Orbital Propagation: Part I

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Welcome to the ***Computers and Satellites*** column of ***Satellite Times***. We are about to embark on an adventure of discovery—an adventure I have been looking forward to for quite some time. I only hope you will enjoy the experience as much as I and want to follow along with each new episode.

Along the way, I hope to enlighten you, the reader—whether novice or expert—on the subtleties involved in the theory and practical application of computers to the process of tracking satellites in earth orbit. Whether you simply want to be able to know where to point your TVRO antenna to pick up your favorite television shows, are curious as to where to look to see the Mir space station on a twilight pass, or want to know when you will be able to DX with the space shuttle on the next SAREX mission, we'll cover it all.

We'll start with some of the basics of orbital propagation (just a fancy way of saying 'predicting where a satellite is going to be') to give you not only an appreciation for why certain orbital models are used but why it is important to use the proper data with those models. Next, we'll talk about the various choices of coordinate systems so that we can be sure we're all talking about the same thing and we'll cover the necessary coordinate transformations to move back and forth among these systems, throwing in an equation or two from time to time to keep things honest. Naturally, I'll include snippets of computer code to illustrate the process and help you build your own algorithmic tool chest.

From time to time, I'll review available computer software which can make it easier for you accomplish your satellite tracking tasks—from commercial products to the finest in public domain software. I'll share information on good places to look to find satellite-related information—both on dial-up BBSs and out on the Information Superhighway. And, of course, we'll cover timely sources for orbital element sets, without which none of us would be able to track satellites.

As your tour guide on this adventure, I'll do my best to anticipate some of the more common questions: How are orbital element sets generated? How often should I get new data? Why do the individual modules of the Mir space station have separate element sets when they are physically docked together? Why can't I find a two-line element set for the sun or the moon? We'll answer these and many more. But for you to get the most out of this experience, you must participate, too. I want you to feel free to ask questions as we go along. I'll do my best to answer each and every one of them. I look forward to hearing from you.

Now, let's get down to the topic of this column: Orbital Propagation. Although the fundamental concepts of how objects move under gravitational attraction has been understood since the time of Isaac Newton, the practical determination of an object's position in orbit is considerably more difficult. In order to be able to select the best method of orbital propagation, it is important that we understand why this is so.

According to Newton's Law of Universal Gravitation, the gravitational attraction between two objects is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. Newton's genius allowed him to deduce this law as the common explanation for why an apple drops to the ground and the moon orbits the earth.

To fully appreciate the implications of this law, we must first develop a model to demonstrate it. A model is simply any representation of some or all of the properties of a system. In this case, our system is two objects and the gravitational force between them (as defined by Newton's Law). For the purpose of this discussion, we will confine ourselves to mathematical models, wherein our representation of the system is described via mathematical equations or computer algorithms. If our two objects are simple point masses, the result of this model will be that the two objects will orbit about their common center of mass moving in elliptical orbits (given the appropriate starting conditions). If one object is considerably more massive than the other, it will appear that the smaller object is following an elliptical orbit around the more massive object.

In a real system, things can become considerably more complicated. Since we are primarily interested in the motion of an artificial satellite around the earth, let's consider that as our system. The primary force affecting the motion of our satellite is still gravitational attraction. However, now we must consider many additional perturbing forces, or perturbations, if we want to be able to accurately predict the position of the satellite. These additional forces will act to change our satellite's orbital path from that of a true ellipse. Knowing which perturbations have the most significant effects and under what conditions will be extremely important.

What are some of these perturbing forces? Let's begin with just the gravitational ones. In our simple model, we assumed our two objects were point masses. The earth, however, is not a point mass. If the earth were perfectly spherical and had a uniform density, however, it would still be possible to treat it as a point mass (it is possible to prove this mathematically). But the earth is not spherical—it bulges at the equator, primarily due to the centrifugal force of its own rotation. Neither does it have uniform density. It is enveloped with oceans which bulge under the tidal forces of the sun and the moon. The lighter material of the continents 'floats' on the denser mantle. While Newton's Law still applies, it is now necessary to consider the gravitational effect of each individual 'piece' of the earth on our satellite. All of a sudden, our simple model has become a lot more complicated.

We also assumed that there were only two gravitational masses in our original model. We cannot, however, ignore the gravitational effects of the sun and the moon, though. And what about the gravitational effects of the planets? Are these to be considered significant, too?

There are non-gravitational forces to consider, as well. The primary one, for some classes of earth orbits, is atmospheric drag. Ignoring this perturbation for a low-earth orbit will result in significant errors. Even tiny forces, such as solar radiation pressure, can have a significant effect on a satellite's orbit under the proper conditions.

Our discussion, to this point, has been purely theoretical. We started with a simple model of gravitation and developed a realization that the real world is more complex (no real surprise here). But since the subject of this column is ***computers*** and satellites, we must develop a mathematical model of an artificial satellite's orbital motion which trades off the accuracy of an extremely complex description of our system against the reduced computational burden of a simpler description. Remember, a model is simply any representation of ***some*** or all of the properties of a system—a good model only selects those properties necessary to achieve the desired degree of accuracy.

Before considering just what effects are most significant for modeling the orbit of an artificial earth satellite, there is another issue that must be addressed from a computational perspective. Up to this point, I have implied that if we knew the current state of a satellite's orbit—for our purposes, the satellite's position and velocity—and the forces acting upon it, we could determine the satellite's state at some future (or past) time. This suggestion is true, regardless of the complexity of the model of our forces. However, if our model is too simple or we take too big of steps (in time), the accuracy of our results will suffer.

Even if we choose the proper level of modeling complexity and the proper size time steps, though, we are faced with one inescapable conclusion resulting from this approach. That conclusion is that if we are given a particular set of starting conditions for our satellite orbit—generally provided in the form of an orbital element set or state vector—we must begin at that point and 'step' along in time until we get to the point we are interested in doing our calculations for. From a computational perspective, this approach can require considerable calculations. We would prefer to have a model which provides an analytical solution, that is, a solution wherein if we know the time of interest we can directly calculate the state of the satellite's orbit at that time without the need to 'step' along in time.

In our next column, we will examine the implications of the material presented above and look at how they affected the historical development of the orbital propagators in use today, particularly those used by NORAD (the North American Aerospace Defense Command). Armed with this background, we will delve into the relationship between orbital data and orbital models and the importance of using the right data with the right model. We'll look at how the data to be used with an orbital model is generated and begin considering examples to illustrate the magnitude of the errors which can result from improper combinations of models and data.

I hope I've given you enough of an idea of some of the adventures ahead to make you want to come back. I look forward to seeing you here next time!

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