# Introduction to Theory of Safe Decision Making

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1st AID Scientific Workshop, Trondheim

# Forewords & Disclaimer

**Objectives:**

- Put in place some common concepts & language
- Identify some key points in safe decision making
- Connect to AI

**Disclaimer:** we are a broad group who needs to get to know each others scientifically.
Apologies if I don't "hit" the right level for all.
I have favored simplicity over "absolute" rigor.

# Outline

# Formally Defining Safety?

Constraints

Violations

Information to Decisions
**Policy $\pi$**

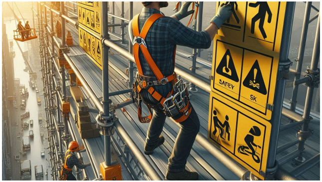
Action

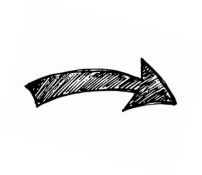
Information

Performance

Utility

**Safety in the real world**





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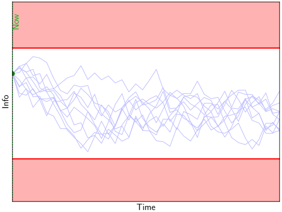

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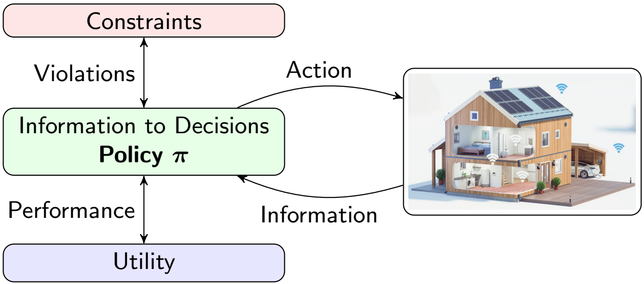


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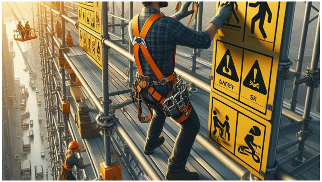

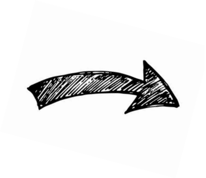

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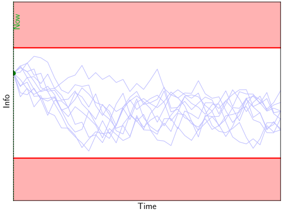

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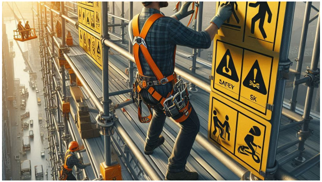

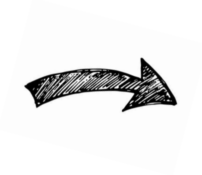

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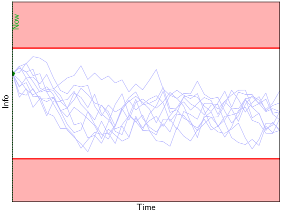

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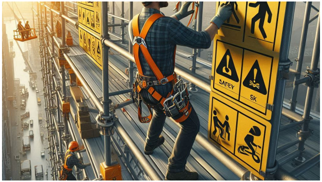

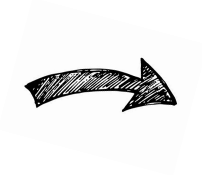

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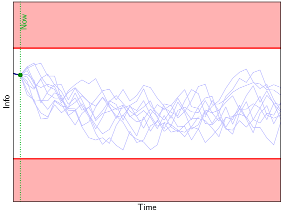

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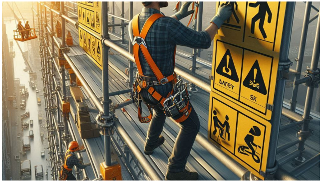

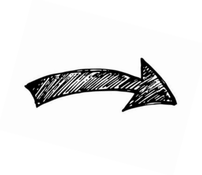

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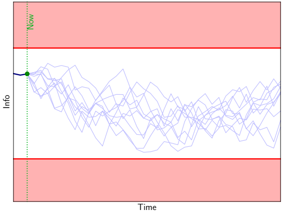

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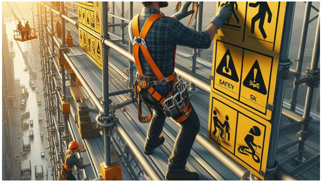


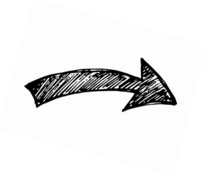

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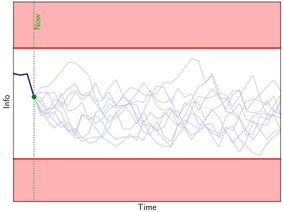

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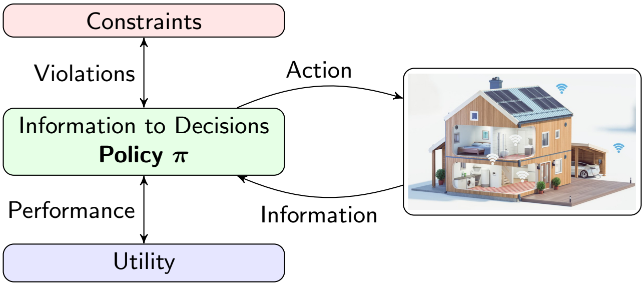


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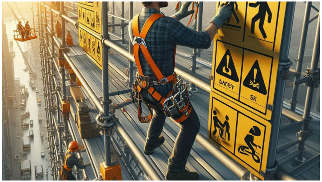

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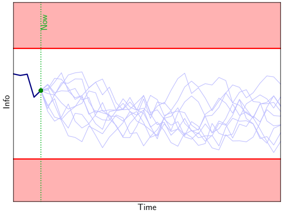

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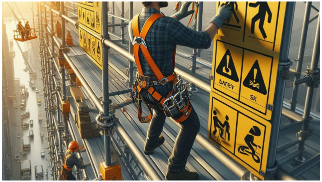

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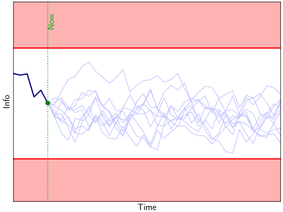

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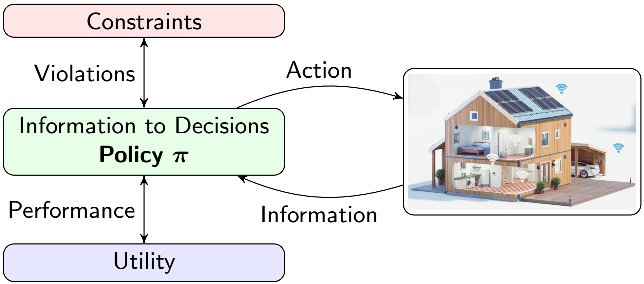


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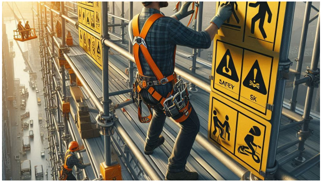

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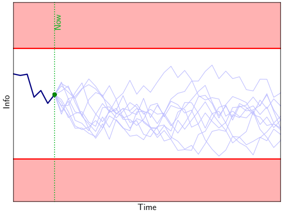

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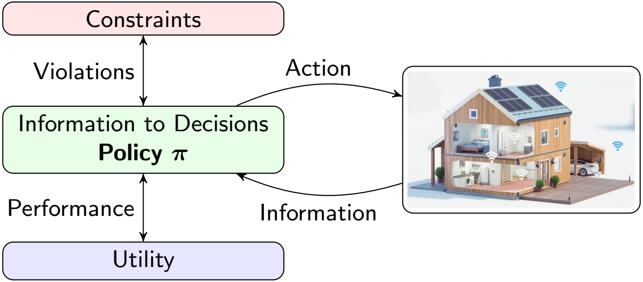


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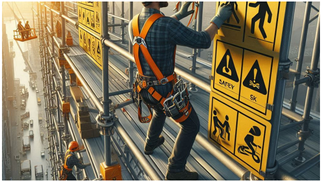

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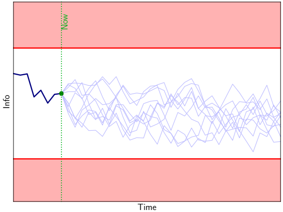
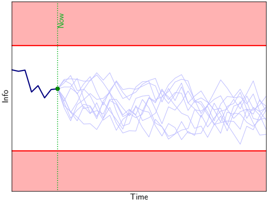


# Constrained MDPs

**In words**

$$\boldsymbol{\pi}^\star = \underset{\boldsymbol{\pi}}{\text{arg max}} \quad \mathrm{E}\left[\sum_{\text{time}} \text{Utility}\right]$$
$$\text{s.t.} \quad \text{Constraints ok at all time}$$



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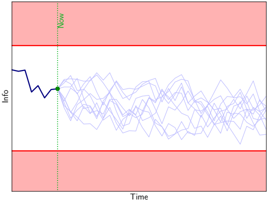


**Formally**

$$\pi^\star = \arg\max_{\pi} \quad \mathrm{E}\left[\sum_{\text{time}} \left(\text{Utility} - \begin{cases} 0 & \text{if} \quad \text{Constraint ok} \\ \infty & \text{if} \quad \text{Constraint not ok} \end{cases}\right)\right]$$

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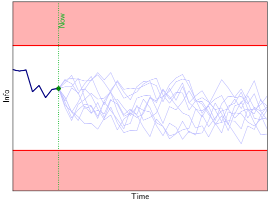


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**Build policy** using

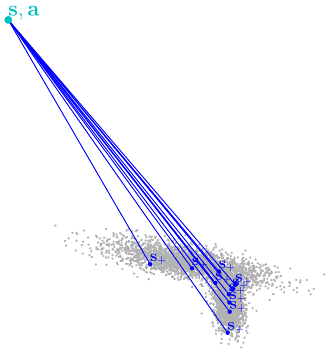
- Perfect model of the real world $\hat{\mathrm{P}}[\mathbf{s}_+|\mathbf{s},\mathbf{a}] = \mathrm{P}[\mathbf{s}_+|\mathbf{s},\mathbf{a}]$
- Model "pessimistic" about the uncertainties

... to evaluate "$\mathrm{E}[\cdot]$"

# Pessimistic Models for Decision Making

- Model must "contain" the uncertainty
- "Container" (set) should be simple for computational reasons
- Trajectories predicted by pessimistic model will "cover" the real world
- Decision policy wants to be safe w.r.t. the "containers"

Distribution of one-step forward



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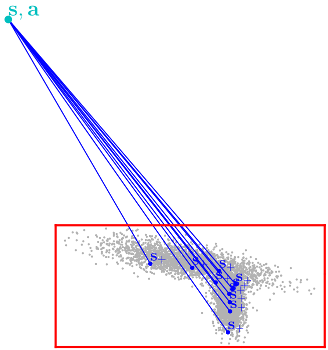
Distribution of one-step forward with "container"



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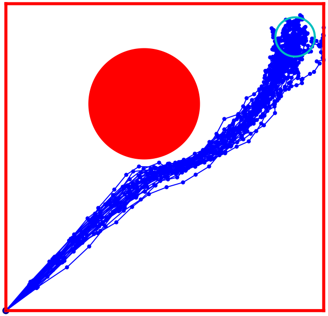
Trajectories



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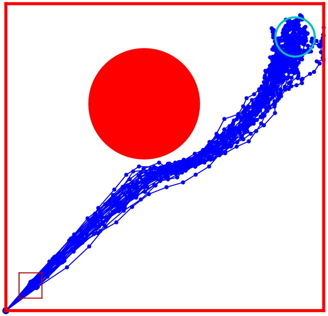

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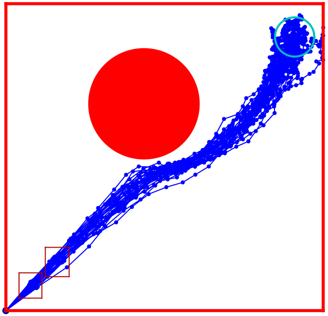

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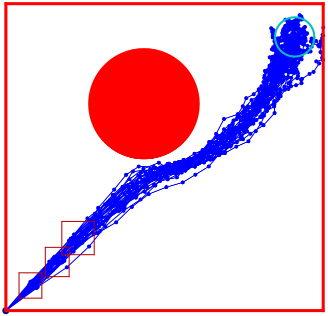

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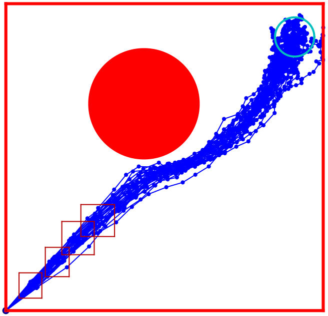

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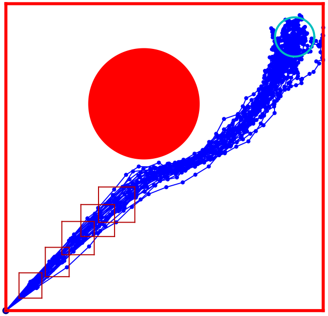

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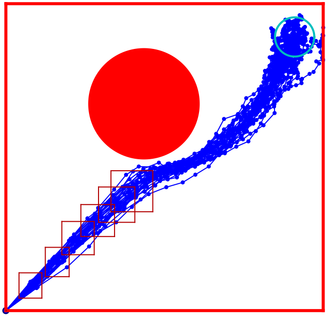

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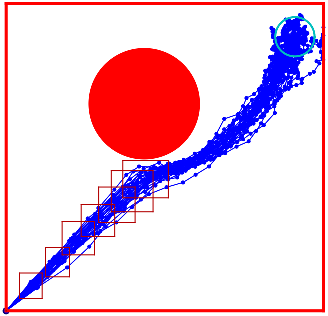

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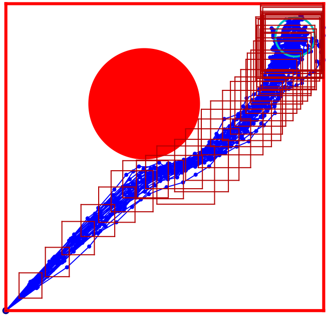

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**Remarks**

- The propagation of the "containers" in the model predictions can be expensive / difficult
- Pessimistic propagations are usually needed $\rightarrow$ pessimistic over pessimistic
- Policy based on worst-case perspective makes the decisions highly conservative
- Often labelled "Robust" decision making

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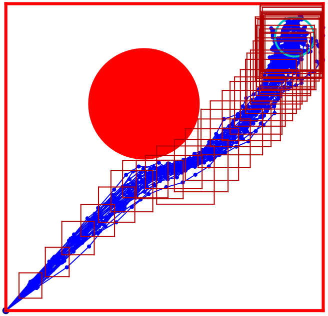

# MDPs with probabilistic safety

**In words**

$$\pi^\star = \arg\max_{\pi} \quad \mathrm{E}\left[\sum_{\text{time}} \text{Utility}\right]$$
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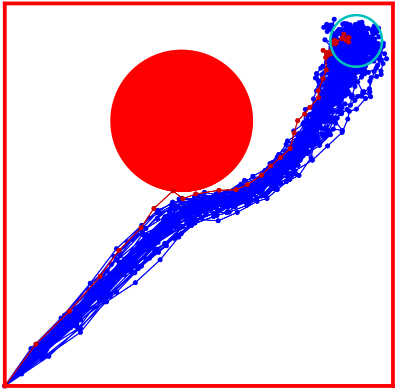

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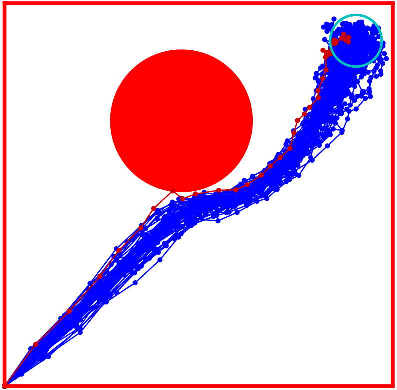
$$\pi^{\star} = \arg\max_{\pi} \quad \mathrm{E}\left[\sum_{k=0}^{\infty} \gamma^{k} L\left(\mathbf{s}_k, \mathbf{a}_k\right)\right]$$
$$\text{s.t.} \quad \mathrm{P}\left[\mathbf{s}_{0,\ldots\infty} \in \mathbb{S}\right] \geq c$$



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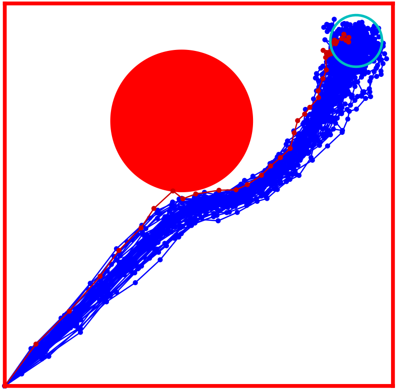
**Remarks**:

- If we can tolerate $c > 0$ (small) it can make a big gain in performance
- Aligned with industrial / practical standards on "large series"
- Problem needs a "termination" (time or goal reached)
- Building decisions can be difficult from a computational point of view



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1. Some Basics of Safe Decision Making
2. Methods
3. Safe Decisions from Data & AI

# Robust Repeated Planning

- Introduction: Robust MPC, Scenario Trees, MC, robust multi-stage stochastic programing, MPPI?
- Difficulties: guarantees for nonlinear systems, persistent safety (recursive feasibility)

# Dynamic Programming & RL with Probabilistic Safety

$$
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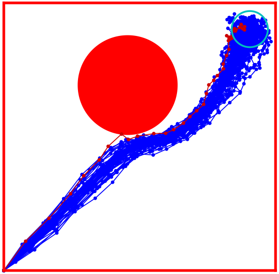
\text{s.t.} \quad & \mathrm{P}\left[\mathbf{s}_{0,\ldots\infty} \in \mathbb{S}\right] \geq c
\end{aligned}
$$

**Expected value form** Illustrate??

$$
\begin{aligned}
\boldsymbol{\pi}^\star = \arg\max_{\boldsymbol{\pi}} \quad & \mathrm{E}\left[\sum_{k=0}^{\infty} \gamma^k L\left(\mathbf{s}_k, \mathbf{a}_k\right)\right] \\
\text{s.t.} \quad & \mathrm{E}\left[\begin{array}{cc} 1 & \text{if no violation occurred} \\ 0 & \text{otherwise} \end{array}\right] \geq c
\end{aligned}
$$



- Expected value form enables classical techniques (ref. 1st lecture), i.e. DP and RL
- Difficulties: estimate expected values (sample based) when $\mathrm{P}\left[\mathbf{s}_{0,\ldots\infty} \in \mathbb{S}\right]$ is close to 1. Illustrate??

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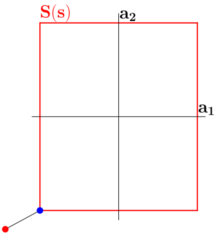

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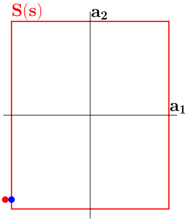

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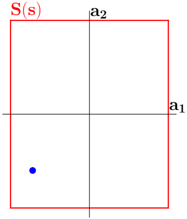

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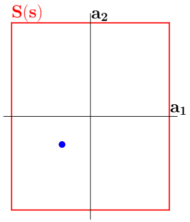

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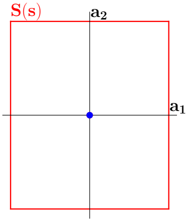

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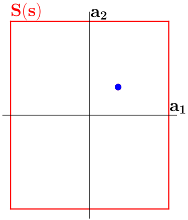

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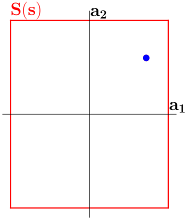

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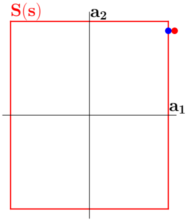

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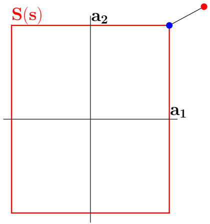

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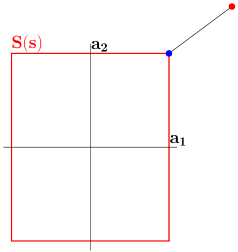

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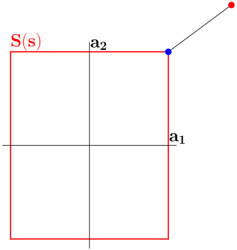
**Remark**: "oracle" can be very hard to build, or not...

Not losing the king



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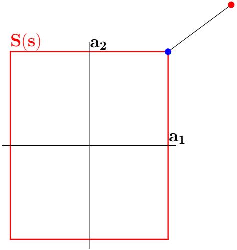
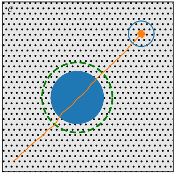
Able to stop within the visible distance



# Control Barrier Functions (CBFs)

**In words**

- Define a safe set $\mathcal{C} = \{x : h(x) \geq 0\}$ using a barrier function $h$.
- Keep the state inside $\mathcal{C}$ by enforcing a simple inequality at each step.
- If a suggested action is unsafe, a small QP adjusts it to the nearest safe action.



Unsafe trajectory: violates $h(x) \geq 0$.

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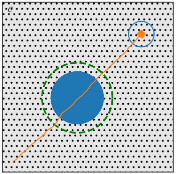
**Formally**

$$\dot{x} = f(x) + g(x)u, \qquad \mathcal{C} = \{x : h(x) \geq 0\}$$

$$u^\star(x) = \arg\min_u \tfrac{1}{2}\|u - u_{\mathrm{des}}(x)\|_2^2 \quad \text{s.t.}$$

$$\underbrace{L_f h(x) + L_g h(x)\, u + \alpha(h(x)) \geq 0}_{\text{CBF constraint}}$$



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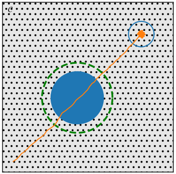

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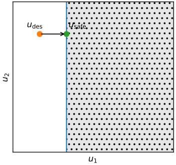


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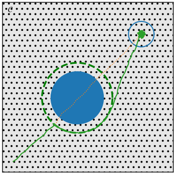
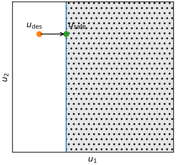


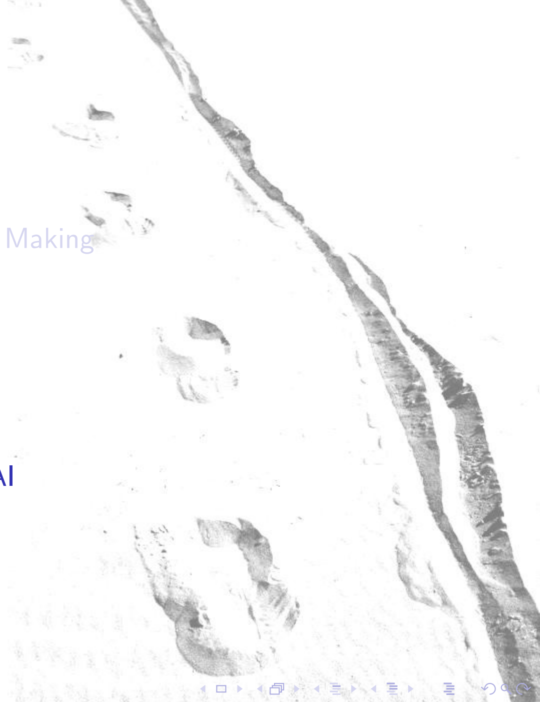
CBF-filtered trajectory.




CBF filter: project $u_{\mathrm{des}}$ to feasible $u_{\mathrm{safe}}$.

- The barrier function and safe set $\mathcal{C}$ are typically **constructed from domain knowledge** (physics, rules, safety envelopes).
- Requires a (possibly simplified, conservative) **system model**: **robust** approach.

# Outline



## Reinforcement Learning

Akhil?

# The Role of Probabilistic AI