Department of Mechanical and Mechatronics Engineering

WKRPT 400
Numerical Analysis of Externally Pressurized Cylinder

A report prepared for:
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Somerville, Massachusetts, United States

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Dear Professor Collins,

Prepared for my 3B work term at ***Altaeros Energies*** , my third and final work term report is entitled ***Numerical Analysis of Externally Pressurized Cylinder***. The objective of this work term report was to perform an in depth structural analysis on the current aerostat's winch system.

Altaeros Energies has one simple mission, to deliver telecommunication in remote areas with increased reliability and more cost effectively. Another major selling point is fully autonomy led by state of the art controls.

As a systems engineering intern under the supervision of Ephraim Lanford, I was primarily responsible for conducting advanced analyses on the aerostat's tether management system.

This report was written entirely by me and has not received any previous academic credit at this or any other institution. I would like to thank Ephraim Lanford for all guidance. Also, John Umina for his expertise in the design of pressure vessels. No other sources of aid were used for this report.

Best Regards,

Sebastien Blanchet
ID:
3B Mechanical Engineering

Signature | Date

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Summary

The objective of this report is

Preliminary results found that the method was not good.

Results

Conclusions and recommendations

Section 1

Introduction

This is the introduction, why we are doing this analysis

1.1 Background

Altaeros Energies does this as per Figure 1.1 seen below.

Figure 1.1: Overview of the buoyant airborne technology

1.2 Purpose

The purpose of this report is to do this.

- Do analysis on drum
- Development numerical algorithm for solving

1.3 Scope

The scope of the report will be as follows.

Section 2

Preliminary Analysis

In this section, the methods used for the numerical approach are discussed in detail.

2.1 Basics: Deflection

The first equation that is presented is the main relation between the internal bending moment on a beam and its curvature by Equation 2.1 as per [1].

$$\frac{1}{\rho} = \frac{M}{EI} \tag{2.1}$$

Where $\frac{1}{\rho}$ is the radius of curvature defined as 2.2

$$\frac{1}{\rho} = \frac{d^2y/dx^2}{(1 + (dy/dx)^2)^f\, rac32} \approx \frac{d^2y}{dx^2} \tag{2.2}$$

2.1.1 Relations

With 2.2, relations based on deflection $y(x)$ for slope 2.3, moment 2.4, shearing force 2.5 and load intensity 2.6 as per [1].

$$\theta(x) = \frac{dy}{dx} \tag{2.3}$$

$$M(x) = EI\,\frac{d^2y}{dx^2} \tag{2.4}$$

$$V(x) = EI\ \frac{d^3y}{dx^3} \tag{2.5}$$

$$q(x) = EI\ \frac{d^4y}{dx^4} \tag{2.6}$$

2.1.2 Boundary Conditions

From these above equations, we may present boundary conditions as follows assuming that the boundary in question is located at x_0.

Free Ends:

$$\begin{aligned} y(x_0) &= y_0 \\ \theta(x_0) &= \theta_0 \\ M(x_0) &= 0 \\ V(x_0) &= 0 \end{aligned} \tag{2.7}$$

End Supported:

$$\begin{aligned} y(x_0) &= y_0 \\ \theta(x_0) &= \theta_0 \\ M(x_0) &= M_0 \\ V(x_0) &= V_0 \end{aligned} \tag{2.8}$$

Fixed:

$$\begin{aligned} y(x_0) &= 0 \\ \theta(x_0) &= 0 \\ M(x_0) &= M_0 \\ V(x_0) &= V_0 \end{aligned} \tag{2.9}$$

In the following section, these preliminary relations will be used to further expand on more complex analysis approaches.

2.2 Capstan Equation

What the results in the previous section reveal is that the solution lies in the added level of complexity in this problem. It is required to understand how one wrap of tether in tension results in an external pressure q applied to the outer surface of the cylinder.

After much research into this problem, the solution reveals itself into the derivation of the well-known Capstan equation 2.10.

$$T_2 = T_1 e^{-\mu\theta} \tag{2.10}$$

From this equation, the free body diagram that leads to this solution is presented below in Figure 2.1.

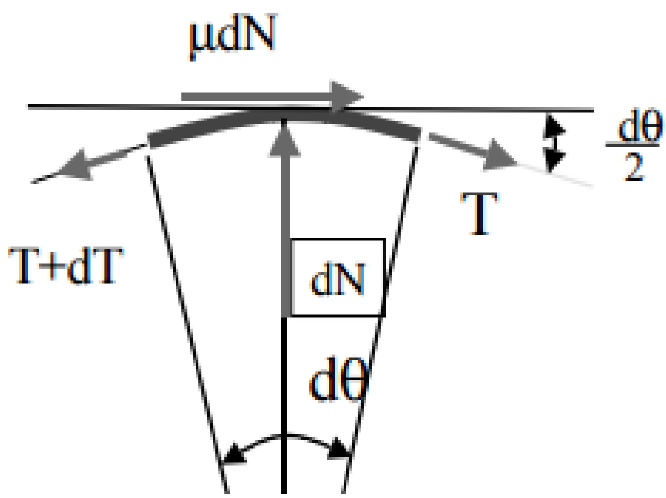

Figure 2.1: Free body diagram of differential capstan problem.

Based on this, what is of question is how to extrapolate a pressure q from the applied tension T. This can be done performing a force balance in the radial direction which reduces to Equation 2.11.

$$dN - (T + dT)\sin\frac{d\theta}{2} + T\sin\frac{d\theta}{2} = 0 \tag{2.11}$$

By assuming a small chance in angle, the substitution of $\sin\theta \approx \theta$ can be made. Applying this relation to 2.11 leaves 2.12.

$$dN = Td\theta \tag{2.12}$$

From the above equation, it is now clear that the overall normal force caused by the tension in the cable T can be solved for by integration or as follows 2.14

$$\int_0^N dN = \int_0^{2\pi} Td\theta \tag{2.13}$$

$$N = 2\pi T \tag{2.14}$$

With this resultant normal force over one revolution of tether, it is now apparent that the pressure q is simply 2.15

$$q = \frac{N}{A} = \frac{2\pi T}{d\pi D} = \frac{2T}{Dd} \tag{2.15}$$

The result concluded in this section will reveal to be very important in the comming sections.

2.3 Theory of Cylindrical Shells

As per Timoshenko's book [2], this section will cover the method of approximating the cylinder as a long thin shell. The following coordinate system is presented as per Figure 2.2 below.

Based on this, the following differential equations are presented as a pressure balance knowing that the differential area can be represented as Equation 2.16

$$dA = dS\ dx = a\ d\varphi\ dx \tag{2.16}$$

The equations of equilibrium may be written as a force projection about the x and z axis and momment balance about y as Equations 2.17, 2.18, 2.19 respectively.

$$\frac{dN_x}{dx}\ a\ d\varphi\ dx = 0 \tag{2.17}$$

$$\frac{dQ_x}{dx}\ a\ d\varphi\ dx + N_\varphi\ a\ d\varphi\ dx + Z\ a\ d\varphi\ dx = 0 \tag{2.18}$$

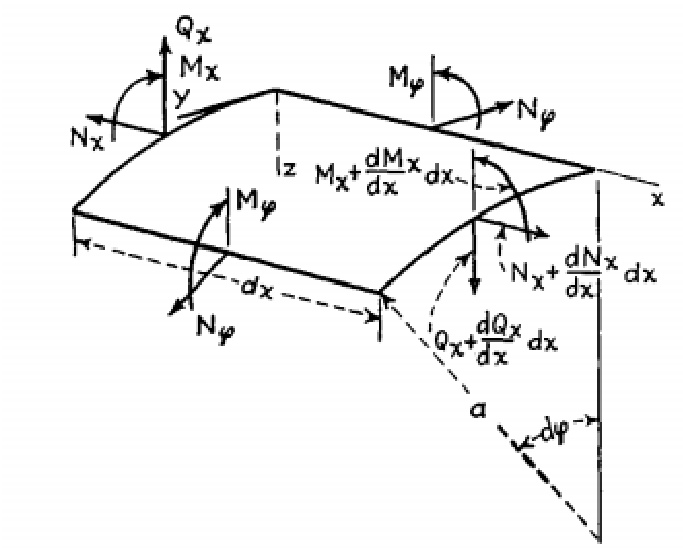

Figure 2.2: Coordinate system adopted for derivation of DEs.

$$\frac{dM_x}{dx}\ a\ d\varphi\ dx - Q_x\ a\ d\varphi\ dx = 0 \tag{2.19}$$

First looking at 2.17, simplifying this equation and taking the integral with respect to x will leave 2.20.

$$N_x = C = 0 \tag{2.20}$$

This above equation simply states that the effects of bending due to the axial forces will be neglected.

Similarly with 2.18 and 2.19, simplifications will lead to 2.21 and 2.22 respectively.

$$\frac{dQ_x}{dx} + \frac{1}{a}\ N_\varphi = -Z \tag{2.21}$$

$$\frac{dM_x}{dx} - Q_x = 0 \tag{2.22}$$

With these equations and using strain relations from Hooke's law, the fine DE for displacement will be determined. In Equation 2.23, the relations between displacement and strain are presented.

$$\begin{aligned}\epsilon_x &= \frac{du}{dx} \\ \epsilon_\varphi &= -\frac{w}{a}\end{aligned} \tag{2.23}$$

From Hooke's law N_x may be also written as Equation 2.24. Substituting 2.23 will leave the final simplification.

$$N_x = \frac{Eh}{1-\nu^2}\ (\epsilon_x + \nu\epsilon_\varphi) = \frac{Eh}{1-\nu^2}\ \left(\frac{du}{dx} - \nu\ \frac{w}{a}\right) \tag{2.24}$$

Solving Equation 2.24 using 2.20 leaves 2.25.

$$\frac{du}{dx} = \nu\ \frac{w}{a} \tag{2.25}$$

Similarly, with N_φ, again applying 2.23 leaves 2.27.

$$N_\varphi = \frac{Eh}{1-\nu^2}\ (\epsilon_\varphi + \nu\epsilon_x) = \frac{Eh}{1-\nu^2}\ \left(-\frac{w}{a} + \nu\ \frac{du}{dx}\right) \tag{2.26}$$

$$N_\varphi = -\frac{Ehw}{a} \tag{2.27}$$

As a result of no change in curvature in the φ direction, we know that $\frac{dM_\varphi}{d\varphi} = 0$ hence no change in the circumferential moments. This relation is translated to axial momments M_x with 2.28.

$$\begin{aligned}M_\varphi &= \nu M_x \\ M_x &= -D\frac{d^2w}{dx^2}\end{aligned} \tag{2.28}$$

Where D is the flexural rigidity of the shell 2.29.

$$D = \frac{Eh^3}{12(1-\nu^3)} \tag{2.29}$$

Simplifying 2.21 to get $Q_x = \frac{dM_x}{dx}$, the following is put in 2.22 to get 2.30

$$\begin{aligned}\frac{d^2M_x}{dx^2} + \frac{1}{a}\ N_\varphi &= -Z \\ D\ \frac{d^4w}{dx^4} + \frac{Eh}{a^2}\ w &= Z \\ \frac{d^4w}{dx^4} + \beta^4\ w &= \frac{Z}{D}\end{aligned} \tag{2.30}$$

Where β^4 is some parameter defined as 2.31.

$$\beta^4 = \frac{Eh}{4a^2D} = \frac{3(1-\nu^2)}{a^2h^2} \tag{2.31}$$

The solution to this common fourth order, linear, non-homogeneous, ordinary differential equation in 2.30 has is known to be Equation 2.32

$$w(x) = e^{\beta x}\left(C_1 \cos \beta x + C_2 \sin \beta x\right) + e^{-\beta x}\left(C_3 \cos \beta x + C_4 \sin \beta x\right) + f(x) \tag{2.32}$$

Where C_1, C_2, C_3, C_4 are intgration constants to be solved based on BCs and $f(x)$ is the particular solution to the ODE.

In the following section, the solution to this differential equation will be explored.

Section 3

Standards

In this section, well known standards will be investigated.

- American Society of Mechanical Engineers:
  - Section III: Part D, 2013 [3]
  - Section VII: Division 1, 2015 [4]
  - Section VII: Division 2, 2015 [5]
- European Standard EN
  - EN 13445-3:2014

3.1 American Society of Mechanical Engineers

3.2 European European Standard EN

3.3 Comparison

Section 4

Finite Element Analysis

Section 5

Discussion

Section 6

Conclusion

In conclusion, ODI must improve the DBM's existing MBA. The 100: 1 reduction Mijno MNT-115-100 gearbox (GB) and Allen Bradley (AB) N-4220-2-H00AA MBS are highly outdated.

Multiple high by 2. thick steel rule samples were bent 45 and from this current, speed and time data were collected. These data were converted to torques. Upon calculation of safety factors (SF) with maximum calculated nominal 280and acceleration 345.2 torques both the MBS and GB failed SF the 1.25 SF requirement.

An ODI supplier for PLC control systems, Brock Solutions was consulted and the new MBS was selected to be an AB MPM-A1153F-MJ72AA, rated for 6. nominal, 19. acceleration torque. As no GB was suggested, the selection process was left to ODI. Constraints were mechanical fit into the DBM and gear ratios between 50 and 100. Existing, GB, GBC , GBM were studied for proper mechanical fit. A possible solution was to keep the same shaft size. Of twelve GBs, none passed the same SF requirements.

Another solution was to increase the shaft size to . The same procedure was followed and the WITTENSTIEN SP-140S-MF2-70-1E1 was selected upon passing aforementioned SF requirements. This 70:1 reduction GB is rated for 360. and 660. nominal and acceleration torques, respectively. To implement this solution, it is required to replace the GBM, bore and broach the GBC and replace the socket head cap screws. The proposed design was validated.

Section 7

Recommendations

Improving testing methods and data extrapolation could be highly beneficial as this could allow more accurate torque values. Due to the lack of data acquisition flexibility, there is possibly a large error in the analyzed data. For instance, the extrapolation methods with AutoCAD were approximated as best as humanly possible. Not to mention the error that is already included in the raw measurements themselves. Given that the extrapolated data is valid, there are other variables to be factored into the equation. Mechanical properties such as steel rule curvature, thickness are likely to be similar for the measured samples but could likely vary for other trials in the future. Hence, the observed torque used to calculate SFs do not represent the entire operation. Theses inconsistencies are normally a result of coiling the steel rule, which varies for all coils. Also, further research could be completed with regards to another possible gearbox solution which does not require any modification to the system. Possible ways to improve the retrofitting process could be to let a more experienced supplier (Brock Solutions) also deal with the selection of the gearbox, given ODI's data.

References

[1] K. Nisbett, R. Budynas, Shigley's Mechanical Engineering Design, McGraw-Hill, 2014.

[2] S. Timoshenko, S. Woinowsky-Krieger, Theory of Plates and Shells, Engineering Societies Monographs, McGraw-Hill, 1959.

[3] ASME's Boiler and Pressure Vessel Code, Section II, Part D, The American Society of Mechanical Engineers, New York, NY, 2015.

[4] ASME's Boiler and Pressure Vessel Code, Section VII, Division 1, The American Society of Mechanical Engineers, New York, NY, 2015.

[5] ASME's Boiler and Pressure Vessel Code, Section VII, Division 2, The American Society of Mechanical Engineers, New York, NY, 2015.

Appendix A