# OC664 Modeling Project Report

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## Summary

We implemented the VDA13 sediment transport model in Python using inputs developed from the SandyDuck'97 dataset. The model outputs were acceptable in comparison to observed bed elevation change. The VDA13 model as published has critical definitional errors which were rectified in our implementation.

## Introduction

Predicting sediment transport rates in coastal zones is a challenging task which requires an understanding of complex interactions between wave-driven flow and sediment particles. Sediment transport rates are key to anticipating morphological changes along shorelines and outcomes of coastal engineering projects, e.g., beach nourishment, and are therefore important to model accurately. We report our comparison of the foundational Meyer-Peter & Müller (MPM) model, a steady flow sediment transport model which can be modified for oscillatory-flow transport, with the more recent SANTOSS formulas proposed by van der A et al. (2013). The SANTOSS formulas link component models of wave orbital velocities, ripple bedforms, wave and current friction factors, sheet flow, entrained sand load, and phase lag to estimate cross-shore sand transport in non-breaking wave conditions. To validate the MPM model and the SANTOSS formulas (referred to in this paper as the VDA13 model) for nearshore wave-dominated coastal sediment transport, a sand transport rate was calculated from bed elevation changes taken from the same field dataset used for the inputs to the analytical models.

The data used for this model validation and comparison is a subset of the data gathered by Elgar et al. (1997) during their "SandyDuck" experiment in Duck, NC. The data subset includes a 3-hour time series of cross-shore velocities recorded by acoustic Doppler velocimeters (ADVs) and water column heights calculated from pressure transducer outputs. In the experiment, eight pairs of ADVs and pressure transducers were arranged in a cross-shore transect approximately

300 m in width. The instruments sampled at a rate of 2 Hz. All instruments were mounted less than ~1 m above the bed. We considered only the output from the most shoreward sensor pair, labeled below as location 1, which was located approximately 185 m offshore. The data also includes transect seabed elevations measured 24 hours apart, which we used for model validation in our analysis. Figure 1 shows the position of each of the instruments and the bathymetry data collected.

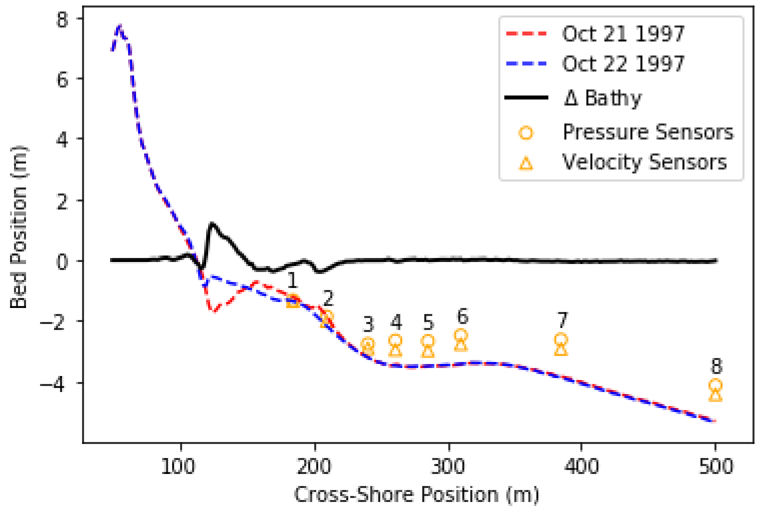


***Figure 1.*** *Comparison of seabed positions along the cross-shore axis at Duck, NC. The black line is the change in vertical position measured on October 21, and 22 1997 (red and blue lines respectively). This report focuses on data gathered from sensors at location 1 (most nearshore sensors). Notice that the bulk of the sediment transport occurs shoreward of sensor 1.*

# Methods

## Hydrodynamic data processing

The model of van der A et al. (2013) requires representative duration parameters and representative near-bed velocity parameters, all of which were extracted from the Elgar et al. (1997) 3-hour time series of velocities at location 1. Two methods were considered to calculate the parameters: (1) a zero up-crossing analysis on the orbital velocity time series data, and (2) a representative wave calculation following Ruessink et al. (2012). The zero-upcrossing analysis produced 1,299 waves of varying amplitude and period, where the representative parameters can be extracted individually from each wave. The method of Ruessink et al. (2012) uses the significant wave height, peak period, peak wave number (based on peak period), and water depth to develop a wave shape with parameters asymmetry, $A_w$, skewness, $S_w$, and amplitude, $U_w$. Following Ruessink et al. (2012), we calculated a representative wave shape with accompanying asymmetry, skewness, and amplitude parameters (Figure 2). We then isolated a single wave from our time series which closely matched that representative shape (Figure 3). For our analysis, we compared the model results using inputs from the isolated representative wave with the results using inputs from each wave in the 1,299-wave series.

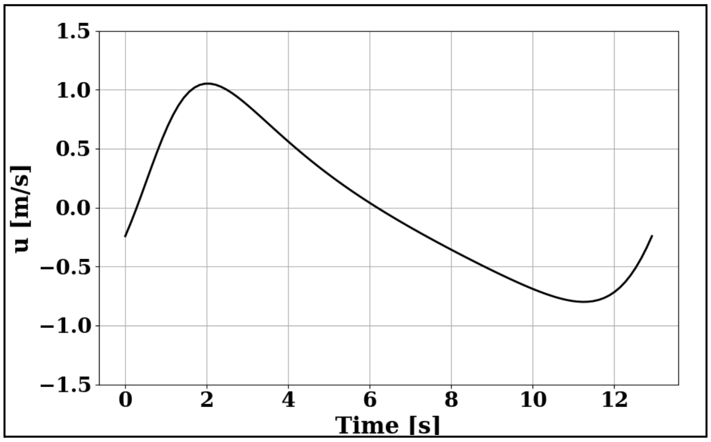


***Figure 2.*** *Representative wave established using Ruessink et al. (2012) method of parameterization. While we did not use this wave to calculate $q_s$, we did compare it to a representative wave in the time series that we then used to calculate $q_s$.*

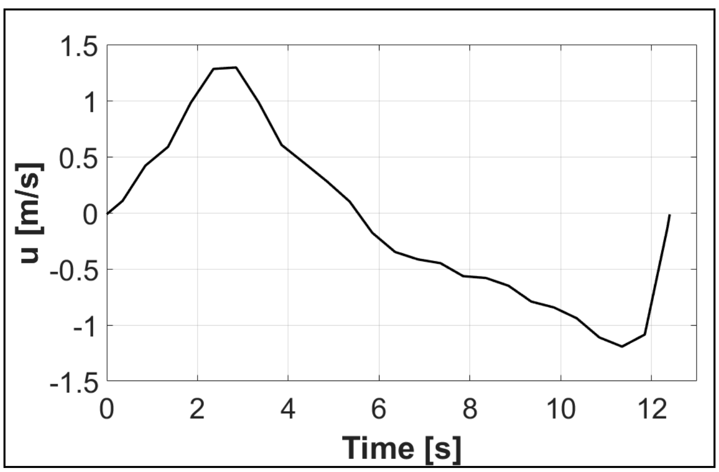


***Figure 3.*** *Representative wave from the time series extracted using zero up-crossing analysis. This wave was applied to the van der A et al. (2013) model to estimate the transport rate, $q_s$.*

We calculated orbital and steady-current velocities with the representative wave portion of the time series and the time series portions associated with each of the 1,299 waves. For both approaches, a velocity time series was decomposed into the time-varying wave component and the steady current component. Then half-cycle orbital velocities were calculated in order to produce wave-current velocities for each half-cycle. Additionally, a representative orbital velocity amplitude and representative orbital excursion amplitude were calculated. These velocity parameters are inputs to most of the sub-process components of the van der A et al. (2013) model.

## van der A (2013) sediment transport formula

van der A et al. (2013) provides a set of formulas which can be used to predict sand transport for non-breaking waves and currents. The model is forced by the bed shear stress, parameterized as the Shields number. The model can be used for both rippled beds and sheet flow conditions, accounts for phase lag and flow acceleration effects, and can be applied to oscillatory flow and surface wave conditions. The van der A et al. (2013) velocity-load equation is given by:

$$\frac{\vec{q}_s}{\sqrt{(s-1)gd_{50}^3}} = \frac{\sqrt{|\theta_c|}T_c\left(\Omega_{cc}+\frac{T_c}{2T_{cu}}\Omega_{tc}\right)\frac{\vec{\theta}_c}{|\theta_c|}+\sqrt{|\theta_t|}T_t\left(\Omega_{tt}+\frac{T_t}{2T_{tu}}\Omega_{ct}\right)\frac{\vec{\theta}_t}{|\theta_t|}}{T}, \tag{1}$$

where $\vec{q}_s$ is the volumetric net transport rate per unit width, *s* is the specific gravity, *g* is the acceleration due to gravity, $d_{50}$ is the median sand grain diameter, $\vec{\theta}$ is the non-dimensional bed shear stress (Shields parameter), , *T* is the wave period, $T_c$ is the duration of the crest (positive) half cycle, $T_t$ is the duration of the trough (negative) half cycle, $T_{cu}$ is the duration of accelerating flow within the crest half cycle, and $T_{tu}$ is the duration of accelerating flow within the trough half cycle (van der A et al. 2013). For all parameters, the subscripts “c” and “t” indicate “crest” and “trough” half-cycles, respectively. The specific gravity and $d_{50}$ were taken as constant values of ~2.59 and 0.2 mm, respectively.

The general structure of the model is as follows: representative orbital velocities, orbital excursion amplitudes, and wave periods are calculated for the whole flow cycle and the crest and trough half-cycles of a wave. These flow parameters and the median grain diameter of the local sand are used to quantify ripple characteristics (length and height) and the consequent bedform roughness, which is developed iteratively with a current-driven friction factor in the boundary layer and a time-averaged Shields parameter. The current-driven friction factor is combined with a half-cycle wave friction factor from a modified version of the Swart equation to develop a half-cycle combined wave/current friction factor. The combined half-cycle wave-current friction factor is used to estimate shear stress at the bed, which forces the model. The magnitude of the bed shear stress is used as an input to the entrained sand load. A full-cycle version of the combined wave-current friction factor is used to calculate turbulent Reynolds stress, which is an addend to the bed shear stress. The half-cycle wave-current friction factor is also used as an input to the sheet flow thickness, which in turn is an input to the phase lag calculation, which determines whether there is an exchange of sediment between half-cycles of the wave orbital. The velocity load equation synthesizes these sub-processes of sediment transport to estimate a sediment transport rate for the input wave and sediment particle characteristics.

## Meyer-Peter and Müller (1948) sediment transport formula

The MPM formula (1948) is an early model for sediment transport and ordinarily is used for river flow. For our previous midterm exam, we modified the MPM formula for oscillatory flow by introducing a time-dependent Shields parameter, $\theta$.

$$q_{MPM} = \begin{cases} 0, & \theta < 0.05 \\ 8\sqrt{(s-1)gd_{50}^3}[\theta - 0.05]^{3/2}, & \theta \geq 0.05 \end{cases} , \tag{2}$$

where the time dependent Shields parameter, $\theta$, is given by

$$\theta(t) = \frac{0.5 f_w u(t)^2}{(s-1)gd_{50}} , \tag{3}$$

and $f_w$ is the friction factor using the Swart (1976) equation and *u(t)* is from the time series of orbital velocities at location 1. Sediment specific gravity, *s*, and median grain size $d_{50}$, used in this formula are the same as used in the van der A. et al. (2013) model.

## Bathymetric change sediment transport formula

For the previous midterm exam, we calculated $q_s$ directly from the change in cross-shore bed elevation measurements from October 21 to October 22, 1997. This rough estimate is based only from the shore to the first gauge, specifically x=100 m and x=185 m, and was calculated to be approximately 6.57 $m^2$/day. Equation (4) was used:

$$\langle q \rangle_{shore} = c_{bed} \int_{100\,m}^{185\,m} \Delta z(x)\, dx , \tag{4}$$

where $c_{bed}$ is the sand bed concentration, taken as a constant of 0.64.

# Results

Table 1 outlines the results of each method used to calculate the net transport rates at Duck, NC from October 21 to 22, 1997. The rate listed on the last row was calculated from empirical data collected by Elgar et al. (1997) during their “SandyDuck" experiment, which is used as the validation data for this model comparison. All model calculations were performed using data from the gauge closest to the shoreline.

***Table 1***: *Net transport rate calculations.*

| **Method** | $q_s\ [m^2/day]$ |
|---|---|
| VDA13 - Representative wave | 2.67 |
| VDA13 - Wave-by-wave (average) | 34.89 |
| VDA13 - Wave-by-wave (median) | 2.29 |
| MPM | 4.32 |
| Bathymetric change from $x = 100 - 185m$ | 6.57 |

The threshold for a successful model result was for the calculated transport rate to be within a factor of 10 of the 24 hour empirical rate. For the VDA13 calculations, the single representative wave yielded a transport rate of 3.09E-5 $m^2/s$ or 2.67 $m^2/day$, the wave-by-wave average transport rate was approximately 4E-4 $m^2/s$, or 34.9 $m^2/day$, and the wave-by-wave median transport rate was approximately 2.65E-5 $m^2/s$ or 2.29 $m^2/day$. The MPM formula resulted in a transport rate of 5.0E-5 $m^2/s$ or 4.32 $m^2/day$. All results met the criteria, but there is a clear outlier in the average rate of the VDA13 wave-by-wave calculations.

Figure 4 shows the distribution of net transport rate calculated per wave using the VDA13 model. The plot on the left has a limit of 5E-3 $m^2/s$, while the limit for the plot on the right was reduced to 1E-3 $m^2/s$. The plotted results indicate that the majority of the calculated transport rates are below 10E-5 $m^2/s$, with some clear outliers visible in the first plot. These outliers likely resulted in the large average net transport rate calculated for the VDA13 model.

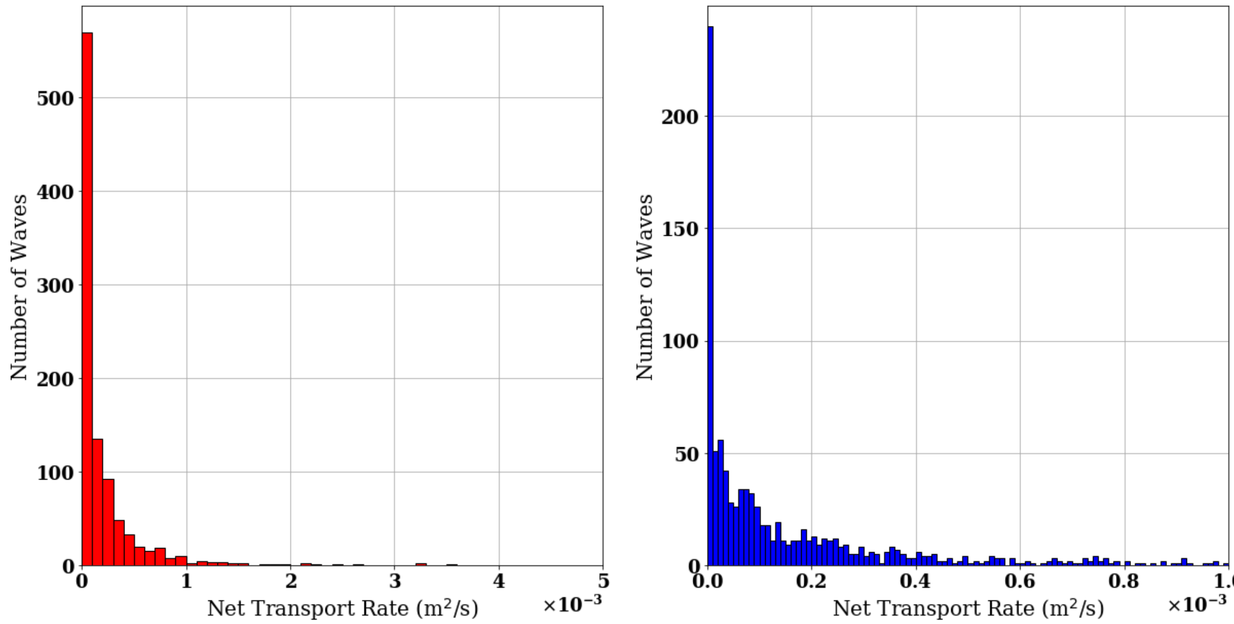


***Figure 4.*** *Histogram of net transport rate per wave calculated using VDA13 (to 5E-3 $m^2/s$ and 1E-3 $m^2/s$).*

Table 2 shows a selection of important parameters calculated with the VDA model. These median values (from the wave-by-wave calculation) are consistent with the results shown in Table 1.

| Parameter [units] | Definition | Median Value |
|---|---|---|
| $u_{cr}$ [m/s] | Representative wave-current velocity vector (crest half-cycle) | 0.76 |
| $u_{tr}$ [m/s] | Representative wave-current velocity vector (trough half-cycle) | -0.45 |
| $\theta_{cx}$ [-] | Non-dimensional bed shear stress (crest half-cycle) | 0.94 |
| $\theta_{tx}$ [-] | Non-dimensional bed shear stress (trough half-cycle) | -0.20 |
| $f_{w\delta c}$ [-] | Combined wave-current friction factor (crest half-cycle) | 0.0098 |
| $f_{w\delta t}$ [-] | Combined wave-current friction factor (trough half-cycle) | 0.0067 |
| $P_c$ [-] | Phase lag parameter (crest half-cycle) | 0.62 |
| $P_t$ [-] | Phase lag parameter (trough half-cycle) | 1.76 |
| $\Omega_c$ [-] | Sand load entrained (crest half-cycle) | 8.48 |
| $\Omega_t$ [-] | Sand load entrained (trough half-cycle) | 1.54 |

# Discussion

## Model validation

The sediment transport rate calculated from our implementation of the VDA13 model was within 59% of the sediment transport rate calculated from the measured change in bed elevation and was within 38% of the sediment transport rate calculated from the MPM model. The consistency in these results is reassuring and indicates that our VDA13 model is implemented correctly and is a useful approximation of sediment transport rates in the field. It is striking that both sediment transport models underestimate the rate of sediment transport calculated from the measured change in bed elevation. The inputs to the VDA13 model were calculated using a representative wave from a 3-hour time series of wave orbital velocities and water elevations, and the inputs to the MPM model were calculated using bulk wave parameters estimated from the same 3-hour time series. The change in bed elevation was calculated as the difference between the bed elevations measured 24 hours apart, where the 24-hour interval surrounded the 3-hour time series of observations of wave orbital velocities and water elevations. Thus, validation of the sediment transport models with the bed elevation change assumes that the wave conditions during the 3-hour observation period were representative of the wave conditions during the 24-hour interval. This assumption is not necessarily valid, and if high wave conditions during the 24-hour interval outside the 3-hour observation period produced greater onshore sediment transport than the wave conditions within the 3-hour observation period, then the sediment transport rate calculated from the bed elevation change would exceed the sediment transport rates calculated from the models, as we see in this analysis. Furthermore, the waves in our dataset are breaking, so wave rollers might bring additional sediment onshore. This process would not be captured by the MPM and VDA13 models, which both assume nonbreaking waves.

From our midterm exam, the sediment transport rate calculated from the MPM model was within 35% of the sediment transport rate calculated from the measured change in bed elevation, which was more accurate than the rate calculated from the VDA13 model. From the calculated results, it would be easy to conclude that the MPM formula is preferable to the VDA13 model, since the MPM formula predicts the measured bed elevation change with greater accuracy and is much less challenging to implement than the VDA13 model. However, the MPM model makes a number of simplifying assumptions which are not necessarily valid. For example, the MPM model assumes a flat bed, a negligible time-average current, and assumes that there is no phase lag, i.e, there is no exchange of sediment between half-cycles of the wave orbital. In the dataset, the bed is approximately flat and the time-average current is minimal, but the phase lag assumption is erroneous: from the VDA13 model, the phase lag parameter associated with the trough half-cycle is non-negligible. Further, while both the MPM model and the VDA13 model are forced by the bed shear stress, the MPM model assumes that the shear stress and the sediment transport rate are related by constants rather than by transport parameters as developed in the VDA13 model. Since we don't have bed elevation change data restricted to the 3-hour observation period, it is plausible that the assumptions in the MPM model result in overestimation of the sediment transport rate in the 3-hour observation period of wave velocities and pressures, and the more nuanced VDA13 model is more accurate. Measurements of bed elevation change at shorter time intervals are necessary to determine whether the MPM model truly outperforms the VDA13 model.

## Representative wave and wave-by-wave approaches

For comparison to the representative wave approach, we calculated the inputs from each individual wave in the time series, where each wave was identified by a zero-upcrossing analysis, used the VDA13 model to estimate the sediment transport rate for each wave, and calculated the median sediment transport rate of the waves. The resulting sediment transport rate was within 10% of the sediment transport rate calculated with the VDA13 model using inputs from a representative wave. This indicates that the representative wave approach can be used interchangeably with an approach where the sediment transport rate is estimated as the median result from a wave-by-wave calculation. We note that the average rate from the wave-by-wave calculation is nearly an order of magnitude greater than the median. This indicates that when the model is used with large wave orbital velocities (which are not considered in the median or representative wave approaches), the calculated sediment transport rates are unrealistically large.

## Definitional errors in VDA13

Several parameter definitions and formulas in van der A et al. (2013) include errors that must be corrected when implementing the model. In section 2 (p. 27), sediment specific gravity is given as $s=(\rho_s - \rho)/\rho$. However, all formulations implement $s$ as ($s-1$), which implies that $s$ must be defined as $s=\rho_s/\rho$ as in Rafati et al. (2021). Further, the formula for the wave Reynolds stress (equation 22, p. 29) mistakenly contains a $\rho$ term, which is not included in the definition of the wave Reynolds stress as given in Nielson (2006) cited by van der A et al. (2013). The erroneous inclusion of the $\rho$ term makes the wave Reynolds stress 3 orders of magnitude too high, which then inflates the bed shear stress (equation 15) and consequently the sediment transport rate (equation 1). Finally, the fraction in the parentheses of phase lag parameter formulas (equations 27 and 28, p. 30) should be corrected to $(1 \pm \frac{\xi \hat{u}_i}{c_w})$ as formulated in equation 31.

# Conclusions

Based on the results from this study, and the observations made:

1. The VDA13 and MPM models of sediment transport provide tolerably accurate estimates of sediment transport rates when compared to bed elevation change over a 24-hour interval. The VDA13 model includes more nuanced parameterizations of sediment transport processes, but is considerably more challenging to implement. Measurements of bed elevation change over shorter time intervals are necessary to develop a more detailed comparison between the models.

2. For the time series of wave orbital velocities and pressures considered in this analysis, the VDA13 model can be used with inputs calculated from a representative wave or with wave-by-wave inputs as long as the sediment transport rate for the wave-by-wave approach is estimated as the median of the output rates. The mean rate output from the wave-by-wave approach overestimates the sediment transport rate calculated by other methods by approximately a factor of 10.

3. The published VDA13 model is conceptually insightful and analytically straightforward but includes several errors which have a significant impact on the results and which must be rectified for the model to be usable.

# Contributions of authors

## Implementation of VDA13 model

Intra-wave velocity time series: **Jacob**; Ripples/wave friction factor (iterative): **Emily, Colin**; Current friction factor: **Maggie**; Sheet flow thickness: **Carson**; Entrained sand load: **Carly**; Phase lag: **Luis**; Main function: **Sebas**; Function organization: **Carly**; GitHub administration: **Sebas**; GitHub coaching: **Carly**; Debugging: **Sebas, Maggie, Luis, Carly**

## Report writing

Introduction: **Emily, Colin**; Methods: **Carly, Jacob, Colin**; Results: **Luis, Sebas**; Discussion: **Maggie, Carson**; Conclusions: **Maggie**; Editing: **Sebas, Maggie, Luis, Carly**

# References

Elgar, S., R. T. Guza, B. Raubenheimer, T. H. C. Herbers, and E. L. Gallagher (1997), Spectral evolution of shoaling and breaking waves on a barred beach, *J. Geophys. Res.*, 102(C7), 15797–15805.

Faria, A. G., Thornton, E. B., Lippmann, T. C., & Stanton, T. P. (2000). Undertow over a barred beach. *Journal of Geophysical Research: Oceans*, *105*(C7), 16999-17010.

Nielsen, P. (2006). Sheet flow sediment transport under waves with acceleration skewness and boundary layer streaming. *Coastal Engineering*, *53*(9), 749-758.

Meyer-Peter, E. and Muller, R. (1948) Formulas for Bed Load Transport. Proceedings of 2nd meeting of the International Association for Hydraulic Structures Research, Delft, 7 June 1948, 39-64.

Rafati, Y., Hsu, T. J., Elgar, S., Raubenheimer, B., Quataert, E., & van Dongeren, A. (2021). Modeling the hydrodynamics and morphodynamics of sandbar migration events. *Coastal Engineering*, *166*, 103885.

Ruessink, B. G., Ramaekers, G., & Van Rijn, L. C. (2012). On the parameterization of the free-stream non-linear wave orbital motion in nearshore morphodynamic models. *Coastal engineering*, *65*, 56-63.

Ruessink, B. G., Houwman, K. T., & Hoekstra, P. (1998). The systematic contribution of transporting mechanisms to the cross-shore sediment transport in water depths of 3 to 9 m. *Marine Geology*, *152*(4), 295-324.

Soulsby, R. (1997). Dynamics of marine sands.

Swart, D.H. (1976). Predictive equations regarding longshore transport.*15th Conference on Coastal Engineering*, 2, pp.1113–1132.

van der A, D.A., Ribberink, J.S., van der Werf, J.J., O’Donoghue, T., Buijsrogge, R.H., & W.M. Krenenburg. (2013). Practical sand transport formula for non-breaking waves and currents. *Coastal Engineering,* 76, 26-42.