Orbital Estimation

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On the morning of 1995 February 9, I awoke a bit earlier than usual. Not that I was any more anxious than usual to get to work that day. Instead, I realized that I had a rare opportunity to watch a celestial event of historical import. What made this event even more exciting was that I was able to predict just when it would be visible from my new home in Montgomery, Alabama.

For several days, I had been watching the television coverage of the first rendezvous of the US Space Shuttle with the Russian Mir Space Station. As amazing as it was to watch these two large spacecraft maneuver around each other on television, I was anxious to have the opportunity to watch this celestial ballet live. After all, this is where "the rubber meets the road" for orbital mechanics. With a fresh set of NORAD two-line orbital elements for both Mir and the Space Shuttle and my trusty TrakStar prediction software using the NORAD SGP4 orbital model, I began calculating when I might have my chance.

Of course, a number of conditions have to be met for you to be able to visually observe an low-earth-orbiting satellite. First, the satellite must pass over your location. That means that the satellite's orbit must have an inclination greater than or (approximately) equal to your latitude (north or south). For many Space Shuttle launches, the inclination is only 28.5 degrees (the latitude of the Kennedy Space Center where the Space Shuttle is launched from) and only a small percentage of the southern United States will even "see" the Space Shuttle pop above the horizon. On this mission, however, the Space Shuttle was launched into a 51.6-degree inclination in order to rendezvous with the Mir Space Station. That meant that our first condition would be met for a large percentage of the Earth's populated landmasses.

The next condition which must be met is for the satellite to pass over your location ***at the right time of day***. This condition is very important. Basically, the observer must be in darkness and the satellite must be illuminated by sunlight to be visible. For the observer to be in darkness, the Sun must be six degrees or more below the horizon. For a satellite at the altitude of Mir or the Space Shuttle, it must be passing near the Earth's terminator (the line on the Earth's surface dividing day and night) within an hour or so after the onset of nightfall or before daybreak. If the satellite passes over at some other time of day, it simply will not be visible to the naked eye.

As luck would have it, the conditions were met for this mission, although they wouldn't come together for me until after the rendezvous was over. Nonetheless, both spacecraft, with their crews of astronauts and cosmonauts, would be chasing each other across the Alabama morning sky. Now, all that was needed was good weather.

My calculations had shown that STS 63 would pop up out of the Earth's shadow at 0554 CST at 14 degrees above the horizon, just north of west. Two minutes later, the Mir Space Station would appear at the same location. Two minutes might not seem like much of a separation, but at 7.5 km/sec, that's a distance of almost 1,000 km. Both spacecraft would rise to just over 45 degrees above the horizon before heading back to the horizon to the southeast. Once again, luck was on my side since I had a nice unobstructed view in this direction.

As I walked outside, it was a brisk -3 degrees Celsius (okay, brisk for Montgomery), but the sky was crystal clear. I started the car and began looking for STS 63. I didn't have to wait long, with STS 63 popping into view exactly when and where TrakStar had said it would. Suddenly, it didn't seem as cold any more. I watched as it arced up toward the apex of its trajectory and then looked back down to the horizon. Again, just as expected, there was the Mir space station at the same place I'd first seen the Space Shuttle. Wow!

I was actually seeing both the Space Shuttle and the Mir Space Station in the sky at the same time! And, they were right on schedule according to my program (that always makes me feel a lot better). I couldn't have asked for a better view, either. With both Venus and Jupiter in the morning sky, it was easy to use them as benchmarks to determine the brightness of the two spacecraft—each was about as bright as Jupiter and pretty tough to miss. It really was wonderful to experience the excitement of calculating a visual satellite pass and watching it unfold, as predicted. It sure brings those numbers to life.

Since this will not be the last time these two spacecraft will meet in space, you will have the opportunity to go out and watch these events for yourself. In fact, the Space Shuttle is scheduled to dock with Mir sometime later this year. So, what do you need to do to prepare to observe?

As was pointed out in my previous columns, the first two things you'll need will be current element sets and the appropriate orbital model. The most widely-available source of orbital data for Mir is the NORAD two-line element sets. These elements are available via Internet at archive.afit.af.mil in the directory pub/space ***[no longer available]*** and via dial-up modem on the Celestial BBS at (334) 409-9280 ***[dial-up BBS no longer available]*** in the Satellite directory of the Files section. These elements are updated every weekday (except holidays). The Mir elements can be found in the file MIR.TLE. These elements are also echoed to many other Internet sites, VANs (Value-Added Networks such as CompuServe and America Online), and BBSs around the world. For the docking of the Space Shuttle to the Mir Space Station later this year, your best bet to be prepared is to predict viewing conditions using the Mir orbital elements until the elements for the Space Shuttle become available (Mir has nowhere near the maneuvering capability of the Space Shuttle).

Why are timely orbital elements important? For Mir, they are important because the orbit is constantly changing due to the effects of changing atmospheric drag and maneuvers to maintain the space station's orbit. Mir also maneuvers to support rendezvous with the Soyuz and Progress spacecraft which supply it and, now, the Space Shuttle. We'll see just how important this is in a little bit.

For the orbital model, any package that implements the NORAD SGP4 model should suffice. I use TrakStar because it does one thing many other tracking programs do not: it calculates visible-only passes. That is, it can be set to output data only when the satellite is illuminated and the observer is in darkness. It doesn't output graphically, but the tabular ASCII output can be easily imported into any spreadsheet software or other plotting package of your choice to produce a trajectory. The key ingredient, however, is the use of the SGP4 orbital model.

Here's why. In an ideal orbit, the basic orbital elements are constants. That is, the orbital altitude, eccentricity, inclination, and orientation of the orbit in space are all fixed. In real life, however, perturbations on the orbit due to the nonuniform density of the Earth and atmospheric drag cause fluctuations in a satellite's orbit. Let's look some specifics for Mir on the day of my observation to illustrate these effects.

The Mir element set I used for my calculations was:

```
1 16609U 86017A   95039.25598067  .00011090  00000-0  14726-3 0  9200
2 16609  51.6461  83.8459 0000831 296.4901  63.6005 15.58764259512771
```

The key fields from this element set are:

Date (year, day of year)	95039.25598067 days
Inclination	51.6461 degrees
Right Ascension of Ascending Node	83.8459 degrees
Eccentricity	0.0000831
Argument of Perigee	296.4901 degrees
Mean Anomaly	63.6005 degrees
Mean Motion	15.58764259 rev/day

Assuming the mean elements of this element set to be constants (which many non-SGP4 programs do), would yield a semi-major axis (half the distance between the satellite's closest and farthest approaches to the Earth) of 6,769 km. However, an examination of the semi-major axis over a time period of six hours shows it to be far from constant, varying by about 12 km over a single orbit (see Figure 1). And, while not immediately obvious from this plot, the altitude is slowly decaying with time. At this altitude, an error of 1 km in altitude corresponds to just over a 1 sec error ***per revolution*** in the orbital period—that's about 20 seconds per day. Larger errors in altitude will produce correspondingly larger errors in time for the position of the satellite.

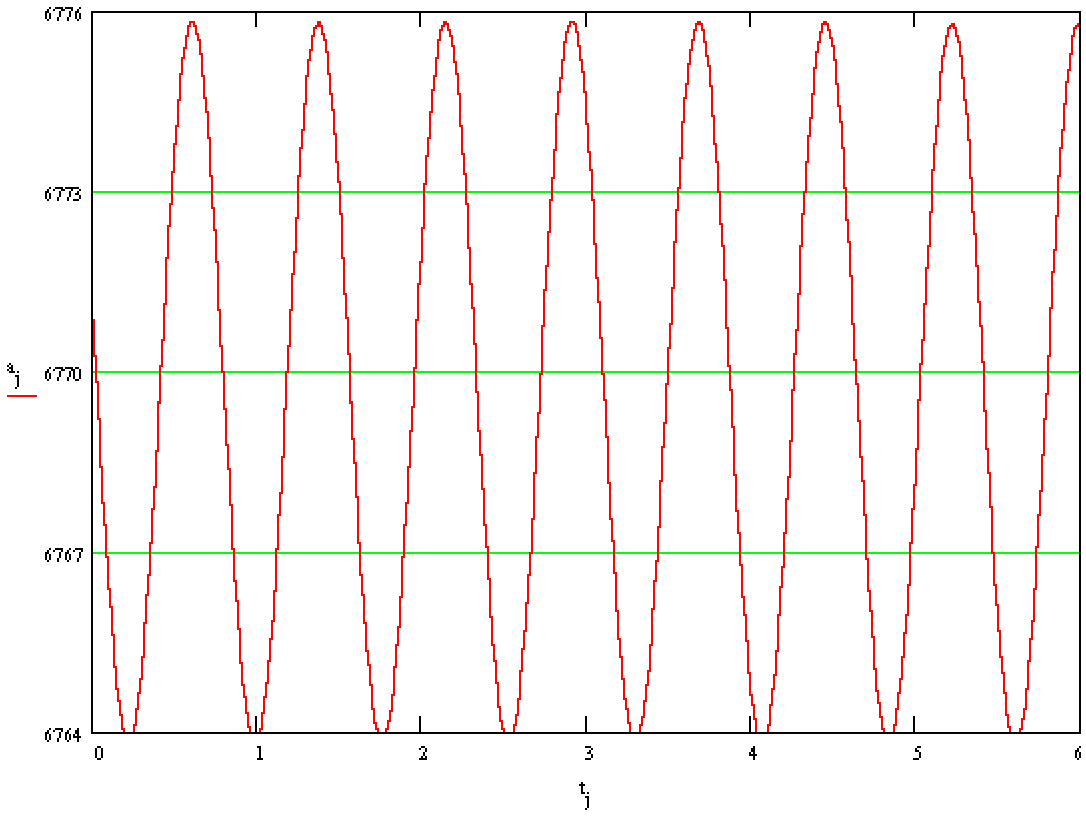

Figure 1. Semi-Major Axis (km) vs. Time (hr)

Looking next at the right ascension of the ascending node, the direction in space where the orbit plane crosses the Earth's equator from south to north, we see that this node precesses at a rate of over 4 degrees per day (see Figure 2). At Mir's altitude, that can cause an error of about 500 km per day! Also, notice that while there is a small periodic effect, the secular (trending) effect is quite pronounced.

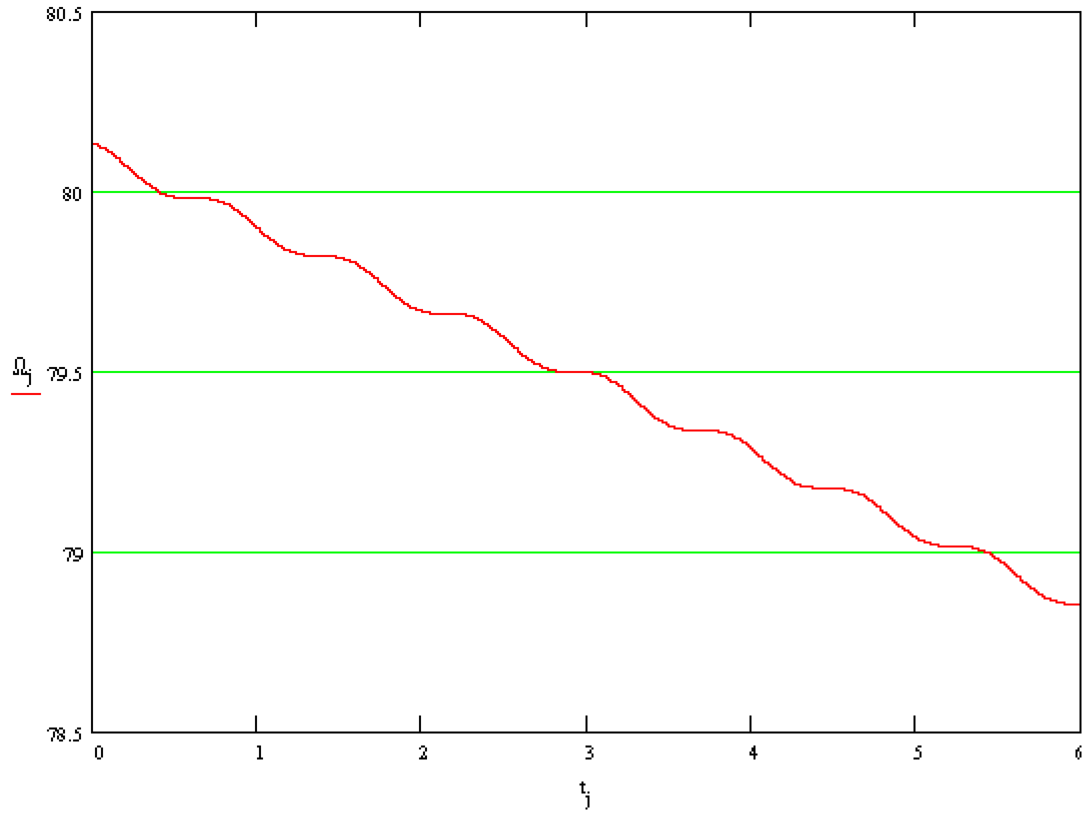

Figure 2. Right Ascension of the Ascending Node (deg) vs. Time (hr)

Finally, to give some idea of the complexity of the fluctuations that can be seen in the orbital elements, let's look at how the argument of perigee for Mir changes over a six-hour period (Figure 3). The argument of perigee is the angle between the right ascension of the ascending node and the direction of the perigee, or closest approach to the Earth's center, measured along the orbit path. As this angle fluctuates, the point where the satellite is lowest also fluctuates, resulting in even more errors. In our example, several periodic effects appear to be present.

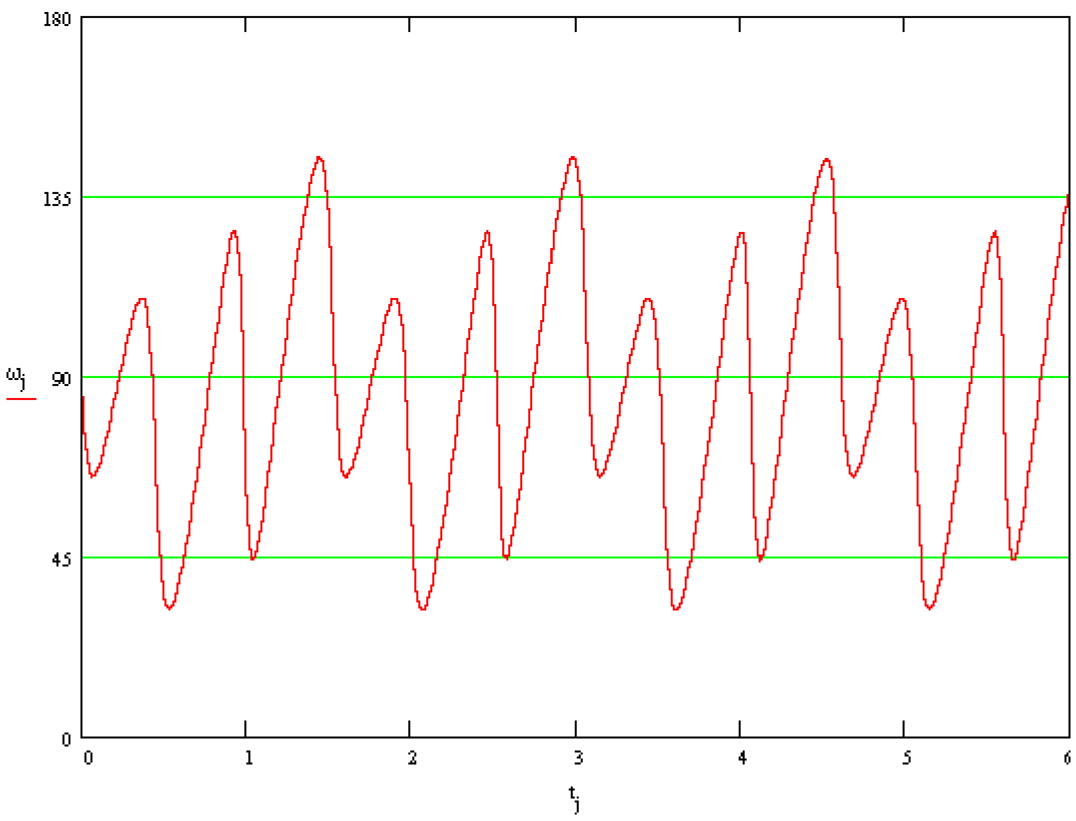

Figure 3. Argument of Perigee (deg) vs. Time (hr)

When we combine all the effects in the SGP4 model and compare it to a simple two-body propagator, we find an error of 150 km after only six hours and almost 400 km by the end of the day! Given element sets which may be several days old, the errors can be on the order of thousands of kilometers! And using a model which incorporates the perturbations in a manner contradictory to SGP4 can make the errors even worse.

In our next column, we'll look at what criteria are used to determine when an element set needs updating and what mathematical methods are used. Once we're done with that, we'll start exploring some of the basics of taking the output of SGP4 to produce calculations like those in TrakStar. See you next time!

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