

Introduction to Theory of Safe Decision Making

Dr. Akhil Anand

1st AID Scientific Workshop, Trondheim

Forewords & Disclaimer

Objectives:

- Put in place some common concepts & language
- Identify some key points in safe decision making
- Connect to AI

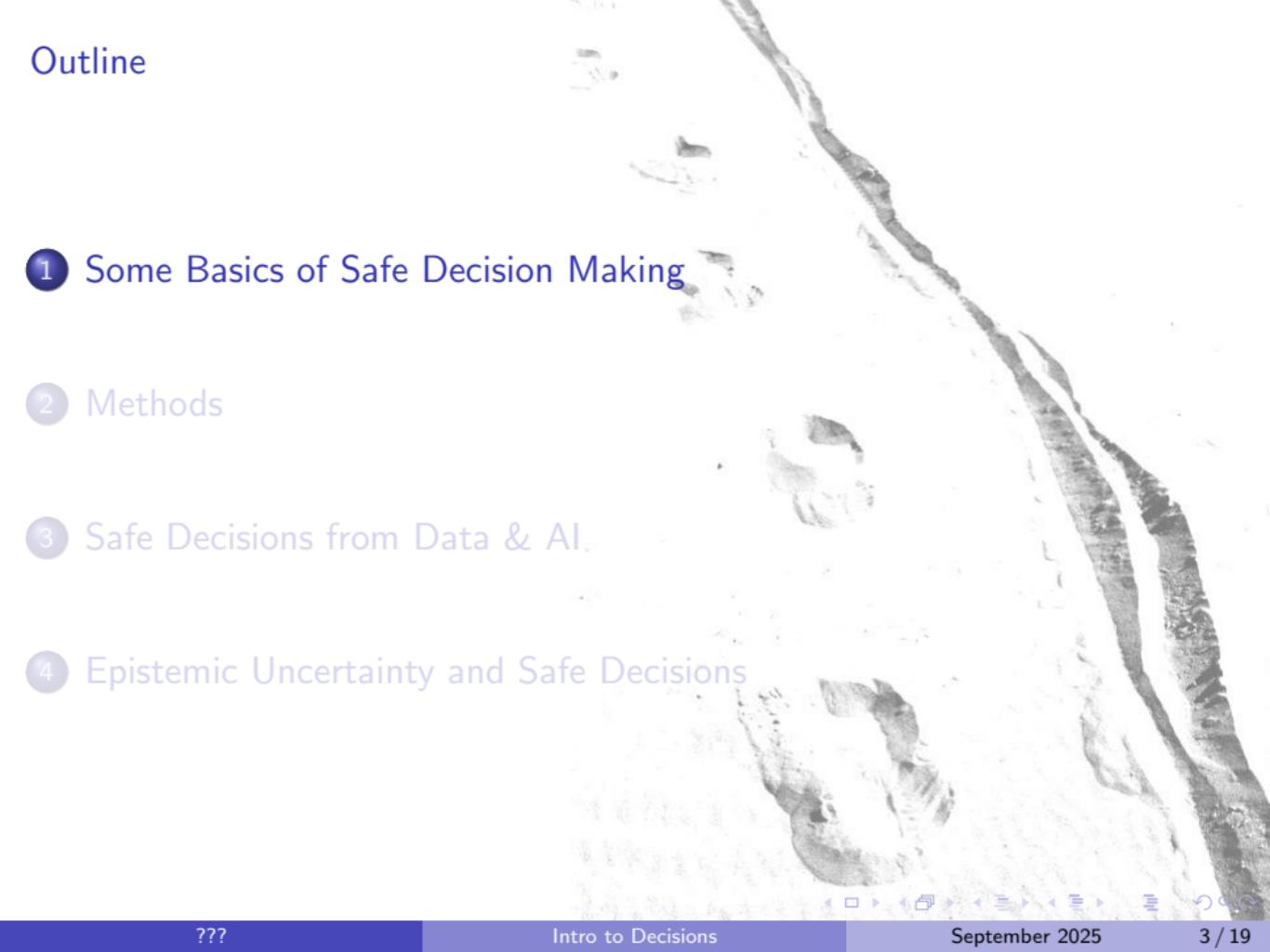
Disclaimer: we are a broad group who needs to get to know each others scientifically.

Apologies if I don't "hit" the right level for all.

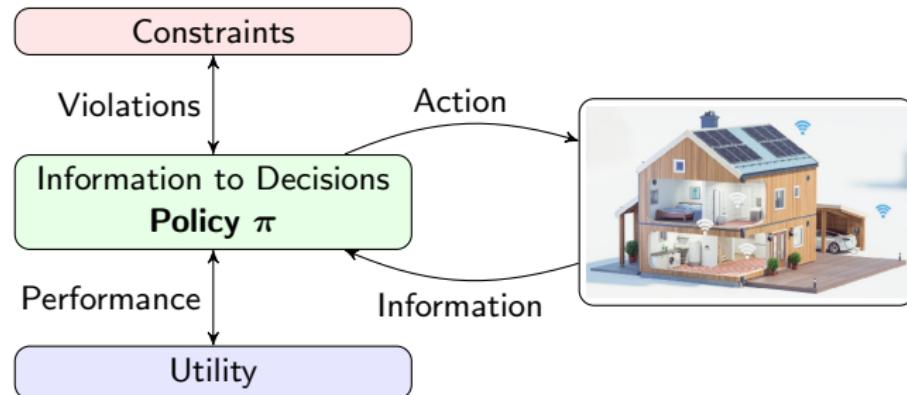
I have favored simplicity over "absolute" rigor.



Outline

- 
- 1 Some Basics of Safe Decision Making
 - 2 Methods
 - 3 Safe Decisions from Data & AI
 - 4 Epistemic Uncertainty and Safe Decisions

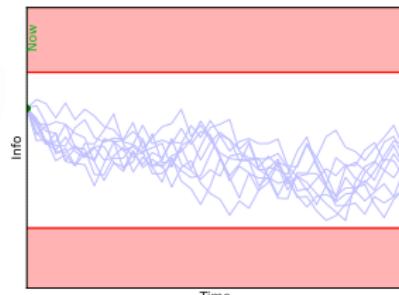
Formally Defining Safety?



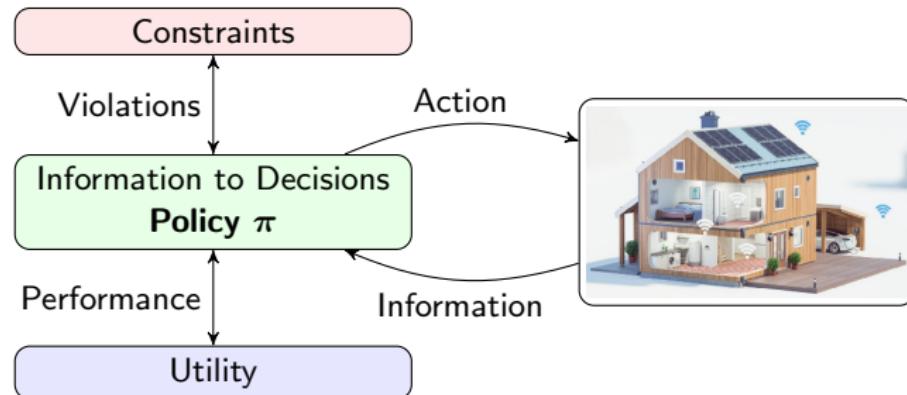
Safety in the real world



Safety in the math world



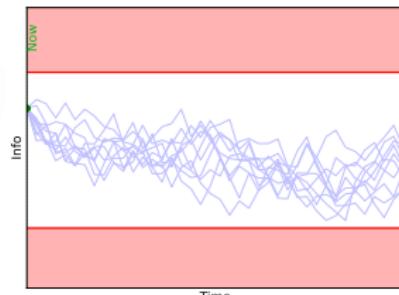
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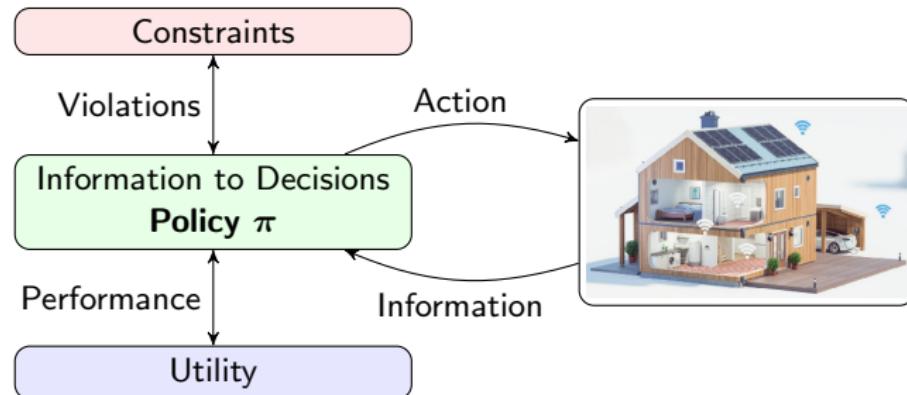
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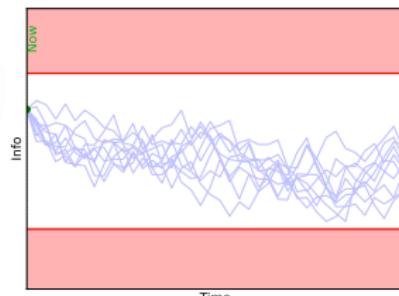
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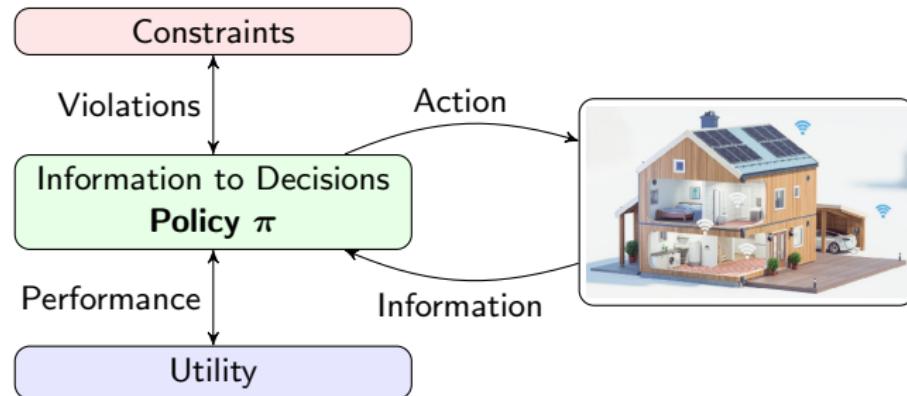
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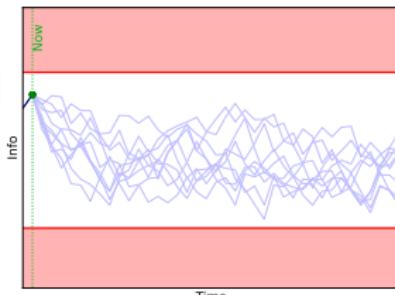
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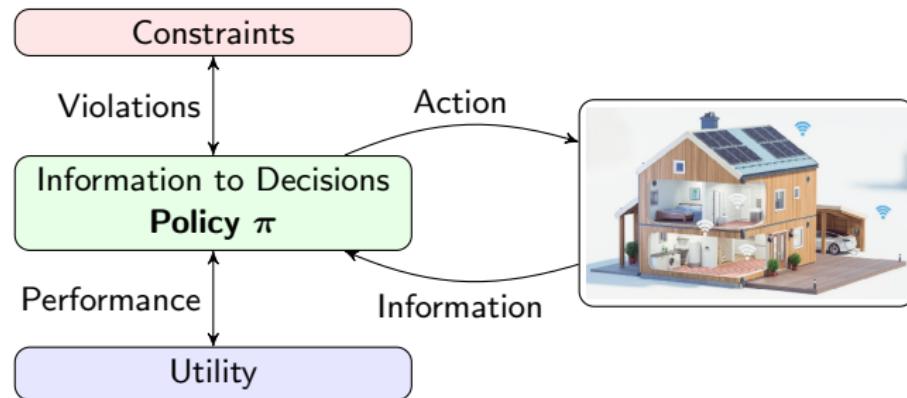
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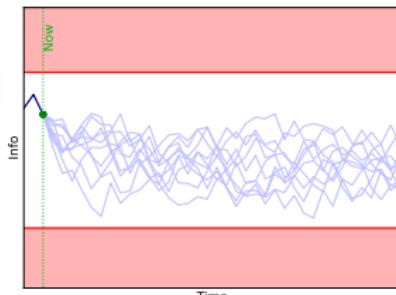
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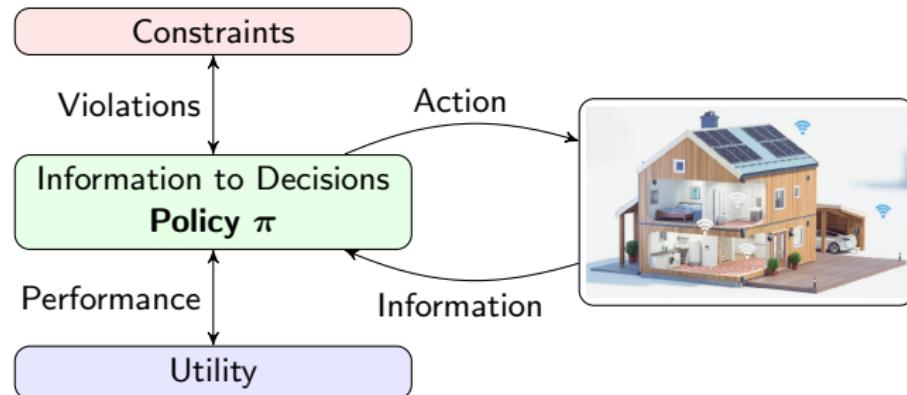
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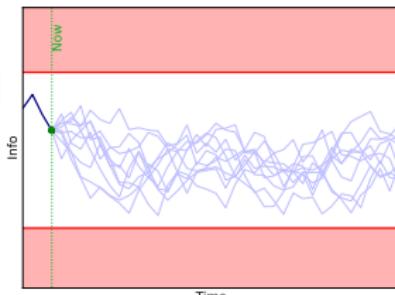
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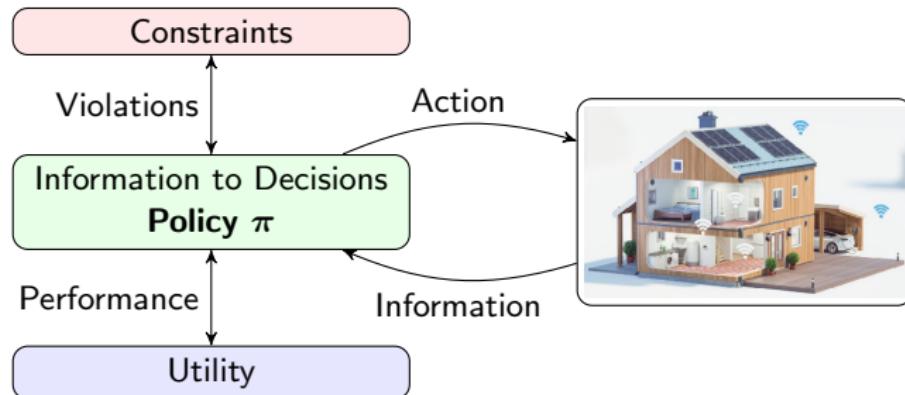
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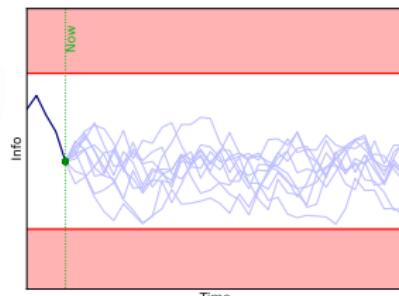
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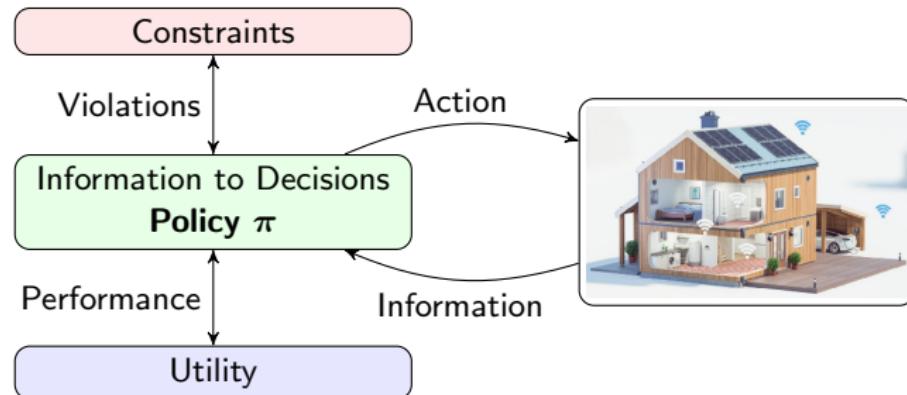
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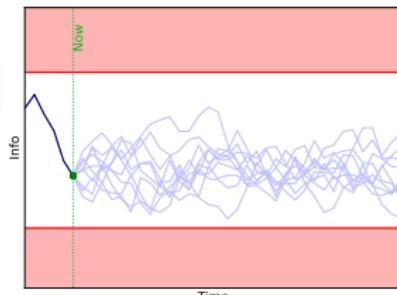
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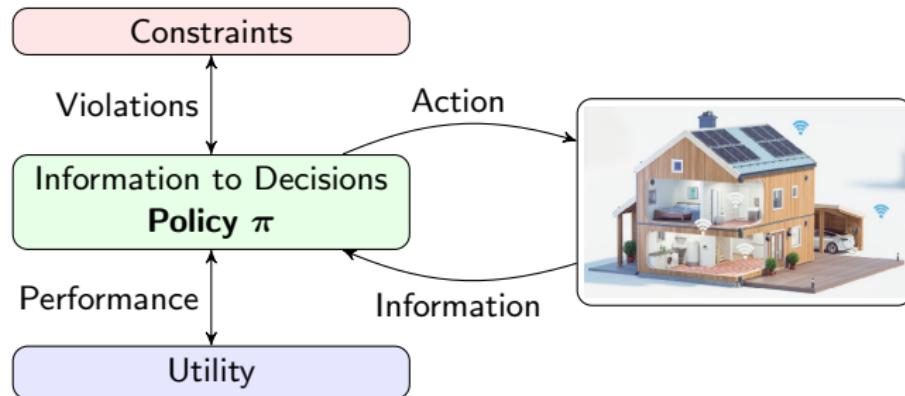
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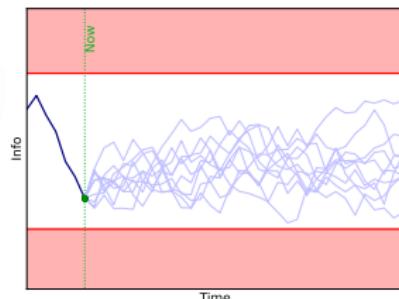
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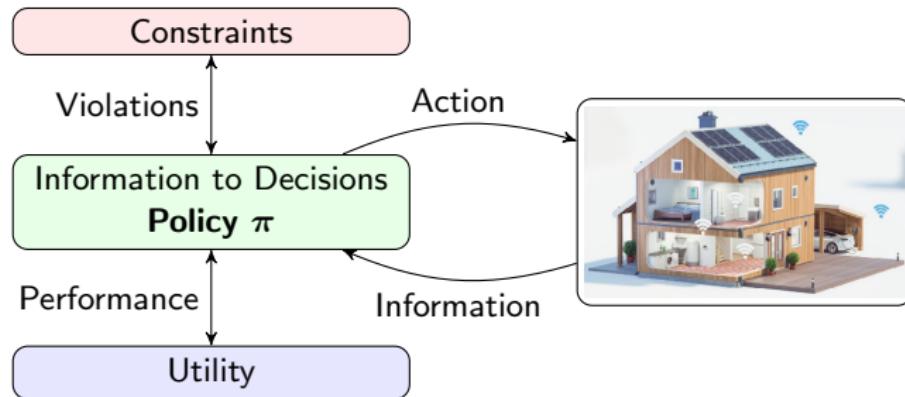
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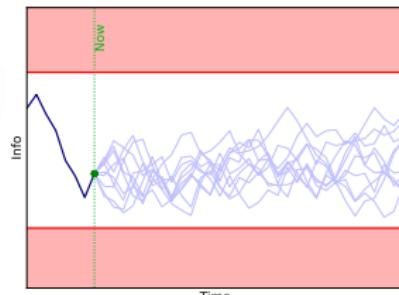
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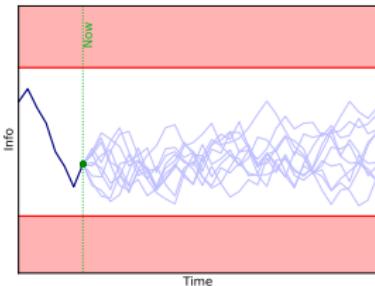


Constrained MDPs

In words

$$\pi^* = \arg \max_{\pi} E \left[\sum_{\text{time}} \text{Utility} \right]$$

s.t. Constraints ok at all time

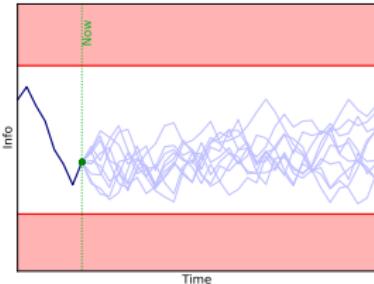


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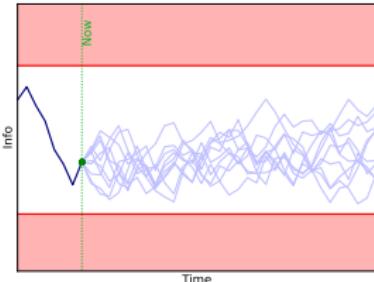
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Build policy using

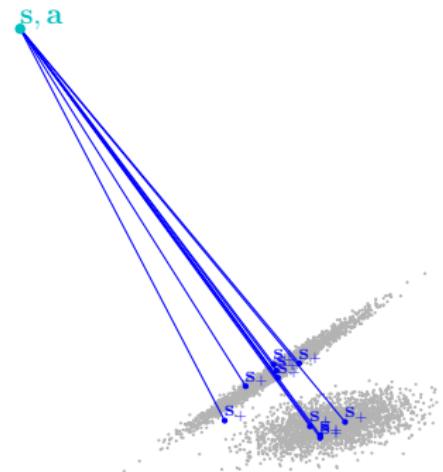
- Perfect model of the real world $\hat{P}[s_+|s, a] = P[s_+|s, a]$
- Model “pessimistic” about the uncertainties

... to evaluate “ $E[\cdot]$ ”

Pessimistic Models for Decision Making

- Model must “contain” the uncertainty
- “Container” (set) should be simple for computational reasons
- Trajectories predicted by pessimistic model will “cover” the real world
- Decision policy wants to be safe w.r.t. the “containers”

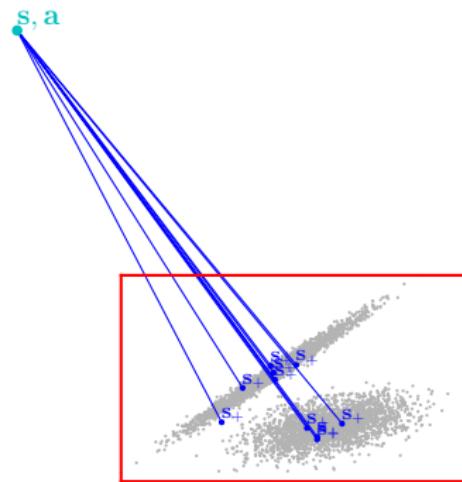
Distribution of one-step forward



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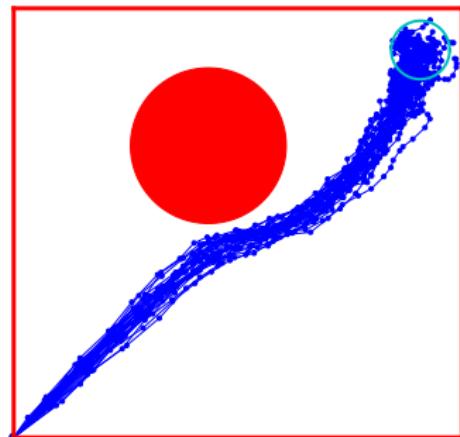
Distribution of one-step forward with “container”



Pessimistic Models for Decision Making

Trajectories

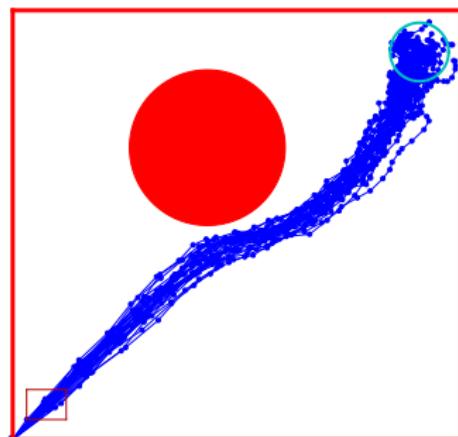
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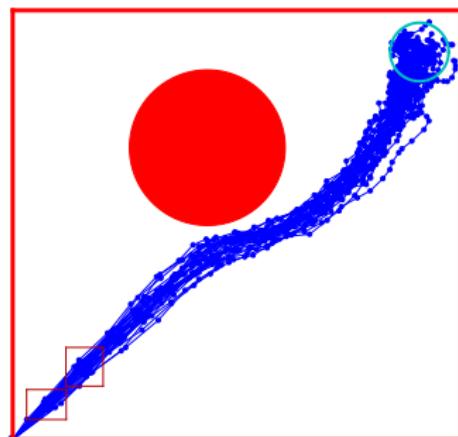
Trajectories with “containers”



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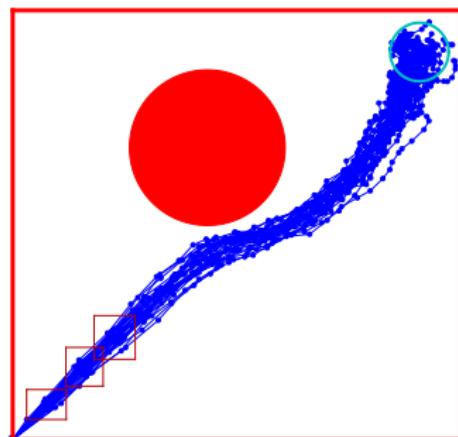
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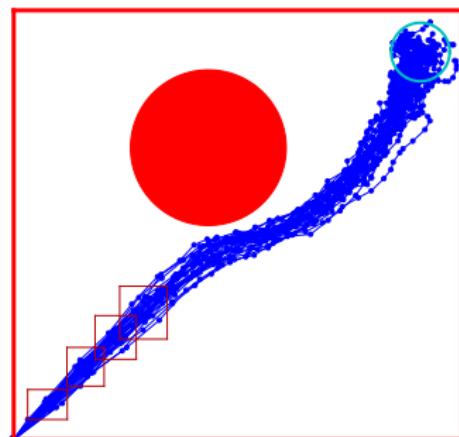
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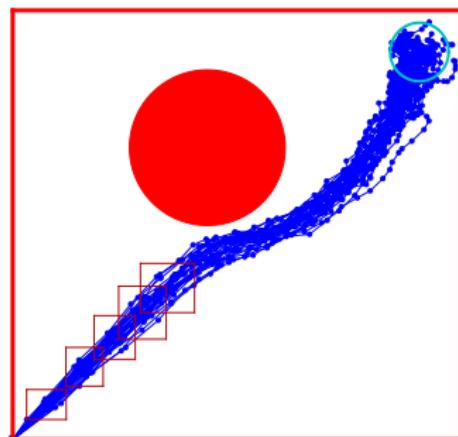
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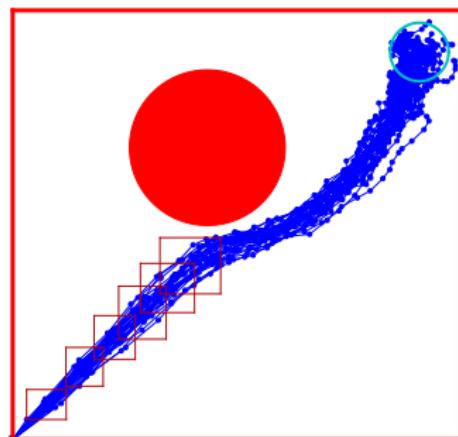
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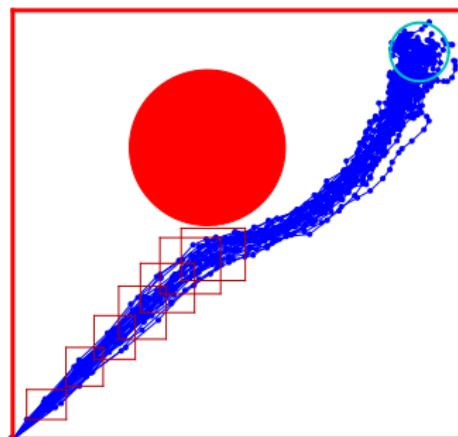
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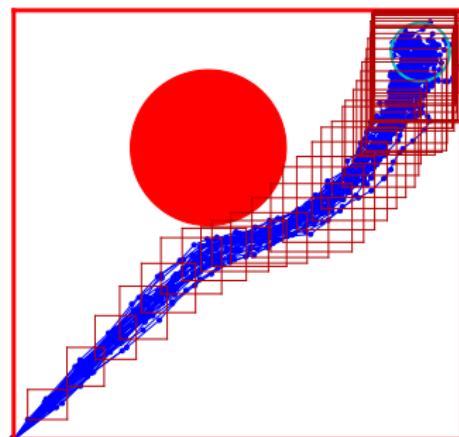
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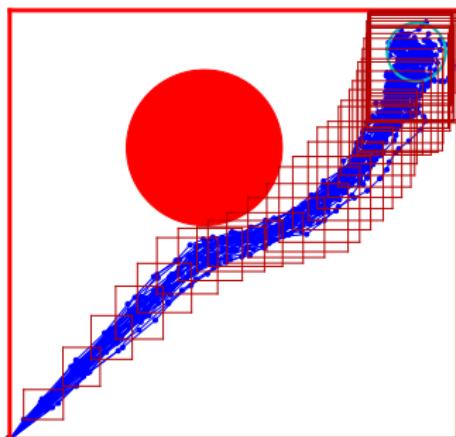
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Trajectories with “containers”



Remarks

- The propagation of the “containers” in the model predictions can be expensive / difficult
- Pessimistic propagations are usually needed → pessimistic over pessimistic
- Policy based on worst-case perspective makes the decisions highly conservative
- Often labelled “Robust” decision making

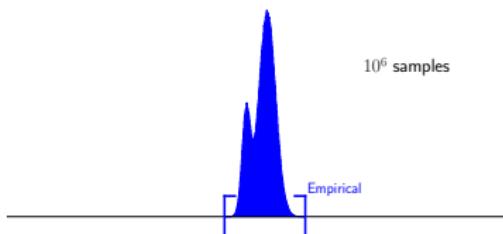
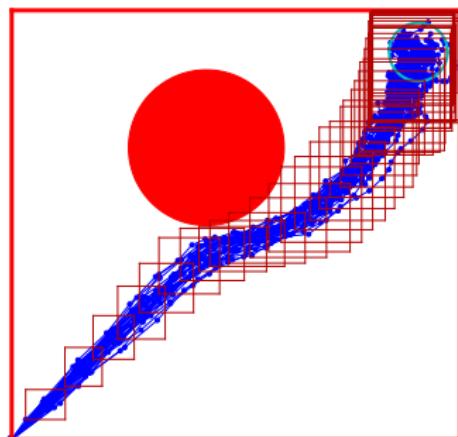
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Epistemic problem: build container...

- from data, but it may expand with more data and safety is “asymptotic”, as data $\rightarrow \infty$

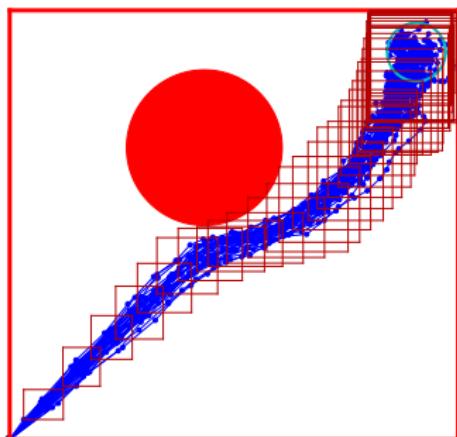
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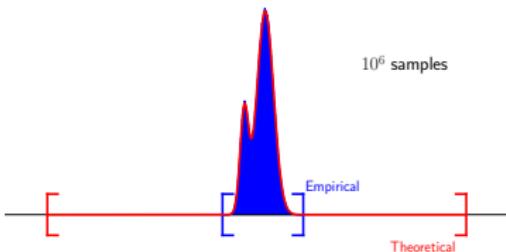
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- “theoretically” (from knowledge), but can be impractically pessimistic

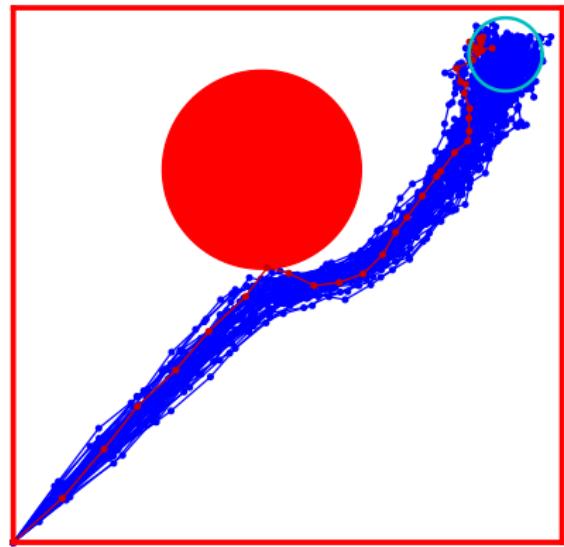


MDPs with probabilistic safety

In words

$$\pi^* = \arg \max_{\pi} E \left[\sum_{\text{time}} \text{Utility} \right]$$

s.t. Probability of no violation $\geq c$



MDPs with probabilistic safety

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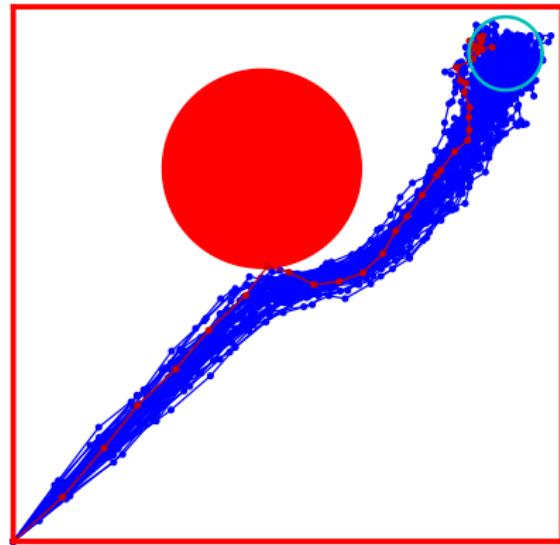
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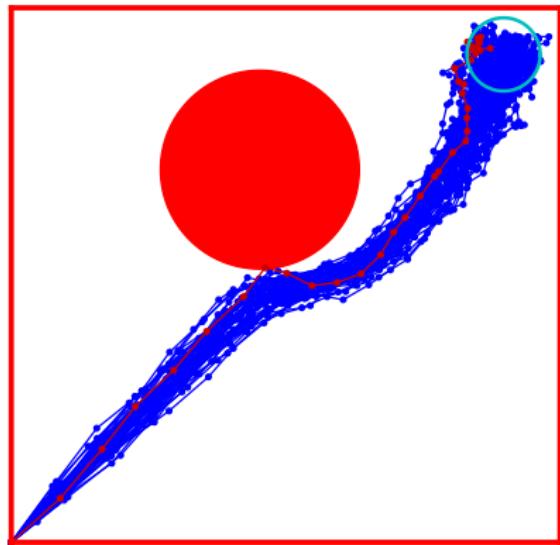
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Remarks:

- If we can tolerate $c > 0$ (small) it can make a big gain in performance
- Aligned with industrial / practical standards on “large series”
- Problem needs a “termination” (time or goal reached)
- Building decisions can be difficult from a computational point of view



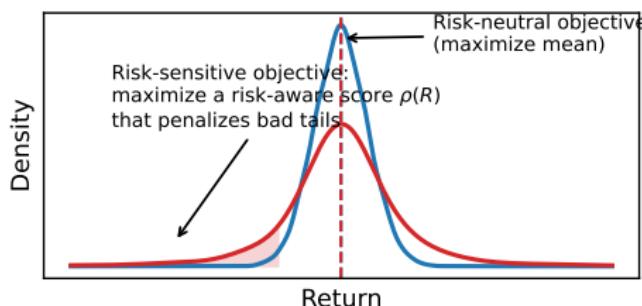
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- Instead of constraints, the utility function itself is altered to contain a risk measure of the return.
- Intuition: a “tail-risk filter” even if outcomes are good most of the time, severe tails influence decisions.

Risk-sensitive MDPs

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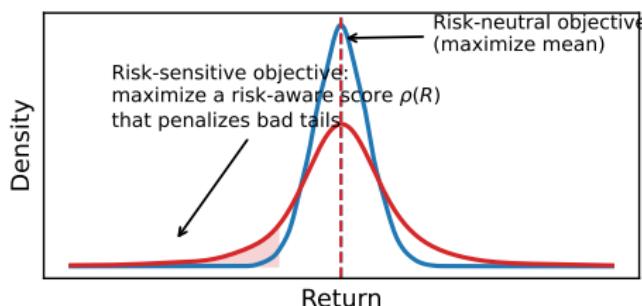
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$$\pi^* = \arg \max_{\pi} \rho \left(\sum_{t=0}^{\infty} \gamma^t r_t \right)$$

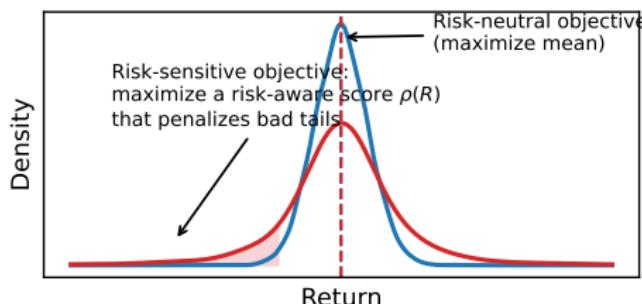
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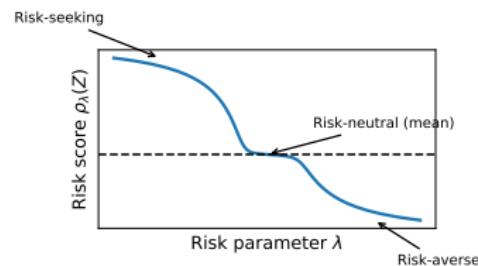
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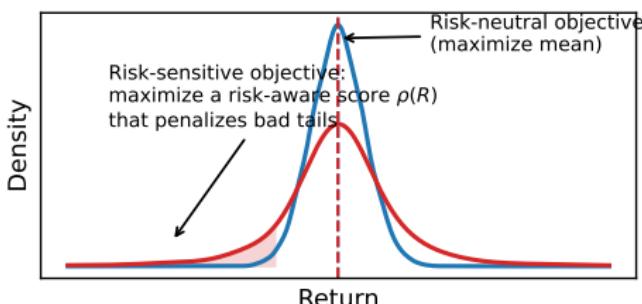
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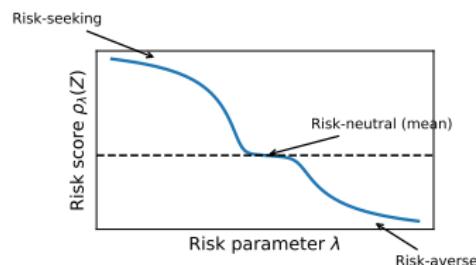
- Views risk from through the lense of utility (not undesirable events).
- Risk-sensitive criteria do *not* guarantee a bound on violation.
- Cares only about the average severity of bad outcomes in the tail and not about the rest

???

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Robust Repeated Planning

- Introduction: Robust MPC, Scenario Trees, MC, robust multi-stage stochastic programming, MPPI?
- Difficulties: guarantees for nonlinear systems, persistent safety (recursive feasibility)

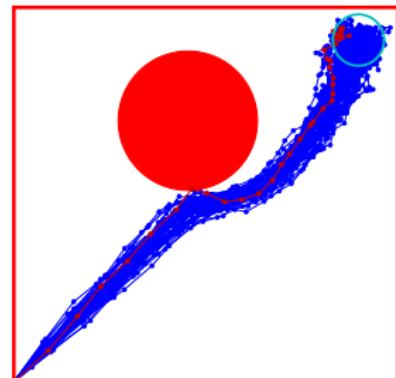
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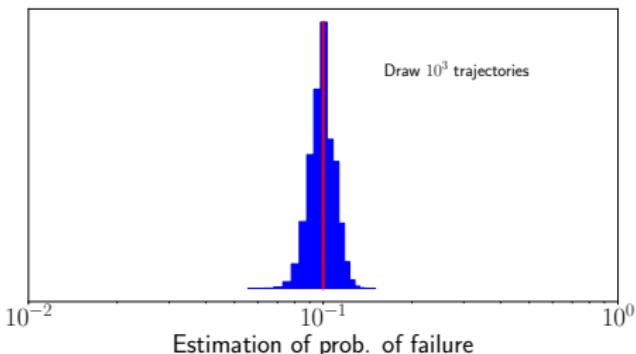
Dynamic Programming & RL with Probabilistic Safety

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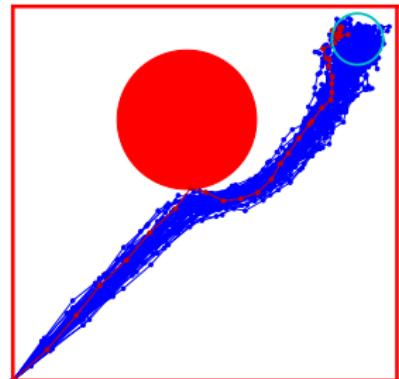
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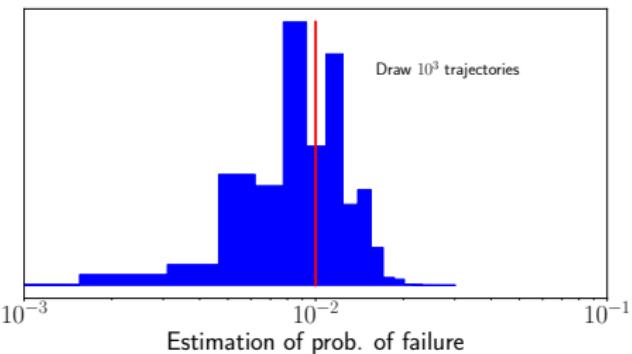
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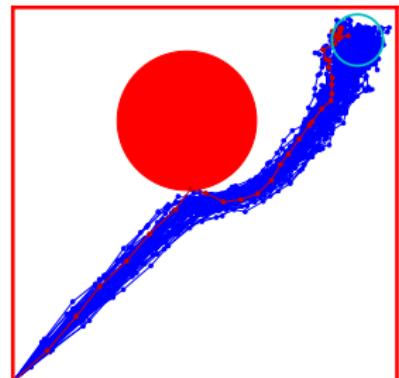
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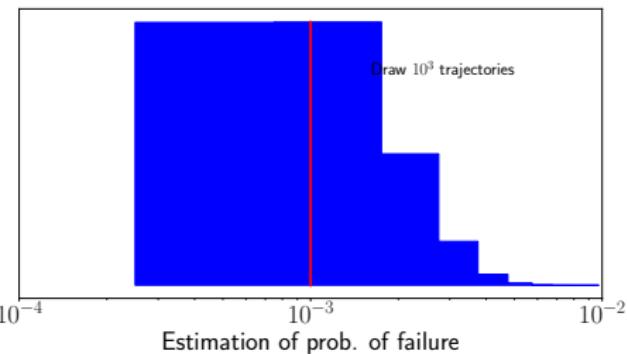
Dynamic Programming & RL with Probabilistic Safety

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s.t. $P[s_0, \dots, \infty \in S] \geq c$



- Classical techniques (e.g. DP and RL) apply but they become very data intensive
- Difficulties: estimate expected values from data (real or not) when $P[s_0, \dots, \infty \in S]$ is close to 1.



Outline

- 
- 1 Some Basics of Safe Decision Making
 - 2 Methods
 - 3 Safe Decisions from Data & AI
 - 4 Epistemic Uncertainty and Safe Decisions

Safety Filters & Reinforcement Learning

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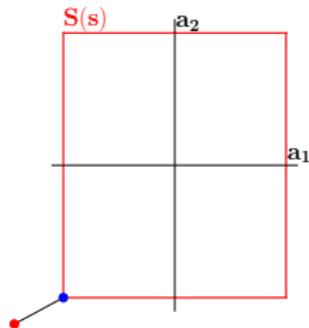
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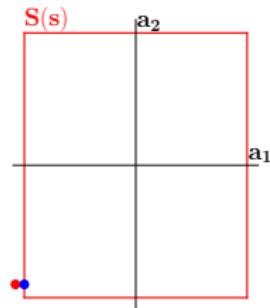
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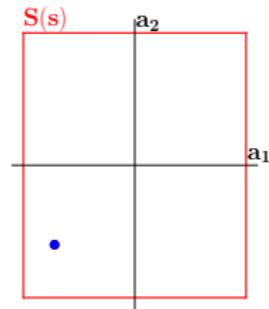
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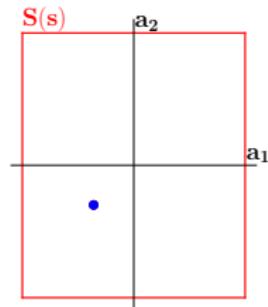
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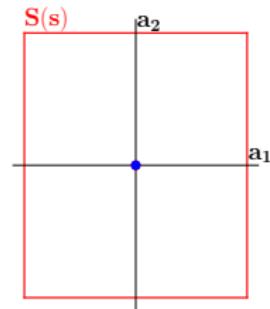
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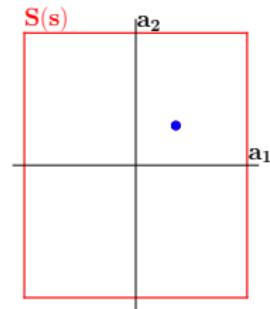
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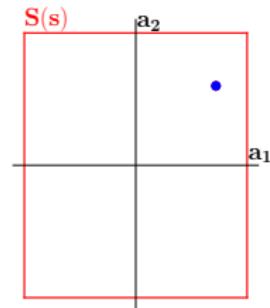
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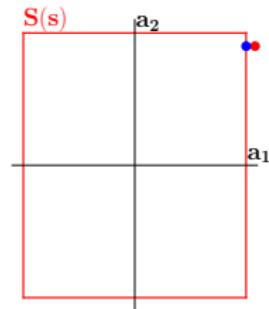
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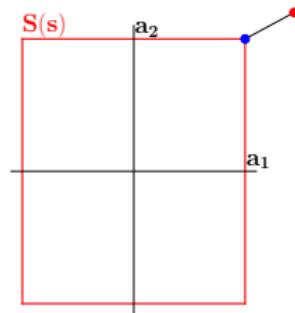
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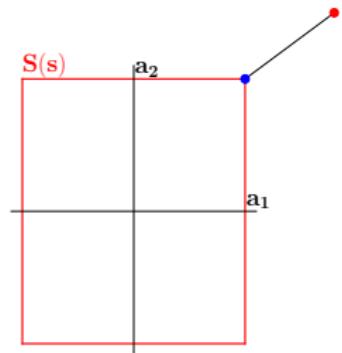
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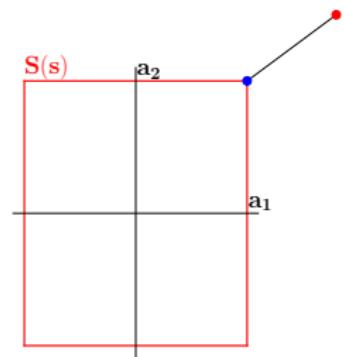
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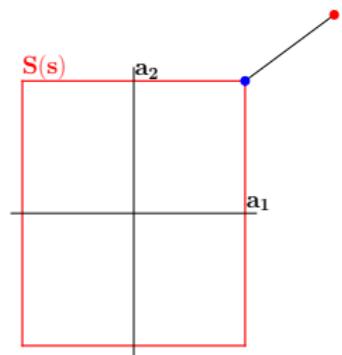
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Able to stop within the visible distance



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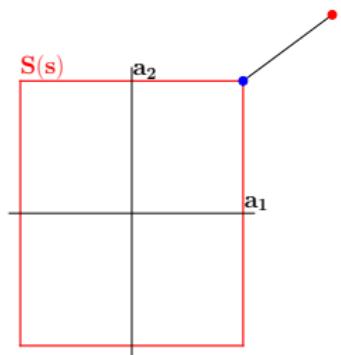
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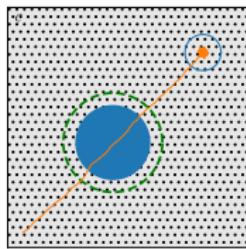
- Often close to practice, e.g., flight envelop protection in modern airplane, semi-autonomous driving in cars, etc
- Oracle \Rightarrow knowledge-based & conservative!



Control Barrier Functions (CBFs)

In words

- CBFs provide a formal, model-based way to build the “oracle” safe set.
- Use a barrier function $h(x)$ so that staying safe means $h(x) \geq 0$.
- At each step, if the proposed action would leave the safe set, a small QP minimally adjusts it to be safe.

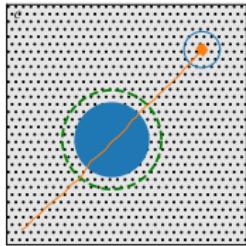


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Formally

$$\dot{x} = f(x) + g(x)u, \quad \mathcal{C} = \{x : h(x) \geq 0\}$$

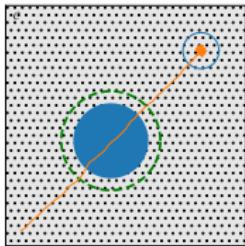
$$u^*(x) = \arg \min_u \frac{1}{2} \|u - u_{\text{des}}(x)\|_2^2 \quad \text{s.t.}$$

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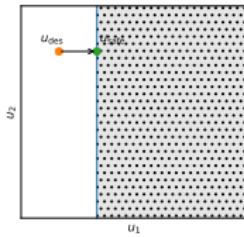
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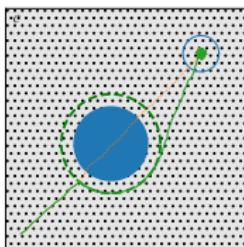


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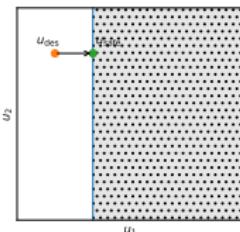
CBF-filtered trajectory.

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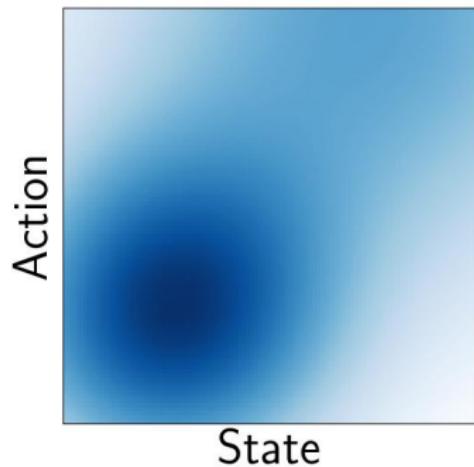
- The barrier function and safe set \mathcal{C} are typically **constructed from domain knowledge** (physics, rules, safety envelopes).
- Requires a (possibly simplified, conservative) **system model: robust approach**.

Akhil can do on 11.9

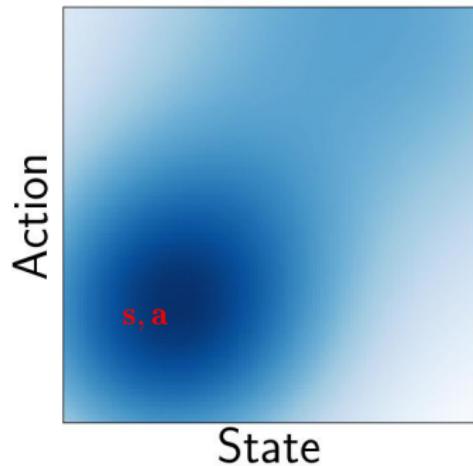
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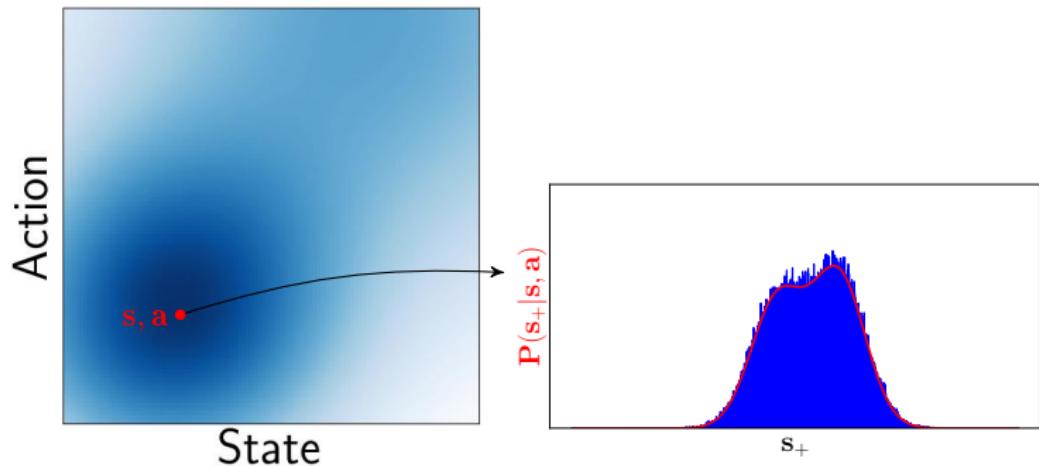
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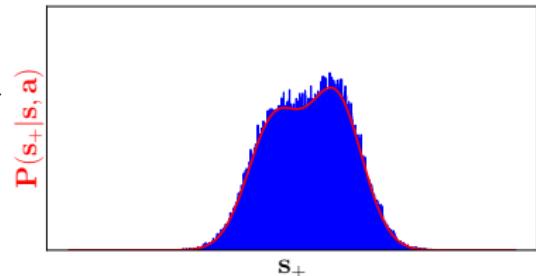
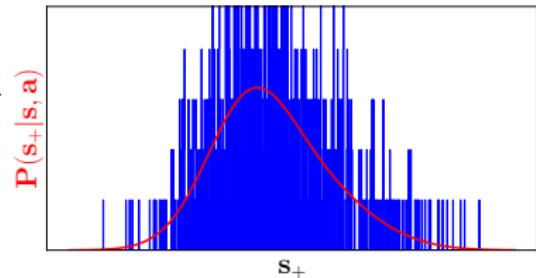
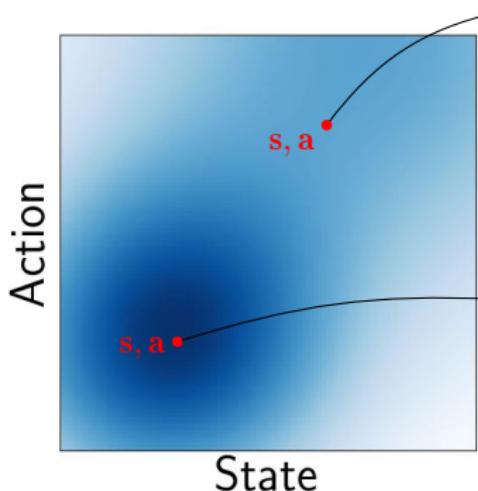
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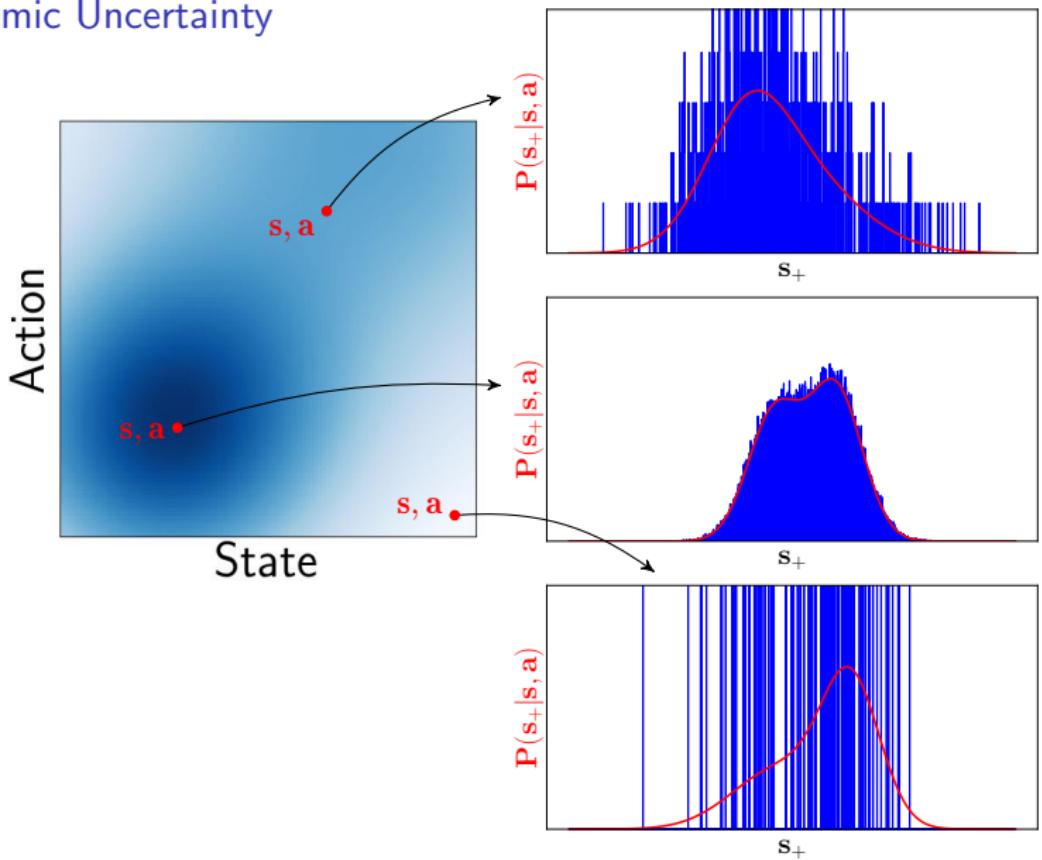
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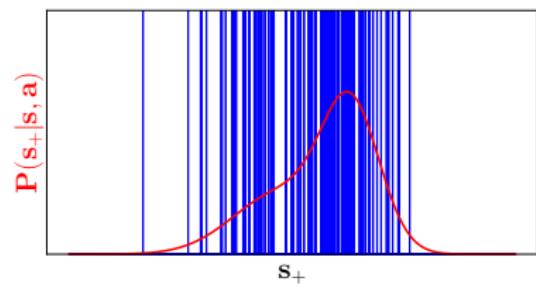
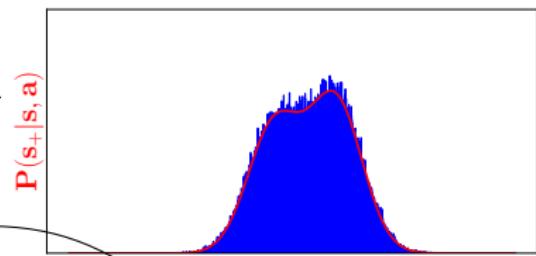
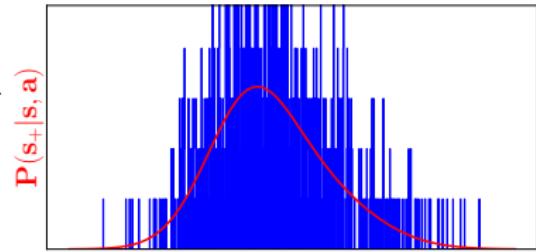
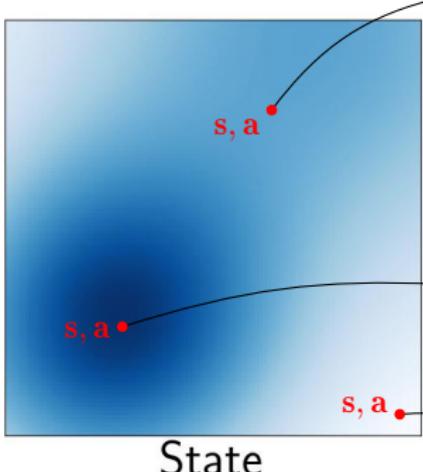
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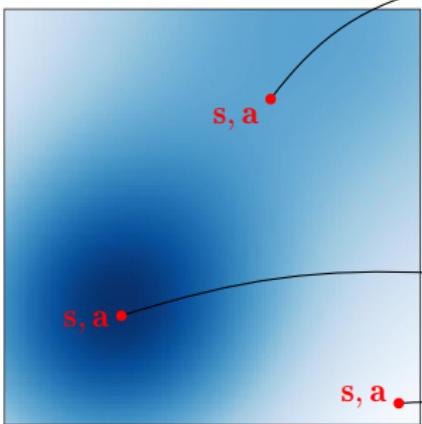
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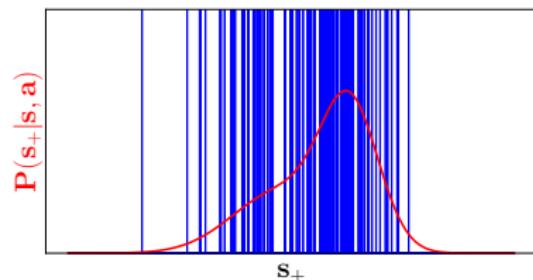
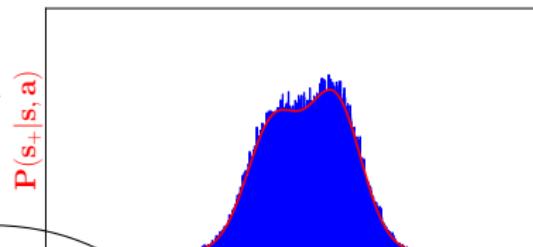
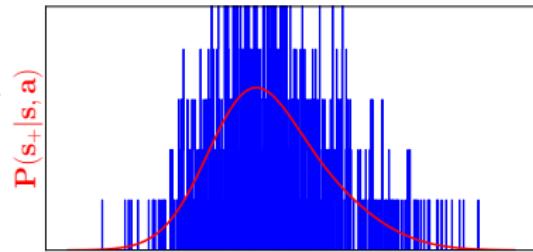
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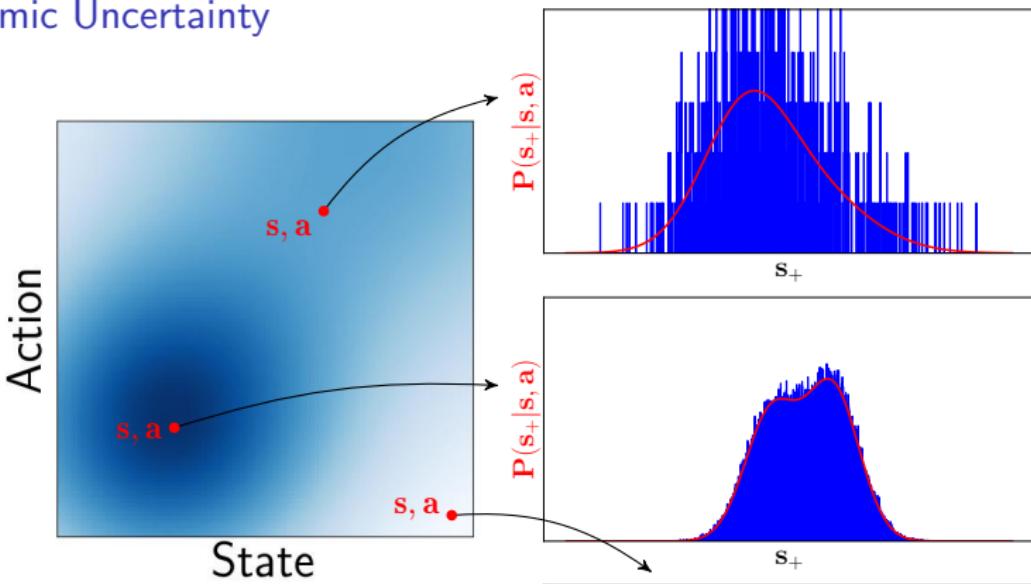
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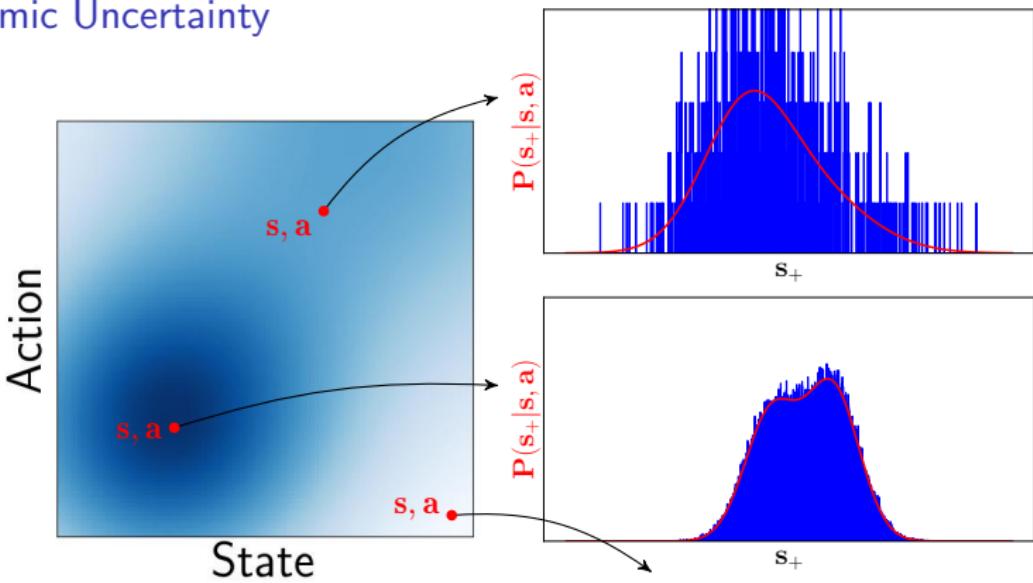
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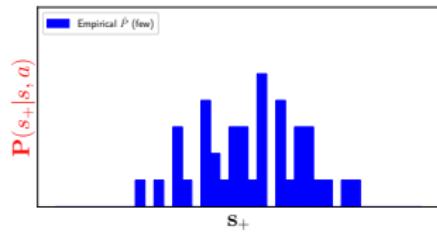
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Distributional View on MDPs

- True dynamics are unknown; only samples or estimates available.

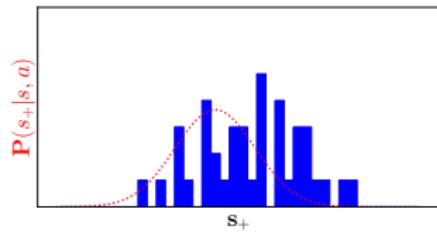
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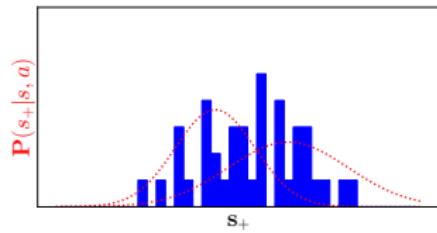
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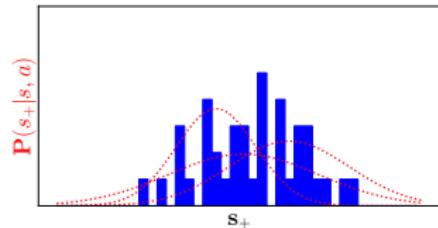
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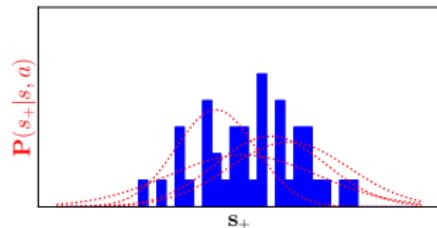
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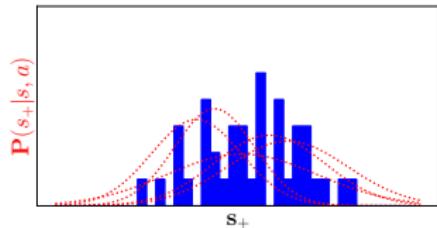
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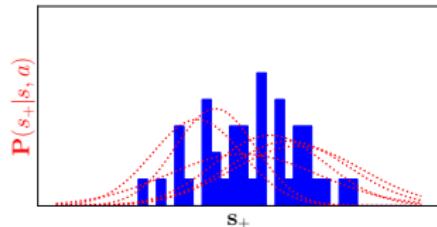
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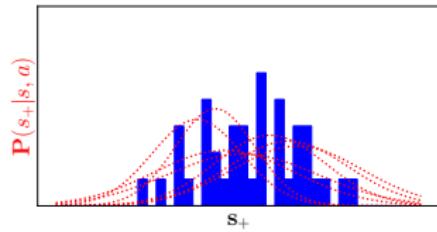
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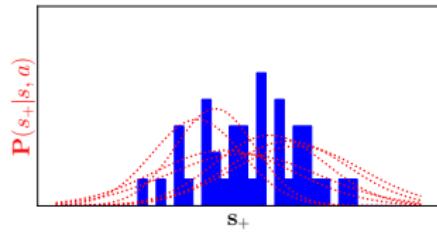
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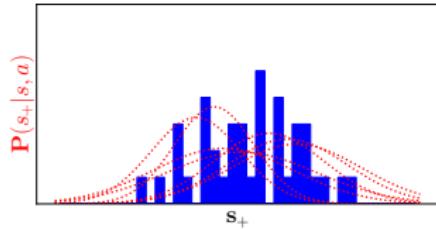
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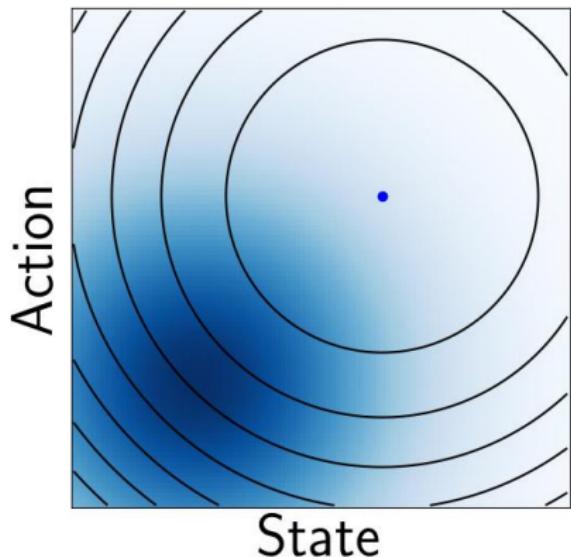
Remarks:

- Accounts for **epistemic uncertainty** from limited data.
- Distributional robust MDP: less pessimistic than robust MDP.
- Probabilistic Safety: confidence level based on model uncertainty??
- Distributional view is applied to risk-sensitive MDP via a distribution of returns (e.g. distributional RL).
- connection to bayesian vs frequentist view??

Out-Of-Distribution (OOD) “Guarding” Methods

Key concepts:

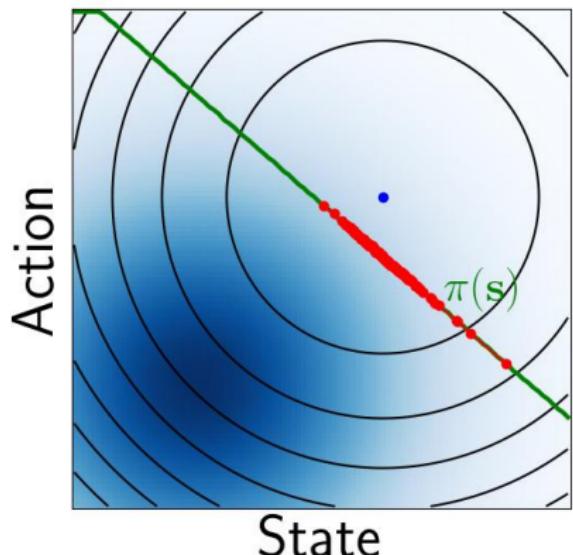
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