École Polytechnique Fédérale de Lausanne

Assignment 1

Network Analytics MGT - 416

Group 06

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Problem 1

The desired result corresponds to the probability of having chosen the left drawer knowing that we randomly picked a knife :

$$P(L|K) = \frac{P(L, K)}{P(K)}$$

We calculate the probability of picking up a knife. 2 possibilities, either the item randomly taken from the right to the left drawer is a fork (probability of 1/6), or it's a knife, (probability of 5/6) :

$$P(K) = \frac{5}{6} * (\frac{1}{2}\frac{1}{9} + \frac{1}{2}\frac{9}{11}) + \frac{1}{6} * (\frac{1}{2}0 + \frac{1}{2}\frac{10}{11}) = \frac{25}{54}$$

$$P(L, K) = \frac{5}{6}\frac{1}{2}\frac{1}{9} + \frac{1}{6}\frac{1}{2}0 = \frac{5}{108}$$

$$P(L|K) = \frac{5}{108}\frac{54}{25} = \frac{1}{10}$$

The probability of having chosen the left drawer knowing that we randomly picked up a knife is then of 1/10.

Problem 2

a)

$$X_1 \sim \mathcal{N}(\mu[0],\, \Sigma[0,0]) = \mathcal{N}(1,\, 1)$$

b)

$$Z = 2X_1 + X_2 + X_3 = b^T * X$$

$$b = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

$$E[Z] = b^T * \mu = 9$$

$$V[Z] = b^T * \Sigma * b = 21$$

$$Z \sim \mathcal{N}(9,\, 21)$$

c)

X_3 is independent of X_1, X_2 (zero-elements of the co-variance matrix of normal random variables) thus:

$$X_1 \sim \mathcal{N}(\mu[2],\, \Sigma[2,2]) = \mathcal{N}(2,\, 9)$$

In general a null covariance does not imply independence, but for normal RVs it is the case

d)

$$X \sim \mathcal{N}(\mu,\, \Sigma)$$

$$Z = \begin{bmatrix} X_2 \\ X_3 \end{bmatrix}$$

$$X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

μ_z : mean vector of Z

$\Sigma_{zx_1}, \Sigma_{x_1 z}, \Sigma_{x_1 x_1}, \Sigma_{zz}$: covariance matrices of $(Z, X_1), (X_1, Z), X_1, Z$ respectively

$$Z|(X_1) \sim \mathcal{N}(\mu^*,\, \Sigma^*)$$

$$\mu^* = \mu_z + \Sigma_{zx_1} \cdot \Sigma_{x_1 x_1}^{-1} \cdot (x_1 - \mu_{x1})$$

$$\Sigma^* = \Sigma_{zz} - \Sigma_{zx_1} \cdot \Sigma_{x_1 x_1}^{-1} \cdot \Sigma_{x_1 z}$$

$$\mu^* = \begin{bmatrix} 5 \cdot X_1 + 5 \\ 2 \end{bmatrix}$$

$$\Sigma^* = \begin{bmatrix} 3, 0 \\ 0, 9 \end{bmatrix}$$

Problem 3

Experiment : Throwing a fair die and flipping a fair coin simultaneously

Set of outcomes of throwing a die : D = {1,2,3,4,5,6} with p = 1/6 (equiprobable elementary events)

Set of outcomes of throwing a coin : C = {Head,Tail} with p = 1/2 (equiprobable elementary events)

Set of outcomes of the experiment :

Ω = {(D,C)} ={(Head,1),(Head,2),(Head,3),(Head,4),(Head,5),(Head,6),(Tail,1),(Tail,2),(Tail,3), (Tail,4),(Tail,5),(Tail,6)}

We can assume that throwing a die and flipping a coin are two independent action, so the probability of each outcome of the experiment is given by : p(d,c) = p(d)∗p(c) = 1/12 for all possible outcomes

Gain functions :

$$W(d,c) = \begin{cases} 2 * d & \text{if } \{c = "Heads"\}, d \in D \\ 0.5 * d & \text{if } \{c = "Tail"\}, d \in D \end{cases}$$

Calculating the expected win :

$$\text{E[W(d, c)]} = \sum_{(d,c)\in\Omega} W(d,c) \cdot p(d,c) = 1/12 \cdot \sum_{(d,c)\in\Omega} W(d,c) = 4.375$$

$$\begin{aligned} E[W] &= P(Head) * E[W|Head] + P(Tail) * E[W|Tail] && (1) \\ &= P(Head) * 2 * E[Die|Head] + P(Tail) * 0.5 * E[Die|Tail] && (2) \\ &= 0.5 * 7 + 0.5 * 1.75 && (3) \\ &= 4.375 && (4) \end{aligned}$$

Problem 4

The demonstration can be done as follow :

$$
\begin{aligned}
P(X, Y|Z) &= \frac{P(X, Y, Z)}{P(Z)} && (5)\\
&= \frac{P(X, Y, Z)}{P(Y, Z)} \frac{P(Y, Z)}{P(Z)} && (6)\\
&= P(X|Y, Z) P(Y|Z) && (7)
\end{aligned}
$$

Problem 5

$$P\left(H|E_1,E_2\right)=\frac{P\left(H,E_1,E_2\right)}{P\left(E_1,E_2\right)}=\frac{P\left(E_1,E_2|H\right)P\left(H\right)}{P\left(E_1,E_2\right)}$$

We can use set (b) to calculate the probability above without conditional independence information

To use the sets (a) and (c), we need to know the conditional independence of $E_1,E_2|H$

More explicitly if :

$$P\left(E_1,E_2|H\right)=P\left(E_1|H\right)P\left(E_2|H\right)$$

Problem 6

a)

We want to find $P(R=T|G=T)$:

$$P(R=T|G=T) = \frac{P(G=T, R=T)}{P(G=T)} = \frac{\sum_S P(G=T, S, R=T)}{\sum_S P(G=T, S, R)}$$

Where,

$$P(G=T, S=T, R=T) = P(G=T|S=T, R=T)P(S=T|R=T)P(R=T)$$
$$= 0.99 \cdot 0.01 \cdot 0.2 = 0.00198$$

$$P(G=T, S=F, R=T) = P(G=T|S=F, R=T)P(S=F|R=T)P(R=T)$$
$$= 0.8 \cdot 0.99 \cdot 0.2 = 0.1584$$

$$P(G=T, S=T, R=F) = 0.288$$

$$P(G=T, S=F, R=F) = 0$$

Then,

$$P(R=T|G=T) = \frac{0.00198 + 0.1584}{0.00198 + 0.288 + 0.1584 + 0} = 0.3577$$

$$P(R, G) = 0.2 * (0.01 * 0.99 + 0.99 * 0.8) = 0.16038$$
$$P(G) = P(R, G) + P(R^c, G) = P(R, G) + 0.8 * (0.4 * 0.9 + 0.6 * 0) = 0.44838$$
$$P(R|G) = \frac{P(R, G)}{P(G)} = 0.35769$$

b)

The event of raining is not dependent on the state of the grass (wet or not). Then,

$$P(R=T|do(G=T)) = P(R=T) = 0.2$$

$$P(R, S) = 0.01$$
$$P(S) = P(R)P(S|R) + P(R^c)P(S|R^c) = 0.2 * 0.01 + 0.8 * 0.4 = 0.322$$
$$P(R|S) = \frac{P(R, S)}{P(S)} = 0.0312$$

c)

$$P(R, G|do(S=T)) = P(R)P(G|R, S=T)$$

Rain is a valid adjustment set for an outside intervention ,which sets the sprinkler to true, because it can be factorized as above. This means that by conditioning on Z we block all non causal path between sprinkler and grass wet.

Problem 7

a)

The mutual information between A;H and B;H is used to tell which one between A and B is a better predictor for H : between B and H is larger than the one between A and H :

$I(A;H) = 2 * \frac{1}{3} * log(\frac{1/3}{1/4}) + 2 * \frac{1}{6} * log(\frac{1/6}{1/4}) = 0.024595$

$I(B;H) = 2 * \frac{19}{40} * log(\frac{19/40}{1/4}) + 1 * \frac{1}{40} * log(\frac{1/40}{1/4}) = 0.214816$

We see that the mutual distribution between B and H is larger than the one between A and H. Then, we can conclude that B is a better predictor of H than A.

b)

We can look at the interventional distribution to understand whether A or B are causing H, we have, according to the SEM given :

$P_S^{H|do(A=1)} = Ber(2/3)$ which is different from $P_S^H = Ber(1/2)$
$P_S^{H|do(B=1)} = Ber(1/2) = P_S^H$

Even though B is a better predictor for H than A, we can clearly see that A has a larger influence over H than B.

c)

From a) and b), we can suppose that a larger influence implies causation, as A has a larger influence over H than B.
The mutual information doesn't give us any information about causation.

Appendices