

SIMPLE INTEGRAL FUSION RINGS OF RANK 4

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A non-trivial perfect integral fusion ring has at least four distinct basic FPdims. In particular, its rank is at least 4. Moreover, every perfect integral fusion ring of rank 4 is simple and has type $[1, d_1, d_2, d_3]$ with $3 \leq d_1 < d_2 < d_3$. The generic form of the fusion data is as follows:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & x_1 & x_2 & x_3 \\ 0 & x_2 & x_4 & x_5 \\ 0 & x_3 & x_5 & x_6 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & x_2 & x_4 & x_5 \\ 1 & x_4 & x_7 & x_8 \\ 0 & x_5 & x_8 & x_9 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & x_3 & x_5 & x_6 \\ 0 & x_5 & x_8 & x_9 \\ 1 & x_6 & x_9 & x_{10} \end{pmatrix}.$$

The matrix ℓ^2 -norm of above matrices are the basic FPdims $1, d_1, d_2, d_3$. The non-negative integers x_i satisfy the following associativity equations:

$$\begin{aligned} x_2^2 - x_1x_4 + x_4^2 + x_5^2 - x_2x_7 - x_3x_8 - 1 &= 0, \\ x_5^2 - x_4x_6 - x_{10}x_8 + x_8^2 - x_7x_9 + x_9^2 - 1 &= 0, \\ x_{10}x_3 - x_3^2 - x_5^2 + x_1x_6 - x_6^2 + x_2x_9 + 1 &= 0, \\ x_3x_4 - x_2x_5 + x_5x_7 - x_4x_8 + x_6x_8 - x_5x_9 &= 0, \\ x_2x_3 - x_1x_5 + x_4x_5 + x_5x_6 - x_2x_8 - x_3x_9 &= 0, \\ x_{10}x_5 - x_3x_5 + x_2x_6 - x_5x_8 + x_4x_9 - x_6x_9 &= 0. \end{aligned}$$

By Frobenius-Perron theorem, FPdim is a ring homomorphism, so we have the dimension equations:

$$\begin{aligned} d_1^2 &= 1 + d_1x_1 + d_2x_2 + d_3x_3, \\ d_2^2 &= 1 + d_1x_4 + d_2x_7 + d_3x_8, \\ d_3^2 &= 1 + d_1x_6 + d_2x_9 + d_3x_{10}, \\ d_1d_2 &= d_1x_2 + d_2x_4 + d_3x_5, \\ d_1d_3 &= d_1x_3 + d_2x_5 + d_3x_6, \\ d_2d_3 &= d_1x_5 + d_2x_8 + d_3x_9. \end{aligned}$$

Below is the exact count $\#$ of simple integral fusion rings of rank 4 with $\text{FPdim} < 10^i$. These data suggest that, in the absence of any bound on FPdim, the total number is likely infinite. However, recall that there is **no** 1-Frobenius example with $\text{FPdim} < 10^{10}$, and no Drinfeld example at all.

FPdim <	10^3	10^4	10^5	10^6	10^7	10^8
$\#$	1	2	6	12	66	298

Below is the complete list of the 298 such rings with $\text{FPdim} < 10^8$. This collection may help in identifying the structure of a potential infinite family.

FPdim	Factors	d_1	d_2	d_3	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
574	$2 \times 7 \times 41$	11	14	16	0	4	4	1	6	3	12	1	9	6
7315	$5 \times 7 \times 11 \times 19$	35	40	67	30	1	2	9	15	25	12	12	25	39
63436	$2^2 \times 15859$	103	149	175	57	13	16	67	23	74	51	44	98	48
65971	37×1783	65	89	232	51	5	2	4	22	56	60	10	79	186
68587	107×641	103	116	211	48	27	12	19	33	79	30	38	79	129
90590	$2 \times 5 \times 9059$	142	180	195	39	0	75	77	60	32	25	87	56	120
113310	$2 \times 3^2 \times 5 \times 1259$	90	172	275	77	2	3	25	40	64	39	75	112	184
310730	$2 \times 5 \times 7 \times 23 \times 193$	312	317	336	62	139	101	48	120	105	168	96	115	130
311343	$3 \times 59 \times 1759$	286	315	361	95	76	85	115	89	141	206	4	241	39
494102	$2 \times 7 \times 29 \times 1217$	396	399	422	40	219	127	20	150	135	248	124	141	162
532159	532159	211	409	566	84	24	30	63	98	129	299	56	332	278
585123	$3 \times 7 \times 11 \times 17 \times 149$	288	397	587	159	30	43	179	59	227	208	40	341	245
1012810	$2 \times 5 \times 101281$	305	622	730	144	32	40	125	140	169	414	125	457	270
1043710	$2 \times 5 \times 29 \times 59 \times 61$	574	587	608	186	165	207	220	186	199	144	220	199	228
1107139	11×100649	256	671	769	12	61	28	23	183	87	582	70	549	261
1152907	7×164701	443	679	704	237	95	38	296	82	340	264	214	421	84
1185558	$2 \times 3 \times 11^2 \times 23 \times 71$	348	722	737	55	32	107	206	124	176	244	371	300	360
1343202	$2 \times 3 \times 7 \times 31981$	203	626	954	72	12	20	65	88	141	451	101	541	569
1411338	$2 \times 3 \times 71 \times 3313$	415	714	854	40	84	112	115	210	185	85	470	219	581
1419779	197×7207	573	720	757	317	66	131	388	126	354	59	335	306	198
1425114	$2 \times 3^4 \times 19 \times 463$	578	727	750	382	31	121	444	106	382	48	316	339	127
1483682	$2 \times 53 \times 13997$	610	645	834	417	39	111	224	270	320	128	236	265	395
1745610	$2 \times 3 \times 5 \times 31 \times 1877$	577	774	902	224	100	140	309	166	345	74	403	322	405
1898038	2×949019	268	842	1057	171	2	23	182	68	208	474	247	628	504
1916226	$2 \times 3^2 \times 13 \times 19 \times 431$	234	715	1162	200	3	5	64	104	169	200	304	507	816
2112179	$71^2 \times 419$	603	763	1080	378	158	14	45	306	379	651	54	554	477
2161715	5×432343	660	925	933	413	32	143	501	135	425	156	408	425	211
2175315	$3 \times 5 \times 145021$	647	879	992	1	199	245	136	323	201	376	357	352	549
2630642	$2 \times 7 \times 41 \times 4583$	384	583	1464	179	17	47	192	72	343	336	48	545	1157
2753395	5×550679	832	919	1103	511	233	48	303	265	575	185	383	400	336
2898555	$3 \times 5 \times 11^2 \times 1597$	709	1092	1097	104	216	176	183	384	213	933	40	804	159
2903702	$2 \times 17 \times 41 \times 2083$	777	984	1154	224	52	328	363	318	285	179	442	393	627
3033052	$2^2 \times 11 \times 29 \times 2377$	943	1009	1061	426	51	411	460	414	184	108	448	215	693
3108087	$3^2 \times 41 \times 8423$	769	793	1374	291	117	200	160	286	492	572	38	611	746
3139986	$2 \times 3 \times 419 \times 1249$	762	1090	1171	219	342	35	76	416	352	793	227	608	376
3185630	$2 \times 5 \times 7 \times 17 \times 2677$	572	1194	1197	39	120	135	310	204	304	52	991	108	944
3325370	$2 \times 5 \times 17 \times 31 \times 631$	983	1074	1098	240	312	360	403	288	379	58	633	197	566
3397890	$2 \times 3 \times 5 \times 191 \times 593$	360	1208	1345	167	23	31	175	160	208	961	175	1008	384
3416428	$2^2 \times 19 \times 44953$	403	1177	1367	191	18	47	139	222	198	148	845	384	978
3483967	109×31963	965	1105	1154	396	240	246	369	370	405	35	716	110	710
3784371	$3 \times 71 \times 109 \times 163$	455	769	1728	133	13	79	108	151	367	204	223	630	1351
3802502	$2 \times 11 \times 307 \times 563$	1106	1119	1152	207	555	324	88	456	352	752	272	417	409
3805498	$2 \times 31 \times 61379$	620	1184	1421	531	19	23	250	300	360	435	515	624	744
3852015	$3 \times 5 \times 256801$	858	1193	1301	198	45	394	565	239	379	56	670	421	665
3939260	$2^2 \times 5 \times 13 \times 109 \times 139$	603	1163	1491	146	169	53	0	402	268	1113	39	970	626
4135971	$3 \times 7 \times 37 \times 5323$	615	1223	1504	373	27	77	460	115	490	228	621	671	758
4245382	2×2122691	919	1252	1354	208	72	416	373	456	215	325	604	384	853
4430367	$3^2 \times 211 \times 2333$	821	1249	1482	588	114	33	593	129	694	101	639	639	559

5146446	$2 \times 3 \times 857741$	926	1412	1515	40	95	453	471	366	308	366	687	548	816
5457862	2×2728931	542	1484	1721	210	25	83	178	306	252	279	983	540	1176
5656406	$2 \times 7 \times 404029$	951	1352	1710	480	220	88	561	186	755	411	432	907	573
6191519	$17 \times 59 \times 6173$	1050	1573	1617	535	242	99	651	231	761	534	588	851	295
6478290	$2 \times 3^2 \times 5 \times 7^2 \times 13 \times 113$	1415	1492	1500	400	472	488	525	440	517	39	950	132	881
6510658	$2 \times 7 \times 11 \times 67 \times 631$	1426	1450	1541	274	643	461	207	552	480	782	437	528	600
6559662	$2 \times 3 \times 31 \times 35267$	1005	1106	2080	496	124	180	535	190	817	460	85	969	1170
6641015	5×1328203	1105	1458	1815	588	108	228	575	360	677	711	250	1038	569
7024422	$2 \times 3 \times 59 \times 19843$	1294	1353	1876	707	273	208	384	468	813	176	584	609	876
7557886	$2 \times 7 \times 539849$	934	1540	2077	444	18	207	594	244	660	237	699	912	1104
7622140	$2^2 \times 5 \times 223 \times 1709$	1433	1555	1775	461	325	500	363	675	438	79	1000	134	1304
7738923	$3 \times 101 \times 25541$	1369	1540	1869	471	399	329	649	301	880	815	122	1219	220
8240068	$2^2 \times 563 \times 3659$	595	1679	2251	71	120	49	405	110	500	1266	201	1500	1000
8712844	$2^2 \times 7 \times 311173$	1313	1615	2093	57	355	514	182	650	489	254	936	485	1412
8847555	$3 \times 5 \times 31 \times 53 \times 359$	768	1591	2393	26	110	165	199	343	487	597	597	1084	1516
8888746	$2 \times 977 \times 4549$	767	1310	2566	96	244	76	13	312	585	513	403	1011	1875
9279060	$2^2 \times 3 \times 5 \times 7 \times 22093$	1403	1825	1995	711	450	75	188	795	623	1628	48	1222	439
9863787	$3 \times 349 \times 9421$	1079	2064	2107	337	344	43	548	344	720	919	841	1064	696
10014627	$3 \times 7 \times 476887$	991	1871	2352	209	335	63	48	609	480	1344	399	1297	1118
10440666	$2 \times 3^2 \times 23 \times 25219$	1315	1884	2272	632	168	256	561	528	729	1121	308	1323	753
10875651	$3 \times 829 \times 4373$	1328	2029	2235	683	236	169	571	547	731	448	1096	709	1157
11355771	$3 \times 7 \times 540751$	1257	1720	2611	24	51	560	274	623	577	319	791	899	1741
11603324	$2^2 \times 2900831$	1467	1785	2503	137	556	383	602	291	1035	1136	110	1536	801
11634299	11634299	1796	1911	2181	939	285	456	703	723	787	537	625	768	860
11952039	$3 \times 11 \times 373 \times 971$	1457	2175	2258	412	536	158	365	706	675	1237	668	1076	786
12126294	$2 \times 3^3 \times 19 \times 53 \times 223$	1858	2015	2148	326	821	555	84	954	483	1848	84	1111	688
12190551	3×4063517	1579	2075	2322	49	43	1002	848	624	340	248	1056	707	1459
12597820	$2^2 \times 5 \times 629891$	967	2143	2659	292	13	235	306	528	456	547	1175	1004	1684
12651822	$2 \times 3^3 \times 234293$	1549	2192	2334	752	116	420	579	834	487	339	1356	365	1668
13167930	$2 \times 3 \times 5 \times 37 \times 11863$	1252	1979	2772	603	45	261	568	468	800	712	648	1305	1479
13895134	$2 \times 11 \times 631597$	764	1466	3341	137	51	121	154	256	624	1347	17	1400	2584
13916994	$2 \times 3 \times 7 \times 13 \times 71 \times 359$	2116	2169	2176	654	719	705	616	796	637	1552	16	1379	182
14158055	$5 \times 1499 \times 1889$	1871	2097	2502	504	468	630	487	810	721	183	1240	452	1584
14292335	5×2858467	1801	2043	2622	1179	171	294	680	756	1010	1192	196	1371	860
14520590	$2 \times 5 \times 7 \times 23 \times 29 \times 311$	858	1445	3420	211	325	25	168	210	763	408	396	1225	2711
14830387	$11 \times 13 \times 137 \times 757$	1036	1253	3491	217	549	46	36	196	952	588	228	1113	2809
14982823	2657×5639	1411	2174	2875	960	15	210	826	435	979	1261	285	1745	1075
15210190	$2 \times 5 \times 467 \times 3257$	1517	1850	3080	664	200	300	737	370	1147	315	559	1332	1715
15233226	$2 \times 3 \times 2538871$	1450	1969	3042	223	179	469	212	716	763	64	1132	895	2099
15286715	$5 \times 43 \times 97 \times 733$	1895	2325	2508	1272	456	48	1135	360	1525	291	1028	1100	336
15987466	$2 \times 11 \times 59 \times 109 \times 113$	1820	2131	2852	720	637	226	216	792	1084	1272	504	1249	1227
16000028	$2^2 \times 11 \times 79 \times 4603$	1957	2277	2643	888	90	714	734	987	578	273	1183	527	1761
16060555	$5 \times 7 \times 23 \times 71 \times 281$	1673	2035	3020	1311	75	150	528	730	1098	1408	130	1543	1372
16119716	$2^2 \times 37 \times 108917$	2077	2181	2655	912	474	522	614	831	986	1057	443	1167	925
16305583	$7^2 \times 332767$	2201	2245	2534	855	981	300	36	1066	996	2072	122	1211	596
16830127	16830127	1639	2354	2933	920	72	344	931	528	1023	866	674	1518	1143
17032558	$2 \times 23 \times 43 \times 79 \times 109$	2110	2336	2669	1137	375	441	1216	486	1336	439	699	1340	440
17036004	$2^2 \times 3 \times 167 \times 8501$	1813	2253	2945	123	1058	231	610	269	1465	1336	326	1838	637
17041446	$2 \times 3^2 \times 17 \times 55691$	612	935	3974	523	3	13	32	136	578	64	200	867	3681
17170734	$2 \times 3 \times 7 \times 227 \times 1801$	1438	2317	3120	830	139	177	972	282	1147	1536	132	2089	1040
17464836	$2^2 \times 3 \times 1455403$	1647	2501	2915	344	590	230	192	915	732	1751	535	1525	1193
17914580	$2^2 \times 5 \times 895729$	1707	2677	2799	544	338	386	930	537	958	249	1755	671	1573
17991027	$3^2 \times 593 \times 3371$	1975	2124	3095	1110	271	366	550	805	1189	425	815	1051	1615
18275599	103×177433	1699	2161	3274	127	589	427	186	693	1020	880	749	1307	1882
18286815	$3 \times 5 \times 17 \times 71713$	2190	2383	2795	162	1628	201	125	485	1619	1711	475	1598	164
18623962	$2 \times 7 \times 89 \times 14947$	1469	2120	3460	840	240	120	911	240	1271	423	653	1618	1929
19603043	$29 \times 41 \times 16487$	1959	2535	3056	928	112	568	1207	552	1137	281	1096	1272	1272
20080974	$2 \times 3 \times 673 \times 4973$	2243	2632	2850	872	536	584	829	884	967	1618	284	1674	543
20250114	$2 \times 3 \times 3375019$	1678	2795	3102	662	287	291	660	762	834	1320	972	1507	1293
20269194	$2 \times 3 \times 11^2 \times 27919$	785	2958	3302	520	48	20	357	372	447	1037	1636	1404	1938
20302684	$2^2 \times 47 \times 79 \times 1367$	2387	2617	2785	558	371	1219	1276	726	660	108	1264	807	1461
20802898	$2 \times 2903 \times 3583$	2198	2573	3058	1209	417	360	1364	402	1601	1056	296	2035	195
20850651	$3^2 \times 353 \times 6563$	1805	2287	3516	344	944	136	445	400	1475	1081	556	1720	1640
21196668	$2^2 \times 3 \times 101 \times 17489$	2169	2641	3085	518	203	987	972	882	720	462	1182	1009	1715
21896812	$2^2 \times 7 \times 151 \times 5179$	2261	2599	3167	94	730	948	316	1075	702	1030	1062	960	1878
21976234	$2 \times 7 \times 1569731$	2210	2687	3142	17	549	1073	388	1172	453	960	1204	833	2111
22143554	$2 \times 17 \times 651281$	2066	2709	3246	111	657	696	580	822	937	824	1204	1181	1664
22325614	$2 \times 53 \times 210619$	1762	2312	3725	483	406	353	531	572	1240	533	853	1512	2200
22441939	13×1726303	1697	2495	3652	909	353	125	1110	237	1477	954	537	2018	1587
22692411	$3^2 \times 7 \times 360197$	2295	2696	3187	1339	510	257	648	1026	1242	1363	661	1398	1110
22739282	2×11369641	1184	3105	3420	570	135	90	736	360	826	736	1896	1259	1991
22917214	2×11458607	1714	2384	3781	102	634	331	554	444	1284	1023	607	1800	2064
23123107	$7 \times 47 \times 67 \times 1049$	2321	2776	3167	203	873	787	586	881	972	1014	1115	1153	1444
23791447	47×506201	2126	2479	3623	510	547	574	894	522	1432	1572	96	2107	1341
23950891	521×45971	1537	3100	3461	671	4	381	1276	232	1160	48	2167	1056	2000
24155679	3×8051893	971	3231	3574	476	120	26	623	282	709	1181	1684	1632	1906
24996567	$3 \times 761 \times 10949$	2593	2841	3194	174	1024	1053	551	985	862	419	1707	523	2029
25216459	$89 \times 293 \times 967$	2060	2957	3497	1537	252	95	690	1010	1150	1749	615	1842	1262
25261910	$2 \times 5 \times 1451 \times 1741$	2868	2894	2943	808	1111	915	687	1062	932	1362	837	1036	1016
25281102	$2 \times 3 \times 7 \times 11 \times 54721$	2299	3084	3238	472	648	680	871	900	959	1684	715	1764	877
25285167	$3^2 \times 89 \times 31567$	2191	2718	3619	936	45	726	1232	693	1231				

26680090	$2 \times 5 \times 103 \times 25903$	1834	3298	3527	1087	363	49	1476	146	1672	1935	507	2748	88
27071982	$2 \times 3^4 \times 7 \times 23873$	2437	3054	3436	1608	432	204	1721	330	1999	739	837	2076	173
27308794	$2 \times 1667 \times 8191$	2930	3022	3097	95	1440	1277	343	1162	588	813	1831	136	2408
27347215	$5 \times 7 \times 61 \times 12809$	2105	2170	4267	579	562	467	498	540	1600	513	597	1600	2664
27403647	$3 \times 179 \times 51031$	1375	3070	4011	1181	49	29	1242	85	1300	1104	1079	2215	1870
28136291	$23 \times 41 \times 29837$	1872	3291	3715	545	143	542	751	921	783	1043	1613	1398	2082
28595788	$2^2 \times 13 \times 79 \times 6961$	1849	2969	4045	726	604	70	90	1015	1072	2150	560	2094	2018
28609603	$11^2 \times 67 \times 3529$	2176	2735	4049	881	903	86	1046	278	1942	1130	522	2233	1497
28765175	$5^2 \times 47 \times 24481$	2457	2959	3738	927	515	598	756	1008	1266	1164	924	1565	1667
28827222	$2 \times 3 \times 127 \times 37831$	2432	3234	3529	103	873	805	669	1014	948	1141	1457	1200	1776
28868435	5×5773687	1697	2049	4668	579	345	255	844	249	1495	1088	115	1908	3287
29135683	19×1533457	1759	1976	4705	127	217	519	211	569	1326	1231	234	1665	3510
29136135	$3 \times 5 \times 7^4 \times 809$	2807	2937	3554	1455	577	591	870	1145	1394	1204	745	1417	1282
29605007	$17 \times 1019 \times 1709$	1602	3529	3819	1169	176	19	1152	342	1278	477	2337	1226	2150
29889310	$2 \times 5 \times 11 \times 163 \times 1667$	1603	3262	4084	112	292	352	513	756	861	1500	1206	2002	2147
30076874	2×15038437	1710	3087	4198	47	331	434	572	702	1017	1568	884	2151	2202
30178674	$2 \times 3^2 \times 1676593$	2416	2809	4056	165	1191	516	352	720	1610	1328	816	1815	1840
30225691	1303×23197	2573	2944	3865	83	631	1177	261	1341	768	723	1518	895	2672
30451098	$2 \times 3 \times 67 \times 211 \times 359$	1298	3783	3802	193	351	28	864	312	978	2176	1304	2379	1101
30467599	30467599	2467	3022	3905	428	561	854	806	931	1207	763	1239	1475	2001
30815694	$2 \times 3^3 \times 7 \times 13 \times 6271$	1690	3552	3917	621	62	405	421	1124	496	157	2897	440	3304
30868135	$5 \times 61 \times 101207$	1982	2099	4747	36	1098	327	416	234	1742	1229	211	1908	3176
31217007	$3 \times 643 \times 16183$	2177	3626	3651	1373	379	103	1153	791	1330	779	2140	1029	1836
31506372	$2^2 \times 3^2 \times 17 \times 51481$	1781	3201	4253	709	251	260	1078	424	1353	2378	168	2897	1506
32148708	$2^2 \times 3 \times 37 \times 61 \times 1187$	2547	3277	3863	282	1718	36	561	552	2055	1971	738	2287	568
32409412	$2^2 \times 7 \times 17 \times 68087$	749	2431	5093	528	22	22	120	297	604	1269	537	2131	3987
32655795	$3 \times 5 \times 2177053$	2644	3453	3707	2217	240	81	1085	1281	1393	1656	900	1701	1129
32693746	2×16346873	1975	3404	4148	1648	24	136	801	952	1129	1137	1479	1737	2185
33598279	$11 \times 13 \times 47 \times 4999$	611	3034	4901	522	5	8	169	273	441	816	1352	2163	3507
34434607	34434607	2203	3161	4426	1218	118	406	925	854	1391	1497	728	2216	2151
35393444	$2^2 \times 31 \times 47 \times 6073$	3147	3397	3735	808	986	1074	1056	1071	1268	2013	369	2159	703
36384207	$3 \times 211 \times 229 \times 251$	2638	3621	4039	99	200	1479	1156	1198	598	877	1705	1310	2474
38805700	$2^2 \times 5^2 \times 388057$	3067	3299	4303	78	221	1961	532	1786	300	574	1710	715	3541
38877762	$2 \times 3 \times 7 \times 11 \times 19 \times 43 \times 103$	2606	3714	4277	1271	445	427	875	1232	1276	2963	119	2860	1016
39749702	2×19874851	2850	3440	4449	484	394	1211	894	1260	1100	771	1491	1480	2600
39804927	$3 \times 23 \times 576883$	1979	3942	4511	970	415	80	1044	635	1389	1199	1939	1969	2181
40239355	$5 \times 13 \times 619067$	2777	3980	4085	1439	600	325	1722	620	1952	1242	1497	2100	712
40247014	2×20123507	2641	3394	4664	496	152	1104	357	1576	869	71	2216	889	3525
40334894	$2 \times 863 \times 23369$	1803	2660	5478	472	688	104	447	432	1559	831	741	2158	3917
40352755	$5 \times 37 \times 59 \times 3697$	2765	3500	4523	798	135	1098	623	1575	875	232	2148	875	3311
41099980	$2^2 \times 5 \times 2054999$	2915	3485	4523	1787	706	183	1403	710	2250	244	1593	1800	1686
41576555	$5 \times 2069 \times 4019$	1617	3779	4968	551	67	296	768	624	1046	1368	1584	2371	2824
42026332	$2^2 \times 127 \times 82729$	2659	4105	4255	1770	410	160	2041	340	2231	1593	1148	2785	174
42665415	$3 \times 5 \times 13 \times 218797$	2125	4133	4590	633	93	607	850	1105	849	1336	2125	1708	2659
43212483	$3^2 \times 41 \times 181 \times 647$	3353	3492	4447	376	1096	1384	477	1432	1185	296	2150	724	2985
43509219	$3 \times 13^2 \times 85817$	3133	3877	4320	986	1150	525	783	1275	1608	1857	1245	1835	1507
43538490	$2 \times 3^2 \times 5 \times 483761$	2861	3708	4648	168	1416	528	535	984	1751	1791	1200	2145	1859
43779570	$2 \times 3 \times 5 \times 1459319$	3593	3852	4004	464	1480	1384	1119	1052	1339	1959	817	2122	761
44538143	$13 \times 23 \times 148957$	1935	3931	5034	309	575	176	0	1290	860	3324	474	3065	2310
44610887	44610887	1806	3005	5685	85	760	145	129	645	1419	1134	948	2299	4019
44722455	$3 \times 5 \times 37 \times 61 \times 1321$	2542	4071	4657	1359	169	498	1638	698	1660	1330	1502	2377	1673
46242526	$2 \times 11 \times 37 \times 56809$	3194	3595	4808	1227	679	799	1256	998	1917	152	1740	1631	2315
46345407	$3 \times 149 \times 103681$	3101	3591	4882	1264	80	1108	1357	1232	1491	437	1458	1736	2658
46780555	$5 \times 151 \times 61961$	2555	4327	4640	1640	32	474	2075	430	1893	689	2250	1992	1740
48151191	$3 \times 11 \times 17 \times 85831$	1155	3923	5606	64	104	152	347	544	743	617	2242	2242	3884
48315799	7×6902257	2339	3379	5606	742	480	377	857	693	1764	697	1259	2331	3465
48412222	$2 \times 4903 \times 4937$	3766	3832	4421	1515	942	1101	1525	1140	1840	1986	301	2600	600
48649916	$2^2 \times 7 \times 1737497$	3493	3825	4671	423	1089	1404	773	1413	1286	2191	760	2146	1952
48878190	$2 \times 3^2 \times 5 \times 53 \times 10247$	3428	4118	4491	1429	941	663	963	1542	1508	2397	843	2168	1352
49317522	$2 \times 3 \times 37 \times 222151$	3521	3684	4832	1696	632	848	1747	892	2223	1247	585	2588	1239
49680695	$5 \times 43 \times 311 \times 743$	2917	3723	5226	450	678	894	575	1290	1499	2711	400	2718	2453
50430388	$2^2 \times 12607597$	2561	4429	4925	288	338	878	394	1773	510	39	3743	141	4533
50712871	$11 \times 499 \times 9239$	3790	3889	4607	179	1260	1907	534	1712	776	2043	1119	1536	2672
51130548	$2^2 \times 3^3 \times 7 \times 47 \times 1439$	2773	3873	5333	1953	426	117	638	1329	1747	2400	738	2646	2503
51546534	$2 \times 3 \times 13 \times 660853$	2329	3804	5626	992	156	448	1213	690	1677	1081	1339	2613	3165
51631566	$2 \times 3 \times 7 \times 107 \times 11489$	1826	4835	4992	413	259	266	832	868	888	3224	1256	3301	1470
51735015	$3^2 \times 5 \times 47 \times 61 \times 401$	3359	3638	5217	1380	1767	42	467	879	2719	1942	882	2457	1753
51915570	$2 \times 3 \times 5 \times 7 \times 173 \times 1429$	1888	3417	6056	801	255	195	1024	408	1597	1008	1040	2703	4033
52766218	$2 \times 37 \times 761 \times 937$	1346	3790	6049	337	12	217	349	622	908	865	1755	2552	4248
53165734	$2 \times 3967 \times 6701$	3520	4303	4718	3023	25	348	1572	1758	1657	504	2292	901	2660
53793190	$2 \times 5 \times 11 \times 37 \times 13217$	2698	4648	4991	166	455	945	1577	798	1444	508	3003	1420	2888
56110115	$5 \times 17 \times 660119$	3303	4112	5319	2134	181	586	1260	1467	1805	1179	1485	2053	2611
56214276	$2^2 \times 3 \times 4684523$	3211	4373	5175	1824	822	166	1882	613	2590	1087	1609	2633	1343
56407491	$3^2 \times 7 \times 895357$	2837	4236	5515	347	1169	383	454	1229	1696	2022	1467	2477	2740
56726970	$2 \times 3 \times 5 \times 19 \times 23 \times 4327$	3432	4333	5116	1834	575	585	2032	800	2362	296	2056	2055	1791
56946147	$3 \times 43 \times 441443$	3771	3971	5192	994	638	1529	677	1903	1205	147	2433	728	3760
57094870	$2 \times 5 \times 7 \times 31 \times 83 \times 317$	3280	3913	5570	983	1803	86	1000	540	2850	1936	800	3033	1761
57168804	$2^2 \times 3 \times 7 \times 11 \times 61871$	3851	4591	4611	1953	423	1164	1610	1878	1009	818	2412	621	3150
59013622	2×29506811	2674	4588	5551	1401	114	519	1						

61330630	$2 \times 5 \times 337 \times 18199$	3676	4742	5033	2066	555	653	1503	1642	1652	708	2703	996	2888
61372380	$2^2 \times 3 \times 5 \times 17 \times 60169$	2025	4075	6377	1295	31	212	1501	325	1750	337	1912	2750	4064
61475779	61475779	1817	4292	6305	66	233	346	574	779	1187	375	2501	2365	4353
61498195	$5 \times 11 \times 1118149$	2248	5117	5501	176	405	470	837	1147	989	1350	3162	1707	3509
62312522	2×31156261	2454	5169	5438	881	367	361	1112	1110	1236	1304	3172	1653	3309
62762295	$3 \times 5 \times 4184153$	1411	5277	5738	692	102	83	715	615	825	657	4073	1380	4266
64319587	64319587	2945	4745	5756	1226	686	314	1755	630	2265	1989	1374	3290	1885
64615362	$2 \times 3 \times 7 \times 1538461$	3514	4181	5898	1675	379	827	992	1562	1914	272	2180	1705	3549
64898236	$2^2 \times 13 \times 311 \times 4013$	2771	5015	5663	184	1382	42	798	1071	1802	2511	1827	2873	2237
65526894	$2 \times 3^7 \times 71 \times 211$	2686	5136	5651	2420	94	41	2474	148	2532	373	3153	2200	2448
65593636	$2^2 \times 1889 \times 8681$	2879	4465	6113	812	588	544	1280	891	1972	1534	1538	2922	3050
65815582	$2 \times 7 \times 19 \times 181 \times 1367$	3770	5075	5084	807	1023	1176	1848	1160	1740	200	3496	725	3070
65817886	2×32908943	2915	3922	6476	1944	48	408	779	1272	1961	569	1680	2332	4181
65912158	$2 \times 13 \times 23 \times 110221$	2288	3098	7147	1740	1	175	414	812	1880	1355	623	2568	5432
66032146	2×33016073	1385	3632	7136	144	96	192	311	528	1079	1975	783	3131	5333
66045206	$2 \times 449 \times 73547$	3330	5172	5311	1684	1030	29	1718	924	2412	1855	2153	2496	1368
67064979	$3 \times 37 \times 604189$	2504	5289	5729	505	798	137	1733	363	2109	3472	920	4281	855
67148754	$2 \times 3 \times 229 \times 48871$	1843	5498	5790	672	144	236	769	974	843	1627	3431	1930	3689
67205487	$3 \times 137 \times 163517$	3939	4586	5537	1517	524	1289	1830	1374	1884	888	1761	2150	2416
67274043	3×22424681	3931	4209	5840	949	769	1453	1280	1393	1949	720	1653	2080	3029
67871795	$5 \times 883 \times 15373$	3903	4336	5817	19	1571	1435	714	1323	1954	1524	1617	2243	2834
68299859	271×252029	3536	5139	5421	936	1251	510	1729	897	2353	2386	1482	3149	901
68397071	68397071	3350	5259	5433	942	1131	390	1582	1014	2128	2276	1912	2783	1427
69293290	$2 \times 5 \times 11 \times 629939$	4432	4732	5221	1387	2372	435	829	1252	2928	2615	1215	2568	408
69720572	$2^2 \times 941 \times 18523$	3927	4681	5691	1060	956	1192	1491	1344	1999	9	2814	1439	3128
72265299	$3 \times 263 \times 91591$	2101	5296	6309	1109	85	259	1368	587	1522	2472	1915	3493	2870
72382027	$37 \times 823 \times 2377$	4091	4936	5593	1695	1783	179	1794	723	3322	2052	1233	3319	234
73911939	$3 \times 1093 \times 22541$	1769	5361	6484	520	88	268	545	988	879	1127	3352	2320	4326
74374083	$3^2 \times 7 \times 503 \times 2347$	3452	3597	7037	1265	1240	439	1096	596	2932	1119	729	2932	4100
74596627	$7 \times 1741 \times 6121$	4049	5092	5681	2082	589	874	2118	1311	2251	2715	621	3601	849
74723483	$17 \times 1511 \times 2909$	3455	4611	6444	1989	1023	54	1372	942	2752	964	1874	2765	2990
74862058	2×37431029	1646	5855	6154	133	233	183	500	1028	619	5088	596	5013	1219
77187918	$2 \times 3 \times 19 \times 239 \times 2833$	2076	4571	7210	331	119	427	432	1008	1314	728	2312	2815	5047
77524346	$2 \times 367 \times 105619$	4635	4796	5748	1992	1816	616	1413	1224	3117	1701	1443	2605	1061
77920343	$23 \times 43 \times 78787$	3883	4002	6843	3264	498	60	599	1638	2891	2624	466	2800	3565
78067366	$2 \times 13 \times 17 \times 347 \times 509$	3121	5182	6440	672	784	556	1329	1062	1997	1947	1959	3091	2985
78467079	$3 \times 229 \times 114217$	3990	5537	5647	2270	1114	123	2302	868	3052	1611	2223	2744	800
79218690	$2 \times 3 \times 5 \times 1481 \times 1783$	3467	4634	6762	1160	868	588	1837	672	2705	2441	561	3905	2699
80417758	$2 \times 311 \times 129289$	3602	5573	6032	955	1529	168	120	2304	1373	4872	576	3665	1826
80473607	19×4235453	3111	5511	6358	653	385	869	952	1683	1227	1306	3179	1932	4083
80473982	$2 \times 79 \times 509329$	3456	5083	6534	219	1589	476	432	1512	2028	2568	1728	2939	3175
80693940	$2^2 \times 3 \times 5 \times 1344899$	2913	5767	6241	2131	395	0	2457	237	2694	2565	1812	3982	1304
81744255	$3^3 \times 5 \times 107 \times 5659$	1278	5807	6811	824	53	40	952	268	1042	296	4520	1903	4993
82812754	$2 \times 19 \times 43 \times 59 \times 859$	3613	5090	6622	1072	664	876	1185	1504	1979	2581	1282	3284	3018
82909716	$2^2 \times 3 \times 1381 \times 5003$	2481	5627	6715	1496	272	136	1225	952	1633	434	3899	2008	4429
83404740	$2^2 \times 3 \times 5 \times 509 \times 2731$	4035	4285	6983	1202	748	1178	1261	1270	2575	306	1713	2500	3961
83503650	$2 \times 3 \times 5^2 \times 556691$	3272	4448	7281	823	821	599	324	1432	2128	2247	1199	3072	4448
85343994	$2 \times 3^2 \times 4741333$	4211	5496	6116	920	928	1432	1261	2012	1417	1365	2844	1555	3743
85431047	$13 \times 107 \times 61417$	3877	5859	6006	651	1575	546	596	2184	1394	4132	1300	3181	2003
86172886	$2 \times 461 \times 93463$	5153	5180	5726	504	1932	2436	1049	1974	1175	3366	697	2773	2160
86931754	$2 \times 7 \times 13 \times 127 \times 3761$	4402	5375	6218	623	927	1874	1188	2122	1241	1584	2436	1767	3812
87057914	2×43528957	4763	5340	5988	3336	600	600	1873	2100	2413	1023	2360	1565	2673
87813055	$5 \times 11 \times 181 \times 8821$	4030	5785	6173	2562	710	293	2495	975	2925	405	3413	1950	2436
87917487	$3 \times 7 \times 4186547$	2707	3914	8079	701	23	661	435	1093	1956	261	1624	2761	6086
88242422	$2 \times 19 \times 2322169$	4056	4457	7206	761	499	1546	876	1686	2143	672	1848	2365	4537
90161223	3×30053741	5270	5541	5629	3419	515	1226	3233	1523	2623	1435	1015	3116	106
90356775	$3 \times 5^2 \times 37 \times 32561$	3851	4642	7347	1	1138	1299	96	1776	2048	2250	1461	2788	4512
91632399	$3 \times 167 \times 182899$	5369	5541	5666	4290	116	909	2691	2509	2054	779	2107	1103	2641
92524047	$3 \times 7 \times 11 \times 17 \times 23561$	3898	3981	7841	1956	1157	378	829	983	3211	790	1208	2879	4783
92677639	92677639	3674	5309	7141	1476	671	632	1379	1361	2337	610	2784	2539	4051
92777543	769×120647	2435	4374	8229	1659	99	177	523	987	1858	1163	1552	3257	5948
92975790	$2 \times 3 \times 5 \times 3099193$	4808	4829	6822	2856	309	1157	2240	1600	2860	1672	656	3237	2515
93732447	$3 \times 2411 \times 12959$	3369	6122	6701	1168	572	584	1767	1176	2001	548	4204	1690	4151
94155486	$2 \times 3 \times 17 \times 761 \times 1213$	4882	5260	6531	2901	1080	611	1744	1720	3040	536	2501	1960	2680
94363486	$2 \times 7 \times 139 \times 48491$	2746	5963	7160	1668	129	306	1068	1348	1506	1600	3224	2761	4283
94841620	$2^2 \times 5 \times 4742081$	4877	5207	6629	2592	1813	257	1944	970	3926	1418	1546	3279	1165
95444270	$2 \times 5 \times 9544427$	3444	4837	7758	1951	59	626	1920	924	2590	1680	1116	3731	4282
95875638	$2 \times 3 \times 23 \times 43 \times 107 \times 151$	4742	4848	7063	2781	655	867	2828	874	3560	1451	433	3964	1952
96034148	$2^2 \times 7 \times 3429791$	4629	5865	6341	814	493	2329	2696	1428	1608	418	3070	1983	3333
96443675	$5^2 \times 3857747$	3912	4569	7763	1825	883	532	1479	987	3063	365	1729	3054	4422
96613075	$5^2 \times 13^3 \times 1759$	4912	5203	6739	2253	855	1278	1539	1981	2451	1091	2053	2174	3274
98026102	$2 \times 467 \times 104953$	3850	5249	7460	1495	741	694	1800	1060	2746	1200	1920	3351	3685
98326194	$2 \times 3 \times 127 \times 129037$	4448	6225	6308	1067	1743	664	1184	1992	2014	3328	2024	2823	2102
98521042	$2 \times 19 \times 89 \times 29131$	4184	4393	7856	3543	15	333	1008	1768	3018	400	1696	2503	4849