Linear approximation of the objective function (upper-level problem)

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$$\underset{y}{\text{maximize}} \sum_m \sum_t q_{i,m,t} \left(\lambda_t - \tilde{c}\right)$$

$$\Rightarrow \sum_m \sum_t \underbrace{q_{i,m,t} \times \lambda_{m,t}}_{\text{product of two continous variables}} - q_{i,m,t} \times \tilde{c} \quad (0)$$

$$y = [c_i, \bar{q}_{i,t}]$$

Linearize binary x continous

Binary v

Nonnegative w

$$z \leq M \cdot v$$

$$z \leq w$$

$$z \geq w + M \cdot v - M$$

Source: Reformulation techniques in mathematical Programming (page 45)

Reformulation of the non-linear term $(q_{i,m,t} \times \lambda_{m,t})$

$$\lambda_{m,t} = \sum_{e \setminus \{i\}} \sigma_{e,m,t} \times c_e + \underbrace{\sigma_{i,m,t} \times c_i}_{\text{error term}} \quad (1)$$

$\Rightarrow$ Insert (1) in (0)

$$\sum_m \sum_t q_{i,m,t} \times \left[\sum_{e \setminus \{i\}} [\sigma_{e,m,t} \times c_e] - \tilde{c}\right] = \sum_m \sum_t \sum_{e \setminus \{i\}} c_e \times \underbrace{q_{i,m,t} \times \sigma_{e,m,t}}_{\text{product of continuous and binary variable } (z_{e,m,t})} - \sum_m \sum_t q_{i,m,t} \times \tilde{c} \quad (2)$$

$$z_{e,m,t} \leq M \times \sigma_{e,m,t} \quad : \forall e \setminus \{i\}, m, t \quad (3)$$

$$z_{e,m,t} \leq q_{i,m,t} \quad : \forall e \setminus \{i\}, m, t \quad (4)$$

$$z_{e,m,t} \geq q_{i,m,t} - (1 - \sigma_{e,m,t}) \times M : \forall e \setminus \{i\}, m, t \quad (5)$$

$$z_{e,m,t} \geq 0 \quad : \forall e \setminus \{i\}, m, t \quad (6)$$

$\boxed{\Rightarrow \sigma_{e,m,t} \text{ is set to one (1) for the marginal exporter only}}$

$$\sum_{e \setminus \{i\}} \sigma_{e,m,t} = 1 \quad : \forall m, t \quad (7)$$

$$\lambda'_{e,m,t} = c_e \times \sigma'_{e,m,t} \quad : \forall e \setminus \{i\}, m, t \quad (8)$$

$$q_{e,m,t} \leq M' \times \sigma'_{e,m,t} \quad : \forall e \setminus \{i\}, m, t \quad (9)$$ (note that M' is a large number)

$$q_{e,m,t} \geq \beta \times \sigma'_{e,m,t} \quad : \forall e \setminus \{i\}, m, t \quad (10)$$ (note that β is a small number)

$$\sigma_{e,m,t} \leq \sigma'_{e,m,t} \quad : \forall e \setminus \{i\}, m, t \quad (11)$$

c_e	$\sigma'_{e,m,t}$	$\lambda'_{e,m,t}$	$\lambda_{m,t}$	$\sigma_{e,m,t}$
5	1	5		0
10	1	10		0
20	1	20	20	1
25	0	0		0

example

$$\lambda_{m,t} \geq \lambda'_{e,m,t} \quad : \forall e \setminus \{i\}, m, t \qquad (12)$$

$$\lambda_{m,t} = \sum_{e \setminus \{i\}} \sigma_{e,m,t} \times c_e \quad : \forall m, t \qquad (13)$$

$$\boxed{\sum_m \sum_t \sum_{e \setminus \{i\}} c_e \times z_{e,m,t} - \sum_m \sum_t q_{i,m,t} \times \tilde{c}} \Rightarrow \text{LINEAR}$$