Department of Mechanical and Mechatronics Engineering

WKRPT 400
Numerical Analysis and Optimization

A report prepared for:
Altaeros Energies
Somerville, Massachusetts, United States

by:
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3B Mechanical Engineering

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Waterloo, Ontario, Canada

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Professor Michael Collins
Associate Chair Undergraduate Studies, Mechanical Engineering
University of Waterloo
Waterloo, Ontario, Canada, N2L 3G1

Dear Professor Collins,

Prepared for my 3B work term at ***Altaeros Energies*** , my third and final work term report is entitled ***Numerical Analysis and Optimization***. The objective of this work term report was to perform an in depth structural analysis on the current aerostat's winch system.

Altaeros Energies has one simple mission, to deliver telecommunication in remote areas with increased reliability and more cost effectively. Another major selling point is fully autonomy led by state of the art controls.

As a systems engineering intern under the supervision of Ephraim Lanford, I was primarily responsible for conducting advanced analyses on the aerostat's tether management system.

This report was written entirely by me and has not received any previous academic credit at this or any other institution. I would like to thank Ephraim Lanford for all guidance. Also, John Umina for his expertise in the design of pressure vessels. No other sources of aid were used for this report.

Best Regards,

Sebastien Blanchet
ID:
3B Mechanical Engineering

Signature   Date

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Summary

The objective of this report is

Preliminary results found that the method was not good.

Results

Conclusions and recommendations

Nomenclature

Characters

Symbol	**Description**	**Unit**
R	the gas constant	$\mathrm{m}^2 \cdot \mathrm{s}^{-2} \cdot \mathrm{K}^{-1}$
C_v	specific heat capacity at constant volume	$\mathrm{m}^2 \cdot \mathrm{s}^{-2} \cdot \mathrm{K}^{-1}$
C_p	specific heat capacity at constant pressure	$\mathrm{m}^2 \cdot \mathrm{s}^{-2} \cdot \mathrm{K}^{-1}$
E	specific total energy	$\mathrm{m}^2 \cdot \mathrm{s}^{-2}$
e	specific internal energy	$\mathrm{m}^2 \cdot \mathrm{s}^{-2}$
h_T	specific total enthalpy	$\mathrm{m}^2 \cdot \mathrm{s}^{-2}$
h	specific enthalpy	$\mathrm{m}^2 \cdot \mathrm{s}^{-2}$
k	thermal conductivity	$\mathrm{kg} \cdot \mathrm{m} \cdot \mathrm{s}^{-3} \cdot \mathrm{K}^{-1}$
T	temperature	K
t	time	s
p	thermodynamic pressure	$\mathrm{kg} \cdot \mathrm{m}^{-1} \cdot \mathrm{s}^{-2}$
$\hat{p}$	hydrostatic pressure	$\mathrm{kg} \cdot \mathrm{m}^{-1} \cdot \mathrm{s}^{-2}$
$\boldsymbol{f}_b$	body force	$\mathrm{kg} \cdot \mathrm{m}^{-2} \cdot \mathrm{s}^{-2}$
S	boundary surface	m^2
V	volume	m^3
$\boldsymbol{V}$	velocity vector	$\mathrm{m} \cdot \mathrm{s}^{-1}$
u	x component of velocity	$\mathrm{m} \cdot \mathrm{s}^{-1}$

v	y component of velocity	$\mathrm{m} \cdot \mathrm{s}^{-1}$
w	z component of velocity	$\mathrm{m} \cdot \mathrm{s}^{-1}$
c	speed of sound	$\mathrm{m} \cdot \mathrm{s}^{-1}$
$\boldsymbol{r}$	position vector	m
$\mathbf{n}$	unit normal vector	1
$\hat{\mathbf{t}}$	unit tangent vector	1
$\tilde{\mathbf{t}}$	unit bitangent vector	1
C_R	coefficient of restitution	1
Re	Reynolds number	1
Pr	Prandtl number	1
Ma	Mach number	1
α	thermal diffusivity	$\mathrm{m}^2 \cdot \mathrm{s}^{-1}$
μ	dynamic viscosity	$\mathrm{kg} \cdot \mathrm{m}^{-1} \cdot \mathrm{s}^{-1}$
ν	kinematic viscosity	$\mathrm{m}^2 \cdot \mathrm{s}^{-1}$
γ	heat capacity ratio	1
ρ	density	$\mathrm{kg} \cdot \mathrm{m}^{-3}$
σ_{ij}	stress tensor	$\mathrm{kg} \cdot \mathrm{m}^{-1} \cdot \mathrm{s}^{-2}$
S_{ij}	deviatoric stress tensor	$\mathrm{kg} \cdot \mathrm{m}^{-1} \cdot \mathrm{s}^{-2}$
τ_{ij}	viscous stress tensor	$\mathrm{kg} \cdot \mathrm{m}^{-1} \cdot \mathrm{s}^{-2}$
δ_{ij}	Kronecker tensor	1
I_{ij}	identity tensor	1

Operators

Symbol	**Description**
Δ	difference
∇	gradient operator

$\delta^{\pm}$	upwind-biased interpolation scheme

Abbreviations

Acronym	**Description**
ANFO	Ammonium Nitrate Fuel Oil
CFD	Computational Fluid Dynamics
CFL	Courant-Friedrichs-Lewy
CJ	Chapman-Jouguet
EOS	Equation of State
JWL	Jones-Wilkins-Lee
TVD	Total Variation Diminishing
WENO	Weighted Essentially Non-oscillatory
ZND	Zel'dovich-von Neumann-Doering

Section 1

Introduction

This is the introduction, why we are doing this analysis

1.1 Background

Altaeros Energies does this as per Figure 1.1 seen below.

Figure 1.1: Overview of the buoyant airborne technology

1.2 Purpose

The purpose of this report is to do this.

- Do analysis on drum
- Development numerical algorithm for solving

1.3 Scope

The scope of the report will be as follows.

Section 2

Preliminary Analysis

The analysis process is discussed

2.1 Simplifications

The results were simplified to do.

2.1.1 Equations

The first equation that is presented , Equation 2.1

$$\begin{aligned} \Sigma F_i &= 0 \\ \Sigma M_i &= 0 \end{aligned} \tag{2.1}$$

As per reference [**?**]

2.1.2 Table

This was used to get the results in the following Table 2.1.

Table 2.1: Test table 1

Case	Method #1	Method #2
1	10	50
2	20	100

2.2 Theory of Cylindrical Shells

As per Timoshenko's book INSERT REF, this section will cover the method of approximating the cylinder as a long thin shell. The following coordinate system is presented as per Figure 2.1 below.

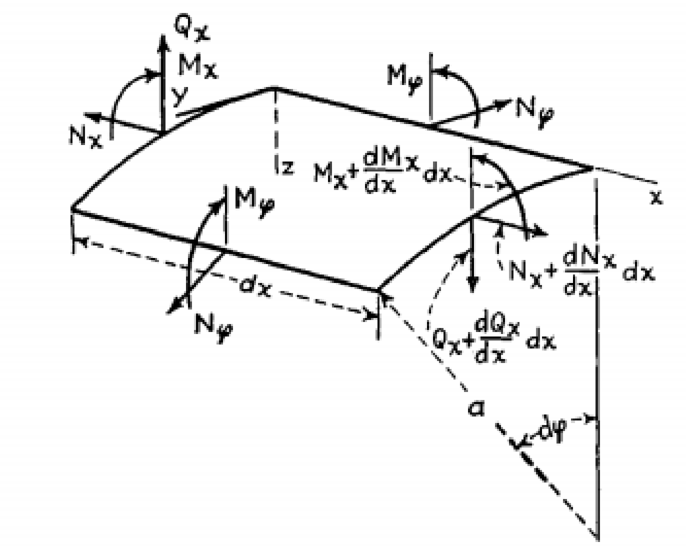


Figure 2.1: Coordinate system adopted for derivation of DEs.

Based on this, the following differential equations are presented as a pressure balance knowing that the differential area can be represented as Equation 2.2

$$dA = dS\ dx = a\ d\varphi\ dx \tag{2.2}$$

The equations of equilibrium may be written as a force projection about the x and z axis and momment balance about y as Equations 2.3, 2.4, 2.5 respectively.

$$\frac{dN_x}{dx}\ a\ d\varphi\ dx = 0 \tag{2.3}$$

$$\frac{dQ_x}{dx}\ a\ d\varphi\ dx + N_\varphi\ a\ d\varphi\ dx + Z\ a\ d\varphi\ dx = 0 \tag{2.4}$$

$$\frac{dM_x}{dx}\ a\ d\varphi\ dx - Q_x\ a\ d\varphi\ dx = 0 \tag{2.5}$$

First looking at 2.3, simplifying this equation and taking the integral with respect to x will leave 2.6.

$$N_x = C = 0 \tag{2.6}$$

This above equation simply states that the effects of bending due to the axial forces will be neglected.

Similarly with 2.4 and 2.5, simplifications will lead to 2.7 and 2.8 respectively.

$$\frac{dQ_x}{dx} + \frac{1}{a}\ N_\varphi = -Z \tag{2.7}$$

$$\frac{dM_x}{dx} - Q_x = 0 \tag{2.8}$$

With these equations and using strain relations from Hooke's law, the fine DE for displacement will be determined. In Equation 2.9, the relations between displacement and strain are presented.

$$\begin{aligned} \epsilon_x &= \frac{du}{dx} \\ \epsilon_\varphi &= -\frac{w}{a} \end{aligned} \tag{2.9}$$

From Hooke's law N_x may be also written as Equation 2.10. Substituting 2.9 will leave the final simplification.

$$N_x = \frac{Eh}{1-\nu^2}\ (\epsilon_x + \nu\epsilon_\varphi) = \frac{Eh}{1-\nu^2}\ \left(\frac{du}{dx} - \nu\ \frac{w}{a}\right) \tag{2.10}$$

Solving 2.10 using 2.6 leaves 2.11.

$$\frac{du}{dx} = \nu\ \frac{w}{a} \tag{2.11}$$

Similarly, with N_φ, again applying 2.9 leaves 2.13.

$$N_\varphi = \frac{Eh}{1-\nu^2}\ (\epsilon_\varphi + \nu\epsilon_x) = \frac{Eh}{1-\nu^2}\ \left(-\frac{w}{a} + \nu\ \frac{du}{dx}\right) \tag{2.12}$$

$$N_\varphi = -\frac{Ehw}{a} \tag{2.13}$$

Section 3

Discussion

Section 4

Conclusion

Section 5

Recommendations

Section 6

Supplemental

6.0.1 Installation

LaTeXis based on open-source code, so it is available on most computing platforms as free software. If encounter some compiling problems after installation, please Google it. For example, MikTeX may complain about ”mathtools.sty”, a solution given on ”StackExchange” is ”The problem is that the package manager has somehow ”desynchronized” (even though it's a fresh install). To fix it, run Miktex Package Manager as administrator—”Package Manager (Admin)”. Go to Repository–Synchronize. When that completes, your TexWorks should automatically find the needed style files again.”

- Linux: TeXLive distribution.
- MacOS: Mactex or TeXLive.
- Windows: MikTeX or TeXLive.

Note: to use LaTeX, you need a text editor for writing and editing ”.tex” files. To open the ”.tex” files in this template, you need a text editor which supports ”UTF-8” encoding. Free options for different platforms are the following:

- Linux: vim.
- MacOS: TeXShop, Macvim.
- Windows: Texmaker, Gvim, Notepad++.

6.0.2 Give a try

After downloading this template and installing a LaTeX distribution. It's time to have a try:

- Linux: run Compile.sh
- MacOS: run Compile.sh
- Windows: run Compile.bat

6.0.3 Include math

LaTeX realization of Equation 6.1 is something like this:

```
\begin{equationa}\label{eq:N-S_equation}
    \frac{\partial (\rho\mathbf{v})}{\partial t} +
    \nabla \cdot (\rho \mathbf{v} \mathbf{v}) =
    -\nabla p + \nabla \cdot\mathbf{T} + \mathbf{f}.
\end{equation}
```

$$\frac{\partial(\rho\mathbf{v})}{\partial t} + \nabla \cdot (\rho\mathbf{v}\mathbf{v}) = -\nabla p + \nabla \cdot \mathbf{T} + \mathbf{f}. \tag{6.1}$$

6.0.4 Include Graphics

Note: inluding figures may seem to be scary by looking at the codes. However, the fact is that you only need to modify the names in each part, the rest are simply copy and paste. These codes are all available in the file "Useful Commands.txt".

Figure 6.1 is an example for including a single figure.

```
\begin{figure}[!htbp]
    \centering
    \includegraphics[width=0.45\textwidth]{ITC_Q_Criteria}
    \caption{An Example for including a single figure}
    \label{fig:ITC_Q_Criteria}
\end{figure}
```

Figure 6.2 is an example for including multiple figuress.

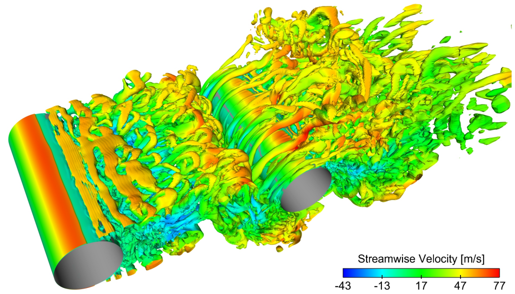
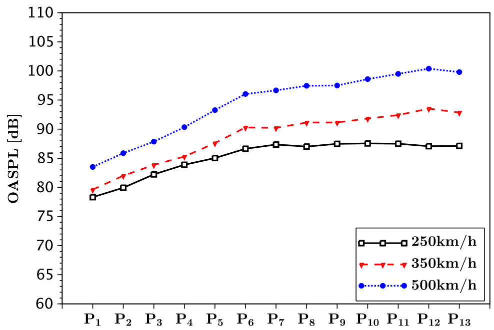
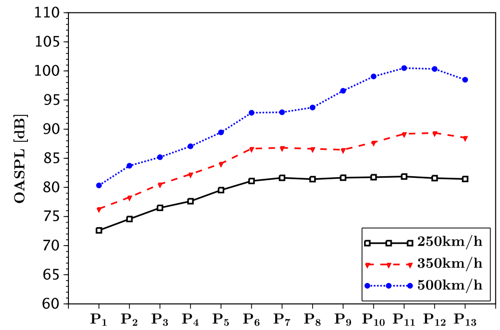


Figure 6.1: An Example for including a single graph

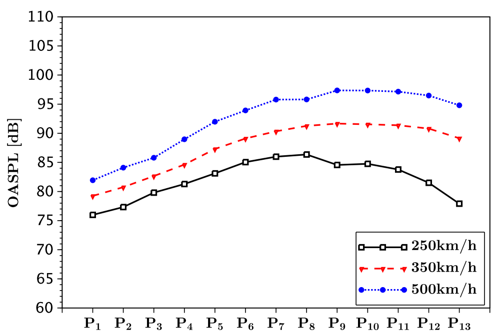
```
\begin{figure}[!htbp]
    \centering
    \begin{subfigure}[b]{0.45\textwidth}
        \includegraphics[width=\textwidth]{HC_OASPL_A}
        \caption{}
        \label{fig:HC_OASPL_A}
    \end{subfigure}%
    ~% add a small space
    \begin{subfigure}[b]{0.45\textwidth}
        \includegraphics[width=\textwidth]{HC_OASPL_B}
        \caption{}
        \label{fig:HC_OASPL_B}
    \end{subfigure}%
    \\% change line
    \begin{subfigure}[b]{0.45\textwidth}
        \includegraphics[width=\textwidth]{HC_OASPL_C}
        \caption{}
        \label{fig:HC_OASPL_C}
    \end{subfigure}%
    ~% add a small space
    \begin{subfigure}[b]{0.45\textwidth}
        \includegraphics[width=\textwidth]{HC_OASPL_D}
        \caption{}
        \label{fig:HC_OASPL_D}
    \end{subfigure}%
    \caption{An Example for including multiple figures}
    \label{fig:HC_OASPL}
\end{figure}
```



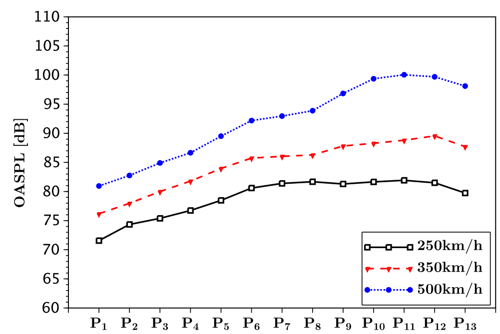
(a)



(b)



(c)



(d)

Figure 6.2: An Example for including multiple figures

6.0.5 Include a citation

Suppose you are going to cite an article named "Document Preparation System", the procedures are:

- Use Google Scholar search "Document Preparation System".
- Open "Cite" and choose "Import to Bibtex" under the target item.
- Copy the citation information of this article into the file "Myrefs.bib"
- Research dominant: cite this article by `\citep{lamport1986document}` like here [**?**]
- Citation dominant: cite this article by `\citet{lamport1986document}` like here **?**]
- References list is generated automatically.

6.0.6 Generate nomenclature

In this template, a simple command for adding nomenclatures is provided. Therefore, packages for automatical nomenclature generation are not included. From my point of view, there is no need to use those packages and make things complicated. However, if you insist, there are a lot of available packages for creating nomenclatures. Recommended options are (Please Google the one you want to know):

- listofsymbols
- nomencl