UNIVERSITÄT ZU LÜBECK

Steganography Based on Pattern Languages

Sebastian Berndt Rüdiger Reischuk

Institut für Theoretische Informatik, Universität zu Lübeck

IM FOCUS DAS LEBEN

And now for something completely different!

- Cryptography: Hide the content of a message

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- Cryptography: Hide the content of a message
- Steganography: Hide that a message is transferred

Sometimes, cryptography is not enough

Overview

Steganography

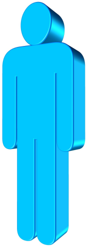
Pattern Languages

The stegosystem

Conclusion

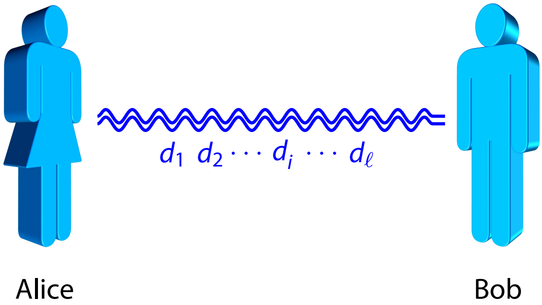
The Prisoners' Problem (not the Dilemma)

Alice



Bob

The Prisoners' Problem (not the Dilemma)



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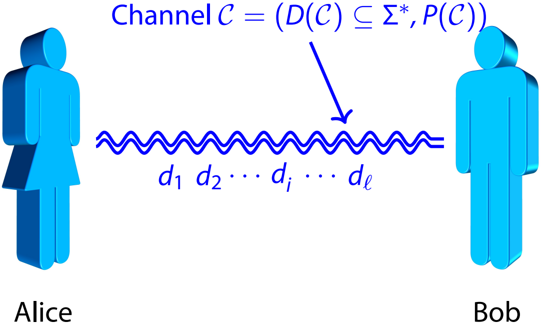

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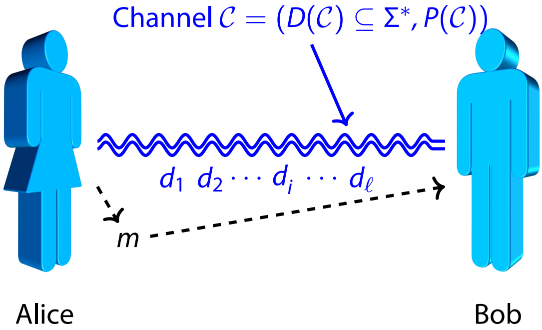

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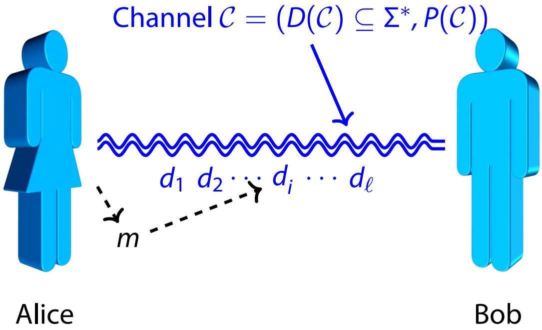

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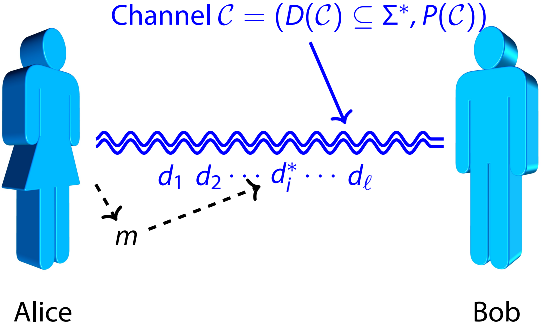

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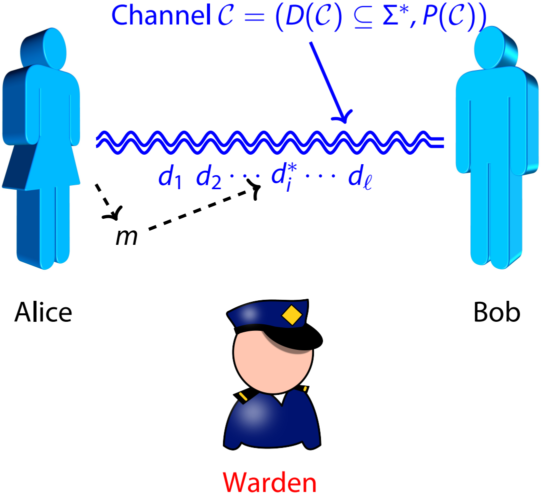

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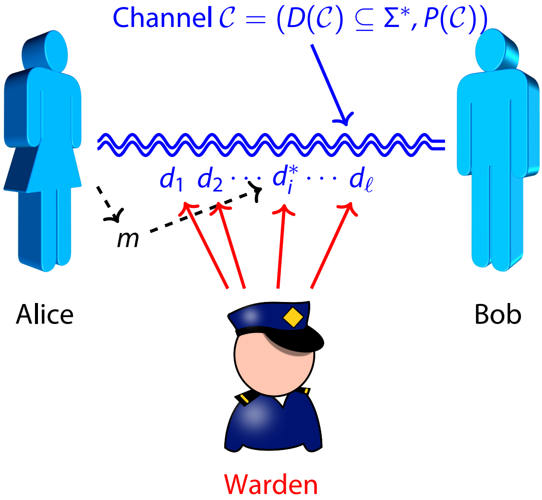

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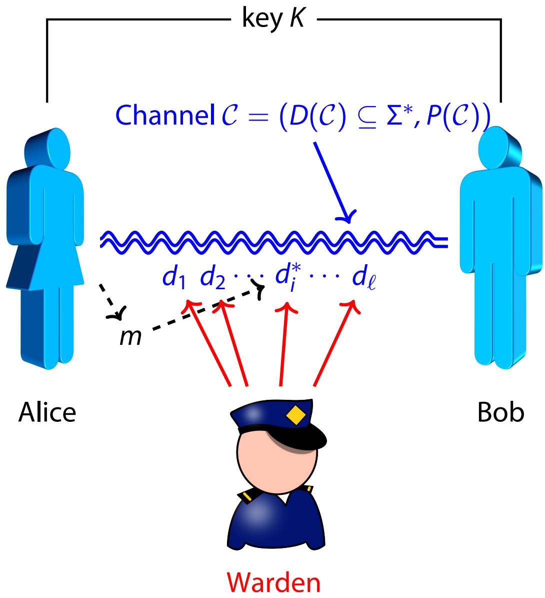

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- Should work for as much channels as possible (*Applicability*)

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Task

Be more specific: Develop stegosystem for large channel-family $\mathcal{F}$!

Previous Work

LB on Rate	UB on Rate	Channels	Authors
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Much more systems exist, but none are provable secure!

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001110
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The system

1. Partition the positions of X into blocks $B_1, \ldots, B_b$ via PRP
2. Replace the different ?'s such that $\sum_{x \in B_i} x = m_i$

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Restrictions

- Simplest non-trivial language
- Simple cross product

Patterns

$\pi = x_1 01 x_2 1 x_1$
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- Much larger class of languages

Pattern Channels

- $D(\mathcal{C}) \subseteq \mathrm{Lang}(\pi)$

$$d_1 \; d_2 \cdots \; d_i \cdots \; d_\ell \; \sim \mathrm{Lang}(\pi)$$

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Pattern Channels

- $D(\mathcal{C}) \subseteq \mathrm{Lang}(\pi)$
- The pattern π is known (or can be learned)
- The documents are of approximately same size
- Substitutions of variables are independent

$$d_1 \ d_2 \cdots \ d_i \cdots \ d_\ell \ \sim \mathrm{Lang}(\pi)$$

The stegosystem

- Expand $\pi = x_1 0 1 x_2 1 x_1$ into $[\pi] = y_1 y_2 y_3 0 1 y_4 1 y_1 y_2 y_3$.

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y_1

y_2

y_3

0

1

y_4

1

y_1

y_2

y_3

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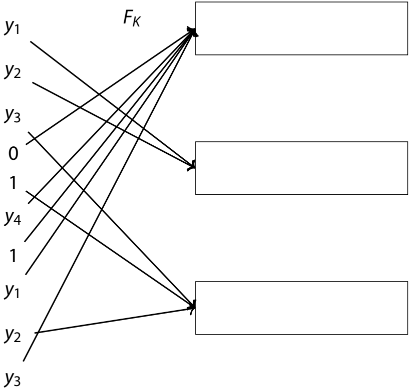

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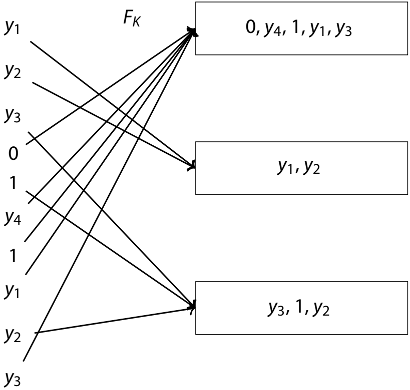

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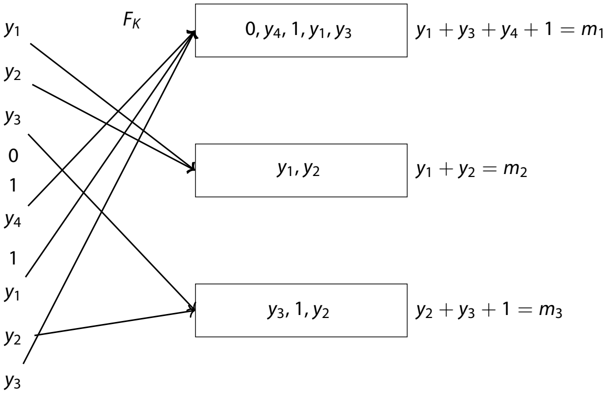

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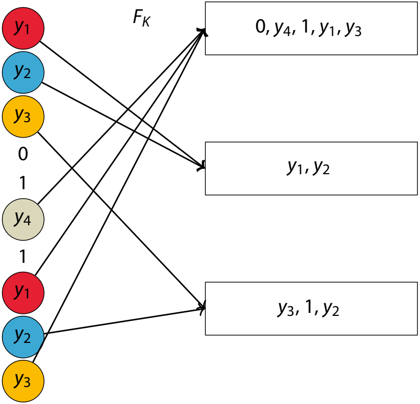

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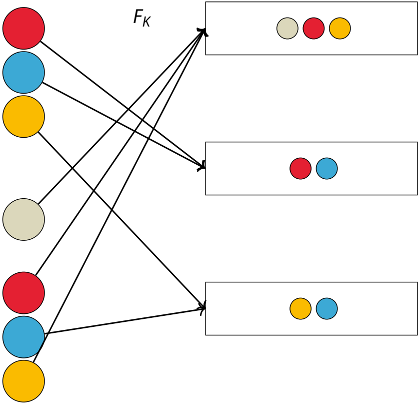
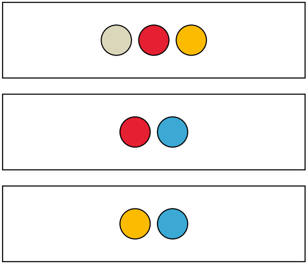


Coloured Balls into Bins



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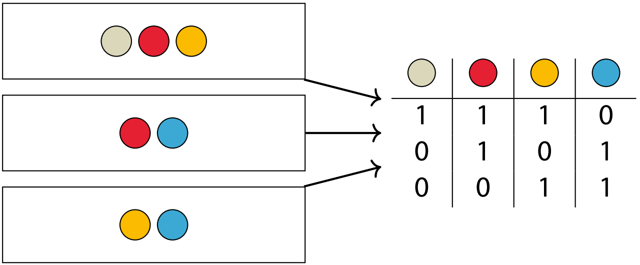

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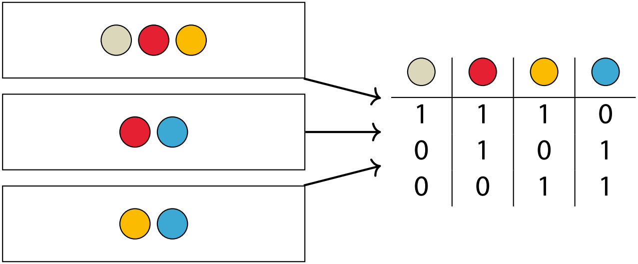

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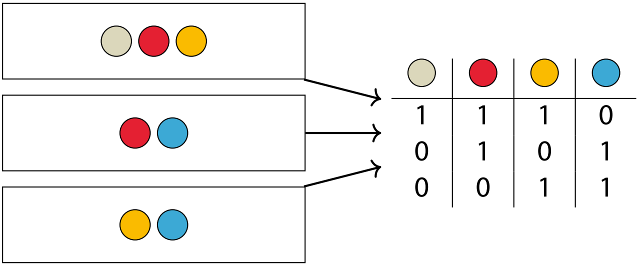

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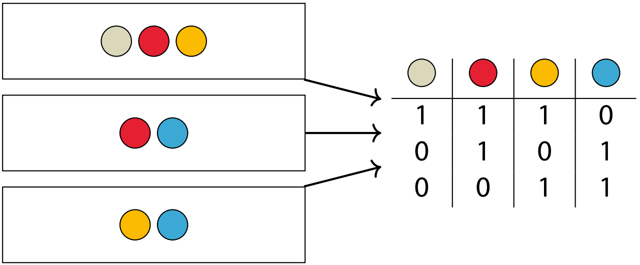

1	1	1	0
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Theorem (Generalization of Poisson Approximation)

W.h.p. there are many colours such that the corresponding rows behave independently!

Some Useful Lemmata

Lemma

Let C be the coloured-balls-into-bin matrix with μ bins, where each colour appears at most ξ times and let X be a matrix of pairwise independent Poisson-distributed RVs.
For every predicate Q of arity l, let $\mathcal{E}_Q(X)$ be the probability that X has a subset of l columns $X_{z_1}, \ldots, X_{z_l}$ such that $Q(X_{z_1}, \ldots, X_{z_l})$ holds. Then:

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Corollary

If $n \geq b^2$, it holds that: $|\Pr[\mathcal{E}_Q(C)] - \Pr[\mathcal{E}_Q(X)]| \leq \exp(-n)$.

Open Questions

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- Steganography for other families of languages ($D(\mathcal{C})$ described by automata, grammars, logics, . . .)?

Conclusion

Secure, rate-efficient steganography is possible on realistic channels!