4 String Matching – What’s behind Ctrl+F?

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Learning Outcomes

1. Know and use typical notions for *strings* (substring, prefix, suffix, etc.).
2. Understand principles and implementation of the *KMP*, *BM*, and *RK* algorithms.
3. Know the *performance characteristics* of the KMP, BM, and RK algorithms.
4. Be able to solve simple *stringology problems* using the *KMP failure function*.

Unit 4: *String Matching*

Outline

4 String Matching

4.1 String Notation

Ubiquitous strings

string = sequence of characters

- universal data type for . . . everything!
  - natural language texts
  - programs (source code)
  - websites
  - XML documents
  - DNA sequences
  - bitstrings
  - . . . a computer's memory $\rightsquigarrow$ ultimately any data is a string

$\rightsquigarrow$ many different tasks and algorithms

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- This unit: finding (exact) **occurrences of a pattern** text.
  - Ctrl+F
  - grep
  - computer forensics (e. g. find signature of file on disk)
  - virus scanner
- basis for many advanced applications

Notations

- *alphabet* Σ: finite set of allowed **characters**; $\sigma = |\Sigma|$ *"a string over alphabet Σ"*
  - letters (Latin, Greek, Arabic, Cyrillic, Asian scripts, ...)
  - "what you can type on a keyboard", Unicode characters $\simeq$ 1306
    comprehensive standard character set including emoji and all known symbols
  - $\{0, 1\}$; nucleotides $\{A, C, G, T\}$; ...

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    comprehensive standard character set including emoji and all known symbols
  - $\{0,1\}$; nucleotides $\{A, C, G, T\}$; ...
- $\Sigma^n = \Sigma \times \cdots \times \Sigma$: strings of **length** $n \in \mathbb{N}_0$ (n-tuples)
- $\Sigma^\star = \bigcup_{n \ge 0} \Sigma^n$: set of **all** (finite) strings over Σ
- $\Sigma^+ = \bigcup_{n \ge 1} \Sigma^n$: set of **all** (finite) **nonempty** strings over Σ
- $\varepsilon \in \Sigma^0$: the *empty* string (same for all alphabets)

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zero-based (like arrays)!

- for $S \in \Sigma^n$, write $S[i]$ (other sources: S_i) for $\boldsymbol{i}$**th** character $(0 \le i < n)$
- for $S, T \in \Sigma^\star$, write $ST = S \cdot T$ for **concatenation** of S and T
- for $S \in \Sigma^n$, write $S[i..j]$ or $S_{i,j}$ for the **substring** $S[i] \cdot S[i+1] \cdots S[j]$ $(0 \le i \le j < n)$
  - $S[0..j]$ is a **prefix** of S; $S[i..n-1]$ is a **suffix** of S
  - $S[i..j) = S[i..j-1]$ (endpoint exclusive) $\rightsquigarrow$ $S = S[0..n)$

Clicker Question

True or false: $\Sigma^\star = \Sigma^+ \cup \{\varepsilon\}$

A True

B False

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Clicker Question

strings of length ≥ 0

$\parallel$

True or false: $\Sigma^\star = \Sigma^+ \cup \{\varepsilon\}$

strings of length ≥ 1

A True ✓

B ~~False~~

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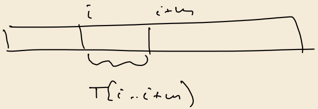
String matching – Definition



Search for a string (pattern) in a large body of text

- **Input:**
  - $T \in \Sigma^n$: The *text* (haystack) being searched within
  - $P \in \Sigma^m$: The *pattern* (needle) being searched for; typically $n \gg m$
- **Output:**
  - the *first occurrence (match)* of P in T: $\min\{i \in [0..n-m) : T[i..i+m) = P\}$
  - or `NO_MATCH` if there is no such i ("P does not occur in T")
- Variant: Find **all** occurrences of P in T.

  $\rightsquigarrow$ Can do that iteratively (update T to $T[i+1..n)$ after match at i)

- **Example:**
  - T = "`Where is he?`"
  - P_1 = "`he`" $\rightsquigarrow$ $i = 1$
  - P_2 = "`who`" $\rightsquigarrow$ `NO_MATCH`
- string matching is implemented in Java in `String.indexOf`, in Python as `str.find`

Clicker Question

0 1 2 3 4 5 6 7 8 9 …

Let T = `COMP526␣is␣fun.`
What is $T[3..7)$?

→ *sli.do/comp526*

Clicker Question

Let T = `COMP526␣is␣fun.`
What is $T[3..7)$?

```
0123456789 01234
```

```
COMP526␣is␣fun.
```

→ *sli.do/comp526*

4.2 Brute Force

Abstract idea of algorithms

String matching algorithms typically use *guesses* and *checks*:

- A **guess** is a position i such that P might start at $T[i]$. Possible guesses (initially) are $0 \leq i \leq n - m$.
- A **check** of a guess is a comparison of $T[i + j]$ to $P[j]$.

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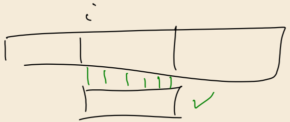

- A **guess** is a position i such that P might start at $T[i]$.
  Possible guesses (initially) are $0 \leq i \leq n - m$.
- A **check** of a guess is a comparison of $T[i + j]$ to $P[j]$.
- Note: need all m checks to verify a single *correct* guess i,
  but it may take (many) fewer checks to recognize an *incorrect* guess.
- Cost measure: #character comparisons

$\rightsquigarrow$ #checks $\leq n \cdot m$ (number of possible checks)

Brute-force method

```
procedure bruteForceSM(T[0..n), P[0..m))
    for i := 0, ..., n − m − 1 do
        for j := 0, ..., m − 1 do
            if T[i + j] ≠ P[j] then break inner loop
        if j == m then return i
    return NO_MATCH
```

- try all guesses i
- check each guess (left to right); stop early on mismatch
- essentially the implementation in Java!

- **Example:**
  T = abbbababbab
  P = abba

a	b	b	b	a	b	a	b	b	a	b
a	b	b	a							
	a									
		a								
			a							
				a	b	b				

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- **Example:**
  T = `abbbababbab`
  P = `abba`

$\rightsquigarrow$ 15 char cmps
(vs $n \cdot m = 44$)
not too bad!

a	b	b	b	a	b	a	b	b	a	b
a	b	b	a							
	a									
		a								
			a							
				a	b	b				
					a					
						a	b	b	a	

Brute-force method – Discussion

Brute-force method can be good enough

- typically works well for natural language text
- also for random strings

but: can be as bad as it gets!

a	a	a	a	a	a	a	a	a	a	a
a	a	a	b							
	a	a	a	b						
		a	a	a	b					
			a	a	a	b				
				a	a	a	b			
					a	a	a	b		
						a	a	a	b	
							a	a	a	b

- Worst possible input: $P = a^{m-1}b$, $T = a^n$
- Worst-case performance: $(n - m + 1) \cdot m$

$\rightsquigarrow$ for $m \le n/2$ that is $\Theta(mn)$

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			a	a	a	b				
				a	a	a	b			
					a	a	a	b		
						a	a	a	b	
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- Bad input: lots of self-similarity in T! $\rightsquigarrow$ can we exploit that?
- brute force does ‘obviously’ stupid repetitive comparisons $\rightsquigarrow$ can we avoid that?

Roadmap

- **Approach 1** (this week): Use *preprocessing* on the **pattern** P to eliminate guesses (avoid 'obvious' redundant work)
  - Deterministic finite automata (**DFA**)
  - **Knuth-Morris-Pratt** algorithm
  - **Boyer-Moore** algorithm
  - **Rabin-Karp** algorithm

- **Approach 2** ($\rightsquigarrow$ Unit ~~6~~ 8): Do *preprocessing* on the **text** T
  Can find matches in time *independent of text size(!)*
  - **inverted indices**
  - **Suffix trees**
  - **Suffix arrays**