Introduction to Theory of Safe Decision Making

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1st AID Scientific Workshop, Trondheim

Forewords & Disclaimer

Objectives:

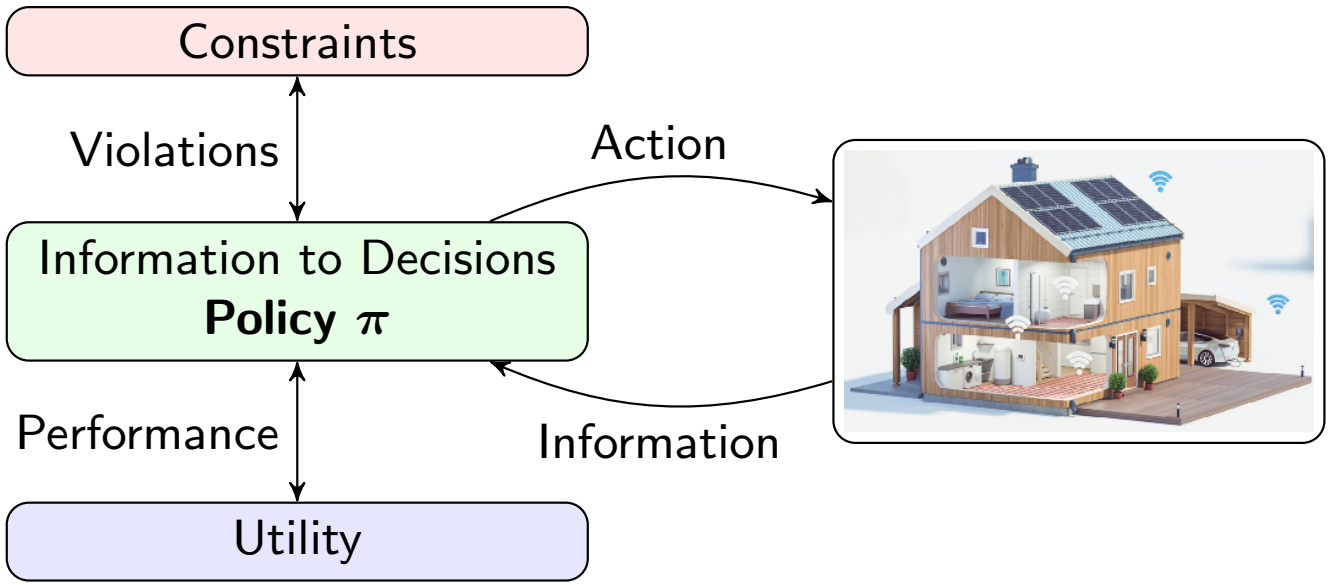
- Put in place some common concepts & language
- Identify some key points in safe decision making
- Connect to AI

Disclaimer: we are a broad group who needs to get to know each others scientifically.
Apologies if I don't "hit" the right level for all.
I have favored simplicity over "absolute" rigor.

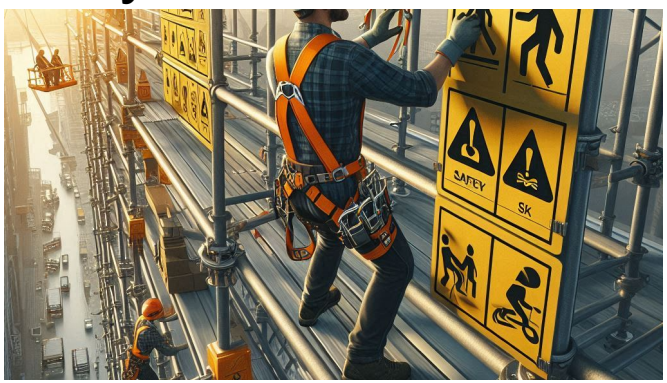
Outline

1. Some Basics of Safe Decision Making
2. Methods
3. Safe Decisions from Data & AI
4. Epistemic Uncertainty and Safe Decisions

Formally Defining Safety?



Safety in the real world



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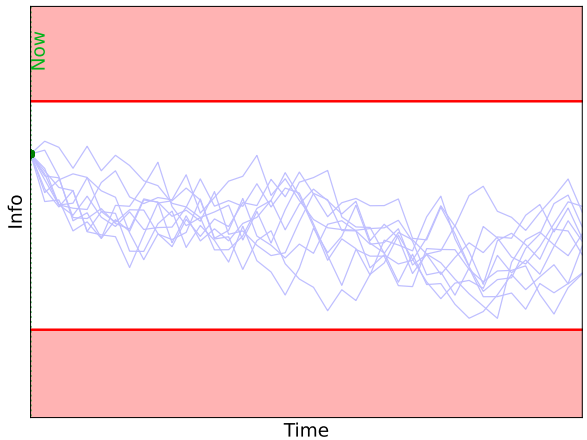

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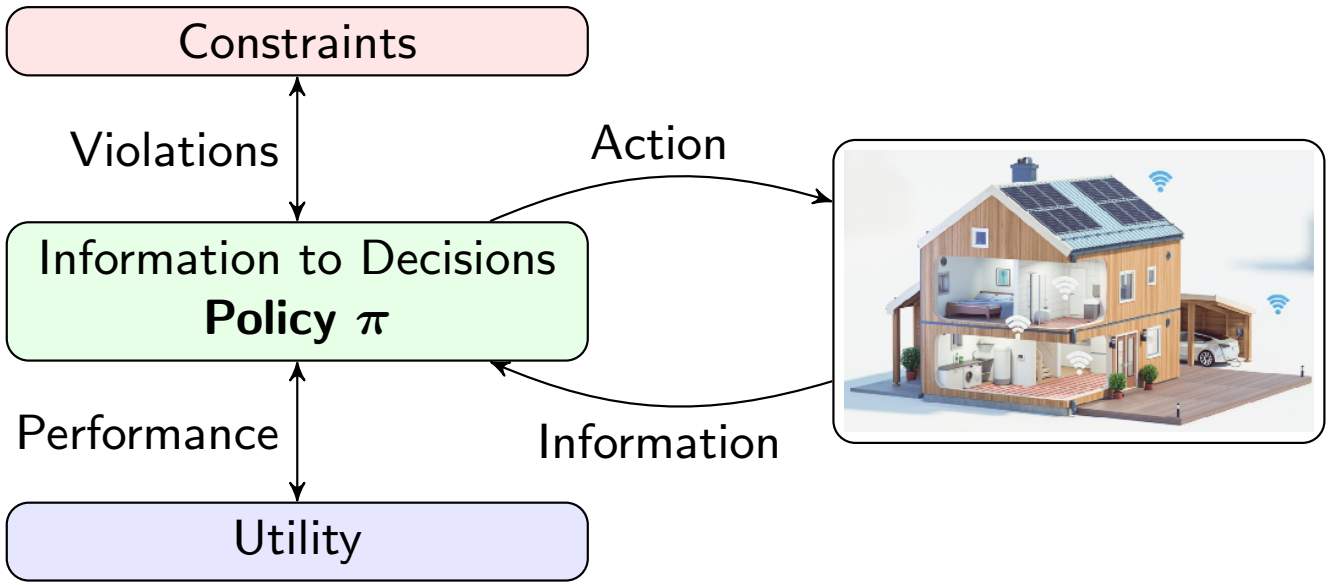

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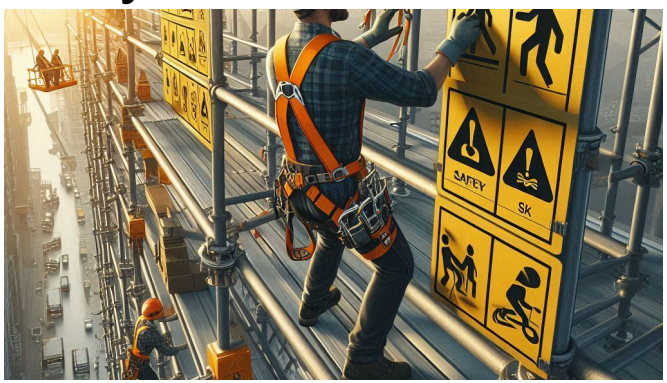

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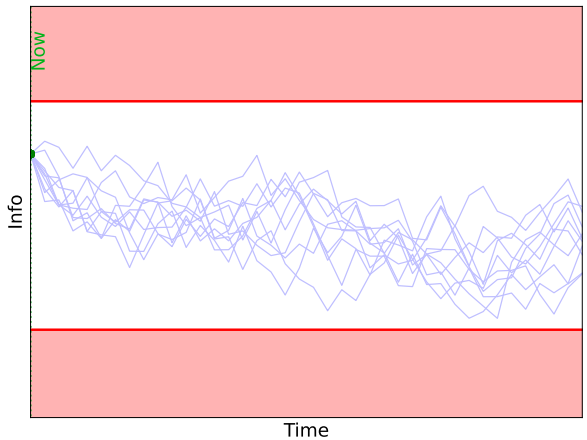

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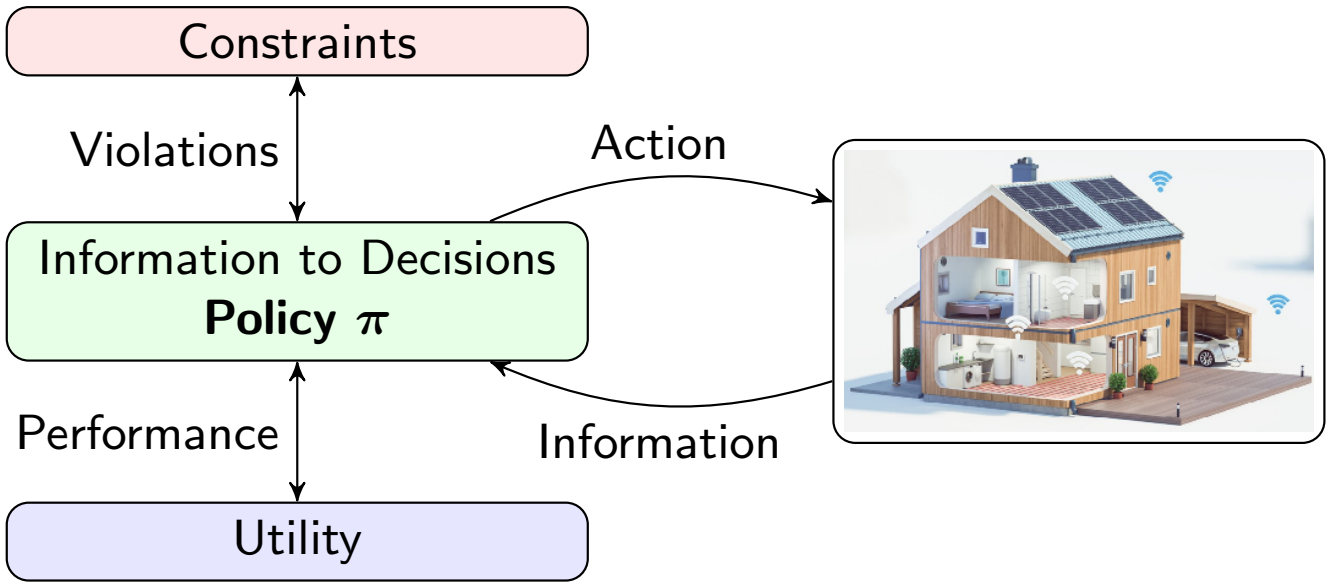

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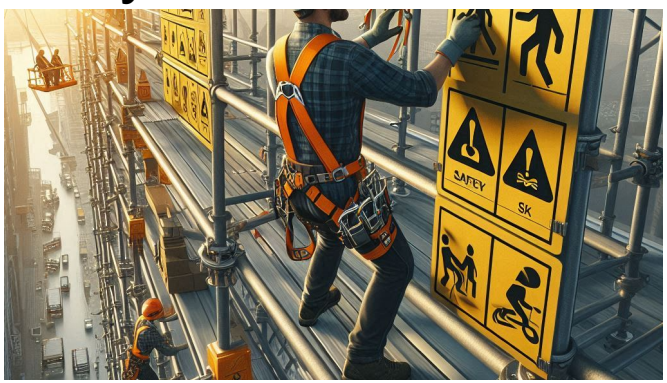

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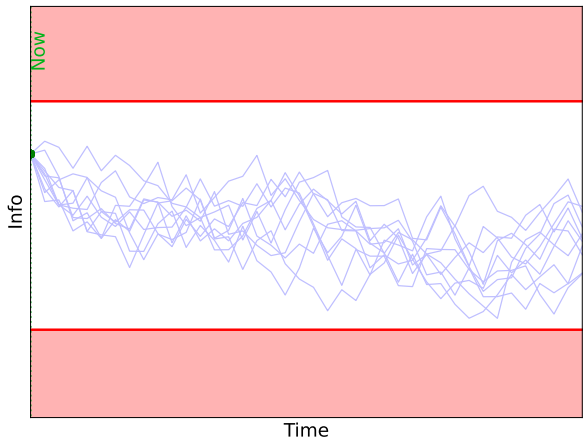

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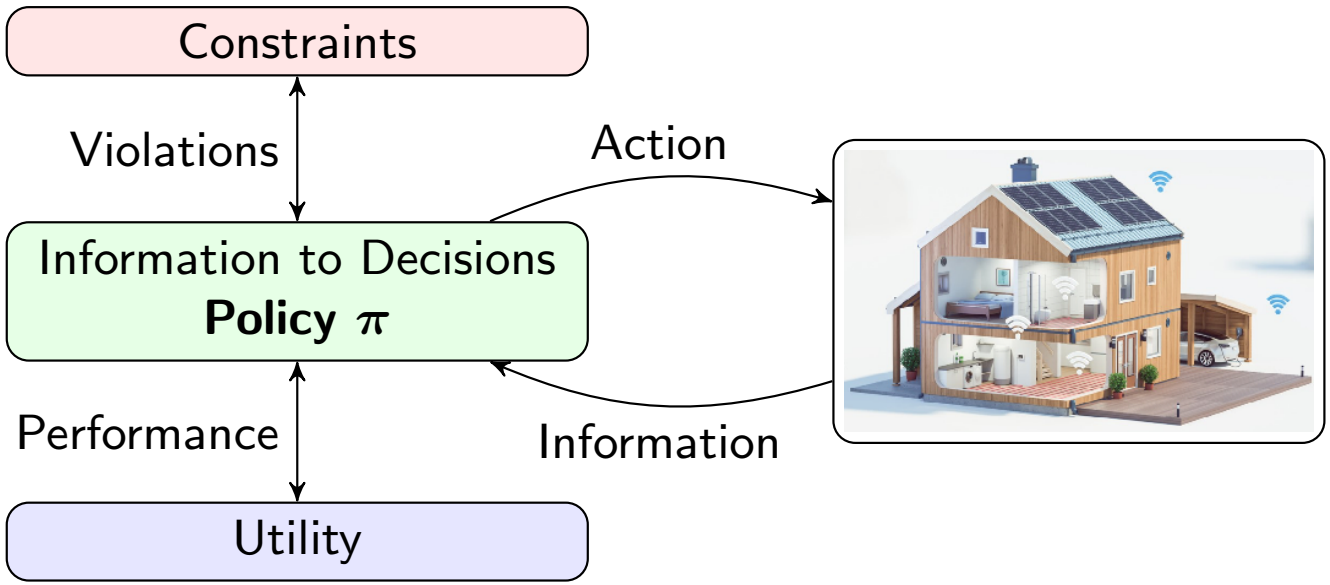

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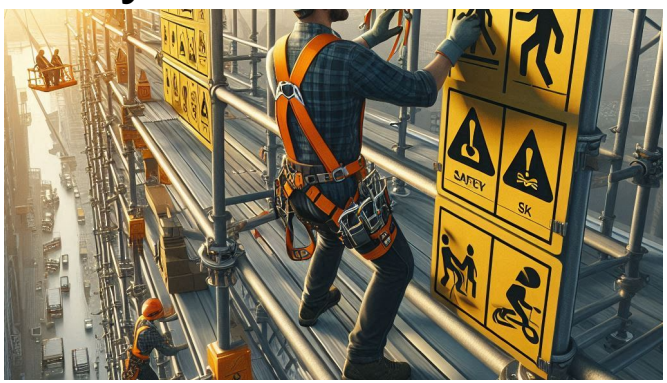

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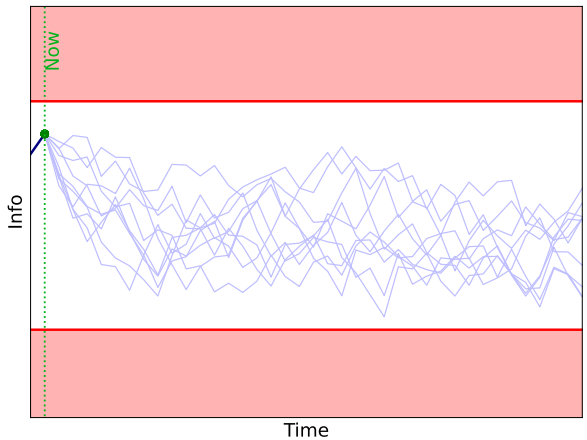

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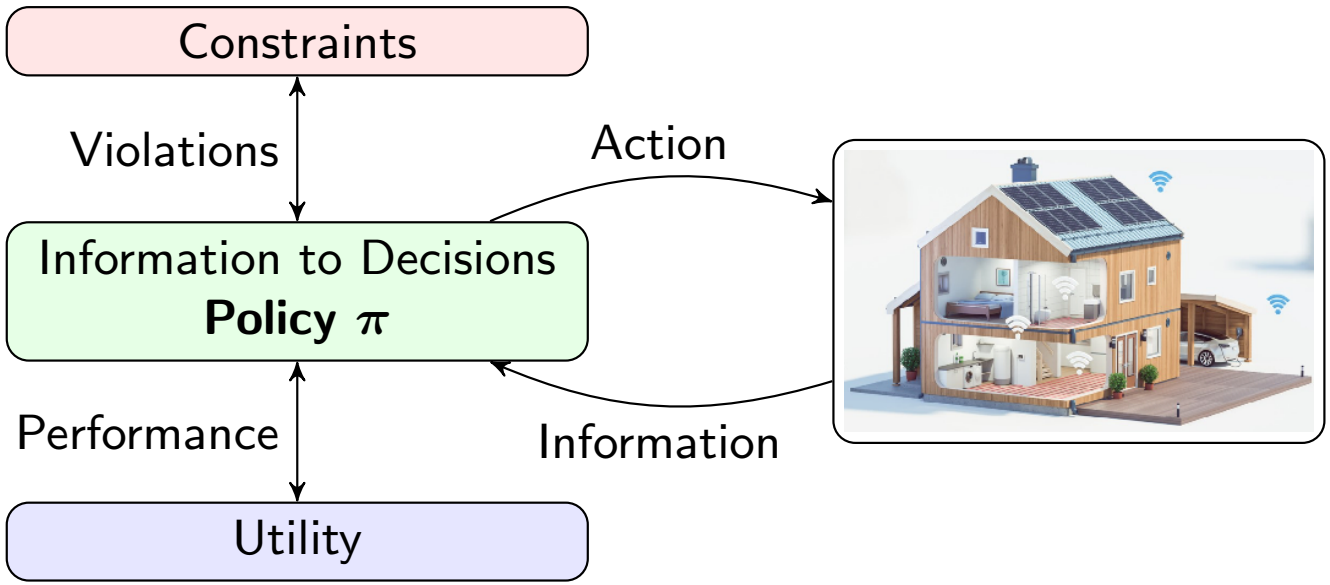

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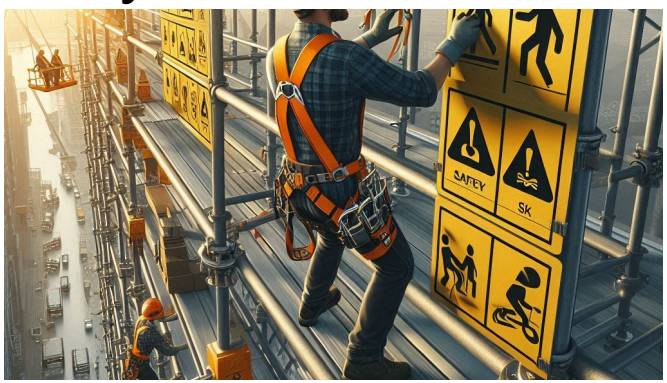

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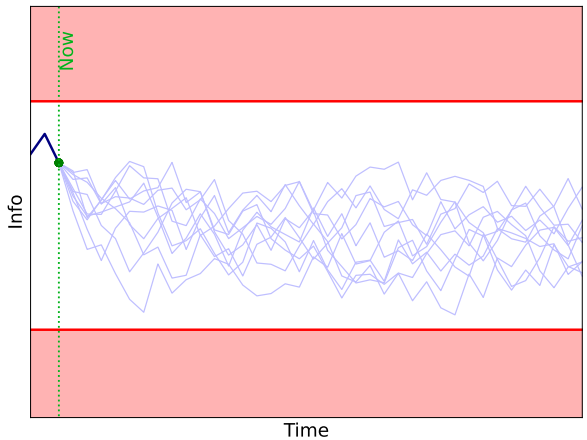

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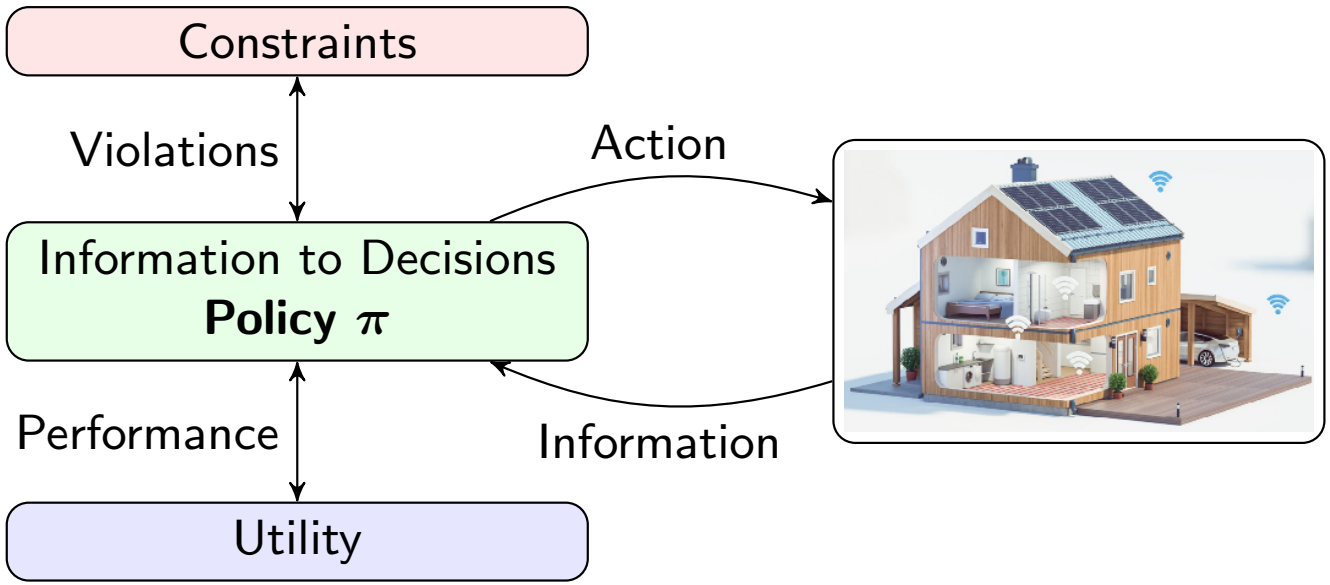

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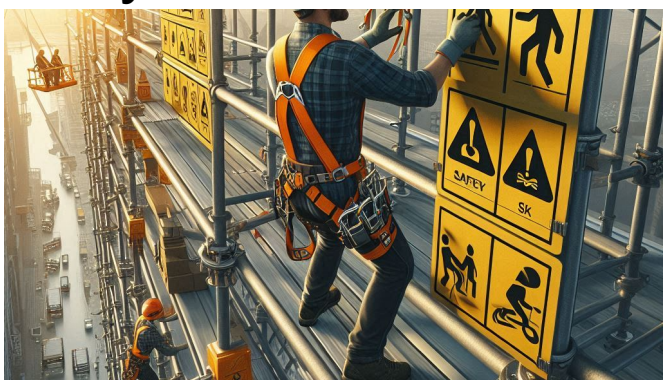

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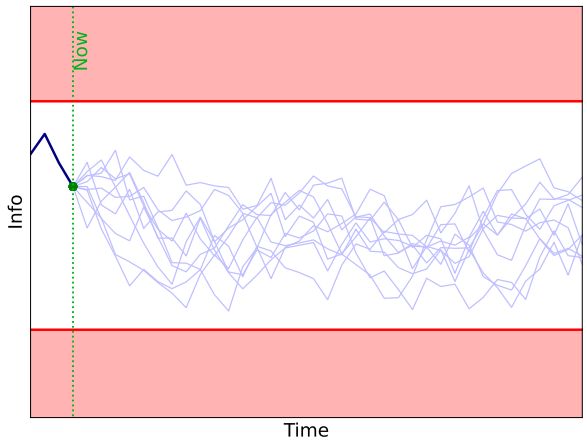

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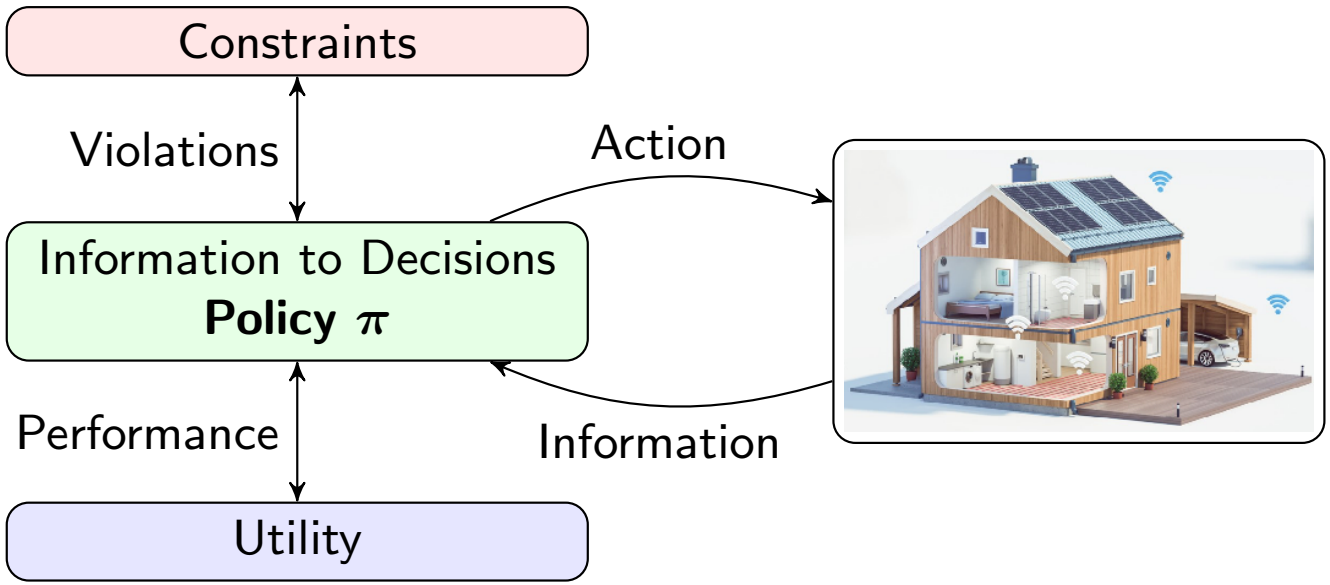

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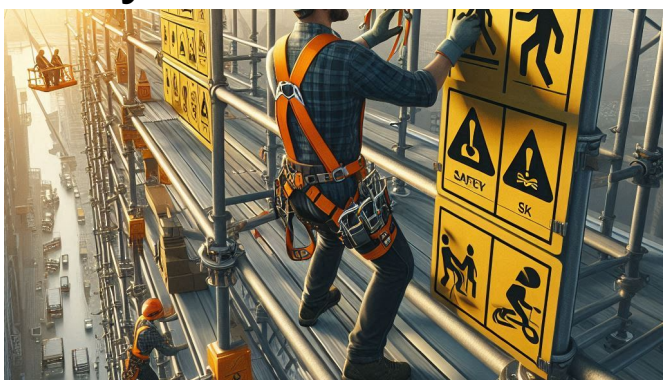

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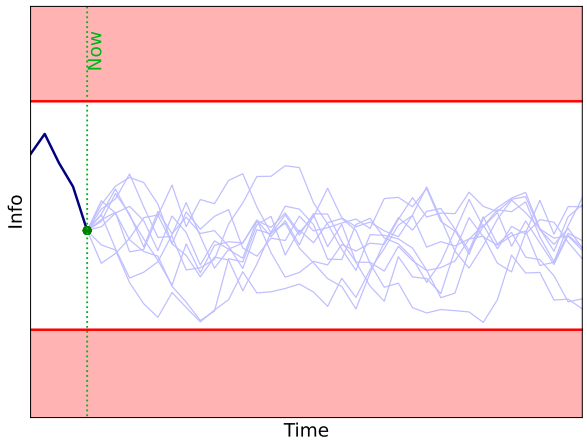

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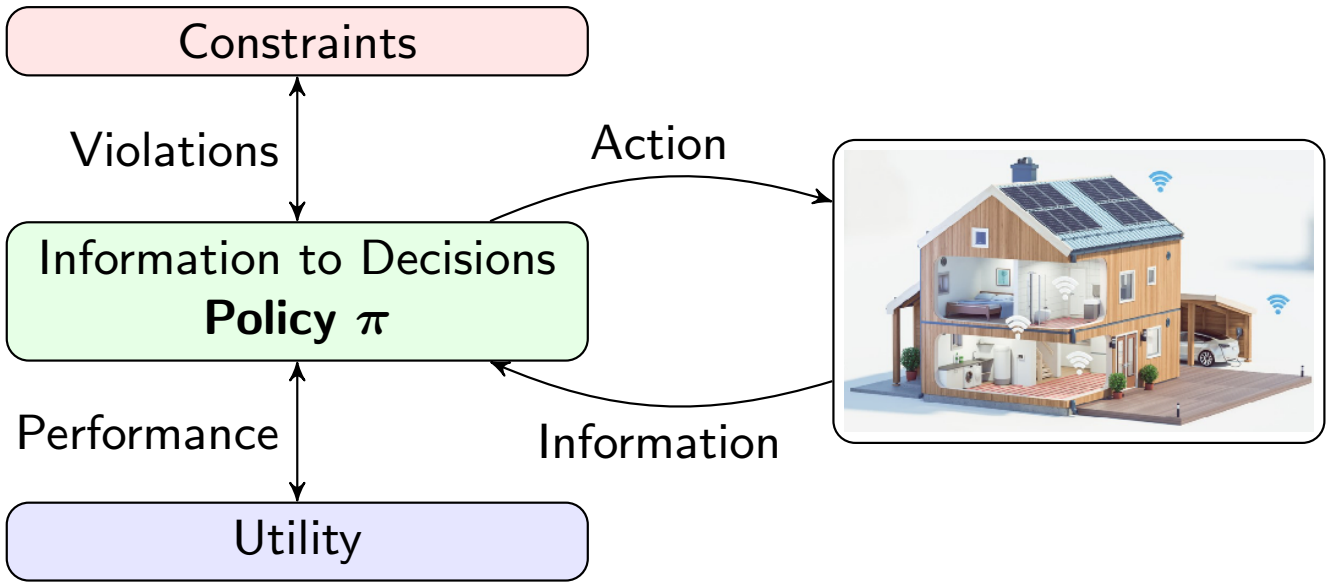

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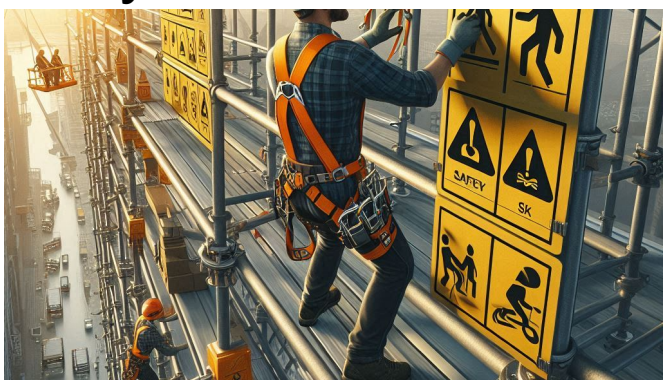

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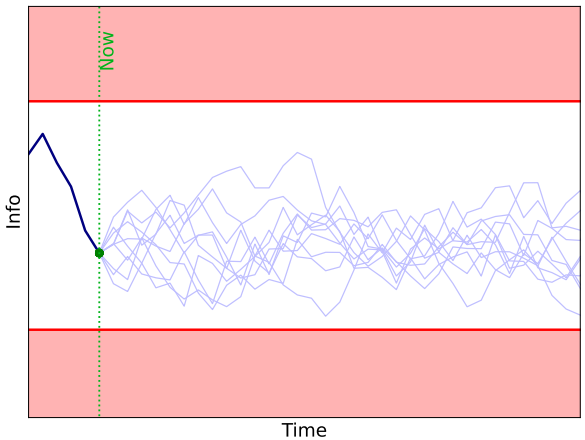

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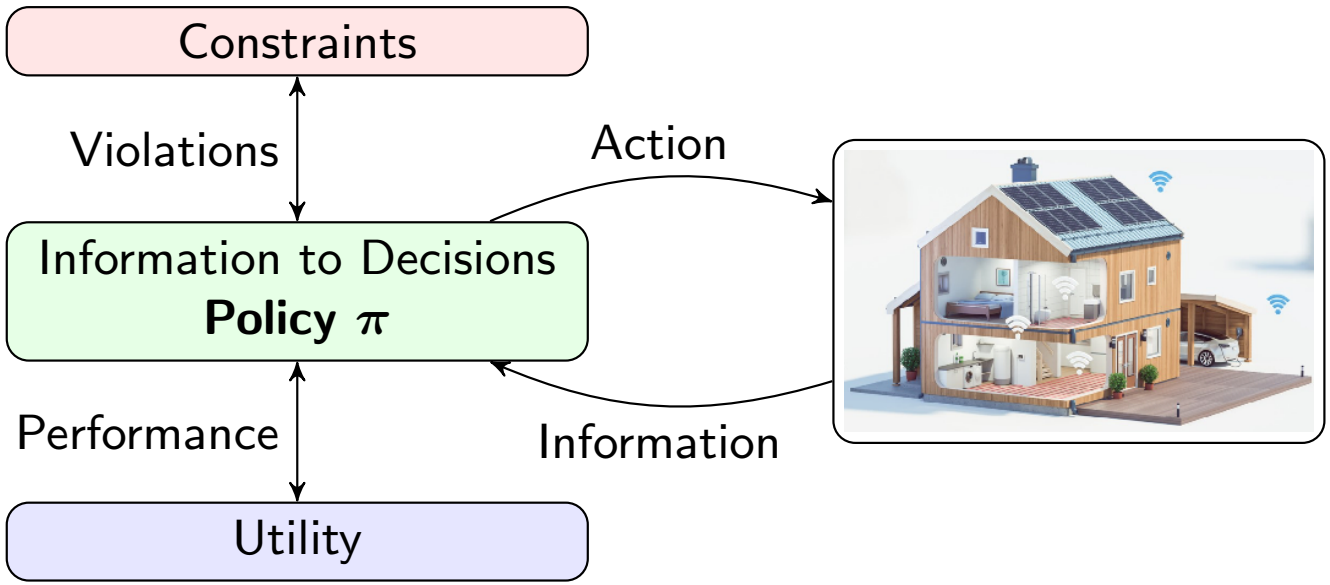

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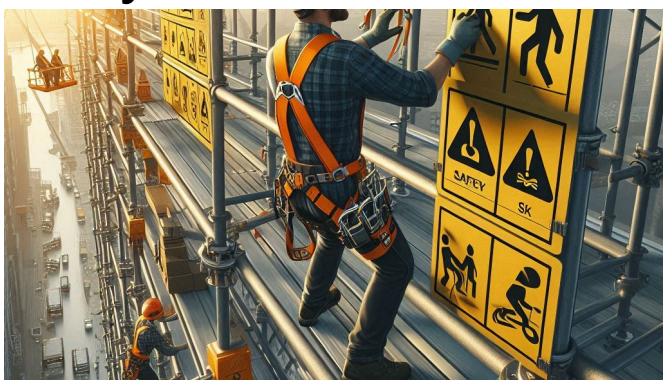

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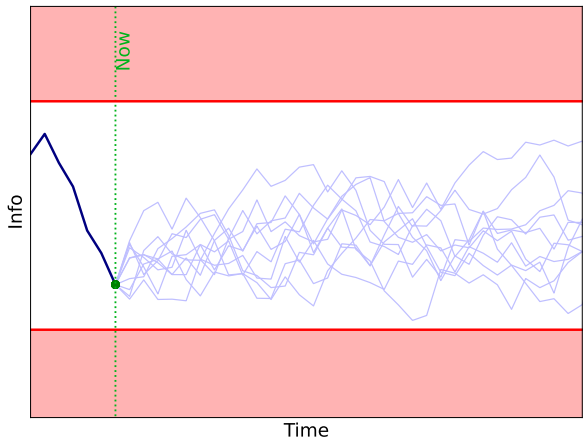

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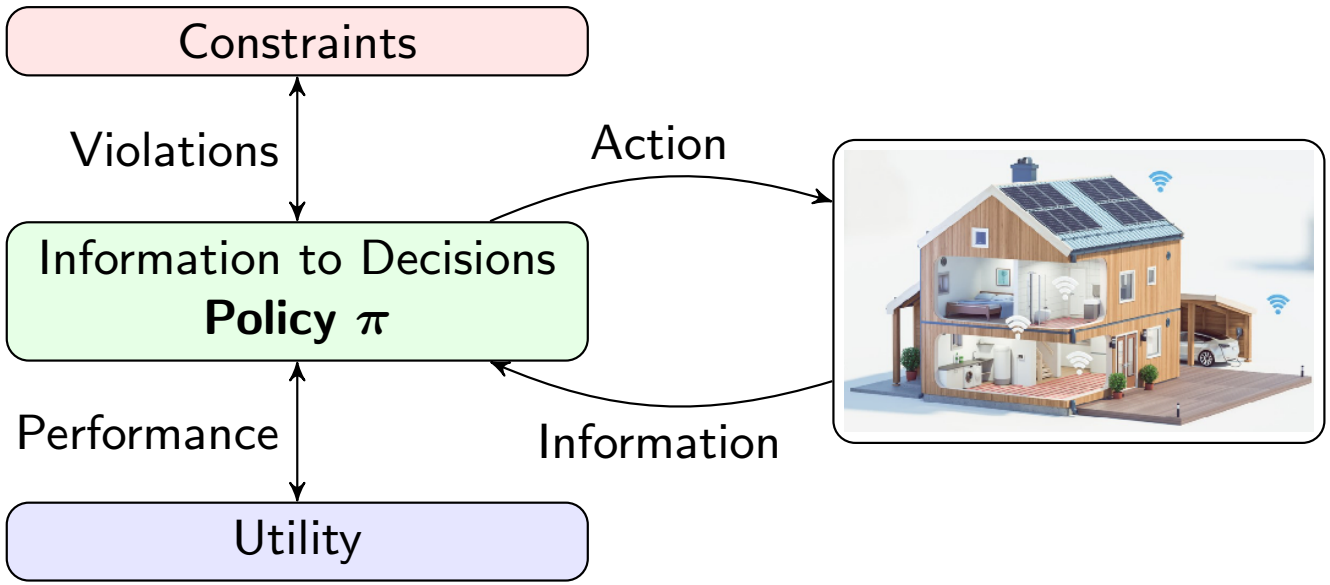

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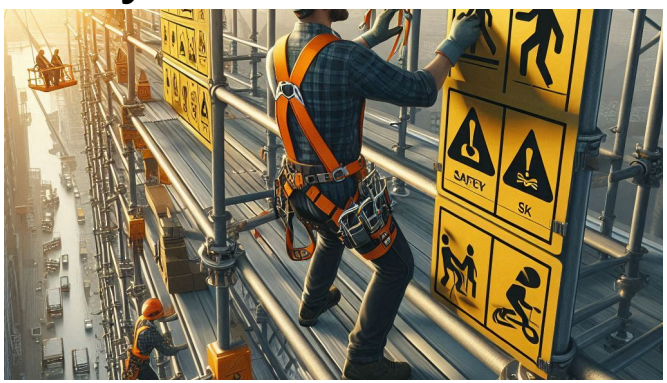

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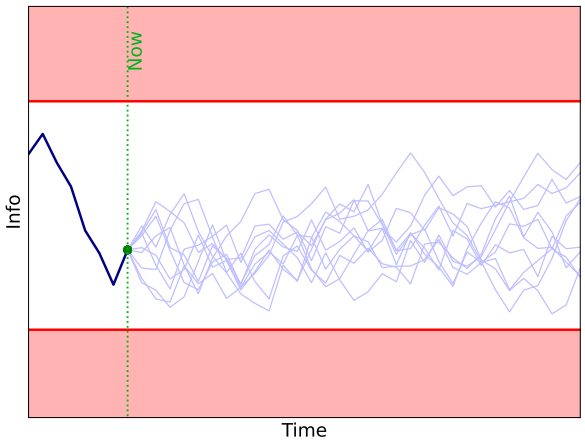
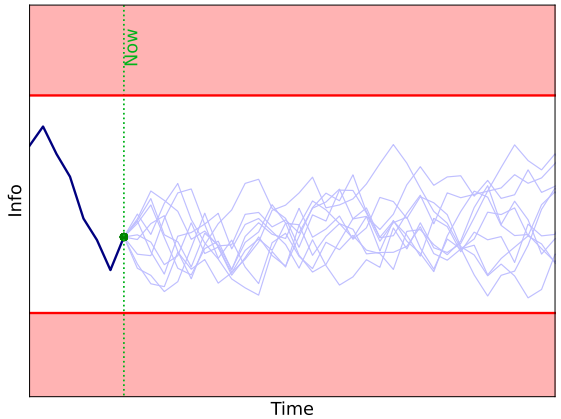


Constrained MDPs

In words

$$\boldsymbol{\pi}^\star = \arg\max_{\boldsymbol{\pi}} \quad \mathrm{E}\left[\sum_{\text{time}} \text{Utility}\right]$$

$$\text{s.t.} \quad \text{Constraints ok at all time}$$



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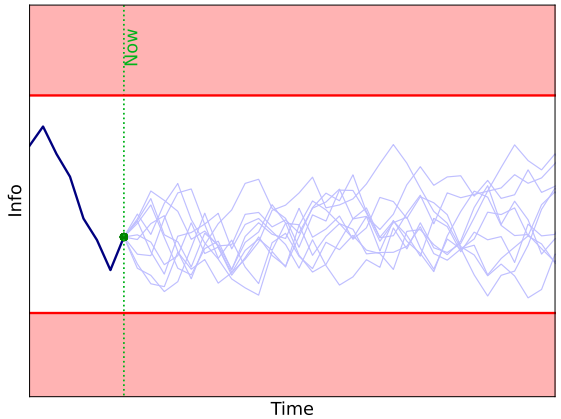

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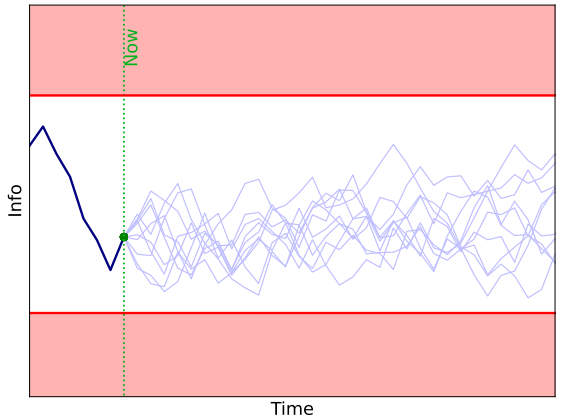

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Build policy using

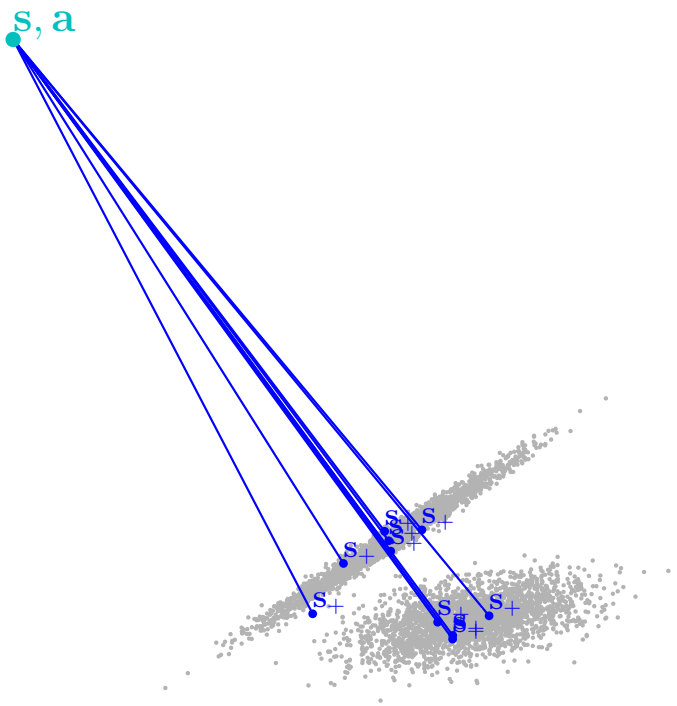
- Perfect model of the real world $\hat{\mathrm{P}}[\mathbf{s}_+|\mathbf{s},\mathbf{a}] = \mathrm{P}[\mathbf{s}_+|\mathbf{s},\mathbf{a}]$
- Model "pessimistic" about the uncertainties

... to evaluate "$\mathrm{E}[\cdot]$"

Pessimistic Models for Decision Making

- Model must "contain" the uncertainty
- "Container" (set) should be simple for computational reasons
- Trajectories predicted by pessimistic model will "cover" the real world
- Decision policy wants to be safe w.r.t. the "containers"

Distribution of one-step forward



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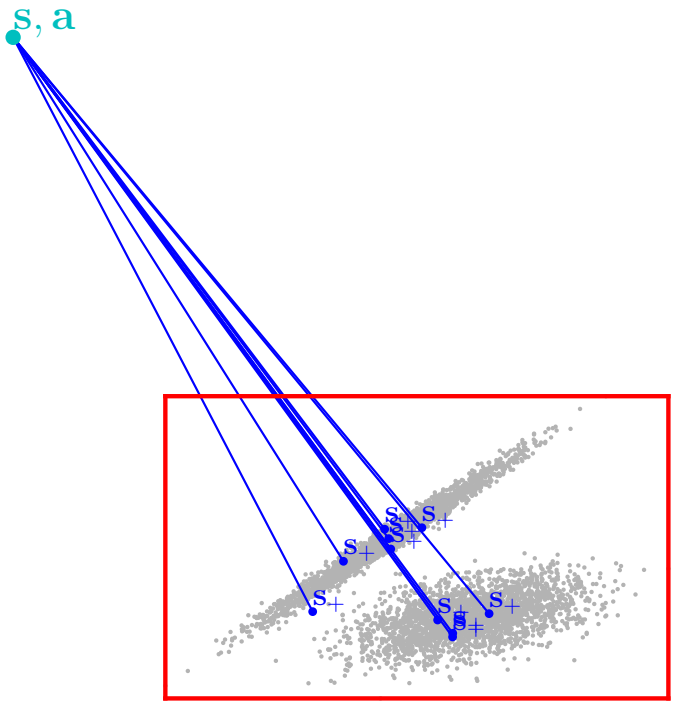
Distribution of one-step forward with "container"



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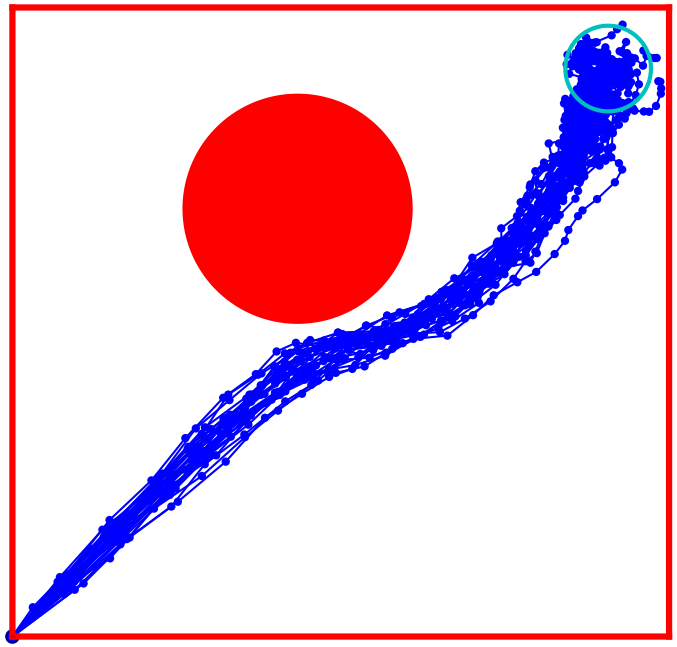
Trajectories



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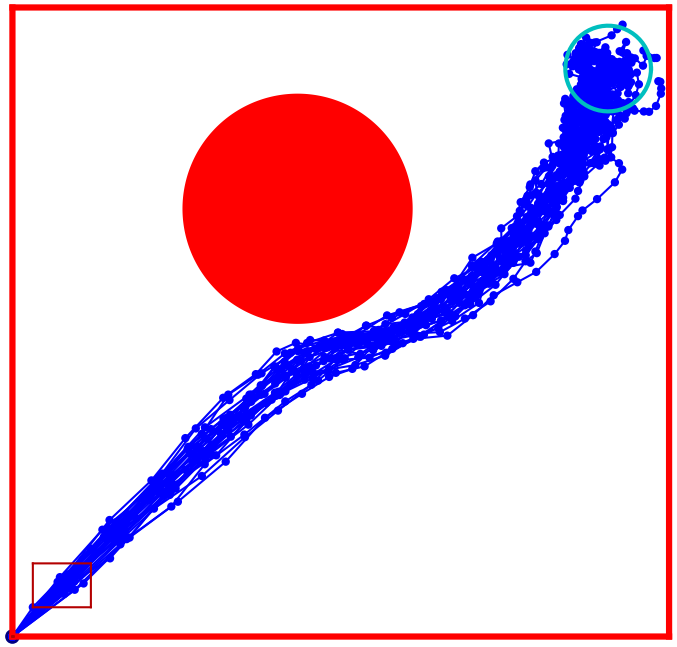

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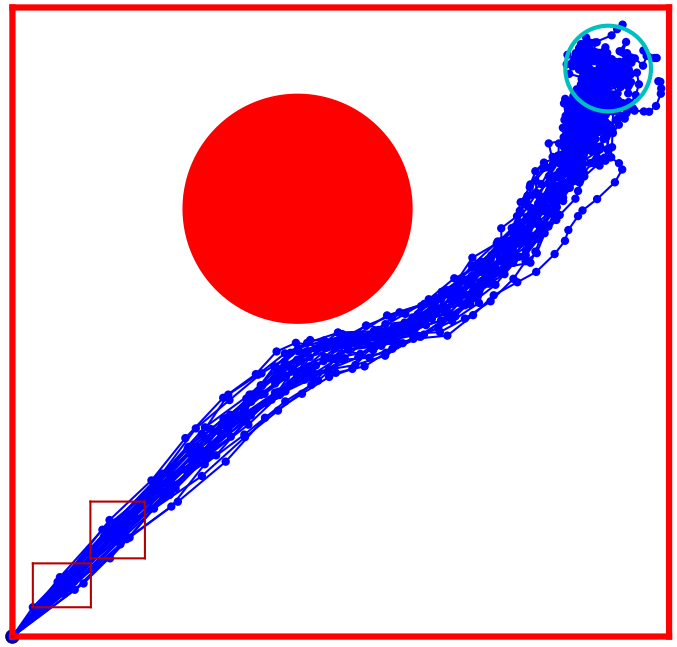

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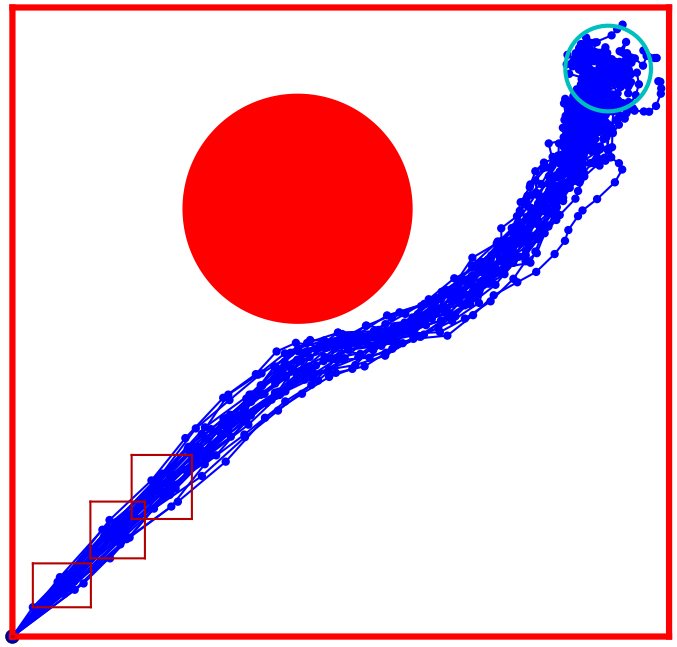

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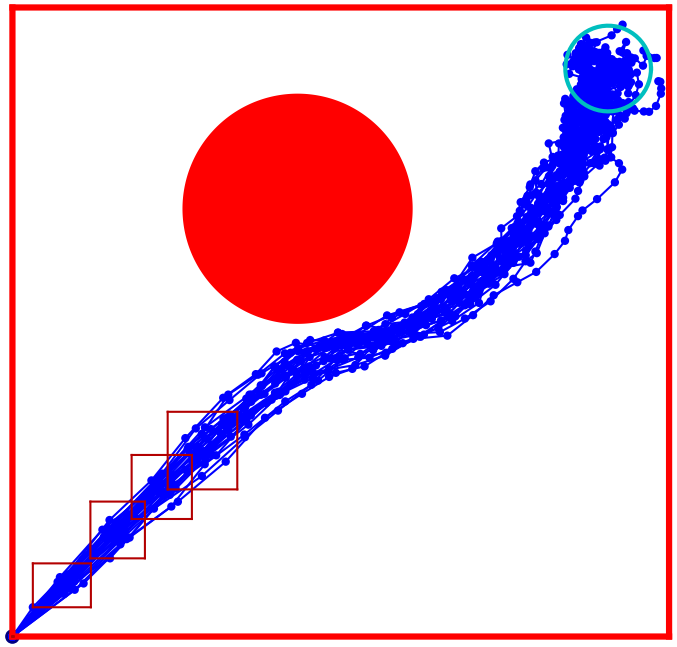

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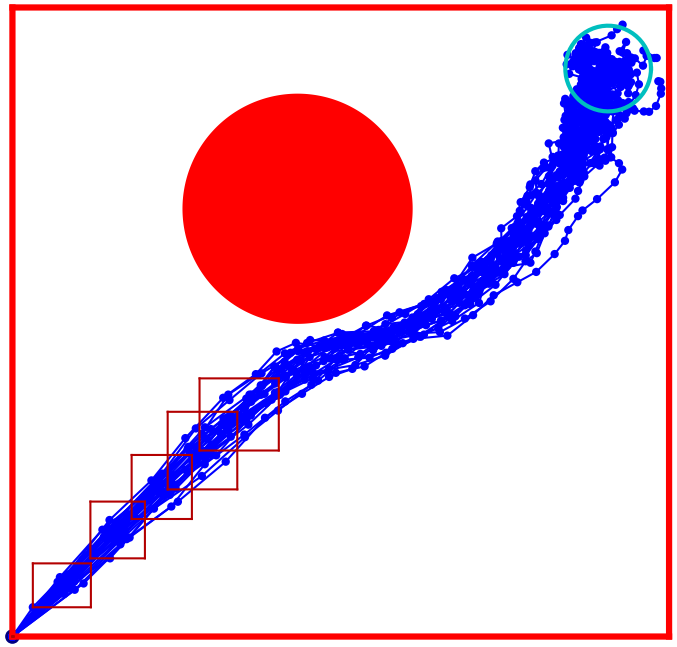

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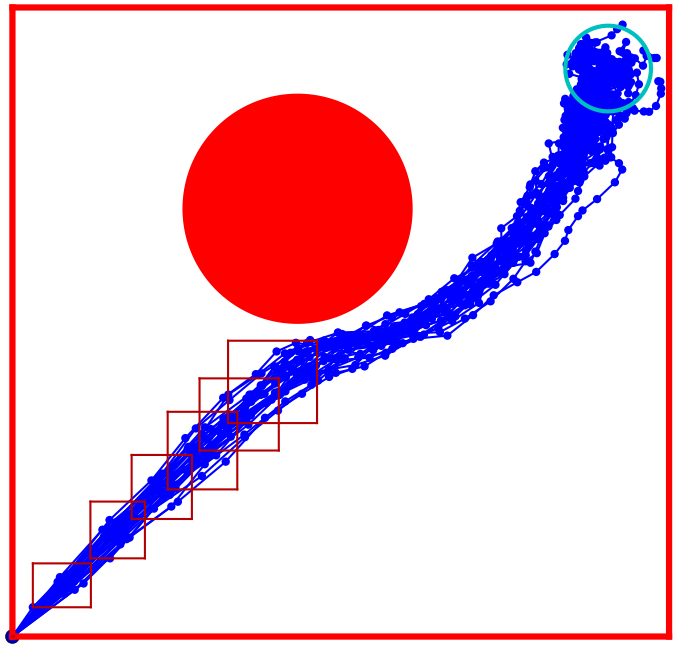

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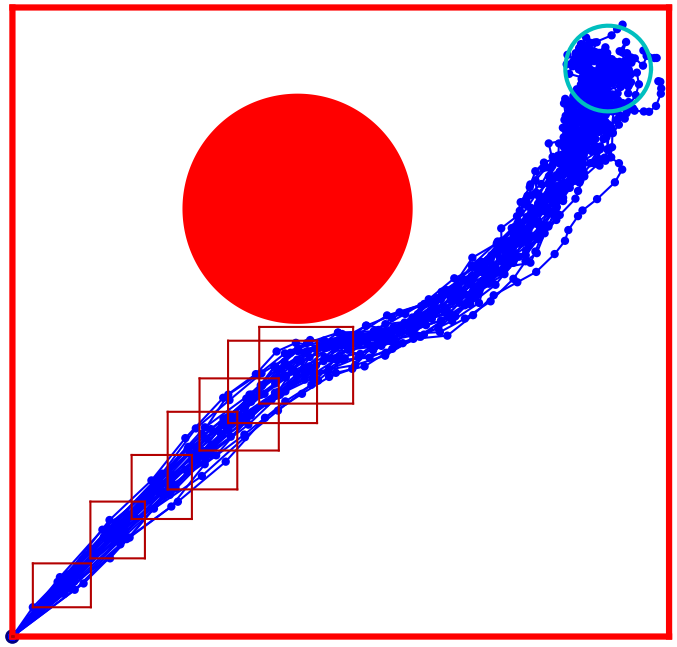

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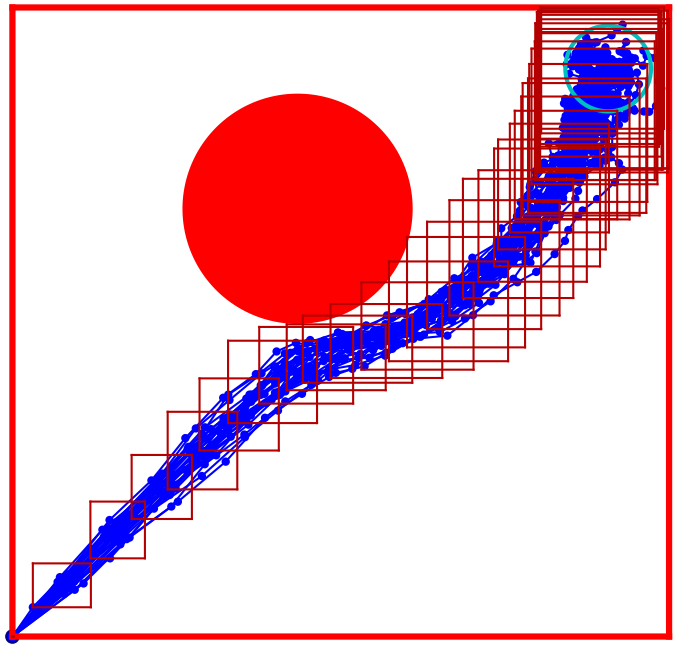
Trajectories with "containers"



Pessimistic Models for Decision Making

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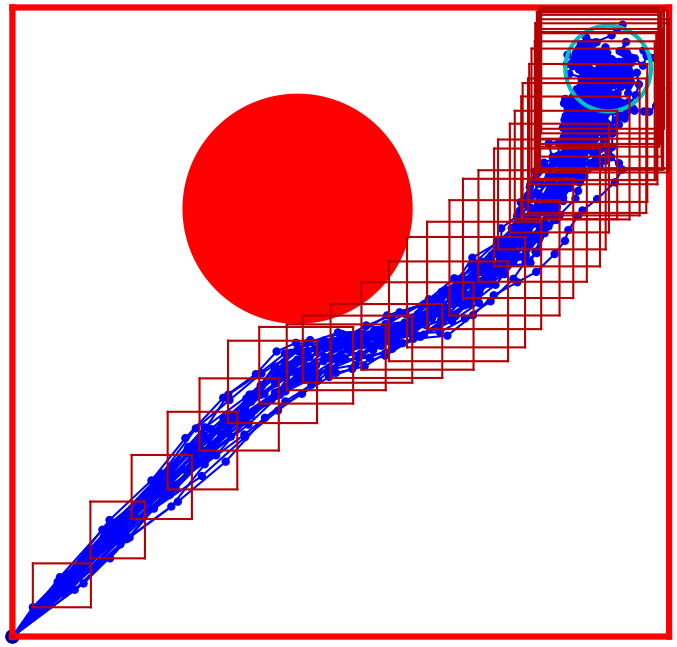

Remarks

- The propagation of the "containers" in the model predictions can be expensive / difficult
- Pessimistic propagations are usually needed $\rightarrow$ pessimistic over pessimistic
- Policy based on worst-case perspective makes the decisions highly conservative
- Often labelled "Robust" decision making

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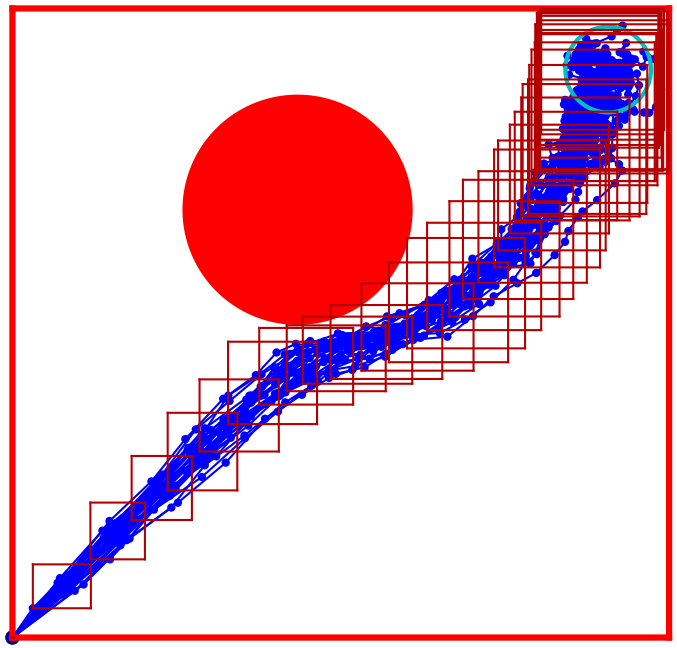

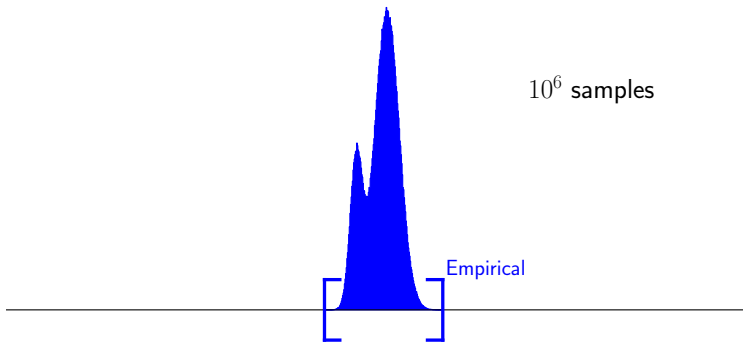

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- "theoretically" (from knowledge), but can be impractically pessimistic

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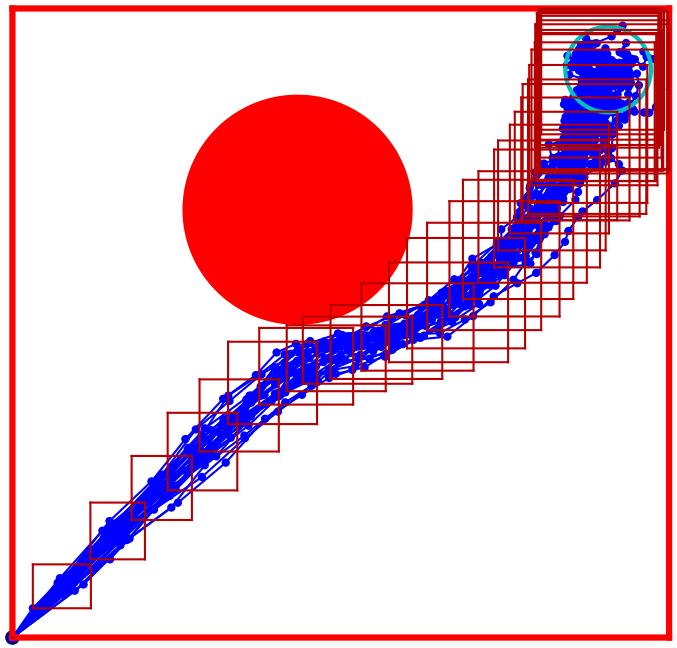
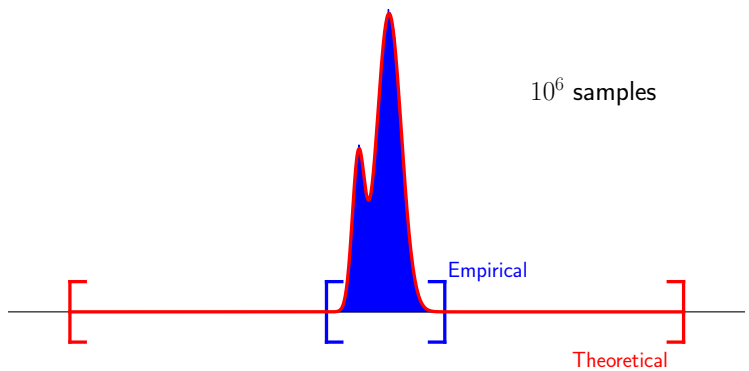


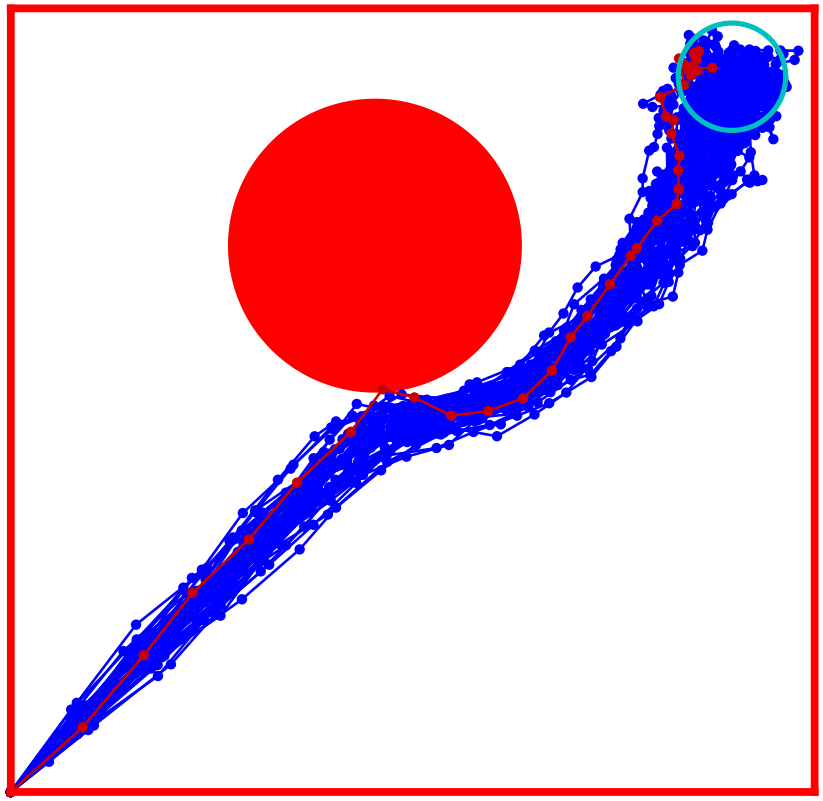


MDPs with probabilistic safety

In words

$$\pi^\star = \arg\max_{\pi} \quad \mathrm{E}\left[\sum_{\text{time}} \text{Utility}\right]$$

$$\text{s.t.} \quad \text{Probability of no violation} \geq c$$



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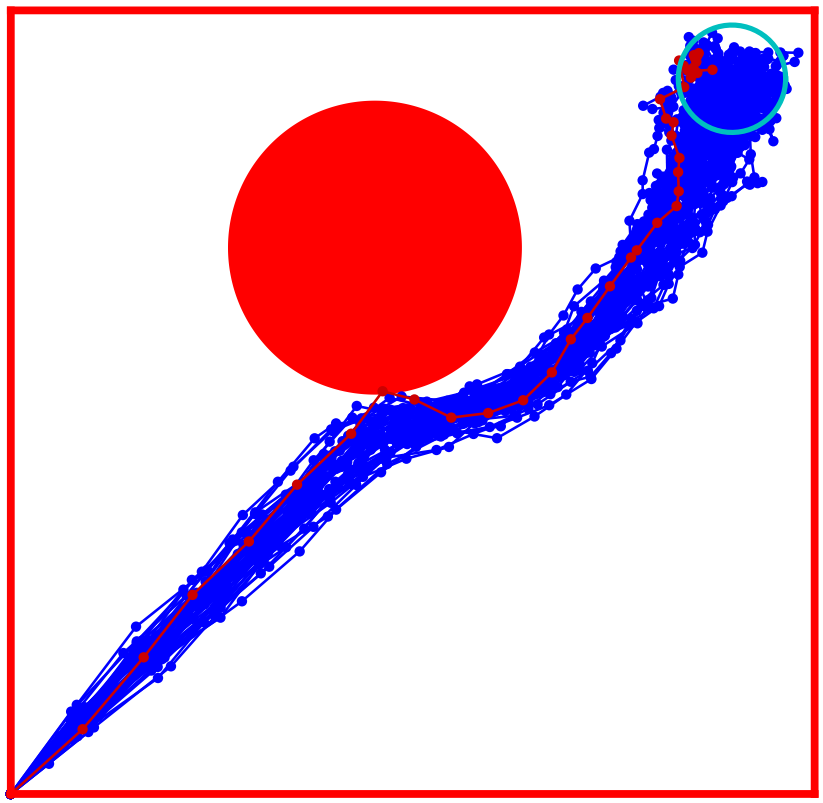

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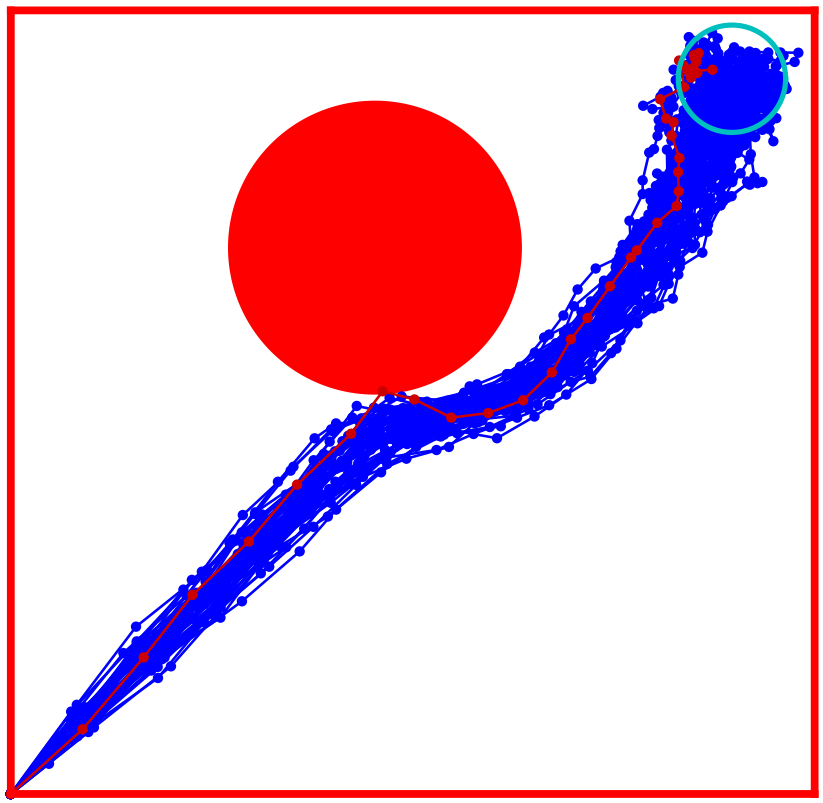
Remarks:

- If we can tolerate $c > 0$ (small) it can make a big gain in performance
- Aligned with industrial / practical standards on "large series"
- Problem needs a "termination" (time or goal reached)
- Building decisions can be difficult from a computational point of view



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- Instead of constraints, the utility function itself is altered to contain a risk measure of the return.
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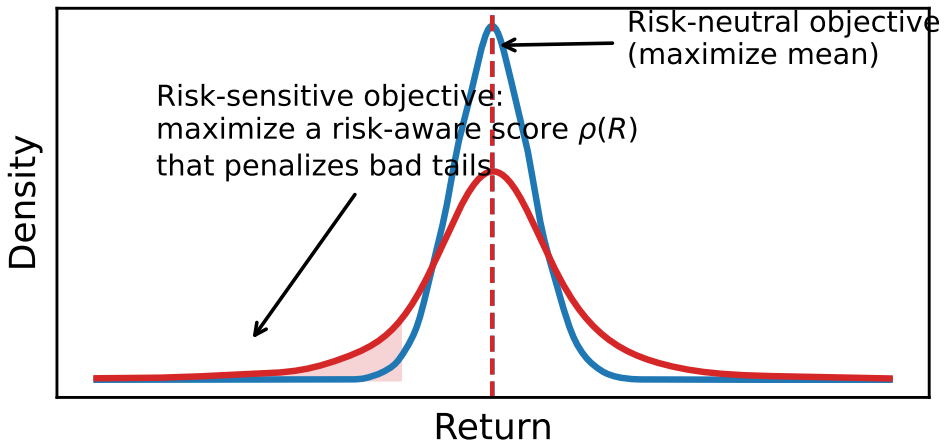

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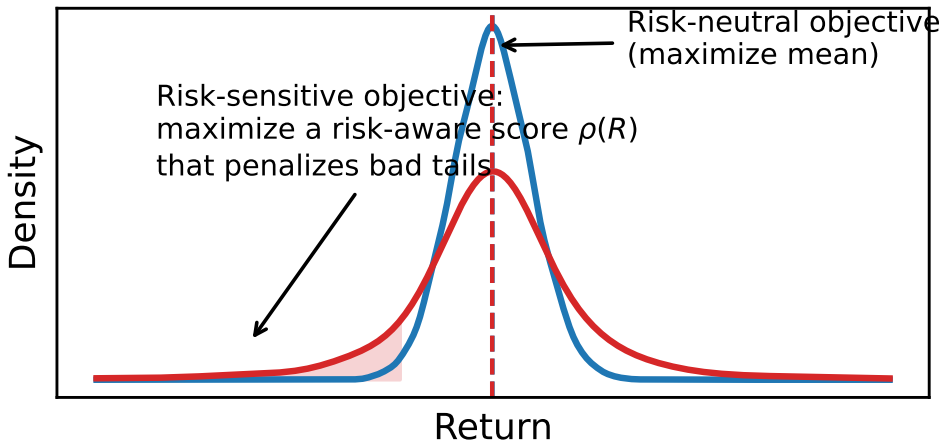

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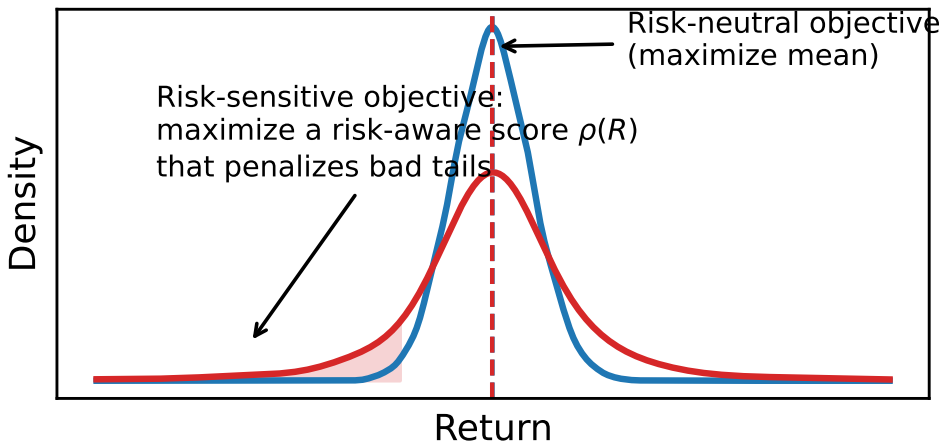

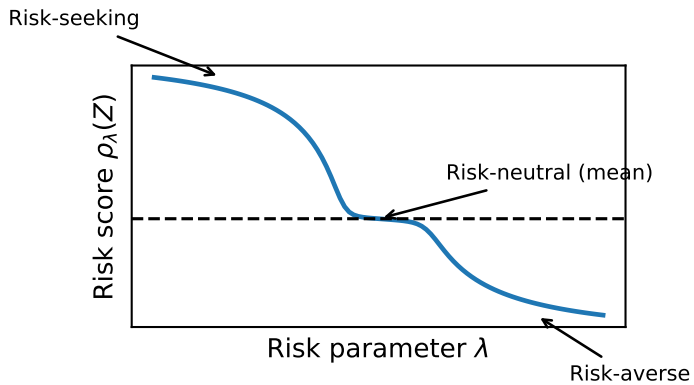

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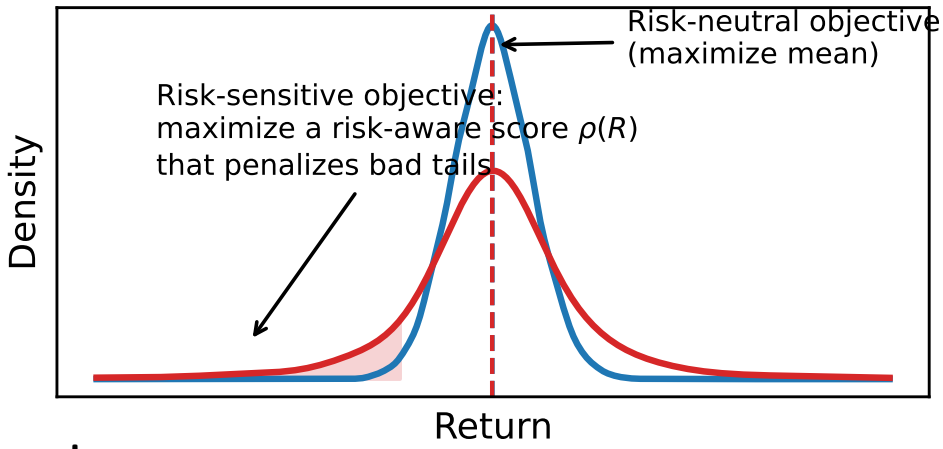

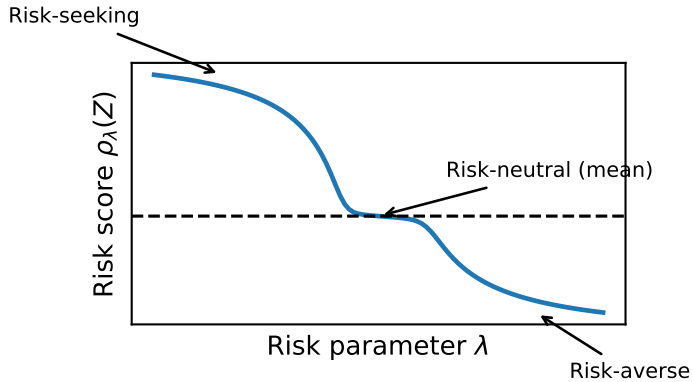

Remarks:

- Views risk from through the lense of utility (not undesirable events).
- Risk-sensitive criteria do *not* guarantee a bound on violation.
- Cares only about the average severity of bad outcomes in the tail and not about the rest.

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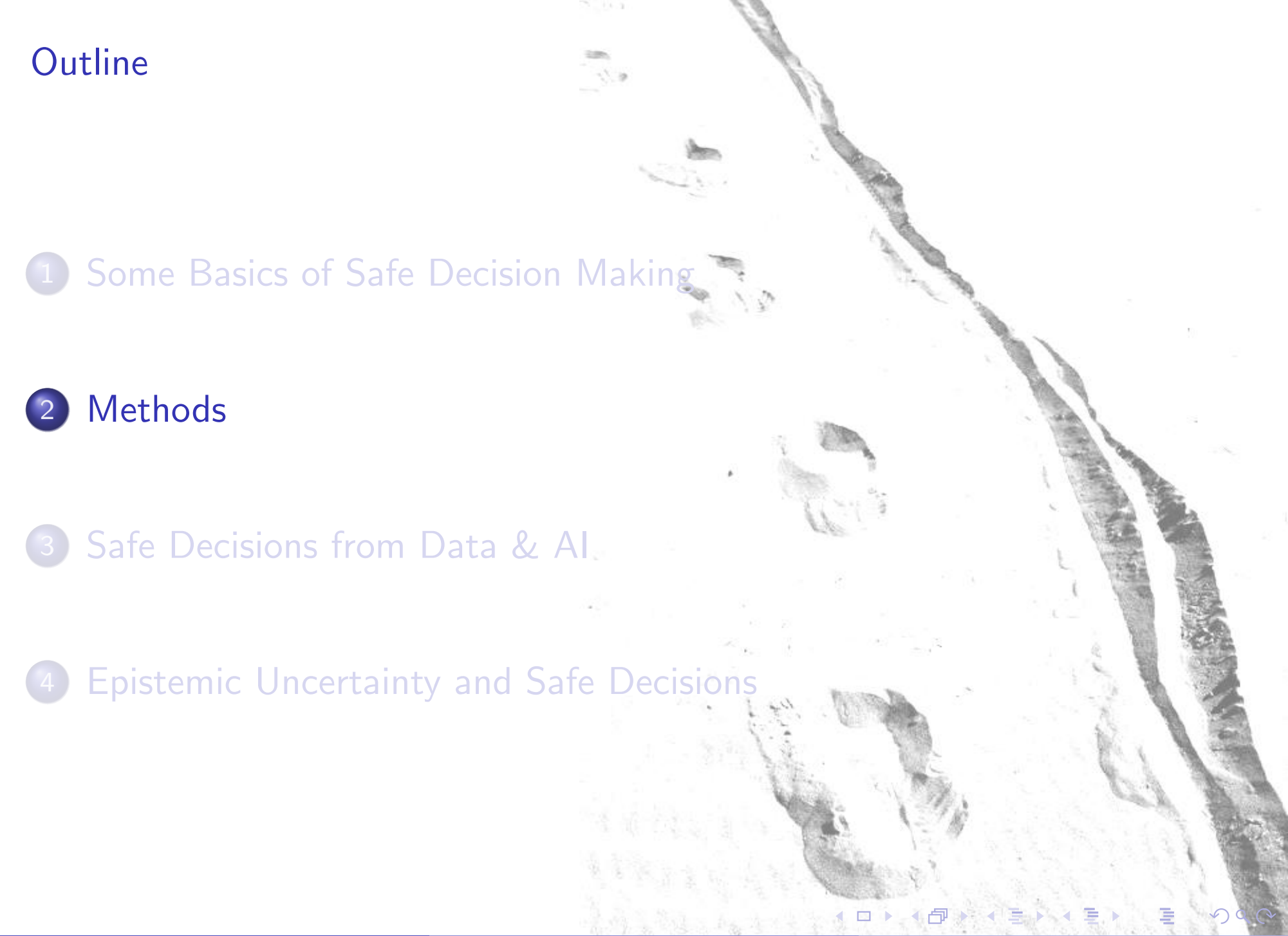
2. Methods
3. Safe Decisions from Data & AI
4. Epistemic Uncertainty and Safe Decisions



Robust Repeated Planning

- Introduction: Robust MPC, Scenario Trees, MC, robust multi-stage stochastic programing, MPPI?
- Difficulties: guarantees for nonlinear systems, persistent safety (recursive feasibility)

TODOTODOTODO

Dynamic Programming & RL with Probabilistic Safety

$$\boldsymbol{\pi}^\star = \arg\max_{\boldsymbol{\pi}} \quad \mathrm{E}\left[\sum_{k=0}^{\infty} \gamma^k L\left(\mathbf{s}_k, \mathbf{a}_k\right)\right]$$
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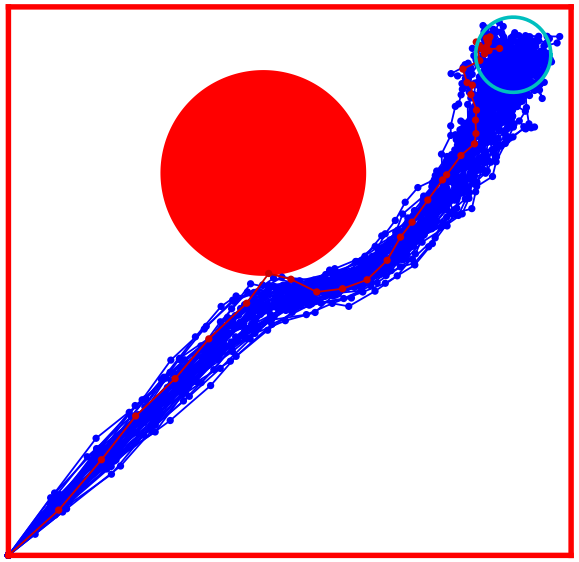

Expected value form

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$$\text{s.t.} \quad \mathrm{E}\left[\begin{array}{ll} 1 & \text{if no violation occurred} \\ 0 & \text{otherwise} \end{array}\right] \geq c$$

- Expected value form enables classical techniques (ref. 1st lecture), i.e. DP and RL
- Difficulties: estimate expected values (sample based) when $\mathrm{P}\left[\mathbf{s}_{0,\ldots\infty} \in \mathbb{S}\right]$ is close to 1. Illustrate?? SG will try on 11.9

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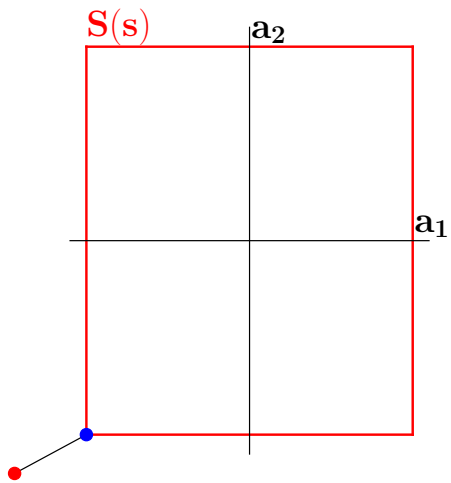

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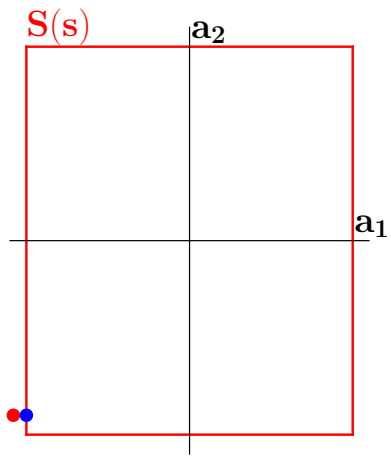

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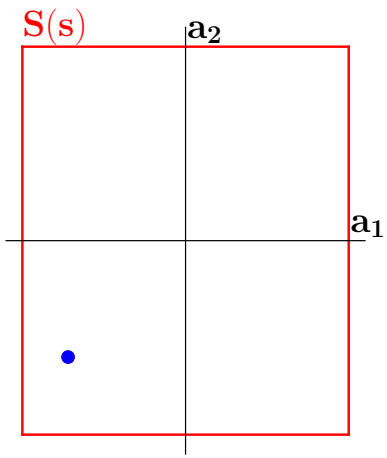

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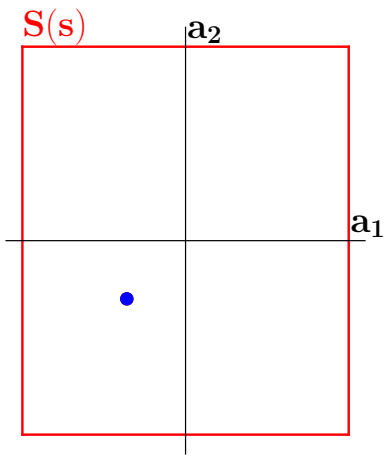

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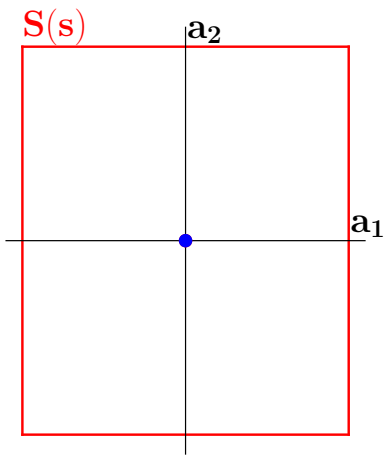

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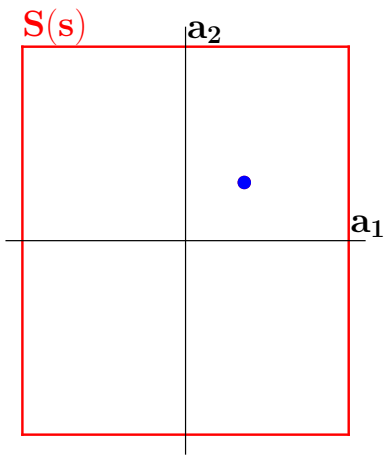

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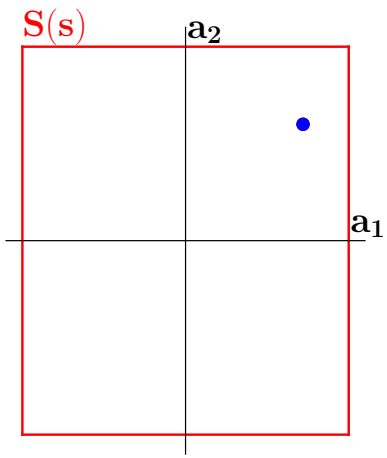

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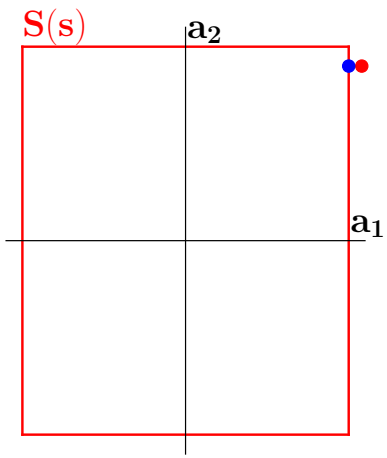

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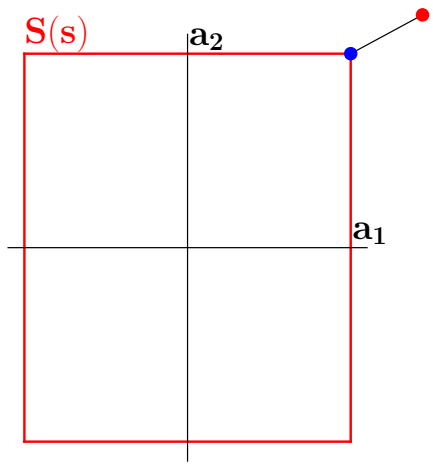

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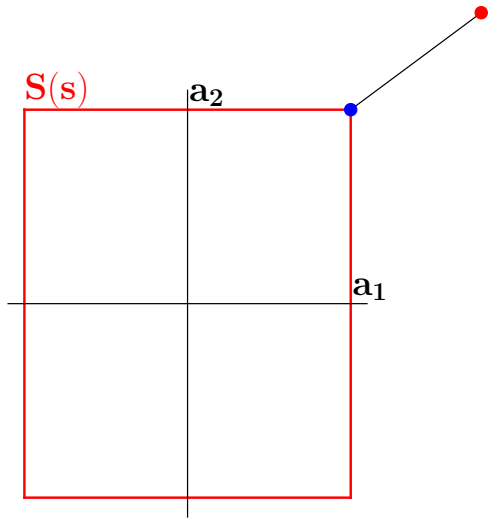

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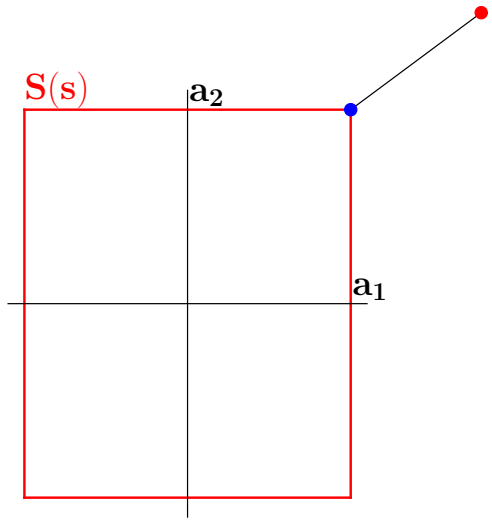
Remark: "oracle" can be very hard to build, or not...

Not losing the king



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$$\boldsymbol{\pi}^{\star} = \arg\max_{\boldsymbol{\pi}} \quad \mathbf{E}\left[\sum_{\text{time}} \text{Utility}\right]$$

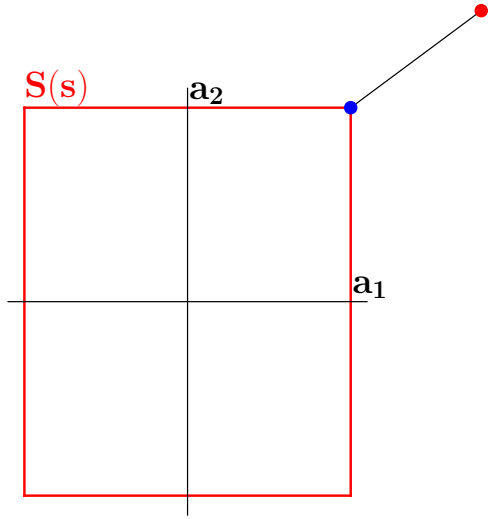
that focuses only on utility

- For state $\mathbf{s}$, take the decisions using: $\text{Action} = \text{Projection}_{\mathbf{S}(\mathbf{s})}\left[\boldsymbol{\pi}^{\star}(\mathbf{s})\right]$

Remark: "oracle" can be very hard to build, or not...

Not losing the king

Able to stop within the visible distance



Safety Filters & Reinforcement Learning

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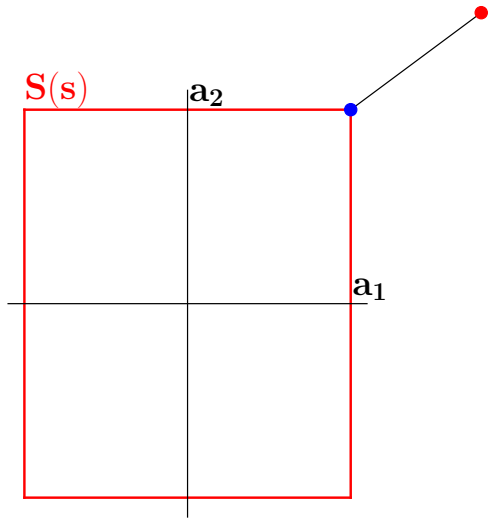
Remarks:

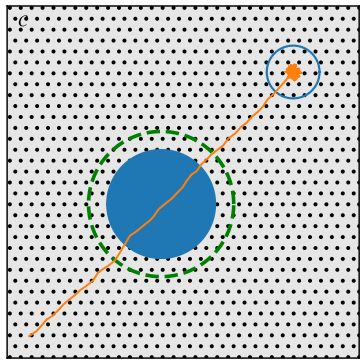
- Often close to practice, e.g., flight envelop protection in modern airplane, semi-autonomous driving in cars, etc
- Oracle $\Rightarrow$ knowledge-based & conservative!



Control Barrier Functions (CBFs)

In words

- CBFs provide a formal, model-based way to build the "oracle" safe set.
- Use a barrier function $h(x)$ so that staying safe means $h(x) \geq 0$.
- At each step, if the proposed action would leave the safe set, a small QP minimally adjusts it to be safe.



Unsafe trajectory: violates $h(x) \geq 0$.

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Formally

$$\dot{x} = f(x) + g(x)u, \qquad \mathcal{C} = \{x : h(x) \geq 0\}$$

$$u^\star(x) = \arg\min_u \tfrac{1}{2}\|u - u_{\text{des}}(x)\|_2^2 \quad \text{s.t.}$$

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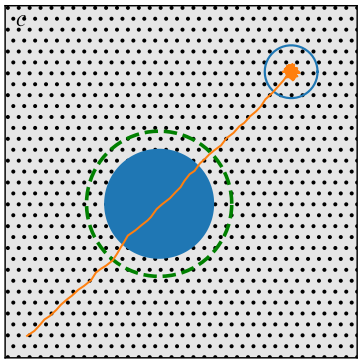

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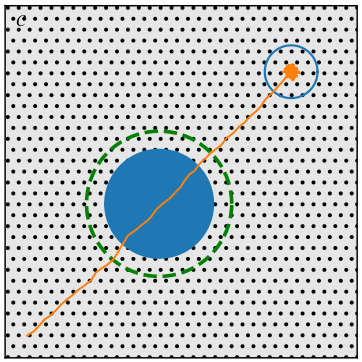

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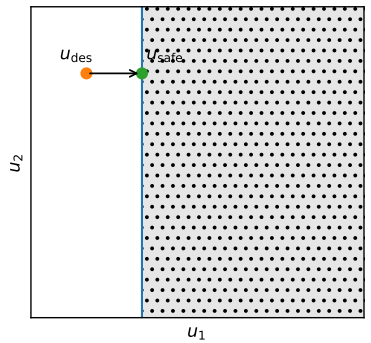

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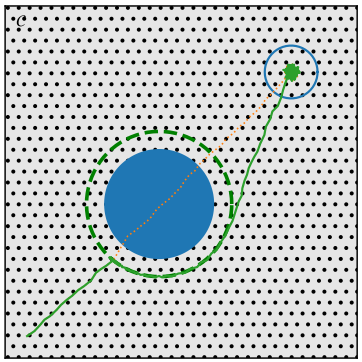
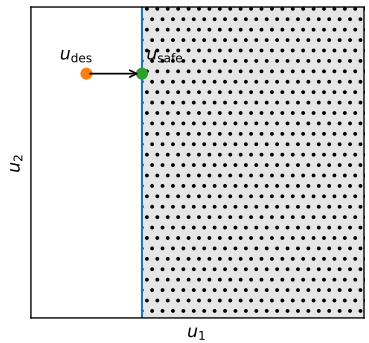


CBF-filtered trajectory.



CBF filter: project u_{des} to feasible u_{safe}.

- The barrier function and safe set $\mathcal{C}$ are typically **constructed from domain knowledge** (physics, rules, safety envelopes).
- Requires a (possibly simplified, conservative) **system model**: **robust** approach

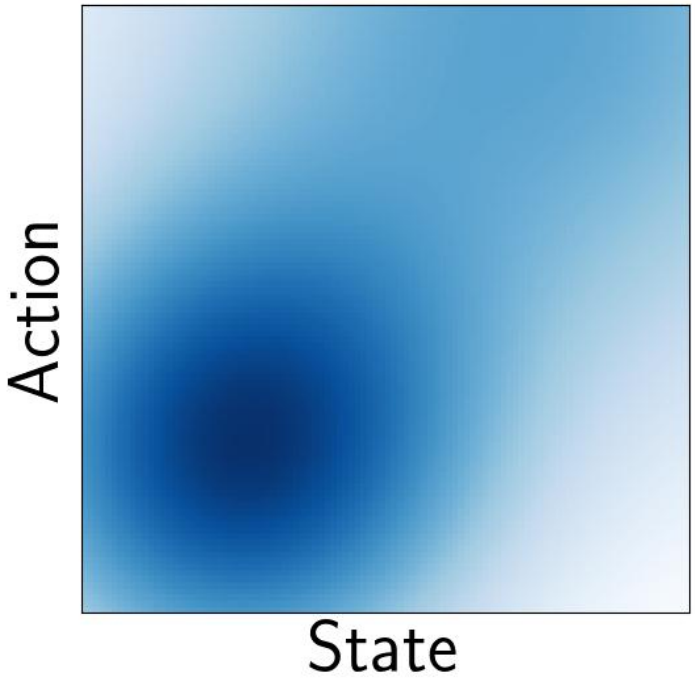
CVaR

Akhil can do on 11.9

Outline

1. Some Basics of Safe Decision Making
2. Methods
3. Safe Decisions from Data & AI
4. Epistemic Uncertainty and Safe Decisions

Data & Epistemic Uncertainty



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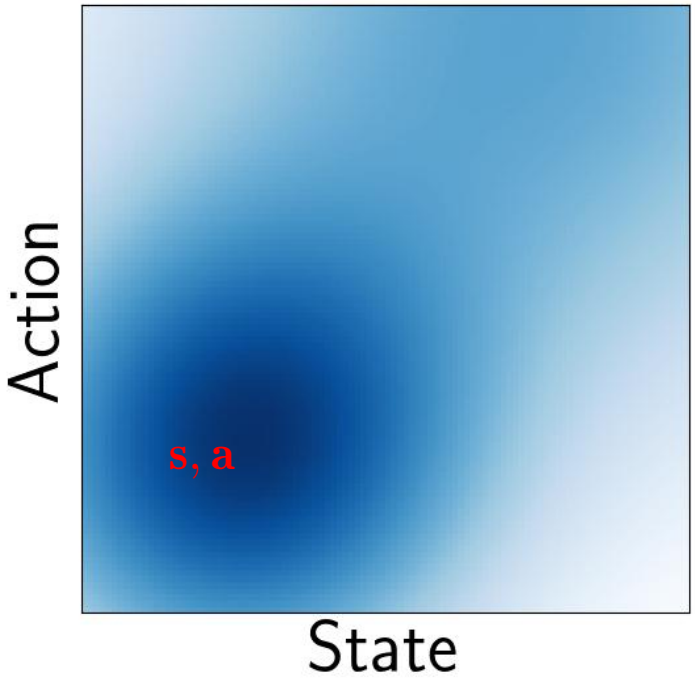

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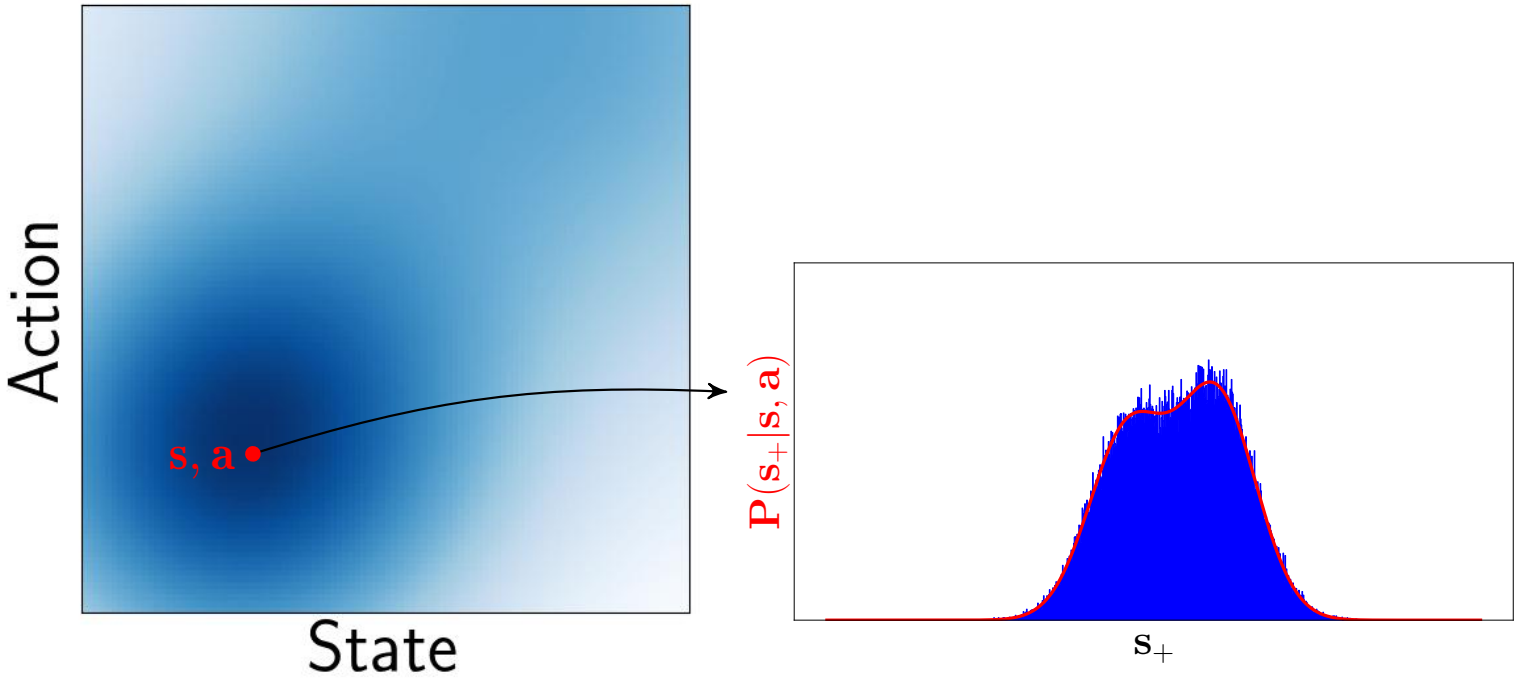

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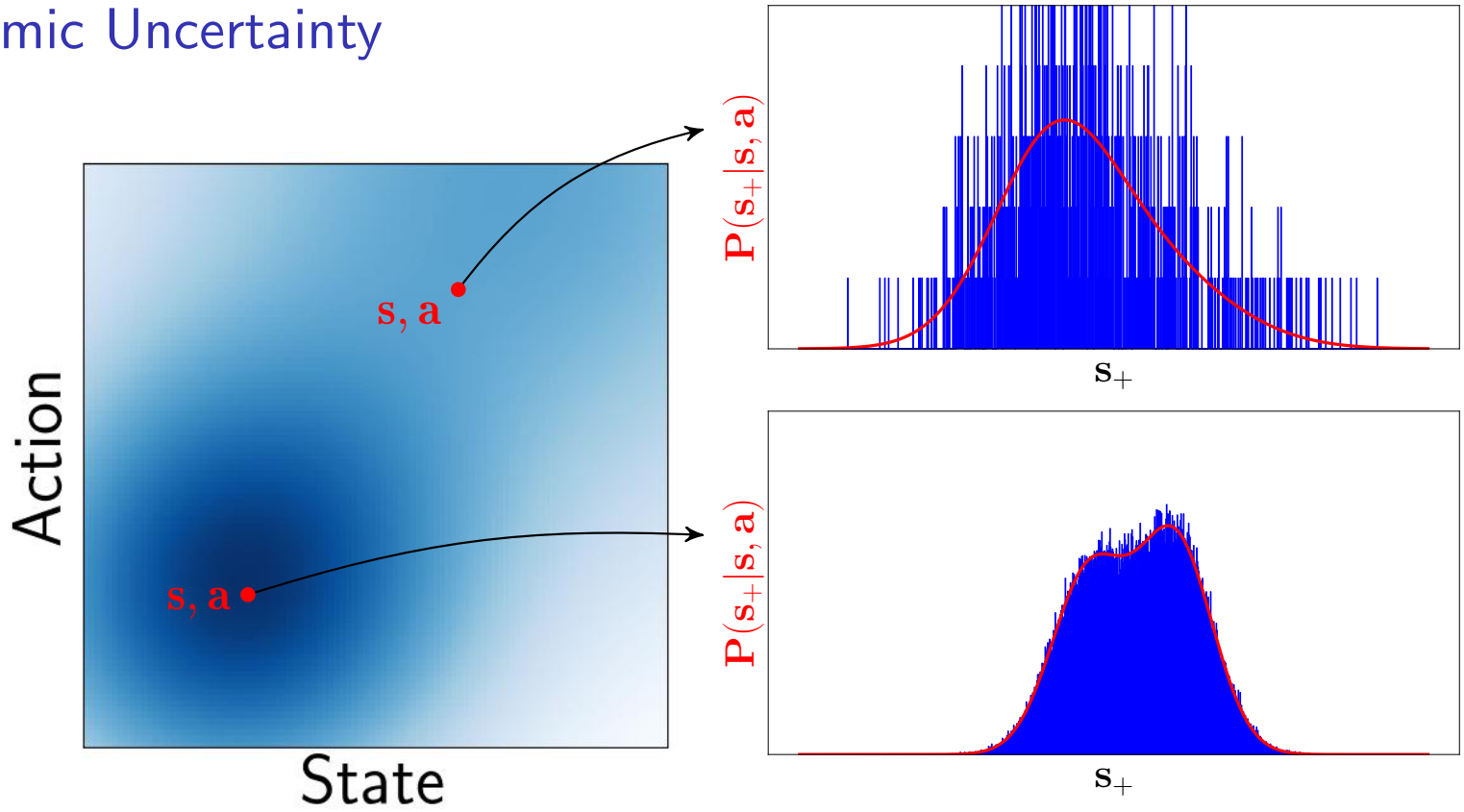

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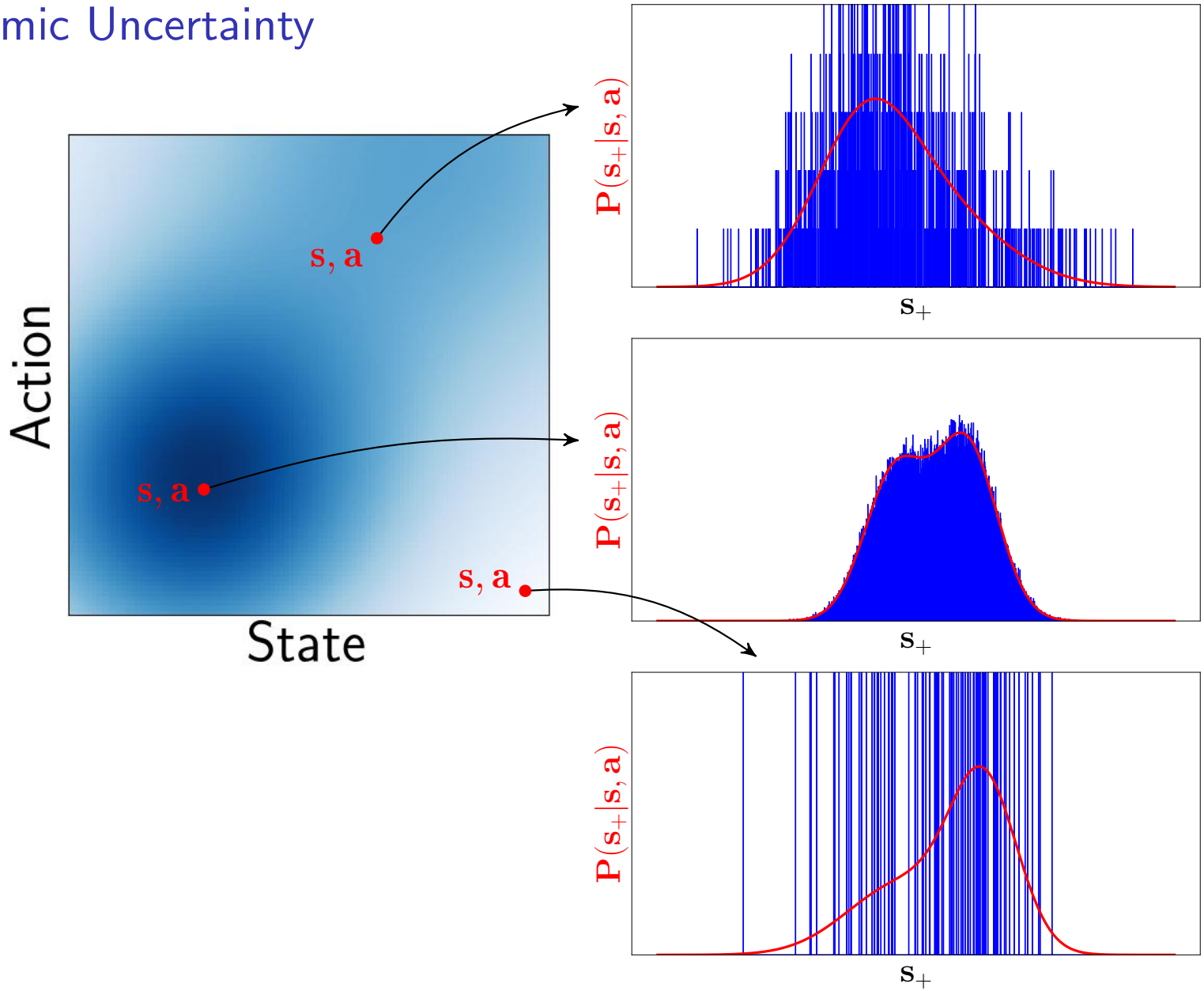

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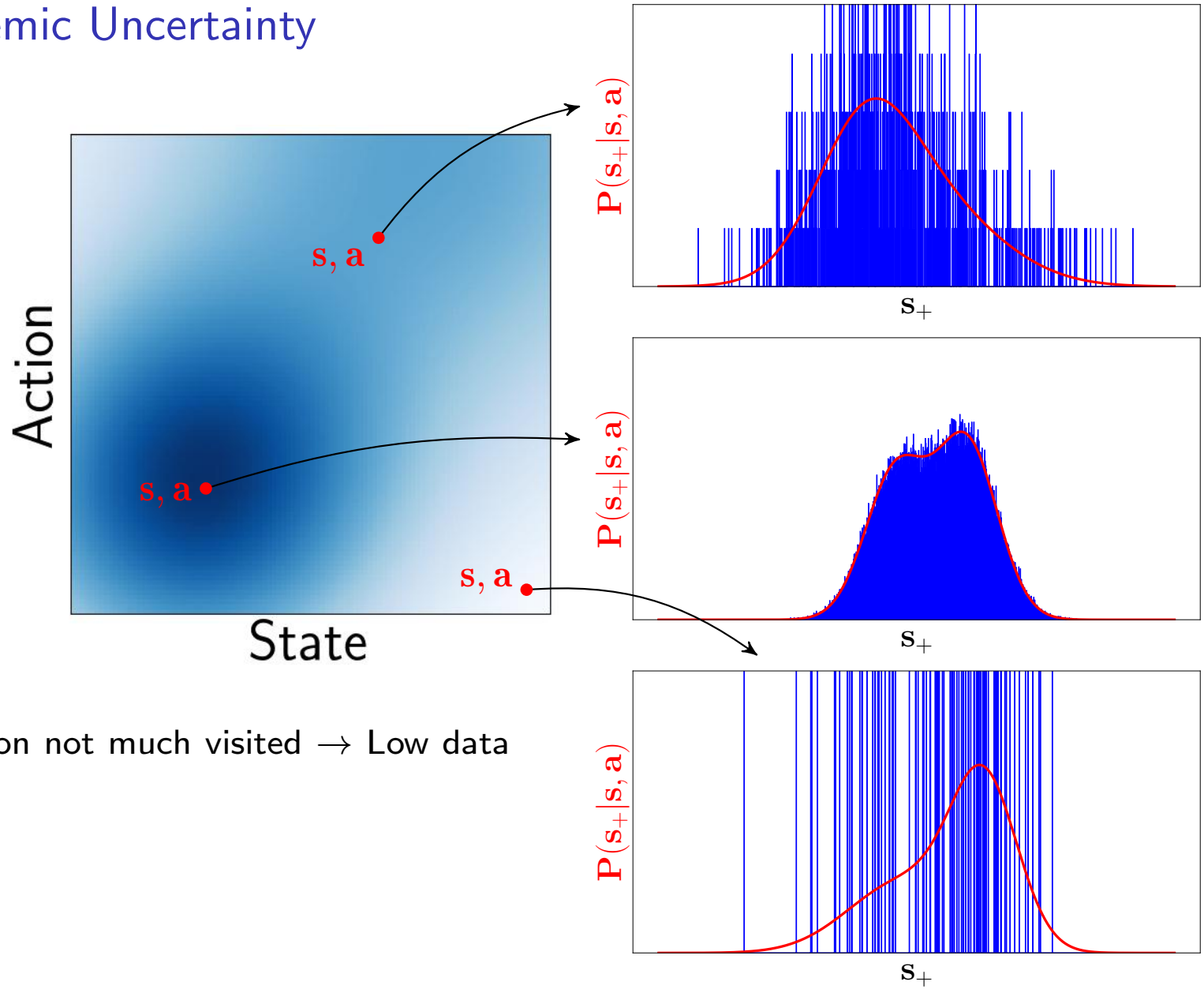

Remarks

- State-action not much visited $\rightarrow$ Low data about s_+

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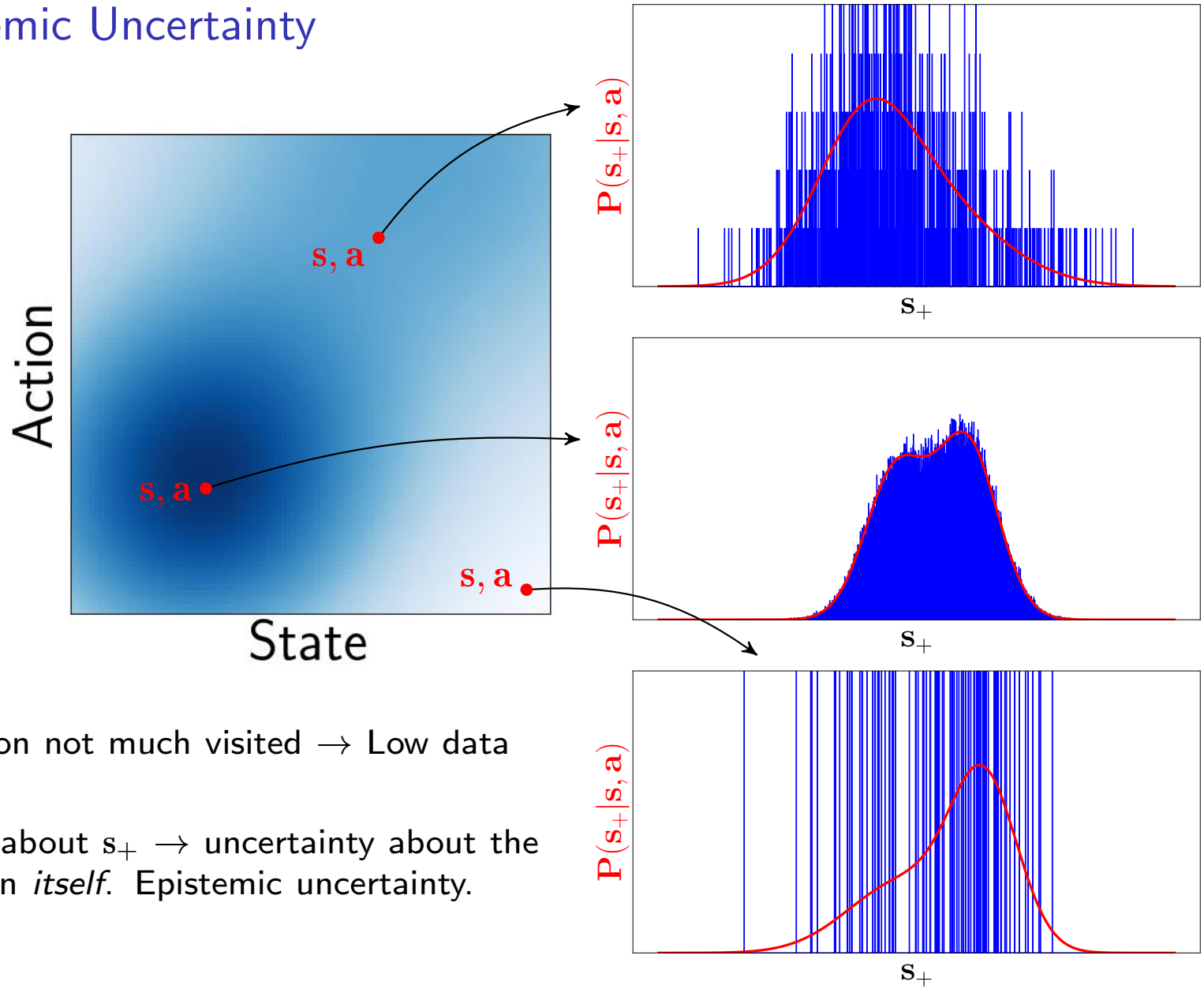

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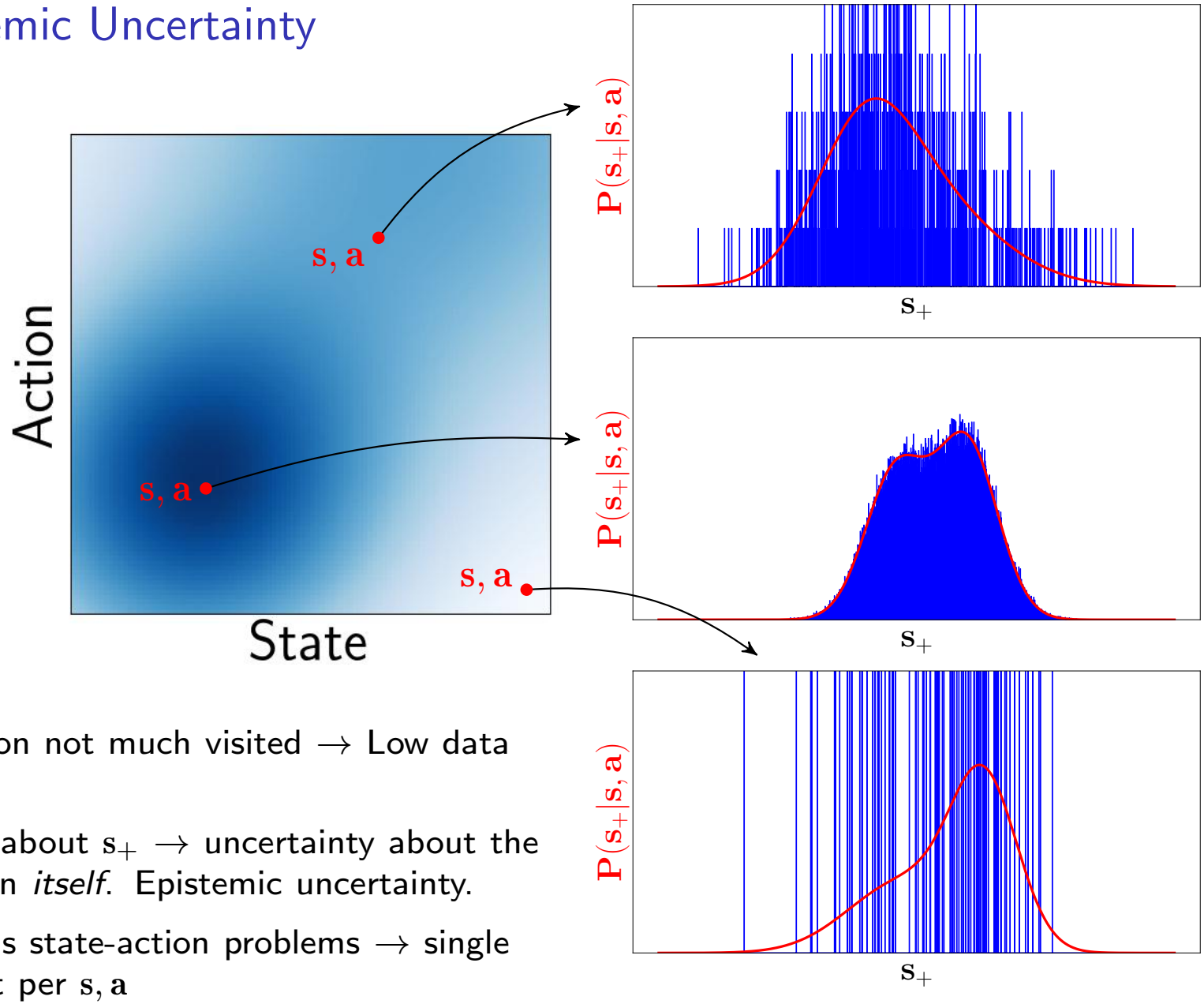

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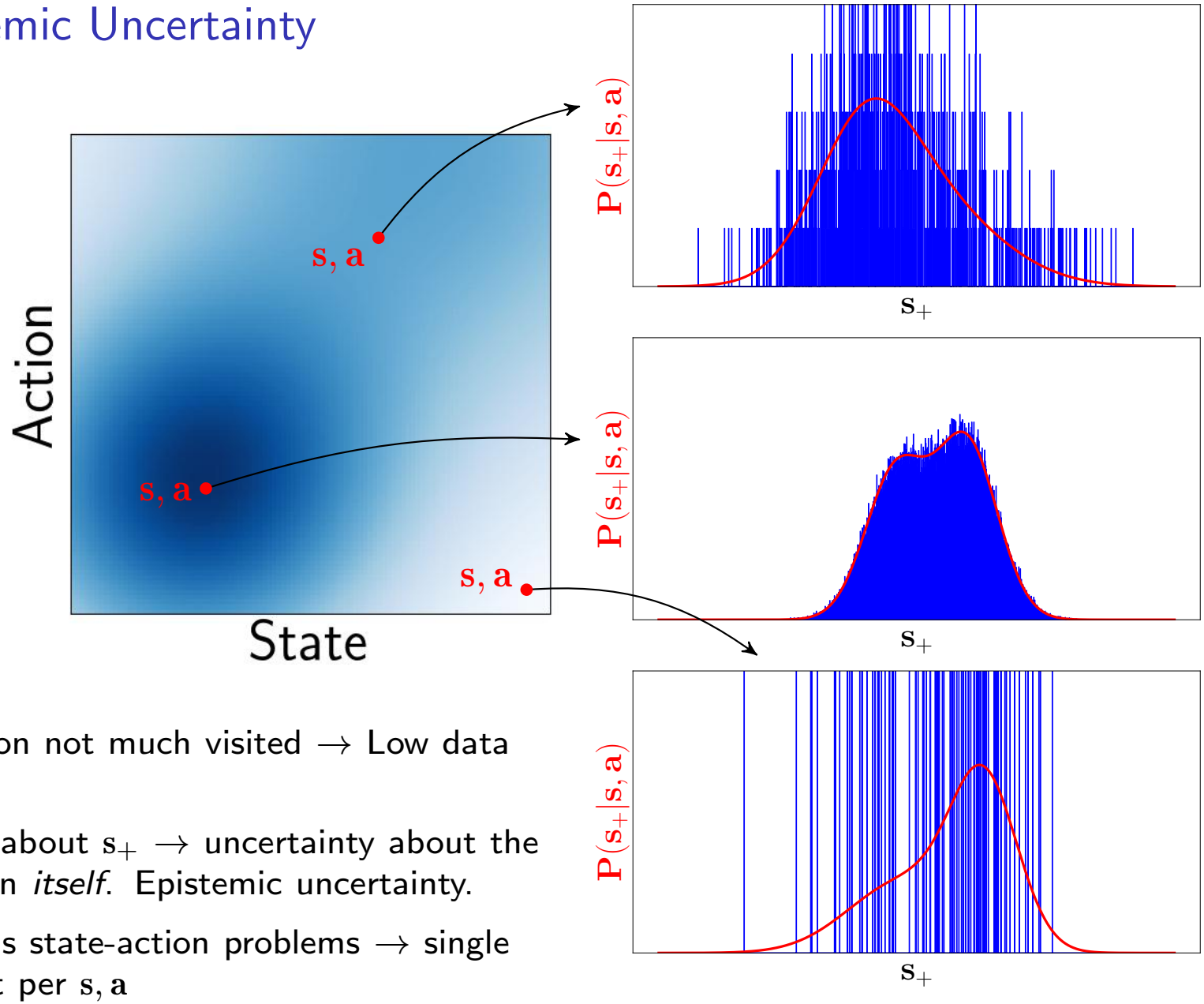

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- Continuous state-action problems $\rightarrow$ single data point per $\mathbf{s}, \mathbf{a}$
- Rely on regularity across $\mathbf{s}, \mathbf{a}$!!

Distributional View on MDPs

- True dynamics are unknown; only samples or estimates available.

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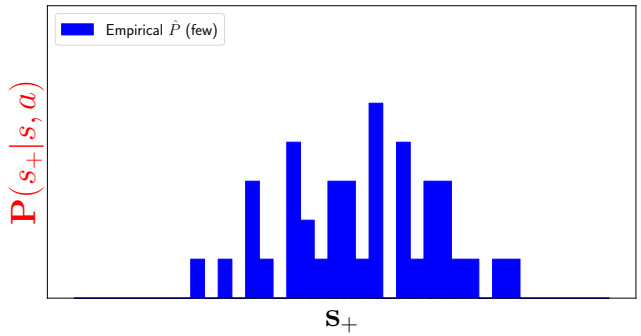

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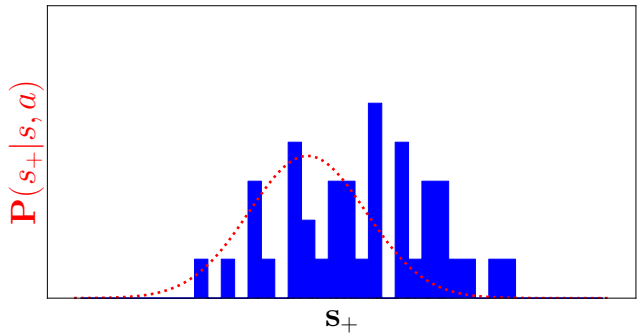

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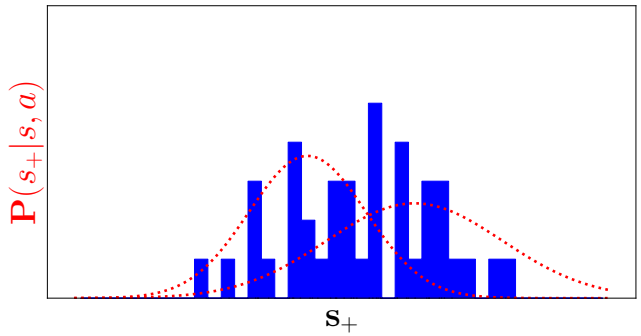

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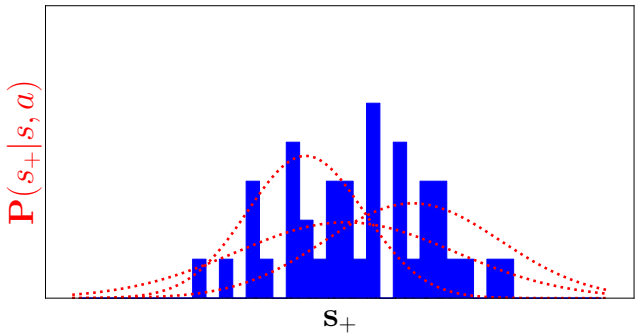

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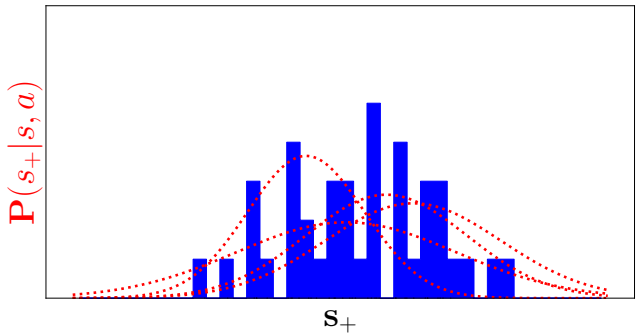

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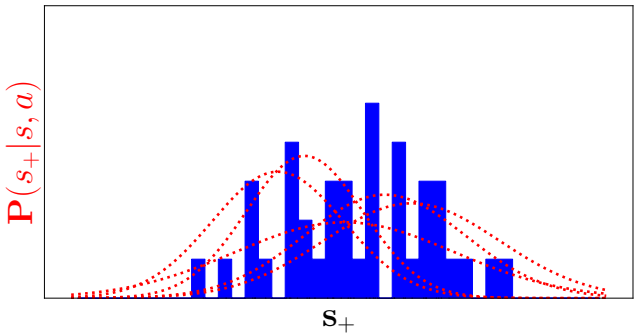

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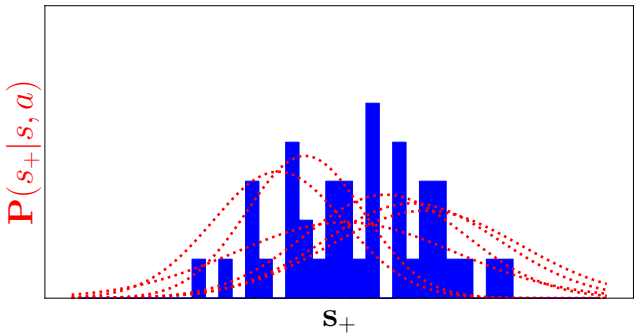

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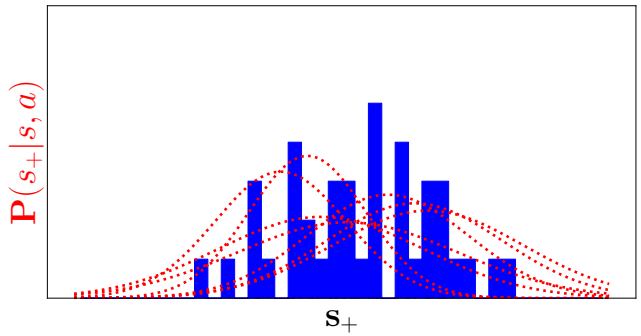

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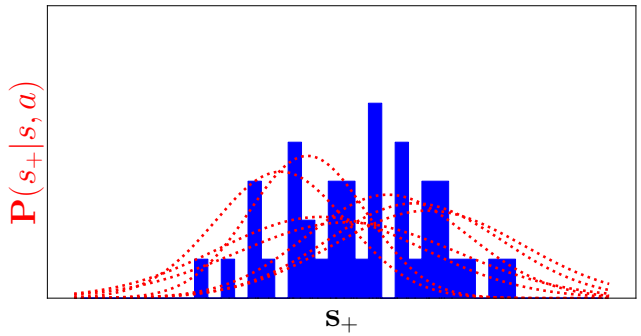

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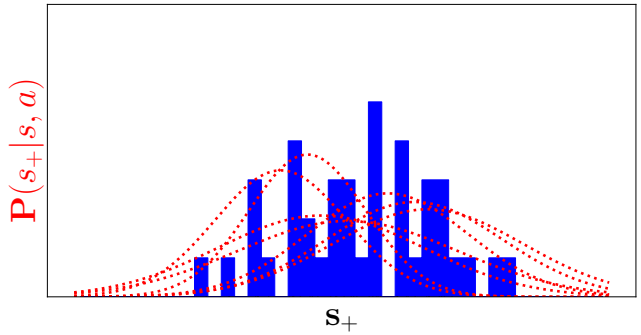


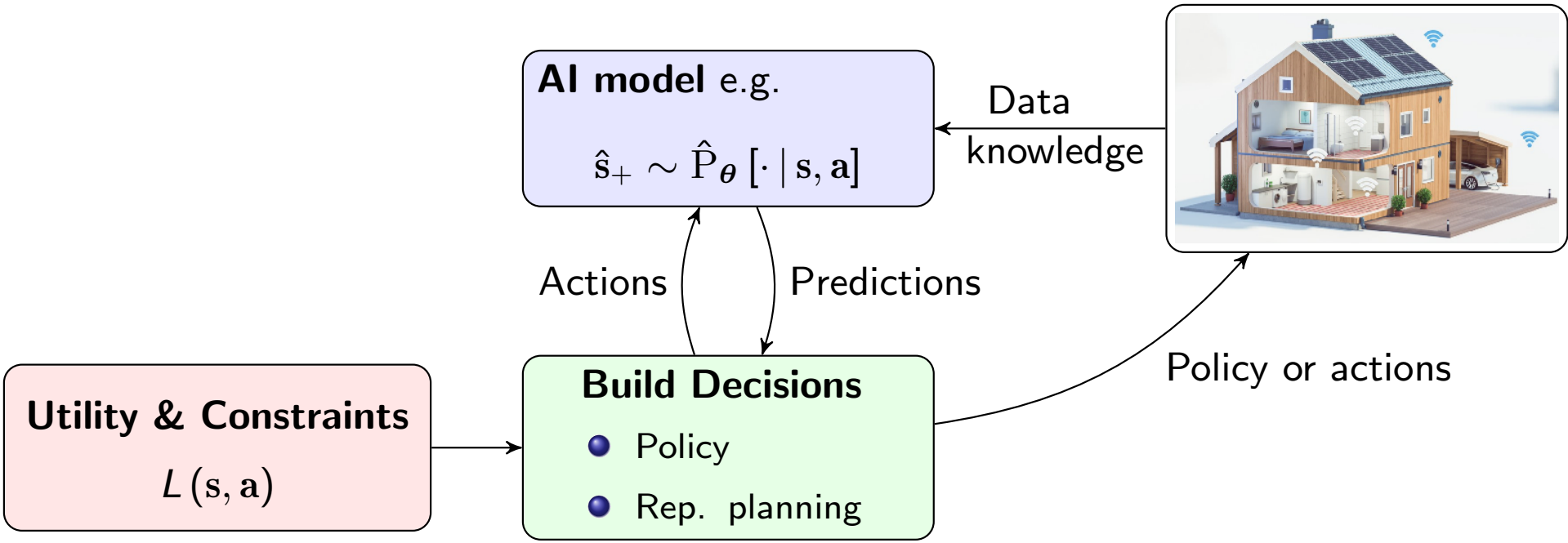
Remarks:

- Accounts for **epistemic uncertainty** from limited data.
- Distributional robust MDP: less pessimistic than robust MDP.
- Probabilistic Safety: confidence level based on model undertainty??
- Distributional view is applied to risk-sensitive MDP via a distribution of returns (e.g distributional RL).
- connection to bayesian vs freequentist view??

Out-Of-Distribution (OOD) "Guard" Methods

SG can do on 11.9

RL over Decision Making



- The model is not $\hat{\mathrm{P}}_{\boldsymbol{\theta}}$, it is the entire "decision box"
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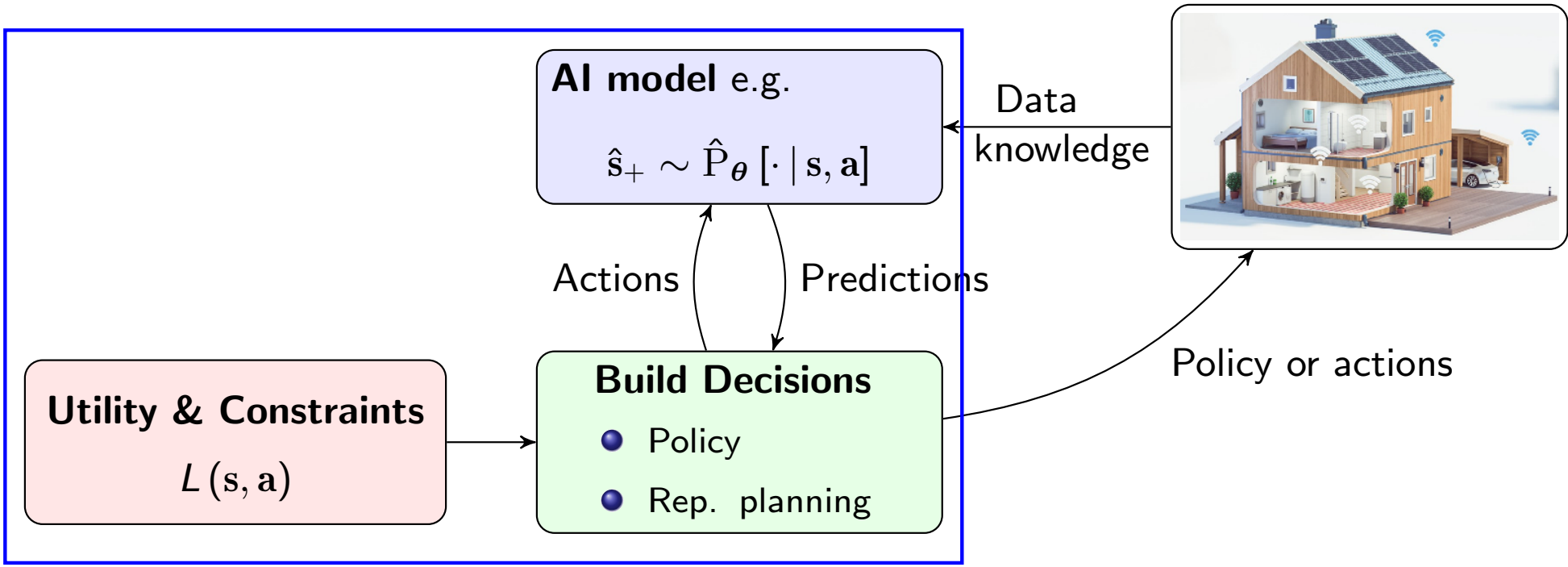

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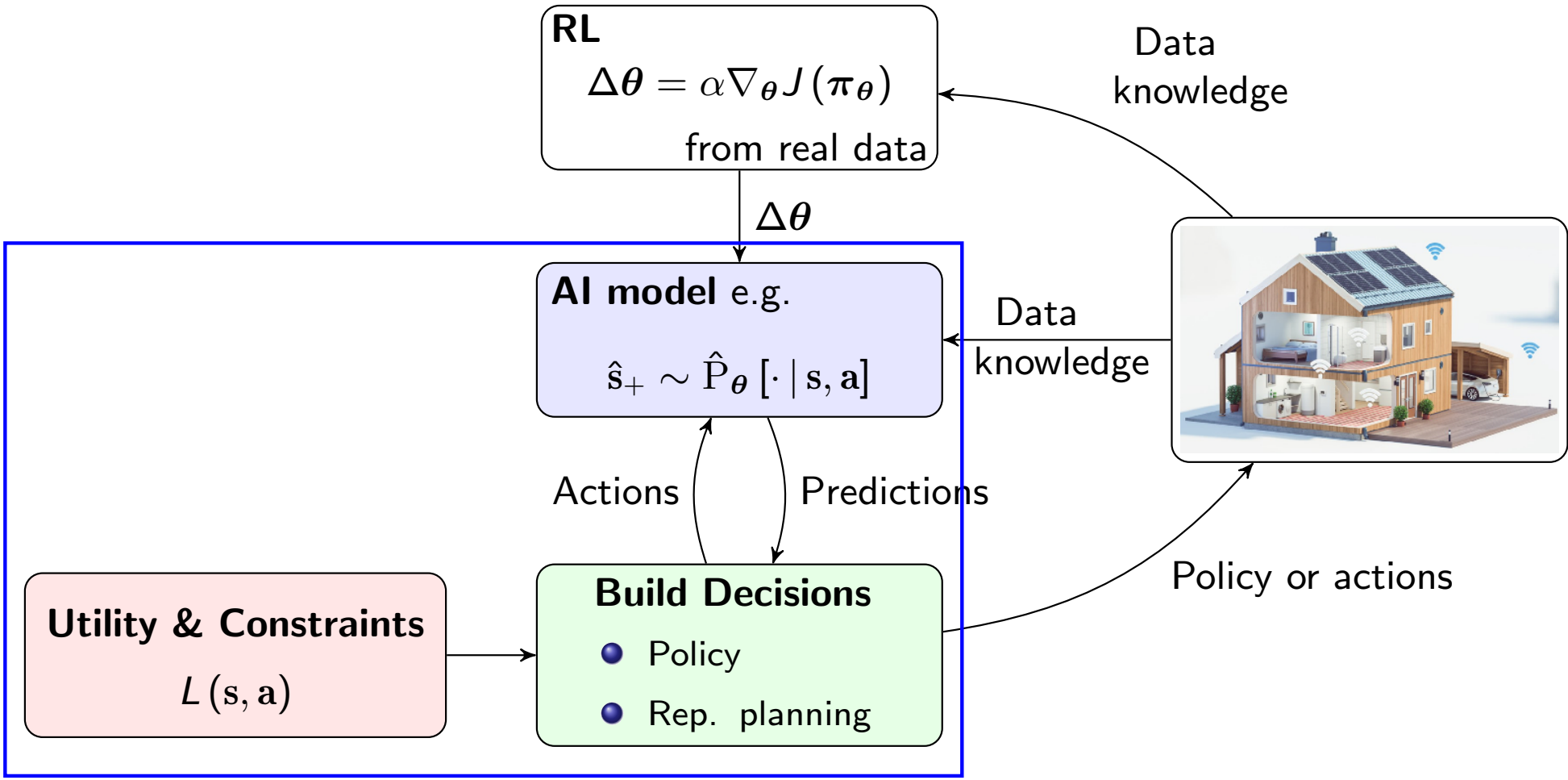

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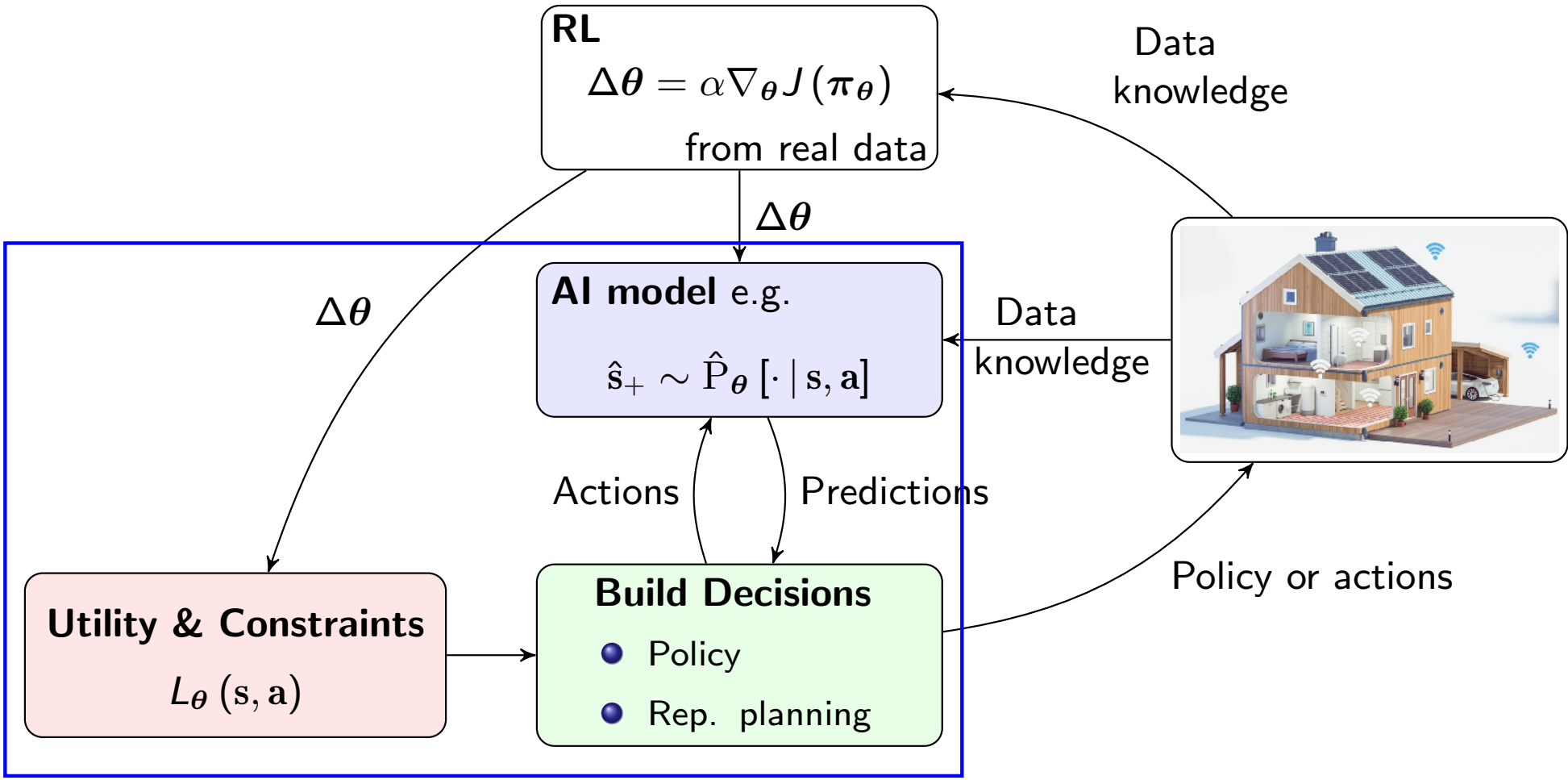

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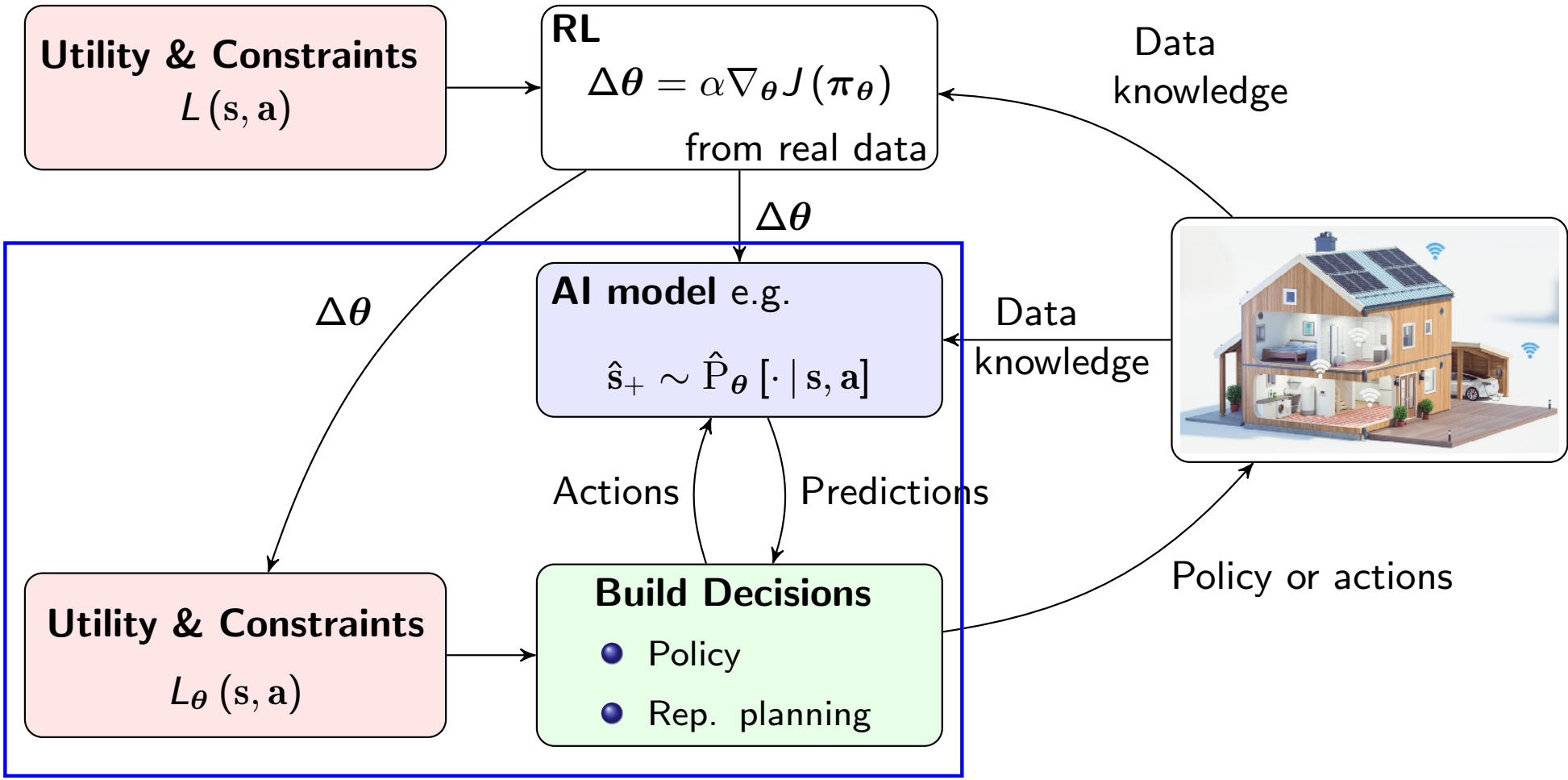

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