1-FROBENIUS SIMPLE INTEGRAL FUSION RINGS

WINFRIED BRUNS AND SÉBASTIEN PALCOUX

1. Complete classification

Within the following bounds,

Rank	4	5	6	7	8	9	10	11	12
FPdim $\leq$	10^{15}	10^7	10^6	10^5	20000	10000	5000	3000	1000
#Types	0	1	1	3	4	10	8	4	0
#Fusion Rings	0	1	1	8	23	94	188	190	0

there are exactly 31 types comprising 505 non-pointed[1] simple integral 1-Frobenius fusion rings. The corresponding fusion data are presented on the following pages.

Rank	FPdim	Factors	Type	#	#3-positive	Note
5	60	$2^2 3^1 5^1$	$[1,3,3,4,5]$	1	1	$\mathrm{PSL}(2,4)$
6	168	$2^3 3^1 7^1$	$[1,3,3,6,7,8]$	1	1	$\mathrm{PSL}(2,7)$
7	210	$2^1 3^1 5^1 7^1$	$[1,5,5,5,6,7,7]$	2	1	$\mathrm{iPSL}(2,6)$
7	360	$2^3 3^2 5^1$	$[1,5,5,8,8,9,10]$	2	1	$\mathrm{PSL}(2,9)$
7	7980	$2^2 3^1 5^1 7^1 19^1$	$[1,19,20,21,42,42,57]$	4	0	
8	660	$2^2 3^1 5^1 11^1$	$[1,5,5,10,10,11,12,12]$	15	2	$\mathrm{PSL}(2,11)$
8	990	$2^1 3^2 5^1 11^1$	$[1,9,10,11,11,11,11,18]$	5	0	
8	1260	$2^2 3^2 57$	$[1,6,7,7,10,15,20,20]$	2	0	
8	1320	$2^3 3^1 5^1 11^1$	$[1,6,6,10,11,15,15,24]$	1	0	
9	504	$2^3 3^2 7^1$	$[1,7,7,7,7,8,9,9,9]$	2	2	$\mathrm{PSL}(2,8)$
9	1092	$2^2 3^1 7^1 13^1$	$[1,7,7,12,12,12,13,14,14]$	30	2	$\mathrm{PSL}(2,13)$
9	1320	$2^3 3^1 5^1 11^1$	$[1,5,5,6,6,10,11,20,24]$	1	0	
9	1512	$2^3 3^3 7^1$	$[1,6,7,8,8,8,8,21,27]$	4	0	
9	2520	$2^3 3^2 5^1 7^1$	$[1,6,10,10,14,14,15,21,35]$	9	1	A_7
9	2730	$2^1 3^1 5^1 7^1 13^1$	$[1,13,13,13,14,15,15,26,30]$	12	0	
9	3420	$2^2 3^2 5^1 19^1$	$[1,9,9,19,20,20,20,20,36]$	12	4	
9	3960	$2^3 3^2 5^1 11^1$	$[1,8,8,9,10,15,15,40,40]$	3	0	
9	7980	$2^2 3^1 5^1 7^1 19^1$	$[1,19,19,20,21,38,38,42,42]$	20	0	
9	8736	$2^5 3^1 7^1 13^1$	$[1,12,12,12,13,14,21,21,84]$	1	0	
10	720	$2^4 3^2 5^1$	$[1,4,4,5,5,9,10,10,10,16]$	2	0	
10	1638	$2^1 3^2 7^1 13^1$	$[1,6,7,7,7,13,14,18,18,21]$	8	0	
10	1680	$2^4 3^1 5^1 7^1$	$[1,7,7,14,14,14,15,16,16,16]$	75	2	$\mathrm{iPSL}(2,15)$
10	1716	$2^2 3^1 11^1 13^1$	$[1,11,11,11,11,11,11,12,13,26]$	1	0	
10	2184	$2^3 3^1 7^1 13^1$	$[1,6,6,12,12,13,14,21,21,24]$	3	0	
10	2640	$2^4 3^1 5^1 11^1$	$[1,11,11,11,15,15,15,16,16,33]$	21	0	
10	3366	$2^1 3^2 11^1 17^1$	$[1,11,11,17,17,18,18,18,22,33]$	48	0	
10	4620	$2^2 3^1 5^1 7^1 11^1$	$[1,11,11,20,20,20,20,20,21,44]$	30	1	
11	720	$2^4 3^2 5^1$	$[1,4,5,5,5,5,6,6,9,15,15]$	2	0	
11	990	$2^1 3^2 5^1 11^1$	$[1,9,9,9,9,9,10,11,11,11,11]$	5	2	$\mathrm{iPSL}(2,10)$
11	2184	$2^3 3^1 7^1 13^1$	$[1,6,6,7,7,12,12,13,14,24,28]$	3	0	
11	2448	$2^4 3^2 17^1$	$[1,9,9,16,16,16,16,17,18,18,18]$	180	8	$\mathrm{PSL}(2,17)$

2. In progress classification

Here are the current exploration bounds (work in progress),

Rank	8	9	10	11	12
FPdim $\leq$	50000	30000	10000	5000	3500

along with the count of newly discovered non-pointed 1-Frobenius simple integral fusion rings[2].

Rank	FPdim	Factors	Type	#	#3-positive	Note
8	46620	$2^2 3^2 5^1 7^1 37^1$	$[1,35,36,37,70,70,105,148]$	20	0	
9	10626	$2^1 3^1 7^1 11^1 23^1$	$[1,21,21,21,22,23,42,42,69]$	1	0	
9	16380	$2^2 3^2 5^1 7^1 13^1$	$[1,12,12,12,13,21,35,84,84]$	1	0	
9	21924	$2^2 3^3 7^1 29^1$	$[1,27,27,28,54,54,54,58,87]$	11	0	
9	28560	$2^4 3^1 5^1 7^1 17^1$	$[1,14,14,14,15,16,48,105,119]$	4	0	
10	5040	$2^4 3^2 5^1 7^1$	$[1,7,8,8,8,8,15,21,42,48]$	4	0	
10	5278	$2^1 7^1 13^1 29^1$	$[1,13,14,14,26,26,29,29,29,29]$	12	0	
10	6720	$2^6 3^1 5^1 7^1$	$[1,10,10,14,15,16,21,24,35,60]$	2	0	
10	7920	$2^4 3^2 5^1 11^1$	$[1,10,10,10,11,16,16,44,45,55]$	2	2	M_{11}
10	7980	$2^2 3^1 5^1 7^1 19^1$	$[1,19,20,21,21,21,21,21,42,57]$	1	0	
11	3960	$2^3 3^2 5^1 11^1$	$[1,10,10,10,10,11,12,12,15,30,45]$	20	0	
11	4620	$2^2 3^1 5^1 7^1 11^1$	$[1,5,5,5,6,14,14,21,35,35,35]$	71	0	
12	2520	$2^3 3^2 5^1 7^1$	$[1,6,6,7,8,8,8,8,15,21,24,30]$	4		
12	2520	$2^3 3^2 5^1 7^1$	$[1,6,7,7,7,7,10,10,14,15,21,35]$	5		+3 non-simple
12	3360	$2^5 3^1 5^1 7^1$	$[1,8,8,14,14,14,15,16,16,21,21,32]$	3		

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[1]The pointed case reduces to prime numbers.

[2]Note: there are at least # such fusion rings, though some dualities may be overlooked