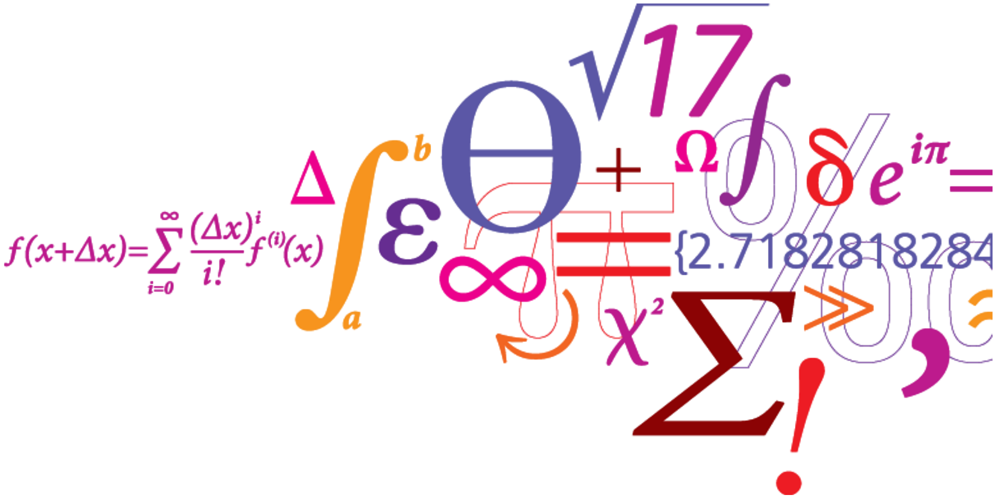
46755 – Renewables in Electricity Markets

Lectures 10-11: Offering strategy of a price-maker actor

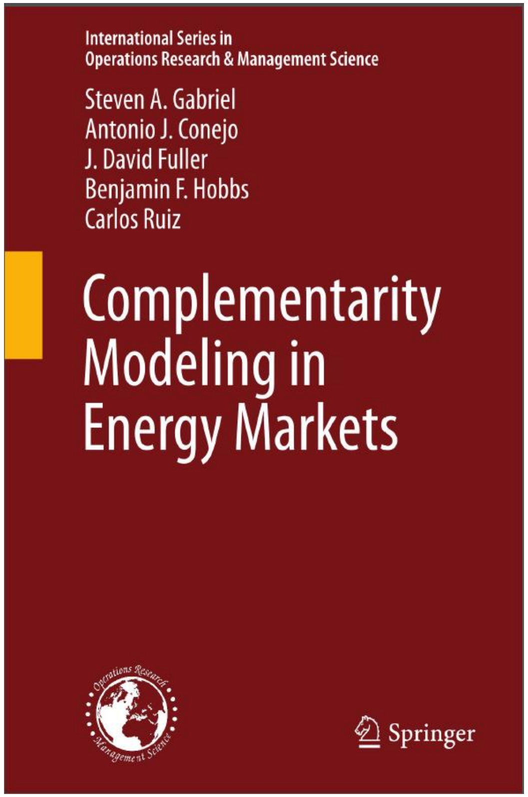
Jalal Kazempour



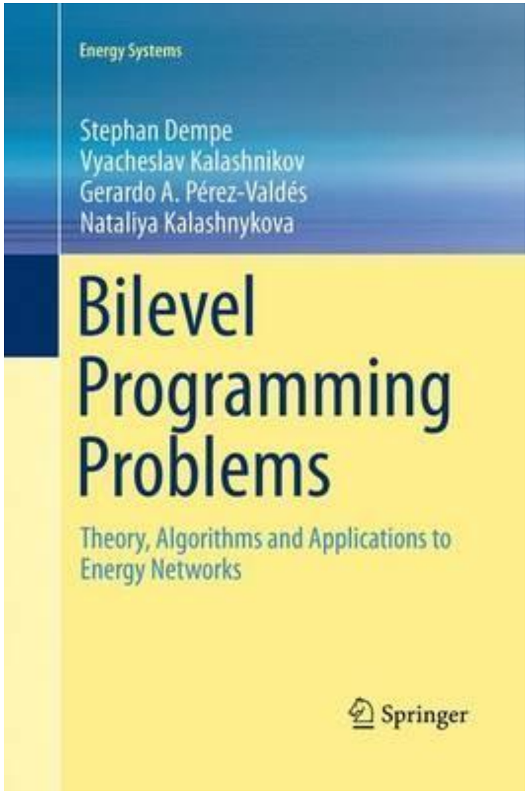
April 17, 2023

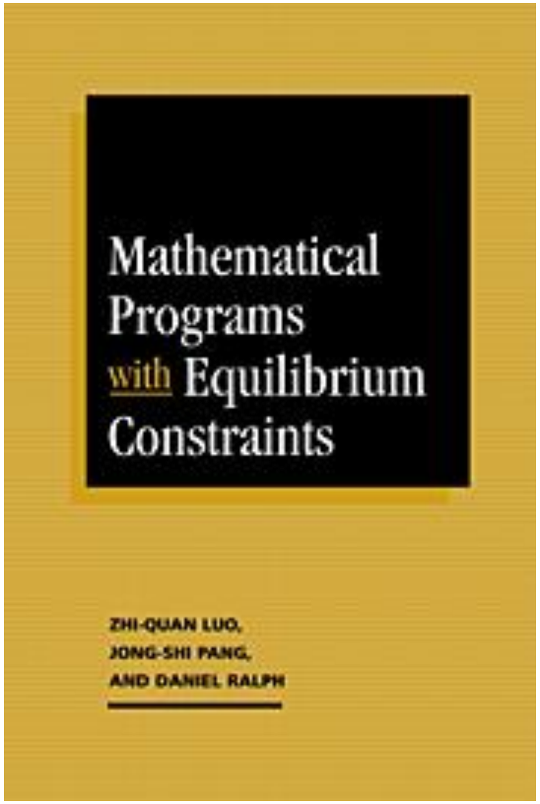
April 24, 2023

Relevant references



Chapter 6





- Pozo, D., Sauma, E., & Contreras, J. (2017). Basic theoretical foundations and insights on bilevel models and their applications to power systems. *Annals of Operations Research*, 254, 303-334.

Assumption of perfect competition

- No one exercises "market power", i.e., every market participant is price-taker!

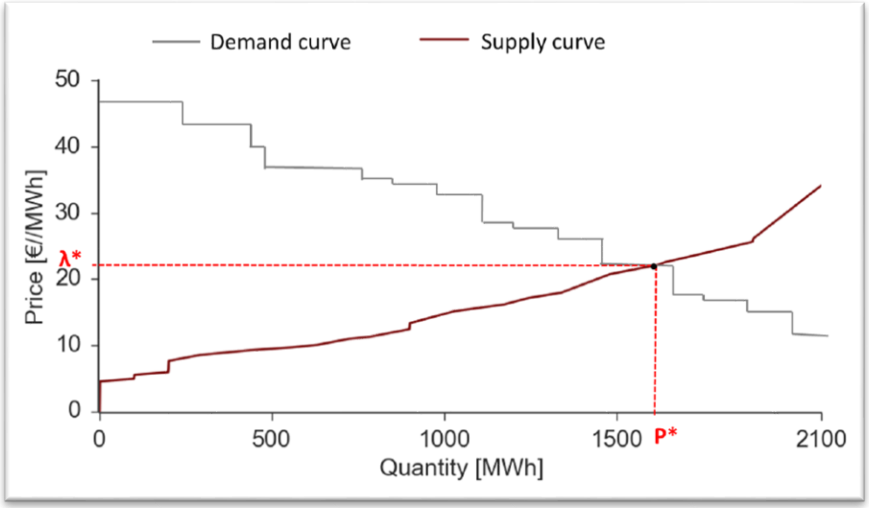
Assumption of perfect competition

- No one exercises "market power", i.e., every market participant is price-taker!

Price-taker market participant does **not** anticipate how her market participation strategy impacts market price formation and thereby market-clearing outcomes.

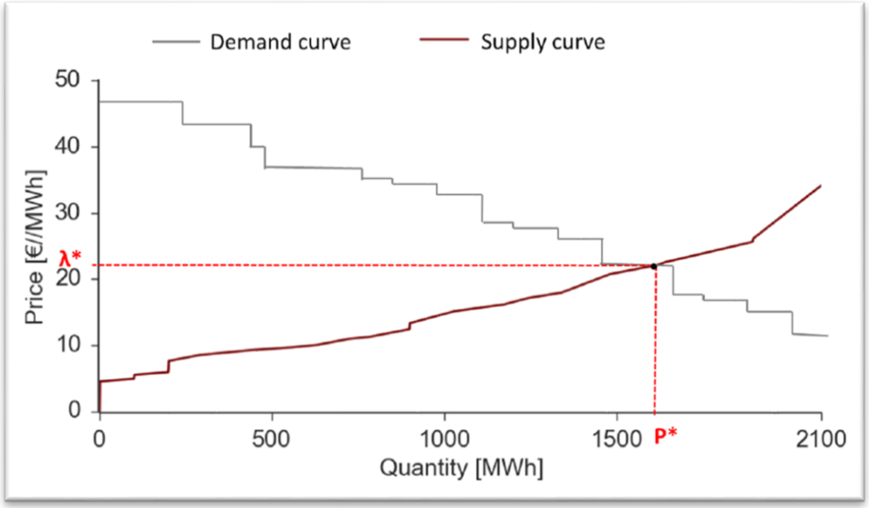
→ She submits offer based on her true cost/utility and does not seek to alter market-clearing outcomes to her own benefit!

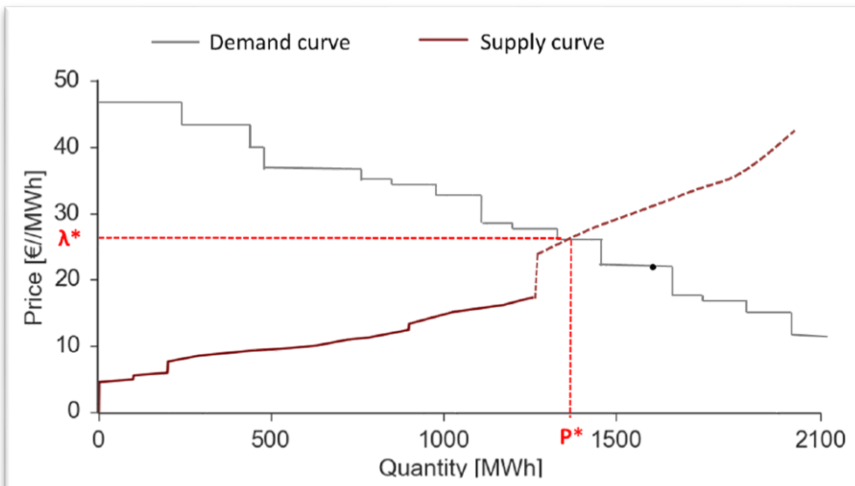
Market-clearing problem as an equilibrium



Let’s assume the marginal producer (the most expensive generator dispatched) offers trustfully at her true production cost. As market-clearing outcomes, the total demand supplied (P*) is around 1600 MW and the market price (lambda*) is around 22 euro/MWh.

Market-clearing problem as an equilibrium





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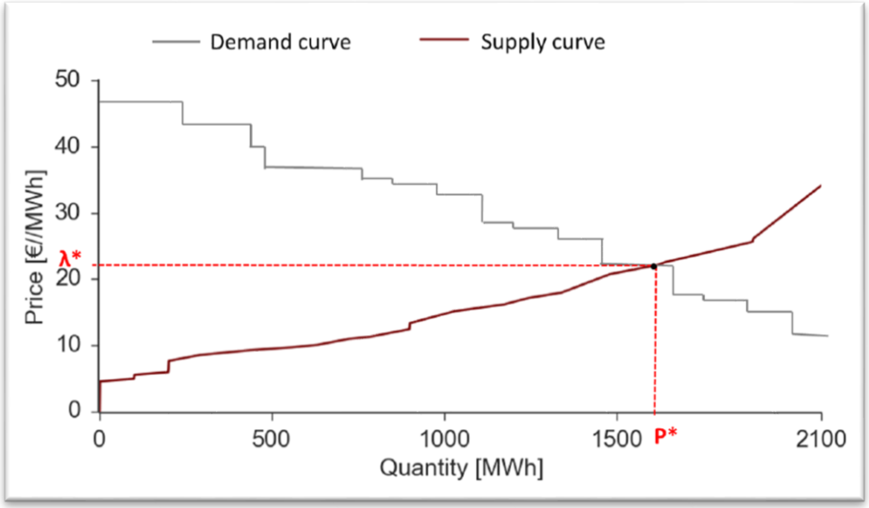

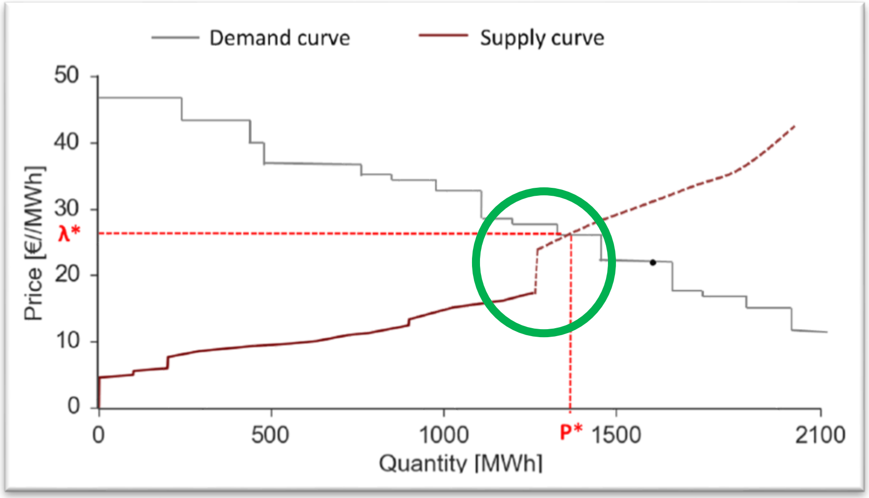

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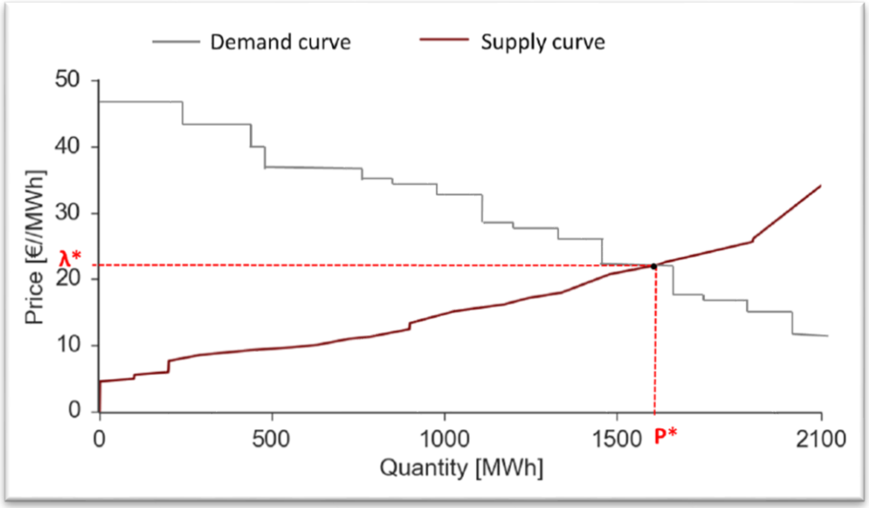

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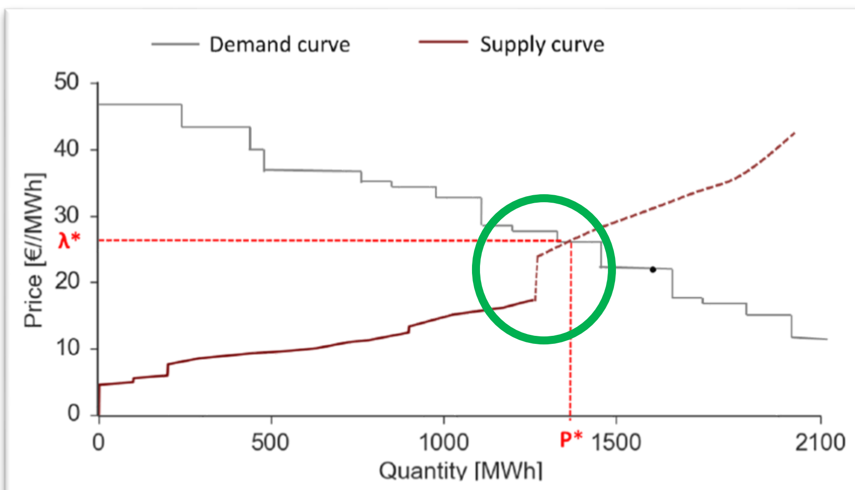
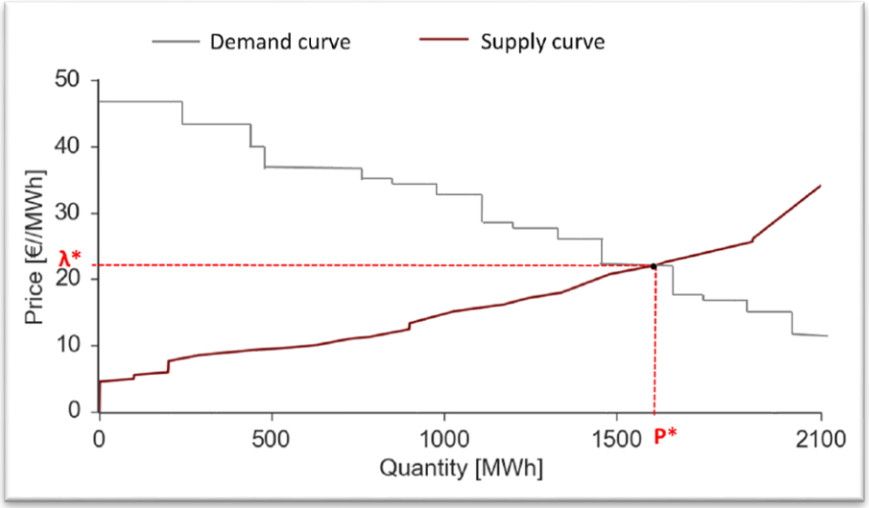
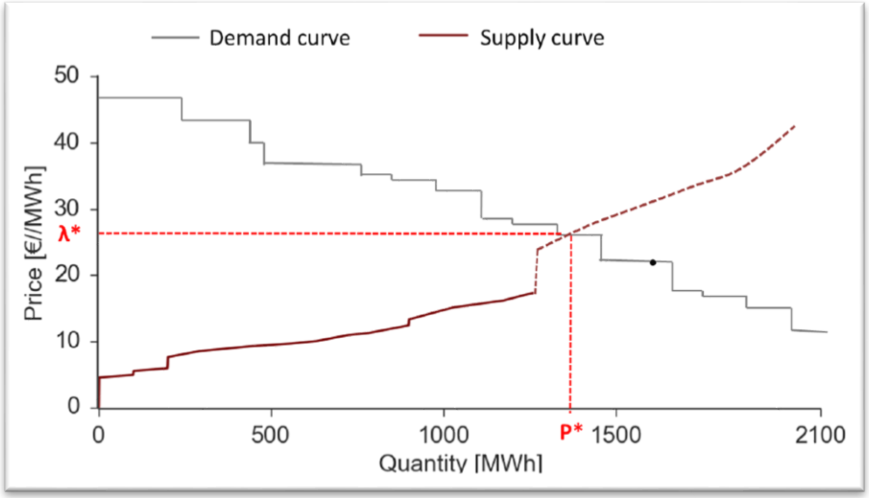


The marginal producer offers at a comparatively higher price (the so-called **strategic offering**) →

- the total demand supplied (P*) reduces to around 1350 MW,
- the market price (lambda*) is now higher, around 28 euro/MWh.

Market-clearing problem as an equilibrium



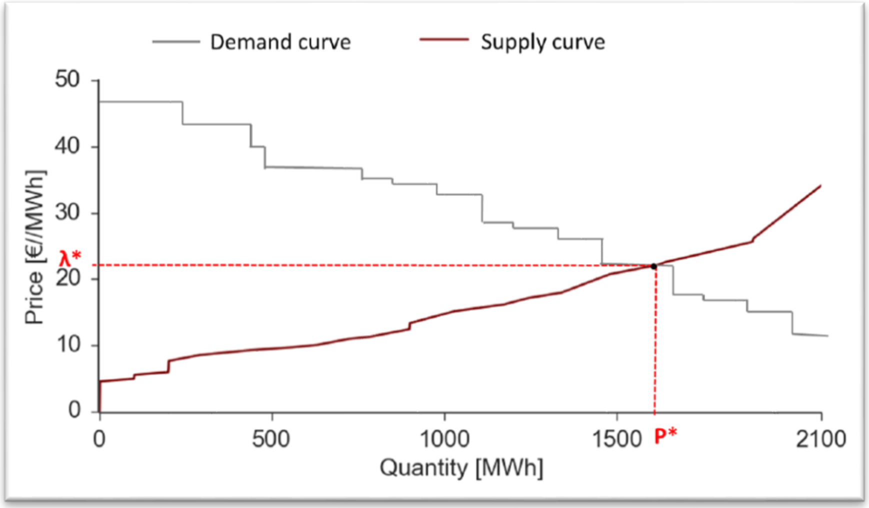


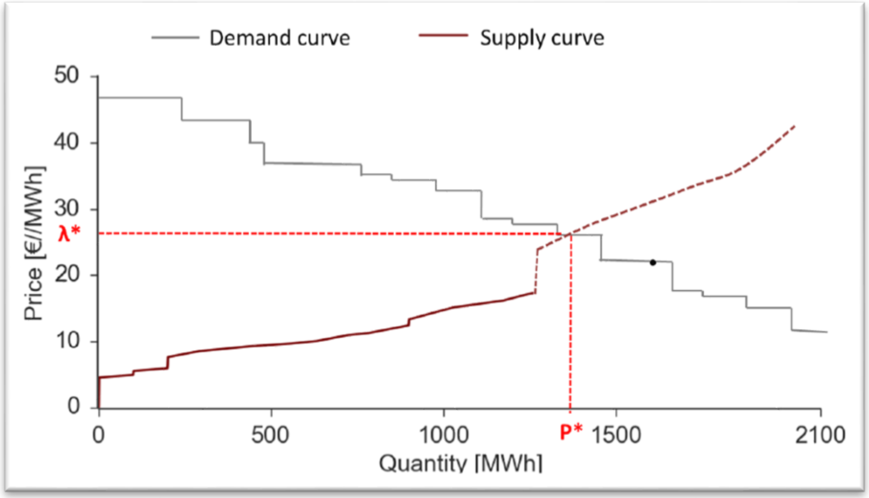
Is strategic offering beneficial to the marginal producer?

Do other producers dispatched earn more?

Does strategic offering impact the social welfare

Market-clearing problem as an equilibrium





Is strategic offering beneficial to the marginal producer? Perhaps, depending on quantity to be produced by the marginal producer and the market price!

Do other producers dispatched earn more? Definitely! They are now paid at a higher price.

Does strategic offering impact the social welfare? Yes, strategic behavior reduces the social welfare (the area between supply and demand curves)!

Market with <u>imperfect</u> competition

Price-maker (strategic) participants are able to exercise “market power”!

Market with imperfect competition

Price-maker (strategic) participants are able to exercise "market power"!

A price-maker (strategic) participant anticipates how her market participation strategy can impact market price formation, and thereby market-clearing outcomes to her own benefit.

→ She submits strategic offer, which is not necessarily equal to her true cost/utility!

How to model imperfect competition?

Common models:

How to model <u>imperfect</u> competition?

Common models:

1. Cournot competition model
2. Bertrand competition model
3. Conjectural variations model
4. Supply function model
5. etc

Cournot competition

- ✓ Each producer assumes that she is able to alter market-clearing outcomes through her production level [1]. In other words, producers compete on quantities produced.
- ✓ Market price is considered as an affine function of the total production.

[1] H. R. Varian. Microeconomic Analysis. Norton & Company, New York, 1992.

Cournot competition

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- ✓ Market price is considered as an <u>affine</u> function of the total production.

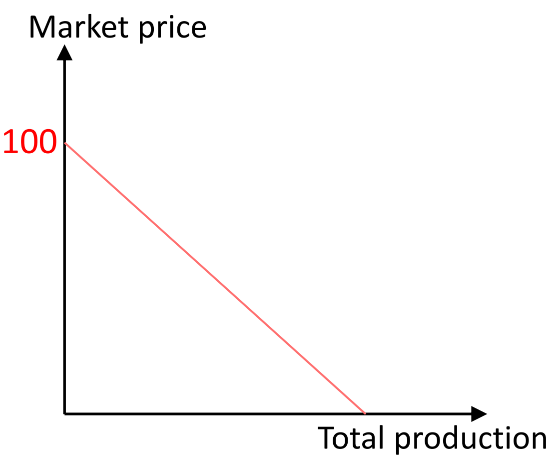
Example with <u>two</u> strategic producers:

Market price = 100 – (production 1 + production 2)

Each producer maximizes her own revenue, i.e.,

*Revenue of producer 1 = production 1 * market price.*

*Revenue of producer 2 = production 2 * market price.*



Note that the market price depends on production strategies of both producers.

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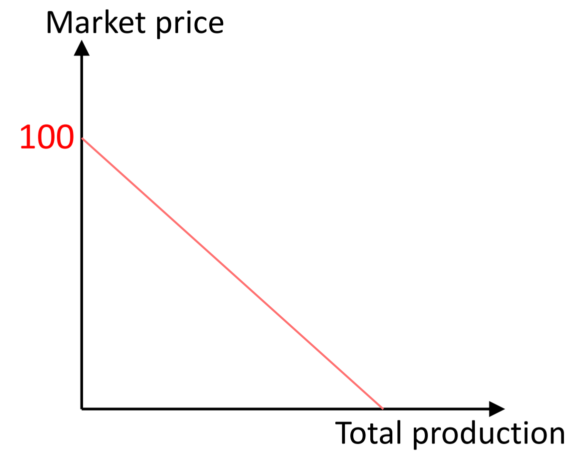

Note that the market price depends on production strategies of both producers.

In an **oligopoly**, there are multiple strategic participants, each maximizing her own benefit. There is **strategic interaction** among those participants.

The above example is a **duopoly** (two strategic producers).

[1] H. R. Varian. Microeconomic Analysis. Norton & Company, New York, 1992.

Other models

Bertrand competition model

- ✓ Similar to Cournot, market price is considered as an affine function of the total production.
- ✓ Unlike Cournot, each producer assumes that she is able to alter market-clearing outcomes through her offer price. In other words, producers compete on pricing.

Conjectural variations model

- ✓ An upgraded version of Cournot model
- ✓ Production strategy of each producer impacts not only the market price but also the production strategy of rivals modeled by given reaction parameters.
- ✓ These reactions parameters model the competitiveness level of the underlying market, ranging from a perfect competition to a monopoly (or a cartel).

Supply function model

- ✓ Each producer submits its supply function offer to the market, containing a price and a production quantity offer.
- ✓ This model constitutes a more accurate description of the functioning of real-world electricity markets if compared with other imperfect competition models such as Cournot, Bertrand or conjectural variations.

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How to model strategic behavior in a market without assuming affine function between market price and total quantity?

How to model strategic behavior in a market without assuming affine function between market price and total quantity?

Note: in models mentioned in previous slides, all participants play simultaneously!

Dynamic game → Stackelberg (leader-follower) game

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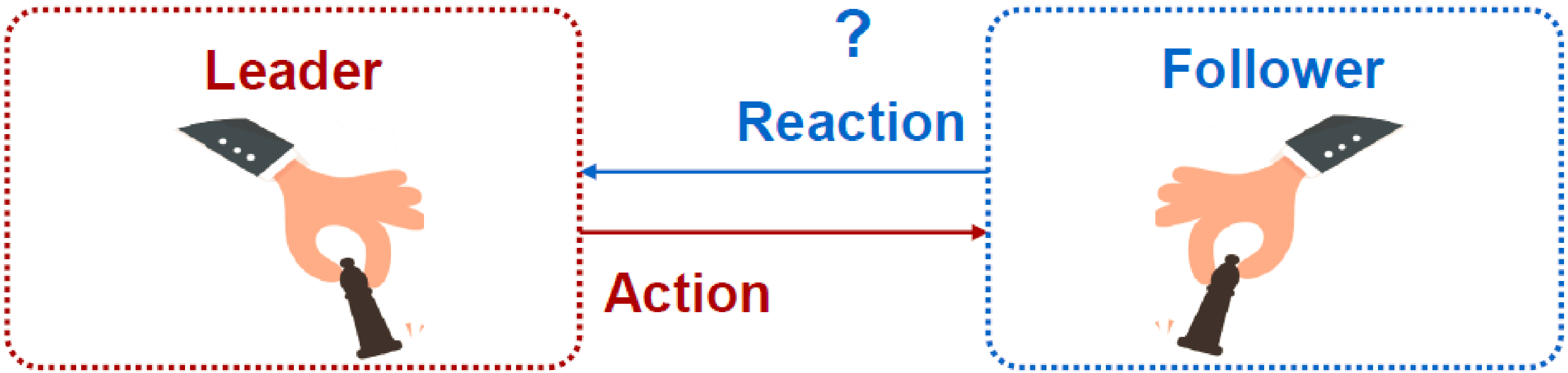
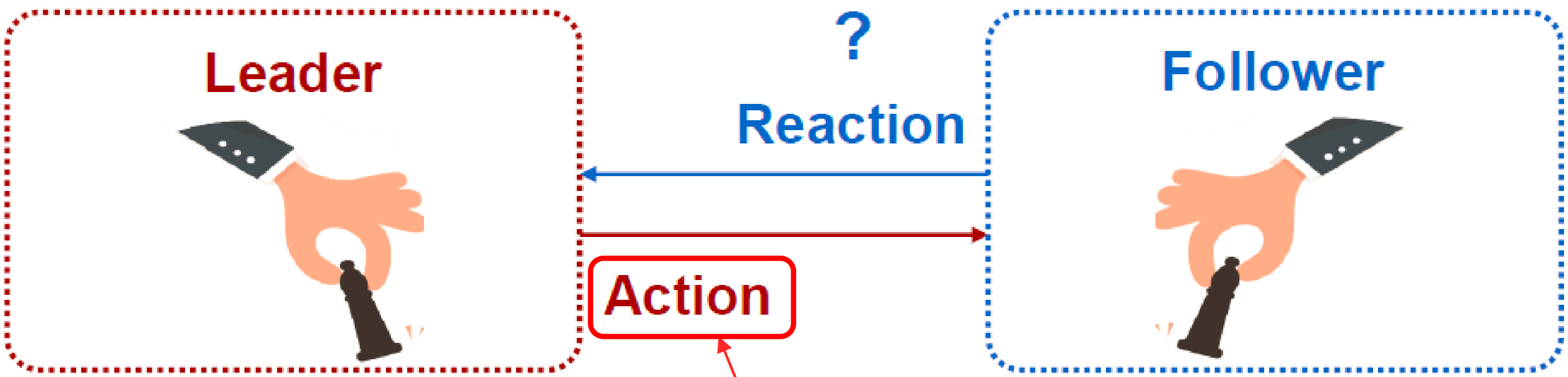
- 2-stage dynamic game



Dynamic game → Stackelberg (leader-follower) game

- 2-stage dynamic game

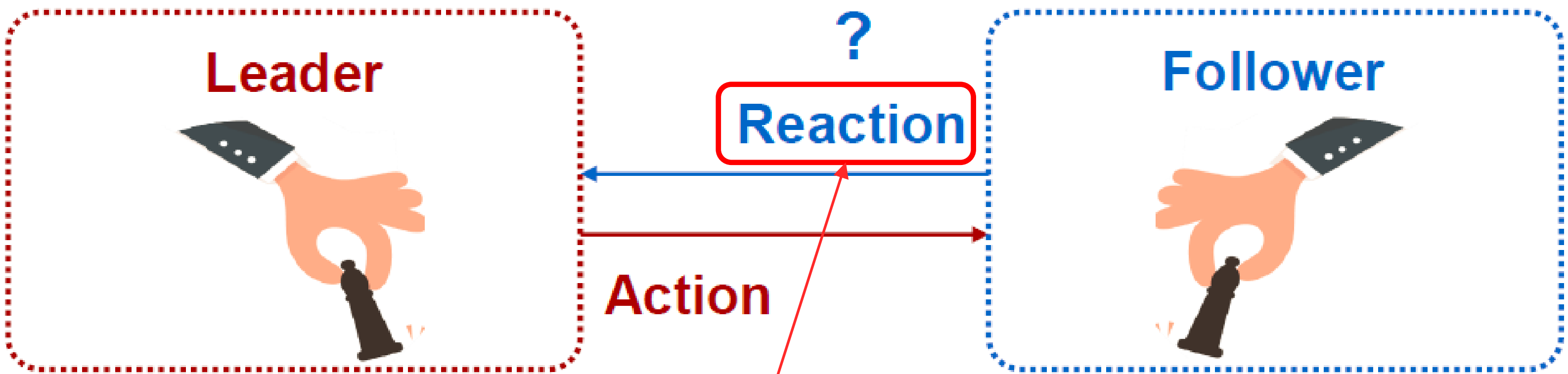


- Has "commitment" power (plays first)

Leader plays first by taking an action!

Dynamic game → Stackelberg (leader-follower) game

- 2-stage dynamic game

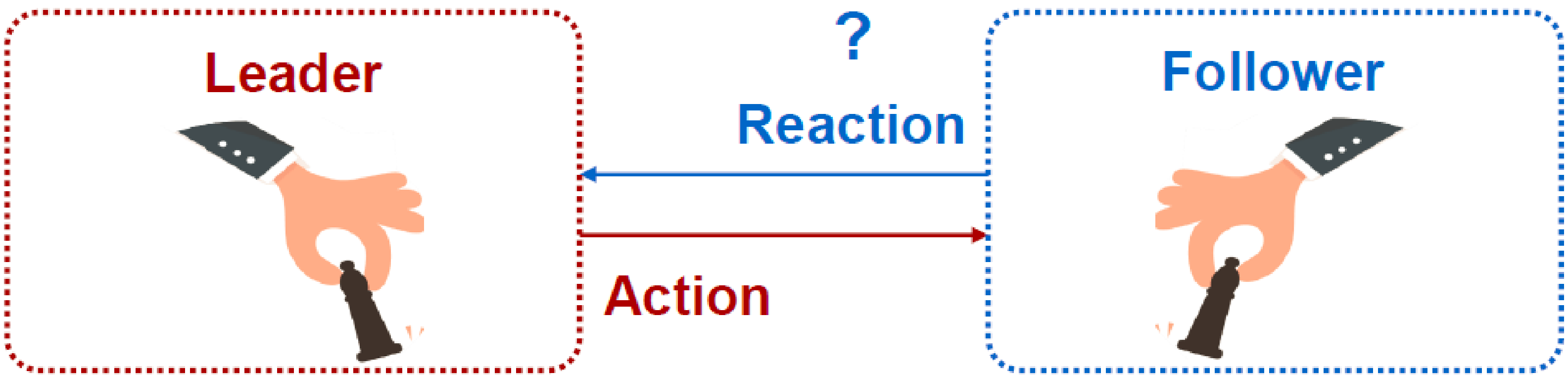


- Has “commitment” power (plays first)
- Plays second

Follower plays second and reacts to the leader’s action!

Dynamic game → Stackelberg (leader-follower) game

- 2-stage dynamic game

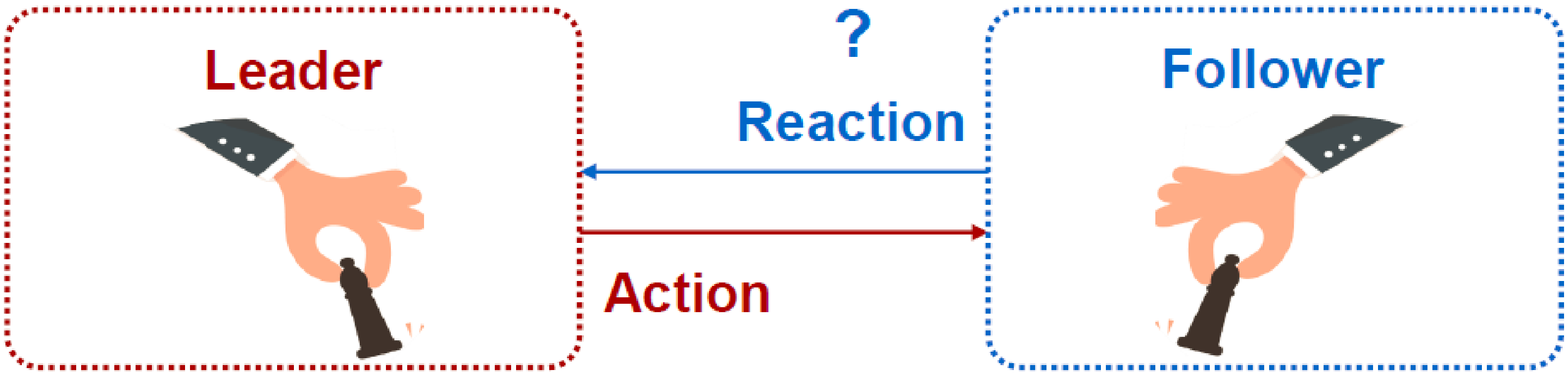


- Has "commitment" power (plays first)
- Action **influences** optimal reaction of follower
- Tries to **anticipate** the follower's reaction

- Plays second

Dynamic game → Stackelberg (leader-follower) game

- 2-stage dynamic game

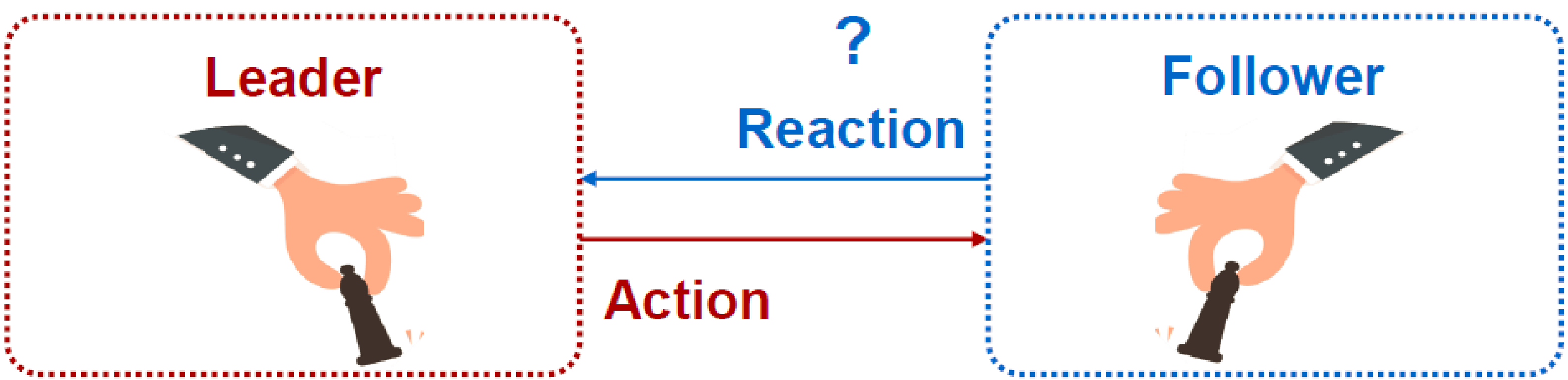


- Has “commitment” power (plays first)
- Action **influences** optimal reaction of follower
- Tries to **anticipate** the follower’s reaction

- Plays second
- Reaction **influences** leader’s profit

Dynamic game → Stackelberg (leader-follower) game

- 2-stage dynamic game



- Has “commitment” power (plays first)
- Action **influences** optimal reaction of follower
- Tries to **anticipate** the follower’s reaction

- Plays second
- Reaction **influences** leader’s profit

Question: How can the leader anticipate the follower’s reaction?

Example

- 2-player game

Leader

$$\min_{x} \; -y$$
$$\text{s.t.} \; -x \leq -2$$

$\xleftarrow{Y(x^*)}$

$\xrightarrow{X^*}$

Follower

$$\min_{y} \; y$$
$$\text{s.t.} \; y \leq 9$$

Example

- 2-player game

Leader

$$\min_{x} \; -y$$
$$\text{s.t.} \; -x \leq -2$$

$Y(x^*)$

X^*

Follower

$$\min_{y} \; y$$
$$\text{s.t.} \; y \leq 9$$

$$-x - y \leq -7$$
$$x - 2y \leq 1$$
$$6x + y \leq 45$$

Constraints linking the action of leader and reaction of follower

Example

- 2-player game

Leader

$$\min_{x,y} \; -y$$
$$\text{s.t.} \; -x \le -2$$

X^* → ← $Y^*(x^*)$

Follower

$$\min_{y} \; y$$
$$\text{s.t.} \; y \le 9$$
$$-x - y \le -7$$
$$x - 2y \le 1$$
$$6x + y \le 45$$

- Reaction of follower constrained by the action of leader **(x is fixed in the follower's problem)**
- Leader tries to anticipate the optimal reaction of the follower **(y(x) is a variable in the leader's problem)**

Example

Leader

$$\min_{x,y} \; -y$$
$$\text{s.t.} \; -x \leq -2$$

Y*(x*)

X*

Follower

$$\min_{y} \; y$$
$$\begin{aligned}\text{s.t.} \quad & y \leq 9 \\ & -x-y \leq -7 \\ & x-2y \leq 1 \\ & 6x+y \leq 45\end{aligned}$$

Example

Leader

$$\min_{x,y} \; -y$$
$$\text{s.t.} \; -x \leq -2$$

$Y^*(x^*)$

X^*

Follower

$$\min_{y} \; y$$
$$\begin{aligned} \text{s.t.} \quad & y \leq 9 \\ & -x - y \leq -7 \\ & x - 2y \leq 1 \\ & 6x + y \leq 45 \end{aligned}$$

Question: How can the leader integrate the follower's optimal reaction in its own startegy?

Example

$$\min_{x,y} \quad -y$$

$$\text{s.t.} \quad -x \le -2$$

X* Y*(x*)

$$\text{s.t.} \quad \min_{y} \quad y$$

$$\text{s.t.} \quad y \le 9$$

$$-x - y \le -7$$

$$x - 2y \le 1$$

$$6x + y \le 45$$

Leader (x,y variable)

Follower (x fixed)

We can <u>constrain</u> the leader's optimization problem by the follower's optimization problem!

Example

$$\min_{x,y} \; -y$$
$$\text{s.t.} \; -x \leq -2$$

Leader

(x,y variable)

X* ↓ ↑ Y*(x*)

$$\text{s.t.} \; \min_{y} \; y$$
$$\text{s.t.} \quad y \leq 9$$
$$-x - y \leq -7$$
$$x - 2y \leq 1$$
$$6x + y \leq 45$$

Follower

(x fixed)

We can <u>constrain</u> the leader's optimization problem by the follower's optimization problem! → Bilevel problem!

Definition

In the bilevel model, an optimization problem is constrained by another optimization problem!

Stackelberg game → bilevel model

Stackelberg game

$$
\begin{aligned}
\min_{x,y}\ & F_0(x,y) \\
\text{s.t. } & H_i(x,y) = 0, && i = 1,\dots,M \\
& G_i(x,y) \le 0, && i = 1,\dots,P
\end{aligned}
$$

Leader's optimization problem

x*

y*(x*)

$$
\begin{aligned}
\min_{y}\ & f_0(x,y) \\
\text{s.t. } & h_i(x,y) = 0, && i = 1,\dots,m \\
& g_i(x,y) \le 0, && i = 1,\dots,p
\end{aligned}
$$

Follower's optimization problem

Bilevel model

$$
\begin{aligned}
\min_{x,y}\ & F_0(x,y) \\
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& G_i(x,y) \le 0, && i = 1,\dots,P \\
& \min_{y}\ f_0(x,y) \\
& \quad \text{s.t. } h_i(x,y) = 0, && : \lambda_i,\ i = 1,\dots,m \\
& \quad\qquad g_i(x,y) \le 0, && : \mu_i,\ i = 1,\dots,p
\end{aligned}
$$

Leader's optimization problem

Follower's optimization problem

Math background: bilevel model

Upper-Level
Problem
(Leader)

Lower-Level
Problem
(Follower)

Math background: bilevel model

Upper-Level Problem (Leader)

minimize Upper-Level Objective Function

subject to

Upper-Level Constraints

Lower-Level Problem (Follower)

minimize Lower-Level Objective Function

subject to

Lower-Level Constraints

Math background: bilevel model

Upper-Level Problem (Leader)

$$\begin{aligned}
&\underset{x,y,\lambda,\mu}{\text{minimize}} \quad f^{\mathrm{U}}(x,y,\lambda,\mu)\\
&\text{subject to}\\
&\quad h^{\mathrm{U}}(x,y,\lambda,\mu)=0\\
&\quad g^{\mathrm{U}}(x,y,\lambda,\mu)\leq 0,
\end{aligned}$$

Lower-Level Problem (Follower)

$$\left\{\begin{aligned}
&\underset{y}{\text{minimize}} \quad f^{\mathrm{L}}(x,y)\\
&\text{subject to}\\
&\quad h^{\mathrm{L}}(x,y)=0 \qquad\qquad : \lambda\\
&\quad g^{\mathrm{L}}(x,y)\leq 0 \qquad\qquad : \mu.
\end{aligned}\right.$$

Math background: bilevel model

Upper-Level Problem (Leader)

$$\begin{aligned} &\underset{x,y,\lambda,\mu}{\text{minimize}} \quad f^{\mathrm{U}}(x,y,\lambda,\mu) \\ &\text{subject to} \\ &\quad h^{\mathrm{U}}(x,y,\lambda,\mu) = 0 \\ &\quad g^{\mathrm{U}}(x,y,\lambda,\mu) \leq 0, \end{aligned}$$

Lower-Level Problem (Follower)

$$\left\{ \begin{aligned} &\underset{y}{\text{minimize}} \quad f^{\mathrm{L}}(x,y) \\ &\text{subject to} \\ &\quad h^{\mathrm{L}}(x,y) = 0 \qquad\qquad : \lambda \\ &\quad g^{\mathrm{L}}(x,y) \leq 0 \qquad\qquad : \mu. \end{aligned} \right.$$

- The upper-level problem is constrained by the lower-level problem!
- In other words, the lower-level problem itself is a constraint for the upper-level problem.

Strategic offering problem of a participant

Strategic offering problem

- ✓ Let's consider a strategic power producer owning multiple generation units, indexed by *i*
- ✓ All generation units belonging to rivals are indexed by *–i*
- ✓ Demands are elastic to price, and indexed by *k*
- ✓ No transmission network, unit commitment constraints, and uncertainty (for simplicity)

Strategic offering problem

Given bid prices of demands and offer prices of producers, the market-clearing problem is

$$\underset{d_k, p_i, p_{-i}}{\text{Maximize}} \quad \sum_k \alpha_k^{\text{bid}} d_k - \sum_i \alpha_i^{\text{offer}} p_i - \sum_{-i} \alpha_{-i}^{\text{offer}} p_{-i}$$

subject to:

$$0 \leq d_k \leq D_k^{\max} \quad : \quad \underline{\mu}_k, \overline{\mu}_k \quad \forall k$$

$$0 \leq p_i \leq P_i^{\max} \quad : \quad \underline{\mu}_i, \overline{\mu}_i \quad \forall i$$

$$0 \leq p_{-i} \leq P_{-i}^{\max} \quad : \quad \underline{\mu}_{-i}, \overline{\mu}_{-i} \quad \forall -i$$

$$\sum_k d_k - \sum_i p_i - \sum_{-i} p_{-i} = 0 \quad : \quad \lambda$$

Strategic offering problem

Given bid prices of demands and offer prices of producers, the market-clearing problem is

Bid price of demand *k* (parameter)

$$\underset{d_k, p_i, p_{-i}}{\text{Maximize}} \quad \sum_k \boxed{\alpha_k^{\text{bid}}} d_k - \sum_i \alpha_i^{\text{offer}} p_i - \sum_{-i} \alpha_{-i}^{\text{offer}} p_{-i}$$

subject to:

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$$\sum_k d_k - \sum_i p_i - \sum_{-i} p_{-i} = 0 \quad : \quad \lambda$$

Strategic offering problem

Given bid prices of demands and offer prices of producers, the market-clearing problem is

Consumption of demand *k* (variable)

$$\underset{d_k, p_i, p_{-i}}{\text{Maximize}} \quad \sum_k \alpha_k^{\text{bid}} \boxed{d_k} - \sum_i \alpha_i^{\text{offer}} p_i - \sum_{-i} \alpha_{-i}^{\text{offer}} p_{-i}$$

subject to:

$$0 \leq d_k \leq D_k^{\max} \quad : \quad \underline{\mu}_k, \overline{\mu}_k \quad \forall k$$

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$$\sum_k d_k - \sum_i p_i - \sum_{-i} p_{-i} = 0 \quad : \quad \lambda$$

Strategic offering problem

Given bid prices of demands and offer prices of producers, the market-clearing problem is

Offer price of generator *i* (parameter) belonging to the strategic producer

$$\underset{d_k, p_i, p_{-i}}{\text{Maximize}} \quad \sum_k \alpha_k^{\text{bid}} d_k - \sum_i \boxed{\alpha_i^{\text{offer}}} p_i - \sum_{-i} \alpha_{-i}^{\text{offer}} p_{-i}$$

subject to:

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Strategic offering problem

Given bid prices of demands and offer prices of producers, the market-clearing problem is

Production of generator *i* (variable) belonging to the strategic producer

$$\underset{d_k, p_i, p_{-i}}{\text{Maximize}} \quad \sum_k \alpha_k^{\text{bid}} d_k - \sum_i \alpha_i^{\text{offer}} \boxed{p_i} - \sum_{-i} \alpha_{-i}^{\text{offer}} p_{-i}$$

subject to:

$$0 \leq d_k \leq D_k^{\max} \quad : \quad \underline{\mu}_k, \overline{\mu}_k \quad \forall k$$

$$0 \leq p_i \leq P_i^{\max} \quad : \quad \underline{\mu}_i, \overline{\mu}_i \quad \forall i$$

$$0 \leq p_{-i} \leq P_{-i}^{\max} \quad : \quad \underline{\mu}_{-i}, \overline{\mu}_{-i} \quad \forall -i$$

$$\sum_k d_k - \sum_i p_i - \sum_{-i} p_{-i} = 0 \quad : \quad \lambda$$

Strategic offering problem

Given bid prices of demands and offer prices of producers, the market-clearing problem is

Offer price of rival generator *-i* (parameter)

$$\underset{d_k, p_i, p_{-i}}{\text{Maximize}} \quad \sum_k \alpha_k^{\text{bid}} d_k - \sum_i \alpha_i^{\text{offer}} p_i - \sum_{-i} \boxed{\alpha_{-i}^{\text{offer}}} p_{-i}$$

subject to:

$$0 \leq d_k \leq D_k^{\max} \quad : \quad \underline{\mu}_k, \overline{\mu}_k \quad \forall k$$

$$0 \leq p_i \leq P_i^{\max} \quad : \quad \underline{\mu}_i, \overline{\mu}_i \quad \forall i$$

$$0 \leq p_{-i} \leq P_{-i}^{\max} \quad : \quad \underline{\mu}_{-i}, \overline{\mu}_{-i} \quad \forall -i$$

$$\sum_k d_k - \sum_i p_i - \sum_{-i} p_{-i} = 0 \quad : \quad \lambda$$

Strategic offering problem

Given bid prices of demands and offer prices of producers, the market-clearing problem is

Production of rival generator *-i* (variable)

$$\underset{d_k, p_i, p_{-i}}{\text{Maximize}} \quad \sum_k \alpha_k^{\text{bid}} d_k - \sum_i \alpha_i^{\text{offer}} p_i - \sum_{-i} \alpha_{-i}^{\text{offer}} \boxed{p_{-i}}$$

subject to:

$$0 \le d_k \le D_k^{\max} \quad : \quad \underline{\mu}_k, \overline{\mu}_k \quad \forall k$$

$$0 \le p_i \le P_i^{\max} \quad : \quad \underline{\mu}_i, \overline{\mu}_i \quad \forall i$$

$$0 \le p_{-i} \le P_{-i}^{\max} \quad : \quad \underline{\mu}_{-i}, \overline{\mu}_{-i} \quad \forall -i$$

$$\sum_k d_k - \sum_i p_i - \sum_{-i} p_{-i} = 0 \quad : \quad \lambda$$

Strategic offering problem

Given bid prices of demands and offer prices of producers, the market-clearing problem is

$$\underset{d_k, p_i, p_{-i}}{\text{Maximize}} \quad \sum_k \alpha_k^{\text{bid}} d_k - \sum_i \alpha_i^{\text{offer}} p_i - \sum_{-i} \alpha_{-i}^{\text{offer}} p_{-i}$$

Social welfare maximization

subject to:

$$0 \leq d_k \leq D_k^{\max} \quad : \quad \underline{\mu}_k, \overline{\mu}_k \quad \forall k$$

Consumption limits of *k*

$$0 \leq p_i \leq P_i^{\max} \quad : \quad \underline{\mu}_i, \overline{\mu}_i \quad \forall i$$

Production limits of *i*

$$0 \leq p_{-i} \leq P_{-i}^{\max} \quad : \quad \underline{\mu}_{-i}, \overline{\mu}_{-i} \quad \forall -i$$

Production limits of -*i*

$$\sum_k d_k - \sum_i p_i - \sum_{-i} p_{-i} = 0 \quad : \quad \lambda$$

Power balance equality

Strategic offering problem

Given bid prices of demands and offer prices of producers, the market-clearing problem is

$$\underset{d_k, p_i, p_{-i}}{\text{Maximize}} \quad \sum_k \alpha_k^{\text{bid}} d_k - \sum_i \alpha_i^{\text{offer}} p_i - \sum_{-i} \alpha_{-i}^{\text{offer}} p_{-i}$$

subject to:

$$0 \leq d_k \leq D_k^{\max} \quad : \quad \underline{\mu}_k, \overline{\mu}_k \quad \forall k$$

$$0 \leq p_i \leq P_i^{\max} \quad : \quad \underline{\mu}_i, \overline{\mu}_i \quad \forall i$$

$$0 \leq p_{-i} \leq P_{-i}^{\max} \quad : \quad \underline{\mu}_{-i}, \overline{\mu}_{-i} \quad \forall -i$$

$$\sum_k d_k - \sum_i p_i - \sum_{-i} p_{-i} = 0 \quad : \quad \boxed{\lambda}$$

Market-clearing price (dual variable)

Strategic offering problem

The market-clearing problem is the follower.

$$\underset{d_k, p_i, p_{-i}}{\text{Maximize}} \quad \sum_k \alpha_k^{\text{bid}} d_k - \sum_i \alpha_i^{\text{offer}} p_i - \sum_{-i} \alpha_{-i}^{\text{offer}} p_{-i}$$

subject to:

$$0 \le d_k \le D_k^{\max} \quad : \quad \underline{\mu}_k, \overline{\mu}_k \quad \forall k$$

$$0 \le p_i \le P_i^{\max} \quad : \quad \underline{\mu}_i, \overline{\mu}_i \quad \forall i$$

$$0 \le p_{-i} \le P_{-i}^{\max} \quad : \quad \underline{\mu}_{-i}, \overline{\mu}_{-i} \quad \forall -i$$

$$\sum_k d_k - \sum_i p_i - \sum_{-i} p_{-i} = 0 \quad : \quad \lambda$$

Market-clearing problem (follower)

Strategic offering problem

The market-clearing problem is the follower.

Question: Who is the leader? What is her objective function?

$$\underset{d_k, p_i, p_{-i}}{\text{Maximize}} \quad \sum_k \alpha_k^{\text{bid}} d_k - \sum_i \alpha_i^{\text{offer}} p_i - \sum_{-i} \alpha_{-i}^{\text{offer}} p_{-i}$$

subject to:

$$0 \leq d_k \leq D_k^{\max} \quad : \quad \underline{\mu}_k, \overline{\mu}_k \quad \forall k$$

$$0 \leq p_i \leq P_i^{\max} \quad : \quad \underline{\mu}_i, \overline{\mu}_i \quad \forall i$$

$$0 \leq p_{-i} \leq P_{-i}^{\max} \quad : \quad \underline{\mu}_{-i}, \overline{\mu}_{-i} \quad \forall -i$$

$$\sum_k d_k - \sum_i p_i - \sum_{-i} p_{-i} = 0 \quad : \quad \lambda$$

Market-clearing problem (follower)

Strategic offering problem

The market-clearing problem is the follower.

Question: Who is the leader? What is her objective function? Strategic producer; profit maximization

$$\underset{d_k, p_i, p_{-i}}{\text{Maximize}} \quad \sum_k \alpha_k^{\text{bid}} d_k - \sum_i \alpha_i^{\text{offer}} p_i - \sum_{-i} \alpha_{-i}^{\text{offer}} p_{-i}$$

subject to:

$$0 \leq d_k \leq D_k^{\max} \quad : \quad \underline{\mu}_k, \overline{\mu}_k \quad \forall k$$

$$0 \leq p_i \leq P_i^{\max} \quad : \quad \underline{\mu}_i, \overline{\mu}_i \quad \forall i$$

$$0 \leq p_{-i} \leq P_{-i}^{\max} \quad : \quad \underline{\mu}_{-i}, \overline{\mu}_{-i} \quad \forall -i$$

$$\sum_k d_k - \sum_i p_i - \sum_{-i} p_{-i} = 0 \quad : \quad \lambda$$

Market-clearing problem (follower)

Strategic offering problem

$$\underset{\alpha_i^{\text{offer}}, d_k, p_i, p_{-i}, \underline{\mu}_k, \overline{\mu}_k, \underline{\mu}_i, \overline{\mu}_i, \underline{\mu}_{-i}, \overline{\mu}_{-i}, \lambda}{\text{Maximize}} \quad \sum_i p_i(\lambda - C_i)$$

subject to:

$$\alpha_i^{\text{offer}} \geq 0$$

Profit-maximization problem of the strategic producer (leader)

$$\underset{d_k, p_i, p_{-i}}{\text{Maximize}} \quad \sum_k \alpha_k^{\text{bid}} d_k - \sum_i \alpha_i^{\text{offer}} p_i - \sum_{-i} \alpha_{-i}^{\text{offer}} p_{-i}$$

subject to:

$$0 \leq d_k \leq D_k^{\max} \quad : \quad \underline{\mu}_k, \overline{\mu}_k \quad \forall k$$

$$0 \leq p_i \leq P_i^{\max} \quad : \quad \underline{\mu}_i, \overline{\mu}_i \quad \forall i$$

$$0 \leq p_{-i} \leq P_{-i}^{\max} \quad : \quad \underline{\mu}_{-i}, \overline{\mu}_{-i} \quad \forall -i$$

$$\sum_k d_k - \sum_i p_i - \sum_{-i} p_{-i} = 0 \quad : \quad \lambda$$

Market-clearing problem (follower)

Strategic offering problem

$$\underset{\boxed{\alpha_i^{\text{offer}}}, d_k, p_i, p_{-i}, \underline{\mu}_k, \overline{\mu}_k, \underline{\mu}_i, \overline{\mu}_i, \underline{\mu}_{-i}, \overline{\mu}_{-i}, \lambda}{\text{Maximize}} \quad \sum_i p_i(\lambda - C_i)$$

Offer price is a variable in the upper level, while it is a parameter in the lower level.

subject to:

$$\alpha_i^{\text{offer}} \geq 0$$

Profit-maximization problem of the strategic producer (leader)

$$\underset{d_k, p_i, p_{-i}}{\text{Maximize}} \quad \sum_k \alpha_k^{\text{bid}} d_k - \sum_i \alpha_i^{\text{offer}} p_i - \sum_{-i} \alpha_{-i}^{\text{offer}} p_{-i}$$

subject to:

$$0 \leq d_k \leq D_k^{\max} \quad : \quad \underline{\mu}_k, \overline{\mu}_k \quad \forall k$$

$$0 \leq p_i \leq P_i^{\max} \quad : \quad \underline{\mu}_i, \overline{\mu}_i \quad \forall i$$

$$0 \leq p_{-i} \leq P_{-i}^{\max} \quad : \quad \underline{\mu}_{-i}, \overline{\mu}_{-i} \quad \forall -i$$

$$\sum_k d_k - \sum_i p_i - \sum_{-i} p_{-i} = 0 \quad : \quad \lambda$$

Market-clearing problem (follower)

Strategic offering problem

$$\underset{\alpha_i^{\text{offer}},\boxed{d_k,p_i,p_{-i},\underline{\mu}_k,\overline{\mu}_k,\underline{\mu}_i,\overline{\mu}_i,\underline{\mu}_{-i},\overline{\mu}_{-i},\lambda}}{\text{Maximize}} \quad \sum_i p_i(\lambda - C_i)$$

All primal and dual variables of the lower-level problem are variables for the upper-level problem.

subject to:

$$\alpha_i^{\text{offer}} \geq 0$$

Profit-maximization problem of the strategic producer (leader)

$$\underset{d_k,p_i,p_{-i}}{\text{Maximize}} \quad \sum_k \alpha_k^{\text{bid}} d_k - \sum_i \alpha_i^{\text{offer}} p_i - \sum_{-i} \alpha_{-i}^{\text{offer}} p_{-i}$$

subject to:

$$0 \leq d_k \leq D_k^{\max} \quad : \quad \underline{\mu}_k, \overline{\mu}_k \quad \forall k$$

$$0 \leq p_i \leq P_i^{\max} \quad : \quad \underline{\mu}_i, \overline{\mu}_i \quad \forall i$$

$$0 \leq p_{-i} \leq P_{-i}^{\max} \quad : \quad \underline{\mu}_{-i}, \overline{\mu}_{-i} \quad \forall -i$$

$$\sum_k d_k - \sum_i p_i - \sum_{-i} p_{-i} = 0 \quad : \quad \lambda$$

Market-clearing problem (follower)

Strategic offering problem

$$\underset{\alpha_i^{\text{offer}}, d_k, p_i, p_{-i}, \underline{\mu}_k, \overline{\mu}_k, \underline{\mu}_i, \overline{\mu}_i, \underline{\mu}_{-i}, \overline{\mu}_{-i}, \lambda}{\text{Maximize}} \quad \underbrace{\sum_i p_i(\lambda - C_i)}_{\text{Profit of the strategic producer}}$$

subject to:

$$\alpha_i^{\text{offer}} \geq 0 \longrightarrow \text{Upper-level constraint}$$

Profit-maximization problem of the strategic producer (leader)

$$\underset{d_k, p_i, p_{-i}}{\text{Maximize}} \quad \sum_k \alpha_k^{\text{bid}} d_k - \sum_i \alpha_i^{\text{offer}} p_i - \sum_{-i} \alpha_{-i}^{\text{offer}} p_{-i}$$

subject to:

$$0 \leq d_k \leq D_k^{\max} \quad : \quad \underline{\mu}_k, \overline{\mu}_k \quad \forall k$$

$$0 \leq p_i \leq P_i^{\max} \quad : \quad \underline{\mu}_i, \overline{\mu}_i \quad \forall i$$

$$0 \leq p_{-i} \leq P_{-i}^{\max} \quad : \quad \underline{\mu}_{-i}, \overline{\mu}_{-i} \quad \forall -i$$

$$\sum_k d_k - \sum_i p_i - \sum_{-i} p_{-i} = 0 \quad : \quad \lambda$$

Market-clearing problem (follower)

Strategic offering problem

$$\underset{\alpha_i^{\text{offer}}, d_k, p_i, p_{-i}, \underline{\mu}_k, \overline{\mu}_k, \underline{\mu}_i, \overline{\mu}_i, \underline{\mu}_{-i}, \overline{\mu}_{-i}, \lambda}{\text{Maximize}} \quad \sum_i p_i(\lambda - C_i)$$

subject to:

$$\alpha_i^{\text{offer}} \geq 0$$

$$\underset{d_k, p_i, p_{-i}}{\text{Maximize}} \quad \sum_k \alpha_k^{\text{bid}} d_k - \sum_i \alpha_i^{\text{offer}} p_i - \sum_{-i} \alpha_{-i}^{\text{offer}} p_{-i}$$

subject to:

$$0 \leq d_k \leq D_k^{\max} \quad : \quad \underline{\mu}_k, \overline{\mu}_k \quad \forall k$$

$$0 \leq p_i \leq P_i^{\max} \quad : \quad \underline{\mu}_i, \overline{\mu}_i \quad \forall i$$

$$0 \leq p_{-i} \leq P_{-i}^{\max} \quad : \quad \underline{\mu}_{-i}, \overline{\mu}_{-i} \quad \forall -i$$

$$\sum_k d_k - \sum_i p_i - \sum_{-i} p_{-i} = 0 \quad : \quad \lambda$$

Profit-maximization problem of the strategic producer (leader)

Market-clearing problem (follower)

Question: How do the leader and the follower interact?

How to solve a bilevel problem?

MPCC

Mathematical **P**rogram with **C**omplementarity **C**onstraint (MPCC)

MPCC ⇐

minimize Upper-Level Objective Function

subject to

Upper-Level Constraints

Optimality (KKT) conditions

of the Lower-Level Problem(s)

MPCC

Mathematical **P**rogram with **C**omplementarity **C**onstraint (MPCC)

MPCC

minimize Upper-Level Objective Function

subject to

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Optimality (KKT) conditions

of the Lower-Level Problem(s)

The resulting MPCC has now a single objective function, as the lower-level objective function disappeared!

MPEC

Mathematical **P**rogram with **C**omplementarity **C**onstraint (MPCC)

Particularly for cases in which lower-level problems refer to "*equilibrium*" problems (e.g., market clearing), MPCC is referred to as

Mathematical **P**rogram with **E**quilibrium **C**onstraint (MPEC).

The resulting MPCC has now a single objective function, as the lower-level objective function disappeared!

MPCC and MPEC

MPCC/MPEC

minimize Upper-Level Objective Function

subject to

Upper-Level Constraints

Optimality (KKT) conditions

of the Lower-Level Problem(s)

MPCC and MPEC

MPCC/MPEC

minimize Upper-Level Objective Function

subject to

Upper-Level Constraints

Optimality (KKT) conditions

of the Lower-Level Problem(s)

Question: Is there any challenge for solving MPCC/MPEC?

MPCC and MPEC

MPCC/MPEC

minimize Upper-Level Objective Function

subject to

Upper-Level Constraints

Optimality (KKT) conditions

of the Lower-Level Problem(s)

Question: Is there any challenge for solving MPCC/MPEC?

Yes! Complementarity conditions contain the product of primal and dual variables!

Linearizing complementarity conditions

Linearizing complementarity conditions

Fortuny-Amat (also known as “Big-M”) approach:

J. Fortuny-Amat and B. McCarl, “A representation and economic interpretation of a two-level programming problem,” *The Journal of the Operational Research Society*, vol. 32, no. 9, pp. 783–792, 1981.

Linearizing complementarity conditions

Fortuny-Amat (also known as “Big-M”) approach:

The complementarity condition

$$0 \leq a \perp b \geq 0$$

can be replaced by

$$a \geq 0, \quad b \geq 0, \quad a \leq \psi M, \quad b \leq (1-\psi)M, \quad \psi \in \{0, 1\}$$

where M is a large enough constant.

J. Fortuny-Amat and B. McCarl, “A representation and economic interpretation of a two-level programming problem,” *The Journal of the Operational Research Society*, vol. 32, no. 9, pp. 783–792, 1981.

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Auxiliary binary variable!

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Linearizing complementarity conditions

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$$a \geq 0, \quad b \geq 0, \quad a \leq \psi M, \quad b \leq (1-\psi)M, \quad \psi \in \{0,1\}$$

Auxiliary binary variable!

where M is a large enough constant.

Tip: After solving the problem using Big-M approach, check and make sure that the original complementarity condition is satisfied!

Linearizing complementarity conditions

Important: the value for constant *M* should be selected carefully!

Linearizing complementarity conditions

Important: the value for constant *M* should be selected carefully!

✓ The choice of proper values for big constants could be challenging, such that a wrong choice may cause sub-optimality or numerical ill-conditioning.

✓ Further details about potential techniques for proper selection of these values are available at:

- Pineda, P., & Morales, J. M. (2019). Solving linear bilevel problems using big-Ms: Not all that glitters is gold. *IEEE Transactions on Power Systems*, 34, pp. 2469-2471.
- Kleinert, T., Labbe, M., Plein, F., & Schmidt, M. (2020). There's no free lunch: On the hardness of choosing a correct big-M in bilevel optimization. *Operation Research*, 68, pp. 1716–1721.

Strategic offering problem

Next step: deriving KKT conditions of the lower-level problem

Lower-level problem

$$\underset{d_k, p_i, p_{-i}}{\text{Maximize}} \quad \sum_k \alpha_k^{\text{bid}} d_k - \sum_i \alpha_i^{\text{offer}} p_i - \sum_{-i} \alpha_{-i}^{\text{offer}} p_{-i}$$

subject to:

$$0 \leq d_k \leq D_k^{\max} \quad : \quad \underline{\mu}_k, \overline{\mu}_k \quad \forall k$$

$$0 \leq p_i \leq P_i^{\max} \quad : \quad \underline{\mu}_i, \overline{\mu}_i \quad \forall i$$

$$0 \leq p_{-i} \leq P_{-i}^{\max} \quad : \quad \underline{\mu}_{-i}, \overline{\mu}_{-i} \quad \forall -i$$

$$\sum_k d_k - \sum_i p_i - \sum_{-i} p_{-i} = 0 \quad : \quad \lambda$$

Strategic offering problem

Next step: deriving KKT conditions of the lower-level problem

Lower-level problem

$$\underset{d_k, p_i, p_{-i}}{\text{Maximize}} \quad \sum_k \alpha_k^{\text{bid}} d_k - \sum_i \alpha_i^{\text{offer}} p_i - \sum_{-i} \alpha_{-i}^{\text{offer}} p_{-i}$$

subject to:

$$0 \leq d_k \leq D_k^{\max} \quad : \quad \underline{\mu}_k, \overline{\mu}_k \quad \forall k$$

$$0 \leq p_i \leq P_i^{\max} \quad : \quad \underline{\mu}_i, \overline{\mu}_i \quad \forall i$$

$$0 \leq p_{-i} \leq P_{-i}^{\max} \quad : \quad \underline{\mu}_{-i}, \overline{\mu}_{-i} \quad \forall -i$$

$$\sum_k d_k - \sum_i p_i - \sum_{-i} p_{-i} = 0 \quad : \quad \lambda$$

Lagrangian function

$$\begin{aligned}\mathcal{L}(d_k, p_i, p_{-i}, \underline{\mu}_k, \overline{\mu}_k, \underline{\mu}_i, \overline{\mu}_i, \underline{\mu}_{-i}, \overline{\mu}_{-i}, \lambda) = \\ & - \sum_k \alpha_k^{\text{bid}} d_k + \sum_i \alpha_i^{\text{offer}} p_i + \sum_{-i} \alpha_{-i}^{\text{offer}} p_{-i} \\ & + \sum_k \overline{\mu}_k \left(d_k - D_k^{\max}\right) - \sum_k \underline{\mu}_k d_k \\ & + \sum_i \overline{\mu}_i \left(p_i - P_i^{\max}\right) - \sum_i \underline{\mu}_i p_i \\ & + \sum_{-i} \overline{\mu}_{-i} \left(p_{-i} - P_{-i}^{\max}\right) - \sum_{-i} \underline{\mu}_{-i} p_{-i} \\ & + \lambda \left(\sum_k d_k - \sum_i p_i - \sum_{-i} p_{-i}\right)\end{aligned}$$

Strategic offering problem

Next step: deriving KKT conditions of the lower-level problem

Lower-level problem

$$\underset{d_k, p_i, p_{-i}}{\text{Maximize}} \sum_k \alpha_k^{\text{bid}} d_k - \sum_i \alpha_i^{\text{offer}} p_i - \sum_{-i} \alpha_{-i}^{\text{offer}} p_{-i}$$

subject to:

$$0 \le d_k \le D_k^{\max} \quad : \quad \underline{\mu}_k, \overline{\mu}_k \quad \forall k$$

$$0 \le p_i \le P_i^{\max} \quad : \quad \underline{\mu}_i, \overline{\mu}_i \quad \forall i$$

$$0 \le p_{-i} \le P_{-i}^{\max} \quad : \quad \underline{\mu}_{-i}, \overline{\mu}_{-i} \quad \forall -i$$

$$\sum_k d_k - \sum_i p_i - \sum_{-i} p_{-i} = 0 \quad : \quad \lambda$$

Lagrangian function

$$\begin{aligned}\mathcal{L}(d_k, p_i, p_{-i}, \underline{\mu}_k, \overline{\mu}_k, \underline{\mu}_i, \overline{\mu}_i, \underline{\mu}_{-i}, \overline{\mu}_{-i}, \lambda) = \\ & - \sum_k \alpha_k^{\text{bid}} d_k + \sum_i \alpha_i^{\text{offer}} p_i + \sum_{-i} \alpha_{-i}^{\text{offer}} p_{-i} \\ & + \sum_k \overline{\mu}_k \left(d_k - D_k^{\max}\right) - \sum_k \underline{\mu}_k d_k \\ & + \sum_i \overline{\mu}_i \left(p_i - P_i^{\max}\right) - \sum_i \underline{\mu}_i p_i \\ & + \sum_{-i} \overline{\mu}_{-i} \left(p_{-i} - P_{-i}^{\max}\right) - \sum_{-i} \underline{\mu}_{-i} p_{-i} \\ & + \lambda \left(\sum_k d_k - \sum_i p_i - \sum_{-i} p_{-i} \right)\end{aligned}$$

KKT conditions

$$\frac{\partial \mathcal{L}(.)}{\partial d_k} = -\alpha_k^{\text{bid}} + \overline{\mu}_k - \underline{\mu}_k + \lambda = 0 \quad \forall k$$

$$\frac{\partial \mathcal{L}(.)}{\partial p_i} = \alpha_i^{\text{offer}} + \overline{\mu}_i - \underline{\mu}_i - \lambda = 0 \quad \forall i$$

$$\frac{\partial \mathcal{L}(.)}{\partial p_{-i}} = \alpha_{-i}^{\text{offer}} + \overline{\mu}_{-i} - \underline{\mu}_{-i} - \lambda = 0 \quad \forall -i$$

$$\sum_k d_k - \sum_i p_i - \sum_{-i} p_{-i} = 0$$

$$0 \le \left(D_k^{\max} - d_k\right) \perp \overline{\mu}_k \ge 0 \quad \forall k$$

$$0 \le d_k \perp \underline{\mu}_k \ge 0 \quad \forall k$$

$$0 \le \left(P_i^{\max} - p_i\right) \perp \overline{\mu}_i \ge 0 \quad \forall i$$

$$0 \le p_i \perp \underline{\mu}_i \ge 0 \quad \forall i$$

$$0 \le \left(P_{-i}^{\max} - p_{-i}\right) \perp \overline{\mu}_{-i} \ge 0 \quad \forall -i$$

$$0 \le p_{-i} \perp \underline{\mu}_{-i} \ge 0 \quad \forall -i$$

Strategic offering problem

MPEC formulation

$$
\begin{aligned}
&\underset{\alpha_i^{\text{offer}}, d_k, p_i, p_{-i}, \underline{\mu}_k, \overline{\mu}_k, \underline{\mu}_i, \overline{\mu}_i, \underline{\mu}_{-i}, \overline{\mu}_{-i}, \lambda}{\text{Maximize}} \quad \sum_i p_i(\lambda - C_i)\\
&\text{subject to:}\\
&\alpha_i^{\text{offer}} \geq 0\\
&-\alpha_k^{\text{bid}} + \overline{\mu}_k - \underline{\mu}_k + \lambda = 0 \quad \forall k\\
&\alpha_i^{\text{offer}} + \overline{\mu}_i - \underline{\mu}_i - \lambda = 0 \quad \forall i\\
&\alpha_{-i}^{\text{offer}} + \overline{\mu}_{-i} - \underline{\mu}_{-i} - \lambda = 0 \quad \forall -i\\
&\sum_k d_k - \sum_i p_i - \sum_{-i} p_{-i} = 0\\
&0 \leq (D_k^{\max} - d_k) \perp \overline{\mu}_k \geq 0 \quad \forall k\\
&0 \leq d_k \perp \underline{\mu}_k \geq 0 \quad \forall k\\
&0 \leq (P_i^{\max} - p_i) \perp \overline{\mu}_i \geq 0 \quad \forall i\\
&0 \leq p_i \perp \underline{\mu}_i \geq 0 \quad \forall i\\
&0 \leq \left(P_{-i}^{\max} - p_{-i}\right) \perp \overline{\mu}_{-i} \geq 0 \quad \forall -i\\
&0 \leq p_{-i} \perp \underline{\mu}_{-i} \geq 0 \quad \forall -i
\end{aligned}
$$

Upper-level problem

KKT conditions of the lower-level problem

Strategic offering problem

MPEC formulation

$$\underset{\alpha_i^{\text{offer}}, d_k, p_i, p_{-i}, \underline{\mu}_k, \overline{\mu}_k, \underline{\mu}_i, \overline{\mu}_i, \underline{\mu}_{-i}, \overline{\mu}_{-i}, \lambda}{\text{Maximize}} \quad \sum_i p_i(\lambda - C_i)$$

subject to:

$$\alpha_i^{\text{offer}} \geq 0$$

Upper-level problem

$$-\alpha_k^{\text{bid}} + \overline{\mu}_k - \underline{\mu}_k + \lambda = 0 \quad \forall k$$

$$\alpha_i^{\text{offer}} + \overline{\mu}_i - \underline{\mu}_i - \lambda = 0 \quad \forall i$$

$$\alpha_{-i}^{\text{offer}} + \overline{\mu}_{-i} - \underline{\mu}_{-i} - \lambda = 0 \quad \forall -i$$

$$\sum_k d_k - \sum_i p_i - \sum_{-i} p_{-i} = 0$$

$$0 \leq (D_k^{\max} - d_k) \perp \overline{\mu}_k \geq 0 \quad \forall k$$

$$0 \leq d_k \perp \underline{\mu}_k \geq 0 \quad \forall k$$

$$0 \leq (P_i^{\max} - p_i) \perp \overline{\mu}_i \geq 0 \quad \forall i$$

$$0 \leq p_i \perp \underline{\mu}_i \geq 0 \quad \forall i$$

$$0 \leq \left(P_{-i}^{\max} - p_{-i}\right) \perp \overline{\mu}_{-i} \geq 0 \quad \forall -i$$

$$0 \leq p_{-i} \perp \underline{\mu}_{-i} \geq 0 \quad \forall -i$$

KKT conditions of the lower-level problem

Question: This problem in nonlinear. Why?

Strategic offering problem

MPEC formulation

$$
\begin{aligned}
&\underset{\alpha_i^{\text{offer}}, d_k, p_i, p_{-i}, \underline{\mu}_k, \overline{\mu}_k, \underline{\mu}_i, \overline{\mu}_i, \underline{\mu}_{-i}, \overline{\mu}_{-i}, \lambda}{\text{Maximize}} \quad \sum_i p_i(\lambda - C_i) \\
&\text{subject to:} \\
&\alpha_i^{\text{offer}} \geq 0
\end{aligned}
$$

Upper-level problem

$$
\begin{aligned}
&-\alpha_k^{\text{bid}} + \overline{\mu}_k - \underline{\mu}_k + \lambda = 0 \quad \forall k \\
&\alpha_i^{\text{offer}} + \overline{\mu}_i - \underline{\mu}_i - \lambda = 0 \quad \forall i \\
&\alpha_{-i}^{\text{offer}} + \overline{\mu}_{-i} - \underline{\mu}_{-i} - \lambda = 0 \quad \forall -i \\
&\sum_k d_k - \sum_i p_i - \sum_{-i} p_{-i} = 0 \\
&0 \leq (D_k^{\max} - d_k) \perp \overline{\mu}_k \geq 0 \quad \forall k \\
&0 \leq d_k \perp \underline{\mu}_k \geq 0 \quad \forall k \\
&0 \leq (P_i^{\max} - p_i) \perp \overline{\mu}_i \geq 0 \quad \forall i \\
&0 \leq p_i \perp \underline{\mu}_i \geq 0 \quad \forall i \\
&0 \leq \left(P_{-i}^{\max} - p_{-i}\right) \perp \overline{\mu}_{-i} \geq 0 \quad \forall -i \\
&0 \leq p_{-i} \perp \underline{\mu}_{-i} \geq 0 \quad \forall -i
\end{aligned}
$$

KKT conditions of the lower-level problem

Question: This problem in nonlinear. Why?

- Complementarity conditions
- The bilinear term (revenue) in the objective function

Strategic offering problem

Next step: Linearization of complementarity conditions (Big-M approach)

Example $$0 \leq (D_k^{\max} - d_k) \perp \overline{\mu}_k \geq 0 \quad \forall k$$

Strategic offering problem

Next step: Linearization of complementarity conditions (Big-M approach)

Example $$0 \leq (D_k^{\max} - d_k) \perp \overline{\mu}_k \geq 0 \quad \forall k$$

$$\Downarrow$$

$$D_k^{\max} - d_k \geq 0 \quad \forall k$$

$$\overline{\mu}_k \geq 0 \quad \forall k$$

$$D_k^{\max} - d_k \leq \overline{\psi}_k M \quad \forall k$$

$$\overline{\mu}_k \leq (1 - \overline{\psi}_k) M \quad \forall k$$

$$\overline{\psi}_k \in \{0, 1\} \quad \forall k$$

Strategic offering problem

The resulting model:

$$\underset{\alpha_i^{\text{offer}}, d_k, p_i, p_{-i}, \underline{\mu}_k, \overline{\mu}_k, \underline{\mu}_i, \overline{\mu}_i, \underline{\mu}_{-i}, \overline{\mu}_{-i}, \lambda, \underline{\psi}_k, \overline{\psi}_k, \underline{\psi}_i, \overline{\psi}_i, \underline{\psi}_{-i}, \overline{\psi}_{-i}}{\text{Maximize}}$$

$$\sum_i p_i(\lambda - C_i) \tag{1}$$

subject to:

$$\alpha_i^{\text{offer}} \geq 0 \tag{2}$$

$$-\alpha_k^{\text{bid}} + \overline{\mu}_k - \underline{\mu}_k + \lambda = 0 \quad \forall k \tag{3}$$

$$\alpha_i^{\text{offer}} + \overline{\mu}_i - \underline{\mu}_i - \lambda = 0 \quad \forall i \tag{4}$$

$$\alpha_{-i}^{\text{offer}} + \overline{\mu}_{-i} - \underline{\mu}_{-i} - \lambda = 0 \quad \forall -i \tag{5}$$

$$\sum_k d_k - \sum_i p_i - \sum_{-i} p_{-i} = 0 \tag{6}$$

$$D_k^{\max} - d_k \geq 0 \quad \forall k \tag{7}$$

$$\overline{\mu}_k \geq 0 \quad \forall k \tag{8}$$

$$D_k^{\max} - d_k \leq \overline{\psi}_k M \quad \forall k \tag{9}$$

$$\overline{\mu}_k \leq (1 - \overline{\psi}_k) M \quad \forall k \tag{10}$$

$$\overline{\psi}_k \in \{0,1\} \quad \forall k \tag{11}$$

$$d_k \geq 0 \quad \forall k \tag{12}$$

$$\underline{\mu}_k \geq 0 \quad \forall k \tag{13}$$

$$d_k \leq \underline{\psi}_k M \quad \forall k \tag{14}$$

$$\underline{\mu}_k \leq (1 - \underline{\psi}_k) M \quad \forall k \tag{15}$$

$$\underline{\psi}_k \in \{0,1\} \quad \forall k \tag{16}$$

$$P_i^{\max} - p_i \geq 0 \quad \forall i \tag{17}$$

$$\overline{\mu}_i \geq 0 \quad \forall i \tag{18}$$

$$P_i^{\max} - p_i \leq \overline{\psi}_i M \quad \forall i \tag{19}$$

$$\overline{\mu}_i \leq (1 - \overline{\psi}_i) M \quad \forall i \tag{20}$$

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$$p_i \geq 0 \quad \forall i \tag{22}$$

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$$\underline{\psi}_i \in \{0,1\} \quad \forall i \tag{26}$$

$$P_{-i}^{\max} - p_{-i} \geq 0 \quad \forall -i \tag{27}$$

$$\overline{\mu}_{-i} \geq 0 \quad \forall -i \tag{28}$$

$$P_{-i}^{\max} - p_{-i} \leq \overline{\psi}_{-i} M \quad \forall -i \tag{29}$$

$$\overline{\mu}_{-i} \leq (1 - \overline{\psi}_{-i}) M \quad \forall -i \tag{30}$$

$$\overline{\psi}_{-i} \in \{0,1\} \quad \forall -i \tag{31}$$

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$$\underline{\mu}_{-i} \leq (1 - \underline{\psi}_{-i}) M \quad \forall -i \tag{35}$$

$$\underline{\psi}_{-i} \in \{0,1\} \quad \forall -i \tag{36}$$

Strategic offering problem

Auxiliary binary variables

$$\underset{\alpha_i^{\text{offer}}, d_k, p_i, p_{-i}, \underline{\mu}_k, \overline{\mu}_k, \underline{\mu}_i, \overline{\mu}_i, \underline{\mu}_{-i}, \overline{\mu}_{-i}, \lambda \boxed{\underline{\psi}_k, \overline{\psi}_k, \underline{\psi}_i, \overline{\psi}_i, \underline{\psi}_{-i}, \overline{\psi}_{-i}}}{\text{Maximize}}$$

$$\sum_i p_i(\lambda - C_i) \tag{1}$$

subject to:

$$\alpha_i^{\text{offer}} \geq 0 \tag{2}$$

$$-\alpha_k^{\text{bid}} + \overline{\mu}_k - \underline{\mu}_k + \lambda = 0 \quad \forall k \tag{3}$$

$$\alpha_i^{\text{offer}} + \overline{\mu}_i - \underline{\mu}_i - \lambda = 0 \quad \forall i \tag{4}$$

$$\alpha_{-i}^{\text{offer}} + \overline{\mu}_{-i} - \underline{\mu}_{-i} - \lambda = 0 \quad \forall -i \tag{5}$$

$$\sum_k d_k - \sum_i p_i - \sum_{-i} p_{-i} = 0 \tag{6}$$

$$D_k^{\max} - d_k \geq 0 \quad \forall k \tag{7}$$

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Strategic offering problem

The resulting model is **mixed-integer nonlinear problem**, for which there is no off-the-shelf solver!

Let's linearize the bilinear term! How?

$$\underset{\alpha_i^{\text{offer}},d_k,p_i,p_{-i},\underline{\mu}_k,\overline{\mu}_k,\underline{\mu}_i,\overline{\mu}_i,\underline{\mu}_{-i},\overline{\mu}_{-i},\lambda,\underline{\psi}_k,\overline{\psi}_k,\underline{\psi}_i,\overline{\psi}_i,\underline{\psi}_{-i},\overline{\psi}_{-i}}{\text{Maximize}} \quad \sum_i p_i(\lambda - C_i) \tag{1}$$

subject to:

$$\alpha_i^{\text{offer}} \geq 0 \tag{2}$$

$$-\alpha_k^{\text{bid}} + \overline{\mu}_k - \underline{\mu}_k + \lambda = 0 \quad \forall k \tag{3}$$

$$\alpha_i^{\text{offer}} + \overline{\mu}_i - \underline{\mu}_i - \lambda = 0 \quad \forall i \tag{4}$$

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Strategic offering problem

Step 1) Let's write the strong duality equality (recall form lecture 1) of the lower-level problem:

$$-\sum_k \alpha_k^{\text{bid}} d_k + \sum_i \alpha_i^{\text{offer}} p_i + \sum_{-i} \alpha_{-i}^{\text{offer}} p_{-i} = -\sum_k \overline{\mu}_k D_k^{\max} - \sum_i \overline{\mu}_i P_i^{\max} - \sum_{-i} \overline{\mu}_{-i} P_{-i}^{\max}$$

Strategic offering problem

Step 1) Let's write the strong duality equality (recall form lecture 1) of the lower-level problem:

$$-\sum_{k} \alpha_k^{\text{bid}} d_k + \sum_{i} \alpha_i^{\text{offer}} p_i + \sum_{-i} \alpha_{-i}^{\text{offer}} p_{-i} = -\sum_{k} \overline{\mu}_k D_k^{\max} - \sum_{i} \overline{\mu}_i P_i^{\max} - \sum_{-i} \overline{\mu}_{-i} P_{-i}^{\max}$$

Step 2) From KKT conditions of the lower-level problem, we have

$$\alpha_i^{\text{offer}} + \overline{\mu}_i - \underline{\mu}_i - \lambda = 0 \quad \forall i$$

Strategic offering problem

Step 1) Let's write the strong duality equality (recall form lecture 1) of the lower-level problem:

$$-\sum_k \alpha_k^{\text{bid}} d_k + \sum_i \alpha_i^{\text{offer}} p_i + \sum_{-i} \alpha_{-i}^{\text{offer}} p_{-i} = -\sum_k \overline{\mu}_k D_k^{\max} - \sum_i \overline{\mu}_i P_i^{\max} - \sum_{-i} \overline{\mu}_{-i} P_{-i}^{\max}$$

Step 2) From KKT conditions of the lower-level problem, we have

$$\alpha_i^{\text{offer}} + \overline{\mu}_i - \underline{\mu}_i - \lambda = 0 \quad \forall i \quad \Longrightarrow \quad \sum_i \alpha_i^{\text{offer}} p_i + \sum_i p_i \overline{\mu}_i - \sum_i p_i \underline{\mu}_i - \sum_i p_i \lambda = 0$$

Strategic offering problem

Step 1) Let's write the strong duality equality (recall form lecture 1) of the lower-level problem:

$$-\sum_k \alpha_k^{\text{bid}} d_k + \sum_i \alpha_i^{\text{offer}} p_i + \sum_{-i} \alpha_{-i}^{\text{offer}} p_{-i} = -\sum_k \overline{\mu}_k D_k^{\max} - \sum_i \overline{\mu}_i P_i^{\max} - \sum_{-i} \overline{\mu}_{-i} P_{-i}^{\max}$$

Step 2) From KKT conditions of the lower-level problem, we have

$$\alpha_i^{\text{offer}} + \overline{\mu}_i - \underline{\mu}_i - \lambda = 0 \quad \forall i \quad \Rightarrow \quad \sum_i \alpha_i^{\text{offer}} p_i + \sum_i p_i \overline{\mu}_i - \sum_i p_i \underline{\mu}_i - \sum_i p_i \lambda = 0$$

$$\begin{aligned} &0 \leq (P_i^{\max} - p_i) \perp \overline{\mu}_i \geq 0 \quad \forall i \\ &0 \leq p_i \perp \underline{\mu}_i \geq 0 \quad \forall i \end{aligned} \quad \Rightarrow \quad \begin{aligned} &\overline{\mu}_i p_i = \overline{\mu}_i P_i^{\max} \quad \forall i \\ &\underline{\mu}_i p_i = 0 \quad \forall i \end{aligned}$$

Strategic offering problem

Step 1) Let's write the strong duality equality (recall form lecture 1) of the lower-level problem:

$$-\sum_k \alpha_k^{\text{bid}} d_k + \sum_i \alpha_i^{\text{offer}} p_i + \sum_{-i} \alpha_{-i}^{\text{offer}} p_{-i} = -\sum_k \overline{\mu}_k D_k^{\max} - \sum_i \overline{\mu}_i P_i^{\max} - \sum_{-i} \overline{\mu}_{-i} P_{-i}^{\max}$$

Step 2) From KKT conditions of the lower-level problem, we have

$$\alpha_i^{\text{offer}} + \overline{\mu}_i - \underline{\mu}_i - \lambda = 0 \quad \forall i$$

$$\boxed{\sum_i \alpha_i^{\text{offer}} p_i + \sum_i p_i \overline{\mu}_i - \sum_i p_i \underline{\mu}_i - \sum_i p_i \lambda = 0}$$

$$0 \leq (P_i^{\max} - p_i) \perp \overline{\mu}_i \geq 0 \quad \forall i$$
$$0 \leq p_i \perp \underline{\mu}_i \geq 0 \quad \forall i$$

$$\boxed{\begin{aligned} &\overline{\mu}_i p_i = \overline{\mu}_i P_i^{\max} \quad \forall i \\ &\underline{\mu}_i p_i = 0 \quad \forall i \end{aligned}}$$

$$\sum_i \alpha_i^{\text{offer}} p_i + \sum_i P_i^{\max} \overline{\mu}_i - \sum_i p_i \lambda = 0$$

Strategic offering problem

Step 1) Let's write the strong duality equality (recall form lecture 1) of the lower-level problem:

$$-\sum_k \alpha_k^{\text{bid}} d_k + \boxed{\sum_i \alpha_i^{\text{offer}} p_i} + \sum_{-i} \alpha_{-i}^{\text{offer}} p_{-i} = -\sum_k \overline{\mu}_k D_k^{\max} - \sum_i \overline{\mu}_i P_i^{\max} - \sum_{-i} \overline{\mu}_{-i} P_{-i}^{\max}$$

Step 2) From KKT conditions of the lower-level problem, we have

$$\alpha_i^{\text{offer}} + \overline{\mu}_i - \underline{\mu}_i - \lambda = 0 \quad \forall i$$

$$0 \leq (P_i^{\max} - p_i) \perp \overline{\mu}_i \geq 0 \quad \forall i$$
$$0 \leq p_i \perp \underline{\mu}_i \geq 0 \quad \forall i$$

$$\boxed{\begin{aligned} &\sum_i \alpha_i^{\text{offer}} p_i + \sum_i p_i \overline{\mu}_i - \sum_i p_i \underline{\mu}_i - \sum_i p_i \lambda = 0 \\ &\overline{\mu}_i p_i = \overline{\mu}_i P_i^{\max} \quad \forall i \\ &\underline{\mu}_i p_i = 0 \quad \forall i \end{aligned}}$$

$$\boxed{\sum_i \alpha_i^{\text{offer}} p_i} + \sum_i P_i^{\max} \overline{\mu}_i - \sum_i p_i \lambda = 0$$

Strategic offering problem

Step 1) Let's write the strong duality equality (recall form lecture 1) of the lower-level problem:

$$-\sum_k \alpha_k^{\text{bid}} d_k + \boxed{\sum_i \alpha_i^{\text{offer}} p_i} + \sum_{-i} \alpha_{-i}^{\text{offer}} p_{-i} = -\sum_k \overline{\mu}_k D_k^{\max} - \sum_i \overline{\mu}_i P_i^{\max} - \sum_{-i} \overline{\mu}_{-i} P_{-i}^{\max}$$

Step 2) From KKT conditions of the lower-level problem, we have

$$\alpha_i^{\text{offer}} + \overline{\mu}_i - \underline{\mu}_i - \lambda = 0 \quad \forall i$$

$$0 \leq (P_i^{\max} - p_i) \perp \overline{\mu}_i \geq 0 \quad \forall i$$
$$0 \leq p_i \perp \underline{\mu}_i \geq 0 \quad \forall i$$

$$\sum_i \alpha_i^{\text{offer}} p_i + \sum_i p_i \overline{\mu}_i - \sum_i p_i \underline{\mu}_i - \sum_i p_i \lambda = 0$$
$$\overline{\mu}_i p_i = \overline{\mu}_i P_i^{\max} \quad \forall i$$
$$\underline{\mu}_i p_i = 0 \quad \forall i$$

$$\boxed{\sum_i \alpha_i^{\text{offer}} p_i} + \sum_i P_i^{\max} \overline{\mu}_i - \sum_i p_i \lambda = 0$$

$$\sum_i p_i \lambda = \underbrace{\sum_k \alpha_k^{\text{bid}} d_k - \sum_{-i} \alpha_{-i}^{\text{offer}} p_{-i} - \sum_k \overline{\mu}_k D_k^{\max} - \sum_{-i} \overline{\mu}_{-i} P_{-i}^{\max}}_{\text{Linear term}}$$

Strategic offering problem

$$\underset{\alpha_i^{\text{offer}},d_k,p_i,p_{-i},\underline{\mu}_k,\overline{\mu}_k,\underline{\mu}_i,\overline{\mu}_i,\underline{\mu}_{-i},\overline{\mu}_{-i},\lambda,\underline{\psi}_k,\overline{\psi}_k,\underline{\psi}_i,\overline{\psi}_i,\underline{\psi}_{-i},\overline{\psi}_{-i}}{\text{Maximize}}$$

$$-\sum_i p_i C_i + \sum_k \alpha_k^{\text{bid}} d_k - \sum_{-i} \alpha_{-i}^{\text{offer}} p_{-i} - \sum_k \overline{\mu}_k D_k^{\max} - \sum_{-i} \overline{\mu}_{-i} P_{-i}^{\max} \qquad (1)$$

subject to:

$$\alpha_i^{\text{offer}} \geq 0 \qquad (2)$$

$$-\alpha_k^{\text{bid}} + \overline{\mu}_k - \underline{\mu}_k + \lambda = 0 \quad \forall k \qquad (3)$$

$$\alpha_i^{\text{offer}} + \overline{\mu}_i - \underline{\mu}_i - \lambda = 0 \quad \forall i \qquad (4)$$

$$\alpha_{-i}^{\text{offer}} + \overline{\mu}_{-i} - \underline{\mu}_{-i} - \lambda = 0 \quad \forall -i \qquad (5)$$

$$\sum_k d_k - \sum_i p_i - \sum_{-i} p_{-i} = 0 \qquad (6)$$

$$D_k^{\max} - d_k \geq 0 \quad \forall k \qquad (7)$$

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Final mixed-integer linear model

$$P_i^{\max} - p_i \geq 0 \quad \forall i \qquad (17)$$

$$\overline{\mu}_i \geq 0 \quad \forall i \qquad (18)$$

$$P_i^{\max} - p_i \leq \overline{\psi}_i M \quad \forall i \qquad (19)$$

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$$\underline{\mu}_{-i} \leq (1 - \underline{\psi}_{-i}) M \quad \forall -i \qquad (35)$$

$$\underline{\psi}_{-i} \in \{0,1\} \quad \forall -i \qquad (36)$$

Discussion

- Is it possible to have multiple lower-level problems?
- What is a trilevel problem?

What if there are more than a single leader (multiple price-makers)?

Math background: EPEC

- **E**quilibrium **P**roblem with **E**quilibrium **C**onstraints (EPEC)

Math background: EPEC

- **E**quilibrium **P**roblem with **E**quilibrium **C**onstraints (EPEC)

Bilevel problem 1

Minimize

Objective function of ULP 1

subject to

Constraints of ULP 1

LLP 1

ULP: Upper-level problem

LLP: Lower-level problem

Math background: EPEC

- **E**quilibrium **P**roblem with **E**quilibrium **C**onstraints (EPEC)

Bilevel problem 1		Bilevel problem *n*		Bilevel problem *m*
Minimize Objective function of ULP 1 subject to Constraints of ULP 1 LLP 1	…	Minimize Objective function of ULP *n* subject to Constraints of ULP *n* LLP *n*	…	Minimize Objective function of ULP *m* subject to Constraints of ULP *m* LLP *m*

ULP: Upper-level problem

LLP: Lower-level problem

Math background: EPEC

- **E**quilibrium **P**roblem with **E**quilibrium **C**onstraints (EPEC)

Bilevel problem 1

Minimize
Objective function of ULP 1
subject to
Constraints of ULP 1
LLP 1

. . .

Bilevel problem *n*

Minimize
Objective function of ULP *n*
subject to
Constraints of ULP *n*
LLP *n*

. . .

Bilevel problem *m*

Minimize
Objective function of ULP *m*
subject to
Constraints of ULP *m*
LLP *m*

Interrelated

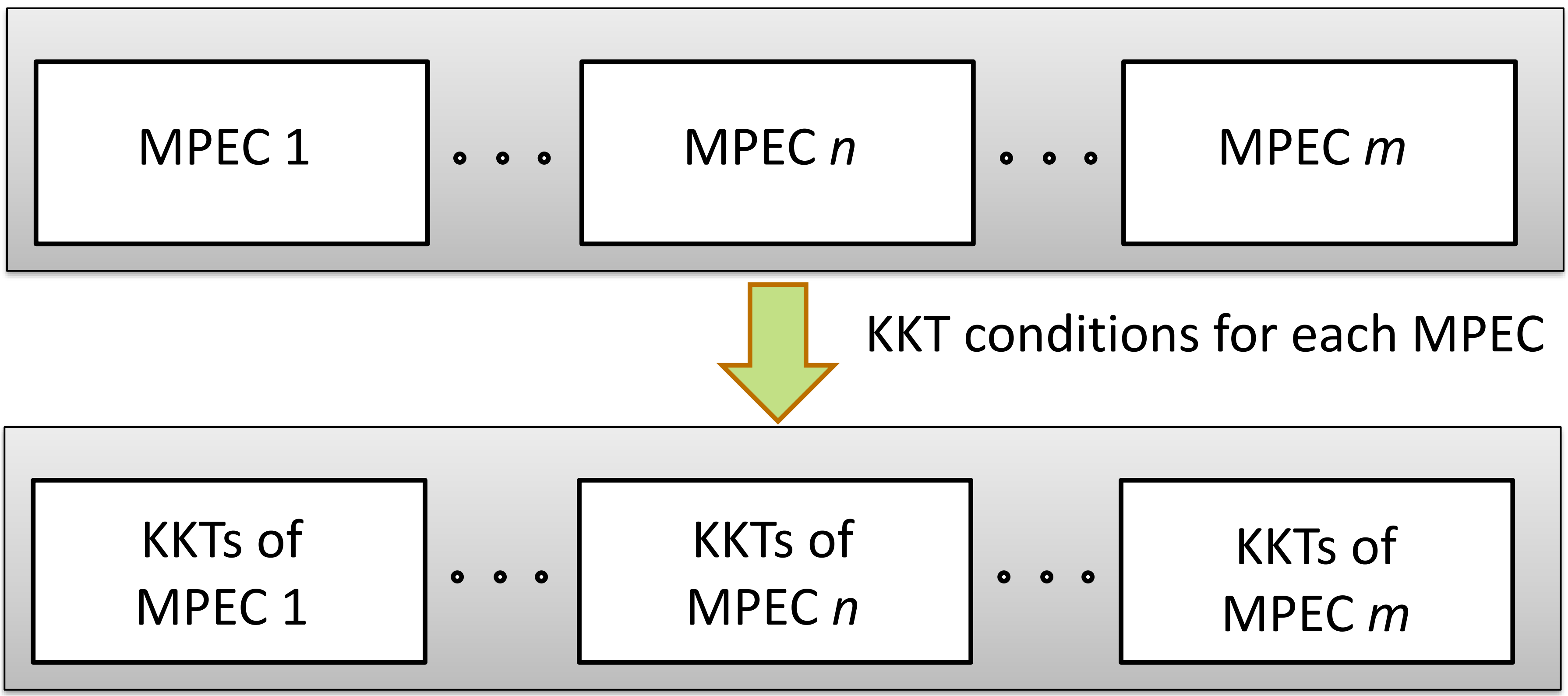
ULP: Upper-level problem

LLP: Lower-level problem

Math background: EPEC

- EPEC problem and its corresponding optimality conditions

EPEC



Optimality conditions of the EPEC

There are many applications for bilevel programming in power and energy systems (a very efficient technique)!

Thanks for your attention!

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