Stochastic Mixed-Integer Programming for Integrated Portfolio Planning in the LNG Supply Chain

Adrian Werner, Kristin Tolstad Uggen**, Marte Fodstad***, Arnt-Gunnar Lium****, and Ruud Egging******

ABSTRACT

We present a new model to support strategic planning by actors in the liquefied natural gas market. The model takes an integrated portfolio perspective and addresses uncertainty in future prices. Decision variables include investments and disinvestments in infrastructure and vessels, chartering of vessels, the timing of contracts, and spot market trades. The model accounts for various contract types and vessels, and it addresses losses. The underlying mathematical model is a multistage stochastic mixed-integer linear problem. Industry-motivated numerical cases are discussed as benchmarks for the potential increases in profits that can be obtained by using the model for decision support. These examples illustrate how a portfolio perspective leads to decisions different than those obtained using the traditional net present value approach. We show how explicitly considering uncertainty affects investment and contracting decisions, leading to higher profits and better utilization of capacity. In addition, model run times are competitive with current business practices of manual planning.

Keywords: Liquefied natural gas supply chain, Decision support system, Strategic planning, Stochastic mixed-integer linear programming

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1. INTRODUCTION

Stochastic mixed-integer programming has been used to examine a number of issues in energy economics, including investment planning in the natural gas industry (Guldmann and Wang 1999, Zheng and Pardalos 2010). The majority of these applications take a cost-minimization approach. In this paper we take the perspective of firms that maximize expected profits. In doing so, we look at the whole value chain in a portfolio perspective. We consider contracting decisions,

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while allowing for arbitrage trades in the spot market, whereby physical shipments can be re-directed to take advantage of geographical price differentials. We illustrate this using a price uncertainty example inspired by a real-life application. In fact, recent developments in the global natural gas markets have made our modeling innovations particularly important.

The increasing importance of liquefied natural gas (LNG) is illustrated by rapid growth of the industry in recent years. In 2000, twelve LNG exporters traded 220 million m^3 to ten LNG importing countries (BP 2001). By 2010, this figure had more than doubled, with 18 LNG exporting countries trading 483 million m^3 to 23 LNG importers (BP 2011, GIIGNL 2010).

The LNG supply chain typically includes production of natural gas, transportation to a liquefaction terminal, the liquefaction process and loading of vessels, shipping of the LNG to regasification terminals, and regasification to natural gas for distribution through the pipeline grid. Figure 1 in Section 3.2 provides an overview of the supply chain elements considered in our model, from liquefaction to transportation, and regasification to markets for natural gas. For a thorough description of this chain, we refer to Fodstad et al. (2010).

Natural gas markets are dynamic and unpredictable both in the long and short term. The US Energy Information Administration (EIA) observed retrospectively, in yearly energy outlooks, that deviations between projections and market outcomes (both volumes and prices) were larger for natural gas than for all other fuels (EIA 2010). Despite significant uncertainty, lower operating and shipping costs and an increasing LNG market liquidity has induced a shift away from risk-reducing long-term contracts over the last decade. Due to a tenfold increase, spot and short-term trade accounted for 25% of total LNG trade in 2011 (GIIGNL 2011). Currently, an increasing share of short-term contracts and cargo re-routing is used to benefit from arbitrage opportunities in spot markets. These developments make it more difficult to devise profitable, yet flexible long-term strategies. It is, therefore, paramount to address uncertainty adequately when developing models to support investment and contract timing decisions.

The main contribution of the LNGPlanner model is to provide a tool to perform integrated analysis of an industry actor's portfolio of both existing and potential investments along the LNG supply chain, which accommodates price uncertainty. It focuses on both physical and economic aspects, allowing for exploitation of flexibility in the supply chain to benefit from market opportunities while meeting operational criteria. The mathematical model forms a multistage stochastic mixed-integer linear programming (SMILP) problem, accommodating uncertainty through a scenario tree approach. The model has been developed for, and in close collaboration with, several partners from the LNG industry to provide decision support for some of their strategic decisions.

2. LITERATURE REVIEW

There is an extensive base of literature on optimal[1] investment strategies; however, integrated approaches for the LNG business that take uncertainty into account are underrepresented. Investment models such as the one in André (2010) tend to focus on deterministic cost minimization rather than stochastic profit maximization. In a recent paper, MirHassani and Noori (2011) explicitly

1. Throughout this paper, we use the word optimal in the sense that it is used in Operations Research literature, namely as a solution that maximizes or minimizes a specified function. For an economist, "optimal" often means "Pareto optimal" or "efficient," or at least utility, profit maximizing, cost, or expenditure minimizing. In particular, in the economics literature, maximization or minimization of an intertemporal objective function also entails specifying "transversality conditions" for the capital stocks or their shadow prices that follow from the underlying consumer or producer objectives and constraints. In our model, the continuation values are specified exogenously.

address the drawbacks of the use of scenario analysis assuming perfect foresight for a realistic problem. Birge and Loveaux (1997) showed that stochastic optimization approaches are needed to make optimal decisions and to represent the hedging behavior of investors facing uncertainty.

Alternative means for evaluating investment opportunities are provided by real-options approaches. Murto and Keppo (2002), Klaassen et al. (2004), and Krey and Minullin (2005) emphasized game-theoretic aspects among the investors. Real option approaches for investment decisions in the oil and gas industry can be found in Smith and McCardle (1999), Bøckman et al. (2008), Kaminski et al. (2008), Thompson et al. (2009), and Lai et al. (2011). Such approaches provide useful insight into the timing of investments. However, they have limitations with regard to capturing the interrelations of multiple investment opportunities within the same modeling framework.

The stochastic dynamic programming (SDP) approaches for energy planning problems discussed in Botterud and Korpås (2007), Fleten (2000), and André (2010) potentially allow for the flexibility needed for optimal capacity and timing decisions that affect each other. However, due to the combinatorial characteristic of the problems, heuristics are needed to provide solutions within acceptable time limits. To circumvent these combinatorial challenges, Pereira and Pinto (1991), Granville et al. (2003), Bezerra et al. (2010), and Aouam and Yu (2008) developed the concept of stochastic dual dynamic programs (SDDP). Although some of these concepts and insights can be transferred to other fields, the applicability of the SDDP approach to the problem studied in this paper is limited. SDDP requires a discretization of the potential values for the decision variables while our approach includes continuous variables that may take a large range of values.

Other approaches have also been used for solving energy planning problems. Egging (2010) developed a stochastic mixed complementarity problem addressing optimal capacity expansion by various actors in the global natural gas market. The approach does not allow for integer variables and does not scale well in terms of the number of scenarios that can be accommodated. In a more operational setting, Tomasgard et al. (2007) presented an integrated operational and financial approach to manage and optimize the various elements of the natural gas supply chain from production to sales, taking into account uncertainty in both demand and prices in a two-stage recourse approach. Zheng and Pardalos (2010) propose a SMILP problem for location of LNG terminals and expansion of pipelines. Their model is highly relevant albeit complementary to the work presented here, as they minimize expected costs while we maximize expected net present value. The authors include pipeline expansions and regasification terminals in their model, while our model includes regasification and liquefaction terminals, vessel investments and charter, contract decisions and spot markets, but not pipeline expansions.

Important stepping-stones to the model presented here are papers by Nygreen et al. (1998), Fodstad et al. (2010), and Grønhaug et al. (2010). Nygreen et al. (1998) developed a model for optimally operating and expanding the pipeline network on the Norwegian continental shelf using a project-based approach for timing the start-up of production fields. The work of Fodstad et al. (2010) and Grønhaug et al. (2010) focused on tactical planning in the LNG business including routing of ships, typically within a yearly horizon, while LNGPlanner has a much longer planning horizon (typically 10–25 years).

The remainder of this paper is organized as follows. The next section presents the model. Sections 4 and 5 discuss two test cases, illustrating selected model features. The first case elaborates on uncertainty, hedging, and spot trading, while the second one discusses the added value of the portfolio approach. Section 6 concludes and provides directions for future research. The appendix provides more detailed results for the test cases.

3. THE MODEL

Our stochastic mixed-integer linear programming model covers the supply chain from liquefaction to shipping and regasification to natural gas markets. The objective of the model is to maximize the expected NPV of a company's portfolio of terminals, vessels, and contracts. This chapter gives a verbal description of the model. The corresponding mathematical formulation can be found in Werner et al. (2012).

3.1 Strategic Decisions

The main decisions are investments and disinvestments that design the supply chain. These constitute the integer and binary variables in the model. Investment opportunities are denoted as "projects" and cover liquefaction terminals, regasification terminals, vessels, and contracts. The timing of investments and disinvestments is chosen by the model, but is limited to a given time interval. The decisions typically make some capacity available in the time periods following the decision and generate a series of cash flows. In some situations, a project depends on other projects being started first. For instance, a terminal cannot be built unless the related feasibility study has been completed and the necessary permits are obtained. The model allows for mutually exclusive projects. For instance, it is not possible to choose two different sizes of terminals for a given location.

Vessel projects represent investments in different vessel types with varying capacities and costs. Furthermore, the fleet can be supplemented by chartering vessels. Unused vessels can be chartered out or sold. In contrast to buying and selling vessels, prematurely ending a charter is not possible.

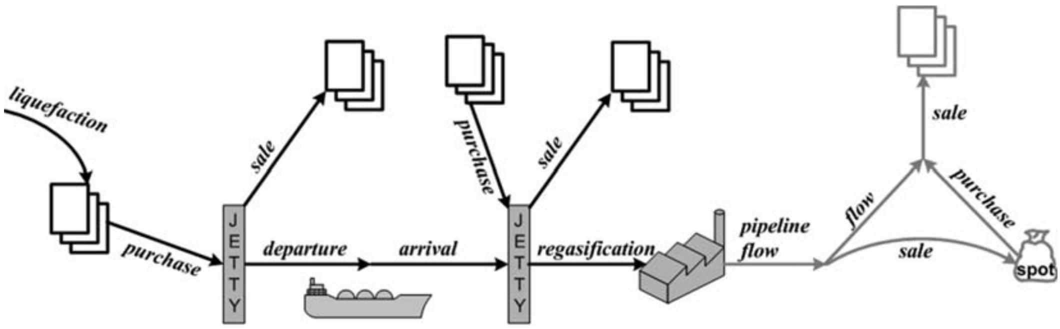
The finite time horizon of a MILP model can often distort decisions close to the end of the horizon. For instance, it is unlikely for this model to invest in new assets in the last part of the time horizon because that would incur investment costs while the model only covers small parts of the operational period of the asset. A common way of handling this issue is to model and solve with a substantially longer time horizon than needed for the analysis. How much longer the time horizon should be depends on both the lifetime of the assets and the discount rate that reduces the importance of distant-future revenues and costs. It should also be kept in mind that for decision support only the first few periods are critical, since the model should be re-optimized as time goes by and new decision points are reached. To alleviate this distortion effect, we include an approximation of the discounted expected remaining value of the assets in the portfolio at the end of the horizon. For terminals, we assume that the remaining value is the investment cost less depreciation. Vessels have a predefined lifetime, and their remaining value is set to the charter rate for the remaining lifetime. Remaining values of contracts are approximated according to market prices. For sales contracts, a contract price increase relative to the market price gives a positive remaining value, and a decrease gives a negative value.

3.2 Operational Decisions

To evaluate operational consequences of strategic decisions, supply chain operations are included in the model. Figure 1 illustrates the operational supply chain as it is modeled; purchase and sales contracts in different parts of the supply chain are represented by stacks of rectangles.

Liquefaction terminals transform natural gas into LNG at a fixed unit cost, at a rate between the terminal's minimum production rate and maximum capacity. The produced volumes should match contractual purchases from the liquefaction terminal. By including both the production rate

Figure 1: Overview of Operational Decisions



and purchase contracts, it is possible to describe liquefaction terminals with different ownerships and levels of control. LNG storage is an important component in the daily operations of an LNG terminal. Typically, storage capacity covers a few days of operations with very limited flexibility to store for later months or peak seasons. Hence, storage has little relevance in a strategic context and is omitted here.

LNG bought at a liquefaction terminal can either be sold under contract at the terminal or shipped by the company's vessels. Vessel routing determines how much transportation capacity a fleet of vessels can give. It is a highly discrete operation that, from an optimization point of view, is known to be combinatorially challenging (Toth and Vigo 2002). To keep the model tractable, a continuous approximation has been chosen, matching transportation demand and total fleet capacity available. The former is determined from the amount of LNG to be transported and the travel time between each pair of terminals while the latter is described by the number of owned and chartered vessels and their capacities, adjusted by a utilization factor reflecting ballast voyages. Additional limitations can be added to specific terminals and vessel types in order to address compatibility issues such as buoy ports requiring vessels with on-board regasification facilities. Some LNG is lost during transportation due to boil-off such that volumes available for regasification are lower than volumes sent out from liquefaction. In short time periods, part of the volumes sent out may arrive at a subsequent period due to long travel times.

LNG arriving at a regasification terminal can either be sold in a contract at the jetty or sent to the regasification facility. LNG can also be bought through a contract with delivery at the jetty. Regasification, converting LNG into natural gas, is subject to lower and upper limits on the production rate and happens at given unit costs. Losses during the process are described as a fraction of the regasification rate. Natural gas is sent to a market hub through pipelines that may also have capacity limitations. Similar to liquefaction terminals, storage facilities at regasification terminals are omitted from the model.

A market hub has a spot market for natural gas trading. The spot price is uncertain and, for a price-taker, independent of the traded volume. Because the real markets are not necessarily well functioning, we include limits on positions that can be taken on the purchases and sales sides.

The model contains both purchase and sale contracts at different places in the supply chain as illustrated in Figure 1. They all have the same structure and the commodity traded is LNG, except for contracts at natural gas hubs, which trade natural gas. All contracts can have uncertain prices. Furthermore, there can be limits on the amounts traded in the contracts. The simplest form of limitation is lower or upper bounds within a time interval. A single contract can have several such limits with overlapping time intervals. Destination and source clauses are limitations linking purchase and sales contracts. A destination clause states a lower or upper limit on the amount that can be delivered from a given purchase contract to a set of sales contracts in a time interval. Similarly, source clauses restrict the sourcing from a set of purchase contracts to a given sales contract.

In the following two sections, we illustrate how our modeling choices with portfolio perspective, stochastic prices, and inclusion of spot markets can affect suggested decisions. Because our data sets are synthetic, our focus is not on the absolute values given by the tests, but rather on how different methods evaluate the projects and portfolios differently.

We implemented the model using off-the-shelf software, more precisely, XPress-MP. Obviously, solution times depend strongly on the size of the solution space (number and range of the variables), the length of the time horizon, and the description of the uncertainty (i.e., the size of the scenario tree). For test cases, such as the ones presented in this paper, solution times were in the range of a few seconds to one minute. For larger realistic cases with up to 1600 scenarios, we experienced solution times of up to one hour. However, while we focused on efficient implementation, solution speed was not the main scope of the development work. Moreover, such investment analyses are not frequently performed and, therefore, speed is not a major issue.

4. TEST CASE 1—VALUING ROBUSTNESS AND FLEXIBILITY

We use this test case to illustrate and discuss two model features, the use of stochastic programming to handle uncertain prices and the use of a supply chain perspective including spot markets where physical supply and trading are integrated.

4.1 Case Set-up

In order to make the discussion easier to follow, we simplify the physical value chain as much as possible. We consider a small system that consists of one liquefaction and one regasification terminal with a related natural gas hub. The existing terminals are assumed to have infinite capacity and there are no terminal investment options. We assume sufficient transportation capacity, no transportation delay, and no natural gas or LNG losses in the supply chain. We also ignore all operational costs. The planning horizon has three periods of equal length and uncertain price information is revealed after each period, resulting in a three-stage problem.

Initially, no contract obligations exist, and all contracts must be sealed one period prior to any delivery. The liquefaction terminal has a single purchase contract option with a constant price of €6/MMBtu and contract limits of [250, 1000] MMBtu in each period. On the downstream side, there is a spot market (S) with an uncertain price that can be either low (€7.95/MMBtu) or high (€9.95/MMBtu) for each time period. Two types of natural gas contracts are also available in the market hub. A long-term contract (LC) can be entered in the first period, with deliveries within the range of [600, 700] MMBtu in the two following periods. Short-term contracts (SC) are available for entry in the two first periods, each with delivery within the range of [500, 525] MMBtu in the next period only. All contract prices are assumed to have a price formula with time lag, such that the price is known one period in advance. Spot and sales contract prices are presented in Figure 2 where all nodes in each time period have equal probability.

4.2 Dealing with Uncertainty

As discussed in Section 2, stochastic programming is often preferred over scenario analysis when uncertainty is an important characteristic of the problem. We investigate how the solutions

Figure 2: Prices in Test Case 1 (€/MMBtu)

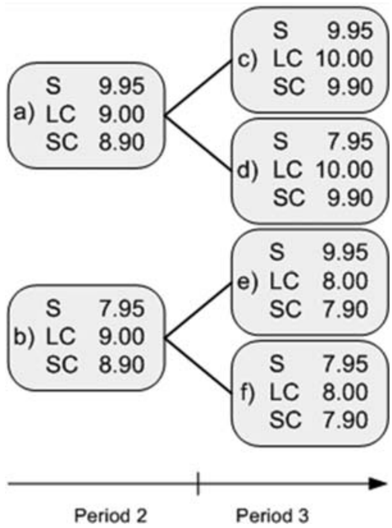


Table 1: Expected Incomes from Spot, Long-term Contract and Short-term Contract, and Profits for Various Planning Approaches (€)*

Planning approach	E[Spot]	E[LC]	E[SC]	E[Profit]	% of STO
STO	5,370	12,600	0	6,150	100.0
DET	5,370	12,600	0	5,970	97.1
Avg DETd	6,116	11,950	0	6,066	98.6
SA_c	19,900	0	0	7,900	128.5
SA_d	13,726	0	5,198	6,924	112.6
SA_e	13,726	0	4,673	6,399	104.1
SA_f	4,770	11,900	0	4,670	75.9
Avg SA	13,031	2,975	2,468	6,473	105.3
EVPI	−2,582	2,975	−69	323	5.3
VSS	9,497	−11,950	2,537	83	1.4

* All approaches have an expected purchase cost of €6,000.

for this test case change with different planning approaches such as stochastic programming (abbreviated to STO in Table 1), scenario analysis (SA_c to SA_f, where the last characters refer to leaf nodes representing scenarios in the scenario tree), and deterministic optimization assuming expected prices (DET). We also include dynamic deterministic optimization as described by Escudero et al. (2007), where the strategy is re-optimized with updated expected prices as time goes by (Avg DETd). This mimics a decision maker who uses a deterministic decision support tool, but re-optimizes his decisions in each period.

Table 1 provides an overview of the objective function values for the different approaches. The bottom rows present the expected values from the scenario analysis (Avg SA), the expected value of perfect information (EVPI), and the value of the stochastic solution (VSS) (Birge and Loveaux 1997). The sale volumes corresponding to the results in this table are given in the appendix.

The table shows that the stochastic solution yields a lower profit than scenario-analysis solutions. This is hardly surprising as the scenario-analysis approach assumes perfect knowledge

Table 2: Contract Commitment Decisions (1 means entering, 0 means not entering the contract)

<table>
<tr><th></th><th colspan="2">1st stage</th><th colspan="2">2nd stage</th></tr>
<tr><th>Solution approach</th><th>LC</th><th>SC_1</th><th>SC_2U</th><th>SC_2L</th></tr>
<tr><td>STO</td><td>0</td><td>0</td><td>1</td><td>0</td></tr>
<tr><td>DET</td><td>1</td><td>0</td><td colspan="2">0</td></tr>
<tr><td>DETd</td><td>1</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>SA_c</td><td>0</td><td>0</td><td>0</td><td>n/a</td></tr>
<tr><td>SA_d</td><td>0</td><td>0</td><td>1</td><td>n/a</td></tr>
<tr><td>SA_e</td><td>0</td><td>1</td><td>n/a</td><td>0</td></tr>
<tr><td>SA_f</td><td>1</td><td>0</td><td>n/a</td><td>0</td></tr>
</table>

about the future and will plan accordingly. That is, for each of the four scenarios, a tailor-made solution is found. It is, however, worth noting that the deterministic solution performs worse than the stochastic solution. This seems somewhat surprising considering that the stochastic solution is not perfectly adapted to any single scenario. The explanation is that the stochastic solution is flexible enough to take advantage of upside variations of the prices while hedging for downside variations. The deterministic solution does not consider such price variations although the periodical contract limits still allow for some flexibility. The dynamic deterministic approach observes price variations as operational decisions are made, but it has limited ability to value future flexibility because future prices are seen as deterministic.

Differences between the strategic decisions are shown in Table 2, where SC 1, SC 2U, and SC 2L refer to short-term contracts in period 1, the upper node in period 2, and the lower node in period 2, respectively. The deterministic model suggests entering the long-term contract, because this is the alternative with the highest price expectation, thereby, locking up more than half of the available gas. The stochastic model, on the other hand, sacrifices revenue from this expected high price, and thereby, keeps the flexibility to choose between spot sales or the short-term contract when the contract prices are revealed in the second period. The dynamic deterministic approach is only partly able to offset the difference between the deterministic and stochastic models by allocating volumes to spot sales or to a long-term contract, depending on prices. This is because the contract obligations limit the volumes that can be sold on spot. The results in Table 2 illustrate a problem that can arise when using scenario analysis. The decisions in the four scenarios are all different, and it is hard to extract a pattern that provides a single optimal decision for the stochastic decision problem.

In our comparison of the stochastic and dynamic deterministic approaches, we evaluated both models with the data from the scenario tree in Table 2. Because the information in this tree was actually available to the stochastic model during optimization, this evaluation favors the stochastic approach. If the actual prices exactly matched the anticipated values, the deterministic model would perform better than the stochastic one. This shows that the performance of different modeling approaches is highly dependent on the prices used for evaluation. Generally, a more balanced evaluation method for the model approaches would be to evaluate the model results on real-world outcomes over several time periods. Alternative evaluation methods use discrete-event simulation or truth trees (for an example, see Lium and Kaut (2006)), both of which rely on the same assumptions of statistical properties for distribution of uncertain parameters as those used when generating the scenario tree. It should be noted, however, that these methods are more challenging in a multi-stage setting than for traditional two-stage problems. Because we study a synthetic case

Table 3: Expected Volumes, Average Prices, and NPV with Different Trading Options

Test	Expected volume (MMBtu)	Expected average price (€/MMBtu)	Expected NPV (€)
Spot	1,000	6.15	6,150
No spot	700	6.00	4,200
Max700	700	6.26	4,380

with a very small scenario tree, neither real outcomes nor evaluations based on the statistical properties in the tree are meaningful. The important aspect is not the quantitative performance of the different models, but rather the different structures of the found decisions. As long as there is uncertainty that can make flexibility valuable, a stochastic model is better suited to determine the most flexible position for realizing this value.

In this test, we used a risk-neutral stochastic model, which prefers to deliver predominantly to the spot market. In real life, many companies are risk averse and would see this strategy as too risky. However, for such companies, stochastic optimization is still a good approach because it can express the value of flexibility in an uncertain environment. Therefore, it allows for a quantified trade-off between risk and profit.

4.3 Trading on the Spot Market

In this section, we illustrate the value of flexibility from the possibility of trading on a spot market as a supplement to contractual deliveries. As a reference, we use the stochastic-programming approach (referred to as Spot) from the previous section. We compare this with a test instance where no spot trade is possible (No spot) and an instance where spot trade is possible but the maximum purchase volume is limited to 700 MMBtu (Max700). The results are summarized in Table 3, while the complete set of results for these tests is presented in Tables 6–8 in the appendix. Removing the opportunity to trade on the spot market forces the stochastic model to enter the long-term contract and deliver at its maximum level in each period, thereby reducing the expected NPV by €1,950. The major difference comes from a reduction from 1,000 MMBtu to 700 MMBtu in the volume delivered, which is caused by a lack of flexibility on the sales side to perfectly balance sales and purchase capacity. If the purchase capacity is restricted to a maximum of 700 MMBtu, the expected NPV is improved by €180 compared to the no-trade situation. Again, the model avoids the long-term contract in order to exploit the price spread between spot and short-term contract sales. This results in an improved average sales price and, thereby, a higher expected profit.

It should be emphasized that not only spot sales, but also spot purchases can add value. If contract obligations can be satisfied by spot purchases rather than own gas, volumes can be redirected to deliveries in other markets or in other time periods where prices are more favorable. Spot purchases enable geographical swaps and time swaps that make it possible to exploit price spreads for arbitrage. A consequence of a geographical swap is that transportation needs can change, which will increase or decrease fleet utilization, transportation costs, and boil-off, depending on the relative distance from the source to each of the alternative markets.

5. TEST CASE 2—HIGHLIGHTING THE INTEGRATED PERSPECTIVE

In this test case, we focus specifically on the added value of the portfolio approach by comparing today's planning praxis with solutions achieved with the LNGPlanner framework.

Figure 3: System Layout in Test Case 2

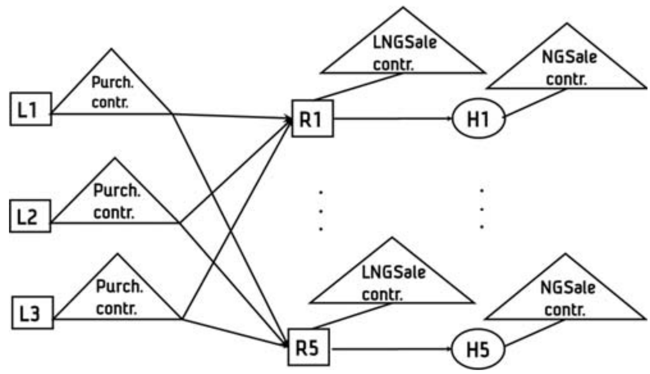


Table 4: Main Characteristics of the Two Investment Options

Investment option	Lifetime (years)	Investment costs (mill. €)	Change in production costs (€/MMBtu)	Aggregated max. production volume (mill. MMBtu)
L1A	15	1,200	0.3 (−40%)	4,260
L1B	10	800	0.6 (+ 20%)	4,230

5.1 Case Set-up

We consider a market with three liquefaction terminals (L1, L2, L3) and five regasification terminals (R1, . . . , R5) over a twenty-period planning horizon where each period is one year. Each regasification terminal is connected to a market hub (H1, . . . , H5), as illustrated in Figure 3. The economic lifetime of terminal L1 ends after period 5. There are two investment options, L1A and L1B, which can extend the terminal's life. These two options differ in lifetime up-front costs and production costs, as shown in Table 4. Although the accumulated production capacities are almost identical, the maximum yearly production capacity of option L1B is approximately 40% higher than that of option L1A.

Terminal L1 is owned by the considered company and has production costs of €0.5/MMBtu. The other two liquefaction terminals are not owned by the company and production costs are internalized in the corresponding contract prices. All regasification terminals have production costs of €0.5/MMBtu.

There are LNG purchase contracts at all liquefaction terminals including extension options, LNG sales supply contracts to all regasification terminals, and natural gas sales contracts at the hubs. The prices and volume limits of these contracts vary with both the location of the respective markets and the time; illustrated in Figures 4 through 7. The plots on the right hand side in Figure 4 show that the purchase contracts at liquefaction terminals allow for ramp-up and -down periods of the corresponding terminal. Natural gas can also be sold spot at the hubs at prices illustrated in Figure 7 with yearly volume limits of 150 MMBtu, 250 MMBtu, 50 MMBtu, 125 MMBtu, and 100 MMBtu, respectively.

Transportation can be carried out using vessels of two sizes: small vessels that have a capacity of 120,000m^3 and an investment cost of €220 million and large vessels that can transport

Figure 4: Prices (€/MMBtu, left) and Volume Limits (mill. MMBtu, right) of LNG Purchase Contracts at Liquefaction Terminals over 20 Time Periods

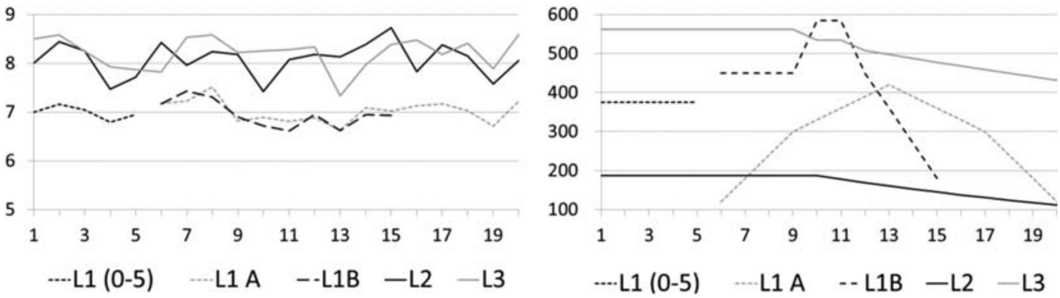


Figure 5: Prices (€/MMBtu, left) and Volume Limits (mill. MMBtu, right) of LNG Sales Contracts to Regasification Terminals over 20 Time Periods

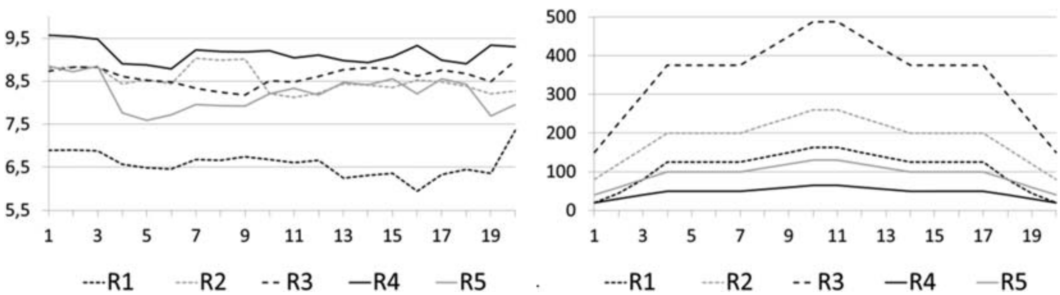
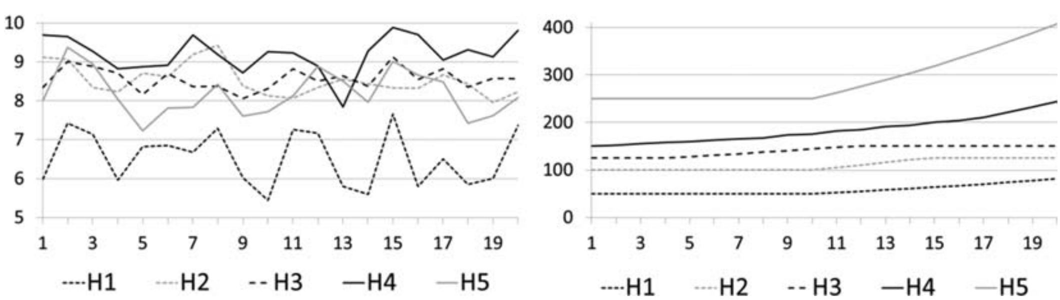


Figure 6: Prices (€/MMBtu, left) and Volume Limits (mill. MMBtu, right) of Natural Gas Sales Contracts over 20 Time Periods



220,000m^3 with an investment cost of €300 million. Vessels can also be chartered in and out at yearly rates that start at €26.88 million and €18.85 million, respectively, and increase yearly by 2%. Initially, the company does not own any vessels and must decide whether to buy or charter vessels. Average transportation costs vary on the different links between liquefaction and regasification terminals, reflecting the terminals' locations. For example, it costs €6.18/m^3 to ship LNG from L1 to R1, €3.18/m^3 to R2, €4.07/m^3 to R3, €20.63/m^3 to R4, and €8.59/m^3 to R5. All costs and prices are in nominal terms and the discount rate is set at 10% annually.

Figure 7: Spot Prices at Hubs over 20 Time Periods (€/MMBtu)

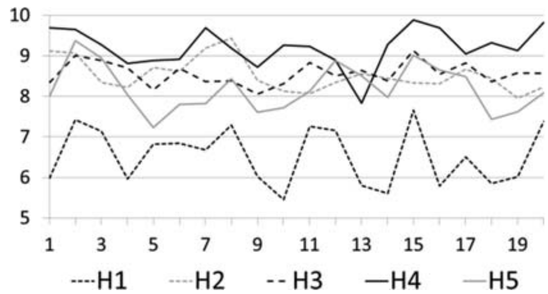


Table 5: Various Planning Approaches: Revenues, Costs, and Profits (Mill. €)

			Isolated		Fixed dec.		Rebalancing	
		No extension	L1A	L1B	L1A	L1B	L1A	L1B
Revenue	Sale	18,404	9,069	11,622	30,629	33,804	29,386	32,354
	Charter	271			172	78	74	516
Cost	Purchase	15,933			25,002	28,639	23,549	25,611
	Operations	951	864	554	1,465	1,592	1,379	1,487
	Charter	535			766	737	599	1,117
	Investments	300	497	745	797	1,045	797	1,045
Profits		956	7,708	10,323	2,771	1,870	3,136	3,611

A decision must be made for which investment option to execute and which contracts to enter into. Optimal fleet size and mix also need to be determined.

In the remainder of this section, we discuss and compare several approaches to solving this planning problem. Table 5 shows revenues, costs, and profits achieved with these approaches, while Tables 9–12 in the appendix provide more detail about the amount of LNG and NG traded and the fleet sizes. For an easier comparison, the NPV of investment options L1A and L1B has also been discounted to the first time period.

5.2 The Current Planning Process

The current practice at a number of energy companies is to evaluate potential investments in several steps before they are implemented. At each step, the level of detail increases and the analysis becomes more precise. If a potential project appears feasible and economically sound, it will be considered for implementation. If there are mutually exclusive projects, the project with the highest NPV is chosen.

Manual planning typically analyzes each long-term investment opportunity individually and, to a lesser extent, it manages how it affects the existing asset portfolio. This implies that the potential new portfolio is not fully re-optimized when investigating how the new investment will complement the existing portfolio. For the test case, excluding the other liquefaction terminals and focusing exclusively on the two options, the NPV of the longer running option, L1A, will be €7,708 million, and that of the shorter option, L1B, will be €10,323 million. Obviously, it is more profitable to invest in the shorter option that allows for higher production volumes in earlier years. Note, however, that this evaluation assumes that all LNG produced at the terminal can be sent to the most profitable market. This assumption is, in general, somewhat optimistic.

Analyzing investment opportunities as complements to existing portfolio of assets, keeping all other decisions fixed, and observing market limitations gives a solution where the purchase contract at the considered liquefaction terminal cannot be utilized to its full potential. In this instance, the NPV of operating the whole system is €2,771 million if option L1A is chosen and €1,870 million for L1B. Choosing not to invest in either option yields a NPV of €956 million. This implies that option L1A would add a value of €1,815 million to the system and option L1B a value of €911 million. Consequently, the longer running option L1A appears more profitable now because it allows for the full potential of the associated contracts to be realized.

In summary, analyzing investment options in a simplistic fashion, instead of re-optimizing the entire portfolio, can lead to an overly optimistic evaluation in some cases, while it may mean assets are not utilized optimally in other cases. In this particular example, we show situations where an investment decision is based on flooding the premium market or where a new purchase contract cannot be fully utilized.

5.3 From Individual Projects to Portfolio Management

An investment in one element of the supply chain is likely to affect other elements of the chain; however, often it is not immediately clear what those effects will be. Not only might it affect the utilization of physical assets such as terminals, vessels, pipelines, or hubs, but it may also impact existing contracts. Furthermore, it will have an effect on the company's ability to sign new contracts and to operate in the spot market. By re-optimizing the whole system considered in this test case instead of just trying to "fit in" the new project, the system's NPV increases to €3,136 million for option L1A and to €3,611 million for option L1B. Hence, the added value of option L1A is €2,180 million while that of the shorter option L1B is €2,655. Again, the shorter option appears more profitable.

In the previous section, we observed that the purchase contract at the terminal could not be fully utilized with the shorter option L1B. This is partially due to limitations from contracts in markets where the LNG could be sent without economic losses. A re-optimization of the decisions over the entire portfolio leads to a change of nearly all deliveries. Almost all sales contracts are affected, either through a changed source purchase contract or through changed volumes. This concerns not only assets directly linked to the two upgrade options, but also seemingly, completely unrelated assets. This re-organization, however, allows for complete and fully optimal utilization of the added LNG volumes.

Another observation is that the longer option, L1A, allows for profitable routing of LNG to more markets than option L1B because the volume is produced over a longer time span. This helps to avoid volume limitations in contracts in the more profitable markets or in transport capacity. But this positive effect is not sufficient to outweigh the lower investment cost associated with L1B. Note that this solution is quite different compared to what manual planners would have found. In particular, since it requires changing the way delivery obligations are satisfied for nearly all contracts, the solution may never have been found with the current planning practice. The value of the best solution without re-optimizing the whole portfolio is €1,815 million. While re-optimizing the system not only leads to a different choice of upgrade option, it also increases the value of implementing this option to €2,655 million. Consequently, in the considered test instance, the evaluation of the investment opportunities using a complete portfolio management approach leads to a solution stipulating an €40 million increase of the system's NPV. Even if the sub-optimal option L1A were chosen, adapting all assets in the system to the new option would yield a €475 million higher NPV for the whole system compared to just fitting the new option to the existing system.

6. CONCLUSIONS AND PERSPECTIVES

We present a stochastic mixed-integer linear programming problem to support strategic planning processes in the LNG value chain. The model focuses on investments and disinvestments into infrastructure and vessels, on chartering decisions, and on decisions about purchases and sales of LNG and natural gas. Selected features of the model were illustrated by numerical case studies motivated by our industry partners. Explicitly taking uncertainty into account (for example, about the future price development) can lead to increased efficiency and higher profits. We also demonstrated that by taking an integrated portfolio perspective, different solutions could be obtained compared to traditional approaches.

Recent developments in liquefied natural gas technology may affect the future market dramatically. For instance, some terminals will become bi-directional, so that both liquefaction and regasification can be performed. Also, larger plants and vessels are built in order to benefit from technological advances and economies of scale (see Spilsbury et al. (2005)). On the other hand, small-scale plants are becoming economically viable. Floating liquefaction (FLNG) allows for production from gas fields that were previously considered too small and too far away (GIIGNL 2010). These developments will undoubtedly challenge traditional planning and modeling approaches, amplifying the need for further work in decision support tools based on mathematical programming approaches.

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APPENDIX

Results of Test Case 1

This section supplements the results for Test case 1. Tables 6–8 present the amounts of LNG sold spot and delivered in the long-term and short-term contracts for each of the model runs presented in Sections 4.2 and 4.3. The solution approach “STO” in Section 4.2 is identical to the test “Spot” in Section 4.3, and therefore, it is only listed once in the tables. The columns in the tables correspond to the nodes in the scenario tree presented in Figure 2. The “DET” approach has only one decision per period, indicated by a single centered value per period.

All model runs give a purchased volume of 1000 MMBtu except for “No spot” and “Max700” that give a purchase of 700 MMBtu.

Table 6: Spot Sale Volume Decisions for Each Solution Approach in Section 4.2 and Test in Section 4.3 (MMBtu)

Solution approach	Period 2		Period 3			
	Node a	Node b	Node c	Node d	Node e	Node f
STO	1000	1000	500	475	1000	1000
DET	300		300			
DETd	400	300	300	300	400	300
SA_c	1000	n/a	1000	n/a	n/a	n/a
SA_d	1000	n/a	n/a	475	n/a	n/a
SA_e	n/a	475	n/a	n/a	1000	n/a
SA_f	n/a	300	n/a	n/a	n/a	300
No spot	0	0	0	0	0	0
Max700	700	700	200	175	700	700

Table 7: Long-term Contract Delivery Volume Decisions for Each Solution Approach in Section 4.2 and Test in Section 4.3 (MMBtu)

Solution approach	Period 2		Period 3			
	Node a	Node b	Node c	Node d	Node e	Node f
STO	0	0	0	0	0	0
DET	700		700			
DETd	600	700	700	700	600	700
SA_c	0	n/a	0	n/a	n/a	n/a
SA_d	0	n/a	n/a	0	n/a	n/a
SA_e	n/a	0	n/a	n/a	0	n/a
SA_f	n/a	700	n/a	n/a	n/a	700
No spot	700	700	700	700	700	700
Max700	0	0	0	0	0	0

Table 8: Short-term Contract Delivery Volume Decisions for Each Solution Approach in Section 4.2 and Test in Section 4.3 (MMBtu)

Solution approach	Period 2		Period 3			
	Node a	Node b	Node c	Node d	Node e	Node f
STO	0	0	500	525	0	0
DET	0		0			
DETd	0	0	0	0	0	0
SA_c	0	n/a	0	n/a	n/a	n/a
SA_d	0	n/a	n/a	525	n/a	n/a
SA_e	n/a	525	n/a	n/a	0	n/a
SA_f	n/a	0	n/a	n/a	n/a	0
No spot	0	0	0	0	0	0
Max700	0	0	500	525	0	0

Results of Test Case 2

This section gives an overview of results of the non-portfolio planning approaches for Test case 2, discussed in Section 5. Table 9 shows how much LNG is purchased at the liquefaction terminals and sold at the regasification terminals in all analyzed time periods. The amounts of natural gas sold in a contract or spot at a hub are indicated in Tables 10 and 11, respectively. Table 12 lists the fleet size and composition needed to transport the LNG from liquefaction to regasification.

Empty cells indicate zero purchases or sales—despite having the possibility. Values placed between two columns apply to both columns. The “n\a” entries in Table 9 mark the validity of the contracts attached to terminal L1 and the extension options L1A and L1B with different lifetimes. We compare the planning approaches of choosing not to extend terminal L1’s lifetime (“no ext.”), evaluating the NPV of both extension options L1A and L1B in isolation, fitting the extension into the existing system (“fixed”), and re-analyzing all decisions when phasing in an extension option (“rebalancing / rebal.”). Note that for the latter two approaches, only the results of selecting the shorter extension option L1B are shown as this option yields a higher profit than option L1A.

Evidently, the solution in the “rebalancing” approach differs slightly from the “no ext.” and “fixed” approach solutions also in periods 1–5 where a potential investment in an extension should not make any difference. This may be due to different profit-maximizing solutions for one of the approaches having the same objective function value despite different variable values during the periods.

Table 9: LNG Purchased at Liquefaction Terminals L1—L3 and Sold at Regasification Terminals R1–R3, Choosing Option L1B for the “Fixed” and “Rebalancing” Approaches [MMBtu]*

<table>
<tr><th></th><th>L1</th><th>L1A</th><th>L1B</th><th colspan="2">L1B</th><th colspan="2">L2</th><th colspan="2">L3</th><th>R1</th><th colspan="3">R2</th><th>R3</th></tr>
<tr><th>Time Period</th><th>All</th><th colspan="2">Isolated</th><th>Fix</th><th>Reb.</th><th>No ext. & Fix</th><th>Reb.</th><th>No ext. & Fix</th><th>Reb.</th><th>All</th><th>No ext.</th><th>Fix</th><th>Reb.</th><th>Reb.</th></tr>
<tr><td>1</td><td>375</td><td colspan="2">n/a</td><td colspan="2">n/a</td><td></td><td>5.1</td><td>9</td><td>6.2</td><td>5</td><td>80</td><td>80</td><td>80</td><td></td></tr>
<tr><td>2</td><td>375</td><td colspan="2">n/a</td><td colspan="2">n/a</td><td></td><td></td><td colspan="2">2.4</td><td>11.3</td><td>30</td><td>30</td><td>30</td><td></td></tr>
<tr><td>3</td><td>375</td><td colspan="2">n/a</td><td colspan="2">n/a</td><td></td><td></td><td></td><td>12</td><td>20</td><td>160</td><td>160</td><td>160</td><td></td></tr>
<tr><td>4</td><td>375</td><td colspan="2">n/a</td><td colspan="2">n/a</td><td>33.5</td><td>85</td><td></td><td></td><td>31.3</td><td colspan="2">200</td><td>50</td><td></td></tr>
<tr><td>5</td><td>375</td><td colspan="2">n/a</td><td colspan="2">n/a</td><td>45.8</td><td>34.6</td><td></td><td></td><td>31.3</td><td colspan="2">135.3</td><td>50</td><td></td></tr>
<tr><td>6</td><td>n/a</td><td>120</td><td>450</td><td colspan="2">450</td><td>82.1</td><td></td><td>14.1</td><td>11.5</td><td>31.3</td><td>50</td><td>145</td><td>50</td><td></td></tr>
<tr><td>7</td><td>n/a</td><td>180</td><td>450</td><td colspan="2">450</td><td>132.9</td><td>129.2</td><td></td><td></td><td>31.3</td><td>50</td><td>200</td><td>198.4</td><td></td></tr>
<tr><td>8</td><td>n/a</td><td>240</td><td>450</td><td colspan="2">450</td><td>132.4</td><td>39.7</td><td></td><td></td><td>34.4</td><td>55</td><td>220</td><td>97.3</td><td></td></tr>
<tr><td>9</td><td>n/a</td><td>300</td><td>450</td><td colspan="2">450</td><td>100.5</td><td></td><td></td><td></td><td>37.5</td><td>62</td><td colspan="2">240</td><td>6.4</td></tr>
<tr><td>10</td><td>n/a</td><td>330</td><td>585</td><td>530.6</td><td>585</td><td>133.6</td><td>28.7</td><td></td><td></td><td>40.6</td><td>65</td><td>260</td><td>79.4</td><td></td></tr>
<tr><td>11</td><td>n/a</td><td>360</td><td>585</td><td>564.5</td><td>585</td><td>133.6</td><td></td><td></td><td>14</td><td>40.6</td><td>65</td><td>260</td><td>65</td><td></td></tr>
<tr><td>12</td><td>n/a</td><td>390</td><td>450</td><td colspan="2">450</td><td>98.5</td><td></td><td></td><td></td><td>37.5</td><td>60</td><td>139.8</td><td>60</td><td></td></tr>
<tr><td>13</td><td>n/a</td><td>420</td><td>360</td><td colspan="2">360</td><td>68.8</td><td></td><td>21.4</td><td>3.4</td><td>34.4</td><td>55</td><td>55</td><td>55</td><td>13.9</td></tr>
<tr><td>14</td><td>n/a</td><td>390</td><td>270</td><td colspan="2">270</td><td>82.1</td><td></td><td>14.1</td><td>16.6</td><td>31.3</td><td>50</td><td>200</td><td>50</td><td>36.4</td></tr>
<tr><td>15</td><td>n/a</td><td>360</td><td>180</td><td colspan="2">180</td><td>82.1</td><td></td><td>37</td><td>24.6</td><td>31.3</td><td>50</td><td>93.2</td><td>50</td><td></td></tr>
<tr><td>16</td><td>n/a</td><td>330</td><td>n/a</td><td colspan="2">n/a</td><td>82.1</td><td>137.8</td><td>14.1</td><td>50.1</td><td>31.3</td><td>50</td><td>50</td><td>50</td><td></td></tr>
<tr><td>17</td><td>n/a</td><td>300</td><td>n/a</td><td colspan="2">n/a</td><td>82.1</td><td>130.9</td><td></td><td>46</td><td>31.3</td><td>50</td><td>50</td><td>50</td><td>3.7</td></tr>
<tr><td>18</td><td>n/a</td><td>240</td><td>n/a</td><td colspan="2">n/a</td><td>60.6</td><td>124.4</td><td>3.7</td><td>16</td><td>20</td><td>40</td><td>40</td><td>40</td><td></td></tr>
<tr><td>19</td><td>n/a</td><td>180</td><td>n/a</td><td colspan="2">n/a</td><td>69.5</td><td>110.6</td><td></td><td></td><td>11.3</td><td>30</td><td>30</td><td>30</td><td>8.3</td></tr>
<tr><td>20</td><td>n/a</td><td>120</td><td>n/a</td><td colspan="2">n/a</td><td>25.3</td><td>72.2</td><td>8.4</td><td></td><td>5</td><td>20</td><td>20</td><td>20</td><td></td></tr>
</table>

* Values for “rebalancing” are in italics if they are the same values but from a different source (purchase contract) compared to “no extension”.

Table 10: Natural Gas Sold in Contracts at Hubs H2–H5, Choosing Option L1B for the "Fixed" and "Rebalancing" Approaches [MMBtu]*

	Hub 2			Hub 3			Hub 4	Hub 5	
Time period	**No ext.**	**Fixed**	**Rebal.**	**No ext.**	**Fixed**	**Rebal.**	**No ext. & Fixed**	**No ext.**	**Fixed**
1	248.8	248.8					8.8		
2				50	50		2.4		
3	57.9	57.9		50	50	50		32.4	32.4
4	72.3	72.3		50	50	50			
5									
6		248.8	169.1		50	50	13.9		
7	50.1	248.8	248.8		50				
8					21.8				10
9									
10					50				
11					52.5	52.5			
12		248.8			55.1				
13		246.7			57.9				
14									
15					63.8				
16							13.9		

* Empty cells indicate no NG sale; there is no NG sale in contracts at hub H1 or after period 16.

Table 11: Natural Gas Sold Spot at Hubs H2–H5, Choosing Option L1B for the "Fixed" and "Rebalancing" Approaches [MMBtu]*

	Hub 2			Hub 3			Hub 4			Hub 5
Time period	**No ext.**	**Fixed**	**Rebal.**	**No ext.**	**Fixed**	**Rebal.**	**No ext.**	**Fixed**	**Rebal.**	**Fixed**
1			248.8	36.1	36.1					
2	248.8	248.8	248.8	29.6	29.6	23.6				
3				50	50	50				
4				50	50	50				
5	248.8	248.8	229.3							
6					50	50				
7					45.4					
8	41.5	248.8	248.8		50					
9		248.8	55.2		17.4					
10		248.8		26.5	50			6.4		
11		248.8		26.5	50	50			14	37.3
12					50					
13					50					
14		116.7					13.9	13.9	16.3	
15					50	3.2	36.4	36.4	24.2	20.5
16										
17										
18							3.7	3.7	4	
19				27.4	27.4	12				
20							8.3	8.3		

* There are no spot sales at hub H1.

Table 12: Fleet Sizes—Number of Vessels of Each Type

	Large vessel			Small vessel		
Time period	**No ext.**	**Fixed**	**Rebal.**	**No ext.**	**Fixed**	**Rebal.**
1	2	2	2	1	1	1
2	2	2	2	1	1	1
3	3	3	3	0	0	0
4	3	3	3	0	0	2
5	3	3	3	0	0	0
6	1	4	3	1	1	1
7	2	5	4	0	0	1
8	2	5	3	0	0	1
9	1	4	3	1	1	1
10	2	5	5	0	1	0
11	2	6	5	0	0	0
12	1	4	4	1	1	0
13	1	4	3	1	0	0
14	1	3	2	1	1	1
15	2	3	2	0	1	0
16	1	1	3	1	1	0
17	1	1	3	1	1	0
18	1	1	1	0	0	2
19	1	1	1	0	0	1
20	0	0	1	1	1	0

REFERENCES

André J. (2010). "Optimization of investments in gas networks." PhD thesis, Université du Littoral Côte d'Opale.

Aouam T. and Z. Yu (2008). "Multistage stochastic hydrothermal scheduling," In IEEE International Conference on Electro/Information Technology, EIT2008, 66–71. IEEE.

Bezerra, B., L. A. Barroso, R. Kelman, B. Flach, M. L. Latorre, N. Campodonico, and M. Pereira (2010). "Integrated electricity–gas operations planning in long-term hydroscheduling based on stochastic models." In Pardalos, P. M. et al., eds., *Handbook of Power Systems I*. Springer. http://dx.doi.org/10.1007/978-3-642-02493-1_7.

Birge J.R. and F. V. Loveaux (1997). *Introduction to Stochastic Programming*. Springer.

Bøckman, T., S. E. Fleten, E. Juliussen, H. J. Langhammer, and I. Revdal (2008). "Investment timing and optimal capacity choice for small hydropower projects." *European Journal of Operational Research*, 190(1): 255–267. http://dx.doi.org/10.1016/j.ejor.2007.05.044.

Botterud, A. and M. Korpås (2007). "A stochastic dynamic model for optimal timing of investments in new generation capacity in restructured power systems." *International Journal of Electrical Power & Energy Systems*, 29(2): 163–174. http://dx.doi.org/10.1016/j.ijepes.2006.06.006.

BP (2001). "Statistical review of world energy." http://www.bp.com/statisticalreview.

BP (2011). "Statistical review of world energy." http://www.bp.com/statisticalreview.

Egging, R. (2010). "Multi-Period Natural Gas Market Modeling—Applications, Stochastic Extensions and Solution Approaches." PhD thesis, Department of Civil and Environmental Engineering, University of Maryland (College Park), Maryland, USA.

EIA (2010). "Annual energy outlook retrospective review: Evaluation of projections in past editions (1982–2009)." http://www.eia.gov/oiaf/analysispaper/retrospective/.

Escudero, L. F., A. Garín, M. Merino, and G. Pérez (2007). "The value of the stochastic solution in multistage problems." *TOP*, 15: 48–64. http://dx.doi.org/10.1007/s11750-007-0005-4.

Fleten, S. E. (2000). "Portfolio management emphasizing electricity market applications. A stochastic programming approach." PhD thesis, Department of Industrial Economics and Technology Management, Norwegian University of Science and Technology, Trondheim, Norway.

Fodstad, M., K. T. Uggen, F. Rømo, A.-G. Lium, G. Stremersch, and S. Hecq (2010). "LNGScheduler: a rich model for coordinating vessel routing, inventories and trade in the liquefied natural gas supply chain." *Journal of Energy Markets*, 3(1): 1–34.

GIIGNL (2010). "The LNG industry." http://www.giignl.org/fr/home-page/lng-industry.

GIIGNL (2011). "The LNG industry." http://www.giignl.org/fr/home-page/lng-industry.

Granville, S., G. C. Oliveira, L. M. Thome, N. Campodonico, M. L. Latorre, M. V. F. Pereira, and L. A. Barroso (2003). "Stochastic optimization of transmission constrained and large scale hydrothermal systems in a competitive framework." In *Power Engineering Society General Meeting,* IEEE.

Grønhaug, R., M. Christiansen, G. Desaulniers, and J. Desrosiers (2010). "A branch-and-price method for a liquefied natural gas inventory routing problem." *Transportation Science*, 44(3): 400–415. http://dx.doi.org/10.1287/trsc.1100.0317.

Guldmann, J.-M. and F. Wang (1999). "Optimizing the natural gas supply mix of local distribution utilities." *European Journal of Operational Research*, 112(3): 598–612. http://dx.doi.org/10.1016/S0377-2217(97)00415-3.

Kaminski, V., Y. Feng, and Z. Pang (2008). "Value, trading strategies and financial investment of natural gas storage assets." *Northern Finance Association Conference,* Kananaskis, Canada. www.northernfinance.org/2008/papers/75.pdf.

Klaassen, G., A. V. Kryazhimskii, and A. M. Tarasyev (2004). "Multiequilibrium game of timing and competition of gas pipeline projects." *Journal of Optimization Theory and Applications*, 120(1): 147–179. http://dx.doi.org/10.1023/B:JOTA.0000012737.20285.33.

Krey, V. and Y. Minullin (2005). "Modeling investment decisions in gas pipeline networks: A game-theoretic mixed complementarity problem formulation and its application to the Chinese gas market." Conference on Applied Infrastructure Research 2005, Berlin, Germany.

http://www.infraday.tu-berlin.de.

Lai, G., M. X. Wang, S. Kekre, A. Scheller-Wolf, and N. Secomandi (2011). "Valuation of storage at a liquefied natural gas terminal." *Operations Research*, 59(3): 602–616. http://dx.doi.org/10.1287/opre.1100.0909.

Lium, A.-G. and M. Kaut (2006). "Scenario generation and obtaining sound solutions." Technical report, Molde University College, Norway. http://work.michalkaut.net/.

MirHassani, S. A. and R. Noori (2011). "Implications of capacity expansion under uncertainty in oil industry." *Journal of Petroleum Science and Engineering*, 77(2): 194–199. http://dx.doi.org/10.1016/j.petrol.2011.03.009.

Murto, P. and J. Keppo (2002). "A game model of irreversible investment under uncertainty." *International Game Theory Review*, 4(2): 127–140. http://dx.doi.org/10.1142/S0219198902000604.

Nygreen, B., M. Christiansen, T. Bjørkvoll, K. Haugen, and Ø. Kristiansen (1998). "Modeling Norwegian petroleum production and transportation." *Annals of Operations Research*, 82: 251–267. http://dx.doi.org/10.1023/A:1018962703587.

Pereira, M. V. F. and L. Pinto (1991). "Multi-stage stochastic optimization applied to energy planning." *Mathematical Programming*, 52(1): 359–375. http://dx.doi.org/10.1007/BF01582895.

Smith, J. E. and K. F. McCardle (1999). "Options in the real world: Lessons learned in evaluating oil and gas investments." *Operations Research*, 47(1): 1–15. http://dx.doi.org/10.1287/opre.47.1.1.

Spilsbury, C., S. McLauchlin, and B. Kennington (2005). "Optimising the LNG liquefaction process." *LNG Journal*, January / February: 40–43.

Thompson, M., M. Davison, and H. Rasmussen (2009). "Natural gas storage valuation and optimization: A real options application." *Naval Research Logistics*, 56(3): 226–238. http://dx.doi.org/10.1002/nav.20327.

Tomasgard, A., F. Rømo, M. Fodstad, and K. Midthun (2007). "Optimization models for the natural gas value chain." In Hasle, G., K. Lie, and E. Quak, eds., *Geometric Modelling, Numerical Simulation, and Optimization*, Springer. http://dx.doi.org/10.1007/978-3-540-68783-2_16.

Toth, P. and D. Vigo (2002). *The Vehicle Routing Problem*. SIAM. http://dx.doi.org/10.1137/1.9780898718515.

Werner, A., K. Tolstad Uggen, M. Fodstad, A.-G. Lium, and R. Egging (2012). "LNGPlanner—mathematical model description." Technical Report A23064, SINTEF.

Zheng, Q. P. and P. M. Pardalos (2010). "Stochastic and risk management models and solution algorithm for natural gas transmission network expansion and LNG terminal location planning." *Journal of Optimization Theory and Applications*, 147(2): 337–357. http://dx.doi.org/10.1007/s10957-010-9725-y.