Orbit Determination

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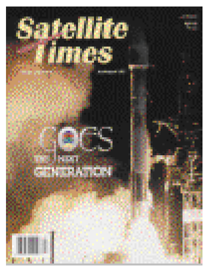

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Up to now, we've talked about orbital models, sources of data, and some of the complexities involved with predicting the motion of artificial earth satellites. The goal here is, of course, to get a handle on the amount of error that can be expected in a particular calculation. After all, the art of celestial mechanics has been refined for centuries and, with the advent of computers, we'd like to think we can depend upon those calculations to be accurate. That way, when we're watching or listening for a satellite to pass overhead, we won't be wondering if we missed the pass because of a bad calculation.

Besides knowing what data to use with which models, the next most important part of the process to understand is how orbital element sets are generated. An understanding of this process will help to answer two questions: How often are the orbital element sets updated and (most importantly) how often do I have to update my data? As with everything else in this field, answering these questions will require a bit of work.

To begin with, let's assume that we already have a basic orbit for our satellite of interest. After all, the overwhelming majority of satellites tracked every day have been on orbit for some time and we are really interested in how we update the orbital element sets for these objects. We will save a description of the process of how to determine an initial element set for a satellite orbit for a future column.

To update any existing orbital element set, observations must first be collected. NORAD, which has responsibility for tracking all man-made objects in earth orbit, uses the Space Surveillance Network (SSN) to collect these observations. This network is comprised of radar sensors for near-earth tracking (below approximately 6,000 kilometers altitude) and optical (actually, electro-optical) sensors for deep-space objects (above 6,000 kilometers altitude). These sensors are geographically distributed around the world to provide global coverage.

Typical observations from a radar site might include azimuth, elevation, range, and range rate while optical sensors provide angles only (azimuth and elevation or right ascension and declination). Each observation is time tagged as it is collected. If you are not familiar with some of these measurement types, don't be concerned—the specific measurement types won't be important to this development and we'll cover these terms in detail when we talk about coordinate systems in our next column.

Of course, since there are thousands of objects orbiting the earth, knowing which observations are associated with which object is no trivial task. NORAD must first process the observations to correlate them with the appropriate object or determine that the observations represent the track of a previously untracked (or lost) object. Once a set of observations has been associated with a particular satellite track, however, the mathematical process of updating the element set can begin. The process used is known as the method of differential corrections. Here's how it works.

As the differential corrections process begins, there are several things we "know." The first is the initial two-line element set, the second is our time-tagged set of observations, and the third is the orbital model we want to use (SGP4, in this case). In order to determine whether our current two-line element set must be adjusted, we must be able to compare it directly to the set of observations we've collected.

While we don't know how to convert observations directly into an element set (after all, that's the goal of this process), we do know how to convert an element set into a set of predicted observations. To generate these predicted observations, we use the current two-line element set with the SGP4 orbital model to generate the satellite's expected position and velocity and then transform this information into expected observations for the particular viewing geometry of the sensor. Now, we can calculate the difference of the actual and expected observations. As might be expected, our goal is to minimize this difference (in reality, the sum of the squares of the differences).

So far, this process sounds simple enough. But how do we know which elements to change and by how much to make the overall difference smaller? Well, as it turns out, we can use some basic calculus to answer this question. Let's say we have one element, x (our independent variable), and one observation, y (our dependent variable). Also, let the transformation from element to observation be given by the transformation function

$$y = f(x).$$

Now, let the actual observation be y_a and the predicted observation be $y_p = f(x_p)$. From calculus, we know that we can approximate the derivative of our transformation at x_p as

$$\frac{x_a - x_p}{y_a - y_p} = \frac{x_a - x_p}{y_a - f(x_p)} = \frac{dx}{dy}.$$

Solving for our "actual" value of the element yields

$$x_a = x_p + (y_a - f(x_p)) \cdot \frac{dx}{df(x)}\bigg|_{x = x_p}.$$

This is a basic Newton iteration for finding a root of a function. For it to work well, our initial value of the element must be "close" to the actual value. Each time we calculate a new "actual" value, we repeat the process by replacing our predicted value with the "actual" value until the difference in the predicted and actual elements between successive calculations is "small."

We are still missing one part, however, in making this process work. The part we're missing is how to calculate the derivative of the observation with respect to the element (in other words, the rate of change in the observation with respect to a change in the element). If our transformation function $f(x)$ were simple enough, we could calculate this value analytically (well, at least some of us could). However, for our real problem, our transformation function takes the values from a two-line element set, runs them through the SGP4 orbital model, and then transforms them for the sensor viewing geometry and coordinate system. Calculating this derivative analytically would be extremely difficult, if not impossible. Instead, we will numerically estimate the derivative by using the same approximation as we did above. That is, we take our initial value of element and some small difference to produce

$$\frac{df(x)}{dx} = \frac{f(x+\Delta x) - f(x)}{(x+\Delta x) - x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}.$$

Once we have an estimate of the derivative at the point of interest, the process of updating the element is relatively easy. It is important to note here the dependence on the orbital model chosen in the transformation function. For the NORAD two-line orbital element sets, the orbital model is SGP4. Using another model would generate different elements because that model would produce a different "best track" through the observations. Therefore, to get the best fit for the NORAD two-line element sets, the SGP4 orbital model ***must*** be used.

In reality, the process of updating the NORAD two-line element set has many orbital elements and observations, so the full differential corrections process must use a multidimensional Newton search. But the basic approach is still the same. Once the process has converged, that is, the difference between the observed position and the calculated position is within some small tolerance, we have a new element set.

Does this mean that a new element set is issued by NORAD? The answer to this question is no. A new element set is issued only when the position predicted by the current element set differs from that predicted by the new element set by more than a certain amount. In the case of the NORAD two-line element sets, that amount is five kilometers (with a 90 percent confidence interval).

So how often are the element sets updated? Well, that depends upon the satellite and its orbit. For near-earth satellites, atmospheric drag can change an orbit in ways that aren't modeled in SGP4, requiring more frequent updates. Also, satellites which maneuver frequently, such as Mir or the US Space Shuttle, will also require more frequent updates since, again, SGP4 is unable to model these changes. You can get a rough idea of how often you need to update your element sets by looking at how long it takes NORAD to issue consecutive element sets for the objects you are interested in. If they issue one element set a week, that means that it takes roughly a week before the orbit has changed enough to require an update. Of course, if you don't need this level of accuracy for your application, then you won't need every update. Remember, five kilometers is less than a second's error at orbital velocity.

Now that we have seen how and when orbital element sets are updated, let's explore why NORAD issues separate element sets for objects which are docked together and why those element sets are not identical. A good example of this practice is with the Mir space station. The Mir complex is actually made up of many objects: the Mir core, Kristall, Kvant-1, Kvant-2, and an assortment of Soyuz and/or Progress modules. Each of these objects was launched individually and tracked by NORAD before docking with the space station. Some of these objects will subsequently undock. As such, NORAD maintains separate tracks of each object.

Of course, when these objects are physically locked together, the observations collected for one object could be used for updating the elements for all objects. But that's not necessarily how it works. Since NORAD has to track over 7,000 objects every day, the process is automated. Now, since all observations have error and each object started with its own independent element set, the element sets generated will remain in tolerance for differing periods of time. How long will depend on the site used to collect the observations, the viewing geometry, changes in atmospheric density or spacecraft attitude, maneuvers, and so on. As such, updates to separate components of a docked structure will be updated at different times.

Okay, so even if NORAD tracks each object separately and maintains separate element sets, why are the element sets different? To understand the answer to this question, you need only look to our last column. Remember, only ideal orbits describe true ellipses and have constant orbital elements. Real orbits experience both periodic and secular (trending) effects in their elements. Because of this, orbital element sets with different epochs will have different values (and different associated errors) for the remaining orbital elements.

Now, you might think that a single element set would result in no error. However, the error is still there—a single element set just seems more consistent. Actually, multiple element sets provide the advantage of redundancy—protecting you from the occasional bad element set.

Well, we covered a lot of territory in this column. If you feel a bit lost, don't feel bad. Remember, when we started, I promised to address issues for the novice to the professional. If you have questions, you are more than welcome to send them to me at TS.Kelso@celestrak.com. I'll do my best to help fill in any gaps.

For our next column, we'll finally start into the discussion of the various coordinate systems used in satellite tracking. Our goal will be to develop an understanding of the various terms used and the importance of each coordinate system. The end result will be to develop methods to transform orbital elements into things like look angles (azimuth and elevation or right ascension and declination) for visual observations and tracking or latitude, longitude, and altitude for plotting on a map or working with satellite imagery. Until then, keep looking up!

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