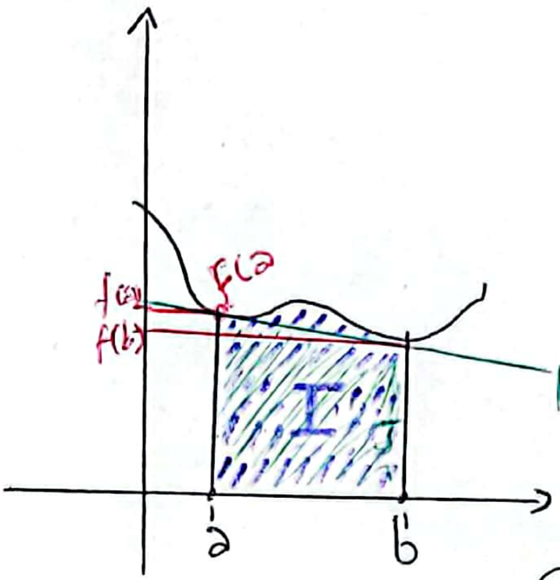
1) Método de trapecio simple

$$I = \int_a^b f(x)\,dx$$

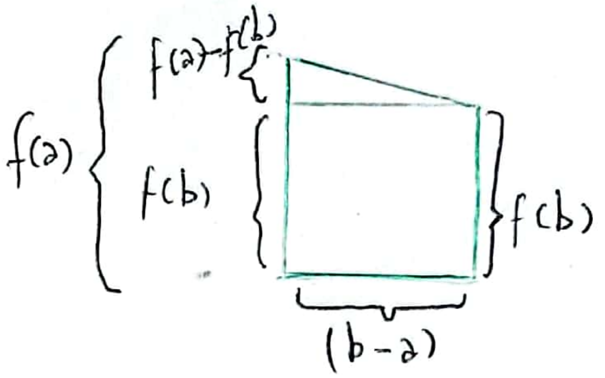


$$\Omega = \{(a, f(a)), (b, f(b))\}$$

$$P_\Omega(x) = \sum_{i=0}^{n-1} f_i(x_i)\, L_i(x)$$

$$P_\Omega(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b)$$

$$\left(\int_a^b P_\Omega(x)dx = J\right) \approx \left(I = \int_a^b f(x)\,dx\right)$$



$$= \int_a^b P_\Omega(x)dx$$

$$A_\square = f(b)\cdot(b-a) + \frac{(f(a)-f(b))\cdot(b-a)}{2} = (b-a)\left(f(b) + \frac{f(a)-f(b)}{2}\right)$$

$$A_\square = \left(\frac{f(a)+f(b)}{2}\right)(b-a) = \int_a^b P_\Omega(x)\,dx$$

2) Error

$f(x) = P_{\underset{a-b}{\Omega_2}}(x) + E(x)$

$h = (b-a)$

$$E(x) = \frac{f''(\xi_2)}{2}(x-a)(x-b)$$

$$\int_a^b \frac{f''(\xi_2)}{2}(x-a)(x-b)\,dx =$$

$$\frac{f''(\xi_2)}{2}\int_a^b (x^2-bx-ax+ab)\,dx =$$

$$\frac{f''(\xi_2)}{2}\left[\frac{x^3}{3}-\frac{bx^2}{2}-\frac{ax^2}{2}+abx\right]_a^b =$$

$$\frac{f''(\xi_2)}{2}\left[\frac{b^3}{3}-\frac{b^3}{2}-\frac{ab^2}{2}+ab^2-\frac{a^3}{3}+\frac{ba^2}{2}+\frac{a^3}{2}+a^2b\right]$$

$$\frac{f'(\xi_2)}{2}\left(\frac{a^3-b^3}{6}+\frac{-ab^2+ba^2}{2}+b^2a+ba^2\right)$$

$$\left(\frac{a^3-b^3}{12}+\frac{-ab^2+ba^2}{4}+\frac{b^2a}{2}+\frac{ba^2}{2}\right)f''(\xi)=O_{(h^3)}$$

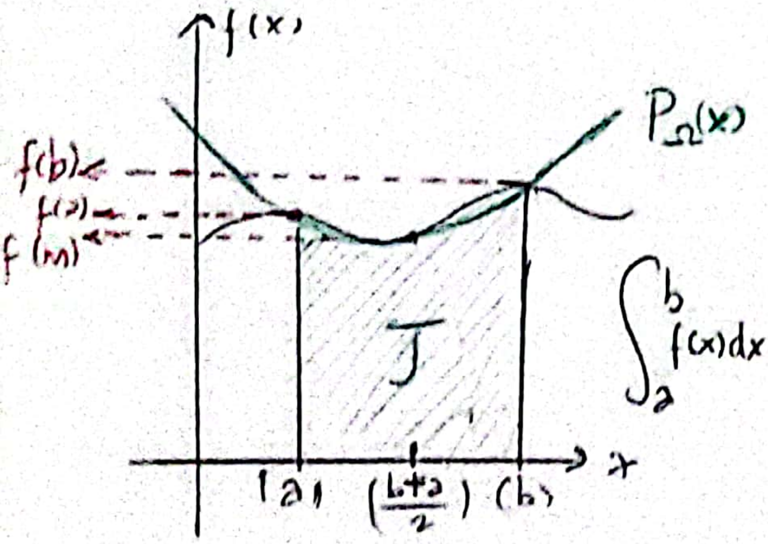
$$h=(b-a)$$

$$\frac{-h^3}{12}=\left(\frac{-b^3}{12}+\frac{3ba^2}{4}+\frac{ab^2}{4}\right)$$

$$\boxed{O \to h^3}$$

3) Método de Simpson simple 1/3

$$I=\int_a^b f(x)\,dx$$



$$\int_a^b f(x)dx \cong \left(J=\int_a^b P_2(x)\,dx\right)$$

4)

$$E = \int_a^b E(x)\,dx = \int_a^b \frac{f'''(\xi_2)}{4!}(x-a)(x-b)\left(x-\left(\frac{a+b}{2}\right)\right)dx$$

$$=$$

$$\frac{f'''(\xi_2)}{24}\int_a^b x^3 + \frac{xb^2}{2} - \frac{ax^2}{2} - \frac{x^2b}{2} + \frac{xa^2}{2} - \frac{a^2b}{2} - \frac{ab^2}{2} + \frac{2abx}{2} - x^2b - ax^2 + abx \quad dx$$

$$=$$

$$\frac{f'''(\xi_2)}{24}\left[\frac{x^4}{4} - \frac{ax^3}{2} - \frac{bx^3}{2} + \frac{a^2x^2}{4} + \frac{b^2x^2}{4} + abx^2 - \frac{ab^2x}{2} - \frac{a^2b}{2}x\right]_a^b$$

$$=$$

$$\frac{f'''(\xi_2)}{24}\left(\cancel{\frac{b^4}{4}} - \cancel{\frac{ab^3}{2}} - \cancel{\frac{b^4}{2}} + \cancel{\frac{a^2b^2}{4}} + \cancel{\frac{b^4}{4}} + \cancel{ab^3} - \cancel{\frac{ab^3}{2}} - \cancel{\frac{a^2b^2}{2}} - \left(\cancel{\frac{a^4}{4}} - \cancel{\frac{a^4}{2}} - \cancel{\frac{ba^3}{2}} + \cancel{\frac{a^4}{4}} + \cancel{\frac{b^2a^2}{4}} + \cancel{a^3b} - \cancel{\frac{a^2b^2}{2}} - \cancel{\frac{a^3b}{2}}\right)\right)$$

$$=$$

$$\frac{f'''(\xi_2)}{24}(0) = 0$$