

# 1-FROBENIUS SIMPLE INTEGRAL FUSION RINGS

WINFRIED BRUNS AND SÉBASTIEN PALCOUX

## 1. COMPLETE CLASSIFICATION

Within the following bounds,

Rank	4	5	6	7	8	9	10	11	12
FPdim $\leq$	$10^{14}$	$10^7$	$10^6$	$10^5$	20000	10000	5000	3000	1000
#Types	0	1	1	3	4	10	8	4	0
#Fusion Rings	0	1	1	8	23	94	188	190	0

there are exactly 31 types comprising 505 non-pointed<sup>1</sup> simple integral 1-Frobenius fusion rings. The corresponding fusion data are presented on the following pages.

Rank	FPdim	Factors	Type	#	#3-positive	Note
5	60	$2^2 3^1 5^1$	[1, 3, 3, 4, 5]	1	1	PSL(2, 4)
6	168	$2^3 3^1 7^1$	[1, 3, 3, 6, 7, 8]	1	1	PSL(2, 7)
7	210	$2^1 3^1 5^1 7^1$	[1, 5, 5, 5, 6, 7, 7]	2	1	iPSL(2, 6)
7	360	$2^3 3^2 5^1$	[1, 5, 5, 8, 8, 9, 10]	2	1	PSL(2, 9)
7	7980	$2^2 3^1 5^1 7^1 19^1$	[1, 19, 20, 21, 42, 42, 57]	4	0	
8	660	$2^2 3^1 5^1 11^1$	[1, 5, 5, 10, 10, 11, 12, 12]	15	2	PSL(2, 11)
8	990	$2^1 3^2 5^1 11^1$	[1, 9, 10, 11, 11, 11, 11, 18]	5	0	
8	1260	$2^2 3^2 5^1 7^1$	[1, 6, 7, 7, 10, 15, 20, 20]	2	0	
8	1320	$2^3 3^1 5^1 11^1$	[1, 6, 6, 10, 11, 15, 15, 24]	1	0	
9	504	$2^3 3^2 7^1$	[1, 7, 7, 7, 7, 8, 9, 9, 9]	2	2	PSL(2, 8)
9	1092	$2^2 3^1 7^1 13^1$	[1, 7, 7, 12, 12, 12, 13, 14, 14]	30	2	PSL(2, 13)
9	1320	$2^3 3^1 5^1 11^1$	[1, 5, 5, 6, 6, 10, 11, 20, 24]	1	0	
9	1512	$2^3 3^3 7^1$	[1, 6, 7, 8, 8, 8, 8, 21, 27]	4	0	
9	2520	$2^3 3^2 5^1 7^1$	[1, 6, 10, 10, 14, 14, 15, 21, 35]	9	1	$A_7$
9	2730	$2^1 3^1 5^1 7^1 13^1$	[1, 13, 13, 13, 14, 15, 15, 26, 30]	12	0	
9	3420	$2^2 3^2 5^1 19^1$	[1, 9, 9, 19, 20, 20, 20, 20, 36]	12	4	
9	3960	$2^3 3^2 5^1 11^1$	[1, 8, 8, 9, 10, 15, 15, 40, 40]	3	0	
9	7980	$2^2 3^1 5^1 7^1 19^1$	[1, 19, 19, 20, 21, 38, 38, 42, 42]	20	0	
9	8736	$2^5 3^1 7^1 13^1$	[1, 12, 12, 12, 13, 14, 21, 21, 84]	1	0	
10	720	$2^4 3^2 5^1$	[1, 4, 4, 5, 5, 9, 10, 10, 10, 16]	2	0	
10	1638	$2^1 3^2 7^1 13^1$	[1, 6, 7, 7, 7, 13, 14, 18, 18, 21]	8	0	
10	1680	$2^4 3^1 5^1 7^1$	[1, 7, 7, 14, 14, 14, 15, 16, 16, 16]	75	2	iPSL(2, 15)
10	1716	$2^2 3^1 11^1 13^1$	[1, 11, 11, 11, 11, 11, 11, 12, 13, 26]	1	0	
10	2184	$2^3 3^1 7^1 13^1$	[1, 6, 6, 12, 12, 13, 14, 21, 21, 24]	3	0	
10	2640	$2^4 3^1 5^1 11^1$	[1, 11, 11, 11, 15, 15, 15, 16, 16, 33]	21	0	
10	3366	$2^1 3^2 11^1 17^1$	[1, 11, 11, 17, 17, 18, 18, 18, 22, 33]	48	0	
10	4620	$2^2 3^1 5^1 7^1 11^1$	[1, 11, 11, 20, 20, 20, 20, 20, 21, 44]	30	1	
11	720	$2^4 3^2 5^1$	[1, 4, 5, 5, 5, 5, 6, 6, 9, 15, 15]	2	0	
11	990	$2^1 3^2 5^1 11^1$	[1, 9, 9, 9, 9, 9, 10, 11, 11, 11, 11]	5	2	iPSL(2, 10)
11	2184	$2^3 3^1 7^1 13^1$	[1, 6, 6, 7, 7, 12, 12, 13, 14, 24, 28]	3	0	
11	2448	$2^4 3^2 17^1$	[1, 9, 9, 16, 16, 16, 16, 17, 18, 18, 18]	180	8	PSL(2, 17)

## 2. IN PROGRESS CLASSIFICATION

Here are the current exploration bounds (work in progress),

Rank	8	9	10	11	12
FPdim $\leq$	50000	30000	10000	5000	3500

along with the count of newly discovered non-pointed 1-Frobenius simple integral fusion rings<sup>2</sup>.

Rank	FPdim	Factors	Type	#	#3-positive	Note
8	46620	$2^2 3^2 5^1 7^1 37^1$	[1, 35, 36, 37, 70, 70, 105, 148]	20	0	
9	10626	$2^1 3^1 7^1 11^1 23^1$	[1, 21, 21, 21, 22, 23, 42, 42, 69]	1	0	
9	16380	$2^2 3^2 5^1 7^1 13^1$	[1, 12, 12, 12, 13, 21, 35, 84, 84]	1	0	
9	21924	$2^2 3^3 7^1 29^1$	[1, 27, 27, 28, 54, 54, 54, 58, 87]	11	0	
9	28560	$2^4 3^1 5^1 7^1 17^1$	[1, 14, 14, 14, 15, 16, 48, 105, 119]	4	0	
10	5040	$2^4 3^2 5^1 7^1$	[1, 7, 8, 8, 8, 8, 15, 21, 42, 48]	4	0	
10	5278	$2^1 7^1 13^1 29^1$	[1, 13, 14, 14, 26, 26, 29, 29, 29, 29]	12	0	
10	6720	$2^6 3^1 5^1 7^1$	[1, 10, 10, 14, 15, 16, 21, 24, 35, 60]	2	0	
10	7920	$2^4 3^2 5^1 11^1$	[1, 10, 10, 10, 11, 16, 16, 44, 45, 55]	2	2	$M_{11}$
10	7980	$2^2 3^1 5^1 7^1 19^1$	[1, 19, 20, 21, 21, 21, 21, 21, 42, 57]	1	0	
11	3960	$2^3 3^2 5^1 11^1$	[1, 10, 10, 10, 10, 11, 12, 12, 15, 30, 45]	20	0	
11	4620	$2^2 3^1 5^1 7^1 11^1$	[1, 5, 5, 5, 6, 14, 14, 21, 35, 35, 35]	71	0	
12	2520	$2^3 3^2 5^1 7^1$	[1, 6, 6, 7, 8, 8, 8, 15, 21, 24, 30]	4		
12	2520	$2^3 3^2 5^1 7^1$	[1, 6, 7, 7, 7, 7, 10, 10, 14, 15, 21, 35]	5		+3 non-simple
12	3360	$2^5 3^1 5^1 7^1$	[1, 8, 8, 14, 14, 14, 15, 16, 16, 21, 21, 32]	3		

*Date:* August 2025.

<sup>1</sup>The pointed case reduces to prime numbers.

<sup>2</sup>Note: there are at least # such fusion rings, though some dualities may be overlooked

*# %attach /home/sebastien/Nutstore Files/SAGE/SimpleIntegralFusionRings.sage*

*# Classification of all the non-pointed simple integral 1-Frobenius fusion rings, under the following bounds:  
# Rank<=r and FPdim<d with (r,d) = (4, 10^12), (5, 10^7), (6, 10^6), (7, 10^5), (8, 20000), (9, 10000), (10, 5000), (11, 3000), (12,1000).*

*#r5d60t[[1, 1], [3, 2], [4, 1], [5, 1]]  
#Finite simple group A5*

S60=[[  
[1,0,0,0,0],[0,1,0,0,0],[0,0,1,0,0],[0,0,0,1,0],[0,0,0,0,1]],  
[0,1,0,0,0],[1,1,0,0,1],[0,0,0,1,1],[0,0,1,1,1],[0,1,1,1,1]],  
[0,0,1,0,0],[0,0,0,1,1],[1,0,1,0,1],[0,1,0,1,1],[0,1,1,1,1]],  
[0,0,0,1,0],[0,0,1,1,1],[0,1,0,1,1],[1,1,1,1,1],[0,1,1,1,2]],  
[0,0,0,0,1],[0,1,1,1,1],[0,1,1,1,1],[0,1,1,1,2],[1,1,1,2,2]]  
]]

*#r6d168t[[1, 1], [3, 2], [6, 1], [7, 1], [8, 1]]  
#Finite simple group PSL(2,7)*

S168=[[  
[1,0,0,0,0,0],[0,1,0,0,0,0],[0,0,1,0,0,0],[0,0,0,1,0,0],[0,0,0,0,1,0],[0,0,0,0,0,1]],  
[0,1,0,0,0,0],[0,0,1,1,0,0],[1,0,0,0,0,1],[0,0,1,0,1,1],[0,0,0,1,1,1],[0,1,0,1,1,1]],  
[0,0,1,0,0,0],[1,1,0,0,0,1],[0,1,0,1,0,0],[0,1,0,0,1,1],[0,0,0,1,1,1],[0,0,1,1,1,1]],  
[0,0,0,1,0,0],[0,0,1,0,1,1],[0,1,0,0,1,1],[1,0,0,2,1,2],[0,1,1,1,2,2],[0,1,1,2,2,2]],  
[0,0,0,0,1,0],[0,0,0,1,1,1],[0,0,0,1,1,1],[0,1,1,1,2,2],[1,1,1,2,2,2],[0,1,1,2,2,3]],  
[0,0,0,0,0,1],[0,1,0,1,1,1],[0,0,1,1,1,1],[0,1,1,2,2,2],[0,1,1,2,2,3],[1,1,1,2,3,3]]  
]]

*#r7d210t[[1, 1], [5, 3], [6, 1], [7, 2]]*

S210=[[  
[1,0,0,0,0,0,0],[0,1,0,0,0,0,0],[0,0,1,0,0,0,0],[0,0,0,1,0,0,0],[0,0,0,0,1,0,0],[0,0,0,0,0,1,0],[0,0,0,0,0,0,1]],  
[0,1,0,0,0,0,0],[1,1,0,1,0,1,1],[0,0,1,0,1,1,1],[0,1,0,0,1,1,1],[0,0,1,1,1,1,1],[0,1,1,1,1,1,1],[0,1,1,1,1,1,1]],  
[0,0,1,0,0,0,0],[0,0,1,0,1,1,1],[1,1,1,0,0,1,1],[0,0,0,1,1,1,1],[0,1,0,1,1,1,1],[0,1,1,1,1,1,1],[0,1,1,1,1,1,1]],  
[0,0,0,1,0,0,0],[0,1,0,0,1,1,1],[0,0,0,1,1,1,1],[1,0,1,1,0,1,1],[0,1,1,0,1,1,1],[0,1,1,1,1,1,1],[0,1,1,1,1,1,1]],  
[0,0,0,0,1,0,0],[0,0,1,1,1,1,1],[0,1,0,1,1,1,1],[0,1,1,0,1,1,1],[1,1,1,1,1,1,1],[0,1,1,1,2,1],[0,1,1,1,1,1,2]],  
[0,0,0,0,0,1,0],[0,1,1,1,1,1,1],[0,1,1,1,1,1,1],[0,1,1,1,1,1,1],[0,1,1,1,1,1,1],[1,1,1,1,2,1],[1,1,1,1,2,0,3],[0,1,1,1,1,3,1]],  
[0,0,0,0,0,0,1],[0,1,1,1,1,1,1],[0,1,1,1,1,1,1],[0,1,1,1,1,1,1],[0,1,1,1,1,1,2],[0,1,1,1,1,3,1],[1,1,1,1,2,1,2]]  
], [  
[1,0,0,0,0,0,0],[0,1,0,0,0,0,0],[0,0,1,0,0,0,0],[0,0,0,1,0,0,0],[0,0,0,0,1,0,0],[0,0,0,0,0,1,0],[0,0,0,0,0,0,1]],  
[0,1,0,0,0,0,0],[1,1,0,1,0,1,1],[0,0,1,0,1,1,1],[0,1,0,0,1,1,1],[0,0,1,1,1,1,1],[0,1,1,1,1,1,1],[0,1,1,1,1,1,1]],  
[0,0,1,0,0,0,0],[0,0,1,0,0,1,1],[1,1,1,0,0,1,1],[0,0,0,1,1,1,1],[0,1,0,1,1,1,1],[0,1,1,1,1,1,1],[0,1,1,1,1,1,1]],  
[0,0,1,0,0,0,0],[0,0,1,0,1,1,1],[1,1,1,0,0,1,1],[0,0,0,1,1,1,1],[0,1,0,1,1,1,1],[0,1,1,1,1,1,1],[0,1,1,1,1,1,1]],  
[0,0,0,1,0,0,0],[0,1,0,0,1,1,1],[0,0,0,1,1,1,1],[1,0,1,1,0,1,1],[0,1,1,0,1,1,1],[0,1,1,1,1,1,1],[0,1,1,1,1,1,1]],  
[0,0,0,0,1,0,0],[0,0,1,1,1,1,1],[0,1,0,1,1,1,1],[0,1,1,0,1,1,1],[1,1,1,1,1,1,1],[0,1,1,1,2,1],[0,1,1,1,1,1,2]],  
[0,0,0,0,0,1,0,0],[0,0,1,1,1,1,1,1],[0,1,0,1,1,1,1],[0,1,1,0,1,1,1],[1,1,1,1,1,1,1],[0,1,1,1,1,2,1],[0,1,1,1,1,1,2]],  
[0,0,0,0,0,1,0],[0,1,1,1,1,1,1],[0,1,1,1,1,1,1],[0,1,1,1,1,1,1],[0,1,1,1,1,2,1],[1,1,1,1,2,1,2],[0,1,1,1,1,2,2]],  
[0,0,0,0,0,0,1],[0,1,1,1,1,1,1],[0,1,1,1,1,1,1],[0,1,1,1,1,1,1],[0,1,1,1,1,1,2],[0,1,1,1,1,2,2],[1,1,1,1,2,2,1]]  
]]

*#r7d360t[[1,1],[5,2],[8,2],[9,1],[10,1]]*

S360=[[  
[1,0,0,0,0,0,0],[0,1,0,0,0,0,0],[0,0,1,0,0,0,0],[0,0,0,1,0,0,0],[0,0,0,0,1,0,0],[0,0,0,0,0,1,0],[0,0,0,0,0,0,1]],  
[0,1,0,0,0,0,0],[1,1,0,0,0,1,1],[0,0,0,1,1,1,0],[0,0,1,1,1,1,1],[0,0,1,1,1,1,1],[0,1,1,1,1,1,1],[0,1,0,1,1,1,2]],  
[0,0,1,0,0,0,0],[0,0,0,1,1,1,0],[1,0,1,0,0,1,1],[0,1,0,1,1,1,1],[0,1,0,1,1,1,1],[0,1,1,1,1,1,1],[0,0,1,1,1,1,2]],  
[0,0,0,1,0,0,0],[0,0,1,1,1,1,1],[0,1,0,1,1,1,1],[1,1,1,1,2,1,2],[0,1,1,2,0,2,2],[0,1,1,2,2,2],[0,1,1,2,2,2,2]],  
[0,0,0,0,1,0,0],[0,0,1,1,1,1,1],[0,1,0,1,1,1,1],[0,1,1,2,0,2,2],[1,1,1,0,3,1,2],[0,1,1,2,1,2,2],[0,1,1,2,2,2,2]],  
[0,0,0,0,0,1,0],[0,1,1,1,1,1,1],[0,1,1,1,1,1,1],[0,1,1,1,2,2,2],[0,1,1,2,1,2,2],[1,1,1,2,2,2,2],[0,1,1,2,2,2,3]],  
[0,0,0,0,0,0,1],[0,1,0,1,1,1,2],[0,0,1,1,1,1,2],[0,1,1,2,2,2,2],[0,1,1,2,2,2,2],[0,1,1,2,2,2,3],[1,2,2,2,2,3,2]]  
], [  
[1,0,0,0,0,0,0],[0,1,0,0,0,0,0],[0,0,1,0,0,0,0],[0,0,0,1,0,0,0],[0,0,0,0,1,0,0],[0,0,0,0,0,1,0],[0,0,0,0,0,0,1]],  
[0,1,0,0,0,0,0],[1,1,0,0,0,1,1],[0,0,0,1,1,1,0],[0,0,1,1,1,1,1],[0,0,1,1,1,1,1],[0,1,1,1,1,1,1],[0,1,0,1,1,1,2]],  
[0,0,1,0,0,0,0],[0,0,0,1,1,1,0],[1,0,1,0,0,1,1],[0,1,0,1,1,1,1],[0,1,0,1,1,1,1],[0,1,1,1,1,1,1],[0,0,1,1,1,1,2]],  
[0,0,0,1,0,0,0],[0,0,1,1,1,1,1],[0,1,0,1,1,1,1],[1,1,1,1,2,1,2],[0,1,1,2,0,2,2],[0,1,1,2,2,2],[0,1,1,2,2,2,2]],  
[0,0,0,0,1,0,0],[0,0,1,1,1,1,1],[0,1,0,1,1,1,1],[0,1,1,2,0,2,2],[1,1,1,0,3,1,2],[0,1,1,2,1,2,2],[0,1,1,2,2,2,2]],  
[0,0,0,0,0,1,0],[0,1,1,1,1,1,1],[0,1,1,1,1,1,1],[0,1,1,1,2,2,2],[0,1,1,2,1,2,2],[1,1,1,2,2,2,2],[0,1,1,2,2,2,3]],  
[0,0,0,0,0,0,1],[0,1,0,1,1,1,2],[0,0,1,1,1,1,2],[0,1,1,2,2,2,2],[0,1,1,2,2,2,2],[0,1,1,2,2,2,3],[1,2,2,2,2,3,2]]  
]]

*#Finite simple group A6*

[1,0,0,0,0,0,0],[0,1,0,0,0,0,0],[0,0,1,0,0,0,0],[0,0,0,1,0,0,0],[0,0,0,0,1,0,0],[0,0,0,0,0,1,0],[0,0,0,0,0,0,1]],  
[0,1,0,0,0,0,0],[1,1,0,0,0,1,1],[0,0,0,1,1,1,0],[0,0,1,1,1,1,1],[0,0,1,1,1,1,1],[0,1,1,1,1,1,1],[0,1,0,1,1,1,2]],  
[0,0,1,0,0,0,0],[0,0,0,1,1,1,0],[1,0,1,0,0,1,1],[0,1,0,1,1,1,1],[0,1,0,1,1,1,1],[0,1,1,1,1,1,1],[0,0,1,1,1,1,2]],  
[0,0,0,1,0,0,0],[0,0,1,1,1,1,1],[0,1,0,1,1,1,1],[1,1,1,1,1,1,2],[0,1,1,1,2,1,2],[0,1,1,1,2,2,2],[0,1,1,2,2,2,2]],  
[0,0,0,0,1,0,0],[0,0,1,1,1,1,1],[0,1,0,1,1,1,1],[0,1,1,1,2,2,2],[1,1,1,1,2,1,2],[0,1,1,2,1,2,2],[0,1,1,2,2,2,2]],  
[0,0,0,0,0,1,0],[0,0,1,1,1,1,1],[0,1,0,1,1,1,1],[0,1,1,1,1,2,2],[1,1,1,1,2,1,2],[0,1,1,2,1,2,2],[0,1,1,2,2,2,2]],  
[0,0,0,0,0,0,1],[0,1,1,1,1,1,1],[0,1,1,1,1,1,1],[0,1,1,1,2,2,2],[0,1,1,2,1,2,2],[1,1,1,2,2,2,2],[0,1,1,2,2,2,3]],  
[0,0,0,0,0,0,1],[0,1,0,1,1,1,2],[0,0,1,1,1,1,2],[0,1,1,2,2,2,2],[0,1,1,2,2,2,2],[0,1,1,2,2,2,3],[1,2,2,2,2,3,2]]  
]]

*#r7d7980t[[1, 1], [19, 1], [20, 1], [21, 1], [42, 2], [57, 1]]*

S7980=[[  
[1,0,0,0,0,0,0],[0,1,0,0,0,0,0],[0,0,1,0,0,0,0],[0,0,0,1,0,0,0],[0,0,0,0,1,0,0],[0,0,0,0,0,1,0],[0,0,0,0,0,0,1]],  
[0,1,0,0,0,0,0],[1,0,0,1,2,2,3],[0,0,1,1,2,2,3],[0,1,1,1,2,2,3],[0,2,2,2,4,4,6],[0,2,2,2,4,4,6],[0,3,3,3,6,6,7]],  
[0,0,1,0,0,0,0],[0,0,1,1,2,2,3],[1,1,1,1,2,2,3],[0,1,1,2,2,3],[0,2,2,2,5,4,6],[0,2,2,2,4,5,6],[0,3,3,3,6,6,8]],  
[0,0,0,1,0,0,0],[0,1,1,1,2,2,3],[0,1,1,2,2,3],[1,1,2,2,0,4,3],[0,2,2,0,9,2,6],[0,2,2,4,2,7,6],[0,3,3,3,6,6,9]],  
[0,0,0,0,1,0,0],[0,2,2,2,4,4,6],[0,2,2,2,5,4,6],[0,2,2,0,9,2,6],[1,4,5,9,2,15,12],[0,4,4,2,15,6,12],[0,6,6,6,12,12,18]],  
[0,0,0,0,0,1,0],[0,2,2,2,4,4,6],[0,2,2,2,4,5,6],[0,2,2,4,2,7,6],[0,4,4,2,15,6,12],[1,4,5,7,6,12,12],[0,6,6,6,12,12,18]],  
[0,0,0,0,0,0,1],[0,3,3,3,6,6,7],[0,3,3,3,6,6,8],[0,3,3,3,6,6,9],[0,6,6,6,12,12,18],[0,6,6,6,12,12,18],[1,7,8,9,18,18,22]]  
], [  
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], [  
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[0,0,0,0,0,0,1],[0,3,3,3,6,6,7],[0,3,3,3,6,6,8],[0,3,3,3,6,6,9],[0,6,6,6,12,12,18],[0,6,6,6,12,12,18],[1,7,8,9,18,18,22]]  
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[0,0,0,0,1,0,0],[0,2,2,2,4,4,6],[0,2,2,2,5,4,6],[0,2,2,0,3,8,6],[1,4,5,3,5,15,12],[0,4,4,8,15,3,12],[0,6,6,6,12,12,18]],  
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*#r8d660t[[1, 1], [5, 2], [10, 2], [11, 1], [12, 2]]*

S660=[[

[illegible]

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[illegible]



[illegible]

[[0,0,0,0,0,0,0,1,0],[0,1,0,0,1,0,1,1,2],[0,0,1,1,1,1,1,2,3],[0,0,1,1,1,1,1,2,3],[0,1,1,1,3,2,2,2,4],[0,0,1,1,1,3,1,3,4],[0,1,1,1,2,2,1,2,2,5],[1,1,2,2,2,3,2,4,6],[0,2,3,3,4,4,5,6,10],[0,0,0,0,0,0,0,0,1],[0,0,1,1,1,1,1,2,3],[0,1,1,1,2,2,2,3,5],[0,1,1,1,2,2,2,3,5],[0,1,2,2,2,3,3,4,7],[0,1,2,2,3,2,3,4,7],[0,1,2,2,3,3,3,5,7],[0,2,3,3,4,4,5,6,10],[1,3,5,5,7,7,7,10,17],[]]





[illegible]

[illegible]

[illegible]

9d3968[[[1, 1], [8, 2], [9, 1], [10, 1], [15, 2], [40, 2]],  
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 ],  
 [[1,0,0,0,0,0,0,0],[0,1,0,0,0,0,0,0],[0,0,1,0,0,0,0,0],[0,0,0,1,0,0,0,0],[0,0,0,0,1,0,0,0],[0,0,0,0,0,1,0,0],[0,0,0,0,0,0,1,0],[0,0,0,0,0,0,0,1]],  
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[illegible]

[0,1,0,0,0,0,0,0],[1,1,2,0,1,0,3,2,2],[0,2,0],[0,0,1,1,2,2,2,2],[0,1,1,1,2,2,2,2],[0,0,2,2,2,3,4,4,4],[0,3,1,2,2,4,2,4,4],[0,2,2,2,2,4,4,4,4],[0,2,2,2,2,4,4,4,4],[0,0,1,0,0,0,0,0],[1,1,2,0,1,1,2,2,2],[1,0,3,0,1,2,1,2,2],[0,1,0,1,1,2,2,2,2],[0,1,1,1,2,2,2,2],[0,2,2,2,2,4,4,4,4],[0,1,1,2,2,2,5,4,4],[0,2,2,2,2,4,4,4,4],[0,0,0,1,0,0,0,0],[0,0,1,1,2,2,2,2],[0,1,0,1,1,2,2,2,2],[1,1,1,1,1,2,2,2,2],[0,1,1,1,2,2,2,2],[0,2,2,2,2,3,4,4,4],[0,2,2,2,2,4,3,4,4],[0,2,2,2,2,4,4,5,4],[0,2,2,2,2,4,4,4,5],[0,0,0,0,1,0,0,0],[0,1,1,1,2,2,2,2],[0,1,1,1,2,2,2,2],[0,1,1,2,2,2,2,2],[0,2,2,2,2,4,4,4,4],[0,2,2,2,2,4,4,4,4],[0,2,2,2,0,4,4,9,2],[0,2,2,4,4,4,2,7],[0,0,0,0,0,1,0,0],[0,0,2,2,2,3,4,4,4],[0,2,2,2,2,3,4,4,4],[0,2,2,2,2,4,4,4,4],[1,3,4,3,4,6,7,8,8],[0,4,2,4,4,7,6,8,8],[0,4,4,4,4,8,8,8,8],[0,4,4,4,4,8,8,8,8],[0,0,0,0,0,1,0,0],[0,3,1,2,2,4,4,4,4],[0,1,1,2,2,5,4,4,4],[0,2,2,2,2,4,3,4,4],[0,2,2,2,2,4,4,4,4],[0,4,2,4,4,7,6,8,8],[1,2,5,3,4,6,7,8,8],[0,4,4,4,4,8,8,8,8],[0,4,4,4,4,8,8,8,8],[0,0,0,0,0,0,1,0],[0,2,2,2,2,4,4,4,4],[0,2,2,2,2,4,4,4,4],[0,2,2,2,2,4,4,5,4],[0,2,2,2,2,4,4,4,9,2],[0,4,4,4,4,8,8,8,8],[1,4,4,5,9,8,8,2,15],[0,4,4,4,2,8,8,15,6],[0,0,0,0,0,0,0,1],[0,2,2,2,2,4,4,4,4],[0,2,2,2,2,4,4,4,4],[0,2,2,2,2,4,4,4,5],[0,2,2,2,2,4,4,4,2,7],[0,4,4,4,4,8,8,8,8],[0,4,4,4,4,8,8,8,8],[0,4,4,4,2,8,8,15,6],[1,4,4,5,7,8,8,6,12]]  
],I  
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[illegible]



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10d1638t[[1, 1], [6, 1], [7, 3], [13, 1], [14, 1], [18, 2], [21, 1]]
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]

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[illegible]

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#r10d1680t[[1, 1], [7, 2], [14, 3], [15, 1], [16, 3]]
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[illegible]

[illegible]

[illegible]



[illegible]



[illegible]

[illegible]

[illegible]

[illegible]

[illegible]





[illegible]

[illegible]

[illegible]



[illegible]



[illegible]

[illegible]

[1,0,0,0,0,0,0,0,0,0]	[0,1,0,0,0,0,0,0,0,0]	[0,0,1,0,0,0,0,0,0,0]	[0,0,0,1,0,0,0,0,0,0]	[0,0,0,0,1,0,0,0,0,0]	[0,0,0,0,0,1,0,0,0,0]	[0,0,0,0,0,0,1,0,0,0]	[0,0,0,0,0,0,0,1,0,0]	[0,0,0,0,0,0,0,0,1,0]	[0,0,0,0,0,0,0,0,0,1]
[0,1,0,0,0,0,0,0,0,0]	[1,1,0,0,0,0,0,0,0,1]	[0,0,0,1,1,1,1,1,0,0]	[0,0,1,1,1,1,1,1,1,2]	[0,0,1,1,1,1,1,1,1,2]	[0,0,1,1,1,1,1,1,1,2]	[0,0,1,1,1,1,1,1,1,2]	[0,0,1,1,1,1,1,1,1,2]	[0,1,1,1,1,1,1,1,1,2]	[0,2,0,2,2,2,2,2,2,5]
[0,0,1,0,0,0,0,0,0,0]	[0,0,0,1,1,1,1,1,0,0]	[0,1,0,0,0,0,0,0,1,2]	[0,1,0,1,1,1,1,1,1,2]	[0,1,0,1,1,1,1,1,1,2]	[0,1,0,1,1,1,1,1,1,2]	[0,1,0,1,1,1,1,1,1,2]	[0,1,0,1,1,1,1,1,1,2]	[0,1,1,1,1,1,1,1,1,2]	[0,2,2,2,2,2,2,2,2,5]
[0,0,0,1,0,0,0,0,0,0]	[0,0,1,1,1,1,1,1,1,2]	[0,1,0,1,1,1,1,1,1,2]	[1,1,1,2,2,2,2,1,4]	[0,1,1,2,2,2,1,2,4]	[0,1,1,2,2,1,2,2,4]	[0,1,1,2,1,2,2,2,4]	[0,1,1,1,2,2,2,2,4]	[0,1,1,2,2,2,2,2,4]	[0,2,2,4,4,4,4,4,4,8]
[0,0,0,0,1,0,0,0,0,0]	[0,1,1,1,1,1,1,1,1,2]	[0,1,0,1,1,1,1,1,1,2]	[0,1,1,2,2,2,1,2,4]	[1,1,1,2,2,1,2,2,4]	[0,1,1,2,2,1,2,2,4]	[0,1,1,2,2,1,2,2,4]	[0,1,1,2,2,1,2,2,4]	[0,1,1,2,2,1,2,2,4]	[0,2,2,4,4,4,4,4,4,8]
[0,0,0,0,0,1,0,0,0,0]	[0,1,1,1,1,1,1,1,1,2]	[0,1,0,1,1,1,1,1,1,2]	[0,1,1,2,2,1,2,2,4]	[1,1,1,2,2,1,2,2,4]	[0,1,1,2,2,2,1,2,4]	[0,1,1,2,2,2,1,2,4]	[0,1,1,2,2,2,1,2,4]	[0,1,1,2,2,2,1,2,4]	[0,2,2,4,4,4,4,4,4,8]
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[illegible]



[illegible]



[illegible]

[illegible]

[1,0,0,0,0,0,0,0,0,0]	[0,1,0,0,0,0,0,0,0,0]	[0,0,1,0,0,0,0,0,0,0]	[0,0,0,1,0,0,0,0,0,0]	[0,0,0,0,1,0,0,0,0,0]	[0,0,0,0,0,1,0,0,0,0]	[0,0,0,0,0,0,1,0,0,0]	[0,0,0,0,0,0,0,1,0,0]	[0,0,0,0,0,0,0,0,1,0]	[0,0,0,0,0,0,0,0,0,1]
[0,1,0,0,0,0,0,0,0,0]	[1,1,0,0,0,0,0,0,0,0]	[0,0,1,1,1,1,1,0,0,0]	[0,0,1,1,1,1,1,1,1,1]	[0,0,1,1,1,1,1,1,1,1]	[0,0,1,1,1,1,1,1,1,1]	[0,0,1,1,1,1,1,1,1,1]	[0,0,1,1,1,1,1,1,1,1]	[0,0,1,1,1,1,1,1,1,1]	[0,0,1,1,1,1,1,1,1,1]
[0,0,1,0,0,0,0,0,0,0]	[0,0,0,1,1,1,1,0,0,0]	[1,0,1,0,0,0,0,1,1,1]	[1,0,1,0,1,1,1,1,1,1]	[1,0,1,0,1,1,1,1,1,1]	[1,0,1,0,1,1,1,1,1,1]	[1,0,1,1,1,1,1,1,1,1]	[1,0,1,1,1,1,1,1,1,1]	[1,0,1,1,1,1,1,1,1,1]	[1,0,1,1,1,1,1,1,1,1]
[0,0,0,1,0,0,0,0,0,0]	[0,0,1,1,1,1,1,1,1,1]	[1,0,1,0,1,1,1,1,1,1]	[1,1,1,2,1,0,4,1,2,2]	[1,1,1,3,2,0,2,2,2]	[1,1,1,0,2,2,2,2,2]	[1,1,1,4,0,2,2,2,2]	[1,1,1,2,2,2,2,2]	[1,1,1,2,2,2,2,2]	[1,1,1,2,2,2,2,2]
[0,0,0,0,1,0,0,0,0,0]	[0,0,1,1,1,1,1,1,1,1]	[1,0,1,0,1,1,1,1,1,1]	[1,1,1,3,2,0,2,2,2]	[1,1,1,2,0,2,2,2,2]	[1,1,1,0,2,2,2,2,2]	[1,1,1,4,0,2,2,2,2]	[1,1,1,2,2,2,2,2]	[1,1,1,2,2,2,2,2]	[1,1,1,2,2,2,2,2]
[0,0,0,0,0,1,0,0,0,0]	[0,0,1,1,1,1,1,1,1,1]	[1,0,1,0,1,1,1,1,1,1]	[1,1,1,2,2,2,2,2]	[1,1,1,2,2,2,2,2]	[1,1,1,2,2,2,2,2]	[1,1,1,2,2,2,2,2]	[1,1,1,2,2,2,2,2]	[1,1,1,2,2,2,2,2]	[1,1,1,2,2,2,2,2]
[0,0,0,0,0,0,1,0,0,0]	[0,0,1,1,1,1,1,1,1,1]	[1,0,1,0,1,1,1,1,1,1]	[1,1,1,4,0,2,2,2,2]	[1,1,1,0,2,2,2,2,2]	[1,1,1,2,2,2,2,2]	[1,1,1,4,0,2,2,2,2]	[1,1,1,2,2,2,2,2]	[1,1,1,2,2,2,2,2]	[1,1,1,2,2,2,2,2]

[illegible]



[illegible]

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[illegible]



[illegible]

[illegible]

[illegible]

BL=[S60,S168,S210,S360,S7980,S660,S990,S1260,S1320,S504,S1092,S1320b,S1512,S2520,S2730,S3420,S3960,S7980b,S8736,S720,S1638,S1680,S1716,S2184,S2640,S3366,S4620,S720b,S990b,S2184b,S2448]

```
sage: for i in range(31):
.....:     for j in range(L[i]):
.....:         if not Burnside(BL[i][j]):
.....:             print([i,j])
.....:
[13, 2]
[13, 3]
[13, 7]
[13, 8]
```



```
[13, 8]
[16, 0]
[16, 1]
[16, 2]
[18, 0]
```

```
sage: %attach /home/sebastien/Nutstore Files/SAGE/AllCriteria.sage
```

```
sage: for i in range(31):
.....:     for j in range(L[i]):
.....:         M=BL[i][j]
.....:         [v,ro]=IntFormalCodegrees(M)
.....:         if v and IsIntDrinfeldInter(M,v,ro):
.....:             print([len(M),i,j])
.....:
[5, 0, 0]
[6, 1, 0]
[7, 2, 0]
[7, 2, 1]
[7, 3, 0]
[7, 3, 1]
[8, 5, 4]
[8, 5, 5]
[8, 5, 10]
[8, 5, 11]
[8, 5, 13]
[8, 5, 14]
[8, 8, 0]
[9, 9, 0]
[9, 9, 1]
[9, 10, 8]
[9, 10, 14]
[9, 10, 20]
[9, 10, 24]
[9, 10, 27]
[9, 10, 29]
[9, 11, 0]
[9, 13, 0]
[9, 13, 4]
[9, 15, 5]
[9, 15, 7]
[9, 15, 10]
[9, 15, 11]
[10, 21, 17]
[10, 21, 19]
[10, 21, 27]
[10, 21, 29]
[10, 21, 32]
[10, 21, 34]
[10, 21, 47]
[10, 21, 49]
[10, 21, 57]
[10, 21, 59]
[10, 21, 62]
[10, 21, 64]
[10, 21, 67]
[10, 21, 69]
[10, 21, 72]
[10, 21, 74]
[10, 26, 0]
[10, 26, 10]
[10, 26, 11]
[10, 26, 13]
[10, 26, 14]
[11, 28, 2]
[11, 28, 4]
[11, 30, 75]
[11, 30, 79]
[11, 30, 132]
[11, 30, 134]
[11, 30, 144]
[11, 30, 149]
[11, 30, 177]
[11, 30, 179]
```

```
% 59 among 505 are Drinfeld
```

```
sage: for i in range(31):
.....:     for j in range(L[i]):
.....:         M=BL[i][j]
.....:         [v,ro]=IntFormalCodegrees(M)
.....:         if v and IsIntDrinfeldInter(M,v,ro):
.....:             if SchurReformulated(M):
.....:                 print([len(M),i,j])
.....:
[5, 0, 0]
[6, 1, 0]
[7, 2, 1]
[7, 3, 1]
[8, 5, 5]
[8, 5, 11]
[9, 9, 0]
[9, 9, 1]
[9, 10, 14]
[9, 10, 29]
[9, 13, 4]
[9, 15, 10]
[9, 15, 11]
[10, 21, 49]
[10, 21, 74]
[10, 26, 0]
[11, 28, 2]
```

[11, 28, 4]  
[11, 30, 75]  
[11, 30, 79]  
[11, 30, 132]  
[11, 30, 134]  
[11, 30, 144]  
[11, 30, 149]  
[11, 30, 177]  
[11, 30, 179]

%among them, 26 are also 3-positive

% the four exotic ones of rank 9 are all open:

sage: SL=[BL[9][0],BL[9][1],BL[10][14],BL[10][29],BL[15][10],BL[15][11]]

sage: CheckingList(SL,60)

[0]

FPdim = 504.00000000000006

FPdims = [1.0, 7.000000000000001, 7.000000000000001, 7.000000000000001, 7.000000000000001, 7.999999999999999, 9.0, 9.0, 9.0]

simple

non-1/2-FrobType, so not modular cat

CC= [504, 9, 9, 9, 8, 7, 7, 7]

Non-pivotal Column= 1

Non-pivotal Column= 2

Non-pivotal Column= 3

Non-pivotal Column= 4

Non-pivotal Column= 5

Non-pivotal Column= 6

Non-pivotal Column= 7

Non-pivotal Column= 8

Isaacs list = [0, -1, -1, -1, -1, -1, -1, -1, -1]

[1]

FPdim = 504.0

FPdims = [1.0, 7.000000000000001, 6.999999999999999, 6.999999999999999, 6.999999999999999, 7.999999999999999, 9.0, 9.0, 9.0]

simple

non-1/2-FrobType, so not modular cat

CC= [504, 9, 9, 9, 9, 8, 7, 7]

Non-pivotal Column= 1

Non-pivotal Column= 2

Non-pivotal Column= 3

Non-pivotal Column= 4

Non-pivotal Column= 5

Non-pivotal Column= 6

Non-pivotal Column= 7

Non-pivotal Column= 8

Isaacs list = [0, -1, -1, -1, -1, -1, -1, -1, -1]

[2]

FPdim = 1091.9999999999998

FPdims = [1.0, 7.000000000000001, 6.999999999999999, 11.999999999999998, 11.999999999999998, 11.999999999999998, 12.999999999999998, 14.0, 14.0]

simple

non-1/2-FrobType, so not modular cat

CC= [1092, 13, 13, 12, 7, 7, 6, 6]

Non-pivotal Column= 1

Non-pivotal Column= 2

Non-pivotal Column= 3

Non-pivotal Column= 4

Non-pivotal Column= 5

Non-pivotal Column= 6

Non-pivotal Column= 7

Non-pivotal Column= 8

Isaacs list = [0, -1, -1, -1, -1, -1, -1, -1, -1]

[3]

FPdim = 1091.9999999999998

FPdims = [1.0, 7.000000000000001, 6.999999999999999, 11.999999999999998, 11.999999999999998, 11.999999999999998, 12.999999999999998, 14.0, 14.0]

simple

non-1/2-FrobType, so not modular cat

CC= [1092, 13, 13, 12, 12, 7, 7, 4]

Non-integral coeff [3, 3, 1] 13/2

Non-pivotal Column= 1

Non-pivotal Column= 2

Non-pivotal Column= 3

Non-pivotal Column= 4

Non-pivotal Column= 5

Non-pivotal Column= 6

Non-pivotal Column= 7

Non-pivotal Column= 8

Isaacs list = [0, -1, -1, -1, -1, -1, -1, -1, -1]

[4]

FPdim = 3420.0

FPdims = [1.0, 9.0, 9.0, 19.0, 20.0, 19.999999999999996, 20.0, 20.0, 36.0]

simple

non-1/2-FrobType, so not modular cat

CC= [3420, 19, 19, 9, 9, 9, 5, 4]

Non-pivotal Column= 1

Non-pivotal Column= 2

Non-pivotal Column= 3

Non-pivotal Column= 4

Non-pivotal Column= 5

Non-pivotal Column= 6

Non-pivotal Column= 7

Non-pivotal Column= 8

Isaacs list = [0, -1, -1, -1, -1, -1, -1, -1, -1]

[5]

FPdim = 3419.9999999999995

FPdims = [1.0, 9.0, 9.0, 19.0, 19.999999999999996, 19.999999999999996, 19.999999999999996, 19.999999999999996, 36.0]

simple

non-1/2-FrobType, so not modular cat

CC= [3420, 19, 19, 9, 9, 9, 5, 4]

Non-pivotal Column= 1

Non-pivotal Column= 2

Non-pivotal Column= 3

Non-pivotal Column= 4

Non-pivotal Column= 5

```
Non-pivotal Column= 6
Non-pivotal Column= 7
Non-pivotal Column= 8
Isaacs list = [0, -1, -1, -1, -1, -1, -1, -1, -1]
```

```
# let us see those only 3-positive:
```

```
sage: c=0
....: for i in range(31):
....:     for j in range(L[i]):
....:         M=BL[i][j]
....:         if PositiveCo(M,3):
....:             c+=1; print([len(M),i,j])
....:
[5, 0, 0]
[6, 1, 0]
[7, 2, 1]
[7, 3, 1]
[8, 5, 5]
[8, 5, 11]
[9, 9, 0]
[9, 9, 1]
[9, 10, 14]
[9, 10, 29]
[9, 13, 4]
[9, 15, 8]
[9, 15, 9]
[9, 15, 10]
[9, 15, 11]
[10, 21, 49]
[10, 21, 74]
[10, 26, 0]
[11, 28, 2]
[11, 28, 4]
[11, 30, 75]
[11, 30, 79]
[11, 30, 132]
[11, 30, 134]
[11, 30, 144]
[11, 30, 149]
[11, 30, 177]
[11, 30, 179]
```

```
sage: c
28
```

```
...
```