UNIVERSITÄT ZU LÜBECK

Fully Dynamic Bin Packing Revisited

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Fully Dynamic Bin Packing =
Online + Removal + Repacking

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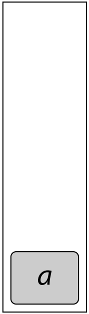
`INS`: $a/0.2$

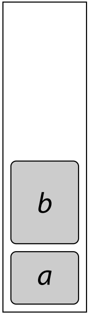
Fully Dynamic Bin Packing = Online + Removal + Repacking

INS: $a/0.2$, $\boxed{\textbf{INS}: b/0.3}$



Fully Dynamic Bin Packing = Online + Removal + Repacking

INS: $a/0.2$, **INS**: $b/0.3$, $\boxed{\textbf{INS}: c/0.4}$



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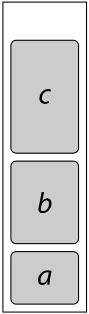
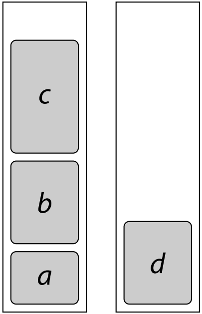
INS: $a/0.2$, **INS**: $b/0.3$, **INS**: $c/0.4$, **INS**: $d/0.2$



Fully Dynamic Bin Packing = Online + Removal + Repacking

INS: $a/0.2$, **INS**: $b/0.3$, **INS**: $c/0.4$, **INS**: $d/0.2$, $\boxed{\textbf{INS}: e/0.3}$



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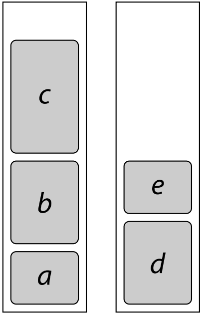
$\mathtt{INS}$: $a/0.2$, $\mathtt{INS}$: $b/0.3$, $\mathtt{INS}$: $c/0.4$, $\mathtt{INS}$: $d/0.2$, $\mathtt{INS}$: $e/0.3$, $\boxed{\mathtt{REM}: c}$



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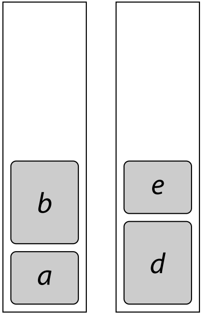

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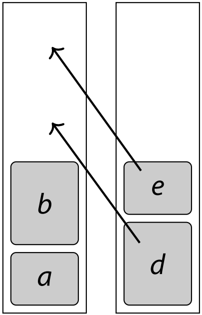

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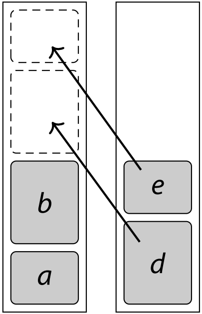

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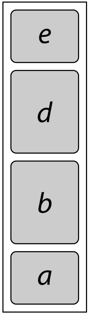

Fully Dynamic Bin Packing = Online + Removal + Repacking

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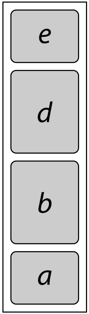

$$\text{Migration Factor}_t = \frac{d + e}{c} = \frac{\text{SIZE(moved)}}{\text{SIZE(new/removed)}}$$

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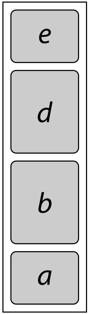

$$\text{Migration Factor}_t = \frac{\boxed{d} + \boxed{e}}{\boxed{c}} = \frac{\text{SIZE(moved)}}{\text{SIZE(new/removed)}}$$

Shifting Moves = number of moved items

Lower Bound

Migration Factor of $\Omega(1/\varepsilon)$ is necessary for ratio $1+\varepsilon$

$L = 1/2 - 1/9$(Migration Factor), $S = 1/3$(Migration Factor)

INS: L, **INS**: L, **INS**: $L, \ldots$

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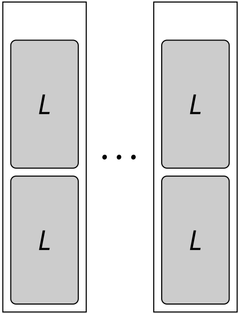
$\boxed{\texttt{INS}: L, \texttt{INS}: L, \texttt{INS}: L, \ldots}$



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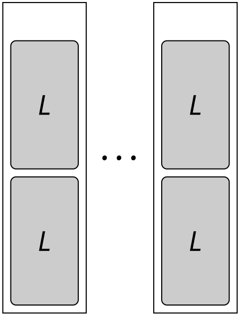

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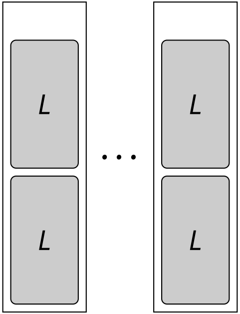

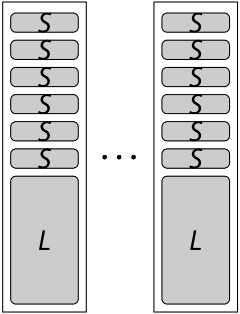

Known Results on upper bounds

Ratio	**REM**?	Shifting M.	Migration F.	Authors

Known Results on upper bounds

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3/2	✗	3	✗	Gambosi, Postiglione, Talamo (2000)
4/3	✗	7	✗	Gambosi, Postiglione, Talamo (2000)

Online Bin Packing with Repacking

Known Results on upper bounds

Ratio	REM?	Shifting M.	Migration F.	Authors
3/2	✗	3	✗	Gambosi, Postiglione, Talamo (2000)
4/3	✗	7	✗	Gambosi, Postiglione, Talamo (2000)
$1+\varepsilon$	✗	poly($\log n$) [am.]	✗	Ivković, Lloyd (1997)
$1+\varepsilon$	✗	$2^{\mathrm{poly}(1/\varepsilon)}$	$2^{\mathrm{poly}(1/\varepsilon)}$	Epstein, Levin (2006)
$1+\varepsilon$	✗	$\mathcal{O}(1/\varepsilon^4)$	$\mathcal{O}(1/\varepsilon^4)$	Jansen, Klein (2013)

Online Bin Packing with Repacking

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Ratio	**REM**?	Shifting M.	Migration F.	Authors
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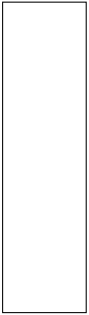
$1+\varepsilon$	✗	$\mathcal{O}(1/\varepsilon^4)$	$\mathcal{O}(1/\varepsilon^4)$	Jansen, Klein (2013)
5/4	✓	poly($\log n$) [am.]	✗	Ivković, Lloyd (1998)
$1+\varepsilon$	✓	$\mathcal{O}(1/\varepsilon^4 \log(1/\varepsilon))$	$\mathcal{O}(1/\varepsilon^4 \log(1/\varepsilon))$	this work

An Overview on the Packing



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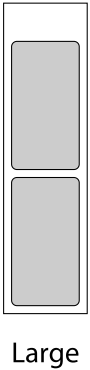

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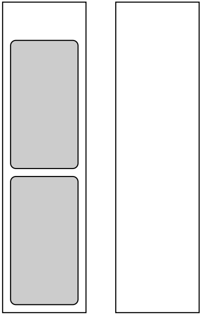
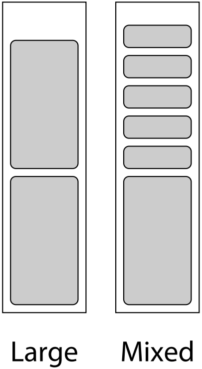


Large

An Overview on the Packing



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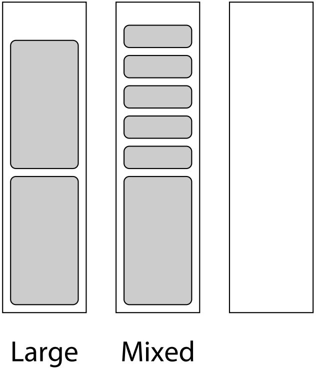

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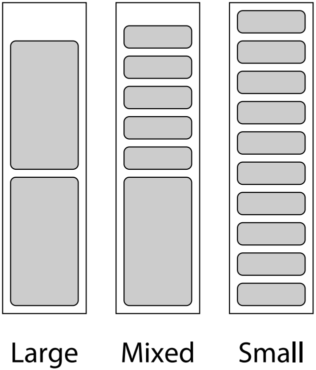

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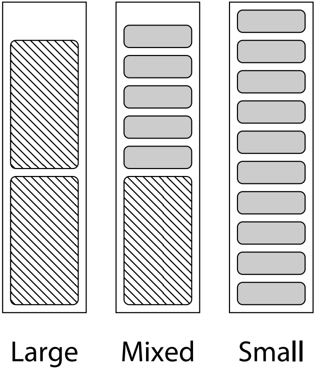
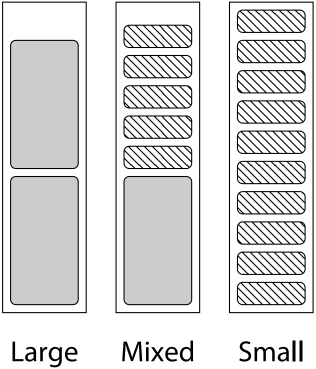


- Pack via LP

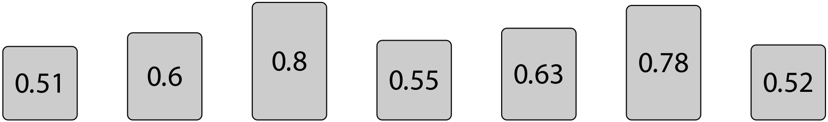
An Overview on the Packing



- Pack via LP
- Pack via “Sorting”

Rounding Large Items Geometrically

1. Find items of size (0.5, 1]



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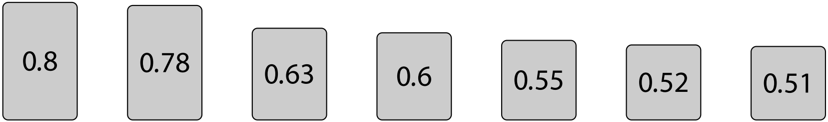
1. Find items of size (0.5, 1]
2. Sort items by size



Rounding Large Items Geometrically

1. Find items of size (0.5, 1]
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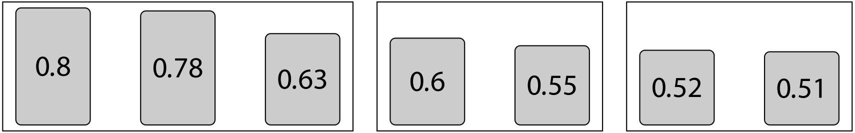
3. Group items in lists



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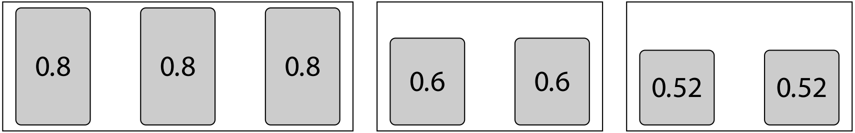

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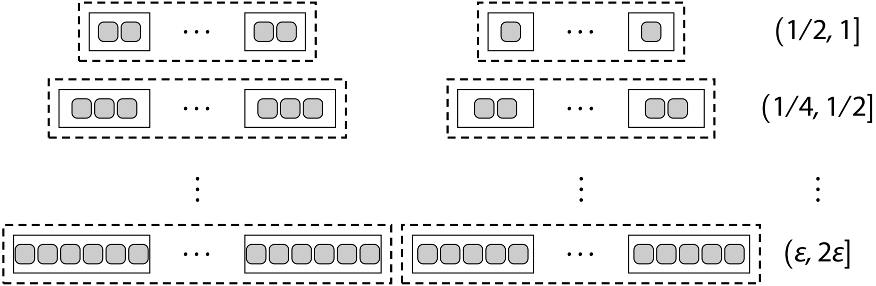

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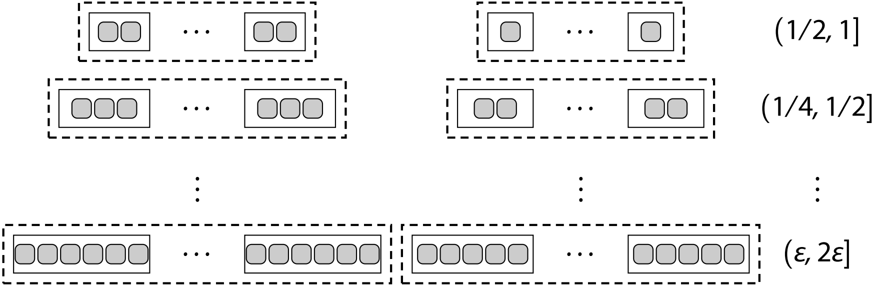

- Pack rounded items via LP
- Size of lists depends on volume of instance $\Rightarrow$ Shifting

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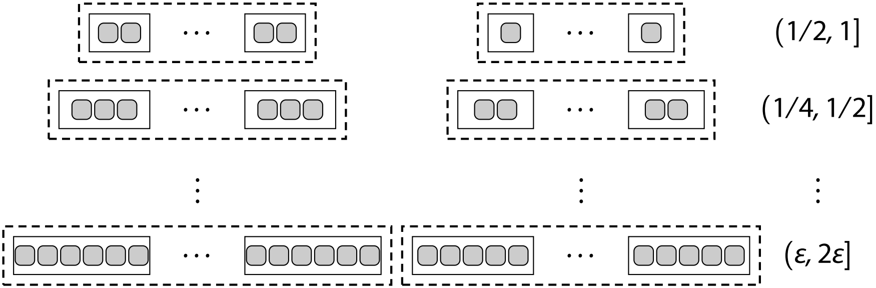

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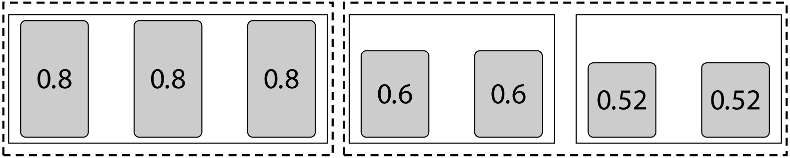

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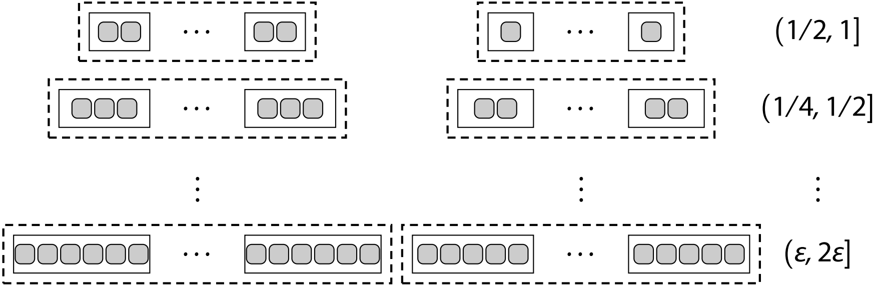

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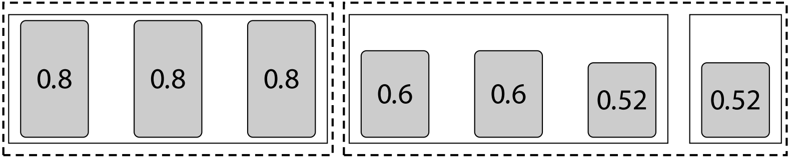

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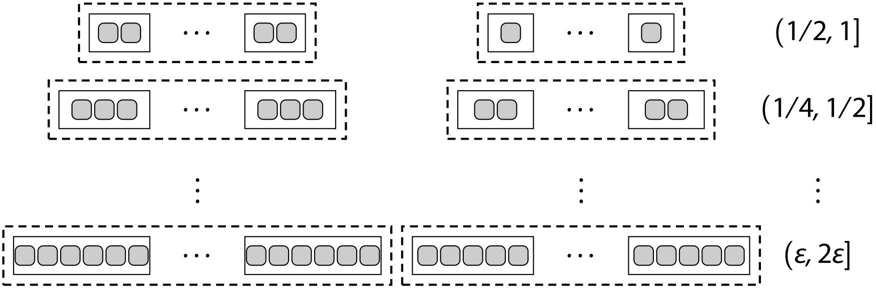

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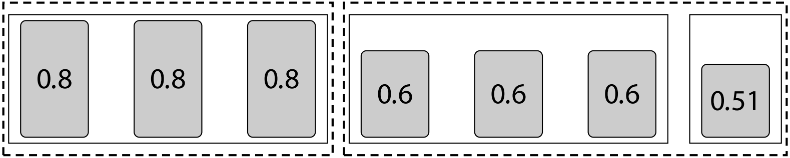

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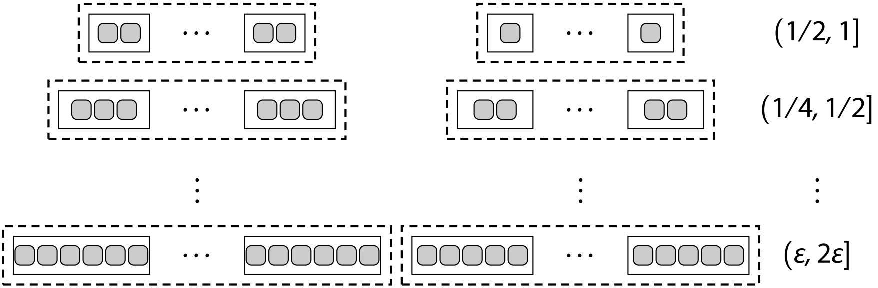

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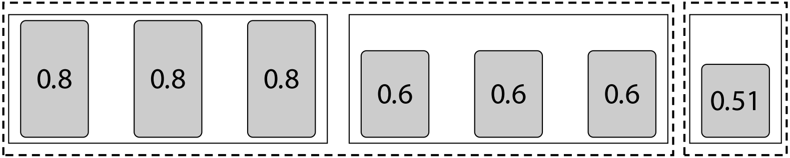

Packing Small Items is Difficult

Greedy fails: $\varepsilon \gg L \gg S$

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$(\texttt{INS}: L, \texttt{INS}: S, \ldots, \texttt{INS}: S)^*$

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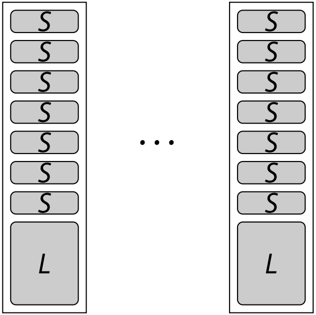

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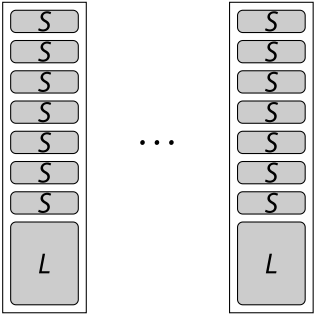

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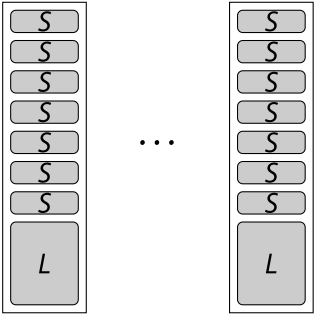

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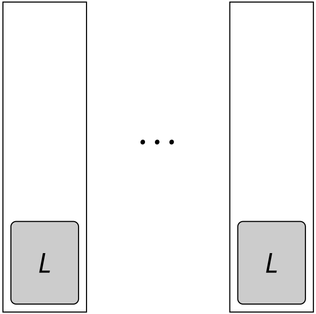

Packing Small Items via Sorting

Idea: “Sort” small items from left to right

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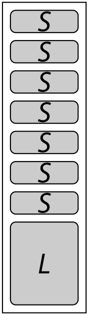

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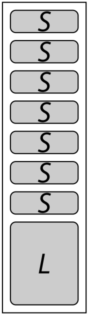

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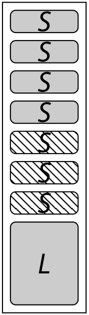

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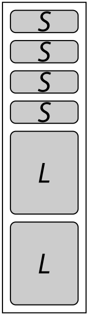

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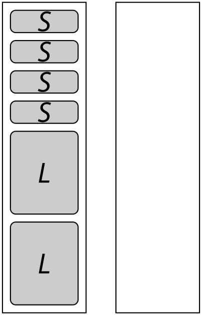

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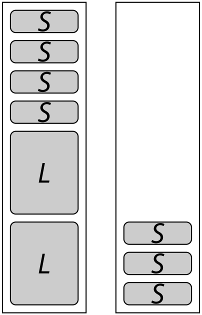
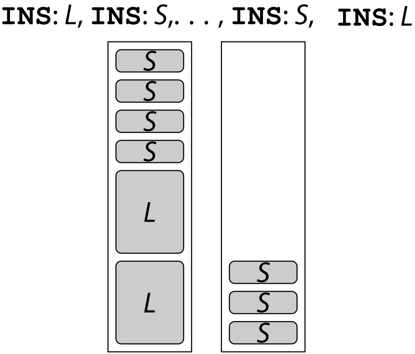


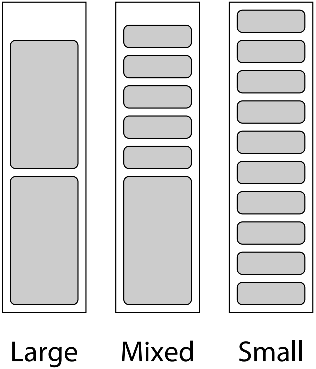
Packing Small Items via Sorting

Idea: "Sort" small items from left to right



Stop at every $1/\varepsilon$-th bin (buffer bin) to bound Migration

An Overview on the Packing



- Pack large items via LP

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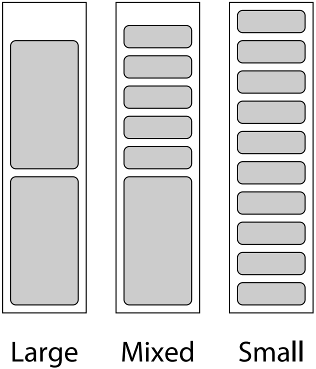

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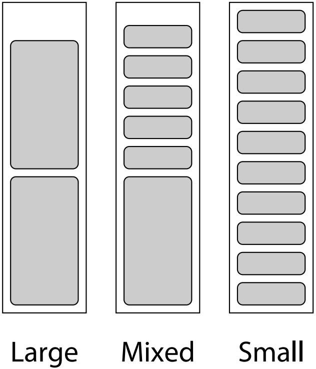

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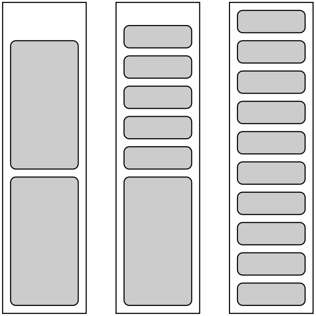

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- Pack small items via "Sorting"
- Small bins $\Rightarrow$ little free space in other bins
- Relate nearly full/empty bins via potential function

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- Can we simplify the handling of the small items?
- Can we adapt our techniques to other problems?