

☐ The language generated by G cannot be recognised by a Turing machine

☐ The grammar G is right-linear (regular)

7. (6 pts) Below you are given four languages described using Kleene closure (*), concatenation and union. Which of these languages contains the string 1010, but does not contain the string 10101?

☐ $\{101\}^* \cup \{0\}^*$

☐ $\{10\}^* \{1, 0\}^*$

☐ $(\{1\}^* \{0\})^* \{\lambda, 1\}$

☐ $(\{1\}^* \{10\})^*$

13. (12 pts) In the following you are asked to construct grammars that generate languages over the alphabet $\{a, b\}$. Remember to give all components of the grammar: its terminal symbols, non-terminal symbols, the starting symbol and the productions of the grammar.

(a) Construct a grammar that generates the language $\{a^{2n}b^n \mid n \geq 0\}$.

Solution: (V, T, S, P) where

- $V = \{a, b, S\}$
- $T = \{a, b\}$
- $P = \{S \to \lambda, S \to aaSb\}$

(b) Construct a grammar that generates the language $\{a^n b^m \mid m \geq n \text{ and } n \geq 0\}$.

Solution: (V, T, S, P) where

- $V = \{a, b, S\}$
- $T = \{a, b\}$
- $P = \{S \to Sb, S \to aSb, S \to \lambda\}$

10. Consider the grammar $G = (V, T, S, P)$, where P is the set of productions

$$P = \{S \to AB, B \to cBd, A \to aAb, A \to e, B \to e\}.$$

Which of the following statements is *true*?

[A] The vocabulary is $V = \{a, b, c, d, e\}$

☑ The language generated by G is $\{a^n e b^n c^m e d^m \mid n \geq 0, m \geq 0\}$

[C] The language generated by G cannot be recognised by a Turing machine

[D] The grammar G is right-linear (regular)

7. (6 pts) Below you are given four languages described using Kleene closure (*), concatenation and union. Which of these languages contains the string 1010, but does not contain the string 10101?

[A] $\{101\}^* \cup \{0\}^*$

[B] $\{10\}^* \{1, 0\}^*$

[C] $(\{1\}^* \{0\})^* \{\lambda, 1\}$

☑ $(\{1\}^* \{10\})^*$