UNIVERSITÄT ZU LÜBECK

Fully Dynamic Bin Packing Revisited

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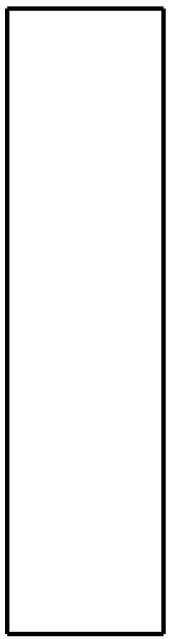
[2] Department of Computer Science, Christian-Albrechts-University to Kiel

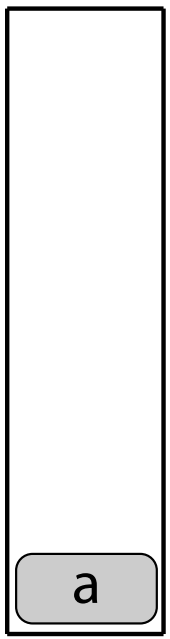
Fully Dynamic Bin Packing = Online + Removal + Repacking

INSERT: *a*/0.2



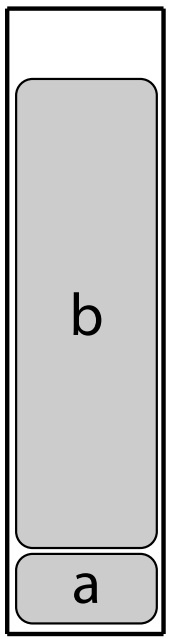
Fully Dynamic Bin Packing = Online + Removal + Repacking

INSERT: *a*/0.2, **INSERT:** *b*/0.7



Fully Dynamic Bin Packing = Online + Removal + Repacking

INSERT: *a*/0.2, **INSERT:** *b*/0.7, **INSERT:** *c*/0.4



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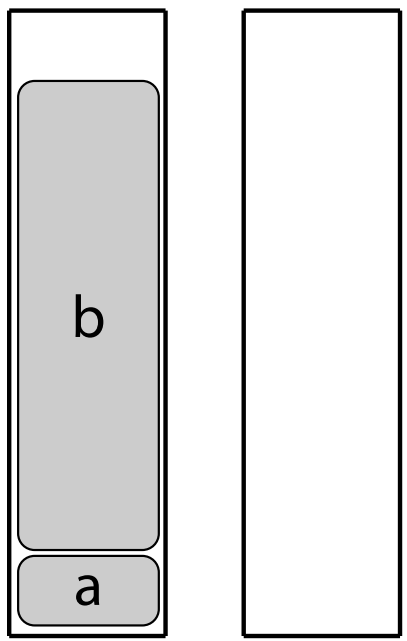
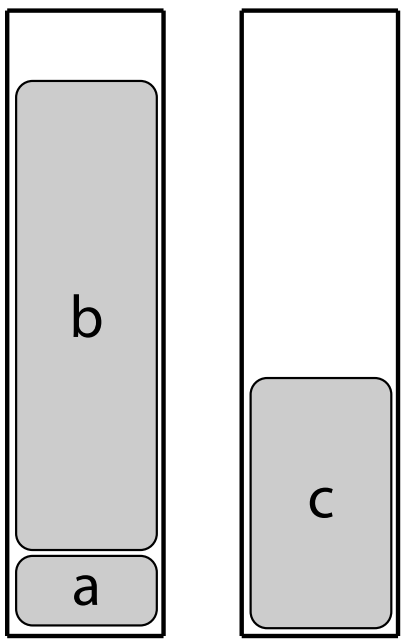
INSERT: *a*/0.2, **INSERT:** *b*/0.7, **INSERT:** *c*/0.4



Fully Dynamic Bin Packing = Online + Removal + Repacking

INSERT: *a*/0.2, **INSERT:** *b*/0.7, **INSERT:** *c*/0.4, **INSERT:** *d*/0.4



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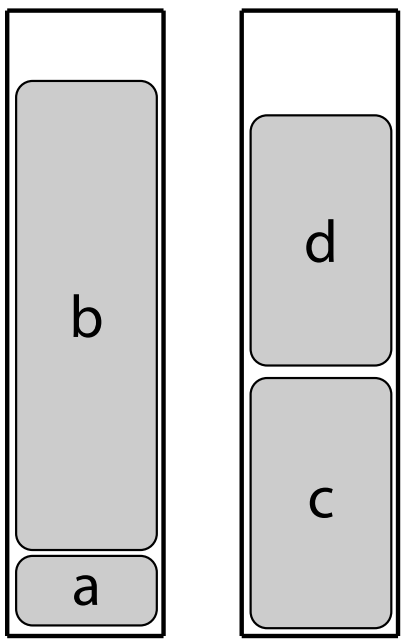
INSERT: *a*/0.2, **INSERT:** *b*/0.7, **INSERT:** *c*/0.4, **INSERT:** *d*/0.4, **REMOVE:** *b*/0.7



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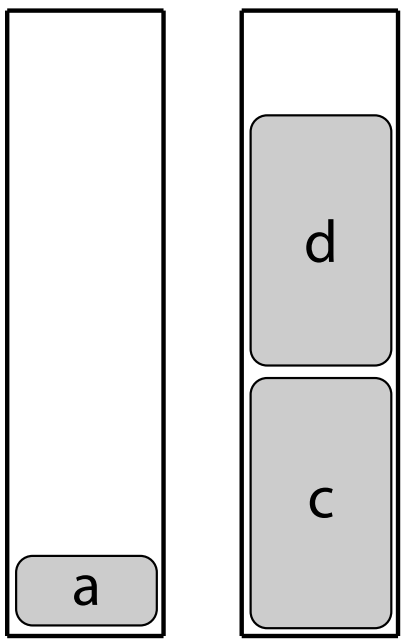

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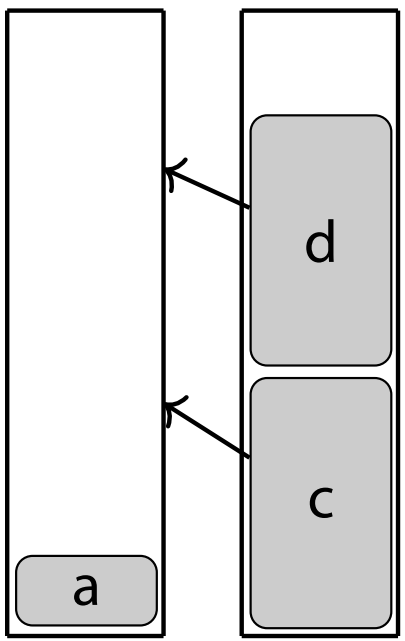

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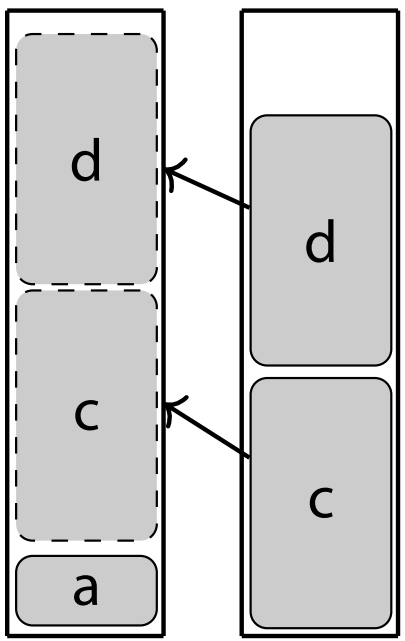

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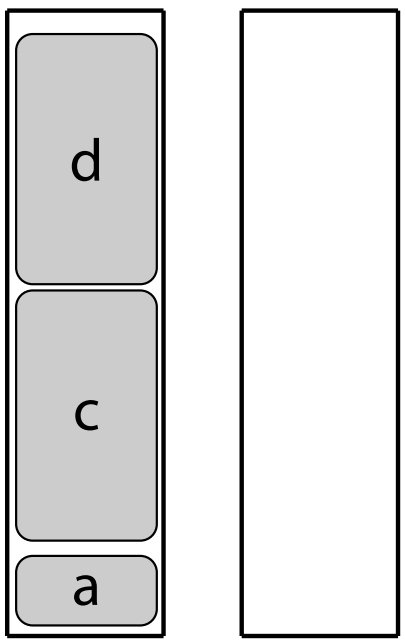

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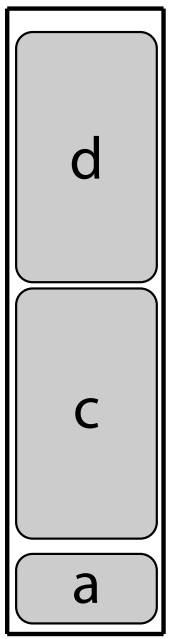

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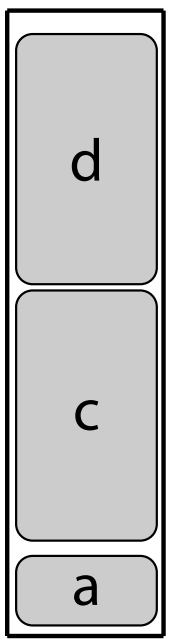

$$\textit{Migration Factor} = \frac{c + d}{b} = \frac{\text{SIZE(moved)}}{\text{SIZE(new/removed)}}$$

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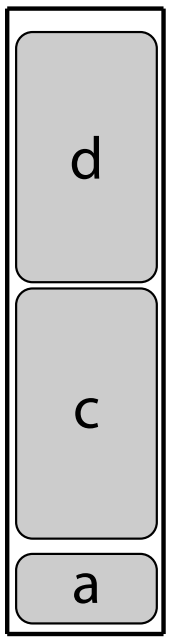

$$\textit{Migration Factor} = \frac{c + d}{b} = \frac{\text{SIZE(moved)}}{\text{SIZE(new/removed)}}$$

Shifting Moves = 2 = # moved

Lower Bound

MF of $\Omega(1/\varepsilon)$ is necessary for rate $1+\varepsilon$

$a = 1/2 - b/3, \quad b = 1/MF$

INSERT: a^+

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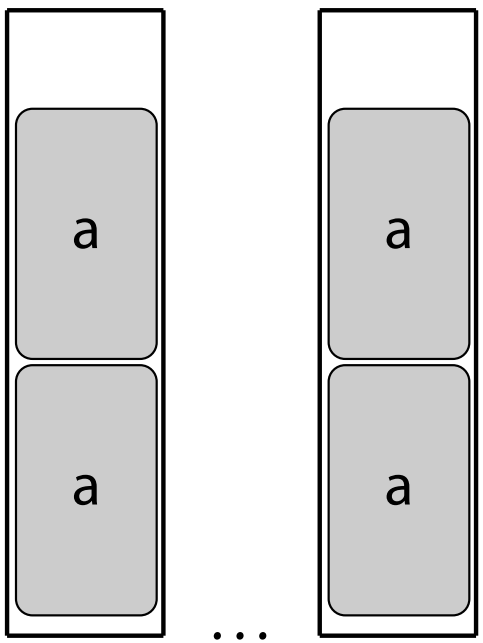

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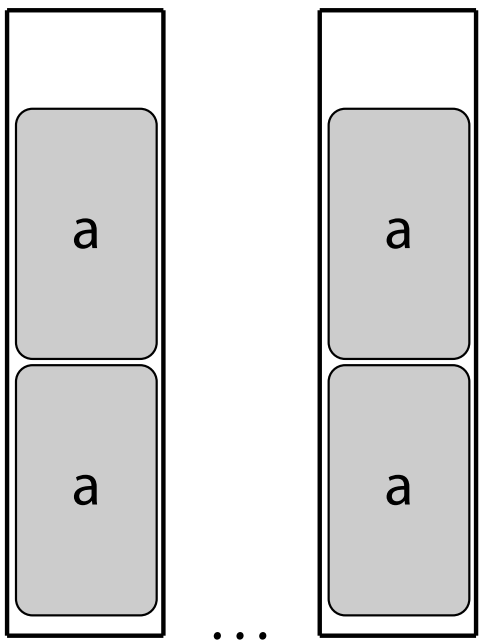
INSERT: b^+



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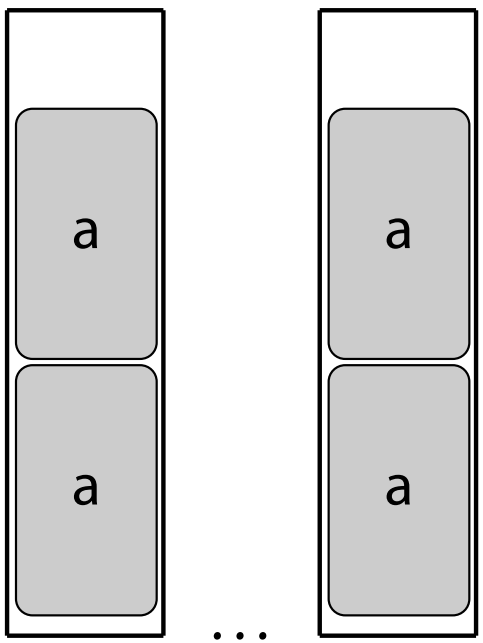

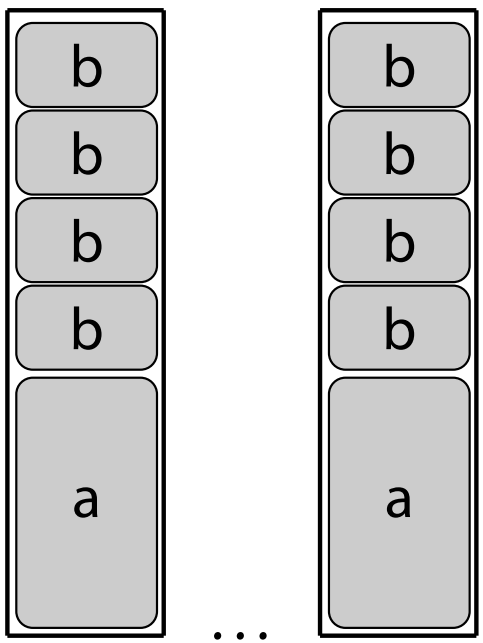

Fully Dynamic Bin Packing

Known Results

Rate	Shifting Moves	Migration Factor	Authors

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5/4	7 (amortized)	∞	Ivković, Lloyd (1998)

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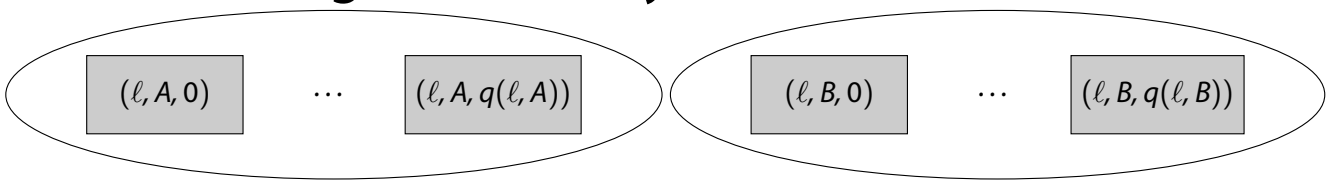
$1+\varepsilon$	$\mathcal{O}(1/\varepsilon^4 \cdot \log(1/\varepsilon))$	$\mathcal{O}(1/\varepsilon^4 \cdot \log(1/\varepsilon))$	*B., Jansen, Klein (2015)*

Large Items

Round Items geometrically:



$s \in [2^{-(\ell+1)}, 2^{-\ell})$

- $|(\ell, A, \cdot)| = 2^{\ell} \cdot \text{SIZE}, \quad |(\ell, B, \cdot)| = 2^{\ell} \cdot (\text{SIZE} - 1)$

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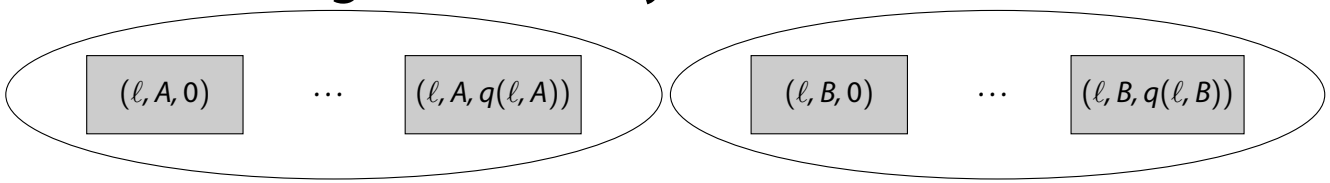

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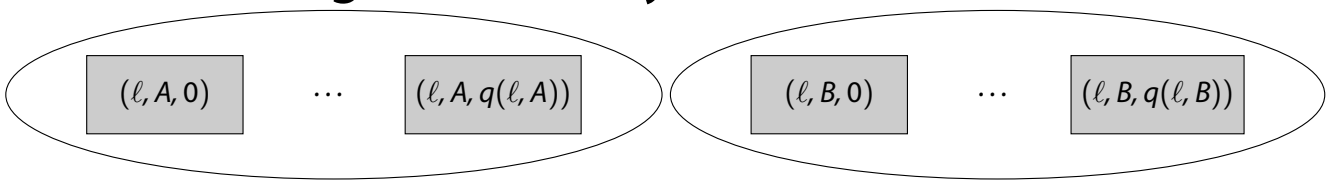

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- Value of SIZE may increase or decrease

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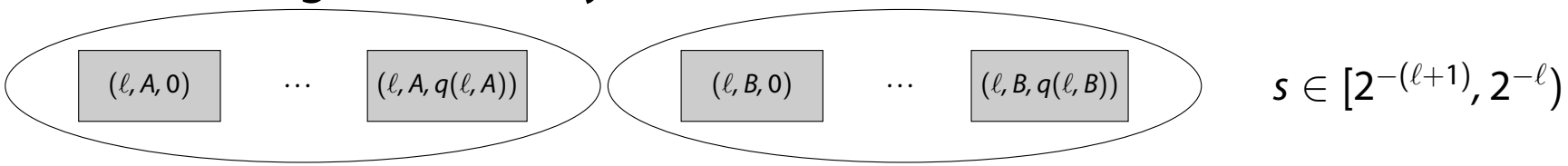

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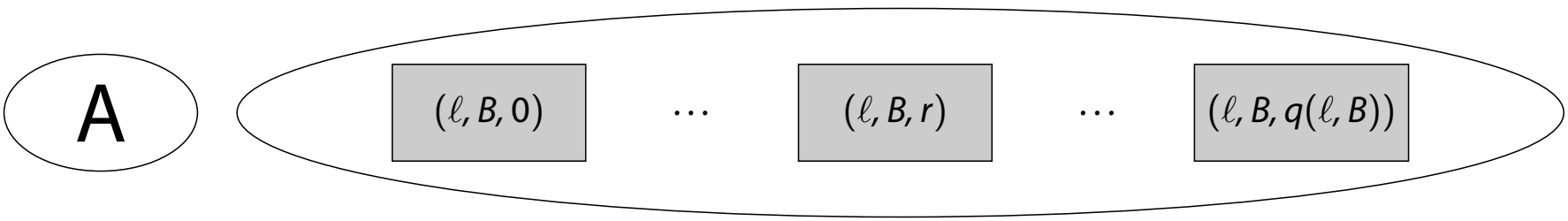

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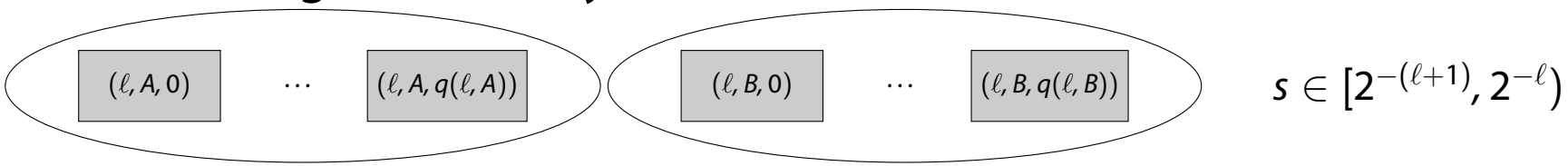

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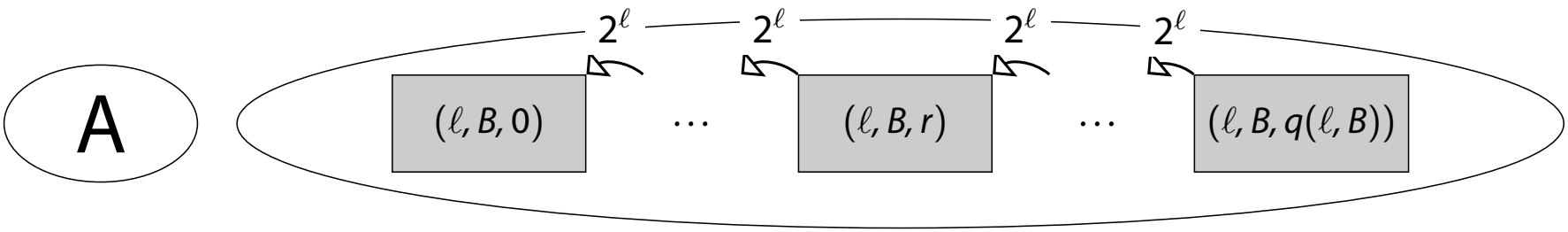

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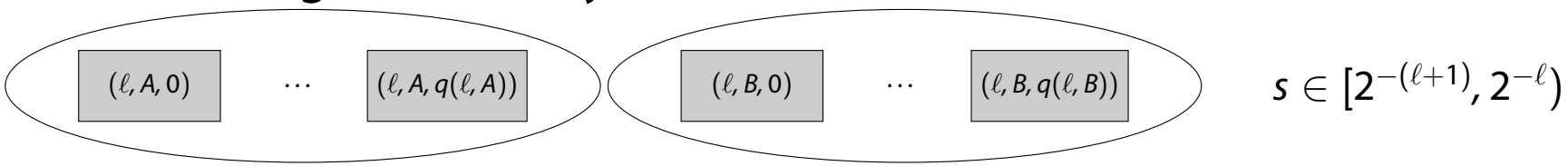

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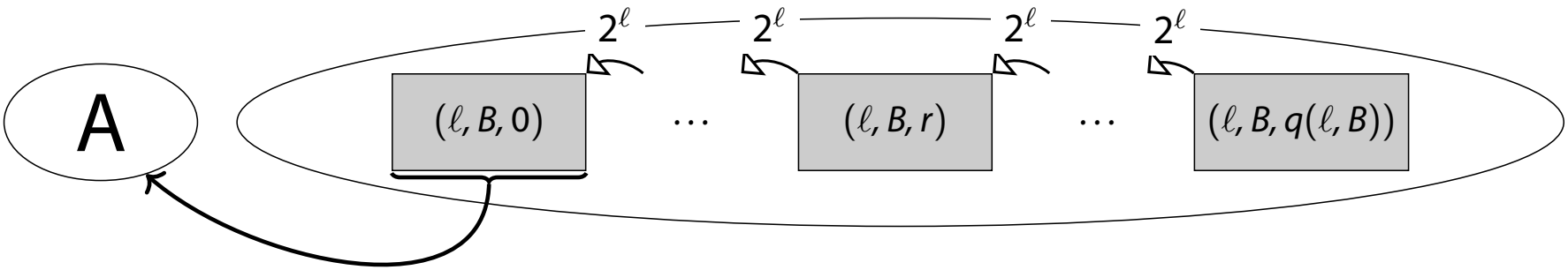

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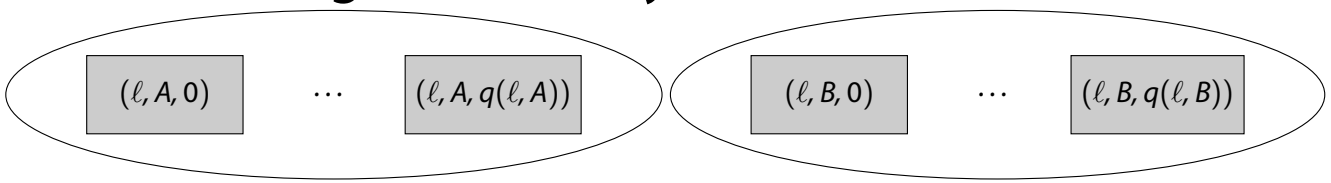

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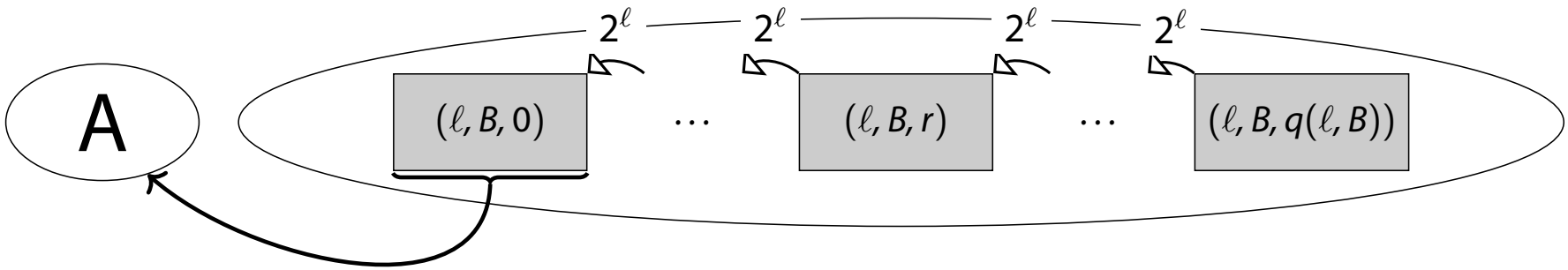

Idea : Shifting Items to the left complies with the packing!

Small Items

Greedy fails: $\varepsilon \gg b \gg a$

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$$(\textbf{INSERT:}\, b, \textbf{INSERT:}\, a, \ldots, \textbf{INSERT:}\, a)^{+}$$

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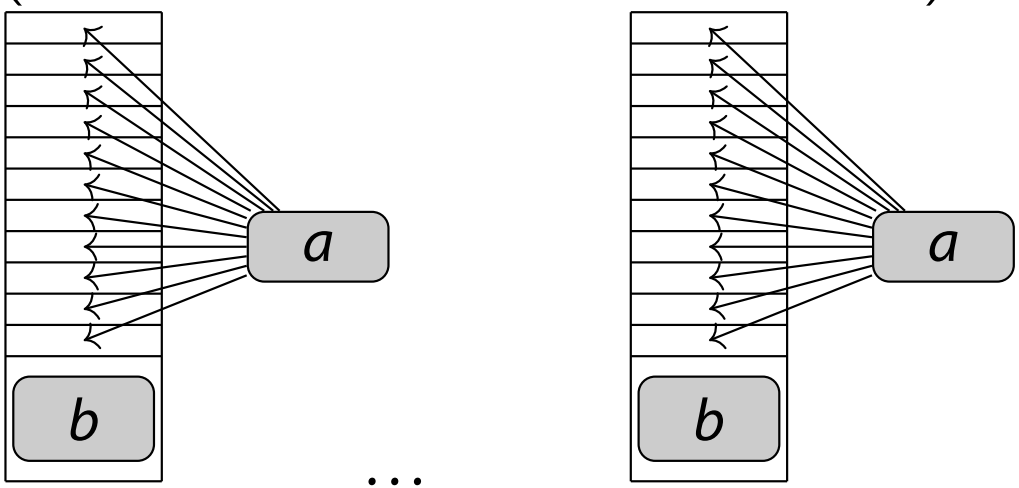

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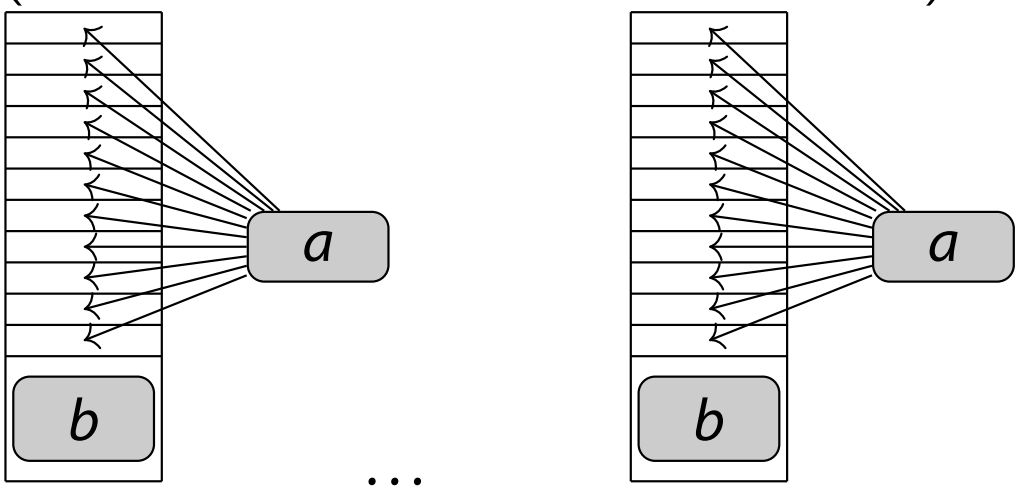
$(\textbf{INSERT:}\, b, \textbf{INSERT:}\, a, \ldots, \textbf{INSERT:}\, a)^+$ $(\textbf{REMOVE:}\, a, \textbf{REMOVE:}\, a, \ldots)^+$



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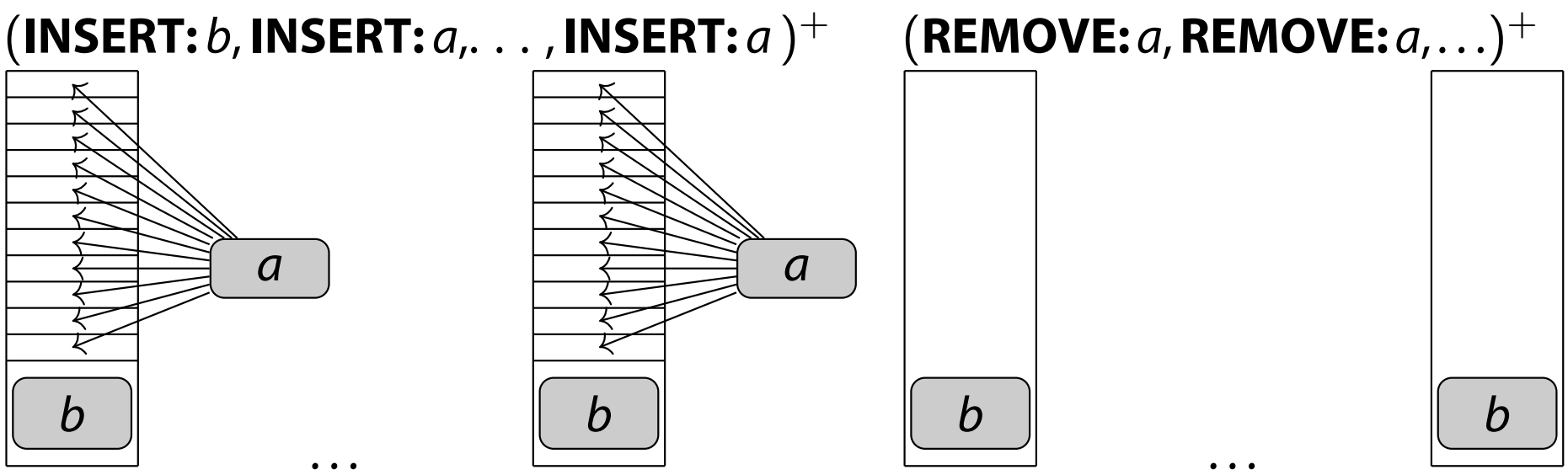
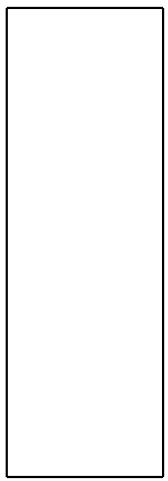


Small Items

Idea: "Sort" small items from left to right



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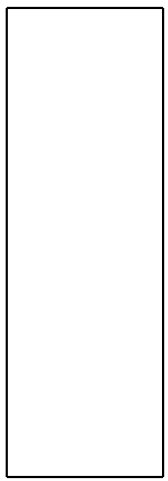

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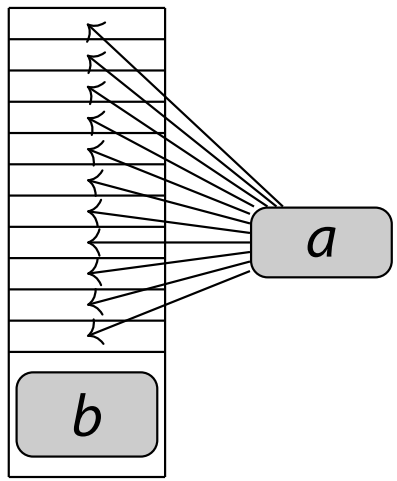

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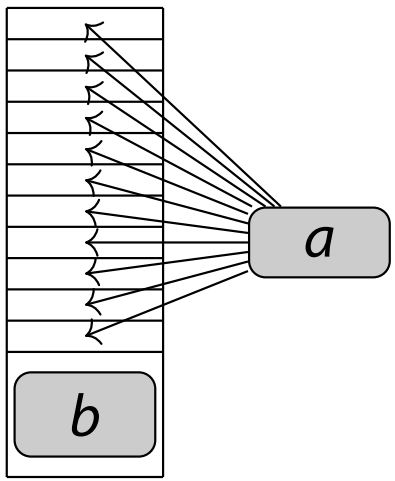

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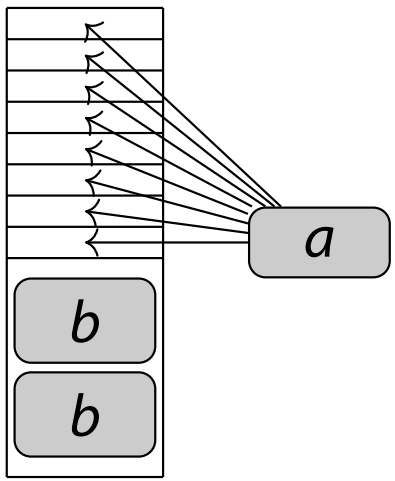

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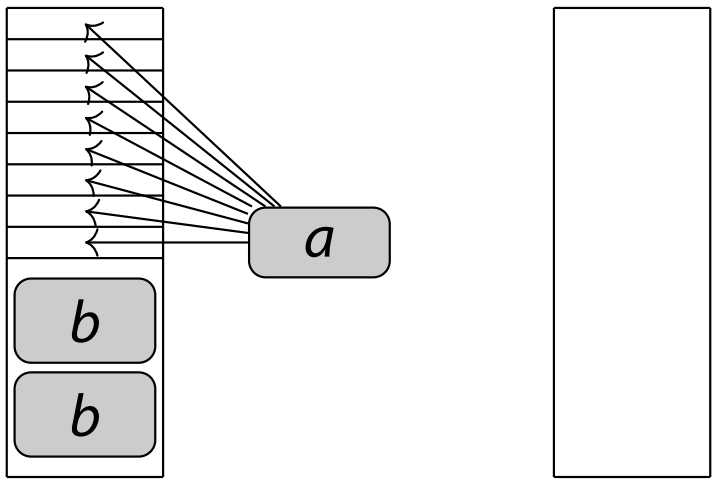

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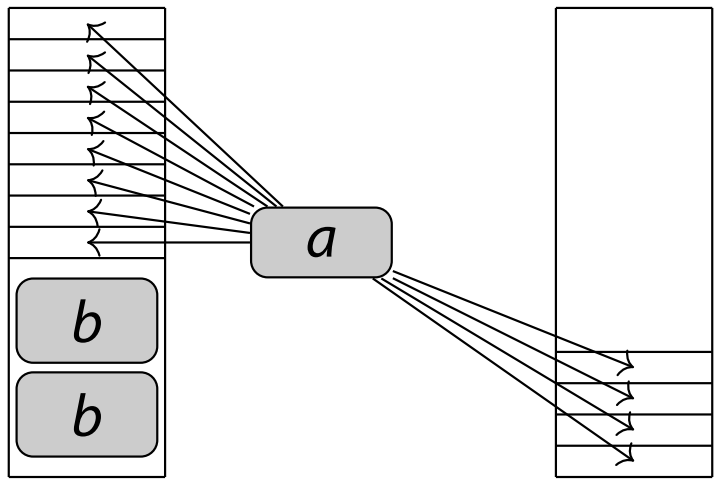

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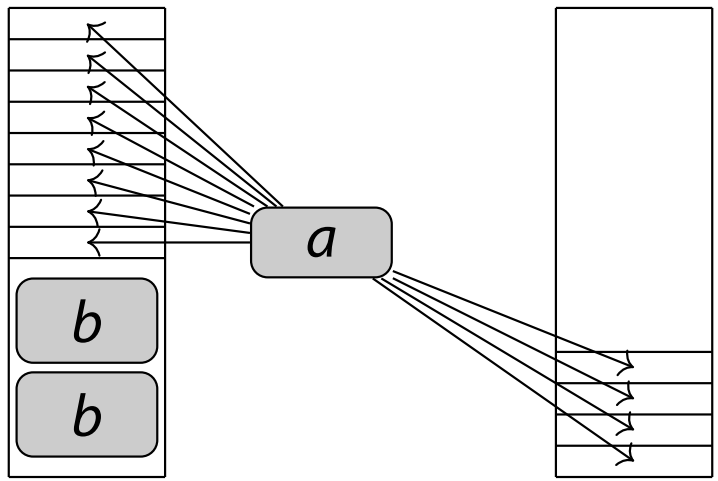
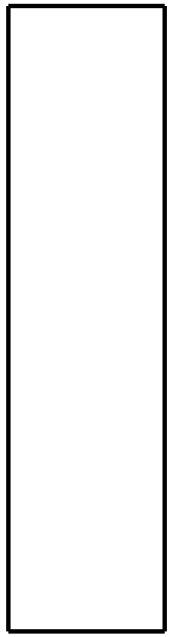


Stop at every $1/\varepsilon$-th bin (buffer bin) to bound MF

Everything



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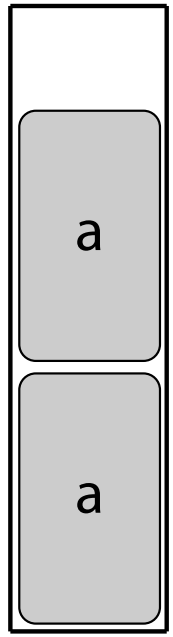

Large

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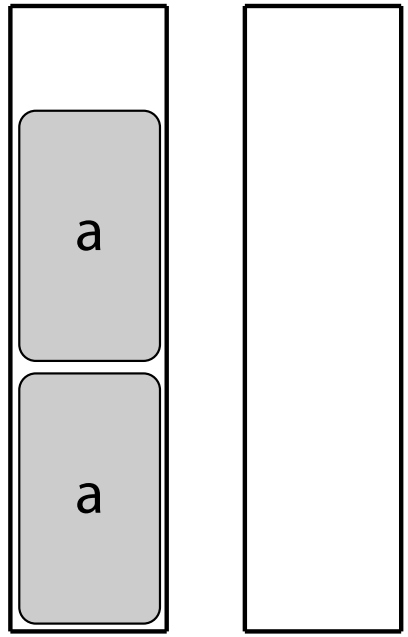

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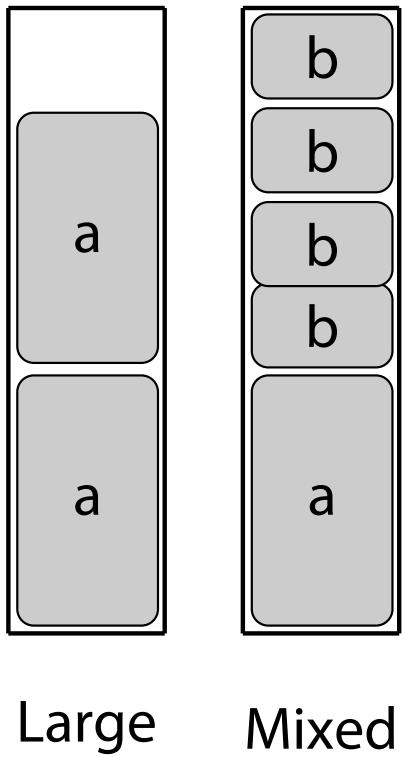

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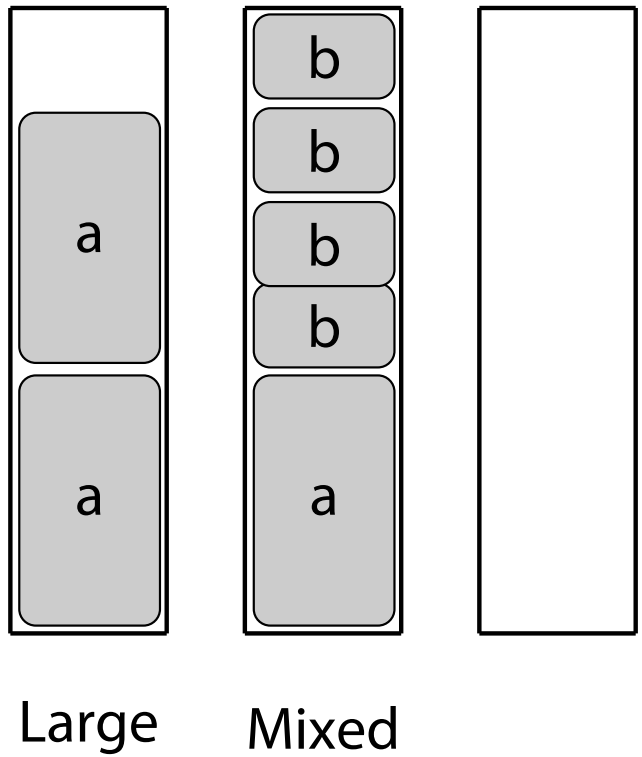
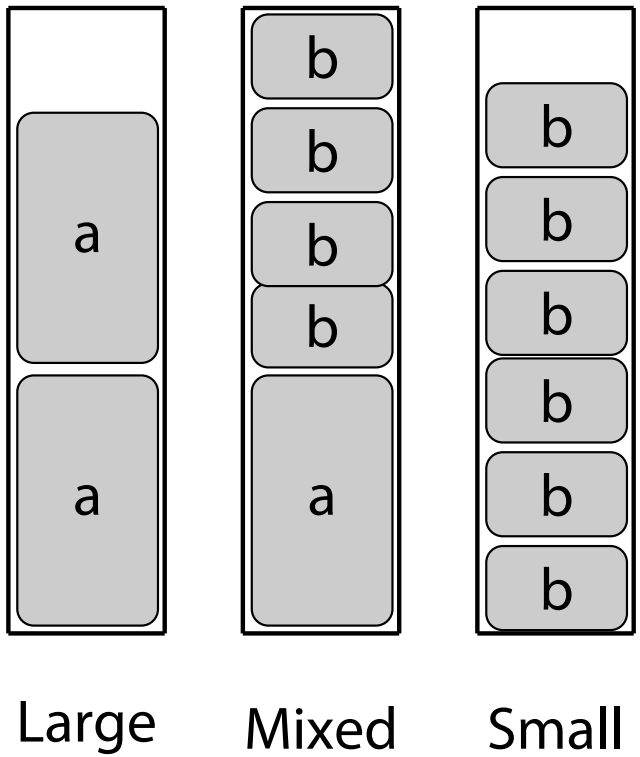
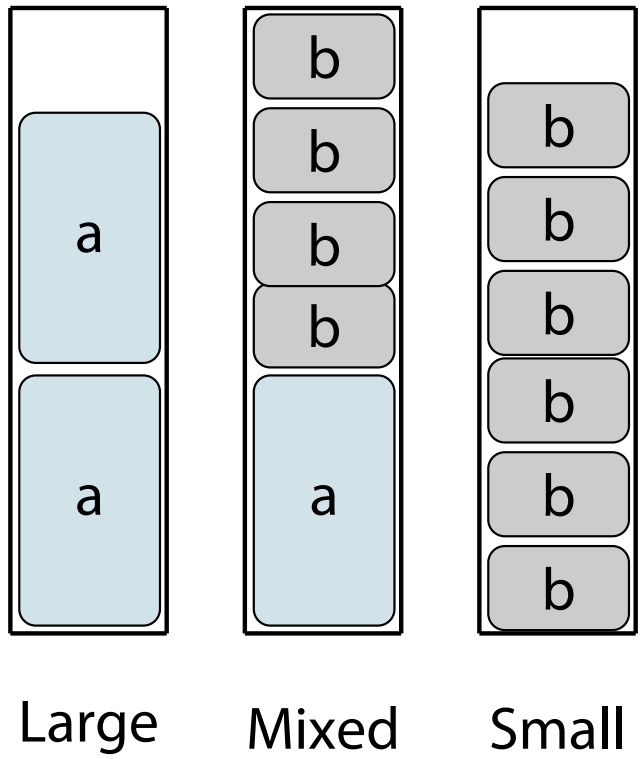


Everything

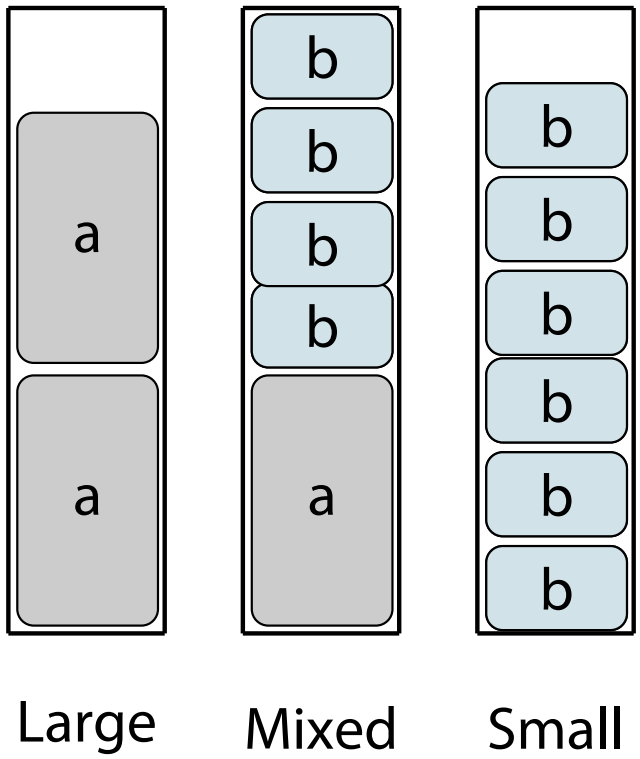


Everything



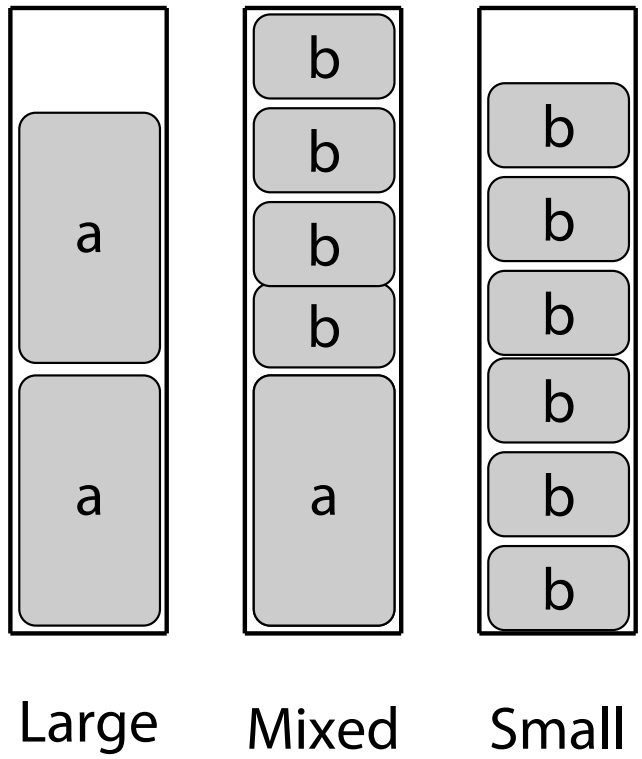
- *Pack via LP*

Everything



- Pack via LP
- *Pack via "Sorting"*

Everything



- Pack via LP
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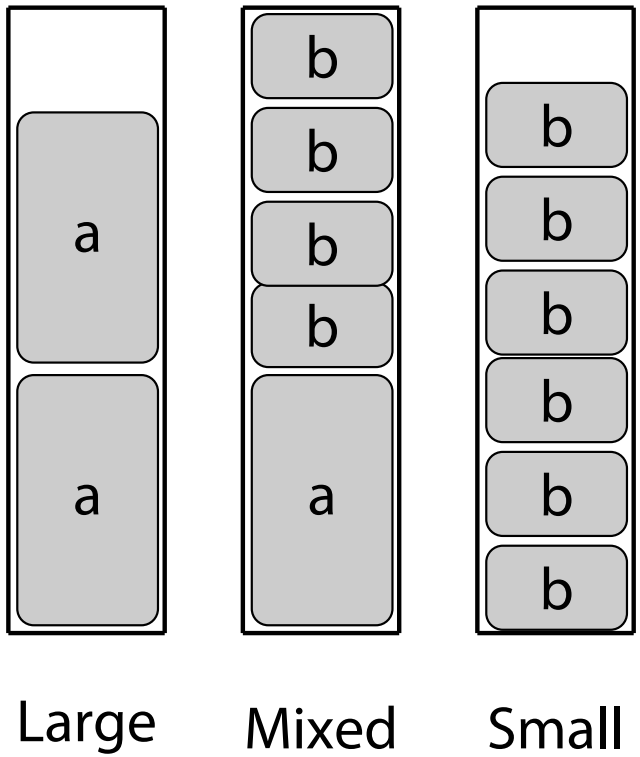
- *Small bins $\Rightarrow$ little free space in other bins*

Everything



- Pack via LP
- Pack via "Sorting"
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- *Relate nearly full / nearly empty bins via potential function*

Lookout

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- Simplify handling of small items
- *Adapt techniques to other problems*