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Representing data with logic

Propositional logic

Propositional logic is a formalism for reasoning about the truth assignments of propositions. Propositions are statements about the world being modelled, and can be true or false.

The propositions are constructed from propositional variables and connectivity operators. Propositional variables denote aspects of the world considered by the model, for instance, sun, rainbow and bob_runs. Variables are used in propositions as literals – a propositional variable or its negation. The connectivity operators compose literals into more complicated statements. Such operators in conjunction (AND operator), disjunction (OR operator), implication (IF-THEN operator) and equivalence (IF-AND-ONLY-IF operator).

A logical theory is a collection of propositions, more precisely, a conjunction of propositions. An interpretation is a truth assignment for each of the propositional variables. An interpretation satisfies a given proposition if it evaluates to true for the given truth assignments to the variables. An interpretation that satisfies the proposition is the model of that proposition.

Example The proposition playful $\wedge$ fluffy $\Rightarrow$ ideal_pet encodes that when something is playful (variable playful is true) and fluffy (variable fluffy is true)then it is an ideal pet (variable ideal_pet is true). Every interpretation in which ideal_pet is true is a model of this proposition.

Once the theory describing the knowledge is available, different inference tasks can be considered. The most fundamental task is that of consistency or satisfiability checking, which checks whether the theory has a model. The task of validity checking checks whether every interpretation is a model. The task of model counting, a generalisation of both aforementioned tasks, counts the number of models of the theory.

Predicate logic

Propositional logic is suitable framework for drawing inferences about individual objects or examples (where each example is an interpretation), but encoding more complex knowledge in form of relations between objects is extremely tedious. For instance, to indicate that two animals are of the same species would require introducing new boolean variables for each pair of animals, such as same_species_dog_cat, and specifying their truth assignments.

To over come this issue, predicate logic extends the propositional logic with objects and relations between them, resulting in a powerful language for representing mathematical formalism. An especially attractive feature of predicate logic is its universal inference algorithm – one can state a set of axioms, or facts, and theorems in predicate logic and rely on resolution Robinson:1965:Resolution inference algorithm to derive new axioms and theorems from the old ones.

The language of predicate logic is similar to the one of the propositional logic. The core building blocks of predicate logic formulas are four types of symbols: constant (referring to individual objects), variable (referring to groups of objects), function and predicate symbols.

The statements in predicate logic, the formulas, are composed of terms, atoms and connectivity operators. A term is defined as: itemize

a variable is a term

i f f/n is a function symbol and $t_1, t_2, ..., t_n$ are terms, then function $f(t_1, ..., t_n)$ is a term.