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Tutorial 2 for COMP 562 – Applied Algorithmics, Winter 2020

Problem 1 (Decreasing function and amortization method)

Consider again the Mod function from last time:

```
1  procedure Mod(n, k)
2  // Input: positive integers n, k.
3  // Output: value of n mod k.
4  t := n
5  while t ≥ k
6      t := (t − k)
7  end while
8  return t
```

a) Apply the *decreasing potential method* to prove that the function $Mod(n, k)$ always terminates.

b) Try to establish the time complexity of this procedure.

Problem 2 (Telescoping recurrence and mathematical induction)

Given a complexity function $T(n)$ recursively defined as

$$T(n) = \begin{cases} 3, & \text{for } n = 0; \\ T(n-1) + 4, & \text{for } n \geq 1. \end{cases} \tag{1}$$

Find a *closed form* (without recursive reference) for $T(n)$ by iterating (inserting the recursive definition) until you can make an educated guess.

Then prove the correctness of your guess by mathematical induction.