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Tutorial 4 for COMP 526 – Applied Algorithmics, Winter 2020

Problem 1 (Fibonacci language and failure function)

The sequence of Fibonacci words $(w_i)_{i\in\mathbb{N}_0}$ is defined recursively:

$$\begin{aligned} w_0 &= \mathtt{a} \\ w_1 &= \mathtt{b} \\ w_n &= w_{n-1}\cdot w_{n-2} \qquad (n \ge 2) \end{aligned}$$

Unfolding the recursion yields $w_2 = \mathtt{ba}$, $w_3 = \mathtt{bab}$, $w_4 = \mathtt{babba}$, an so on.

(Note that the lengths $|w_0|, |w_1|, |w_2|, \ldots$ are *Fibonacci numbers*, hence the name. More precisely, we have $|w_n| = F_{n+1}$, with the Fibonacci numbers defined as $F_0 = 0$, $F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$, for $n \ge 2$.)

Construct the failure function F and the draw the KMP automaton with failure links for w_6.

Problem 2 (How KMP uses itself)

Recall the example $T = \mathtt{abababaabab}$ and $P = \mathtt{ababaca}$ used in the lecture to illustrate the KMP failure link automaton.

Now consider the string $S = S[0..m+n] = P\,\$\,T$ over the extended alphabet $\Sigma' = \Sigma \cup \{\$\} = \{\mathtt{a}, \mathtt{b}, \$\}$ and construct the failure-links array $\mathit{fail}[0..n+m]$.

Compare the result with the sequence of states from the matching T with the KMP automaton for P; what do you observe?

Bonus: Can you compute the values $\mathit{fail}[0..n+m]$ using only $\Theta(P)$ extra space? Here, it is enough to have the values available at some during the computation; we (obviously) cannot store all of them explicitly in the allowed space.