2 Fundamental Data Structures

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ADT = abstract data type

- list of supported operations
  - what should happen
  - <u>not</u> how to do this

    <u>not</u> how data is stored

VS

data structures

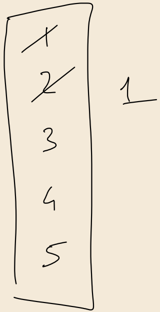
- how data is stored in memory
- algorithms to work on data

ex: stack

pop() → removes topmost element

push(v) → adds v to top of stack

pop pop push(1)

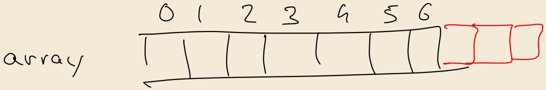


Outline

2 Fundamental Data Structures

2.1 Stacks & Queues

2.2 Resizable Arrays



- arrays have fixed size
  - ↳ if we need more space, allocate new array & copy old data
- doubling arrays: when array is full double its size
- if array becomes too empty (deletions)
  - ↳ if $\leq \frac{1}{4}$ full $\rightsquigarrow$ halve size

$\rightarrow$ space $\Theta(n)$ $\quad n = \#$ elements stored

Java Generics

Stack<String>

Stack<Integers>

implement $\overset{\text{ONCE}}{\vee}$ stack with type parameter

use stack with <u>many</u> different types

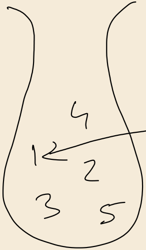
Iterators

ADT abstracts linear scan over collection of items

- hasNext()
- next move ahead & return element

2.3 Priority Queues

ADT

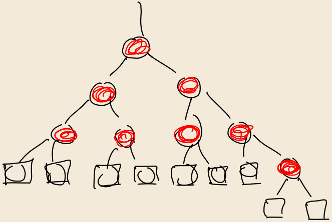


- isEmpty
- delMin
- insert
- decreaseKey
  changeKey

Heaps

- binary tree

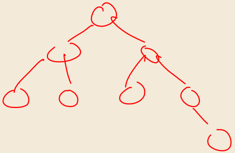
extended binary trees



every node has 0 or 2 children

$\longleftrightarrow$

binary trees

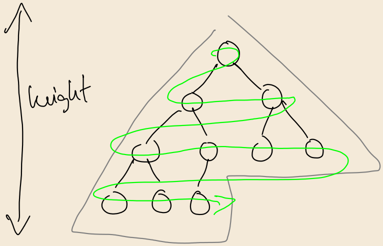


nodes have left / right children (both can be missing)

- complete binary tree + flush-left last level – lowest level as far to the left as possible

"heap-shaped trees"

Why heap-shaped trees?



- minimal height among binary trees with n nodes

n nodes → 1 heap-shape

⇒ easy

can use array: store nodes in level-order starting at 1

⇝ find parent of node k at $\lfloor \frac{k}{2} \rfloor$

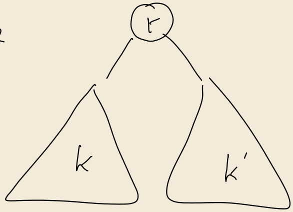
left child at $2k$

right " $2k+1$

heap-order

entire subtree version

every node



$k \geq r$

$k' \geq r$

all keys in subtrees are $\leq$ key in root

$\Rightarrow$ root stores minimum

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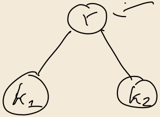
children-version

every node

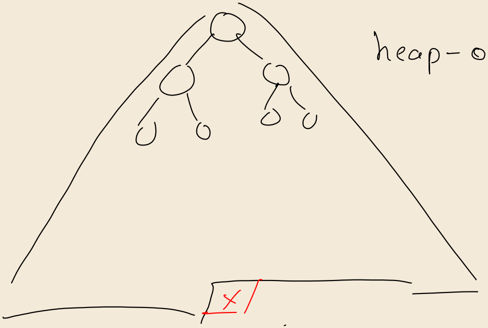


$k_1 \geq r$

$k_2 \geq r$

Insertion

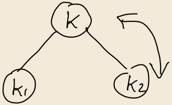
insert new key x



heap-ordered

1. store x in lowest level, next free position

2. repair heap-order

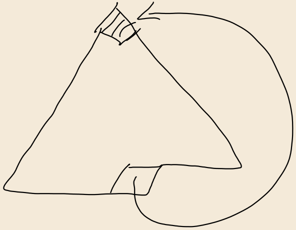


$\boxed{k_2 < k}$ violation

→ swap them!

repeat up the tree

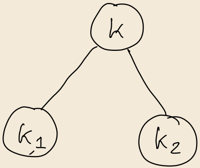
delete Min



① delete root (and return its key)

② move "last" element to root
  rightmost on lowest level

③ repair heap-order



$k > k_1$ and/or $k > k_2$

(a) find $\min\{k_1, k_2\}$

(b) swap k with smaller child

continue along the changed child link

Analysis

worst case in both cases: follow one path

$\Rightarrow$ cost = # levels = height of tree

$\sim \lg n$

PQ = 2 ADT

MinPQ | MaxPQ

insert

delMin | delMax

2 separate ADTs

min-oriented binary heaps

max-oriented binary heaps

2.4 Binary Search Trees

ADT : Symbol table

aka dictionaries

associative arrays   A['hello'] = 17

maps

$\approx$ partial mathematical functions

$\boxed{\text{dynamic}}$ — change function over time

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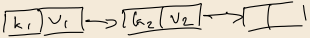
primitive implementations

① unordered list   sequential search   easy add



= check every   hard find

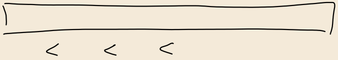
$O(n)$ $n = \#$ keys

(2) sorted array

binary search

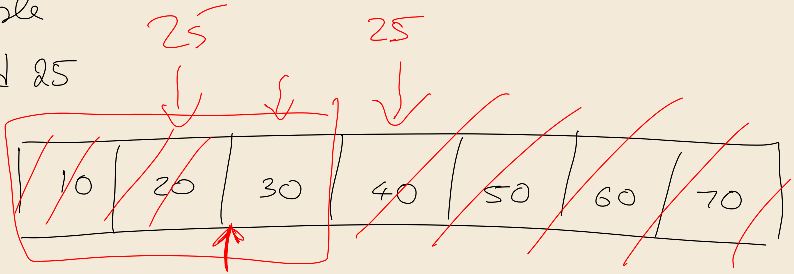
good find

hard to change



Example

find 25



$\sim \lg n$

BSTs $\hat{=}$ dynamic sorted "array"

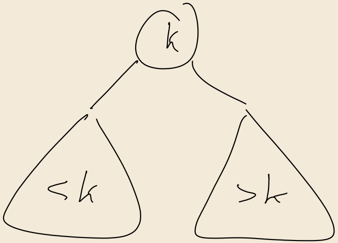
- binary tree = nodes have <u>left / right child</u>

  both can be empty

$\oplus$

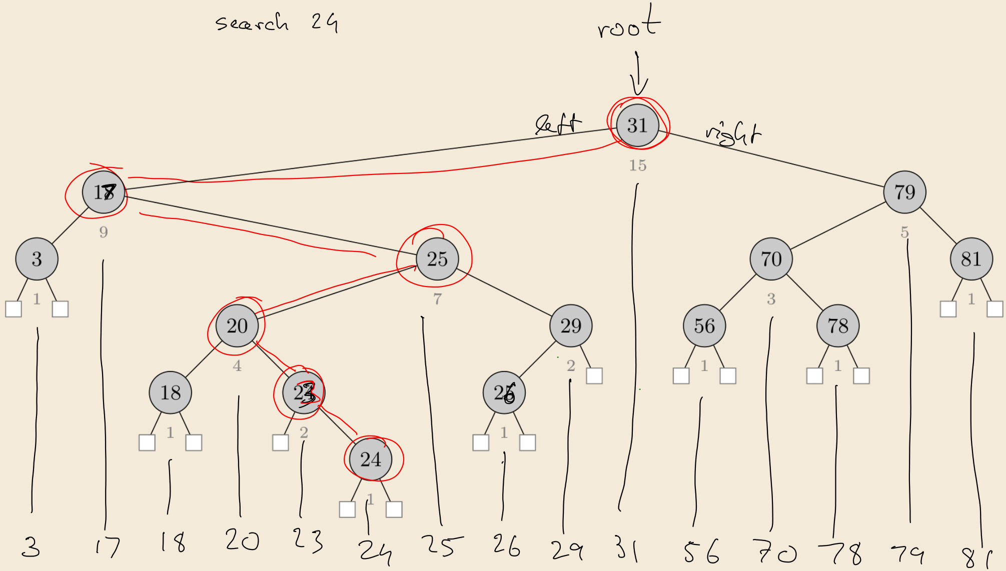
- search-tree property

  (symmetric order)



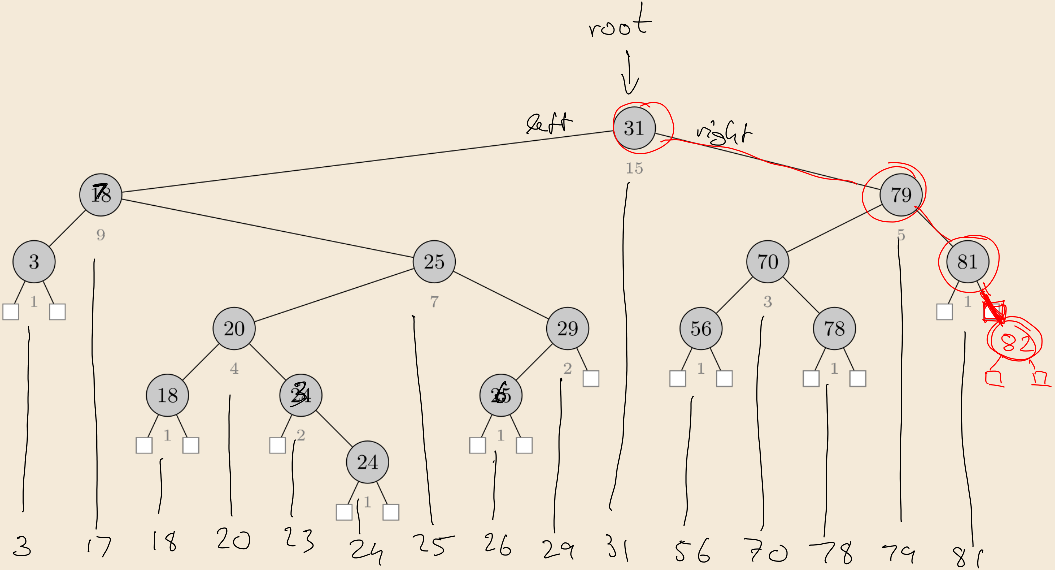
in Java : Node { left, right }

BST { root }



search 24
root
left
right
31
15
17
9
79
5
3
1
25
7
70
3
81
1
20
4
29
2
56
1
78
1
18
1
23
2
26
1
24
1
3 17 18 20 23 24 25 26 29 31 56 70 78 79 81

insert 82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70
78
79
81

delete 3 easy leaf

delete 29 easy unary node

delete 79 hard binary

find inorder predecessor
(left, right, right, ...)
swap with 79
delete 78