Problem 1

a) What is a dominant strategy? **See Watson p.50**
b) Which strategies are dominated in the following games:

1 \ 2	L	C	R
U	5, 9	0, 1	4, 3
M	3, 2	0, 9	1, 1
D	2, 8	0, 1	8, 4

1 \ 2	W	X	Y	Z
U	3, 6	4, 10	5, 0	0, 8
M	2, 6	3, 3	4, 10	1, 1
D	1, 5	2, 9	3, 0	4, 6

Game on the left : 'L' dominates 'R'
Game on the right: (2/3, 0, 1/3) dominates 'M'. 'X' dominates 'Z'

Problem 2

a) What is a rationalizable strategy? **See Watson p.70**
b) Find the set of rationalizable strategies in both games:

1 \ 2	L	C	R
U	6, 3	5, 1	0, 2
M	0, 1	4, 6	6, 0
D	2, 1	3, 5	2, 8

1 \ 2	X	Y	Z
A	8, 6	0, 1	8, 2
B	1, 0	2, 6	5, 1
C	0, 8	1, 0	4, 4

Game on the left: R = {(U, L)}. Game on the right: R = {A, B} × {X, Y}.

Problem 3

a) What is a Nash-Equilibrium? **See Watson p. 97**
b) Find all pure and mixed-strategy Nash equilibria in the following games:

1 \ 2	A	B
A	1, 4	2, 0
B	0, 8	3, 9

1 \ 2	L	M	R
U	8, 1	0, 2	4, 3
C	3, 1	4, 4	0, 0
D	5, 0	3, 3	1, 4

Game on the left: NE: {(A, A), (B, B)}, MNE: {(1/5, 4/5), (1/2, 1/2)}.
Game on the right: MNE = {(x, 1/5, y), (0, 1/2, 1/2)} where x, y ≥ 0 and x + y = 4/5

Problem 4

Consider an asymmetric Cournot-duopoly game, where the two firms have different costs of production. Firm 1 selects quantity q_1 at a production cost of $2q_1$. Firm 2 selects quantity q_2 and pays the production cost $4q_2$. The market price is given by $p = 12 - q_1 - q_2$.

a) What are the firms' payoff functions?

$$u_1(q_1, q_2) = (12 - q_1 - q_2)q_1 - 2q_1 and\ u_2(q_1, q_2) = (12 - q_1 - q_2)q_2 - 4q_2$$

b) Calculate the firms' best-response functions.

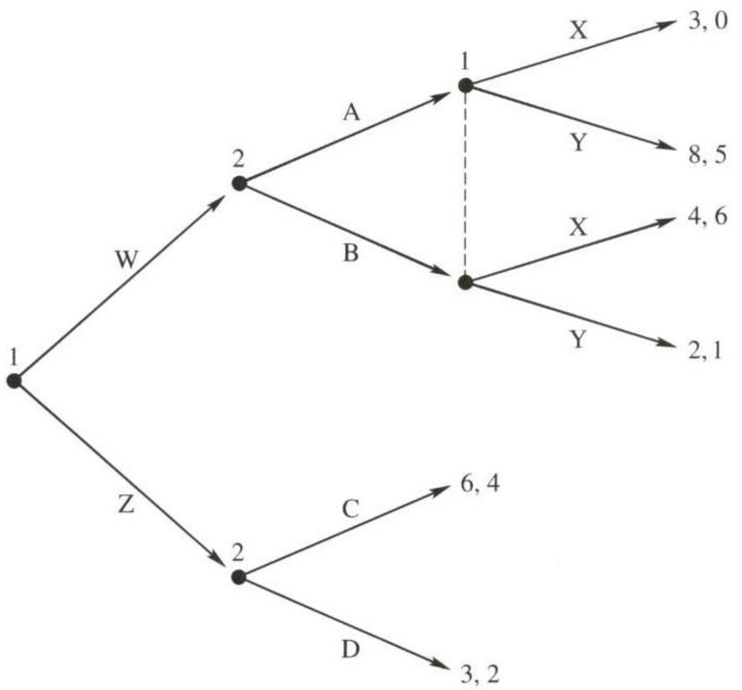
$$BR_1(q_2) = 5 - \frac{1}{2} * q_2\ and\ BR_2(q_1) = 4 - \frac{1}{2} * q_1$$

c) Find the Nash-equilibrium of the Cournot-game.

NE: $q_1^* = 4$ **and** $q_2^* = 2$

Problem 5

Find all Nash-equlibria and Subgame-perfect Nash equilibria in the following game:

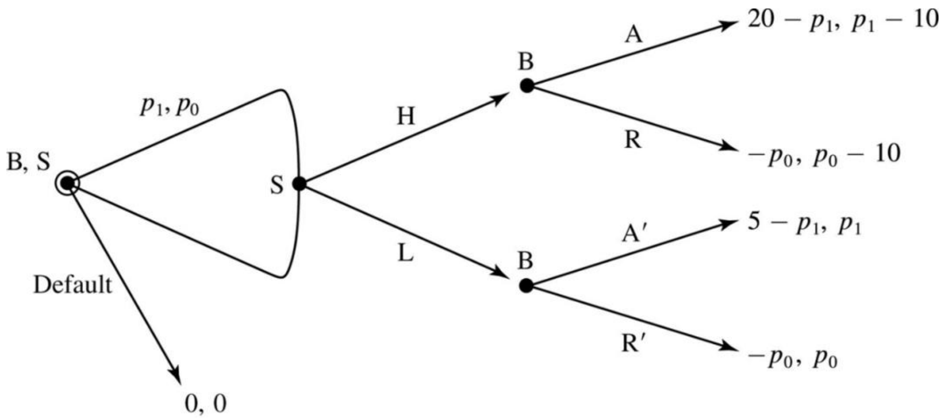


(a)

NE: {(WY, AC), (ZX, BC), (WY, AD), (ZY, BC) , (WX, BD)}
SPNE: {(WY, AC), (ZX, BC)}

Problem 6

Consider the following sequential move game over the delivery of a good between a buyer and seller:



The buyer (B) and seller (S) first negotiate on prices p_1 and p_0 that the buyer has to pay depending on whether he accepts the delivery (A) or rejects it (R). Prior to trade, the seller can choose to deliver a high (H) or low (L) quality good (high quality costs the seller 10).

a) What is the efficient outcome of the game? **S plays H, B plays A**
b) Fully describe the negotiation equilibrium of the game, under the assumption that the parties have equal bargaining weights. **The surplus is 10. Each player gets 5. Thus, p_1 is 15 and p_0 between -5 and 5. The seller chooses H. The buyer chooses A if H and R if L.**

Problem 7

a) Consider a repeated game in which the stage game below is played in each of two periods, and that there is no discounting. Describe a subgame-perfect equilibrium in which the players select (U, L) in the first period.

1 \ 2	L	M	R
U	8, 8	0, 9	0, 0
C	9, 0	0, 0	3, 1
D	0, 0	1, 3	3, 3

The game has three Nash equilibria (NE = {(D, M), (C, R), (D, R)}). The efficient strategy profile which is not a Nash equilibrium is (U, L). Because there is more than one Nash equilibrium in this game, reputational equilibria might exist in which the efficient strategy profile is played in the first period. Notice the following strategy description for example:

Strategy for player 1: Play U in t = 1 and D in t = 2 if player 2 played L in t = 1, otherwise play C in t = 2.

Strategy for player 2: Play L in t = 1 and R in t = 2 if player 1 played U in t = 1, otherwise play M in t =2.

To see that this constitutes a SPNE, notice that if each player adheres to the described strategy, his or her payoff from the game is equal to 8 + 3 = 11. If either player decides to deviate in t = 1 and play his or her best response in the first period, then the payoff from the game equals 9 + 1 = 10, which is strictly less than adhering to the prescribed strategy. Thus, (UD, LR) is a subgame perfect Nash equilibrium.

b) Assume now that the following stage game (prisoners' dilemma) is played twice. Is there a subgame perfect Nash equilibrium where (C,C) is played in the first period?

1 \ 2	C	D
C	2, 2	0, 3
D	3, 0	1, 1

(D, D) is the only one Nash equilibrium in this prisoners' dilemma (stage) game. Since the only Nash equilibrium has to be played in the final round, (D, D) will also be played in the first (previous) period. This can be easily seen by noticing that, if one player plays C in the first period, the other player can gain 3 by playing D in the same period. This strategy yields a higher final payoff than playing C as well (2 + 1 = 3 by playing C in t=1 and D in t = 2; 3 + 1 = 4 by playing D in both periods). Cooperation cannot be sustained in the finitely repeated prisoners' dilemma game because there are no other Nash equilibria that either player can induce after observing that the other player did not cooperate.

c) Will the answer in b) change if the stage game is played 10 times instead of two? Explain.

Changing the horizon of the game to ten periods has no effect on the possibility to sustain mutual cooperation (C, C) in the prisoners dilemma game. The explanation given in b) is also valid for the last two periods of the ten period game. By the logic of backward induction, deviation will then also occur in the 8 th period, and subsequently in all previous periods such that mutual defection is the outcome for each round.

Problem 8

Consider the following game in which two players can choose to invest (I) or not invest (N)

1 \ 2	I	N
I	6, 8	0, 9
N	7, 0	0, 0

a) What is the Nash equilibrium in this game? **Answer: NE = {(I, N), (N, N), (N, I)}**
Assume now that the players can write a contract whereby they agree to invest, and that a court of law can verify the players' behavior.

b) How does the induced game look like if the court applies expectation damagers in case of a breach Can (I, I) be enforced?

$1/2$	I	N
I	$6, 8$	$6, 3$
N	$-1, 8$	$0, 0$

In the induced game (I, I) is a NE and thus enforceable. Since (I, I) is the efficient outcome of the underlying game, under expectation damages, (I, I) is enforceable.

c) How does the induced game look like if the court applies reliance damages in case of a breach? Can (I, I) be enforced?

In the induced game (I, I) is not enforceable. The damage caused through a breach by either party is not sufficiently large enough.

$1/2$	I	N
I	$6, 8$	$0, 9$
N	$7, 0$	$0, 0$

Assume now that the court applies expectation damages, but that the court only can verify whether or not there has been a breach (N), not who has breached the contract.

d) Is there a contract that makes it possible that both invests, i.e. that they play (I,I)?

For (I, I) to be enforceable it must hold that $7 + \alpha \leq 6$ and $9 - \alpha \leq 8$. There exists no α to satisfy both conditions. Hence, the contract is not enforcable under limited verifiability.