Counting & Probability (CP)

	Chapter	Lecture	Assignment
Counting & Discrete Probability	9	9, 10	5

6. (6 pts) You have three coins. Only one of them is *fair* and comes up heads 50% of the time, whereas the other two coins are *biased* and each comes up heads 75% of the time. You pick one of the coins at random, flip it, and observe that it comes up heads. What is the probability that the coin that you flipped was a biased coin?

☐ $\frac{3}{4}$

☐ $\frac{3}{7}$

☐ $\frac{4}{7}$

☐ $\frac{2}{3}$

4. (6 pts) A toy factory has two machines A and B. Machine A produces 75% of the factory output and machine B produces 25% of the factory output. 2% of the toys produced by machine A are defective and 6% of those produced by B are defective. If a toy is chosen at random from the factory output and turns out to be defective, what is the likelihood it was produced by machine B.

☐ 70%

☐ 50%

☐ 40%

☐ 22.33%

6. (6 pts) You have three coins. Only one of them is *fair* and comes up heads 50% of the time, whereas the other two coins are *biased* and each comes up heads 75% of the time. You pick one of the coins at random, flip it, and observe that it comes up heads. What is the probability that the coin that you flipped was a biased coin?

☒ $\frac{3}{4}$

B $\frac{3}{7}$

C $\frac{4}{7}$

D $\frac{2}{3}$

Solution: Let H denote the event that the flipped coin comes up heads, and F the event that the flipped coin is fair. Then we know the following probabilities:

$$P(H|F) = \frac{1}{2} \qquad P(F) = \frac{1}{3}$$
$$P(H|F^c) = \frac{3}{4} \qquad P(F^c) = \frac{2}{3}$$

We need to find the probability $P(F^c|H)$. To this end we will use Bayes' theorem:

$$\begin{aligned} P(F^c|H) &= \frac{P(H|F^c)\cdot P(F^c)}{P(H|F^c)\cdot P(F^c) + P(H|F)\cdot P(F)} \\ &= \frac{\frac{3}{4}\cdot\frac{2}{3}}{\frac{3}{4}\cdot\frac{2}{3}+\frac{1}{2}\cdot\frac{1}{3}} \\ &= \frac{3}{4} \end{aligned}$$

4. (6 pts) A toy factory has two machines A and B. Machine A produces 75% of the factory output and machine B produces 25% of the factory output. 2% of the toys produced by machine A are defective and 6% of those produced by B are defective. If a toy is chosen at random from the factory output and turns out to be defective, what is the likelihood it was produced by machine B.

A 70%

☒ 50%

C 40%

D 22.33%

Solution: Let $P(B)$ be the probability that a toy is produced by machine B and $P(A)$ be the probability that a toy is produced by machine A. Let $P(D)$ be the probability that a toy is defective. We want to calculate the conditional probability $P(B|D)$. Bayes' formula say that

$$P(B|D) = \frac{P(D|B)P(B)}{P(D)}$$

But we have

$$P(D) = P(D|A)P(A) + P(D|B)P(B) = 0.02\cdot 0.75 + 0.06\cdot 0.25 = 0.03$$

Now

$$P(B|D) = \frac{0.06\cdot 0.25}{0.03} = \frac{0.015}{0.03} = 0.5$$

4. (6 pts) You have a standard deck of 52 cards: It has 13 cards of each of the four suits ($\heartsuit$,$\spadesuit$,$\clubsuit$,$\diamondsuit$). The 13 cards of each suit have different ranks (A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K)[1]. How many ways are there to have a hand of 5 cards so that 3 cards have the suit $\heartsuit$ and 3 cards have the rank *king*? For example, $A\heartsuit, 2\heartsuit, K\heartsuit, K\diamondsuit, K\clubsuit$ is such a hand. Note that the order of the cards does not matter, i.e. $K\diamondsuit, K\clubsuit, K\heartsuit, A\heartsuit, 2\heartsuit$ is considered to be the same hand as $A\heartsuit, 2\heartsuit, K\heartsuit, K\diamondsuit, K\clubsuit$.

☐ 198

☐ 286

☐ 468

☐ 1144

5. (6 pts) You have three *six-sided* dice. One of them is *loaded*. The chance of rolling a 6 with the loaded die is 25%. The other two dice are *fair*, i.e. each side is equally likely to come up. You pick up one of the three dice at random, roll it, and observe that you rolled a 6. What is the probability that the die that you rolled was the loaded die?

☐ $\frac{1}{2}$

☐ $\frac{3}{5}$

☐ $\frac{2}{3}$

☐ $\frac{3}{7}$

4. (6 pts) You have a standard deck of 52 cards: It has 13 cards of each of the four suits ($\heartsuit$,$\spadesuit$,$\clubsuit$,$\diamondsuit$). The 13 cards of each suit have different ranks (A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K)[1]. How many ways are there to have a hand of 5 cards so that 3 cards have the suit $\heartsuit$ and 3 cards have the rank *king*? For example, $A\heartsuit, 2\heartsuit, K\heartsuit, K\diamondsuit, K\clubsuit$ is such a hand. Note that the order of the cards does not matter, i.e. $K\diamondsuit, K\clubsuit, K\heartsuit, A\heartsuit, 2\heartsuit$ is considered to be the same hand as $A\heartsuit, 2\heartsuit, K\heartsuit, K\diamondsuit, K\clubsuit$.

☒ 198

[B] 286

[C] 468

[D] 1144

Solution: One of the 5 cards must be $K\heartsuit$. Of the other 4 cards, 2 must have suit $\heartsuit$ and 2 must have rank *king*. There are $\binom{12}{2}$ many ways of choosing the $\heartsuit$ cards and $\binom{3}{2}$ many ways of choosing the king cards. By the multiplication rule we thus have the following number of possible choices:

$$\binom{12}{2} \cdot \binom{3}{2} = 198$$

5. (6 pts) You have three *six-sided* dice. One of them is *loaded*. The chance of rolling a 6 with the loaded die is 25%. The other two dice are *fair*, i.e. each side is equally likely to come up. You pick up one of the three dice at random, roll it, and observe that you rolled a 6. What is the probability that the die that you rolled was the loaded die?

[A] $\frac{1}{2}$

[B] $\frac{3}{5}$

[C] $\frac{2}{3}$

☒ $\frac{3}{7}$

Solution: Let L be the event of rolling the loaded die, and X be the event of rolling a 6. We need to calculate $P(L|X)$. According to the text we have the following probabilities: $P(X|L) = \frac{1}{4}$, $P(X|L^c) = \frac{1}{6}$, and $P(L) = \frac{1}{3}$. Hence, $P(L^c) = 1 - P(L) = \frac{2}{3}$. According to Bayes' Theorem, we can calculate $P(L|X)$ as follows:

$$P(L|X) = \frac{P(X|L) \cdot P(L)}{P(X|L) \cdot P(L) + P(X|L^c) \cdot P(L^c)} = \frac{\frac{1}{4} \cdot \frac{1}{3}}{\frac{1}{4} \cdot \frac{1}{3} + \frac{1}{6} \cdot \frac{2}{3}} = \frac{3}{7}$$

[1] A = ace, J = jack, Q = queen, K = king

5. (6 pts) The exam for the course *Digital Media* at the *Imaginary Technical University* is structured as follows: It consists of 7 multiple choice questions and 3 open ended questions. The students are asked to choose 5 questions to answer, but they have to at least choose one open ended question. How many different choices are there?

- [ ] 229
- [ ] 230
- [ ] 231
- [ ] 232

6. (6 pts) You have three coins. Only one of them is *fair* and comes up heads 50% of the time, whereas the other two coins are *biased* and each comes up heads 75% of the time. You pick one of the coins at random, flip it, and observe that it comes up heads. What is the probability that the coin that you flipped was a biased coin?

- [ ] $\frac{3}{4}$
- [ ] $\frac{3}{7}$
- [ ] $\frac{4}{7}$
- [ ] $\frac{2}{3}$

5. (6 pts) The exam for the course *Digital Media* at the *Imaginary Technical University* is structured as follows: It consists of 7 multiple choice questions and 3 open ended questions. The students are asked to choose 5 questions to answer, but they have to at least choose one open ended question. How many different choices are there?

A 229

B 230

- [x] 231

D 232

Solution: There are $\binom{10}{5}$ ways to pick 5 arbitrary questions out of the 10 questions. Out of those, $\binom{7}{5}$ only have multiple choice questions. Hence, according to the difference rule (Theorem 7 on page 14) there are

$$\binom{10}{5} - \binom{7}{5} = 252 - 21 = 231$$

different ways to choose 5 questions with at least one open-ended question.

6. (6 pts) You have three coins. Only one of them is *fair* and comes up heads 50% of the time, whereas the other two coins are *biased* and each comes up heads 75% of the time. You pick one of the coins at random, flip it, and observe that it comes up heads. What is the probability that the coin that you flipped was a biased coin?

- [x] $\frac{3}{4}$

B $\frac{3}{7}$

C $\frac{4}{7}$

D $\frac{2}{3}$

Solution: Let H denote the event that the flipped coin comes up heads, and F the event that the flipped coin is fair. Then we know the following probabilities:

$$P(H|F) = \frac{1}{2} \qquad P(F) = \frac{1}{3}$$
$$P(H|F^c) = \frac{3}{4} \qquad P(F^c) = \frac{2}{3}$$

We need to find the probability $P(F^c|H)$. To this end we will use Bayes' theorem:

$$\begin{aligned} P(F^c|H) &= \frac{P(H|F^c) \cdot P(F^c)}{P(H|F^c) \cdot P(F^c) + P(H|F) \cdot P(F)} \\ &= \frac{\frac{3}{4} \cdot \frac{2}{3}}{\frac{3}{4} \cdot \frac{2}{3} + \frac{1}{2} \cdot \frac{1}{3}} \\ &= \frac{3}{4} \end{aligned}$$

4. (6 pts) A train company has found that 10% of their trains are delayed. Furthermore, their statistics show that 60% of the delayed trains travelled during rain, while of the punctual trains (i.e. trains that were on time) only 20% travelled during rain. Given that it is raining, what is the probability that a train is delayed?

☐ 25%

☐ 30%

☐ 60%

☐ 75%

5. (6 pts) A team of seven software developers have to divide a set of roles among them. They need three *backend programmers*, two *testers*, and two *UI programmers*. One of the members of the team does not have the experience to be a backend programmer, but otherwise all other assignments of roles are allowed. In how many ways can they constitute their team?

☐ 720

☐ 120

☐ 100

☐ 54

4. (6 pts) A train company has found that 10% of their trains are delayed. Furthermore, their statistics show that 60% of the delayed trains travelled during rain, while of the punctual trains (i.e. trains that were on time) only 20% travelled during rain. Given that it is raining, what is the probability that a train is delayed?

☒ 25%

B 30%

C 60%

D 75%

Solution: Let D be the event that a given train is delayed and R the event that a given train travels during rain. We are given that $P(D) = 0.1$, $P(R|D) = 0.6$, and $P(R|D^c) = 0.2$. We need to calculate the conditional probability $P(D|R)$. To this end, we apply Bayes' Theorem:

$$P(D|R) = \frac{P(R|D) \cdot P(D)}{P(R|D) \cdot P(D) + P(R|D^c) \cdot P(D^c)}$$

$$\frac{0.6 \cdot 0.1}{0.6 \cdot 0.1 + 0.2 \cdot 0.9}$$

$$= 0.25$$

Here we also used Theorem 8 to calculate $P(D^c)$ as follows:

$$P(D^c) = 1 - P(D) = 1 - 0.1 = 0.9$$

5. (6 pts) A team of seven software developers have to divide a set of roles among them. They need three *backend programmers*, two *testers*, and two *UI programmers*. One of the members of the team does not have the experience to be a backend programmer, but otherwise all other assignments of roles are allowed. In how many ways can they constitute their team?

A 720

☒ 120

C 100

D 54

Solution: There are $\binom{6}{3}$ ways of choosing the three backend programmers. After we choose the 3 backend programmers there are 4 people left, so there are $\binom{4}{2}$ ways of choosing 2 testers. The remaining two people are the UI programmers. Hence, there are $\binom{6}{3} \cdot \binom{4}{2} = 120$ different ways to constitute the team.

4. (6 pts) You play the following game using a fair coin (i.e. heads and tails are equally likely): You toss the coin four times. You win the game if among the four outcomes you obtain at least one heads and at least one tails. What is the probability of you winning this game?

☐ $\frac{1}{2}$

☐ $\frac{3}{4}$

☐ $\frac{7}{8}$

☐ $\frac{15}{16}$

5. (6 pts) A TV production company is making a cooking show where three teams compete against each other. Each team consists of two people: one professional chef and one celebrity. For the first episode of the show, the producers have to choose among 5 professional chefs and 6 celebrities to form the three teams. How many different casts (i.e. combinations of teams) can the producers choose between?

☐ 200

☐ 600

☐ 1200

☐ 7200

4. (6 pts) You play the following game using a fair coin (i.e. heads and tails are equally likely): You toss the coin four times. You win the game if among the four outcomes you obtain at least one heads and at least one tails. What is the probability of you winning this game?

[A] $\frac{1}{2}$

[B] $\frac{3}{4}$

☑ $\frac{7}{8}$

[D] $\frac{15}{16}$

Solution: The question can be rephrased as follows: What is the probability that not all four coin tosses come up identical (i.e. four heads or four tails). Let A be the event that all four coin tosses come up identical. Then

$$P(A) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$$

After we throw the first coin, the remaining three coin tosses have to come up like the first. The probability for each of those individual coin tosses is $\frac{1}{2}$. We then use the multiplication rules to obtain the probability that all three coin tosses come up like the first.

Finally, the probability we are interested in is $P(A^c)$, which we calculate using Theorem 8:

$$P(A^c) = 1 - P(A) = 1 - \frac{1}{8} = \frac{7}{8}$$

5. (6 pts) A TV production company is making a cooking show where three teams compete against each other. Each team consists of two people: one professional chef and one celebrity. For the first episode of the show, the producers have to choose among 5 professional chefs and 6 celebrities to form the three teams. How many different casts (i.e. combinations of teams) can the producers choose between?

[A] 200

[B] 600

☑ 1200

[D] 7200

Solution: First the producers choose three chefs, for which there are $\binom{5}{3}$ different choices. Then the producers pair the chefs with 3 celebrities. This can be seen as an r-combination with $r = 3$, i.e. there are $P(6,3)$ different choices. Using the multiplication rule we obtain:

$$\binom{5}{3} \cdot P(6,3) = 10 * 120 = 1200$$

Alternatively, the producers first choose the 3 celebrities and then the 3 chefs to pair with the celebrities:

$$\binom{6}{3} \cdot P(5,3) = 20 * 60 = 1200$$

5. (6 pts) You decided that you want to read 5 books over the summer vacation. So you ask your friends for recommendations on what books to read. In total you have received recommendations for 11 different books. You have to pick 5 books out of those 11 books for your summer reading. However, 3 books out of the 11 recommended books are part of a trilogy. So you only want to pick all 3 books of the trilogy or none of them. Given this restriction, how many different choices do you have to pick your 5 books?

- [ ] 28
- [ ] 56
- [ ] 84
- [ ] 162

6. (6 pts) A toy factory has two machines: A and B. Machine A produces 75% of the factory output and machine B produces 25% of the factory output. 1% of the toys produced by machine A are defective and 2% of those produced by B are defective.
If a toy is chosen at random from the factory output and turns out to be defective, what is the likelihood it was produced by machine B.

- [ ] 30%
- [ ] 33%
- [ ] 40%
- [ ] 50%

5. (6 pts) You decided that you want to read 5 books over the summer vacation. So you ask your friends for recommendations on what books to read. In total you have received recommendations for 11 different books. You have to pick 5 books out of those 11 books for your summer reading. However, 3 books out of the 11 recommended books are part of a trilogy. So you only want to pick all 3 books of the trilogy or none of them. Given this restriction, how many different choices do you have to pick your 5 books?

- A 28
- B 56
- [x] 84
- D 162

Solution: This can be broken down into a two step process: First decide whether to pick the trilogy and then pick the remaining books from the 8 books that are not part of the trilogy.
If you pick the trilogy, you can pick 2 more books from the other 8 books. There are $\binom{8}{2} = 28$ different ways of choosing these two books. If you do not pick the trilogy, you only pick 5 books of the 8 books that are not part of the trilogy. There are $\binom{8}{5} = 56$ different ways of choosing these two books. So, in total, you have $28 + 56 = 84$ ways off picking those 5 books.

6. (6 pts) A toy factory has two machines: A and B. Machine A produces 75% of the factory output and machine B produces 25% of the factory output. 1% of the toys produced by machine A are defective and 2% of those produced by B are defective.
If a toy is chosen at random from the factory output and turns out to be defective, what is the likelihood it was produced by machine B.

- A 30%
- B 33%
- [x] 40%
- D 50%

Solution: Let A be the event that the toy was produced by machine A, B the event that the toy was produced by machine B, and D the event the toy is defect. Then we have

$$
\begin{aligned}
P(A) &= 0.75 \qquad & P(D|A) &= 0.01 \\
P(B) &= 0.25 \qquad & P(D|B) &= 0.02
\end{aligned}
$$

We need to calculate $P(B|D)$, which according to Bayes' Theorem can be done as follows:

$$P(B|D) = \frac{P(D|B)P(B)}{P(D|A)P(A) + P(D|B)P(B)} = 0.4$$

4. (6 pts) You have three *six-sided* dice. One of them is *loaded*. The chance of rolling a 6 with the loaded die is 50%. The other two dice are *fair*, i.e. each side is equally likely to come up. You pick up one of the three dice at random, roll it, and observe that you rolled a 6. What is the probability that the die that you rolled was the loaded die?

- ☐ $\frac{1}{2}$
- ☐ $\frac{3}{5}$
- ☐ $\frac{2}{3}$
- ☐ $\frac{3}{4}$

5. (6 pts) Anna, Bjørn, Camilla, David and Emily participate in a 5×5km relay race. They need to decide in which order they will run, i.e. who will go first, second, third, forth and fifth. However, Anna does not want to go first unless Bjørn will run right after her. For example, the order ABDCE (Anna goes first and Bjørn second) and the order EACDB (Anna does not go first) is allowed, but the order ADBCE (Anna goes first, but Bjørn does not go second) is not allowed. How many different choices do the five friends have if they want to accommodate Anna's preference?

- ☐ 72
- ☐ 96
- ☐ 102
- ☐ 120

4. (6 pts) You have three *six-sided* dice. One of them is *loaded*. The chance of rolling a 6 with the loaded die is 50%. The other two dice are *fair*, i.e. each side is equally likely to come up. You pick up one of the three dice at random, roll it, and observe that you rolled a 6. What is the probability that the die that you rolled was the loaded die?

- [A] $\frac{1}{2}$
- ☑ $\frac{3}{5}$
- [C] $\frac{2}{3}$
- [D] $\frac{3}{4}$

Solution: Let L be the event of rolling the loaded die, and X be the event of rolling a 6. We need to calculate $P(L|X)$. According to the text we have the following probabilities: $P(X|L) = \frac{1}{2}$, $P(X|L^c) = \frac{1}{6}$, and $P(L) = \frac{1}{3}$. Hence, $P(L^c) = 1 - P(L) = \frac{2}{3}$. According to Bayes' Theorem, we can calculate $P(L|X)$ as follows:

$$P(L|X) = \frac{P(X|L) \cdot P(L)}{P(X|L) \cdot P(L) + P(X|L^c) \cdot P(L^c)} = \frac{\frac{1}{2} \cdot \frac{1}{3}}{\frac{1}{2} \cdot \frac{1}{3} + \frac{1}{6} \cdot \frac{2}{3}} = \frac{3}{5}$$

5. (6 pts) Anna, Bjørn, Camilla, David and Emily participate in a 5×5km relay race. They need to decide in which order they will run, i.e. who will go first, second, third, forth and fifth. However, Anna does not want to go first unless Bjørn will run right after her. For example, the order ABDCE (Anna goes first and Bjørn second) and the order EACDB (Anna does not go first) is allowed, but the order ADBCE (Anna goes first, but Bjørn does not go second) is not allowed. How many different choices do the five friends have if they want to accommodate Anna's preference?

- [A] 72
- [B] 96
- ☑ 102
- [D] 120

Solution: There are $P(5) = 5! = 120$ ways of ordering the five friends. However, we have to subtract the orderings that have Anna first, but someone other than Bjørn second. By the multiplication rule there are $3 \cdot 3! = 18$ such orderings (pick Anna first, then pick from the remaining friends but not Bjørn, then order the remaining three friends). Hence, there are $120 - 18 - 102$ ways to order the five friends.
Alternative solution: First, we count how many orderings there are that start with Anna and Bjørn. There are $3! = 6$. Then we count how many orders there are that have someone other than Anna go first. There are $4 \cdot 4! = 96$. Hence, there are $6 + 96 = 102$ possible orderings in total.

6. (2 pts) Assume that ITU user names must consist of either four or five characters, where each character is any of the 26 characters from `a` to `z`. To make user names easier to remember ITU is considering to introduce a policy where at least two of the characters in the user name must be identical:

- Examples of allowed user names are `azbz`, `azzz`, `zazz`, `zzzz`, `zazbz` and `zzzzz`.
- Examples of disallowed user names are `abcd` and `cegjk`.

What is the total number of allowed user names according to this policy?

☐ $2\,490\,519 = \frac{26^4}{4} + \frac{26^5}{5}$

☐ $80\,730 = \binom{26}{4} + \binom{26}{5}$

☐ $8\,252\,400 = 26 \cdot 25 \cdot 24 \cdot 23 + 26 \cdot 25 \cdot 24 \cdot 23 \cdot 22$

☐ $4\,085\,952 = 26^4 - 26 \cdot 25 \cdot 24 \cdot 23 + 26^5 - 26 \cdot 25 \cdot 24 \cdot 23 \cdot 22$

7. (2 pts) Suppose that the probability of snow in Denmark is 10%. If it is snowing then 60% of all trains will be delayed. Otherwise, if it is not snowing then only 20% of the trains will be delayed. Given that a train is delayed, what is the probability of it is snowing?

☐ 25%

☐ 30%

☐ 60%

☐ 75%

6. Assume that ITU user names must consist of either four or five characters, where each character is any of the 26 characters from `a` to `z`. To make user names easier to remember ITU is considering to introduce a policy where at least two of the characters in the user name must be identical:

- Examples of allowed user names are `azbz`, `azzz`, `zazz`, `zzzz`, `zazbz` and `zzzzz`.
- Examples of disallowed user names are `abcd` and `cegjk`.

What is the total number of allowed user names according to this policy?

[A] $2\,490\,519 = \frac{26^4}{4} + \frac{26^5}{5}$

[B] $80\,730 = \binom{26}{4} + \binom{26}{5}$

[C] $8\,252\,400 = 26 \cdot 25 \cdot 24 \cdot 23 + 26 \cdot 25 \cdot 24 \cdot 23 \cdot 22$

☑ $4\,085\,952 = 26^4 - 26 \cdot 25 \cdot 24 \cdot 23 + 26^5 - 26 \cdot 25 \cdot 24 \cdot 23 \cdot 22$

7. Suppose that the probability of snow in Denmark is 10%. If it is snowing then 60% of all trains will be delayed. Otherwise, if it is not snowing then only 20% of the trains will be delayed. Given that a train is delayed, what is the probability of it is snowing?

☑ 25%

[B] 30%

[C] 60%

[D] 75%

5. (6 pts) You throw two fair six-sided dice without looking. Given that one of them cam up 6 what is the probability that the other one also came up 6?

- [ ] $\frac{1}{9}$
- [ ] $\frac{1}{10}$
- [ ] $\frac{1}{11}$
- [ ] $\frac{1}{12}$

5. (6 pts) You throw two fair six-sided dice without looking. Given that one of them came up 6 what is the probability that the other one also came up 6?

[A] $\frac{1}{9}$

[B] $\frac{1}{10}$

- [x] $\frac{1}{11}$

[D] $\frac{1}{12}$

Solution: Let A be the event of getting two sixes when rolling two dice and let B be the event of getting at least one six when rolling two dice. We need to calculate the conditional probability $P(A|B)$. By the definition of conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Moreover, $A \cap B = A$ since $A \subseteq B$. To calculate $P(A \cap B)$ and $P(B)$ we use the probability of rolling a six with a single die. Let S be the event of rolling a six with a single die. $P(S) = \frac{1}{6}$. By the multiplication rules we have that

$$P(A \cap B) = P(A) = P(S) \cdot P(S) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

In order to get at least one six with two dice, one either needs to roll a six with the first die, or roll something other than a six with the first and then a six with the second. Hence,

$$\begin{aligned} P(B) &= P(S) + P(S^C) \cdot P(S) \\ &= P(S) + (1 - P(S)) \cdot P(S) \\ &= \frac{1}{6} + \frac{5}{6} \cdot \frac{1}{6} \\ &= \frac{11}{36} \end{aligned}$$

Hence,

$$P(A|B) = \frac{\frac{1}{36}}{\frac{11}{36}} = \frac{1 \cdot 36}{36 \cdot 11} = \frac{1}{11}$$

Alternative solution: Let A be the event that the first die shows a six and B the event that the second die shows a six, i.e. $A = \{(6, n) \mid 1 \leq n \leq 6\}$ and $B = \{(n, 6) \mid 1 \leq n \leq 6\}$. Our sample space is $A \cup B$ and we need to calculate the probability of the event $A \cap B$. Since all outcomes are equally likely, we can calculate the probability as follows:

$$P(A \cap B) = \frac{N(A \cap B)}{N(A \cup B)}$$

Clearly, $A \cap B = \{(6, 6)\}$. Moreover we may use the fact that

$$N(A \cup B) = N(A) + N(B) - N(A \cap B)$$

Since $N(A) = N(B) = 6$ and $N(A \cap B) = 1$, we have that

$$P(A \cap B) = \frac{1}{6 + 6 - 1} = \frac{1}{11}$$