Introduction to Theory of Safe Decision Making

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1st AID Scientific Workshop, Trondheim

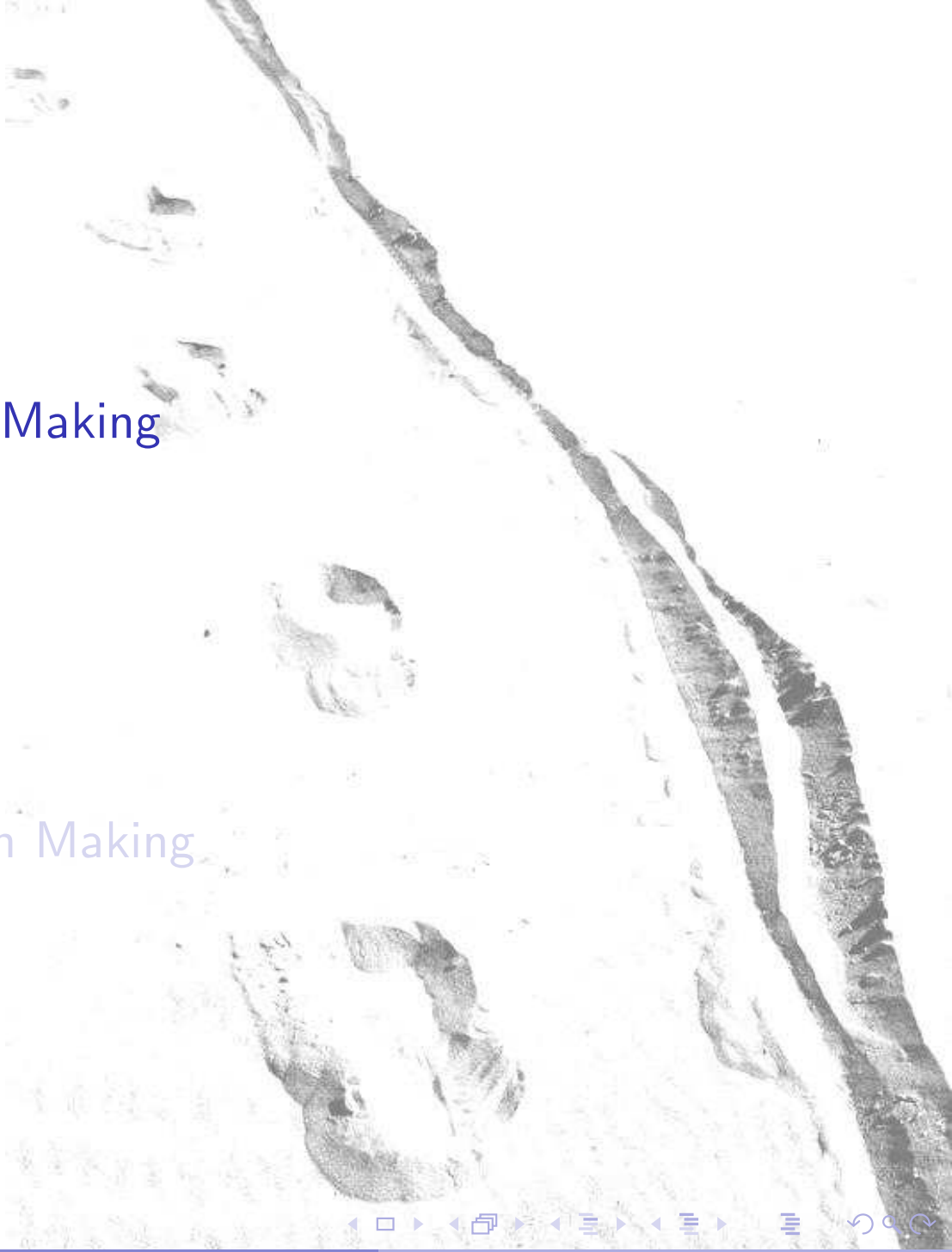
Forewords & Disclaimer

Objectives:

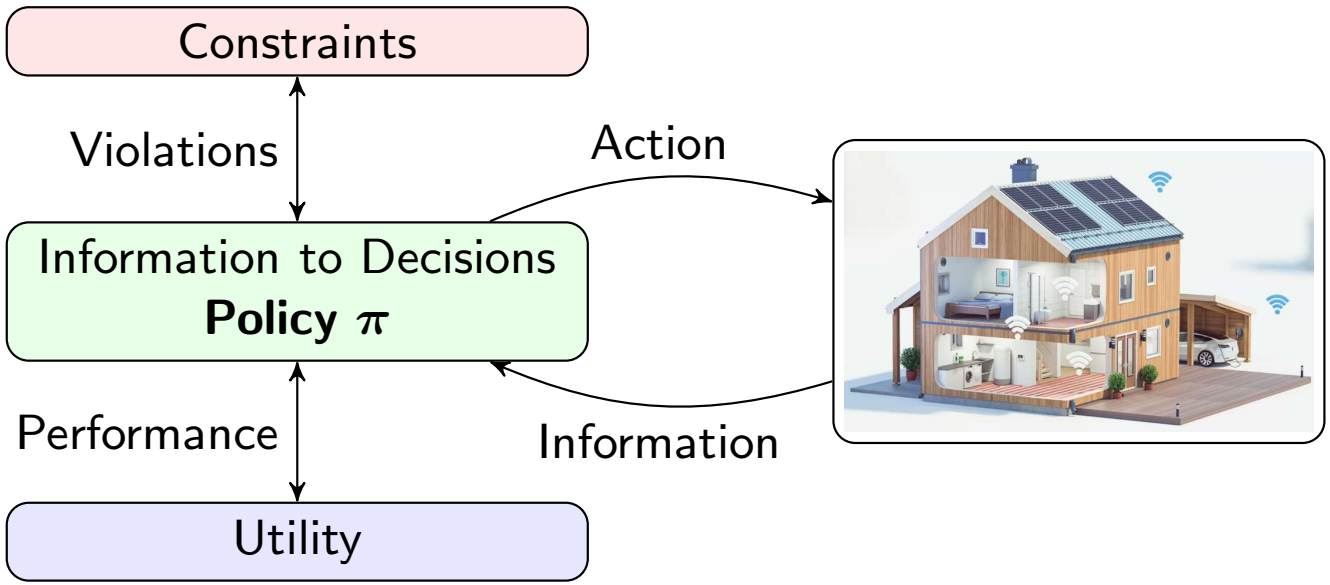
- Put in place some common concepts & language
- Identify some key points in safe decision making
- Connect to AI

Disclaimer: we are a broad group who needs to get to know each others scientifically.
Apologies if I don't "hit" the right level for all.
I have favored simplicity over "absolute" rigor.

Outline



Formally Defining Safety?



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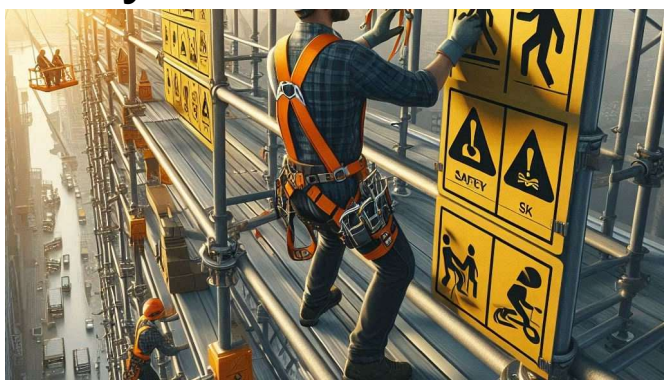

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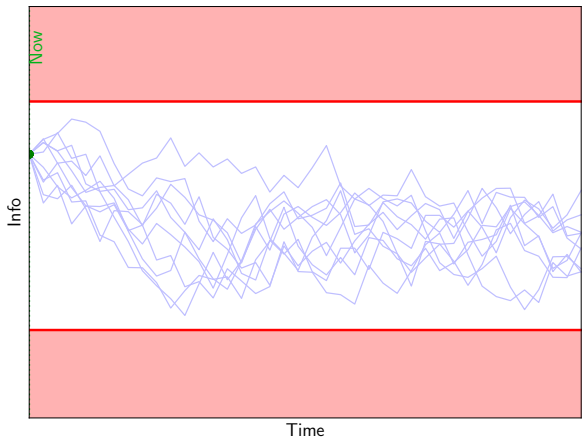

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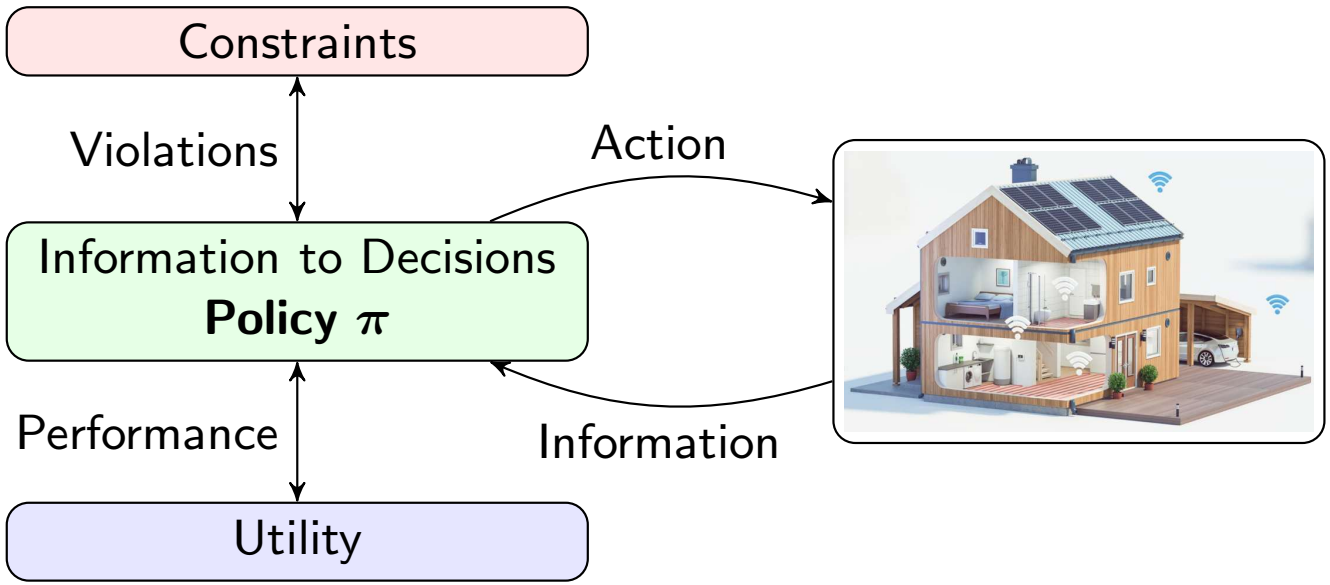

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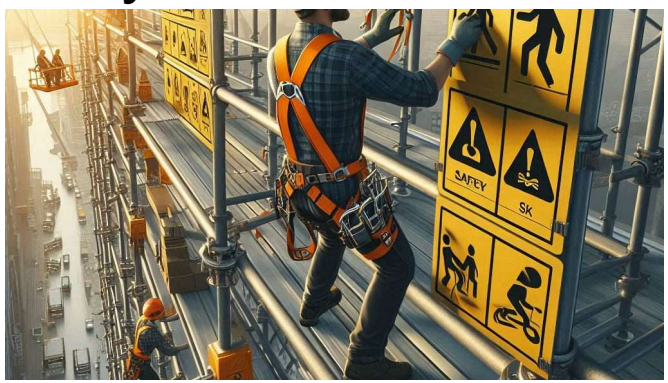

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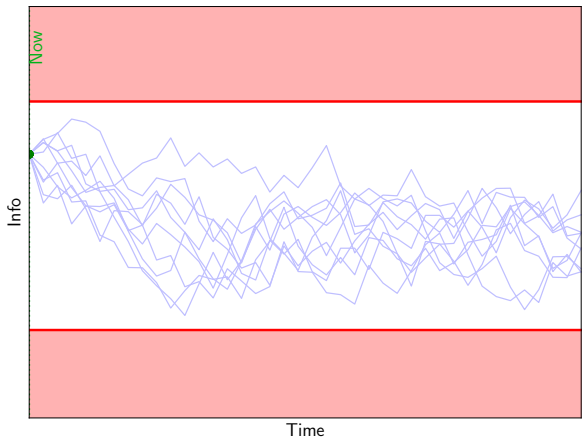

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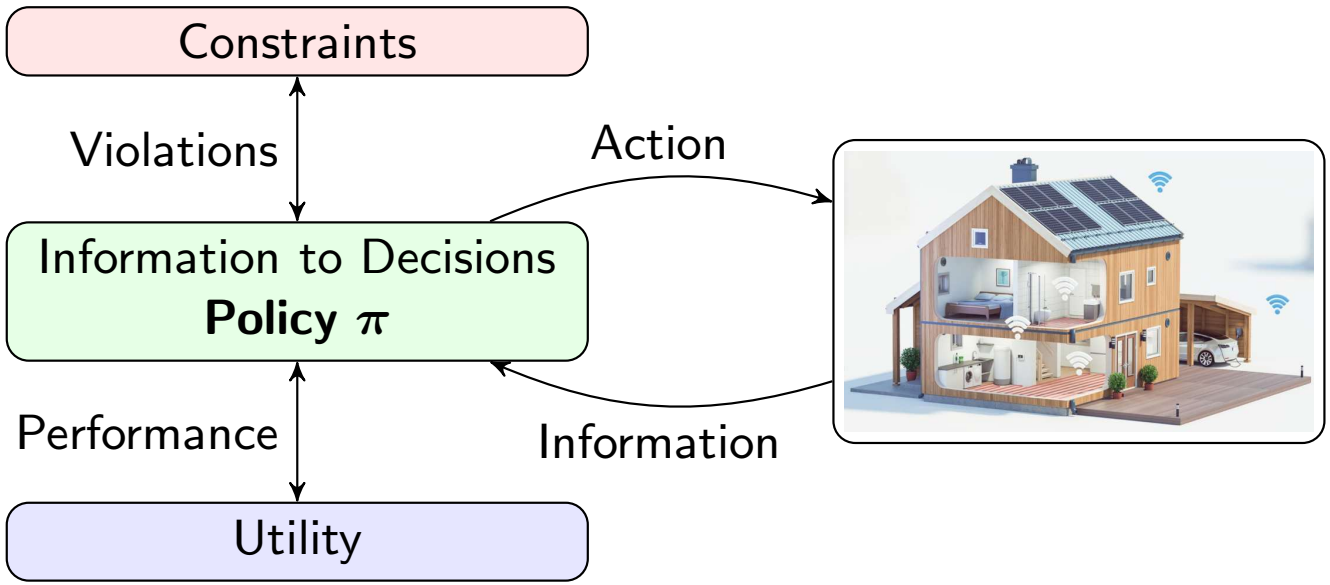

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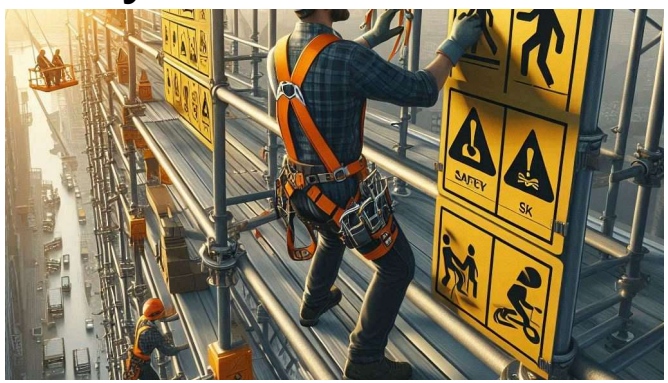

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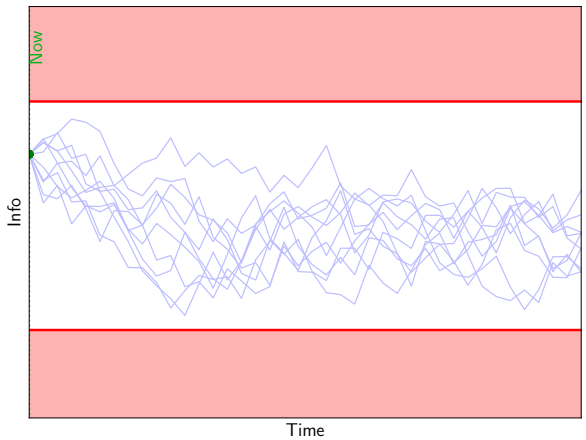

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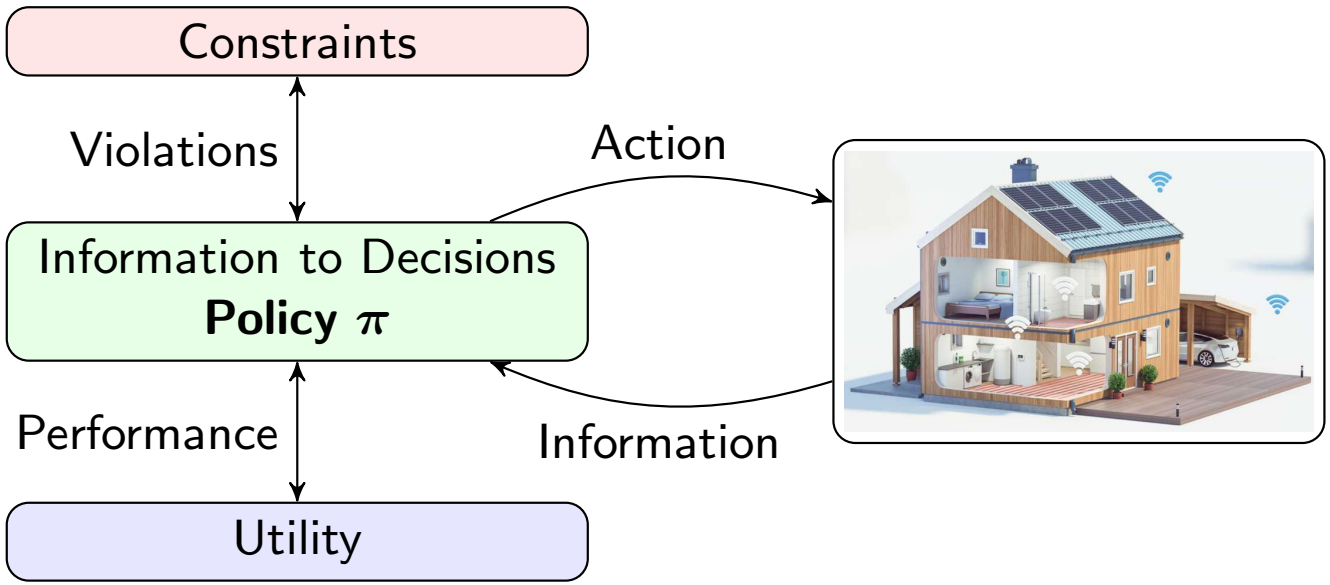

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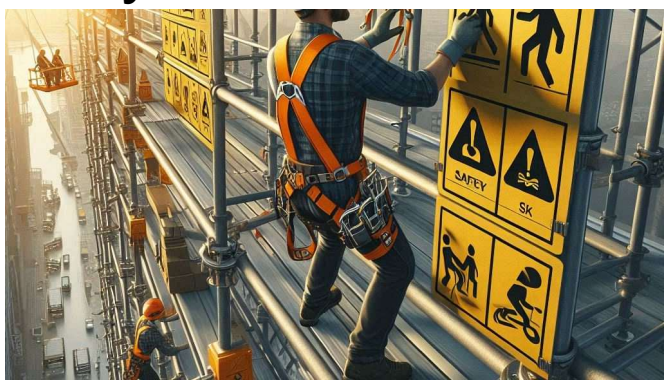

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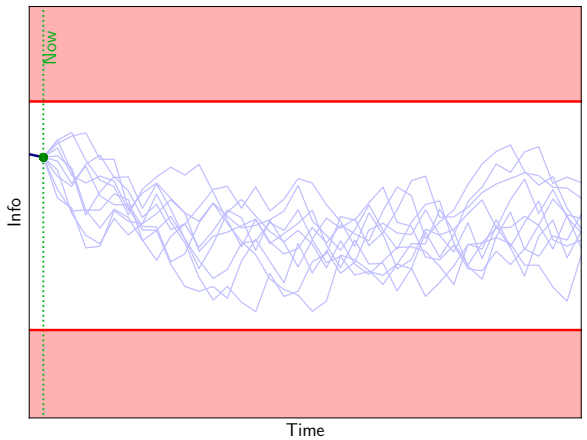

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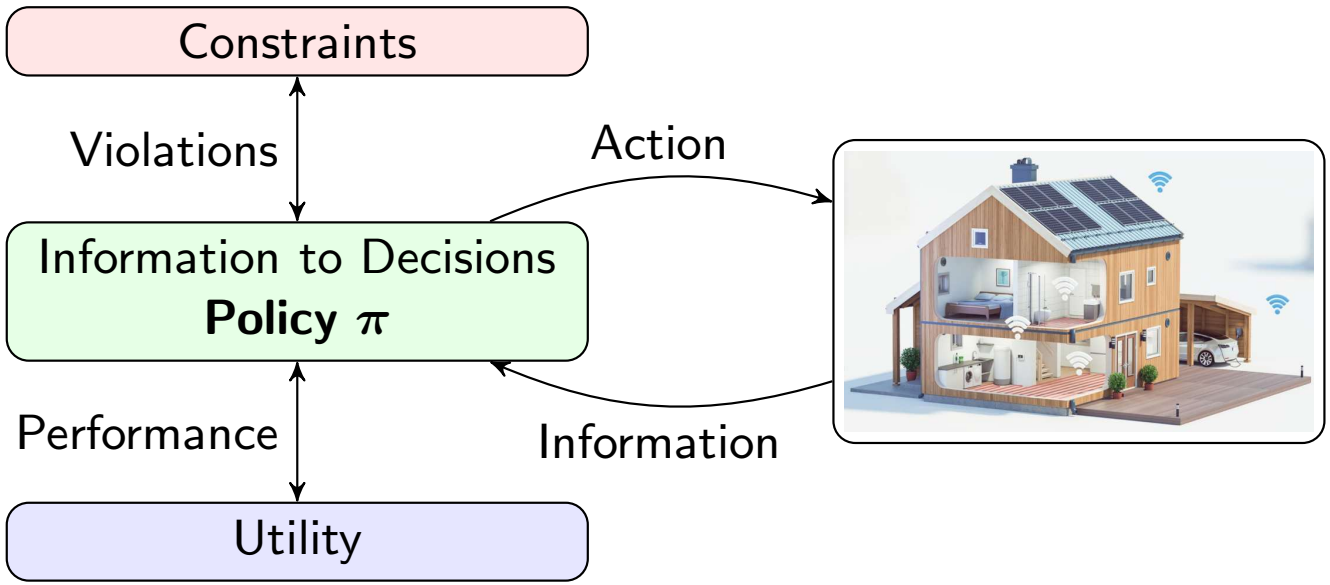

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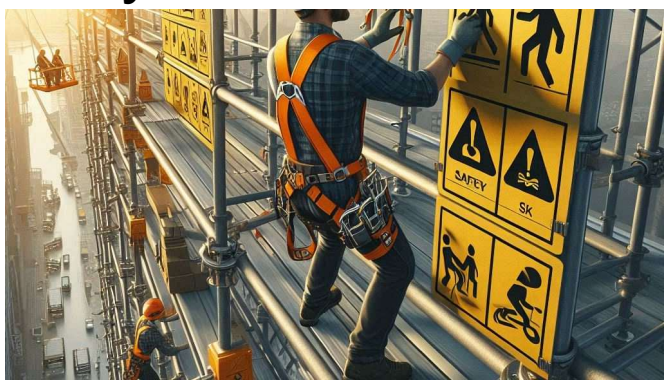

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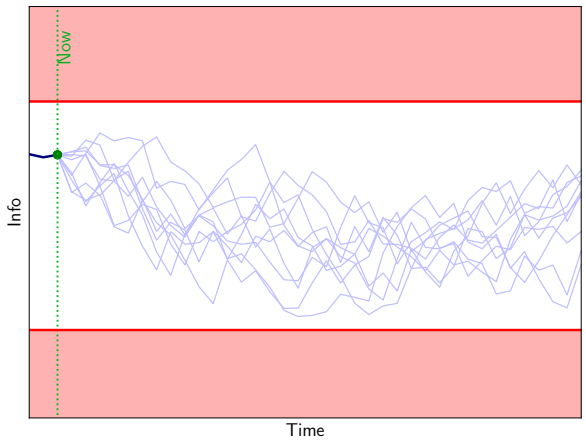

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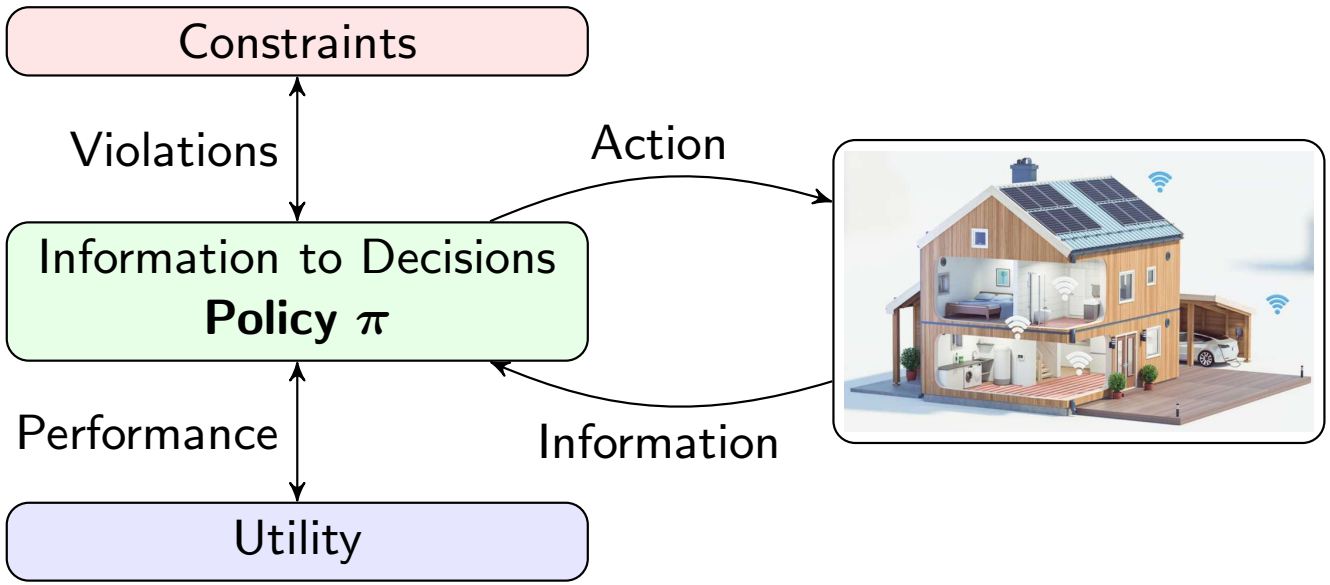

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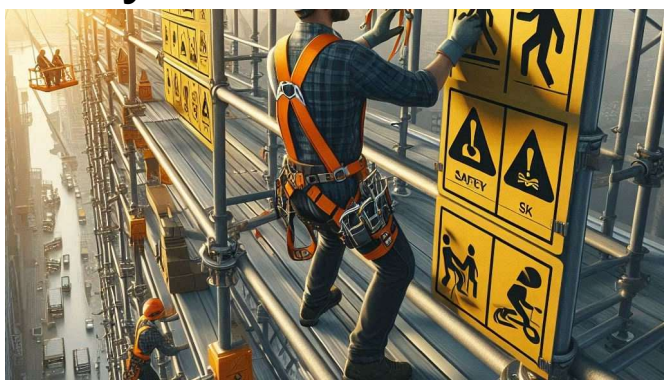

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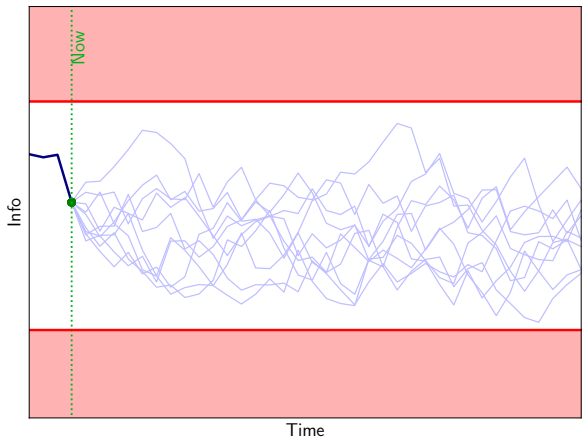

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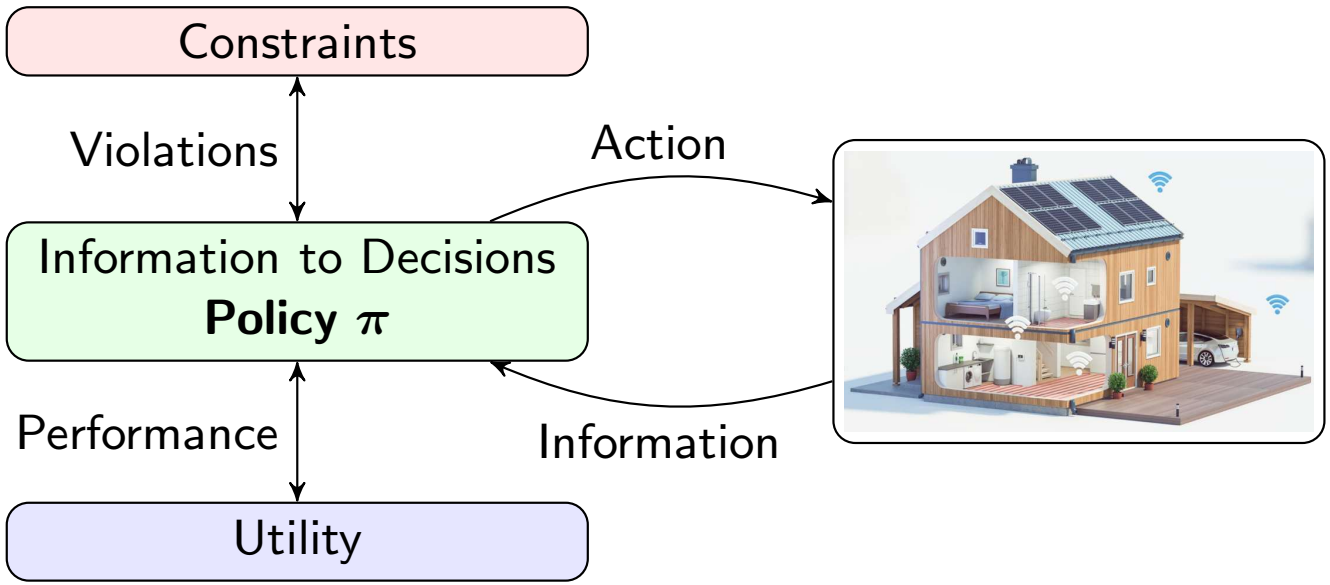

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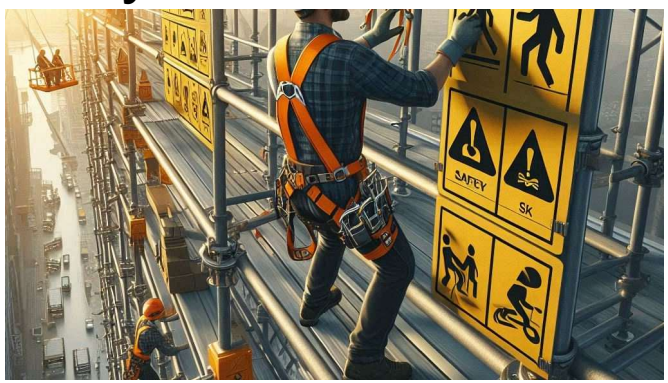

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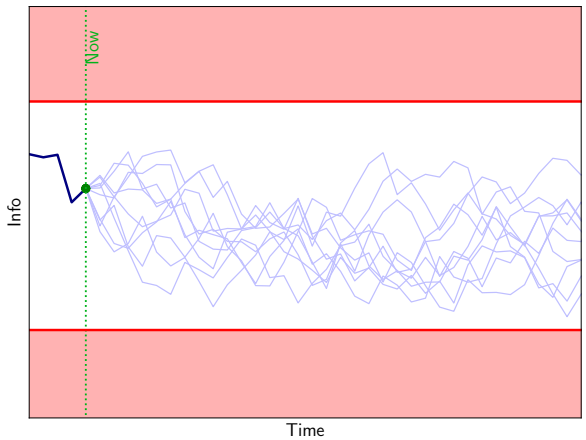

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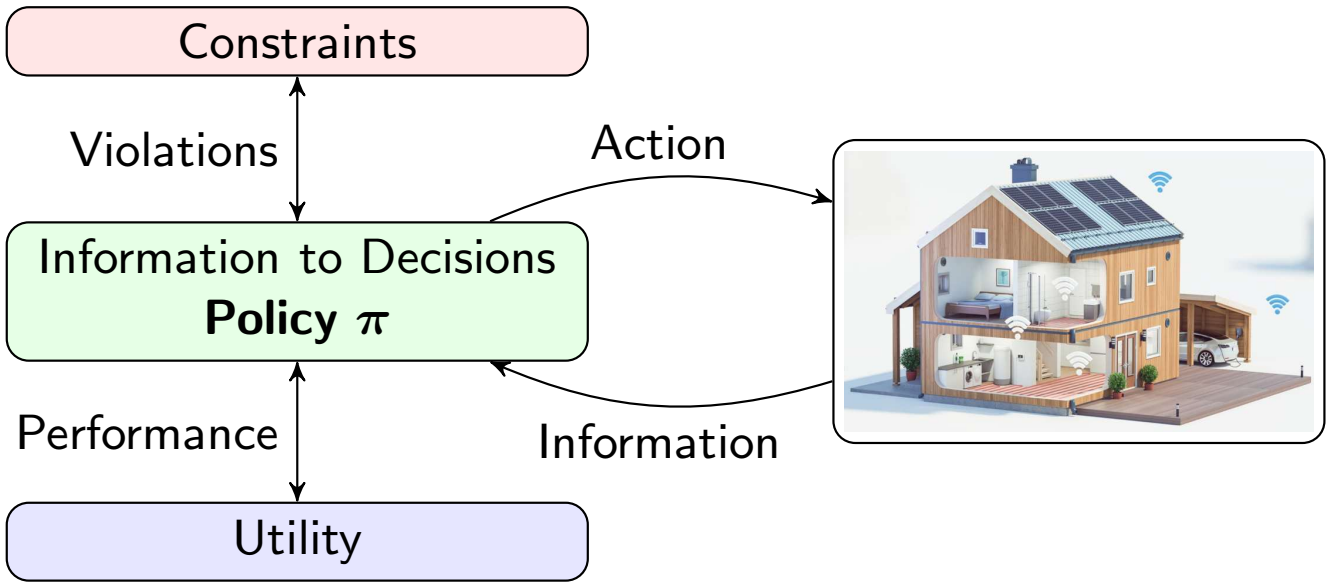

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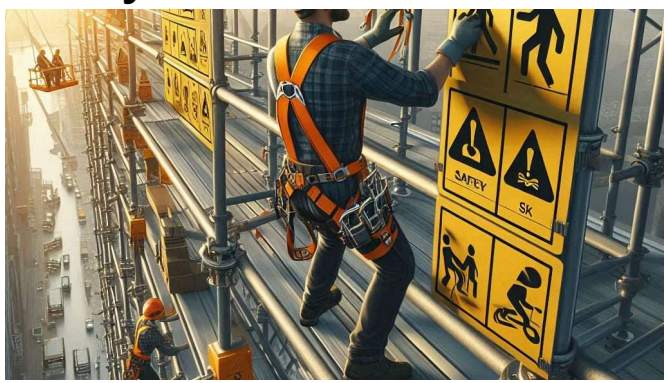

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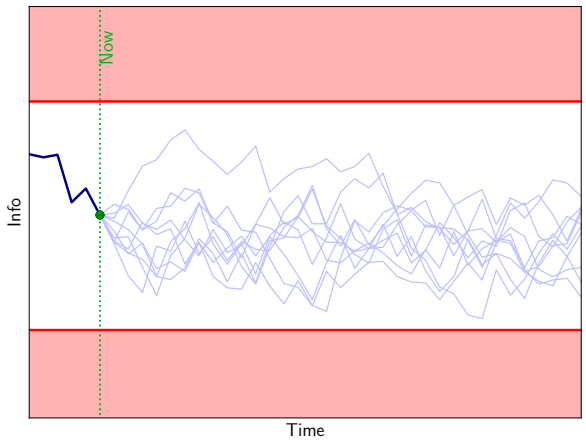

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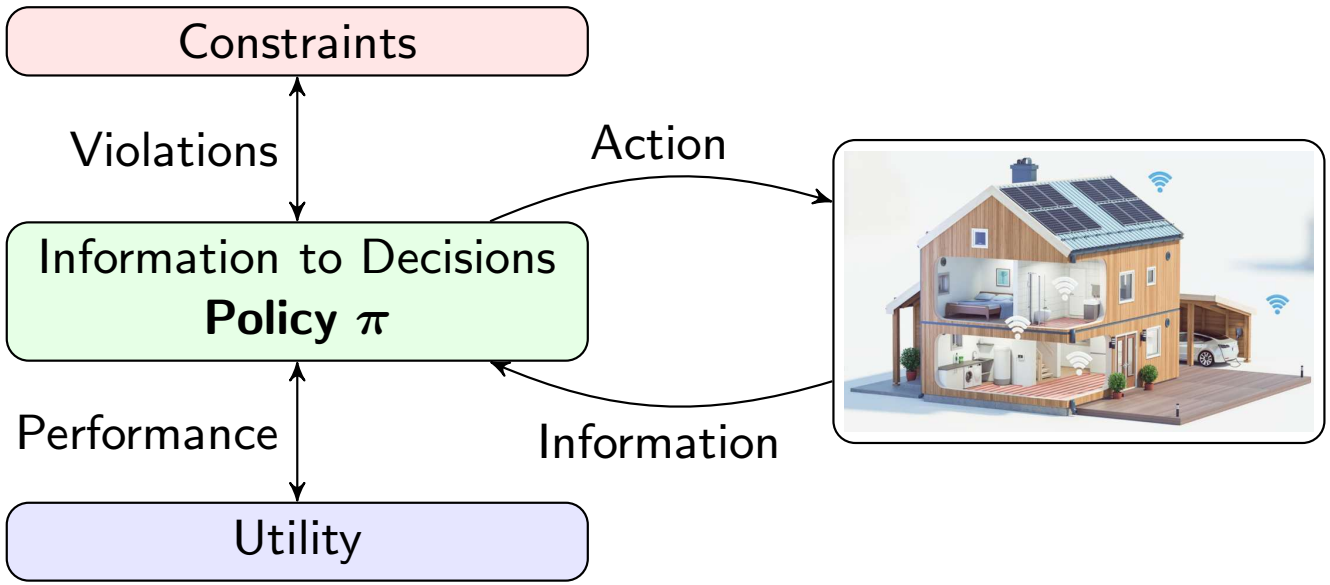

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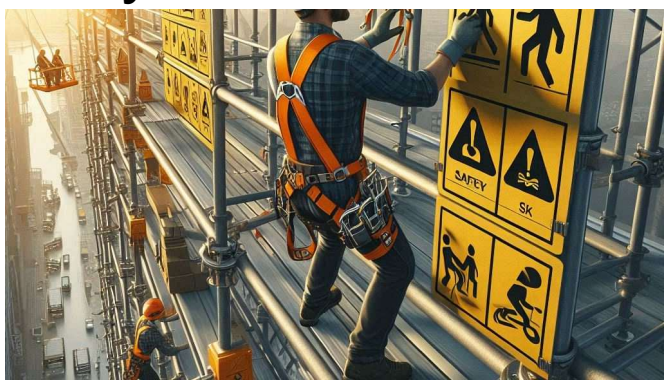

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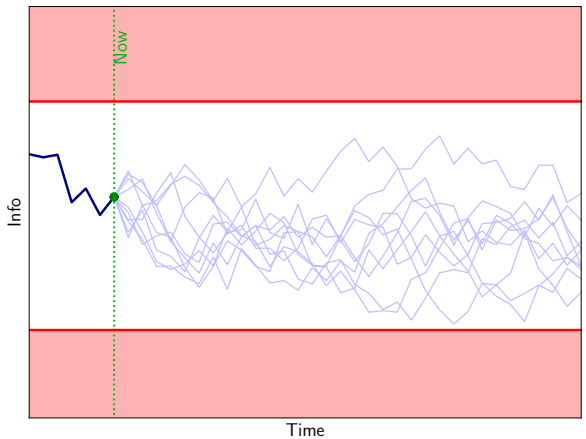

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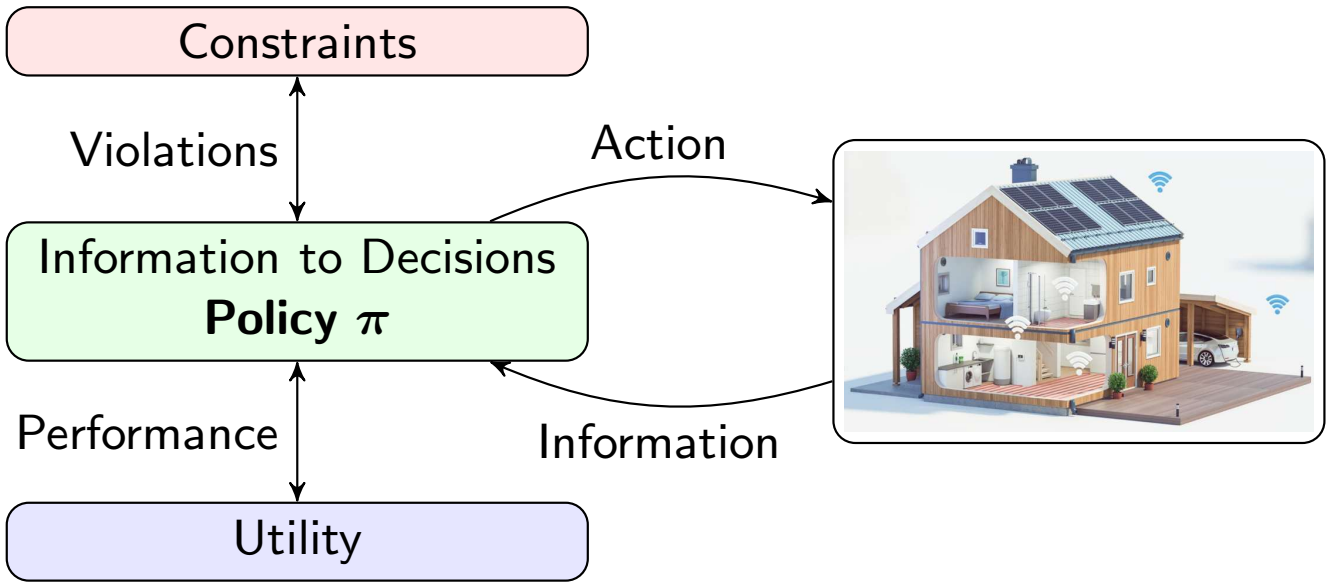

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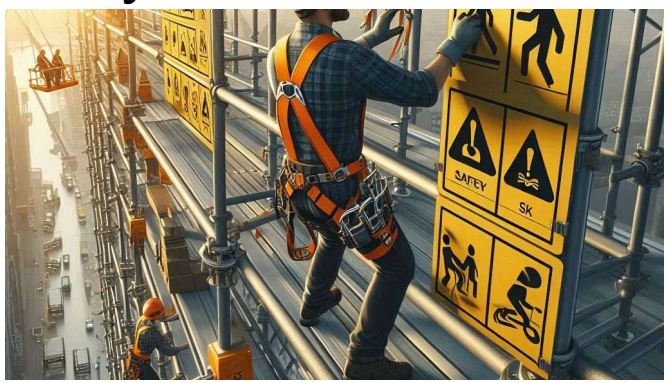

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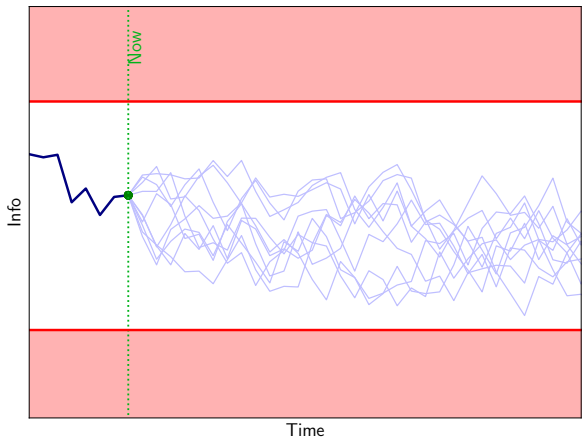

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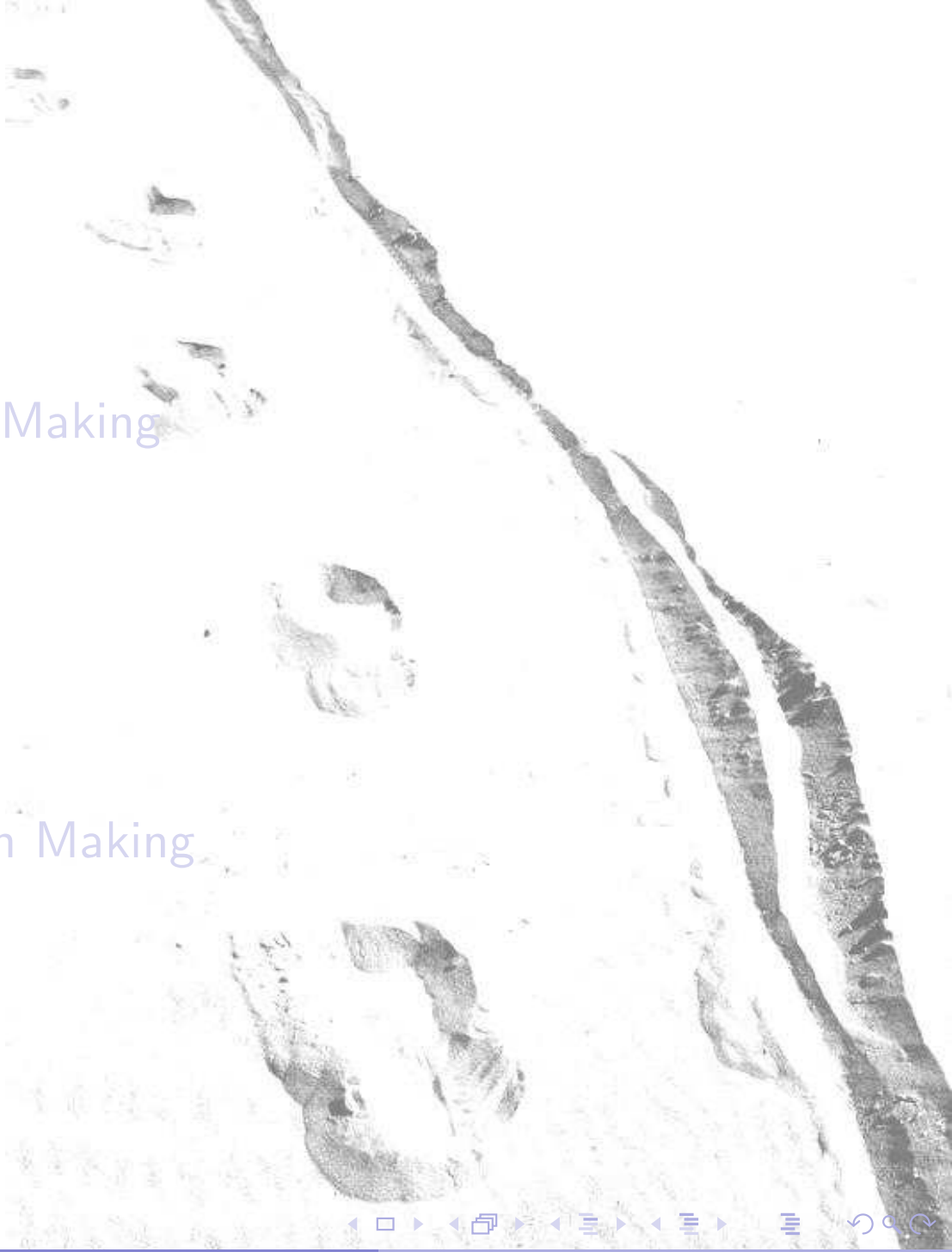
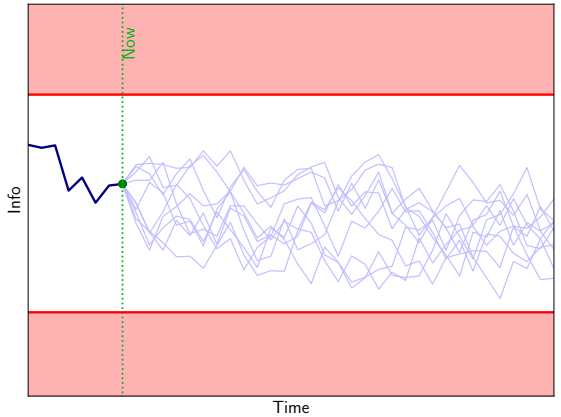


Constrained MDPs

$$\boldsymbol{\pi}^\star = \arg\max_{\boldsymbol{\pi}} \quad \mathrm{E}\left[\sum_{\text{time}} \text{Utility}\right]$$

$$\text{s.t.} \quad \text{Constraints ok at all time}$$



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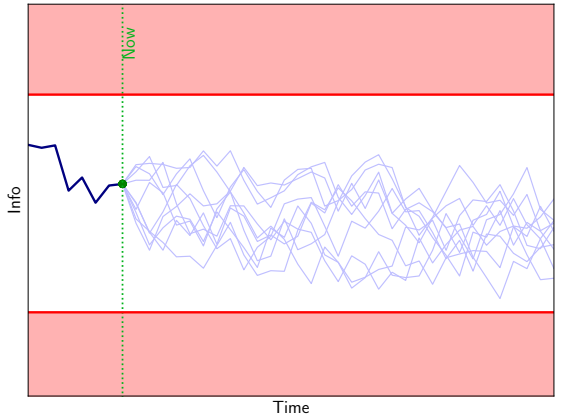

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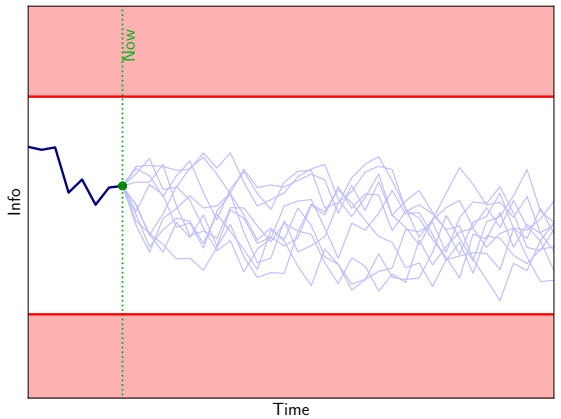

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Build policy using

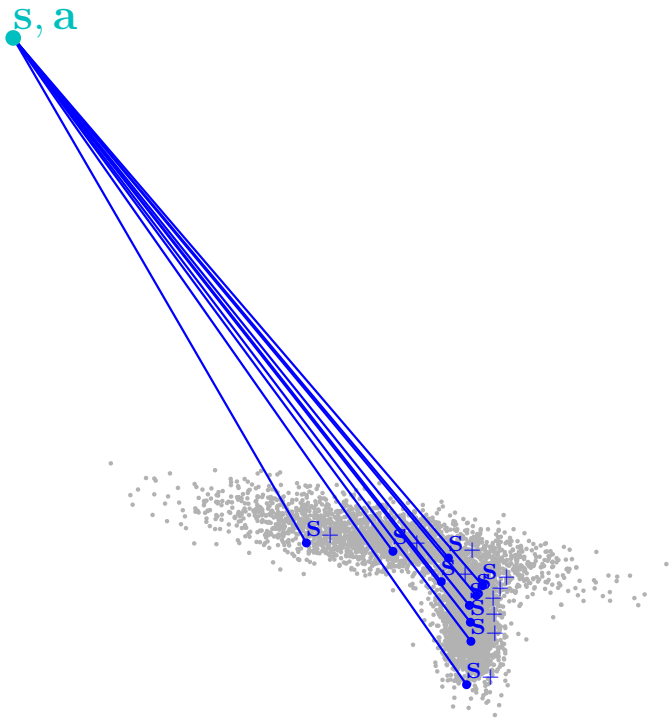
- Perfect model of the real world $\hat{\mathrm{P}}[\mathbf{s}_+|\mathbf{s},\mathbf{a}] = \mathrm{P}[\mathbf{s}_+|\mathbf{s},\mathbf{a}]$
- Model "pessimistic" about the uncertainties

... to evaluate "$\mathrm{E}[\cdot]$"

Pessimistic Models for Decision Making

- Model must "contain" the uncertainty
- "Container" (set) should be simple for computational reasons
- Trajectories predicted by pessimistic model will "cover" the real world
- Decision policy wants to be safe w.r.t. the "containers"

Distribution of one-step forward



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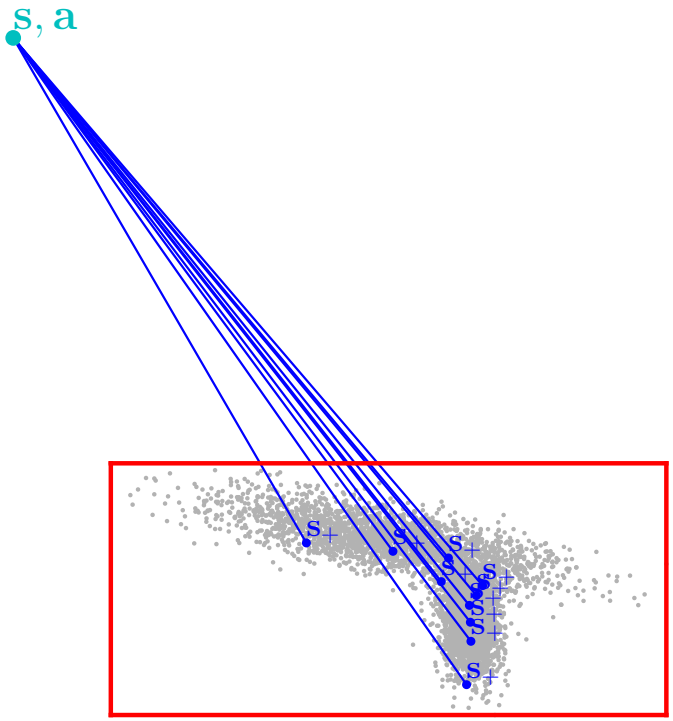
Distribution of one-step forward with "container"



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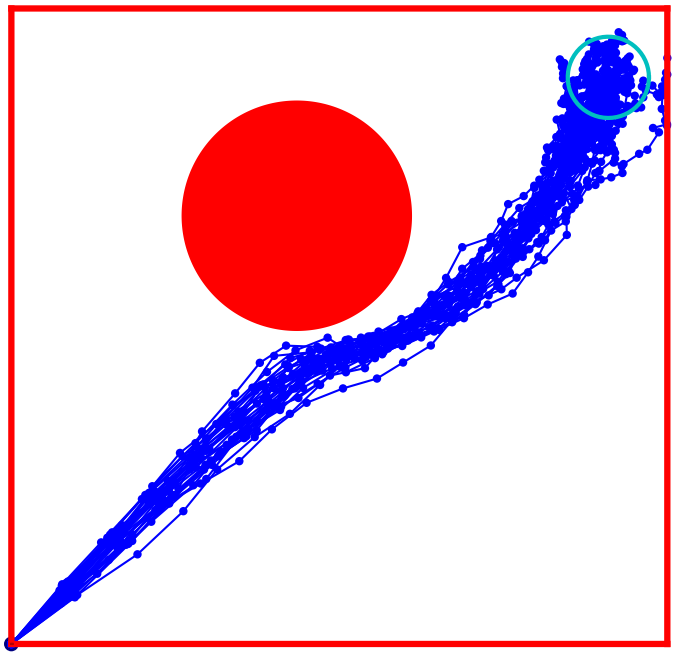

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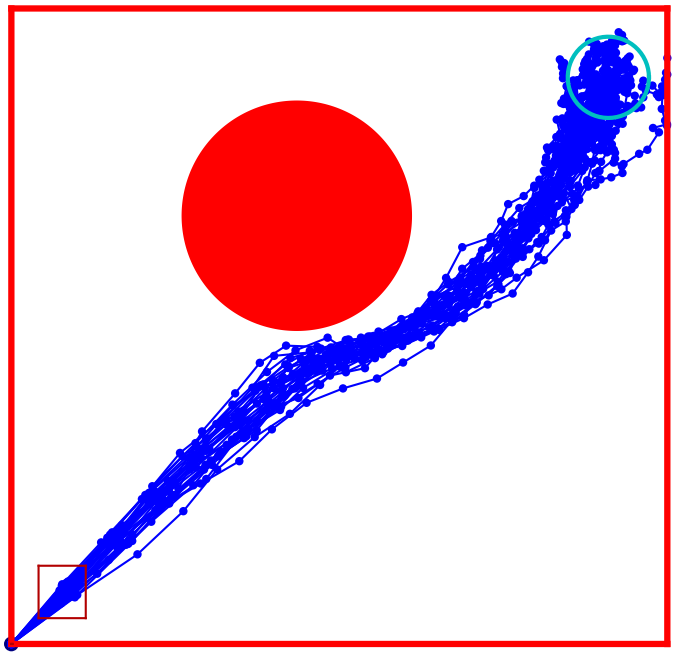

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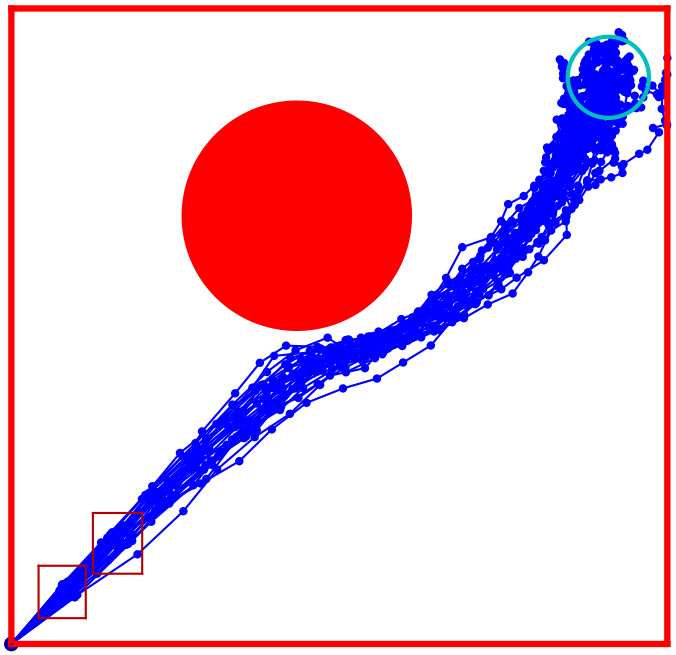

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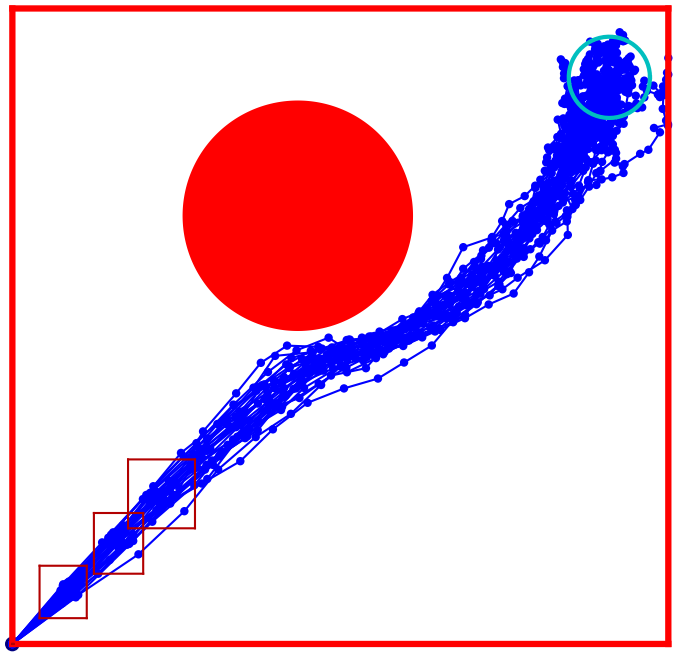

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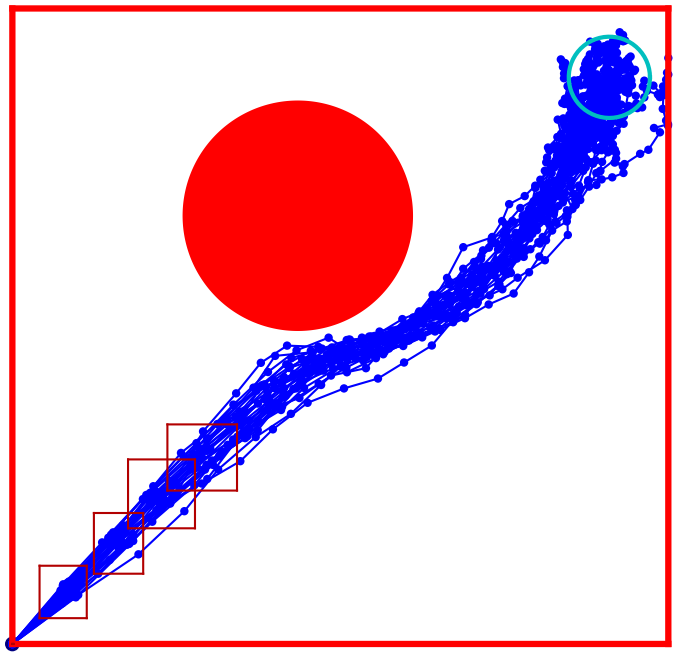

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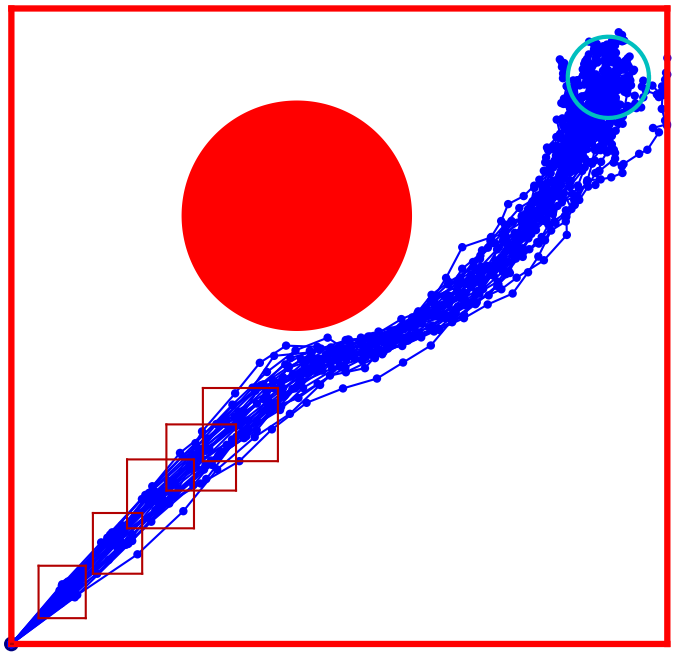

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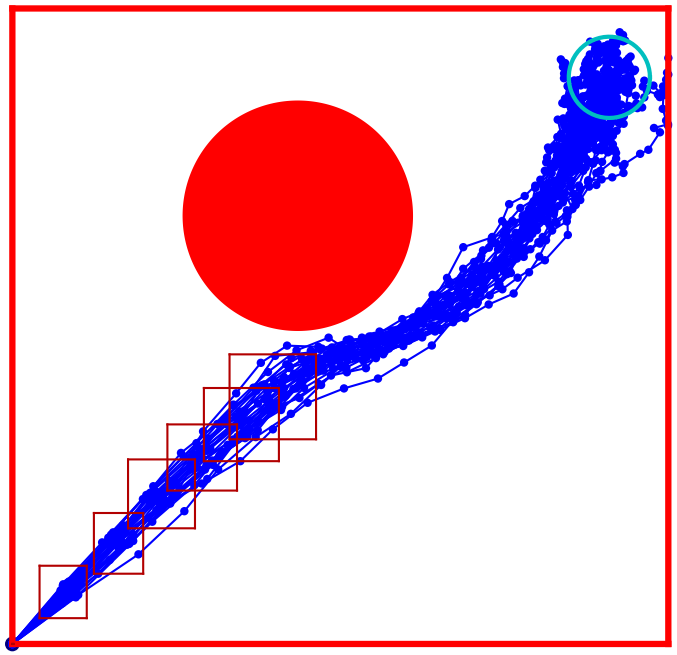

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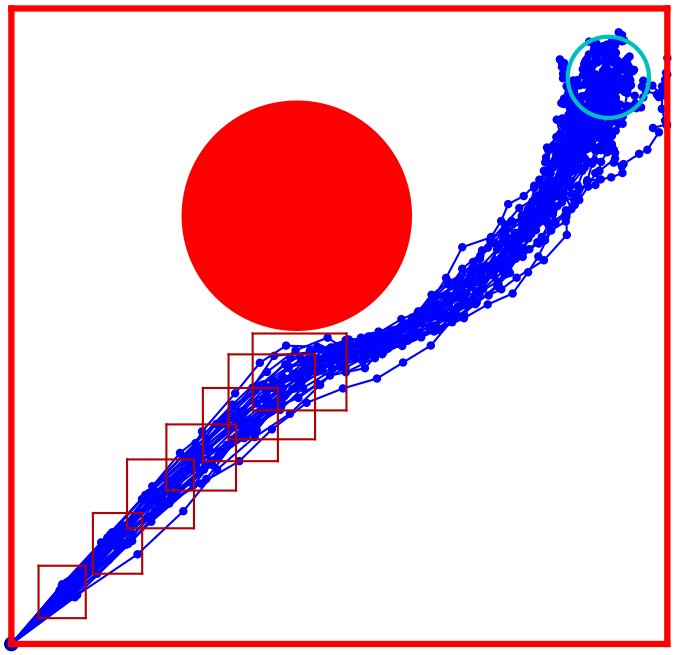

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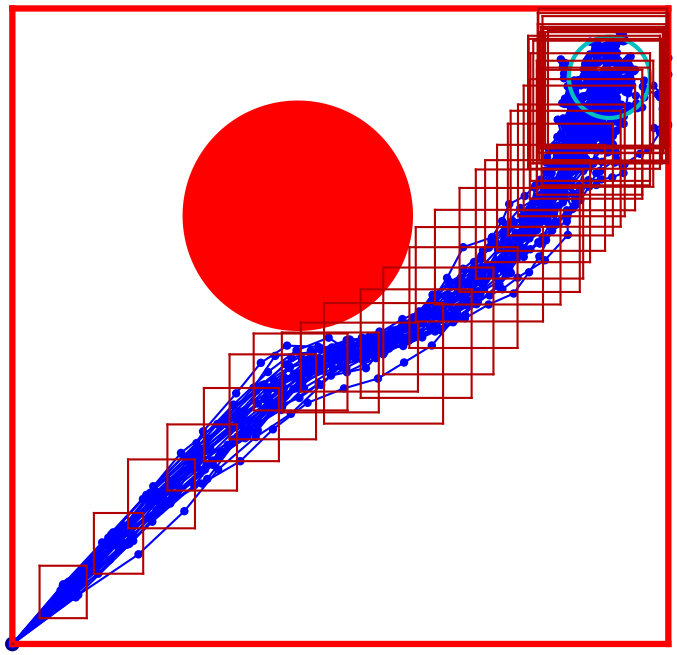

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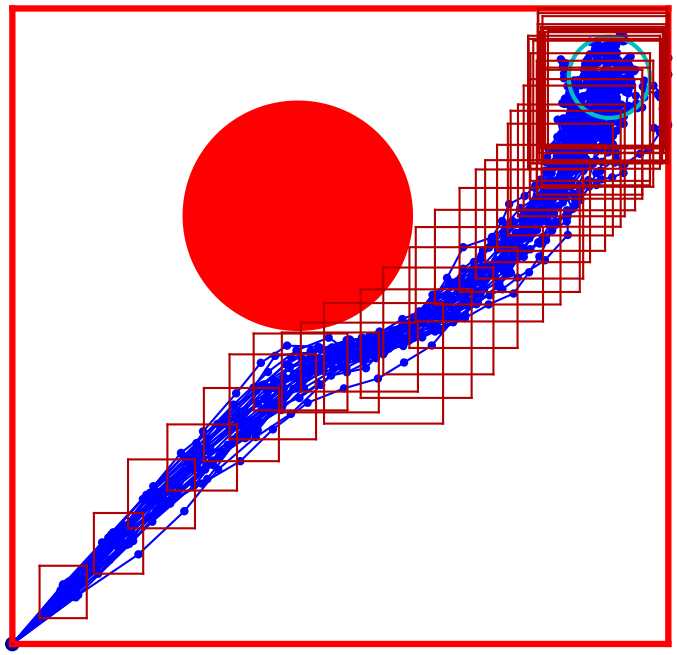

Remarks

- The propagation of the "containers" in the model predictions can be expensive / difficult
- Policy based on worst-case perspective makes the decisions highly conservative
- Often labelled "Robust" decision making

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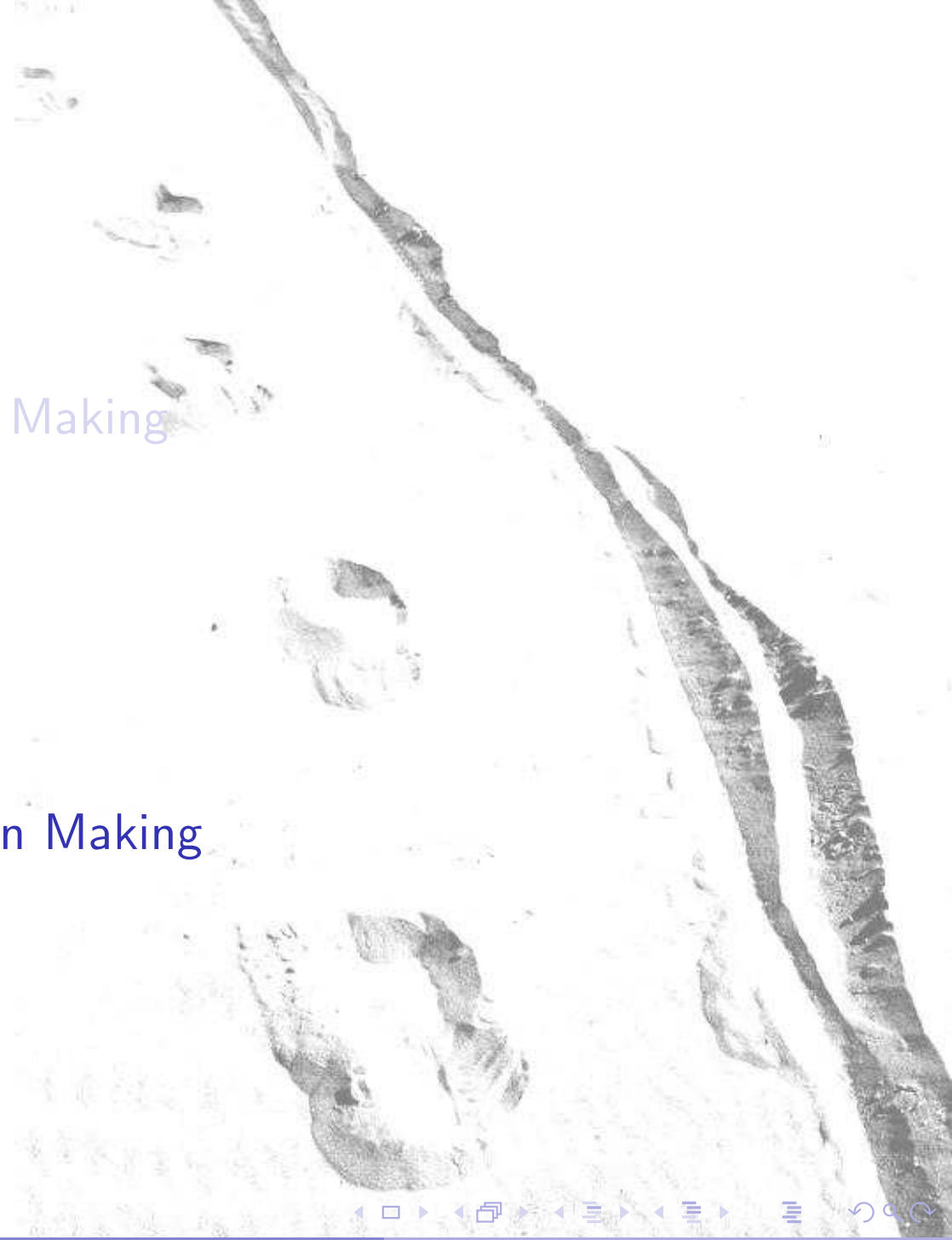

MDPs with probabilistic safety

In words

$$\boldsymbol{\pi}^\star = \arg\max_{\boldsymbol{\pi}} \quad \mathrm{E}\left[\sum_{\text{time}} \text{Utility}\right]$$
$$\text{s.t.} \quad \text{Probability of no violation} \geq c$$

Formally

$$\boldsymbol{\pi}^\star = \arg\max_{\boldsymbol{\pi}} \quad \mathrm{E}\left[\sum_{k=0}^{\infty} \gamma^k L\left(\mathbf{s}_k, \mathbf{a}_k\right)\right]$$
$$\text{s.t.} \quad \mathrm{P}\left[\mathbf{s}_{0,\ldots\infty} \in \mathbb{S}\right] \geq c$$