Introduction to Theory of Safe Decision Making

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1st AID Scientific Workshop, Trondheim

Forewords & Disclaimer

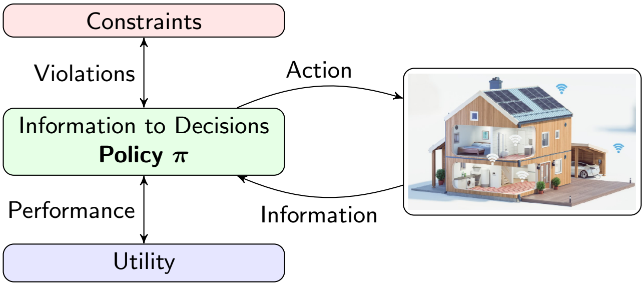
Objectives:

- Put in place some common concepts & language
- Identify some key points in safe decision making
- Connect to AI

Disclaimer: we are a broad group who needs to get to know each others scientifically.
Apologies if I don't "hit" the right level for all.
I have favored simplicity over "absolute" rigor.

Outline

Formally Defining Safety?



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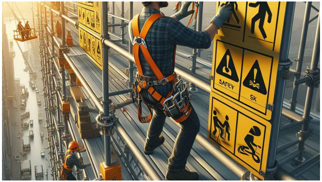

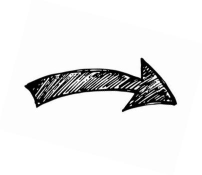

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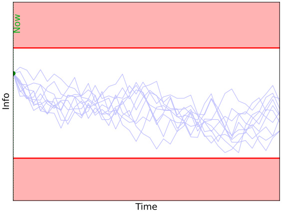

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Constraints

Violations

Information to Decisions
Policy π

Performance

Utility

Action

Information

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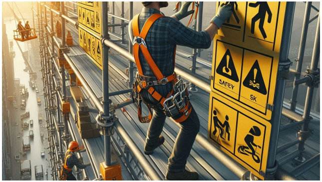

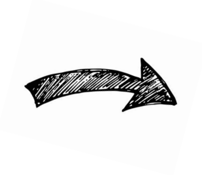

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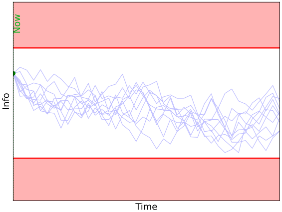

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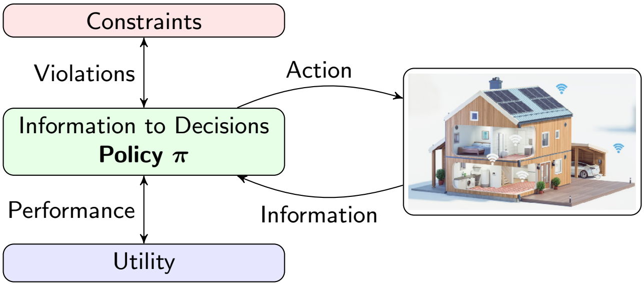

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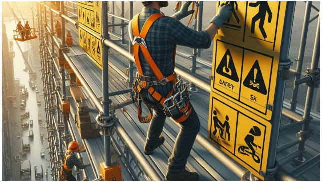

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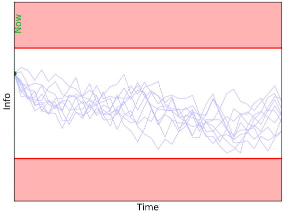

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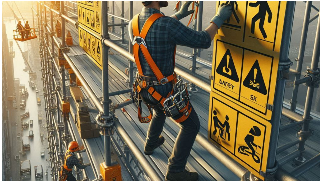

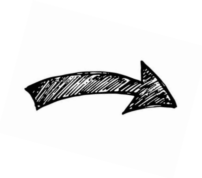

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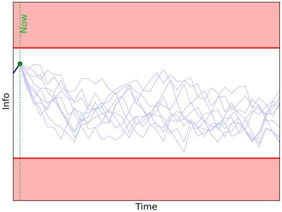

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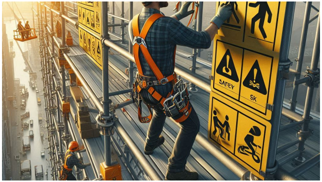

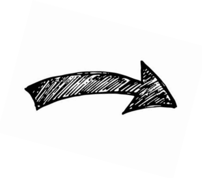

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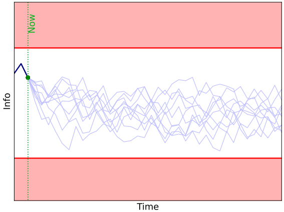

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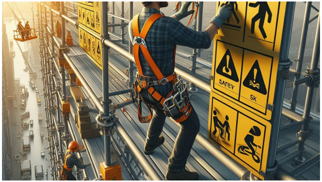

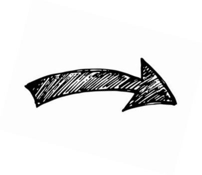

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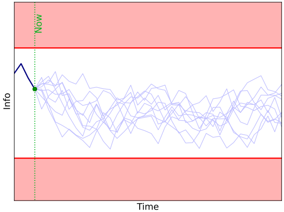

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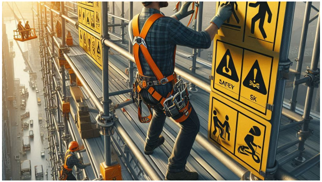

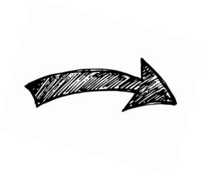

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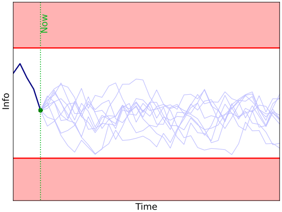

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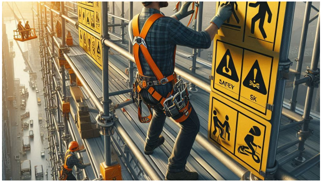

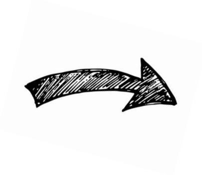

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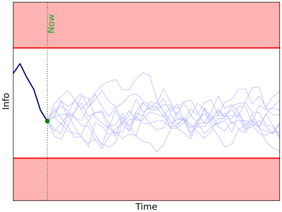

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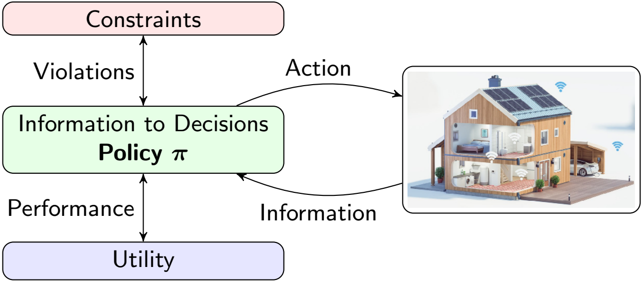

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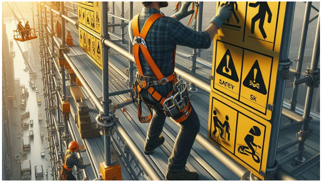

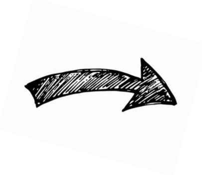

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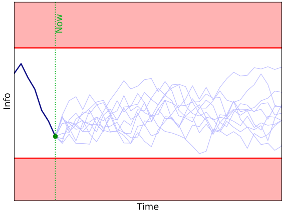

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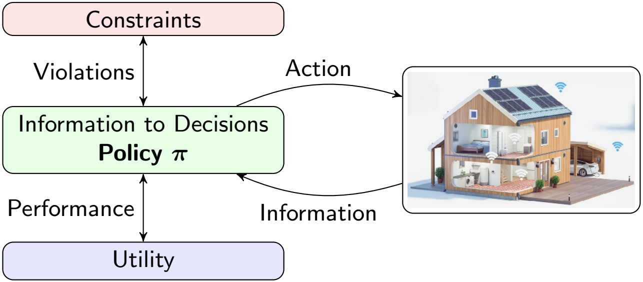

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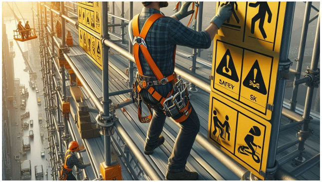

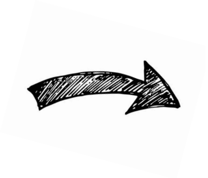

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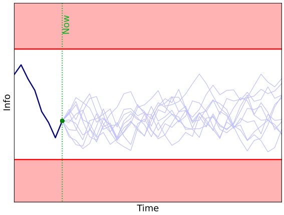
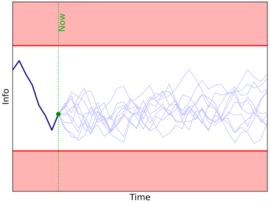


Constrained MDPs

In words

$$\boldsymbol{\pi}^\star = \arg\max_{\boldsymbol{\pi}} \quad \mathrm{E}\left[\sum_{\text{time}} \text{Utility}\right]$$
$$\text{s.t.} \quad \text{Constraints ok at all time}$$



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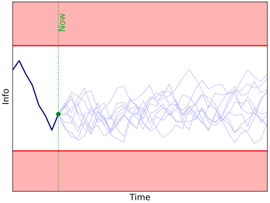

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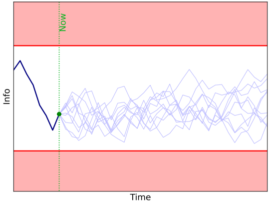

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Build policy using

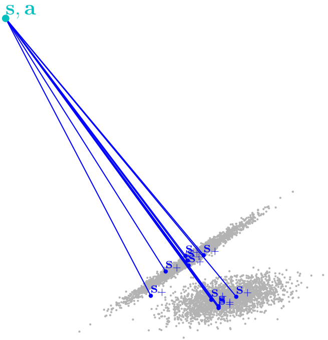
- Perfect model of the real world $\hat{\mathrm{P}}[\mathbf{s}_+|\mathbf{s},\mathbf{a}] = \mathrm{P}[\mathbf{s}_+|\mathbf{s},\mathbf{a}]$
- Model "pessimistic" about the uncertainties

... to evaluate "$\mathrm{E}[\cdot]$"

Pessimistic Models for Decision Making

- Model must "contain" the uncertainty
- "Container" (set) should be simple for computational reasons
- Trajectories predicted by pessimistic model will "cover" the real world
- Decision policy wants to be safe w.r.t. the "containers"

Distribution of one-step forward



Illustrate: wide & thin-tailed distribution requires a big box, to contain transitions that almost surely won't happen. SG can do on 11.9

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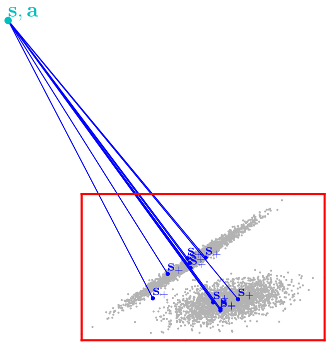
Distribution of one-step forward with "container"



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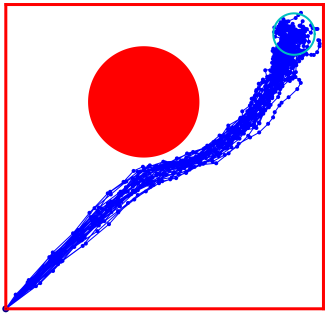
Trajectories



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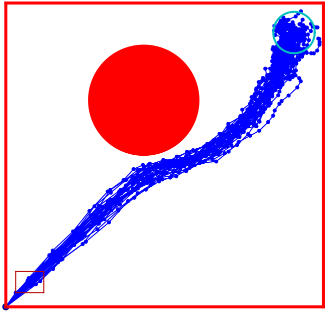

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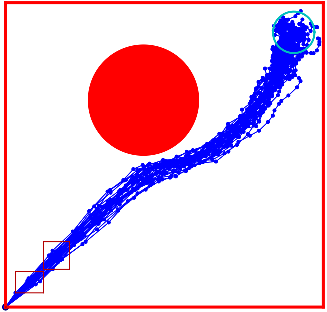

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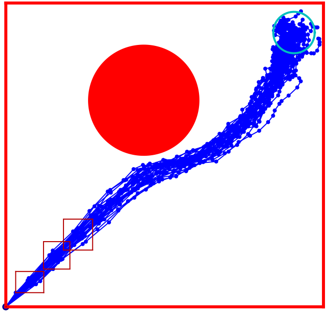

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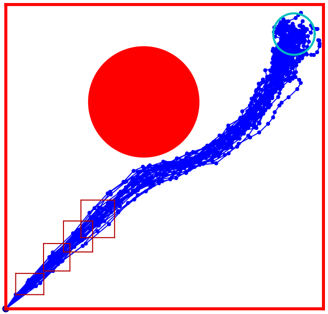

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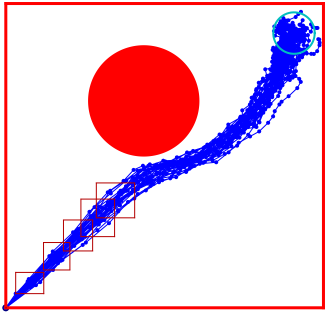

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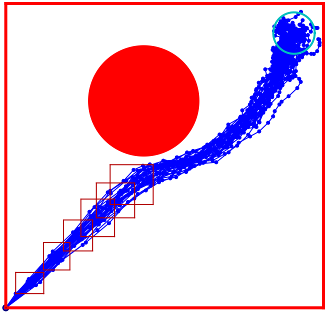

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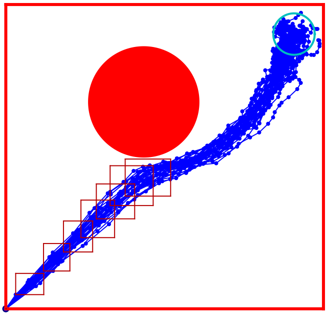

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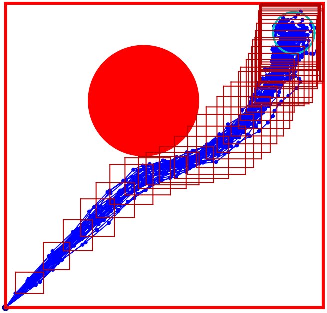

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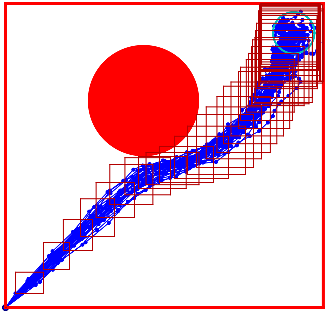

Remarks

- The propagation of the "containers" in the model predictions can be expensive / difficult
- Pessimistic propagations are usually needed → pessimistic over pessimistic
- Policy based on worst-case perspective makes the decisions highly conservative
- Often labelled "Robust" decision making

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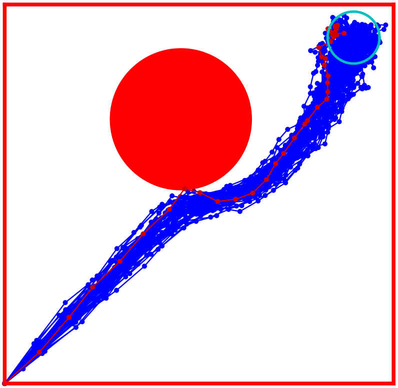
MDPs with probabilistic safety

In words

$$\begin{aligned} \boldsymbol{\pi}^\star = \arg\max_{\boldsymbol{\pi}} \quad & \mathrm{E}\left[\sum_{\text{time}} \text{Utility}\right] \\ \text{s.t.} \quad & \text{Probability of no violation} \geq c \end{aligned}$$



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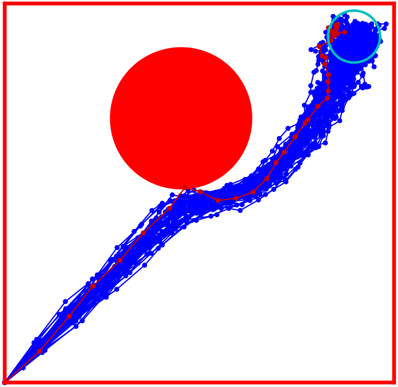

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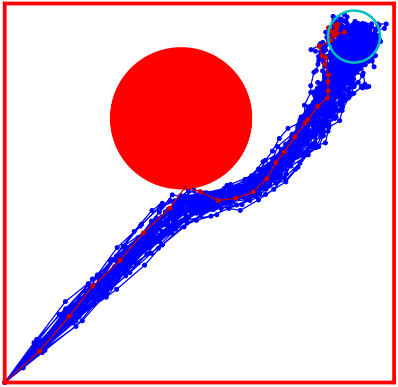
Remarks:

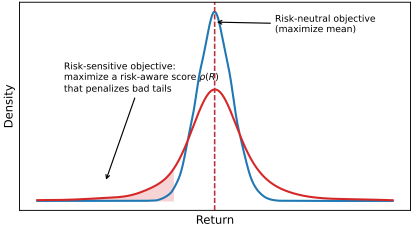
- If we can tolerate $c > 0$ (small) it can make a big gain in performance
- Aligned with industrial / practical standards on "large series"
- Problem needs a "termination" (time or goal reached)
- Building decisions can be difficult from a computational point of view



Risk-sensitive MDPs

In words

- Risk-sensitive MDPs optimizes a **risk measure** to account for the severity of rare outcomes.
- Intuition: a "tail-risk filter" even if outcomes are good most of the time, severe tails influence decisions.



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Mean-optimal can ignore tails; risk-sensitive downweights heavy losses.

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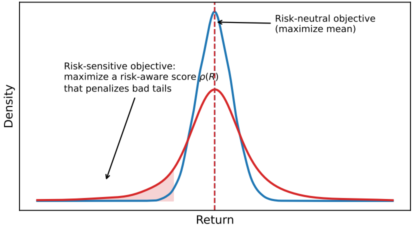

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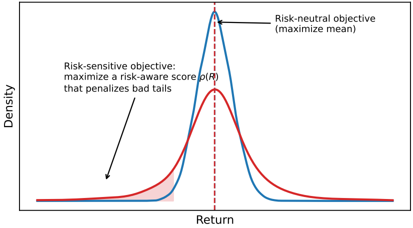

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Remarks:

- Often possile to solve using classical methods such as DP.
- Risk-sensitive criteria do *not* guarantee a bound on violation.

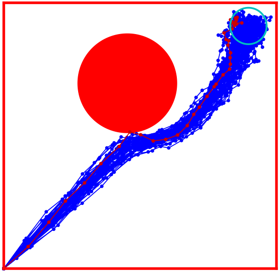
Outline

Robust Repeated Planning

- Introduction: Robust MPC, Scenario Trees, MC, robust multi-stage stochastic programing, MPPI?
- Difficulties: guarantees for nonlinear systems, persistent safety (recursive feasibility)

Dynamic Programming & RL with Probabilistic Safety

$$\begin{aligned} \boldsymbol{\pi}^\star = \arg\max_{\boldsymbol{\pi}} \quad & \mathrm{E}\left[\sum_{k=0}^{\infty} \gamma^k L(\mathbf{s}_k, \mathbf{a}_k)\right] \\ \text{s.t.} \quad & \mathrm{P}\left[\mathbf{s}_{0,\ldots\infty} \in \mathbb{S}\right] \geq c \end{aligned}$$



Expected value form

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- Expected value form enables classical techniques (ref. 1st lecture), i.e. DP and RL
- Difficulties: estimate expected values (sample based) when $\mathrm{P}\left[\mathbf{s}_{0,\ldots\infty} \in \mathbb{S}\right]$ is close to 1. Illustrate?? SG will try on 11.9

Outline

Safety Filters & Reinforcement Learning

- For any state s, an "oracle" tells you which actions will not jeopardize safety in the long run $\rightarrow$ "Safe set" $\mathrm{S}(\mathrm{s})$

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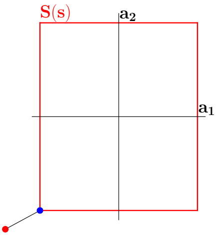

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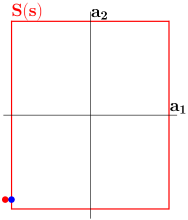

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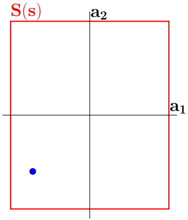

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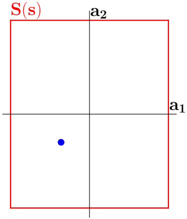

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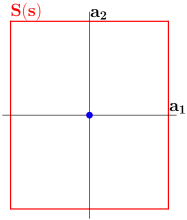

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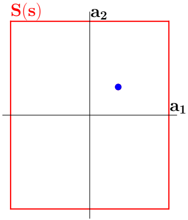

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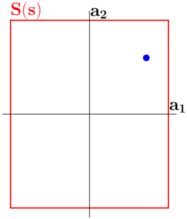

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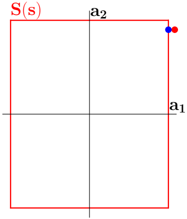

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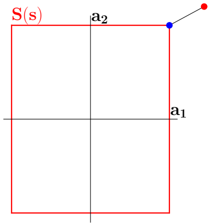

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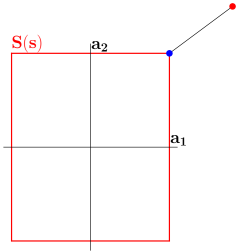

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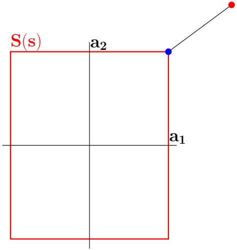
Not losing the king



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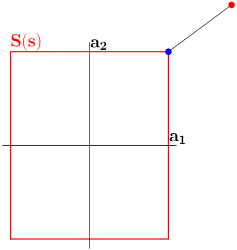
Able to stop within the visible distance



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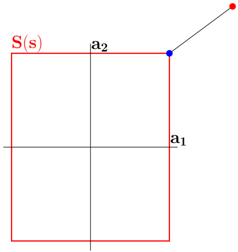
Remarks:

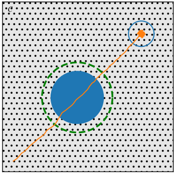
- Often close to practice, e.g., flight envelop protection in modern airplane, semi-autonomous driving in cars, etc
- Oracle ⇒ knowledge-based & conservative!



Control Barrier Functions (CBFs)

In words

- CBFs provide a formal, model-based way to build the "oracle" safe set.
- Use a barrier function $h(x)$ so that staying safe means $h(x) \geq 0$.
- At each step, if the proposed action would leave the safe set, a small QP minimally adjusts it be safe.



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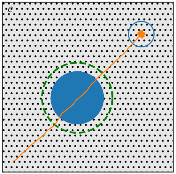
$$u^\star(x) = \arg\min_u \tfrac{1}{2}\|u - u_{\mathrm{des}}(x)\|_2^2 \quad \text{s.t.}$$

$$\underbrace{L_f h(x) + L_g h(x)\, u + \alpha(h(x)) \geq 0}_{\text{CBF constraint}}$$



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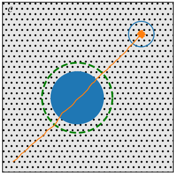

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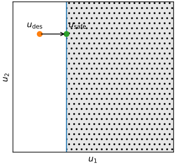

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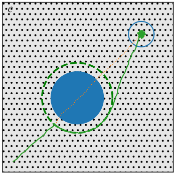
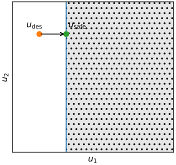


CBF-filtered trajectory.

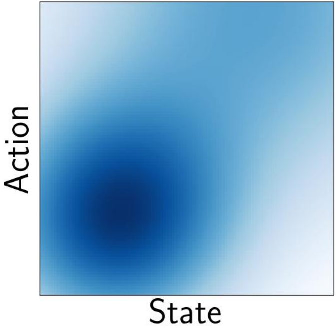


CBF filter: project u_{des} to feasible u_{safe}.

- The barrier function and safe set $\mathcal{C}$ are typically **constructed from domain knowledge** (physics, rules, safety envelopes).
- Requires a (possibly simplified, conservative) **system model**: **robust** approach.

Outline

Data & Epistemic Uncertainty



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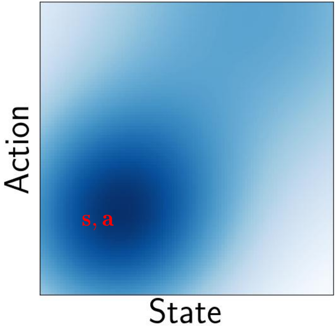

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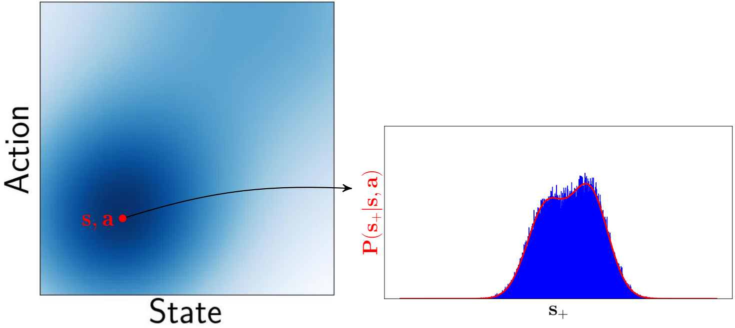

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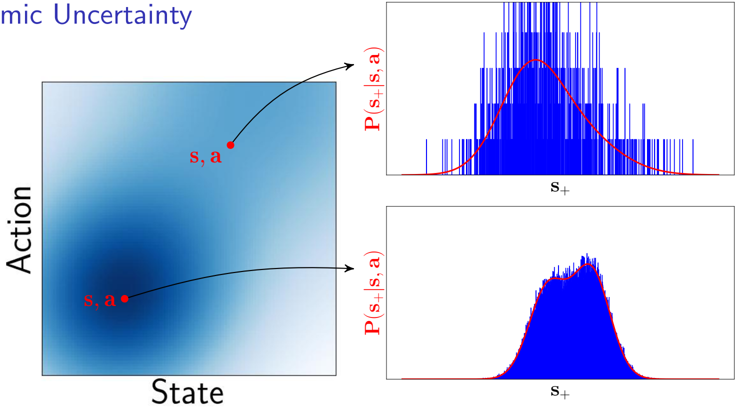

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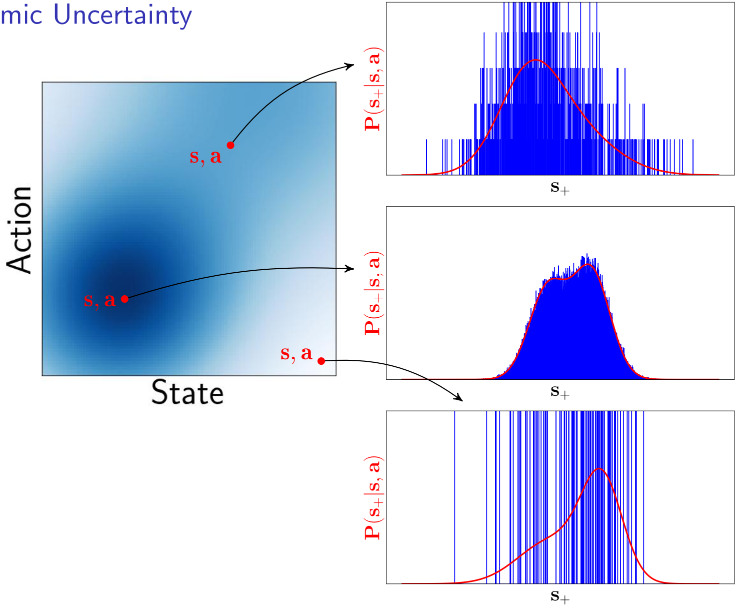

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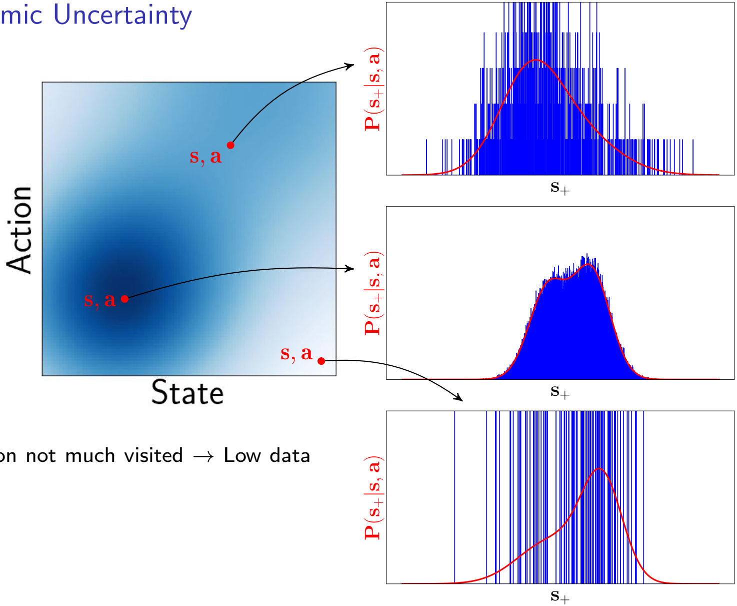

Remarks

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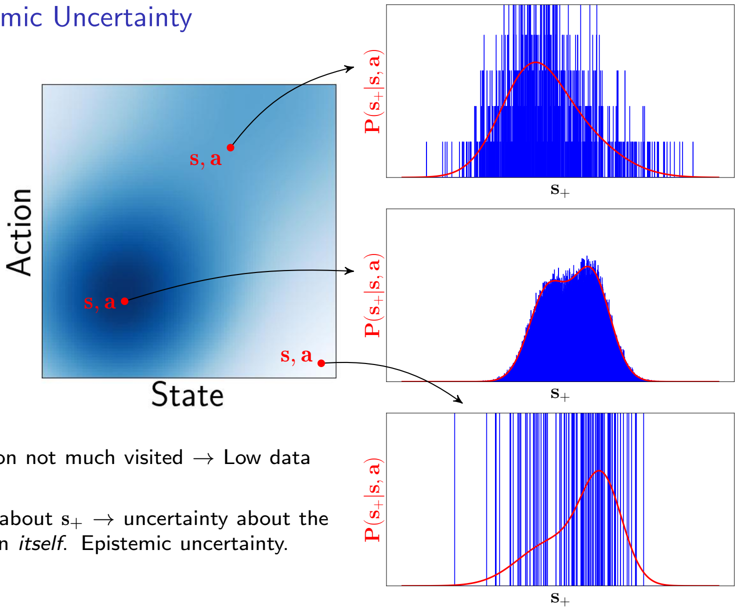
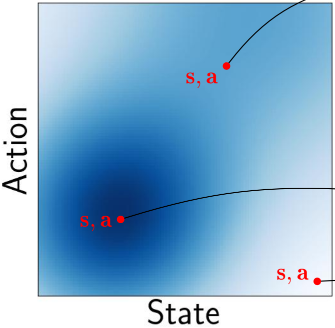
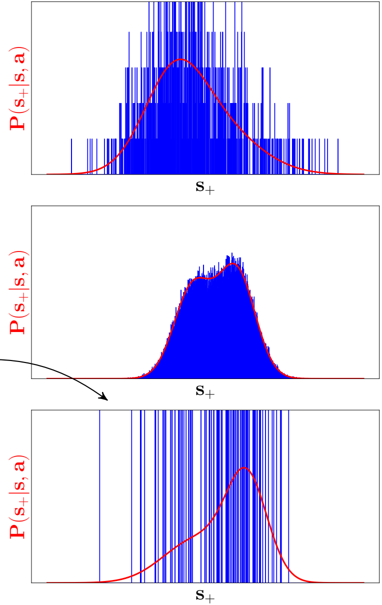


Remarks

- State-action not much visited $\rightarrow$ Low data about s_+
- Low data about s_+ $\rightarrow$ uncertainty about the distribution *itself*. Epistemic uncertainty.

Data & Epistemic Uncertainty





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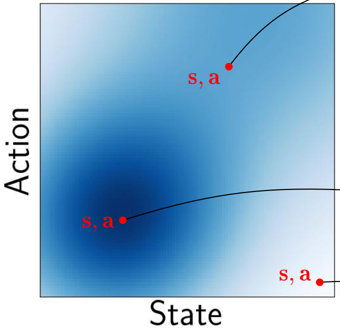

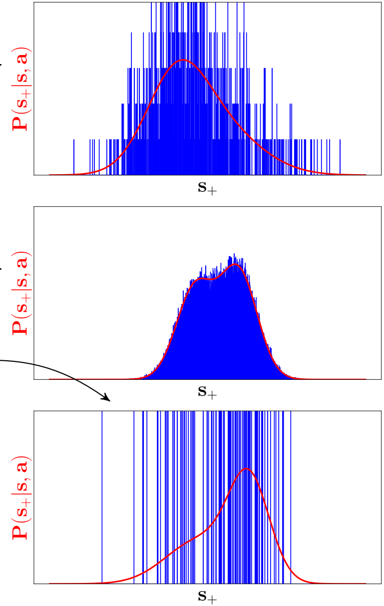

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- Low data about s_+ $\rightarrow$ uncertainty about the distribution *itself*. Epistemic uncertainty.
- Continuous state-action problems $\rightarrow$ single data point per $\mathrm{s}, \mathbf{a}$
- Rely on regularity across $\mathrm{s}, \mathbf{a}$!!

Distributionally Robust Methods

AKHIL?

Out-Of-Distribution (OOD) “Guard” Methods

SG can do on 11.9