Orbital Coordinate Systems, Part I

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September/October 1995

By this point, I hope to have helped you develop an understanding of two key aspects of practical orbital mechanics. The first has to do with why we use the orbital models we do for predicting the position of earth-orbiting artificial satellites. As with any computer model, orbital models must trade off accuracy for computational speed. Which model you decide to use will depend upon which of these factors is most important to you.

Of course, from a practical perspective, the choice of orbital model is also strongly influenced by the availability of data (element sets). Knowing that orbital element sets are generated by fitting observations to a trajectory based upon a particular orbital model is the second of our key aspects. Accuracy of our predictions will depend upon using that same orbital model.

Up to now, however, all we've really talked about are orbital element sets. But how do we get from the data in these orbital element sets to something we can use, such as knowing where to look (or point an antenna) when a satellite passes over? To answer this question requires an understanding of the various coordinate systems involved and how to transform coordinates (typically position and velocity) from one system to another. The correct application of these coordinate transformations is every bit as important to our overall accuracy as the selection of the orbital model itself.

Where do we start? Let's start with the orbital element sets themselves and discuss some terminology. The two most common forms of orbital element sets are ***state vectors*** and ***Keplerian orbital elements*** (e.g., the NORAD two-line element sets). A ***state vector*** is a collection of values (states) that if known, together with the ***state transformation rules*** (how the state vector changes over time), the state vector for any past or future time can be computed. For a satellite in Earth orbit, if we ignore atmospheric drag and maneuvering, the state vector would be comprised of the satellite's position and velocity. Knowing the position alone would not be sufficient, since a satellite with zero velocity would fall to Earth while one with orbital velocity would not, even if the satellites start at the same physical location.

We cannot, however, talk about position and velocity without discussing the coordinate system that these values are measured relative to. For most state vectors, this is the ***Earth-Centered Inertial*** (ECI) coordinate system. The first part of this designation should seem fairly obvious. That is, since we're studying objects that revolve around the center of the Earth, it seems natural to have the center (origin) of our coordinate system at the center of the Earth. Inertial, in this context, simply means that the coordinate system is not accelerating (rotating). In other words, it is 'fixed' in space relative to the stars. We shall see that this is an ideal definition of the ECI coordinate system, but we won't worry about the slight rotations involved until later.

The ECI coordinate system (see Figure 1) is typically defined as a ***Cartesian*** coordinate system, where the coordinates (position) are defined as the distance from the origin along the three orthogonal (mutually perpendicular) axes. The ***z*** axis runs along the Earth's rotational axis pointing North, the ***x*** axis points in the direction of the ***vernal equinox*** (more on this in a moment), and the ***y*** axis completes the right-handed orthogonal system. As seen in Figure 1, the vernal equinox is an imaginary point in space which lies along the line representing the intersection of the Earth's equatorial plane and the plane of the Earth's orbit around the Sun or the ***ecliptic***. Another way of thinking of the ***x*** axis is that it is the line segment pointing from the center of the Earth towards the center of the Sun at the beginning of Spring, when the Sun crosses the Earth's equator moving North. The ***x*** axis, therefore, lies in both the equatorial plane and the ecliptic. These three axes defining the Earth-Centered Inertial coordinate system are 'fixed' in space and do not rotate with the Earth.

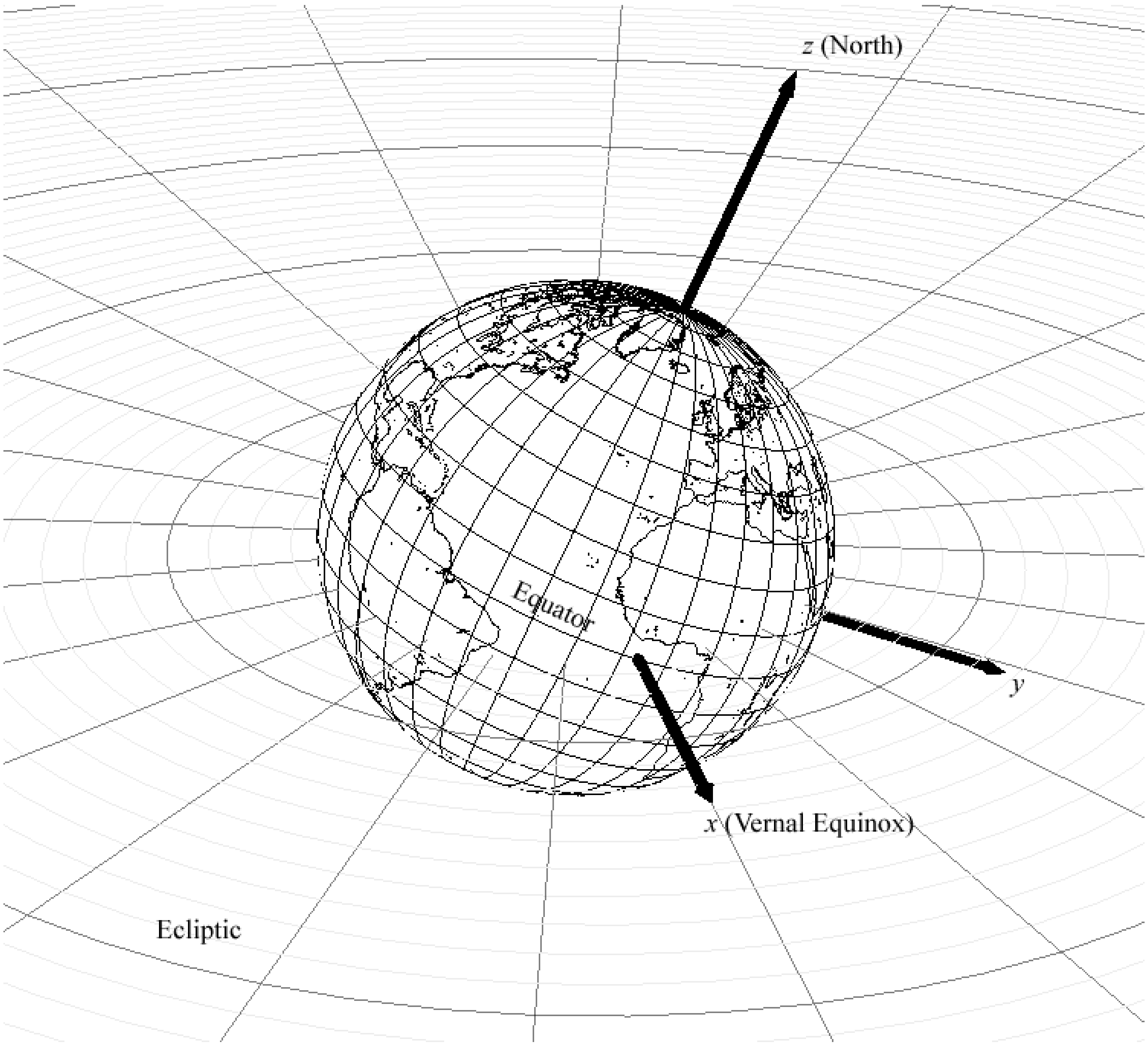

Figure 1. Earth-Centered Inertial (ECI) Coordinate System

Now, while state vectors are normally used with numerical integration models for highly accurate calculations, Keplerian orbital elements are used for the vast majority of orbital predictions. But, the ECI coordinate system is still often used as the common coordinate system when performing coordinate transformations. For example, before a calculation can be made of the distance between a satellite and an observer on the ground, both the satellite and the observer's position must be defined in a common coordinate system. Since the satellite's position is typically represented by a Keplerian orbital element set and the observer's position is given in latitude, longitude, and altitude above the Earth's surface, we cannot perform the calculation directly without first converting to a common coordinate frame.

As it turns out, the NORAD SGP4 orbital model takes the standard two-line orbital element set and the time and produces an ECI position and velocity for the satellite. In particular, it puts it in an ECI frame relative to the "true equator and mean equinox of the epoch" of the element set. This specific distinction is necessary because the direction of the Earth's true rotation axis (the North Pole) wanders slowly over time, as does the true direction of the vernal equinox. Since observations of satellites are made by stations fixed to the Earth's surface, the elements generated will be referenced relative to the true equator. However, since the direction of vernal equinox is not tied to the Earth's surface, but rather to the Earth's orientation in space, an approximation must be made of its true direction. The approximation made in this case is to account for the precession of the vernal equinox but to ignore the ***nutation*** (nodding) of the Earth's ***obliquity*** (the angle between the equatorial plane and the ecliptic). We'll address how to use this level of detail in a future column.

So, we now know that whether we're using state vectors or Keplerian orbital element sets, our calculations will likely yield ECI position and velocity. Let's begin working now to answer two common questions in satellite tracking. The first question is: Where do I look or point my antenna to acquire a particular satellite? The second question is: What is the latitude, longitude, and altitude of that satellite? These questions come up frequently, whether the goal is to watch the US Space Shuttle and Russian Mir Space Station pass overhead, to acquire an amateur radio satellite, or to determine the longitude of a geostationary TVRO satellite. But, to be able to answer these questions, we will need to determine either the position of an observer on the Earth relative to the ECI coordinate frame or the position of a satellite relative to the Earth. In either case, we will need to know the rotation angle between the Greenwich Meridian (zero degrees longitude) and the vernal equinox and, hence, the orientation of the Earth relative to the ECI coordinate frame.

Let's start by calculating the position of an observer in the ECI coordinate frame. For our initial discussions, we'll assume a spherical Earth. This assumption is not a particularly good one, as we'll see in our next column, but will make the initial development easier to follow. The calculation of the z coordinate is straightforward, as can be seen in Figure 2. This figure shows a side cutaway of the Earth with North up. For an observer at latitude φ, the z coordinate is shown in Figure 2, where R_e is the Earth's equatorial radius. To calculate the x and y coordinates, we will also need the value of R from Figure 2. If we wanted to calculate z and R for distances above mean sea level, we would simply replace R_e with $R_e + h$, where h is the distance above mean sea level.

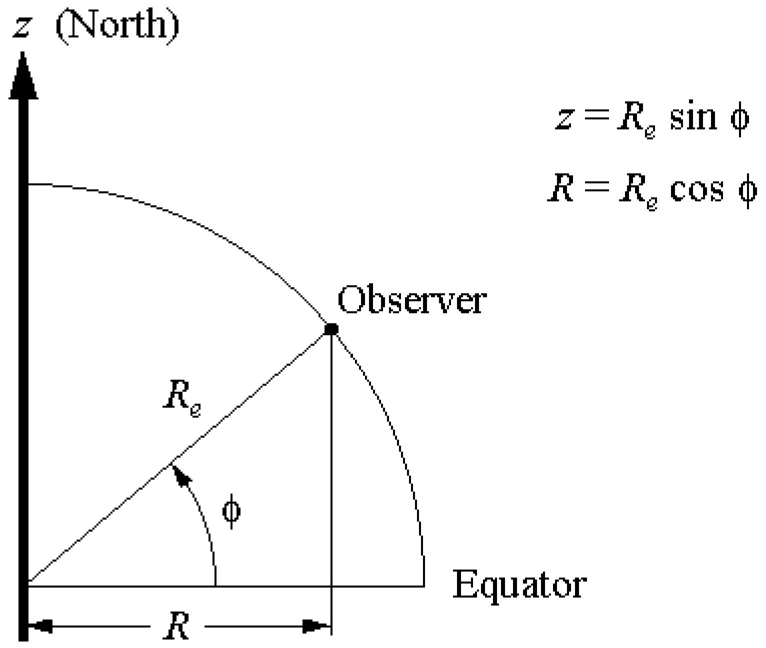

Figure 2. Latitude to ECI Conversion

Computing the ***x*** and ***y*** coordinates requires a bit more work. Since the Earth rotates in the ***x-y*** plane (i.e., about the ***z*** axis), the ***x*** and ***y*** coordinates of a point on the Earth's surface will vary with time, unlike the ***z*** coordinate. However, if we know the angle between the observer's longitude and the ***x*** axis (the vernal equinox), we can specify the ***x*** and ***y*** coordinates as a function of time. In fact, if we designate the angle between the ***x*** axis and the observer's longitude as $\theta(\tau)$, where τ is the time of interest, $\boldsymbol{x}(\tau)$ and $\boldsymbol{y}(\tau)$ are given in Figure 3. This figure shows a slice through the Earth, parallel to the equatorial plane and through the observer's location.

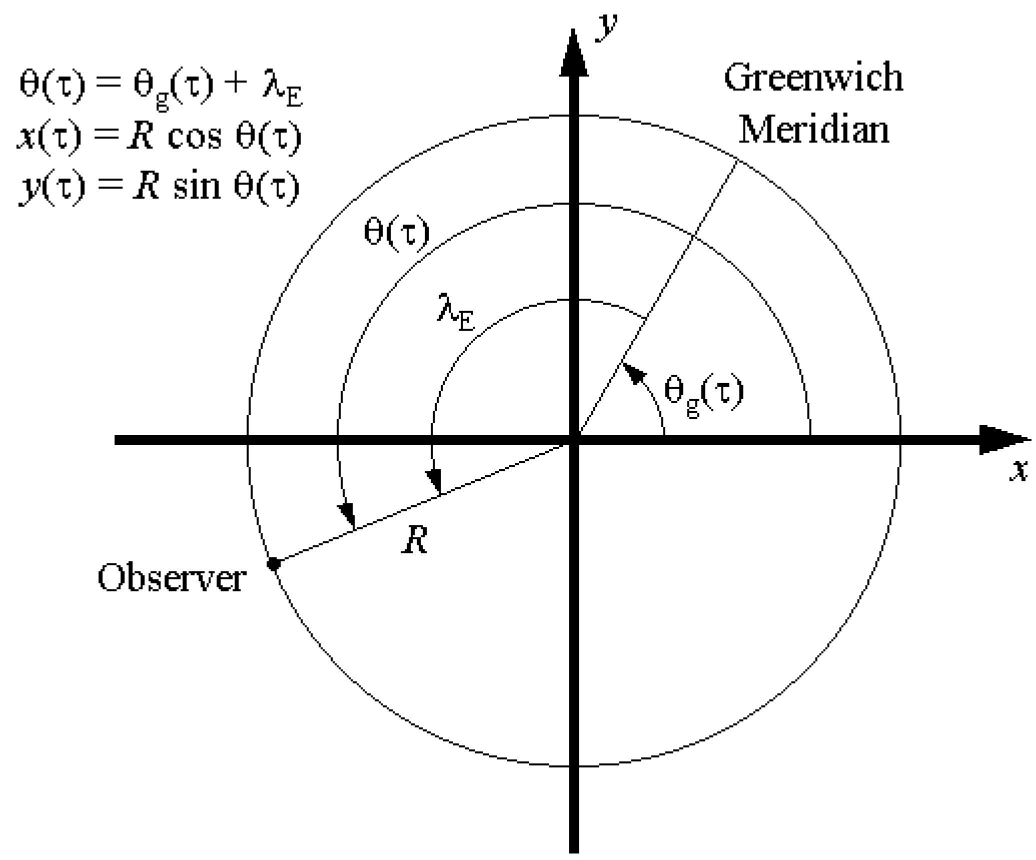

Figure 3. Longitude to ECI Conversion

Upon first inspection, these equations would seem straightforward enough. But just what is $\theta(\tau)$ and how is it calculated? The function $\theta(\tau)$ is what astronomers refer to as the ***local sidereal time***. ***Sidereal time*** is simply time measured relative to the stars. In our day-to-day lives, we are used to measuring time relative to the position of the Sun because of its obvious position in the heavens. This time scale is referred to as ***mean solar time***. As with any time system, time is defined as the angle between the observer and some reference direction. With mean solar time, the reference direction is the direction of the mean sun; with sidereal time, the direction is the vernal equinox—just the direction we need for our calculation. So what causes the difference between these two time scales?

As seen in Figure 4, the position of the Sun moves with respect to the stars because of the Earth's orbit around it. Let's say we noted the position of the Sun relative to the stars when it crosses our meridian (longitude) on one day. By definition, that passage is called local noon. However, when that same position relative to the stars crosses our meridian on the following day, the Sun will not yet have reached our meridian. That is to say, the position will cross our meridian before local noon. The interval of time between two successive meridian crossings of a fixed position in inertial space is referred to as one ***sidereal day***. Sidereal midnight occurs when the vernal equinox crosses the meridian. The interval of time between two successive meridian crossings of the mean sun is referred to as one mean solar day. As seen in Figure 4, the Earth must rotate a bit more for a mean solar day than for a sidereal day. In fact, a sidereal day is only $23^h56^m04^s.09054$ of mean solar time. This difference, while small, is extremely important.

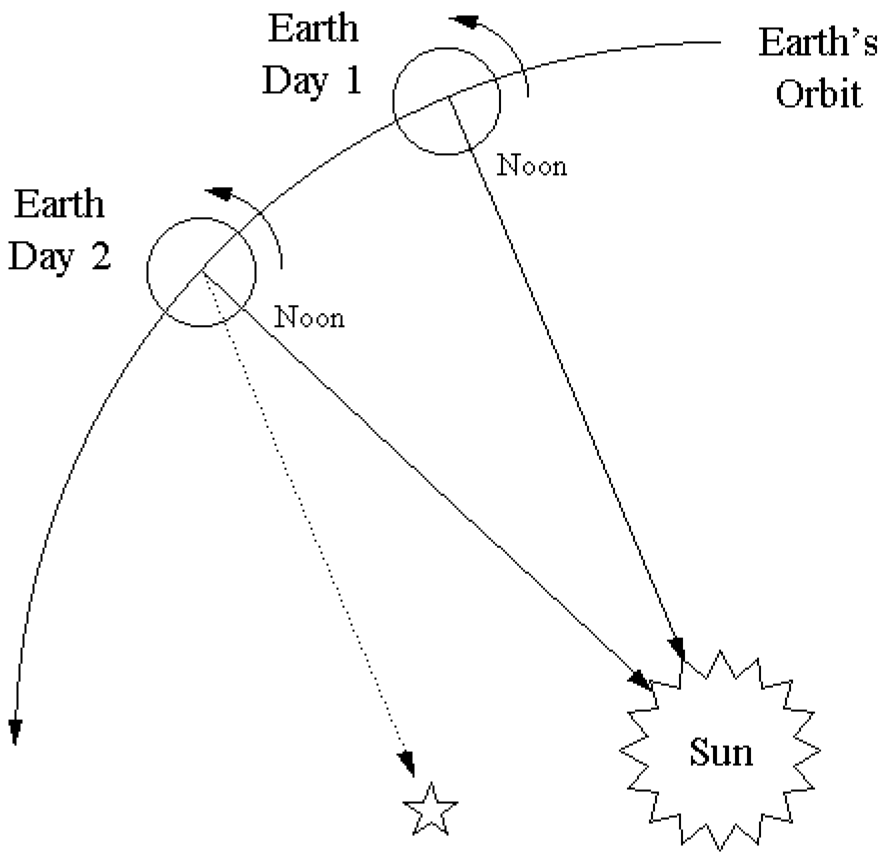

Figure 4. Sidereal versus Solar Time

Now, since all of our common time measurements are based on UTC (Coordinated Universal Time) which is mean solar time, how do we calculate our local sidereal time? Well, as shown in Figure 3, the local sidereal time can be calculated by adding the observer's ***east*** longitude, λ_E, to the Greenwich sidereal time (GST), $\theta_g(\tau)$. Oftentimes, GST (or more specifically, Greenwich Mean Sidereal Time or GMST), can be found in references such as the US Naval Observatory's ***Astronomical Almanac***. If GMST is known for 0^h UTC, $\theta_g(0^h)$, on a particular date, then $\theta_g(\Delta\tau) = \theta_g(0^h) + \omega_e \cdot \Delta\tau$, where $\Delta\tau$ is the UTC time of interest and $\omega_e = 7.29211510 \times 10^{-5}$ radians/second is the Earth's rotation rate. Unfortunately, this approach requires a table of reference times to do the calculations. Another approach is to calculate $\theta_g(0^h)$ using the equation from Page 50 of the ***Explanatory Supplement to the Astronomical Almanac***:

$$\theta_g(0^h) = 24110^s.54841 + 8640184^s.812866\, \boldsymbol{T_u} + 0^s.093104\, \boldsymbol{T_u}^2 - 6.2 \times 10^{-6}\, \boldsymbol{T_u}^3$$

where $\boldsymbol{T_u} = \boldsymbol{d_u}/36525$ and $\boldsymbol{d_u}$ is the number of days of Universal Time elapsed since JD 2451545.0 (2000 January 1, 12h UT1).

While we've covered a lot of ground in this column, we obviously still have a bit more to go before we can answer the questions raised above. For our computer implementation, we will first need to develop a procedure for calculating the Julian Date in our last equation. Then, we will need to refine our conversion from latitude, longitude, and altitude to ECI coordinates to incorporate an oblate (flattened) Earth. When we make this refinement, we will also see the magnitude of error which can occur if this factor is ignored. At this point, we will have finished our first coordinate transformation and will be able to calculate the vector from the Earth observer to the satellite. We will then begin the process of developing our second coordinate transformation, that from ECI to the topocentric-horizon or azimuth-elevation coordinate system. It is this system which will allow us to measure the position of a satellite relative to the Earth's surface.

We will also begin to include snippets of computer code to illustrate the theory we're developing here. If you'd like to look ahead, these routines can be found in the file sgp4-plb26a.zip on the ***CelesTrak WWW*** site.

As always, if you have questions or comments on this column, feel free to send me e-mail at TS.Kelso@celestrak.com or write care of ***Satellite Times***. Until next time, keep looking up!

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