# Appendices

## A Mathematical formulation of the bi-level optimization problem

### A.1 Lower-level problem: market clearing at minimized total cost

#### A.1.1 Primal decision variables

$$x = [q_{e,m,t}, \bar{q}_{e',t}, \bar{q}^{add}_{e',t}, \bar{q}^{retire}_{e',t}, q^{arb}_{M1,M2,t}, q^{del}_{1,m,t}, q^{arb}_{1,m,t}, q^{stock,in/out}_{M1,t}, q^{stock,stored}_{M1,t}] \tag{22}$$

#### A.1.2 Objective function

$$\min_{x} \underbrace{\sum_{e}\sum_{m}\sum_{t} c^{gen}_{e,t} \times q_{e,m,t}}_{\text{Generation cost of all exporters}} + \underbrace{\sum_{e'}\sum_{t} c^{main}_{e'} \times \bar{q}_{e',t}}_{\text{Maintenance cost of fringe exporters}} + \underbrace{\sum_{t} c^{stock} \times q^{stock,stored}_{M1,t}}_{\text{Stockpiling cost of European market}} \tag{23}$$

#### A.1.3 Constraints (primal problem)

**Equality constraints**

$$q^{del}_{1,m,t} + q^{arb}_{1,m,t} - q_{1,m,t} = 0 \quad : \forall m, t \quad (\lambda^4_{m,t}) \tag{24}$$

$$\left[\sum_{e'} q_{e',M1,t}\right] - q^{arb}_{M1,M2,t} + q^{del}_{1,M1,t} + q^{arb}_{1,M2,t} + q^{stock,in/out}_{M1,t} - d_{M1,t} = 0 \quad : \forall t \quad (\lambda^5_t) \tag{25}$$

$$\left[\sum_{e'} q_{e',M2,t}\right] + q^{arb}_{M1,M2,t} + q^{del}_{1,M2,t} + q^{arb}_{1,M1,t} - d_{M2,t} = 0 \quad : \forall t \quad (\lambda^6_t) \tag{26}$$

$$q_{e,M1,t} = 0 \quad : \forall \underline{\boldsymbol{e}}, \quad (\lambda^7_{\underline{\boldsymbol{e}},t}) \tag{27}$$

$$q^{stock,stored}_{M1,2025} = 0 \quad (\lambda^9) \tag{28}$$

$$q^{stock,stored}_{M1,t} - q^{stock,stored}_{M1,t-1} + q^{stock,in/out}_{M1,t} = 0 \quad : \forall t' \quad (\lambda^{10}_{t'}) \tag{29}$$

**Inequality constraints**

$$\left[\sum_{m} q_{e,m,t}\right] - \bar{q}_{e,t} \leq 0 \quad : \forall e, t \quad (\mu^3_{e,t}) \tag{30}$$

$$q^{del}_{1,M1,t} + q^{arb}_{1,M2,t} - \alpha \times d_{M1,t} \leq 0 \quad : \forall t \quad (\mu^8_t) \tag{31}$$

$$-q_{e,m,t} \leq 0 \quad : \forall e, m, t \quad (\mu^{11}_{e,m,t}) \tag{32}$$

$$-\bar{q}_{e',t} \leq 0 \quad : \forall e', t \quad (\mu^{12}_{e',t}) \tag{33}$$

$$-q^{del}_{1,m,t} \leq 0 \quad : \forall m, t \quad (\mu^{13}_{m,t}) \tag{34}$$

$$-q^{arb}_{1,m,t} \leq 0 \quad : \forall m, t \quad (\mu^{14}_{m,t}) \tag{35}$$

$$-q^{stock,stored}_{M1,t} \leq 0 \quad : \forall t \quad (\mu^{15}_{t}) \tag{36}$$

#### A.1.4 Dual decision variables

$$\lambda = [\lambda^4_{m,t}, \lambda^5_t, \lambda^6_t, \lambda^7_{\underline{e},t}, \lambda^9, \lambda^{10}_{t'}] \tag{37}$$

$$\mu = [\mu^3_{e,t}, \mu^8_t, \mu^{11}_{e,m,t}, \mu^{12}_{e',t}, \mu^{13}_{m,t}, \mu^{14}_{m,t}, \mu^{15}_t] \tag{38}$$

### A.1.5 Lagrangian function

$$
\begin{aligned}
\mathcal{L}(x,\lambda,\mu) = & \sum_{e}\sum_{m}\sum_{t} c^{gen}_{e,t} \times q_{e,m,t} + \sum_{e'}\sum_{t} c^{main}_{e'} \times \bar{q}_{e',t} + \sum_{t} c^{stock} \times q^{stock,stored}_{M1,t} \\
& + \sum_{m}\sum_{t} \lambda^4_{m,t} \times \left\{ q^{del}_{1,m,t} + q^{arb}_{1,m,t} - q_{1,m,t} \right\} \\
& + \sum_{t} \lambda^5_{t} \times \left\{ \left[ \sum_{e'} q_{e',M1,t} \right] - q^{arb}_{M1,M2,t} + q^{del}_{1,M1,t} + q^{arb}_{1,M2,t} + q^{stock,in/out}_{M1,t} - d_{M1,t} \right\} \\
& + \sum_{t} \lambda^6_{t} \times \left\{ \left[ \sum_{e'} q_{e',M2,t} \right] + q^{arb}_{M1,M2,t} + q^{del}_{1,M2,t} + q^{arb}_{1,M1,t} - d_{M2,t} \right\} \\
& + \sum_{\underline{e}}\sum_{t} \lambda^7_{\underline{e},t} \times \left\{ q_{\underline{e},M1,t} \right\} \\
& + \lambda^9 \times \left\{ q^{stock,stored}_{M1,2025} \right\} \\
& + \sum_{t'} \lambda^{10}_{t'} \times \left\{ q^{stock,stored}_{M1,t'} - q^{stock,stored}_{M1,t'-1} + q^{stock,in/out}_{M1,t'} \right\} \\
& + \sum_{e}\sum_{t} \mu^3_{e,t} \times \left\{ \left[ \sum_{m} q_{e,m,t} \right] - \bar{q}_{e,t} \right\} \\
& + \sum_{t} \mu^8_{t} \times \left\{ q^{del}_{1,M1,t} + q^{arb}_{1,M2,t} - \alpha \times d_{M1,t} \right\} \\
& + \sum_{e}\sum_{m}\sum_{t} \mu^{11}_{e,m,t} \times \left\{ -q_{e,m,t} \right\} \\
& + \sum_{e'}\sum_{t} \mu^{12}_{e',t} \times \left\{ -\bar{q}_{e',t} \right\} \\
& + \sum_{m}\sum_{t} \mu^{13}_{m,t} \times \left\{ -q^{del}_{1,m,t} \right\} \\
& + \sum_{m}\sum_{t} \mu^{14}_{m,t} \times \left\{ -q^{arb}_{1,m,t} \right\} \\
& + \sum_{t} \mu^{15}_{t} \times \left\{ -q^{stock,stored}_{M1,t} \right\}
\end{aligned}
\tag{39}
$$

### A.1.6 Karush–Kuhn–Tucker conditions

$$
\frac{\partial \mathcal{L}}{\partial q_{e,m,t}} = \begin{cases}
c^{gen}_{1,t} - \lambda^4_{m,t} + \mu^3_{1,t} - \mu^{11}_{1,m,t} = 0 & : \forall m,t \text{ if } 1 \notin \underline{\mathcal{E}} \\
c^{gen}_{1,t} - \lambda^4_{m,t} + \lambda^7_{1,t} + \mu^3_{1,t} - \mu^{11}_{1,m,t} = 0 & : \forall m,t \text{ if } 1 \in \underline{\mathcal{E}} \\
c^{gen}_{e',t} + \lambda^5_{t} + \mu^3_{e',t} - \mu^{11}_{e',M1,t} = 0 & : \forall e' \notin \underline{\mathcal{E}}, t \\
c^{gen}_{e',t} + \lambda^5_{t} + \lambda^7_{e',t} + \mu^3_{e',t} - \mu^{11}_{e',M1,t} = 0 & : \forall e' \in \underline{\mathcal{E}}, t \\
c^{gen}_{e',t} + \lambda^6_{t} + \mu^3_{e',t} - \mu^{11}_{e',M2,t} = 0 & : \forall e' \notin \underline{\mathcal{E}}, t \\
c^{gen}_{e',t} + \lambda^6_{t} + \lambda^7_{e',t} + \mu^3_{e',t} - \mu^{11}_{e',M2,t} = 0 & : \forall e' \in \underline{\mathcal{E}}, t
\end{cases}
\tag{40}
$$

$$
\frac{\partial \mathcal{L}}{\partial \bar{q}_{e',t}} = c^{main}_{e',t} + \mu^3_{e',t} - \mu^{12}_{e',t} = 0 \quad : \forall e', t
\tag{41}
$$

$$\frac{\partial \mathcal{L}}{\partial q^{arb}_{M1,M2,t}} = -\lambda^5_t + \lambda^6_t = 0 \quad : \forall t \tag{42}$$

$$\frac{\partial \mathcal{L}}{\partial q^{del}_{1,m,t}} = \begin{cases} \lambda^4_{M1,t} + \lambda^5_t + \mu^8_t - \mu^{13}_{M1,t} = 0 & : \forall t \\ \lambda^4_{M2,t} + \lambda^6_t - \mu^{13}_{M2,t} = 0 & : \forall t \end{cases} \tag{43}$$

$$\frac{\partial \mathcal{L}}{\partial q^{arb}_{1,m,t}} = \begin{cases} \lambda^4_{M1,t} + \lambda^6_t - \mu^{14}_{M1,t} = 0 & : \forall t \\ \lambda^4_{M2,t} + \lambda^5_t + \mu^8_t - \mu^{14}_{M2,t} = 0 & : \forall t \end{cases} \tag{44}$$

$$\frac{\partial \mathcal{L}}{\partial q^{stock,in/out}_{M1,t}} = \begin{cases} \lambda^5_{2025} - \mu^{15}_{2025} = 0 \\ \lambda^5_{t'} + \lambda^{10}_{t'} - \mu^{15}_{t'} = 0 & : \forall t' \end{cases} \tag{45}$$

$$\frac{\partial \mathcal{L}}{\partial q^{stock,stored}_{M1,t}} = \begin{cases} c^{stock} + \lambda^9 - \lambda^{10}_{2026} = 0 \\ c^{stock} + \lambda^{10}_{t'} - \lambda^{10}_{t'+1} = 0 & : \forall t' \setminus \{2040\} \\ c^{stock} + \lambda^{10}_{2040} = 0 \end{cases} \tag{46}$$

$$0 \leq \mu^3_{e,t} \quad \perp \quad \left[\sum_m q_{e,m,t}\right] - \bar{q}_{e,t} \leq 0 \quad : \forall e, t \tag{47}$$

$$0 \leq \mu^8_t \quad \perp \quad q^{del}_{1,M1,t} + q^{arb}_{1,M2,t} - \alpha \times d_{M1,t} \leq 0 \quad : \forall t \tag{48}$$

$$0 \leq \mu^{11}_{e,m,t} \quad \perp \quad -q_{e,m,t} \leq 0 \quad : \forall e, m, t \tag{49}$$

$$0 \leq \mu^{12}_{e',t} \quad \perp \quad -\bar{q}_{e',t} \leq 0 \quad : \forall e', t \tag{50}$$

$$0 \leq \mu^{13}_{m,t} \quad \perp \quad -q^{del}_{1,m,t} \leq 0 \quad : \forall m, t \tag{51}$$

$$0 \leq \mu^{14}_{m,t} \quad \perp \quad -q^{arb}_{1,m,t} \leq 0 \quad : \forall m, t \tag{52}$$

$$0 \leq \mu^{15}_t \quad \perp \quad -q^{stock,stored}_{M1,t} \leq 0 \quad : \forall m, t \tag{53}$$

### A.1.7 Complementarity condition linearization

The complementarity conditions in Equations 47 to 53 are linearized using the well-known linear expressions (see [19]) as follows, where $u$ is a binary decision variable and $M$ is a parameter large enough to ensure complementarity (both indexed accordingly).

$$\begin{aligned} 0 \leq \mu^3_{e,t} \leq M^3 \times u^3_{e,t} \quad &: \forall e, t \\ 0 \leq \left[\sum_m q_{e,m,t}\right] - \bar{q}_{e,t} \leq M^3 \times \left(1 - u^3_{e,t}\right) \quad &: \forall e, t \end{aligned} \tag{54}$$

$$\begin{aligned} 0 \leq \mu^8_t \leq M^8 \times u^8_t \quad &: \forall t \\ 0 \leq q^{del}_{1,M1,t} + q^{arb}_{1,M2,t} - \alpha \times d_{M1,t} \leq M^8 \times \left(1 - u^8_t\right) \quad &: \forall t \end{aligned} \tag{55}$$

$$\begin{aligned} 0 \leq \mu^{11}_{e,m,t} \leq M^{11} \times u^{11}_{e,m,t} \quad &: \forall e, m, t \\ 0 \leq q_{e,m,t} \leq M^{11} \times \left(1 - u^{11}_{e,m,t}\right) \quad &: \forall e, m, t \end{aligned} \tag{56}$$

$$\begin{aligned} 0 \leq \mu^{12}_{e',t} \leq M^{12} \times u^{12}_{e',t} \quad &: \forall e', t \\ 0 \leq \bar{q}_{e',t} \leq M^{12} \times \left(1 - u^{12}_{e',t}\right) \quad &: \forall e', t \end{aligned} \tag{57}$$

$$\begin{aligned} 0 \leq \mu^{13}_{m,t} \leq M^{13} \times u^{13}_{m,t} \quad &: \forall m, t \\ 0 \leq q^{del}_{1,m,t} \leq M^{13} \times \left(1 - u^{13}_{m,t}\right) \quad &: \forall m, t \end{aligned} \tag{58}$$

$$\begin{aligned} 0 \leq \mu^{14}_{m,t} \leq M^{14} \times u^{14}_{m,t} \quad &: \forall m, t \\ 0 \leq q^{arb}_{1,m,t} \leq M^{14} \times \left(1 - u^{14}_{m,t}\right) \quad &: \forall m, t \end{aligned} \tag{59}$$

$$\begin{aligned} 0 \leq \mu^{15}_t \leq M^{15} \times u^{15}_t \quad &: \forall t \\ 0 \leq q^{stock,stored}_{M1,t} \leq M^{15} \times \left(1 - u^{15}_t\right) \quad &: \forall t \end{aligned} \tag{60}$$

## A.2 Upper-level problem: profit maximization of the major exporter

### A.2.1 Decision variables

$$\mathcal{Y} = [c_{1,t}, \bar{q}_{1,t}] \tag{61}$$

### A.2.2 Objective function

$$\max_{\mathcal{Y}} \sum_m \sum_t q_{1,m,t} \times (\lambda_t - \tilde{c}) \tag{62}$$

### A.2.3 Constraints

$$0 \leq \bar{q}_{1,t} \leq \tilde{q}_1 \quad : \forall t \tag{63}$$

### A.2.4 Linear reformulation

- Of the non-linear term $q_{1,m,t} \times \lambda_t$ (see Equation 62)
- With $m$,$n$ in [($M1$, 5), ($M2$, 6)] (see Equations 25 and 26)
- With the following new variables: $\sigma_{e',m,t}$ (binary), $\tilde{\lambda}^n_{e',t}$ (continuous), $\tilde{\sigma}_{e',m,t}$ (binary)
- With the following new parameters: $\tilde{\beta}$ (large enough), $\epsilon$ (small enough)

$$\lambda_t = \lambda^n_t = \sum_{e'} c_{e'} \times \sigma_{e',m,t} \quad : \forall t, m, n \tag{64}$$

$$\sum_{e'} \sigma_{e',m,t} = 1 \quad : \forall t, m \tag{65}$$

$$\tilde{\lambda}^n_{e',t} = c_{e'} \times \tilde{\sigma}_{e',m,t} \quad : \forall e', t, m, n \tag{66}$$

$$q_{e',m,t} \leq \tilde{\beta} \times \tilde{\sigma}_{e',m,t} \quad : \forall e', m, t \tag{67}$$

$$q_{e',m,t} \geq \epsilon \times \tilde{\sigma}_{e',m,t} \quad : \forall e', m, t \tag{68}$$

$$\sigma_{e',m,t} \leq \tilde{\sigma}_{e',m,t} \quad : \forall e', m, t \tag{69}$$

$$\lambda^n_t \geq \tilde{\lambda}^n_{e',t} \quad : \forall e', t, n \tag{70}$$

- Introducing the following new variable $z_{e',m,t}$ (continuous) and parameter $\beta$ (large enough)

$$z_{e',m,t} \leq \beta \times \sigma_{e',m,t} \quad : \forall e', m, t \tag{71}$$

$$z_{e',m,t} \leq q_{1,m,t} \quad : \forall e', m, t \tag{72}$$

$$z_{e',m,t} \geq q_{1,m,t} - \left(1 - \sigma_{e',m,t}\right) \times \beta \quad : \forall e', m, t \tag{73}$$

$$z_{e',m,t} \geq 0 \quad : \forall e', m, t \tag{74}$$

### A.3 Completed optimization problem

$$
\begin{aligned}
\max_{x,\lambda,\mu,u,y,\sigma,z} \sum_{e'} \sum_{m} \sum_{t} c_{e'} \times z_{e',m,t} - \sum_{m} \sum_{t} q_{1,m,t} \times \tilde{c} \\
\text{s.t. } (40) - (46) \\
(54) - (60) \\
(64) - (74) \\
(63) \\
(18)
\end{aligned}
\tag{75}
$$

## B Data

MAX

### B.1 Assumptions regarding the potential of recycling platinum