

$$\begin{aligned}
 \min_x \quad & \sum_e \sum_m \sum_t c_e \times q_{e,m,t} \quad \text{mit } x = [q_{e,m,t}, \bar{q}_{e,t}, q_{e,m,t}^{\text{del}}, q_{e,m,t}^{\text{arb}}] \\
 \text{s.t.:} \quad & -\sum_m q_{e,m,t} \leq 0 : \forall e,t \quad (\mu_{e,t}^1) \\
 & \left[\sum_m q_{e,m,t} \right] - \bar{q}_{e,t} \leq 0 : \forall e,t \quad (\mu_{e,t}^4) \\
 & q_{e,m,t}^{\text{del}} + q_{e,m,t}^{\text{arb}} - q_{e,m,t} = 0 : \forall e,m,t \quad (\lambda_{e,m,t}^5) \\
 & \left[\sum_e q_{e,m,t}^{\text{del}} + q_{e,m,t}^{\text{arb}} \right] - d_{m,t} = 0 : \forall t \quad (\lambda_t^6) \\
 & \left[\sum_e q_{e,m,t}^{\text{del}} + q_{e,m,t}^{\text{arb}} \right] - d_{m,t} = 0 : \forall t \quad (\lambda_t^7) \\
 & q_{e,m,t} = 0 : \forall e \in \Sigma_{\text{outgoing}}, t \quad (\lambda_{e,t}^8) \\
 & q_{e,m,t}^{\text{del}} + q_{e,m,t}^{\text{arb}} - \mathcal{L} d_{m,t} \leq 0 : \forall e,t \quad (\mu_{e,t}^9) \\
 & -q_{e,m,t} \leq 0 : \forall e,m,t \quad (\mu_{e,m,t}^{10}) \\
 & -\bar{q}_{e,t} \leq 0 : \forall e \setminus \{i\}, t \quad (\mu_{e,t}^{11}) \\
 & -q_{e,m,t}^{\text{del}} \leq 0 : \forall e,m,t \quad (\mu_{e,m,t}^{12}) \\
 & -q_{e,m,t}^{\text{arb}} \leq 0 : \forall e,m,t \quad (\mu_{e,m,t}^{13})
 \end{aligned}$$

STANDARD FORM OF THE MODEL

$$\begin{aligned}
 \min_x \quad & f(x) \\
 \text{s.t.:} \quad & h(x) = 0 : \lambda \\
 & g(x) \leq 0 : \mu
 \end{aligned}$$

$$\lambda = [\lambda_{e,m,t}^5, \lambda_t^6, \lambda_t^7, \underbrace{\lambda_{e,t}^8}_{\forall e \in \Sigma_{\text{outgoing}}}]$$

DUAL VARIABLE VECTORS

$$\mu = [\mu_{e,t}^1, \mu_{e,t}^4, \mu_{e,t}^9, \mu_{e,m,t}^{10}, \underbrace{\mu_{e,t}^{11}, \mu_{e,m,t}^{12}, \mu_{e,m,t}^{13}}_{\forall e \setminus \{i\}, t}]$$

LAGRANGIAN FUNCTION

$$\begin{aligned}
 \mathcal{L}(x, \lambda, \mu) = & \sum_e \sum_m \sum_t c_e \times q_{e,m,t} + \lambda_{e,m,t}^5 \{ q_{e,m,t}^{\text{del}} + q_{e,m,t}^{\text{arb}} - q_{e,m,t} \} \\
 & + \lambda_t^6 \left\{ \sum_e [q_{e,m,t}^{\text{del}} + q_{e,m,t}^{\text{arb}}] - d_{m,t} \right\} \\
 & + \lambda_t^7 \left\{ \sum_e [q_{e,m,t}^{\text{del}} + q_{e,m,t}^{\text{arb}}] - d_{m,t} \right\} \\
 & + \lambda_{e,t}^8 \{ q_{e,m,t} \} \\
 & + \mu_{e,t}^1 \left\{ - \sum_m q_{e,m,t} \right\} \\
 & + \mu_{e,t}^4 \left\{ \sum_m [q_{e,m,t}] - \bar{q}_{e,t} \right\} \\
 & + \mu_{e,t}^9 \{ q_{e,m,t}^{\text{del}} + q_{e,m,t}^{\text{arb}} - \mathcal{L} d_{m,t} \} \\
 & + \mu_{e,m,t}^{10} \{ -q_{e,m,t} \} \\
 & + \mu_{e,t}^{11} \{ -\bar{q}_{e,t} \} \\
 & + \mu_{e,m,t}^{12} \{ -q_{e,m,t}^{\text{del}} \} \\
 & + \mu_{e,m,t}^{13} \{ -q_{e,m,t}^{\text{arb}} \}
 \end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial q_{e,m,t}} = \begin{cases} c_e - \lambda_{e,m,t}^5 + \lambda_t^6 - \mu_{e,t}^1 + \mu_{e,t}^4 - \mu_{e,m,t}^{10} = 0 & \text{if } m=M \\ c_e - \lambda_{e,m,t}^5 - \mu_{e,t}^1 + \mu_{e,t}^4 - \mu_{e,m,t}^{10} = 0 & \text{otherwise} \end{cases}$$

KKT CONDITIONS

$$\nu_{e,t} - \nu_{e,t} + \nu_{e,t} - \nu_{e,m,t}^{10} = 0 \text{ otherwise}$$

$$\frac{\partial \mathcal{L}}{\partial \bar{q}_{e,t}} = -\nu_{e,t}^4 - \nu_{e,t}^{11} = 0 \quad \forall e \in \mathcal{E} \setminus \{i\}, t$$

$$\frac{\partial \mathcal{L}}{\partial q_{e,m,t}^{del}} = \begin{cases} \lambda_{e,m,t}^5 + \lambda_t^6 + \nu_{e,t}^9 - \nu_{e,m,t}^{12} = 0 & \text{if } m = M1 \\ \lambda_{e,m,t}^5 + \lambda_t^7 + \nu_{e,t}^9 - \nu_{e,m,t}^{12} = 0 & \text{otherwise} \end{cases}$$

$$\frac{\partial \mathcal{L}}{\partial q_{e,m,t}^{arb}} = \begin{cases} \lambda_{e,m,t}^5 + \lambda_t^7 - \nu_{e,m,t}^{13} = 0 & \text{if } m = M1 \\ \lambda_{e,m,t}^5 + \lambda_t^6 + \nu_{e,t}^9 - \nu_{e,m,t}^{13} = 0 & \text{otherwise} \end{cases}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_{e,m,t}^5} = q_{e,m,t}^{del} + q_{e,m,t}^{arb} - q_{e,m,t} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t^6} = \sum_e [q_{e,M1,t}^{del} + q_{e,M2,t}^{arb}] - d_{M1,t} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t^7} = \sum_e [q_{e,M2,t}^{del} + q_{e,M1,t}^{arb}] - d_{M2,t} = 0$$

TORTUOUS - AMAT (BIG M)

$$0 \leq \sum_m q_{e,m,t} \perp \nu_{e,t}^3 \geq 0 \Leftrightarrow \nu_{e,t}^3 \geq \psi_{e,t}^3 \cdot M^3; \sum_m q_{e,m,t} \leq (1 - \psi_{e,t}^3) M^3; \psi_{e,t}^3 \in [0, 1]$$

$$0 \leq \bar{q}_{e,t} - \sum_m q_{e,m,t} \perp \nu_{e,t}^4 \geq 0 \Leftrightarrow \nu_{e,t}^4 \geq \psi_{e,t}^4 \cdot M^4; \bar{q}_{e,t} - \sum_m q_{e,m,t} \leq (1 - \psi_{e,t}^4) M^4; \psi_{e,t}^4 \in [0, 1]$$

$$d \times d_{e,t} - q_{e,M1,t}^{del} - q_{e,M2,t}^{arb} \geq 0 \perp \nu_{e,t}^9 \geq 0 \Leftrightarrow \nu_{e,t}^9 \geq \psi_{e,t}^9 \cdot M^9; d \times d_{e,t} - q_{e,M1,t}^{del} - q_{e,M2,t}^{arb} \leq (1 - \psi_{e,t}^9) M^9; \psi_{e,t}^9 \in [0, 1]$$

$$q_{e,m,t} \geq 0 \perp \nu_{e,m,t}^{10} \geq 0 \Leftrightarrow \nu_{e,m,t}^{10} \geq \psi_{e,m,t}^{10} \cdot M^{10}; q_{e,m,t} \leq (1 - \psi_{e,m,t}^{10}) M^{10}; \psi_{e,m,t}^{10} \in [0, 1]$$

$$\bar{q}_{e,t} \geq 0 \perp \nu_{e,t}^{11} \geq 0: \forall e \in \mathcal{E} \setminus \{i\}, t \Leftrightarrow \nu_{e,t}^{11} \geq \psi_{e,t}^{11} \cdot M^{11}; \bar{q}_{e,t} \leq (1 - \psi_{e,t}^{11}) M^{11}; \psi_{e,t}^{11} \in [0, 1]$$

$$q_{e,m,t}^{del} \geq 0 \perp \nu_{e,m,t}^{12} \geq 0 \Leftrightarrow \nu_{e,m,t}^{12} \geq \psi_{e,m,t}^{12} \cdot M^{12}; q_{e,m,t}^{del} \leq (1 - \psi_{e,m,t}^{12}) M^{12}; \psi_{e,m,t}^{12} \in [0, 1]$$

$$q_{e,m,t}^{arb} \geq 0 \perp \nu_{e,m,t}^{13} \geq 0 \Leftrightarrow \nu_{e,m,t}^{13} \geq \psi_{e,m,t}^{13} \cdot M^{13}; q_{e,m,t}^{arb} \leq (1 - \psi_{e,m,t}^{13}) M^{13}; \psi_{e,m,t}^{13} \in [0, 1]$$