



# Modeling of the strategic raw material supply for the European hydrogen target 2030

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# Topics based on the review by Ben and Marzia

1. Nonlinear term in the objective function of the upper-level problem
2. Representation of the capacity constraint ("function  $f$ ")
3. Demand elasticity
4. Arbitrage between markets
5. Storage

# 1. Nonlinear term in the objective function (1/4)

The solution difficulty arises from having  $q_{i,t\_bar}$  and  $c_i$  both as decision variables? **But maybe only one instrument is needed?** I.e., given the step-wise fringe supply curve that results from the subproblem (Section 3.1), the optimal (revenue maximizing) **solution for the leader will always be on the vertical part of a step** (never in the middle of a horizontal part, unless the  $q_{i\_bar}$  is binding). So actually, only  $c_i$  is needed as an upper level variable, and not  $q_{i,t\_bar}$  (omit the latter, and just insert  $q_{i,t\_bar} = q_{i\_bar}$  as the upper bound. The best solution for the leader will be where  $c_i$  cuts a vertical face of the fringe supply, unless  $i$ 's output =  $q_{i\_bar}$ .

- Use the step-wise fringe supply curve (two possible options to make the model linear)
  - A. Keep only  $c_1$  („marginal supply cost of major exporter 1) as decision variable (as suggested) and make  $\bar{q}_{1,t}$  a parameter
  - B. Reformulation and linear approximation of the term  $q_{1,m,t} * \lambda_t$  in the objective function
- Note: both options build upon the idea that the solution for the leader will always be on the vertical part of a step!



# 1. Nonlinear term in the objective function (2/4)

$$\underset{y}{\text{maximize}} \sum_m \sum_t q_{i,m,t} (\lambda_t - \tilde{c})$$

$$\Rightarrow \sum_m \sum_t \underbrace{q_{i,m,t} \times \lambda_{m,t}}_{\text{product of two continuous variables}} - q_{i,m,t} \times \tilde{c} \quad (0)$$

$$y = [c_i, \bar{q}_{i,t}]$$

Linearize binary x continuous
Binary $V$
Nonnegative $w$
$z \leq M \cdot V$
$z \leq w$
$z \geq w + M \cdot V - M$

Source: Reformulation techniques in mathematical programming (page 45)

Reformulation of the non-linear term  $(q_{i,m,t} \times \lambda_{m,t})$

$$\lambda_{m,t} = \underbrace{\sum_{e \in I_i} \delta_{e,m,t} \times c_e}_{\text{error term}} + \underbrace{\delta_{i,m,t} \times c_i}_{\text{error term}} \quad (1)$$

$\Rightarrow$  Insert (1) in (0)

# 1. Nonlinear term in the objective function (3/4)

$$\sum_m \sum_t q_{i,m,t} \times \left[ \sum_{e \in \{i\}} [\sigma_{e,m,t} \times c_e] - \tilde{c} \right] = \sum_m \sum_t \sum_{e \in \{i\}} c_e \times \underbrace{q_{i,m,t} \times \sigma_{e,m,t}}_{\substack{\text{product of continuous} \\ \text{and binary variable} \\ (z_{e,m,t})}} - \sum_m \sum_t q_{i,m,t} \times \tilde{c} \quad (2)$$

$$z_{e,m,t} \leq M \times \sigma_{e,m,t} : \forall e \in \{i\}, m, t \quad (3)$$

$$z_{e,m,t} \leq q_{i,m,t} : \forall e \in \{i\}, m, t \quad (4)$$

$$z_{e,m,t} \geq q_{i,m,t} - (1 - \sigma_{e,m,t}) \times M : \forall e \in \{i\}, m, t \quad (5)$$

$$z_{e,m,t} \geq 0 : \forall e \in \{i\}, m, t \quad (6)$$

$\Rightarrow \sigma_{e,m,t}$  is set to one (1) for the marginal exporter only

$$\sum_{e \in \{i\}} \sigma_{e,m,t} = 1 : \forall m, t \quad (7)$$

$$\lambda'_{e,m,t} = c_e \times \sigma'_{e,m,t} : \forall e \in \{i\}, m, t \quad (8)$$

$$q_{e,m,t} \leq M' \times \sigma'_{e,m,t} : \forall e \in \{i\}, m, t \quad (9) \quad (\text{note that } M' \text{ is a large number})$$

$$q_{e,m,t} \geq \beta \times \sigma'_{e,m,t} : \forall e \in \{i\}, m, t \quad (10) \quad (\text{note that } \beta \text{ is a small number})$$

$$\sigma_{e,m,t} \leq \sigma'_{e,m,t} : \forall e \in \{i\}, m, t \quad (11)$$



# 1. Nonlinear term in the objective function (3/3)

$$\lambda_{m,t} \geq \lambda'_{e,m,t} \quad : \forall e \in \{i\}, m, t \quad (12)$$

$$\lambda_{m,t} = \sum_{e \in \{i\}} \delta_{e,m,t} \times C_e \quad : \forall m, t \quad (13)$$

$$\boxed{\sum_m \sum_t \sum_{e \in \{i\}} C_e \times z_{e,m,t} - \sum_m \sum_t q_{1,m,t} \times \tilde{z}} \Rightarrow \text{LINEAR}$$

- Reformulation and linear approximation of the term  $q_{1,m,t} * \lambda_t$  in the objective function

## 2. Representation of the capacity constraint ("function f")

*Would a better representation of the capacity constraint for  $e$  be  $q_{e,t\_bar} = q_{e,t-1\_bar} + q_{ADDED\_e,t\_bar} - q_{RETIRED\_e,t\_bar}$  with  $q_{ADD\_e,t\_bar} = \text{some convex function of } \lambda_{t-1}$ ? (add more capacity if price is higher?) And have some maintenance cost associated with  $q_{e,t\_bar}$  so there is an incentive to retire capacity if prices are low?*

- $\bar{q}_{e,t} = \bar{q}_{e,t-1} + q_{e,t}^{add} - q_{e,t}^{retire}$
- With  $q_{e,t}^{add} = f_e(\lambda_t)$  and function  $f_e$  is a convex function

- Modify capacity constraint as suggested
- Add maintenance cost to the objective function of the lower-level problem

$$\min_x \sum_e \sum_m \sum_t c_e^{gen} * q_{e,m,t} + c_e^{main} * \bar{q}_{e,t}$$

### 3. Demand elasticity

*Demand could be elastic (depend on  $\lambda$ )...include in objective function with a quadratic concave benefit (or piecewise linear)*

- To be discussed





## 4. Arbitrage between markets

*Arbitrage: will be easier if you have a separate arbitrage player with just  $(q_{M1,M2,t}^{ARB})$  being a single unrestricted variable, and being subtracted from the market clearing for M1 and added to the market clearing for M2. Don't need one for each player. This will make the KKTs much simpler.*

- Agree that this approach (separate arbitrage player) would make the model clearer and certainly easier
  - However, due to the diversification restriction, it is necessary in my opinion that the arbitrage is carried out separately for each exporter
  - Otherwise, the 65% limit of the supply share per exporter cannot be guaranteed
- Maintain current formulation of the model regarding individual arbitrage per exporter

## 5. Storage

*Could add storage (one per country, or just one storage player...this would result in some arbitrage over time, unless there's a limit to the amount that can be put in storage*

- Undisputedly an important point in the topic of critical raw material supply (keyword stockpiling)
  - However, there are several arguments against explicitly including storage in our analysis
    - 5-year time steps in the model (i.e., would require significant storage capacity, which is uncertain)
    - Modified capacity constraint (as suggested by Ben & Marzia) takes into account delay in production capacity change
- Storage are not explicitly included in the model but mentioned in “future work”

# Next steps for the model

- **Mathematics:** Adapt the mathematical formulation of the model
  - Linear approximation of the objective function in the upper-level problem
  - Modify capacity constraint introducing  $q_{e,t}^{add}, q_{e,t}^{retire}$
  - Adding maintenance cost in the objective function in the lower-level problem
  - Remove equation (3)
  - Major exporter  $e = \{1\}$
- **Documentation:** Update the methodology section in Overleaf
- **Coding:** Implementing the model in Python
- **Data:** Prepare input data sheets (.xlsx) that can be easily filled in with the data you have collected

Collect all the materials (model, presentation slides, etc.) in a GitHub repository

*IAMC template*

	A	B	C	D	E	F	G	H	
1	<b>Model</b>	<b>Scenario</b>	<b>Region</b>	<b>Variable</b>	<b>Unit</b>	<b>2005</b>	<b>2010</b>	<b>2015</b>	
2	MESSAGE	CD-LINKS 400	World	Primary Energy	EJ/y	462.5	500.7	...	