

Tutorial 1 for COMP 562 – Applied Algorithmics, Winter 2020

Problem 1 (Orders of magnitude)

Order the following functions with respect to their asymptotic order of magnitude (i.e., their Θ -class).

$$n, \sqrt{n}, n^{1.5}, n^2, n \lg n, n \lg \lg n, n \lg^2 n, n \lg(n^2), \frac{2}{n}, 2^n, 2^{n/2}, 37, n^3, n^2 \lg n.$$

Problem 2 (Loop invariants)

There are two integral¹ parts of integer division: *the quotient* and *the remainder*. For two integers $n, k > 0$ the quotient (or result) of the integer division “ $n \operatorname{div} k$ ” is defined as the largest integer m with $m \cdot k \leq n$. The remainder of the division is defined as $r = n - m \cdot k$. Note that $0 \leq r < k$. The value r is also known as the result of *modulo* operation, written “ $r = n \bmod k$ ”.

Example: $10 \operatorname{div} 3 = 3$ and $10 \bmod 3 = 1$,
 $13 \operatorname{div} 5 = 2$ and $13 \bmod 5 = 3$.

Apply the *invariant method* to prove the correctness of the following function $\operatorname{Mod}(n, k)$, which is supposed to compute $n \bmod k$, where n and k are two positive integer input parameters of the function.

```
1  procedure  $\operatorname{Mod}(n, k)$ 
2    // Input: positive integers  $n, k$ .
3    // Output: value of  $n \bmod k$ .
4     $t := n$ 
5    while  $t \geq k$ 
6       $t := (t - k)$ 
7    end while
8    return  $t$ 
```

¹pun intended