

Vector fun homework

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1 Vector Basics

(a) A point is given in a coordinate system: $B = (7, 8)$. What is the vector going from $(0, 0)$ to B?

$$\vec{A} = [7, 8]$$

A vector between two points can be found by subtracting one point from the other. E.g. the vector going from Copenhagen (C) to Kolding (K) would be found by K to C.

(b) Two points, A and B, in a coordinate system are given by $A = (4, 3)$, $B = (7, 8)$. The vector \vec{A} goes from A to B. What is \vec{A} ?

$$\vec{A} = [3, 4]$$

(c) Given vector $\vec{A} = [3, 5]$, what is its magnitude $|\vec{A}|$ (its length)?

$$\sqrt{3^2 + 5^2} = 5,83$$

Two guys are pulling a boat on a river, each on their side, as seen on Figure 1. The pulls from our heroes are given as \vec{A} and \vec{B} respectively.

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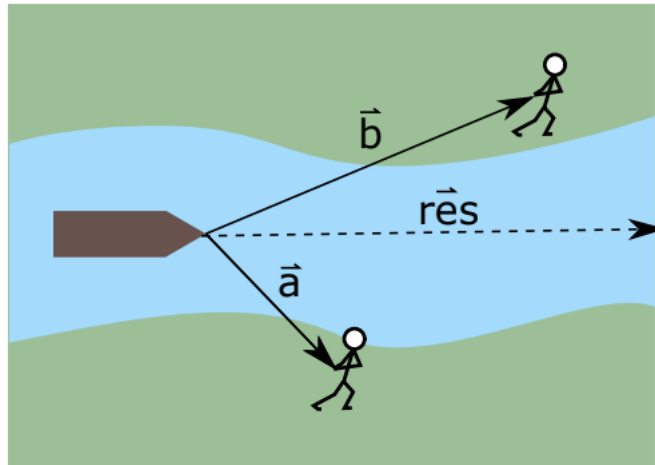


Figure 1: Two bachelor students pulling a boat

$$\vec{a} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}, \vec{b} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

(d) What is the resulting vector of their combined pull \vec{res} ?

$$\vec{res} = [7, 0]$$

Given three vectors, $\vec{A} = [3, 2]$ $\vec{B} = [5, 1]$ $\vec{C} = [-2, 6]$ calculate

$$\vec{a} + \vec{b}$$

$$\vec{res} = [8, 3]$$

$$\vec{b} + \vec{c}$$

$$\vec{res} = [3, 7]$$

$$(\vec{a} + \vec{b}) + \vec{c}$$

$$\vec{res} = [6, 9]$$

$$\vec{a} + (\vec{b} + \vec{c})$$

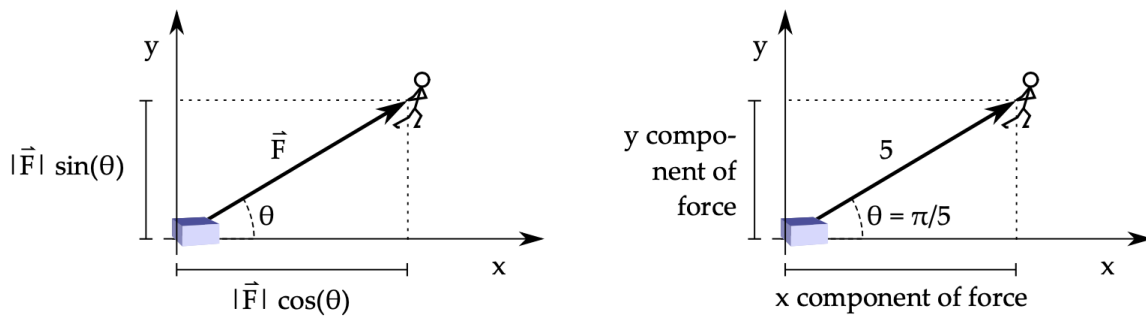
$$\vec{res} = [6, 9]$$

$$\vec{b} + \vec{a}$$

$$r\vec{e}_s = [8, 3]$$

2 Vector Decomposition

- (f) In Figure 3, what is the x-component of the force applied? What is the y-component?
- (g) In Figure 3, what is the vector of the force \vec{f} ?
- (h) Having found the x-component (let's call it F_x) and the y-component (let's call it F_y), find $\sqrt{f_x^2 + f_y^2}$



3 Multiplication of a Vector with a Scalar

When multiplying with a scalar, each component is multiplied with the scalar.

- (i) Given $\vec{a} = [-4, 5]$ and $\vec{b} = \vec{a} * 5$, what is \vec{b} ?

$$\vec{b} = [-20, 25]$$

- (j) What is the length of ?

$$\sqrt{-4^2 + 5^2} = 6,40$$

- (k) What is the length of ?

$$\sqrt{-20^2 + 25^2} = 32,01$$

- (l) What is the result of $[1, 0] \cdot 4$?

$$r\vec{e}_s = [4, 0]$$

- (m) What is the result of $[1, 3] \cdot 0,5$?

$$r\vec{e}_s = [0.5, 1.5]$$

4 unit vectors

Unit vectors have a magnitude of 1. The unit vector of a vector \mathbf{a} is denoted $\hat{\mathbf{a}}$, and points exactly in the same direction as \mathbf{a} – it just has the magnitude of 1.

- (n) Given any vector, how can you find its unit vector?
- (o) What is the unit vector of $\mathbf{a} = 2$?
- (p) What is the unit vector of $\mathbf{a} = 2$?

5 Dot Product / Scalar Product

The product (multiplication) of two vectors is called the dot product or the scalar product (confusing – I know! We will stick to the term “dot product”). The dot product of two vectors \mathbf{a} and \mathbf{b} is given by

6 python

Using Python and numpy, implement a `mag(vec)` function to return the magnitude (length) of a 2-dimensional vector (as a numpy array).

```
def mag(vec):  
    return np.linalg.norm(vec)
```