

Tutorial 4 for COMP 526 – Applied Algorithmics, Spring 2021

Problem 1 (Parallel And)

We consider the problem of computing the logical *and* (\wedge) of an array $B[0..n-1]$ of n Boolean values (n bits), i.e., the result should be *true* if and only if all n entries are true. (We assume here that each bit is stored as a full word.)

- Design a CREW-PRAM parallel algorithm for computing the “logical and” of $B[0..n-1]$. Your algorithm should have $\mathcal{O}(\log n)$ time (span) and $\mathcal{O}(n \log n)$ work.
- Can you make the algorithm work-efficient?
- Now consider a CRCW-PRAM; you can choose a write-conflict resolution rule that is convenient for your purposes. Design a *constant-time* parallel algorithm for computing the logical and.

Problem 2 (Sorted with outliers)

In this task, we consider a different type of presortedness:

Given an array $A[0..n]$, we say A is *d-deletion-sortable* if there are indices $0 \leq i_1 < i_2 < \dots < i_d < n$, so that after deleting the positions i_1, \dots, i_d from A , the resulting sequence is sorted. For example

2, 4, 1, 6, 7, 5, 8, 12, 0

is 3-deletion-sortable (by removing elements 1, 5, 0), but it is not 2-deletion-sortable.

In the following, we always assume that we are given an array $A[0..n]$ that is *d-deletion-sortable*. For simplicity, you may assume that the elements in A are pairwise different.

- Design an adaptive sorting algorithm for A when you are also *given* (a sorted array $D[0..d]$ of) the *indices* i_1, \dots, i_d to delete.

Assuming $d \ll n$, your algorithm should run in time $o(n \log n)$; more precisely, a full solution would sort a \sqrt{n} -deletion-sortable $A[0..n]$ in $O(n)$ time.

Describe your algorithm (in clear prose or pseudocode) and analyze its running time (as a Θ -class).

- b) How large can d be before your solution requires $\omega(n)$ time?
How large can d be before your solution requires $\Omega(n \log n)$ time?
- c) *Bonus problem:*
Design an algorithm as in a), but this time you are neither given d , nor the indices to delete.
- Hint:* Can you find a set of indices I , so that A is sorted after removing those indices (without making I too big)? You may not be able to achieve $|I| \leq d$ easily, but it is sufficient to be “not too far” from d .