

C1. Wave friction factor

$$f_m = \begin{cases} 0.00251 \exp \left[5.21 \left(\frac{2T_m}{T_c} \right)^{-0.19} \right] & \text{for } \frac{\delta}{\lambda} > 1.587 \\ 0.3 & \text{for } \frac{\delta}{\lambda} \leq 1.587 \end{cases} \quad (21)$$

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$$\langle \theta \rangle = \frac{J_f u_w^2}{(s-1)g d_{50}} + \frac{J_w u^2}{(s-1)g d_{50}} \quad (A.3)$$

C2. Current friction factor

$$J_f = 2 \left[\frac{0.4}{\ln(3)} \right]^{1/2} \quad (20)$$

$$J_w = \frac{u_w}{u} \quad (19)$$

$$J_w = \frac{u_w}{u} \quad (18)$$

$$C_{w0} = \frac{u_w}{u} \quad (22)$$

$$C_{w0} = \frac{u_w}{u} \quad (15)$$

A. Intra-wave velocity time-series

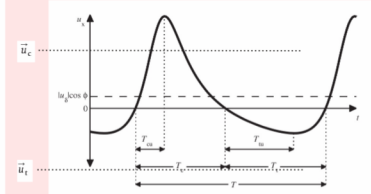


Fig. 1. Definition sketch of near-bed velocity time-series in wave direction. The parameters T_p and T_n are the positive (crest) and negative (trough) flow durations. Similarly, T_{p0} and T_{n0} are the durations of flow acceleration in positive and negative x-directions.

$$u = \sqrt{\frac{2}{T} \int_0^T u_w^2(t) dt} \quad (8)$$

$$\bar{u} = \frac{\bar{u} T}{2\pi} \quad (9)$$

$$\bar{u}_{cr} = \frac{1}{2} \sqrt{2} u_c \quad (10)$$

$$\bar{u}_{cr} = \frac{1}{2} \sqrt{2} u_c \quad (11)$$

$$u_x(t) = u_w(t) + |u_c| \cos \varphi \quad (4)$$

$$u_y = |u_c| \sin \varphi \quad (5)$$

$$\bar{u}_{cr} = \{u_{crx}, u_{cry}\} = \{\bar{u}_{cr} + |u_c| \cos \varphi, |u_c| \sin \varphi\} \quad (12)$$

$$\bar{u}_{cr} = \{u_{crx}, u_{cry}\} = \{-\bar{u}_{cr} + |u_c| \cos \varphi, |u_c| \sin \varphi\} \quad (13)$$

F. Sheet flow thickness

$$\delta_s = \frac{u_w^2}{(s-1)g d_{50}} \quad (C.2)$$

$$\frac{\delta_s}{d_{50}} = \begin{cases} 25\delta_0 & \text{if } d_{50} \leq 0.15 \text{ mm} \\ \left[25 - \frac{12(d_{50} - 0.15)}{(0.20 - 0.15)} \right] & \text{if } 0.15 \text{ mm} < d_{50} < 0.20 \text{ mm} \\ 13\delta_0 & \text{if } d_{50} \geq 0.20 \text{ mm} \end{cases} \quad (C.1)$$

B. Entrained sand load

$$\Omega_s = \begin{cases} 0 & \text{if } \theta_0 \leq \theta_{cr} \\ m(\theta_0 - \theta_{cr})^n & \text{if } \theta_0 > \theta_{cr} \end{cases} \quad (2)$$

E. Phase lag

$$\Omega_c = \begin{cases} \Omega_c & \text{if } P_c \leq 1 \\ \frac{1}{P_c} \Omega_c & \text{if } P_c > 1 \end{cases} \quad (23)$$

$$\Omega_a = \begin{cases} 0 & \text{if } P_c \leq 1 \\ \left(1 - \frac{1}{P_c}\right) \Omega_c & \text{if } P_c > 1 \end{cases} \quad (24)$$

$$\Omega_n = \begin{cases} \Omega_c & \text{if } P_c \leq 1 \\ \frac{1}{P_c} \Omega_c & \text{if } P_c > 1 \end{cases} \quad (25)$$

$$\Omega_c = \begin{cases} 0 & \text{if } P_c \leq 1 \\ \left(1 - \frac{1}{P_c}\right) \Omega_c & \text{if } P_c > 1 \end{cases} \quad (26)$$

$$W_{sc} = W_s - W_{min}(r_c) \quad (29)$$

$$W_H = \max(W_s - W_{max}(r_s), 0) \quad (30)$$

$$P_c = \begin{cases} \alpha \left(\frac{1 - \xi \theta_s}{\theta_c} \right) \frac{\eta}{2(T_c - T_{cr}) W_{sc}} & \text{if } \eta > 0 \text{ (ripple regime)} \\ \alpha \left(\frac{1 - \xi \theta_s}{\theta_c} \right) \frac{\theta_s}{2(T_c - T_{cr}) W_{sc}} & \text{if } \eta = 0 \text{ (sheet flow regime)} \end{cases} \quad (27)$$

$$P_t = \begin{cases} \alpha \left(\frac{1 - \xi \theta_s}{\theta_c} \right) \frac{\eta}{2(T_t - T_{cr}) W_{st}} & \text{if } \eta > 0 \text{ (ripple regime)} \\ \alpha \left(\frac{1 - \xi \theta_s}{\theta_c} \right) \frac{\theta_s}{2(T_t - T_{cr}) W_{st}} & \text{if } \eta = 0 \text{ (sheet flow regime)} \end{cases} \quad (28)$$

$$\Phi = \frac{\bar{u}_s}{\sqrt{(s-1)g d_{50}}} = \sqrt{\frac{\Omega_c}{\Omega_c + \frac{1}{P_c} \Omega_c}} \sqrt{\frac{\Omega_a}{\Omega_a + \frac{1}{P_c} \Omega_a}} \quad (1)$$

G. Main function

The colored circles and lines show variables that come from formulas. Each color is specific to a variable. The quantities identified by black rectangles are the following physical constants or tuning parameters:

$s = (\rho_s - \rho) / \rho$ according to section 2 of VDA13

g is the constant of gravity

d_{50} is the median sand grain size

$\delta = 0.2$ m is the wave boundary layer thickness assumed in the VDA13 model

$\alpha_w = 0.424$ is a scale factor for the wave Reynolds stress