11 Greedy Algorithms

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Learning Outcomes

Unit 11: ***Greedy Algorithms***

1. Describe informally what greedy algorithms are.
2. Know exemplary problems and their greedy solutions: Change-Making Problem, MSTs, SSSPP, Assignment Problem.
3. Be able to design and proof correctness of greedy algorithms for (simple) algorithmic problems.
4. Be able to explain the matroid properties and its relation to greedy algorithms.

Outline

11 Greedy Algorithms

11.1 Introduction

Myopic Optimization

- In a *"greedy" algorithm*,
  we assemble a solution to an **optimization** problem **step by step**
  always picking the next step to maximize **current** gain,
  and we **never take back** earlier steps.

"Take what you can, give nothing back!"

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- reminiscent of *gradient-descent* algorithms
  but discrete and even more unwilling to undo mistakes

$\rightsquigarrow$ greedy algorithms only yield optimal solutions for certain problems

  - but where they do, their speed is usually unbeatable

  $\rightsquigarrow$ it is understanding where they succeed

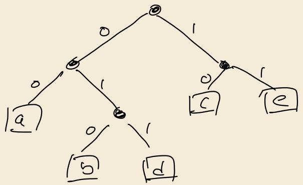
- even where they are not optimal, greedy approaches can be efficient $\overset{\text{(unknown quality)}}{\text{heuristics}}$ or $\underbrace{\text{approximation algorithms}}_{c\text{-approximation = at most factor } c \text{ worse than optimum}}$

Plan for the Unit

- We will first see a few examples (known and new) of greedy algorithms to make the vague generic description concrete
  - in particular minimum spanning trees and shortest paths in graphs
- Unlike other algorithm design techniques, greedy algorithms have a formal basis: *matroids* (and *greedoids*)
  - The second part will introduce these and how they can unify correctness proofs

A First Example: Reunion With An Old Friend

- We have seen an example of a Greedy Algorithm in Unit 7: *Huffman Codes!*
- Recall the problem:
  - **Given:** Set of symbols $\Sigma = [0..\sigma)$, weights $w : \Sigma \to \mathbb{R}_{\geq 0}$
  - **Goal:** prefix code E (= code trie) that minimizes $\sum_{c\in\Sigma} w(c) \cdot |E(c)|$



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$\rightsquigarrow$ Since only *code tries* are valid, all solutions consist in repeatedly merging tries (starting from single-leaf tries, until single trie left)

- each merge contributes the subtree's total weight to overall cost (since all leaves in merged tries move one level down / all codewords get one extra bit)
- **Huffman's Algorithm:** Always choose current cheapest merge.

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- **Huffman's Algorithm:** Always choose current cheapest merge.
- In the correctness proof, we had to show:
  There is always an optimal code trie where the two lowest-weight symbols are siblings.

This is typical: To show that Greedy is optimal, we need a structural insight into optimal solutions.

11.2 How Can Greed Succeed?

Greed For Change

The Change-Making Problem (a. k. a. Coin-Exchange Problem)

- **Given:** a set of integer denominations of coins $w_1 < w_2 < \cdots < w_k$ with $w_1 = 1$, target value $n \in \mathbb{N}_{\geq 1}$

  (we have sufficient supply of all coins . . .)
- **Goal:** "fewest coins to give change n", i. e., multiplicities $c_1, \ldots, c_k \in \mathbb{N}_{\geq 0}$ with $\sum_{i=1}^{k} c_i \cdot w_i = n$ minimizing $\sum_{i=1}^{k} c_i$

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For Euro coins, denominations are 1¢, 2¢, 5¢, 10¢, 20¢, 50¢, 1€, and 2€.

$$\text{formally:}\quad \begin{array}{cccccccc} 1, & 2, & 5, & 10, & 20, & 50, & 100, & \text{and } 200. \\ w_1 & w_2 & w_3 & w_4 & w_5 & w_6 & w_7 & w_8 \end{array}$$

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$\rightsquigarrow$ Simple greedy algorithm:
largest coins first

- optimal time ($O(k)$ if coins sorted)
- is $\sum c_i$ minimal?

```
procedure greedyChange($w[1..k]$, $n$):
    // Assumes $1 = w[1] < w[2] < \cdots < w[k]$
    for $i := k, k-1, \ldots, 1$:
        $c[i] := \lfloor n / w[i] \rfloor$
        $n := n - c[i] \cdot w[i]$
    // Now $n == 0$
    return $c[1..k]$
```