Applied Finite Elements
STUDENT PROJECT

DESCRIPTION

This document contains all student projects for AFEM. Your project is assigned to you below.

Each project outlines a specific problem which must be solved using FEM. You are allowed to use any code used within the first five weeks of the course. You are also allowed to work with one another on derivations and compare results. However, **everyone must submit their own codes**. Codes are to be submitted with your written report.

Projects are expected to take the remaining 4 weeks of the course and require a similar amount of time per week as the weekly homework. Please make sure to stay on task and allocate an appropriate amount of time to finishing your project.

Tutorial will provide an opportunity for everyone to work together on projects. This is also an excellent time to get feedback on your project, suggestions or hints.

REMINDER: PASSING YOUR ORAL PRESENTATION AND WRITTEN REPORT ARE BOTH MANDATORY FOR PASSING AFEM.

MESHES

Meshes for the projects below are provided in different zip folders available on KEATS. For each project, there are four meshes of varying mesh size stored in separate *.mat files. You may use any of these meshes for your project.

Each mat file contains a domain variable structure for both a linear interpolation of the domain as well as a quadratic interpolation of the domain (NOTE: structures are missing the basis object which you must add to use in previously written codes).

Within each domain variable structure you will also find a *.b array which stores the boundary data information for your project. The columns of this array list element, nodes on the boundary face, patch index. Here the element refers to a global element which contains a face on the boundary, the nodes on the boundary face are the list of nodes which are found of the boundary and patch index corresponds to which part of the boundary (Γ) the face belongs.

PROBLEM MESHES

Brain Waves Project	*brainwave.zip*
Torso Waves Project	*torsolung.zip*
Valve Flow Project	*jetflow.zip*
Heart EP Project	*heart-sa.zip*

NOTE: FOR ALL MESHES, YOU MUST BUILD THE ELEMENT OBJECT (*.e), e.g.

```
Omega.e = fem_get_basis(Omega.p, quad, 'triangle')
```

Applied Finite Elements
BRAIN WAVES PROJECT

DESCRIPTION

Magnetic Resonance Elastography (MRE) is an imaging technique whereby small amplitude waves are generated on the surface of the body near a region of interest and the propagation of these waves through the body is recorded (similar to the affect you get standing near a loud speaker). The way in which these elastic waves propagate within the body gives an indication of the material properties (or stiffness) present within the tissue. This is of value as diseased tissue commonly exhibits varied stiffness compared to its surroundings.

Below is a transverse cross-section of the brain of a patient with a brain tumor. The tumor introduces a region 3 times stiffer than the surrounding tissue. Supposing the right side of the brain is stimulated with a periodic shear displacement, use the conservation of mass and momentum for a linear viscoelastic material to solve the transient propagation of the resulting waves. Assuming the body reaches a periodic steady-state, the wave system can be written in terms of a complex displacement, $\boldsymbol{u}$, where the observed transient wave is then $\boldsymbol{u}_{obs} = \mathrm{Re}\{\boldsymbol{u}\, e^{i\omega t}\}$.

THE QUESTION

Compute the wave behavior in the brain and show the difference in the observed wave propagation with and without the tumor.

THE MODELING PROBLEM

Find $(\boldsymbol{u}, p)$ such that

$$
\begin{aligned}
-\rho\omega^2\boldsymbol{u} - \nabla\cdot G\nabla\boldsymbol{u} + \nabla p &= \boldsymbol{0}, \quad \text{on } \Omega \\
\nabla\cdot\boldsymbol{u} &= 0, \quad \text{on } \Omega \\
\boldsymbol{u} &= \boldsymbol{u}_0, \quad \text{on } \Gamma_1 \\
(G\nabla\boldsymbol{u} - p\boldsymbol{I})\cdot\boldsymbol{n} &= \boldsymbol{0}, \quad \text{on } \Gamma_2
\end{aligned}
$$

where $\boldsymbol{u}$ and p are the complex displacement and pressure solutions for the observed wave $\boldsymbol{u}_{obs} = \mathrm{Re}\{\boldsymbol{u}\, e^{i\omega t}\}$ and $p_{obs} = \mathrm{Re}\{p\, e^{i\omega t}\}$, G is the complex modulus of the tissue given by:

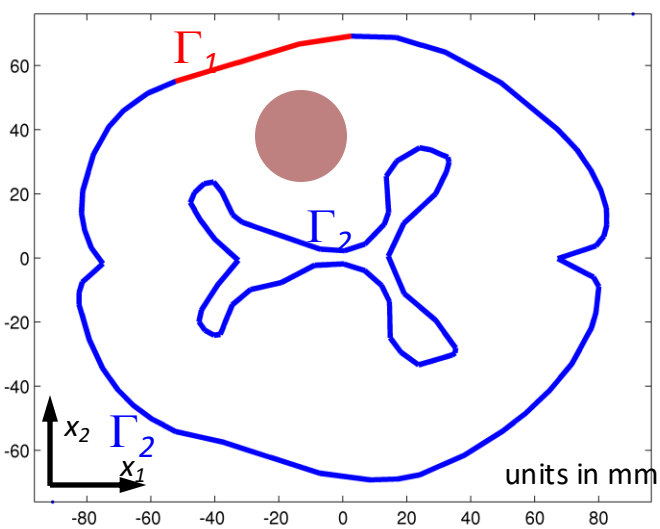
$$
G = \begin{cases} 2000 + G_o & R \le 16 \\ G_o & R > 16 \end{cases}
$$

with R being the distance from the center of the tumor: $R = \left((x_1 + 15)^2 + (x_2 - 38)^2\right)^{1/2}$

and G_o stiffness of healthly brain (in *Pa*): $G_o = 1000 + 500i$

denoting elastic and viscous stiffness of the material (real and imaginary components of G, respectively). $\boldsymbol{u_0}$ Is the boundary shear applied, ρ = 1.016 e-3 $g\ mm^{-3}$ is the density, and $\omega = 2\pi f$ (with driving frequency f=30 Hz):

$$
\boldsymbol{u}_0 = (0.1,\ 0)^T mm
$$



Applied Finite Elements
TORSO WAVES PROJECT

DESCRIPTION

Characterization of lung tissue is challenging in vivo, due to the complexity of the organ which contains air, blood and lung tissue. Imaging through classical methods, such as MRI, is challenging due to the lack of clear signal. It has recently been proposed that vibrational echoes through the lungs could be analyzed to provide some insight into the stiffness of these tissue, enabling better understanding of the health (or disease) state of the lung. As wave propagation is routinely used to assess tissue biomechanics, it is suggested that short burst transient waves could unlock the key to assessment of lung stiffness *in vivo*.

Below is a cross-section of the torso of a patient with lung disease where lungs are 3 times stiffer than normal. Supposing that the back is stimulated with a short burst 60Hz displacement, use the conservation of mass and momentum for a linear viscoelastic material to solve the transient propagation of the resulting waves.

THE QUESTION

Compute the wave behavior in the torso for both healthy and the patient's unhealthy lungs. Monitoring the displacements along the chest wall, characterize the impact of altered lung stiffness on the displacement at the chest.

THE MODELING PROBLEM

$$\text{Find } (\boldsymbol{u}, \boldsymbol{v}, p) \text{ such that} \quad \begin{aligned} \rho \frac{\partial \boldsymbol{v}}{\partial t} - \nabla \cdot \boldsymbol{\sigma} &= \boldsymbol{0}, \quad \text{on } \Omega \\ \nabla \cdot \boldsymbol{u} &= 0, \quad \text{on } \Omega \\ \boldsymbol{u} &= \boldsymbol{u}_0, \quad \text{on } \Gamma_1 \\ \boldsymbol{\sigma} \cdot \boldsymbol{n} &= 0, \quad \text{on } \Gamma_2 \end{aligned}$$

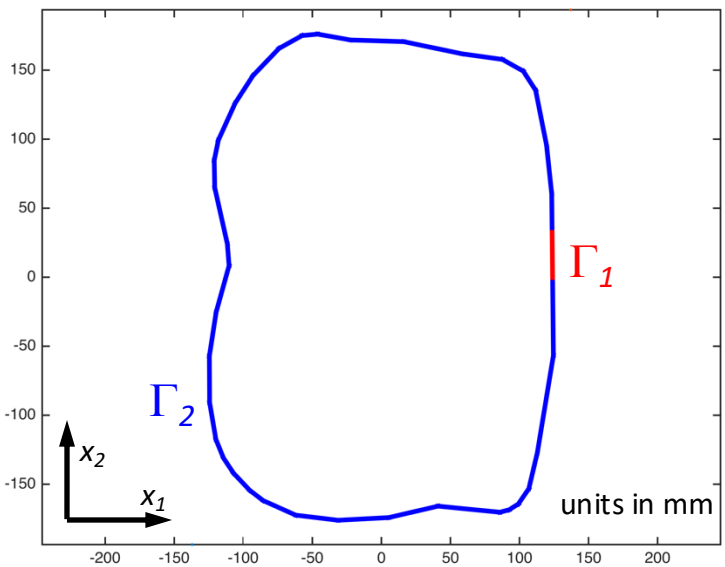
where ***u***, ***v*** and *p* are the displacement, velocity and pressure solutions for the observed wave (note: ***v*** = d***u***/dt), ρ = 1.016 e-3 *g mm*$^{-3}$ is the density, $\boldsymbol{\sigma}$ is the Cauchy stress given by:

$$\boldsymbol{\sigma}(\boldsymbol{u}, \boldsymbol{v}, p) = G(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T) + \eta(\nabla \boldsymbol{v} + \nabla \boldsymbol{v}^T) - p\boldsymbol{I}$$

G and η are the tissue stiffness and viscosity, respectively. Each element is assigned a stiffness according to whether its surrounding tissue (1 – *G =1000, η = 200*), right lung (2 – G = 200, *η = 20*) or left lung (3 – G = 600, *η = 60*). These values can be found in the mesh files as the field *Reg*.

A boundary 'punch' is applied in the negative x_1 direction with driving frequency *f=60 Hz* and amplitude *A = 1 mm:*

$$\boldsymbol{u}_0 = -A \sin^2(2\pi f t)\boldsymbol{e}_1, \qquad \text{for } t \leq 1/(2\pi f)$$



Applied Finite Elements
VALVE FLOW PROJECT

DESCRIPTION

Aortic valve stenosis results from the build up of calcified materials in the leaflets of the aortic valve. These mineral deposits yield poor mobility of the valve, often limiting the degree to which the valve can open and close. This results in the formation of an aortic outflow jet – or strong stream of flow – that requires excessive force (or pressure) generation from the heart.

Below is an idealized schematic of the aortic sinus and downstream aorta. Here the valve is in its maximally dilated position during the peak flow where the change in momentum in the fluid is zero. A small flow is observed within the coronary arteries during this phase of the cycle. Using the conservation of momentum and conservation of mass we can predict the flow of blood through the stenosed valve along with the pressure required to drive the flow.

THE QUESTION

Compute the drop of pressure through the stenosed valve.

THE MODELING PROBLEM

Find $(\boldsymbol{v}, p)$ such that

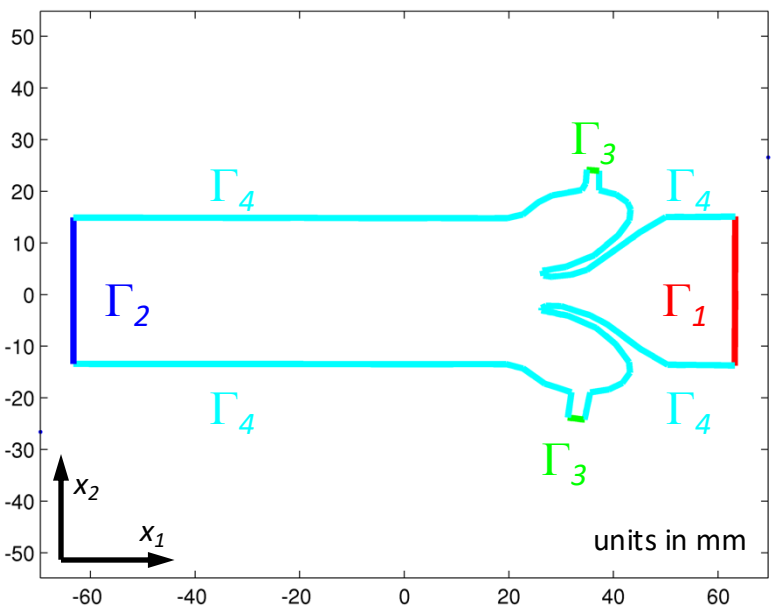
$$
\begin{aligned}
\rho \boldsymbol{v} \cdot \nabla \boldsymbol{v} - \mu \Delta \boldsymbol{v} + \nabla p &= \boldsymbol{0}, && \text{on } \Omega \\
\nabla \cdot \boldsymbol{v} &= 0, && \text{on } \Omega \\
\boldsymbol{v} &= \boldsymbol{v}_0, && \text{on } \Gamma_1 \\
(\mu \nabla \boldsymbol{v} - p \boldsymbol{I}) \cdot \boldsymbol{n} &= \boldsymbol{0}, && \text{on } \Gamma_2 \\
\boldsymbol{v} &= \boldsymbol{v}_1, && \text{on } \Gamma_3 \\
\boldsymbol{v} &= \boldsymbol{0}, && \text{on } \Gamma_4
\end{aligned}
$$

where $\boldsymbol{v}$ and p are the flow velocity and pressure, ρ = 1 e-3 *g mm*$^{-3}$ and μ = 4 e-2 *g (mm s)*$^{-1}$ are the density and viscosity of the fluid, and $\boldsymbol{v}_0$, $\boldsymbol{v}_1$ are the given boundary data, *i.e.*

$$\boldsymbol{v}_0 = (-200,\ 0)^T mm\ s^{-1}$$

$$\boldsymbol{v}_1 = \begin{cases} (0,\ 10)^T mm\ s^{-1} & x_2 > 0 \\ (0,\ -10)^T mm\ s^{-1} & x_2 < 0 \end{cases}$$

NOTES:
The pressure drop is the mean inlet pressure (computed as the integral of pressure over the inlet divided by the length) minus the mean outlet pressure (computed as the integral of pressure over the outlet Γ_2 by the length of the outlet segments).



Applied Finite Elements
Heart Electrophysiology Project

DESCRIPTION

The human heart relies on electrical rhythm in order to activate the heart muscle, or myocardium. The electrophysiology of the heart plays an important role in normal heart function and can pose serious complications in disease. One condition, known as Left Bundle Branch Block (LBBB) occurs when electrical conduction to the left ventricle through the Purkinje network is blocked. One method of therapy is cardiac resynchronization therapy (CRT), where pacing in the heart is restored using electrical leads placed on the heart muscle.

Below is cross-section of a patient's heart with LBBB who requires resynchronization therapy. Activation through the right Purkinje system is preserved, but the left relies on stimulation through the muscle, prolonging activation time in the heart and weakening the hearts contraction. CRT therapy relies on improving the speed of activation in the muscle.

THE QUESTION

Compute the time it takes for cardiac activation and determine which of the positions, (42,165), (91,179), or (138,148) yield the optimal lead placement.

THE MODELING PROBLEM

Find (u, z) such that,

$$\frac{\partial u}{\partial t} - \nabla \cdot (\boldsymbol{D}\nabla u) = ku(1-u)(u-a) - uz, \quad \text{on } \Omega \times [0,T]$$

$$\frac{dz}{dt} = -e(ku(u-a-1)+z), \quad \text{on } \Omega \times [0,T]$$

$$u(\cdot,0) = u_0, \quad \text{on } \Omega$$

$$\boldsymbol{D}\nabla u \cdot \boldsymbol{n} = 0, \quad \text{on } \Gamma \times [0,T]$$

where u is the normalised potential, z the normalised repolarization, $\{k, a, e\}=\{8, 0.15, 0.01\}$ are tissue specific parameters, $\boldsymbol{D}$ is the conductivity tensor:

$$\boldsymbol{D} = \frac{3}{4}\boldsymbol{f}\boldsymbol{f}^T + \frac{1}{4}\boldsymbol{I}$$

where $\boldsymbol{f}$ is a vector field denoting the direction of muscle fibers (given in the field *Fibers*). The normalised potential, u, is related to the actual membrane potential, $v = 125\,u - 80$ (mV). The muscle activation time occurs when the membrane potential $v > 20$ mV.

The initial condition u_0 is zero at all nodes except those which on the boundary of Γ_2, that are stimulated by the right ventricular Purkinje system (all **boundary points** within 7mm of points (83, 75), (19, 44), and (124, 10)).

Stimulation sites provided further stimulate boundary points on Γ_3 that are within 7mm of the site.