

1-FROBENIUS SIMPLE INTEGRAL FUSION RINGS

WINFRIED BRUNS AND SÉBASTIEN PALCOUX

1. COMPLETE CLASSIFICATION

Within the following bounds,

Rank	4	5	6	7	8	9	10	11	12
FPdim \leq	10^{14}	10^7	10^6	10^5	20000	10000	5000	3000	1000
#Types	0	1	1	3	4	10	8	4	0
#Fusion Rings	0	1	1	8	23	94	188	190	0

there are exactly 31 types comprising 505 non-pointed¹ simple integral 1-Frobenius fusion rings. The corresponding fusion data are presented on the following pages.

Rank	FPdim	Factors	Type	#	#3-positive	Note
5	60	$2^2 3^1 5^1$	[1, 3, 3, 4, 5]	1	1	PSL(2, 4)
6	168	$2^3 3^1 7^1$	[1, 3, 3, 6, 7, 8]	1	1	PSL(2, 7)
7	210	$2^1 3^1 5^1 7^1$	[1, 5, 5, 5, 6, 7, 7]	2	1	iPSL(2, 6)
7	360	$2^3 3^2 5^1$	[1, 5, 5, 8, 8, 9, 10]	2	1	PSL(2, 9)
7	7980	$2^2 3^1 5^1 7^1 19^1$	[1, 19, 20, 21, 42, 42, 57]	4	0	
8	660	$2^2 3^1 5^1 11^1$	[1, 5, 5, 10, 10, 11, 12, 12]	15	2	PSL(2, 11)
8	990	$2^1 3^2 5^1 11^1$	[1, 9, 10, 11, 11, 11, 11, 18]	5	0	
8	1260	$2^2 3^2 5^1 7^1$	[1, 6, 7, 7, 10, 15, 20, 20]	2	0	
8	1320	$2^3 3^1 5^1 11^1$	[1, 6, 6, 10, 11, 15, 15, 24]	1	0	
9	504	$2^3 3^2 7^1$	[1, 7, 7, 7, 7, 8, 9, 9, 9]	2	2	PSL(2, 8)
9	1092	$2^2 3^1 7^1 13^1$	[1, 7, 7, 12, 12, 12, 13, 14, 14]	30	2	PSL(2, 13)
9	1320	$2^3 3^1 5^1 11^1$	[1, 5, 5, 6, 6, 10, 11, 20, 24]	1	0	
9	1512	$2^3 3^3 7^1$	[1, 6, 7, 8, 8, 8, 8, 21, 27]	4	0	
9	2520	$2^3 3^2 5^1 7^1$	[1, 6, 10, 10, 14, 14, 15, 21, 35]	9	1	A_7
9	2730	$2^1 3^1 5^1 7^1 13^1$	[1, 13, 13, 13, 14, 15, 15, 26, 30]	12	0	
9	3420	$2^2 3^2 5^1 19^1$	[1, 9, 9, 19, 20, 20, 20, 20, 36]	12	4	
9	3960	$2^3 3^2 5^1 11^1$	[1, 8, 8, 9, 10, 15, 15, 40, 40]	3	0	
9	7980	$2^2 3^1 5^1 7^1 19^1$	[1, 19, 19, 20, 21, 38, 38, 42, 42]	20	0	
9	8736	$2^5 3^1 7^1 13^1$	[1, 12, 12, 12, 13, 14, 21, 21, 84]	1	0	
10	720	$2^4 3^2 5^1$	[1, 4, 4, 5, 5, 9, 10, 10, 10, 16]	2	0	
10	1638	$2^1 3^2 7^1 13^1$	[1, 6, 7, 7, 7, 13, 14, 18, 18, 21]	8	0	
10	1680	$2^4 3^1 5^1 7^1$	[1, 7, 7, 14, 14, 14, 15, 16, 16, 16]	75	2	iPSL(2, 15)
10	1716	$2^2 3^1 11^1 13^1$	[1, 11, 11, 11, 11, 11, 11, 12, 13, 26]	1	0	
10	2184	$2^3 3^1 7^1 13^1$	[1, 6, 6, 12, 12, 13, 14, 21, 21, 24]	3	0	
10	2640	$2^4 3^1 5^1 11^1$	[1, 11, 11, 11, 15, 15, 15, 16, 16, 33]	21	0	
10	3366	$2^1 3^2 11^1 17^1$	[1, 11, 11, 17, 17, 18, 18, 18, 22, 33]	48	0	
10	4620	$2^2 3^1 5^1 7^1 11^1$	[1, 11, 11, 20, 20, 20, 20, 20, 21, 44]	30	1	
11	720	$2^4 3^2 5^1$	[1, 4, 5, 5, 5, 5, 6, 6, 9, 15, 15]	2	0	
11	990	$2^1 3^2 5^1 11^1$	[1, 9, 9, 9, 9, 9, 10, 11, 11, 11, 11]	5	2	iPSL(2, 10)
11	2184	$2^3 3^1 7^1 13^1$	[1, 6, 6, 7, 7, 12, 12, 13, 14, 24, 28]	3	0	
11	2448	$2^4 3^2 17^1$	[1, 9, 9, 16, 16, 16, 16, 17, 18, 18, 18]	180	8	PSL(2, 17)

2. IN PROGRESS CLASSIFICATION

Here are the current exploration bounds (work in progress),

Rank	8	9	10	11	12
FPdim \leq	50000	30000	10000	5000	3500

along with the count of newly discovered non-pointed 1-Frobenius simple integral fusion rings².

Rank	FPdim	Factors	Type	#	#3-positive	Note
8	46620	$2^2 3^2 5^1 7^1 37^1$	[1, 35, 36, 37, 70, 70, 105, 148]	20	0	
9	10626	$2^1 3^1 7^1 11^1 23^1$	[1, 21, 21, 21, 22, 23, 42, 42, 69]	1	0	
9	16380	$2^2 3^2 5^1 7^1 13^1$	[1, 12, 12, 12, 13, 21, 35, 84, 84]	1	0	
9	21924	$2^2 3^3 7^1 29^1$	[1, 27, 27, 28, 54, 54, 54, 58, 87]	11	0	
9	28560	$2^4 3^1 5^1 7^1 17^1$	[1, 14, 14, 14, 15, 16, 48, 105, 119]	4	0	
10	5040	$2^4 3^2 5^1 7^1$	[1, 7, 8, 8, 8, 8, 15, 21, 42, 48]	4	0	
10	5278	$2^1 7^1 13^1 29^1$	[1, 13, 14, 14, 26, 26, 29, 29, 29, 29]	12	0	
10	6720	$2^6 3^1 5^1 7^1$	[1, 10, 10, 14, 15, 16, 21, 24, 35, 60]	2	0	
10	7920	$2^4 3^2 5^1 11^1$	[1, 10, 10, 10, 11, 16, 16, 44, 45, 55]	2	2	M_{11}
10	7980	$2^2 3^1 5^1 7^1 19^1$	[1, 19, 20, 21, 21, 21, 21, 21, 42, 57]	1	0	
11	3960	$2^3 3^2 5^1 11^1$	[1, 10, 10, 10, 10, 11, 12, 12, 15, 30, 45]	20	0	
11	4620	$2^2 3^1 5^1 7^1 11^1$	[1, 5, 5, 5, 6, 14, 14, 21, 35, 35, 35]	71	0	
12	2520	$2^3 3^2 5^1 7^1$	[1, 6, 6, 7, 8, 8, 8, 8, 15, 21, 24, 30]	4		
12	2520	$2^3 3^2 5^1 7^1$	[1, 6, 7, 7, 7, 7, 10, 10, 14, 15, 21, 35]	5		+3 non-simple
12	3360	$2^5 3^1 5^1 7^1$	[1, 8, 8, 14, 14, 14, 15, 16, 16, 21, 21, 32]	3		

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¹The pointed case reduces to prime numbers.

²Note: there are at least # such fusion rings, though some dualities may be overlooked