Tutorial 5 for
COMP 526 – Applied Algorithmics, Spring 2021

Problem 1 (How KMP uses itself)

Recall the example $T = \texttt{ababababaabab}$ and $P = \texttt{ababaca}$ used in the lecture to illustrate the KMP failure-link automaton.

Now consider the string $S = S[0..m + n] = P\,\$\,T$ over the extended alphabet $\Sigma' = \Sigma \cup \{\$\} = \{\texttt{a}, \texttt{b}, \texttt{c}, \$\}$ and construct the failure-links array $fail[0..n + m]$.

Compare the result with the sequence of states from simulation the failure-link automaton for P on T; what do you observe?

Bonus: Can you compute the values $fail[0..n + m]$ using only $\Theta(P)$ extra space? Here, it is enough to have the values available at some time during the computation; we (obviously) cannot store all of them explicitly in the allowed space.

Problem 2 (Periodicity lemma)

Prove the periodicity lemma:

If string $S = S[0..n - 1]$ has periods p and q with $p + q \leq n$, then it has also period $\gcd(p, q)$.