3 Methodology

A deterministic bi-level optimization problem is proposed to answer the research questions. The lower-level problem considers market clearing under minimized total supply cost to satisfy the demand, and the upper-level problem is profit maximization of the major exporter with market power. The main link between the lower-level problem and the upper-level problem is the export price and quantity offered by the major exporter to the market clearing (i.e., decision variables from the upper-level problem serve as parameters for the lower-level problem) and the cleared quantity and price (i.e., decision variables from the follower and parameters for the upper-level problem). In the lower-level problem, the market clearing is treated separately for the European and global markets $M1$ and $M2$ (by having two separate supply-demand equilibrium constraints), but the total cost of both is minimized. As described in detail below, this allows for the consideration of customized conditions (such as diversification of exporters), especially in finding the optimality of the European market clearing. Against this background and also taking into account the market power of the major exporter, it is assumed that there is no price discrimination between the two markets under consideration in the long run. Therefore, the mathematical formulation of the lower-level problem ensures that the market clearing prices are uniform.

3.1 Lower-level problem: market clearing at minimized total cost

As is typical for bi-level optimization problems, the original formulation (or primal problem) of the lower-level problem is transformed into its dual problem using the Karush-Kuhn-Tucker (KKT) solution formalism. The complete formulation of the dual problem of the lower-level problem can be found in the Appendix A. The main text focuses on the original formulation and the primal problem of the lower-level problem, as it is more readable even for those unfamiliar with bi-level optimization techniques.

3.1.1 Objective and decision variables

The objective of the lower-level problem is to minimize the total cost of supply when satisfying the demand of the European and global markets. Equation 1 shows the objective function while $\mathcal{X}$ is a vector and contains all the decision variables of the lower-level problem.

$$\min_{\mathcal{X}} \sum_{e} \sum_{m} \sum_{t} c_e \times q_{e,m,t} \tag{1}$$

The decision variables $q_{t,e,m}$ is the supply quantity of exporter e, market m, and timestep t. c_e is a parameter and describes the marginal supply cost per exporter e. The vector of decision variables is described in Equation 2

$$\mathcal{X} = [q_{e,m,t}, \underbrace{\bar{q}_{e,t}}_{\forall e \setminus \{i\}, t}, q^{del}_{e,m,t}, q^{arb}_{e,m,t}] \tag{2}$$

where $\bar{q}_{e,t}$ is the available supply capacity per e and t, $q^{del}_{e,m,t}$ the cleared quantity delivered to m per e and t. $q^{arb}_{e,m,t}$ is a decision variable used solely to ensure uniform clearing prices between markets, as mentioned above. Essentially, this decision variable prevents exporters from engaging in price arbitrage. Note, that $\bar{q}_{e,t}$ for the major export (index i) is a parameter for the lower-level problem, while it is a decision variable for the upper-level problem (see Section 3.2 in detail).

3.1.2 Constraints

The constraints of the primal problem are described below. For each constraint, the equation is given together with its applicability and the variable of the dual problem in parentheses. To help the reader understand the mathematical formulation of the model, the dual variables are numbered consecutively according to the equation number (see superscript of the dual variables). A distinction is also made between equality and inequality equations. Lambda (λ) is used for equality equations and mu (μ) for inequality equations. The equations 3 and 4 ensure that the supply quantity is greater than or equal to zero and less than the available supply capacity per e and t.

$$0 \leq \sum_{m} q_{e,m,t} \quad : \forall e, t \quad (\mu^{3}_{e,t}) \tag{3}$$

$$\sum_{m} q_{e,m,t} \leq \bar{q}_{e,t} \quad : \forall e, t \quad (\mu^{4}_{e,t}) \tag{4}$$

The equation 5 separates the supply quantity into the cleared supply quantity delivered and is used to avoid arbitrage between markets and to unify prices (arbitrage). The equations 6 and 7 show the supply balances for each of the markets ($M1$ and $M2$). For example, the demand $d_{t,M1}$ is satisfied by the cleared supply quantity delivered and the quantity transferred from market $M2$. However, this rather general division between delivered and transferred quantities ultimately ensures that λ^{6}_{t} is equal to λ^{7}_{t}. Consequently, market clearing prices are uniform.

$$q_{e,m,t} = q^{del}_{e,m,t} + q^{arb}_{e,m,t} \quad : \forall e, m, t \quad (\lambda^{5}_{e,m,t}) \tag{5}$$

$$d_{M1,t} = \sum_{e} q^{del}_{e,M1,t} + q^{arb}_{e,M2,t} \quad : \forall t \quad (\lambda^{6}_{t}) \tag{6}$$

$$d_{M2,t} = \sum_{e} q^{del}_{e,M2,t} + q^{arb}_{e,M1,t} \quad : \forall t \quad (\lambda^{7}_{t}) \tag{7}$$

The equations 8 and 9 consider customized conditions for the clearing of the European market $M1$. The first equation takes into account the embargo of specific exporters and the second equation the diversification of shares of exporters satisfying the demand. The latter limits the maximum share of an exporter to α of the total demand.

$$q_{e,M1,t} = 0 \quad : \forall e \in \mathcal{E}_{embargo}, t \quad (\lambda^{8}_{e,t}) \tag{8}$$

$$q^{del}_{e,M1,t} + q^{arb}_{e,M2,t} \leq \alpha \times d_{M1,t} \quad : \forall e, t \quad (\mu^{9}_{e,t}) \tag{9}$$

The equations 10 to 13 ensure the non-negativity of the decision variables.

$$0 \leq q_{e,m,t} \quad : \forall e, m, t \quad (\mu^{10}_{e,m,t}) \tag{10}$$

$$0 \leq \bar{q}_{e,t} \quad : \forall e \setminus \{i\}, t \quad (\mu^{11}_{e,t}) \tag{11}$$

$$0 \leq q^{del}_{e,m,t} \quad : \forall e, m, t \quad (\mu^{12}_{e,m,t}) \tag{12}$$

$$0 \leq q^{arb}_{e,m,t} \quad : \forall e, m, t \quad (\mu^{13}_{e,m,t}) \tag{13}$$

The dual variables λ^6_t and λ^7_t (below just λ_t)are particularly important with regard to the following description of the upper-level problem. Both variables are directly incorporated in the objective function of the upper-level problem and thus in maximizing the profit of the major exporter. In order to take into account the impact of the market price not only on the decision and resulting outcome of the major exporter in the upper-level problem, the following approach is also chosen.

- The available supply capacity $\bar{q}_{e,t}$ of all exporters except the major exporter i (i.e., $\forall e \setminus \{i\}$) is assumed to be a function of the market clearing price λ_t.
- The equation 14 describes this relation. For reasons of simplification, it is assumed that more exporter's capacities are economically viable and can be made available to the market when market prices rise (for example as a result of the profit maximization strategy of the major exporter).
- However, there is no optimal capacity expansion. The available supply capacity is determined solely by the market clearing price of the previous one.

$$\bar{q}_{e,t} = f(\lambda_t) \quad \text{with } f = a + b \times \lambda_t \quad : \forall e \setminus \{i\}, t \tag{14}$$

The available supply capacity of the major exporter is not affected by the market clearing price, nor is it a decision variable in the lower-level problem.

Before a detailed description of the upper-level problem is given, a few thoughts are added here to underscore the proposed approach, especially the relationship between the market-clearing price and the available supply capacity of exporters. Here we could use examples from other markets, such as the oil market, to show empirically how prices have jumped due to the market power of large exporters and the increase/decrease in available production capacity of other exporters. One reference I have in mind is Wirl's 2008 paper "Why do oil prices jump" [1]. In this paper, he shows that the price setter seeks hysteresis in the clearing price to maximize profits by crowding out other exporters.

3.2 Upper-level problem: profit maximization of the major exporter

4 Results and discussion

5 Conclusion

Declaration of Competing Interest

The authors report no declarations of interest.

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References

[1] Franz Wirl. Why do oil prices jump (or fall)? *Energy Policy*, 36(3):1029–1043, 2008. doi: https://doi.org/10.1016/j.enpol.2007.11.024.

Appendices

A Karush–Kuhn–Tucker conditions of the lower-level problem