STANDARD FORM

$$\min_{x} \; f(x)$$

$$\text{s.t.}: \; h(x) = 0 \quad : \lambda$$

$$g(x) \leq 0 \quad : \mu$$

OUR MODEL

Decision variable vector $x = [q_{e,m,t}, \underbrace{\bar{q}_{e,t}}_{\forall e \setminus \{i\}, t}, q^{del}_{e,m,t}, q^{arb}_{e,m,t}]$

$$\min_{x} \sum_{e} \sum_{m} \sum_{t} c_e \times q_{e,m,t}$$

$$\text{s.t.}: \; -\sum_{m} q_{e,m,t} \leq 0 \quad : \forall e,t \quad (\mu^3_{e,t})$$

$$\left[\sum_{m} q_{e,m,t}\right] - \bar{q}_{e,t} \leq 0 \quad : \forall e,t \quad (\mu^4_{e,t})$$

$$q^{del}_{e,m,t} + q^{arb}_{e,m,t} - q_{e,m,t} = 0 \quad : \forall e,m,t \quad (\lambda^5_{e,m,t})$$

$$\left[\sum_{e} q^{del}_{e,M1,t} + q^{arb}_{e,M2,t}\right] - d_{M1,t} = 0 \quad : \forall t \quad (\lambda^6_t)$$

$$\left[\sum_{e} q^{del}_{e,M2,t} + q^{arb}_{e,M1,t}\right] - d_{M2,t} = 0 \quad : \forall t \quad (\lambda^7_t)$$

$$q_{e,M1,t} = 0 \quad : \forall e \in \mathcal{E}_{embargo}, t \quad (\lambda^8_{e,t})$$

$$q^{del}_{e,M1,t} + q^{arb}_{e,M2,t} - \alpha \, d_{M1,t} \leq 0 \quad : \forall e,t \quad (\mu^9_{e,t})$$

$$-q_{e,m,t} \leq 0 \quad : \forall e,m,t \quad (\mu^{10}_{e,m,t})$$

$$-\bar{q}_{e,t} \leq 0 \quad : \forall e \setminus \{i\}, t \quad (\mu^{11}_{e,t})$$

$$-q^{del}_{e,m,t} \leq 0 \quad : \forall e,m,t \quad (\mu^{12}_{e,m,t})$$

$$-q^{arb}_{e,m,t} \leq 0 \quad : \forall e,m,t \quad (\mu^{13}_{e,m,t})$$

DUAL VARIABLE VECTORS

$$\lambda = [\lambda^5_{e,m,t}, \lambda^6_t, \lambda^7_t, \lambda^8_{e,t}]$$

$$\lambda^8_{e,t} \quad \forall e \in \mathcal{E}_{embargo}, t$$

$$\mu = [\mu^3_{e,t}, \mu^4_{e,t}, \mu^9_{e,t}, \mu^{10}_{e,m,t}, \mu^{11}_{e,t}, \mu^{12}_{e,m,t}, \mu^{13}_{e,m,t}]$$

$$\mu^{11}_{e,t} \quad \forall e \in \mathcal{E} \setminus \{i\}, t$$

LAGRANGIAN FUNCTION

$$\mathcal{L}(x, \lambda, \mu) = f(x) + \lambda^T h(x) + \mu^T g(x)$$

$$\begin{aligned}
\mathcal{L}(x, \lambda, \mu) = \sum_{e} \sum_{m} \sum_{t} c_e \times q_{e,m,t} &+ \sum_{e} \sum_{m} \sum_{t} \lambda^5_{e,m,t} \left\{ q^{del}_{e,m,t} + q^{arb}_{e,m,t} - q_{e,m,t} \right\} + \\
&+ \sum_{t} \lambda^6_t \left\{ \sum_{e} \left[q^{del}_{e,M1,t} + q^{arb}_{e,M2,t} \right] - d_{M1,t} \right\} + \\
&+ \sum_{t} \lambda^7_t \left\{ \sum_{e} \left[q^{del}_{e,M2,t} + q^{arb}_{e,M1,t} \right] - d_{M2,t} \right\} + \\
&+ \sum_{e \in \mathcal{E}_{embargo}} \sum_{t} \lambda^8_{e,t} \left\{ q_{e,M1,t} \right\} + \\
&+ \sum_{e} \sum_{t} \mu^3_{e,t} \left\{ -\sum_{m} q_{e,m,t} \right\} + \\
&+ \sum_{e} \sum_{t} \mu^4_{e,t} \left\{ \sum_{m} \left[q_{e,m,t} \right] - \bar{q}_{e,t} \right\} + \\
&+ \sum_{e} \sum_{t} \mu^9_{e,t} \left\{ q^{del}_{e,M1,t} + q^{arb}_{e,M2,t} - \alpha \times d_{M1,t} \right\} +
\end{aligned}$$

$$+\sum_e\sum_t \mu^4_{e,t}\left\{\sum_m [q_{e,m,t}] - \bar{q}_{e,t}\right\} +$$

$$+\sum_e\sum_t \mu^9_{e,t}\left\{ q^{del}_{e,M1,t} + q^{arb}_{e,M2,t} - \alpha\times d_{M1,t}\right\}+$$

$$+\sum_e\sum_m\sum_t \mu^{10}_{e,m,t}\left\{-q_{e,m,t}\right\}+$$

$$+\sum_e\sum_t \mu^{11}_{e,t}\left\{-\bar{q}_{e,t}\right\}+$$

$$+\sum_e\sum_m\sum_t \mu^{12}_{e,m,t}\left\{-q^{del}_{e,m,t}\right\}+$$

$$+\sum_e\sum_m\sum_t \mu^{13}_{e,m,t}\left\{-q^{arb}_{e,m,t}\right\}$$

KKT CONDITIONS

$$\frac{\partial\mathcal{L}}{\partial q_{e,m,t}} = \begin{cases} c_e - \lambda^5_{e,m,t} + \lambda^8_{e,t} - \mu^3_{e,t} + \mu^4_{e,t} - \mu^{10}_{e,m,t} = 0 & \text{if } m = M1 \wedge e \in \mathcal{E}_{embargo} \\ c_e - \lambda^5_{e,m,t} - \mu^3_{e,t} + \mu^4_{e,t} - \mu^{10}_{e,m,t} = 0 & \text{elif } m = M1 \\ c_e - \lambda^5_{e,m,t} - \mu^3_{e,t} + \mu^4_{e,t} - \mu^{10}_{e,m,t} = 0 & \text{else} \Leftrightarrow m = M2 \end{cases}$$

$$\frac{\partial\mathcal{L}}{\partial \bar{q}_{e,t}} = -\mu^4_{e,t} - \mu^{11}_{e,t} = 0 \qquad \forall e \in \mathcal{E}\setminus\{i\}, t$$

$$\frac{\partial\mathcal{L}}{\partial q^{del}_{e,m,t}} = \begin{cases} \lambda^5_{e,m,t} + \lambda^6_t + \mu^9_{e,t} - \mu^{12}_{e,m,t} = 0 & \text{if } m = M1 \\ \lambda^5_{e,m,t} + \lambda^7_t + \mu^9_{e,t} - \mu^{12}_{e,m,t} = 0 & \text{else} \Leftrightarrow (m = M2) \end{cases}$$

$$\frac{\partial\mathcal{L}}{\partial q^{arb}_{e,m,t}} = \begin{cases} \lambda^5_{e,m,t} + \lambda^7_t - \mu^{13}_{e,m,t} = 0 & \text{if } m = M1 \\ \lambda^5_{e,m,t} + \lambda^6_t + \mu^9_{e,t} - \mu^{13}_{e,m,t} = 0 & \text{else} \Leftrightarrow (m = M2) \end{cases}$$

$$\frac{\partial\mathcal{L}}{\partial \lambda^5_{e,m,t}} = q^{del}_{e,m,t} + q^{arb}_{e,m,t} - q_{e,m,t} = 0$$

$$\frac{\partial\mathcal{L}}{\partial \lambda^6_t} = \sum_e \left[q^{del}_{e,M1,t} + q^{arb}_{e,M2,t}\right] - d_{M1,t} = 0$$

$$\frac{\partial\mathcal{L}}{\partial \lambda^7_t} = \sum_e \left[q^{del}_{e,M2,t} + q^{arb}_{e,M1,t}\right] - d_{M2,t} = 0$$

$$\frac{\partial\mathcal{L}}{\partial \lambda^8_{e,t}} = q_{e,M1,t} = 0 \qquad : \forall e \in \mathcal{E}_{embargo}$$

FORTUNY-AMAT ("BIG M")

$$0 \le \sum_m q_{e,m,t} \perp \mu^3_{e,t} \ge 0 \Leftrightarrow \boxed{\mu^3_{e,t} \ge \psi^3_{e,t}\cdot M^3;\ \sum_m q_{e,m,t} \le (1-\psi^3_{e,t})M^3;\ \psi^3_{e,t} \in [0,1]}$$

$$0 \le \bar{q}_{e,t} - \sum_m q_{e,m,t} \perp \mu^4_{e,t} \ge 0 \Leftrightarrow \mu^4_{e,t} \ge \psi^4_{e,t}\cdot M^4;\ \bar{q}_{e,t} - \sum_m q_{e,m,t} \le (1-\psi^4_{e,t})M^4;\ \psi^4_{e,t} \in [0,1]$$

$$\alpha \times d_{M1,t} - q^{del}_{e,M1,t} - q^{arb}_{e,M2,t} \ge 0 \perp \mu^9_{e,t} \ge 0 \Leftrightarrow \mu^9_{e,t} \ge \psi^9_{e,t}\cdot M^9;\ \alpha\times d_{M1,t} - q^{del}_{e,M1,t} - q^{arb}_{e,M2,t} \le (1-\psi^9_{e,t})\cdot M^9;\ \psi^9_{e,t} \in [0,1]$$

$$q_{e,m,t} \ge 0 \perp \mu^{10}_{e,m,t} \ge 0 \Leftrightarrow \mu^{10}_{e,m,t} \ge \psi^{10}_{e,m,t}\cdot M^{10};\ q_{e,m,t} \le (1-\psi^{10}_{e,m,t})\cdot M^{10};\ \psi^{10}_{e,m,t} \in [0,1]$$

$$\bar{q}_{e,t} \ge 0 \perp \mu^{11}_{e,t} \ge 0 : \forall e \in \mathcal{E}\setminus\{i\}, t \Leftrightarrow \mu^{11}_{e,t} \ge \psi^{11}_{e,t}\cdot M^{11};\ \bar{q}_{e,t} \le (1-\psi^{11}_{e,t})M^{11};\ \psi^{11}_{e,t} \in [0,1]$$

$$q^{del}_{e,m,t} \ge 0 \perp \mu^{12}_{e,m,t} \ge 0 \Leftrightarrow \mu^{12}_{e,m,t} \ge \psi^{12}_{e,m,t}\cdot M^{12};\ q^{del}_{e,m,t} \le (1-\psi^{12}_{e,m,t})M^{12};\ \psi^{12}_{e,m,t} \in [0,1]$$

$$q^{arb}_{e,m,t} \ge 0 \perp \mu^{13}_{e,m,t} \ge 0 \Leftrightarrow \mu^{13}_{e,m,t} \ge \psi^{13}_{e,m,t}\cdot M^{13};\ q^{arb}_{e,m,t} \le (1-\psi^{13}_{e,m,t})M^{13};\ \psi^{13}_{e,m,t} \in [0,1]$$

$$\ldots_{e,m,t} \geq \psi_{e,m,t} \cdot M^{13}; \quad q^{arb}_{e,m,t} \leq \left(1-\psi^{13}_{e,m,t}\right) M^{13}; \quad \psi^{13}_{e,m,t} \in [0,1]$$

$$\underset{\lambda,\nu}{\text{maximize}} \left[\min_{x} \mathcal{L}(x,\lambda,\nu) \right]$$

$$\begin{aligned}
\mathcal{L}(x,\lambda,\nu) = & \sum_{e}\sum_{m}\sum_{t} c_e \times q_{e,m,t} + \sum_{e}\sum_{m}\sum_{t} \lambda^{5}_{e,m,t} \left\{ q^{del}_{e,m,t} + q^{arb}_{e,m,t} - q_{e,m,t} \right\} + \\
& + \sum_{t} \lambda^{6}_{t} \left\{ \sum_{e} \left[q^{del}_{e,M1,t} + q^{arb}_{e,M2,t} \right] - d_{M1,t} \right\} + \\
& + \sum_{t} \lambda^{7}_{t} \left\{ \sum_{e} \left[q^{del}_{e,M2,t} + q^{arb}_{e,M1,t} \right] - d_{M2,t} \right\} + \\
& + \sum_{e \in E_{analogue}} \sum_{t} \lambda^{8}_{e,t} \left\{ q_{e,M1,t} \right\} + \\
& + \sum_{e}\sum_{t} \nu^{3}_{e,t} \left\{ - \sum_{m} q_{e,m,t} \right\} + \\
& + \sum_{e}\sum_{t} \nu^{4}_{e,t} \left\{ \sum_{m} \left[q_{e,m,t} \right] - \bar{q}_{e,t} \right\} + \quad e = i,t \\
& + \sum_{e}\sum_{t} \nu^{9}_{e,t} \left\{ q^{del}_{e,M1,t} + q^{arb}_{e,M2,t} - \alpha \times d_{M1,t} \right\} + \\
& + \sum_{e}\sum_{m}\sum_{t} \nu^{10}_{e,m,t} \left\{ - q_{e,m,t} \right\} + \\
& + \sum_{e}\sum_{t} \nu^{11}_{e,t} \left\{ - \bar{q}_{e,t} \right\} + \\
& + \sum_{e}\sum_{m}\sum_{t} \nu^{12}_{e,m,t} \left\{ - q^{del}_{e,m,t} \right\} + \\
& + \sum_{e}\sum_{m}\sum_{t} \nu^{13}_{e,m,t} \left\{ - q^{arb}_{e,m,t} \right\}
\end{aligned}$$

STRONG DUALITY

$$\underset{\lambda,\nu}{\text{maximize}} \left(- \sum_{t} \lambda^{6}_{t} d_{M1,t} - \sum_{t} \lambda^{7}_{t} d_{M2,t} - \sum_{t} \nu^{4}_{i,t} \bar{q}_{i,t} \right)$$

$$\sum_{e}\sum_{m}\sum_{t} c_e q_{e,m,t} = - \sum_{t} \lambda^{6}_{t} d_{M1,t} - \sum_{t} \lambda^{7}_{t} d_{M2,t} - \sum_{t} \nu^{4}_{i,t} \bar{q}_{i,t}$$

$$\nu^{4}_{e,t} \left\{ \sum_{m} \left[q_{e,m,t} \right] - \bar{q}_{e,t} \right\} = 0 \quad \overset{\text{with } e=i}{\Longleftrightarrow} \quad \nu^{4}_{i,t} \sum_{m} q_{i,m,t} = \nu^{4}_{i,t} \bar{q}_{i,t}$$

$$\boxed{\sum_{e}\sum_{m}\sum_{t} c_e q_{e,m,t} = - \sum_{t} \lambda^{6}_{t} d_{M1,t} - \sum_{t} \lambda^{7}_{t} d_{M2,t} - \sum_{t} \nu^{4}_{i,t} \sum_{m} q_{i,m,t}}$$