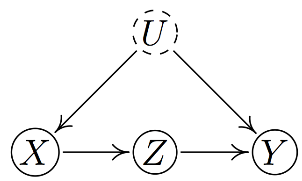
Homework #2

Due to 21-04

Problem 1. Let $\mathcal{S}$ be an SEM with corresponding graph



Compute the observational equivalent of $p_{do(X=x)}(y, z)$.

Problem 2. Program a subroutine (in Python, Matlab, ...) that takes as inputs the adjacency matrix of a DAG and three subsets of its nodes, X, Y, and Z and check whether X and Y are d-separated given Z. If so it should return 1 and 0 otherwise. Note that this subroutine should also check that these three given subsets are disjoint.
- We have seen in the course that it is possible to check if a selected subset Z satisfies backdoor criterion by adding a parent I to node X and check the next two conditions

$$Y \text{ d-sep } I|X, Z \qquad\qquad Z \text{ d-sep } I.$$

Write a function that uses the above tests and find all subsets Z that satisfy the backdoor criteria (backdoor sets).
- Use your codes to find all the backdoor sets for $P(Y|do(X))$ in the following DAG.

$$A = \begin{array}{c} \begin{array}{cccccc} & & X & & Y & \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array} \begin{array}{c} \\ \\ X \\ \\ Y \\ \\ \end{array}$$

- In this adjacency $A_{1,2} = 1$ that means there is an arrow from node 1 to node 2.
- Note that your code should be able to find all the backdoor sets for arbitrary DAGs.

Problem 3.
Use conditional independence tests and Meek rules to learn the DAG in the Figure 1 (plot the intermediate steps.) Can you learn the structure completely?

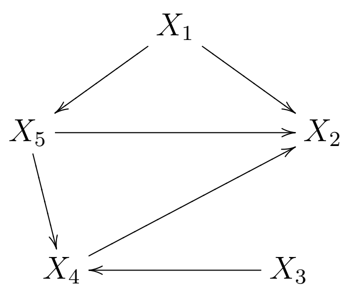


Figure 1

Problem 4. Consider the DAG in Figure 2.
(I) Find all the valid adjustment sets for the following pairs: (Hint: Use backdoor-criterion and towards necessity)
-(X_3, X_5) $(P(X_5|do(X_3)))$,
-(X_2, X_5) $(P(X_5|do(X_2)))$.
(II) Find the observational only equivalents of the following expressions (Hint: Use do-calculus and valid adjustments to get rid of the do operations):
-$P(X_5|do(X_3), do(X_4))$,
-$P(X_5|do(X_2), do(X_3))$.

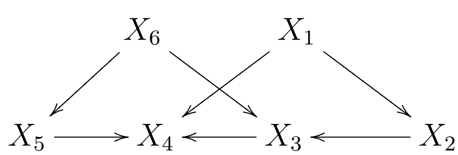


Figure 2