Problem 1

a) Explain the concept of *strategy* and provide a formal definition.

A strategy is a complete contingent plan for a player in the game. A player's strategy describes a players actions at all possible decisions nodes, i.e. at all his information sets, where an information set specify the players' information at a decision node.

b) Explain the concept of *Nash equilibrium* and provide a formal definition.

If the players play a strategy profile where no-one have incentives to deviate, they are in a Nash equilibrium. A strategy profile is a Nash equilibrium if and only if each players' strategy is best response to the other players' strategies. Formally: A strategy profile $s\in S$ **is a Nash equilibrium if and only if** $s_i \in BR_i(s_{-i})$ **for each player i. That is,** $u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i}) \forall s_i \in S_i$ **for each player i.**

Problem 2

The normal-form game pictured below represents a situation in tennis, whereby the server (player 1) decides whether to serve to the opponent’s forehand (F), center (C), or backhand (B) side. Simultaneously, the receiver (player 2) decides whether to favor the forehand, center, or backhand side.

1 \ 2	F	C	B
F	0, 5	2, 3	2, 3
C	2, 3	0, 5	3, 2
B	5, 0	3, 2	2, 3

a) A player’s strategy can be stricttly dominated by another strategy. Formally define what this means.

A pure strategy s_i **of player** ***i*** **is dominated if there is a strategy (pure or mixed)** $\sigma_i \in \Delta S_i$ **such that** $u_i(\sigma_i, s_{-i}) > u_i(s_i, s_{-i})$**, for all strategy profiles** $s_{-i} \in S_{-i}$ **of the other players.**

b) Are any strategies strictly dominated in the tennis game by another (pure or mixed) strategy?

Strategy F of player 1 is dominated by the mixed strategy $\sigma_1 = (0, 1-q, q), q \in \left(\frac{2}{3}, 1\right)$**.**

After removing F from the strategy set of player 1, strategy F of player 2 is dominated by strategy C. No other strategies are strictly dominated.

c) Find the mixed strategy Nash equilibria of this game.

Player one is indifferent between choosing strategy C or B if $q*0+(1-q)*3=q*3+(1-q)*2$**, yielding** $q=\frac{1}{4}$**. Player 2 is indifferent between choosing C or B if** $p*5+(1-p)*2=p*2+(1-p)*3$**, yielding** $p=\frac{1}{4}$**. Thus, the mixed strategy Nash equilibrium is** $\left\{\left(0,\frac{1}{4},\frac{3}{4}\right),\left(0,\frac{1}{4},\frac{3}{4}\right)\right\}$

d) What is the probability that player 1's payoff exceeds player 2's payoff in the equilibrium?

For the strategy profiles (C, B) and (B, C) player 1's expected payoff exceeds the payoff of player 2. This occurs with probability $\frac{1}{4}\times\frac{3}{4}=\frac{3}{16}$ **and** $\frac{1}{4}\times\frac{3}{4}=\frac{3}{16}$**, repsectively. Thus, player 1's expected payoff exceeds player 2's expected payoff with a probability of** $\frac{6}{16}=\frac{3}{8}$**.**

Problem 3

Consider the following game describing a contractual relationship in which both players, 1 and 2 have written a contract that specifies the play of (I, I).

1 \ 2	I	N
I	z_1, z_2	y_1, x_2
N	x_1, y_2	0, 0

a) Use the game to explain the legal principles of expectation damages and reliance damages.

These legal principles guide damage awards in case a contract is breached. They define a transfer level α**from player 2 to player 1 in case player 1 breaches the contract, a transfer level** β**from player 2 to player to player 1 in case player 2 breaches the contract, and a transfer level** γ**from player 2 to player to player 1 in case player 2 breaches the contract. The induced game is as follows:**

$\frac{1}{2}$	I	N
I	z_1, z_2	$y_1+\beta, x_2-\beta$
N	$x_1+\alpha, y_2-\alpha$	$\gamma, -\gamma$

Under the legal principle of expectation damages, the court forces the defendant to transfer to the plaintiff the sum of money needed to give the plaintiff the payoff that he or she would have received had the contract been fulfilled. This implies that the following transfer rules are applied in the induced game:

1\2	I	N
I	z_1, z_2	$z_1, x_2+y_1-z_1$
N	$x_1+y_2-z_2, z_2$	$0, 0$

This specific transfers are $\alpha=y_2-z_2$ **,** $\beta=z_1-y_1$ **and** $\gamma=0$

Under the legal principle of reliance damages, the court imposes a transfer that returns the plaintiff to the state in which he or she would have been but for the contract. That is, the court determines the payoff that the plaintiff would have received had no contract been written, and then the court forces a transfer sufficient to give the plaintiff this total payoff. This implies that the following transfer rules are applied in the induced game:

1\2	I	N
I	z_1, z_2	$0, x_2+y_1$
N	$x_1+y_2, 0$	$0, 0$

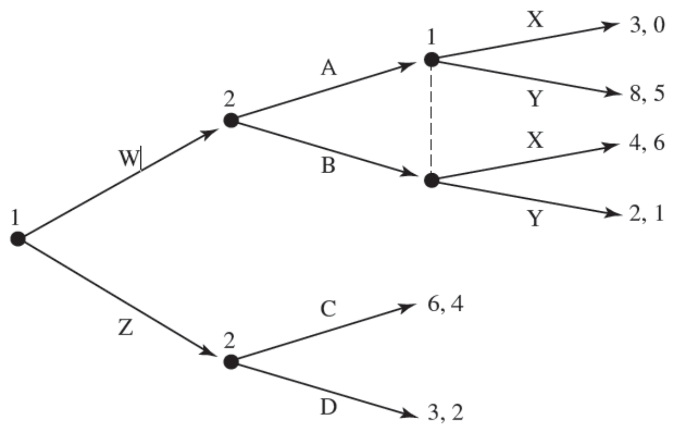
This specific transfers are $\alpha=y_2-z_2$ **,** $\beta=-y_1$ **and** $\gamma=0$

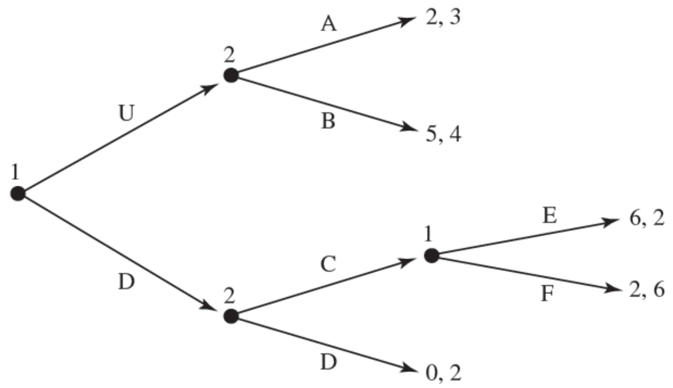
b) Show when and why expectation damages encourage the play of (I, I).

Both players play I if neither player has an incentive to deviate, i.e. if $z_1 \geq x_1+y_2-z_2$ **and** $z_1 \geq x_1+y_2-z_1$**, for player 1 and 2 repectively. Re-aranging both equation yields** $z_1 \geq x_1+y_2-z_2$ **and** $z_1+z_2 \geq x_1+y_2$**, which shows that under expectation damages, (I, I) is supported as a Nash equilibrium only if the outcome is also (I, I) efficient.**

Problem 4

Compute the pure strategy Nash equilibria and subgame perfect equilibria for the two following games. Do so by writing the normal-form matrices for each game and its subgames. Which Nash equilibria are not subgame perfect?





Top: NE = {(WY, AC), (ZX, BC), (WY, AD), (ZY, BC), (WX, BD)}, SPNE = {(WY, AC), (ZX, BC)}.

Bottom: NE = {(UE, BD), (DE, BC), (UF, BD), (DE, AC)}, SPNE = {(UE, BD), (DE, BC)}.

Problem 5

Suppose that a cattle rancher (R) and a corn farmer (F) are located next to each other. Currently, there is no fence between the ranch and the farm, so R's cattle enter F's field and destroy some of F's corn. This results in a loss of 300 to F. Under the current situation, the value of production for R is 1,000, and the value of production for F is 500 (including the loss of 300). A fence that will keep the cattle out of the cornfield would cost 100 for R to build (assume that only R can build the fence).

a) Suppose that R is not legally required to prevent his cattle from entering F's cornfield. The players negotiate from the legal rule (property right), but the legal rule determines the default outcome. Assume the outcome of negotiation is given by the Nash standard bargaining solution with equal bargaining weights. What is the outcome of negotiation?

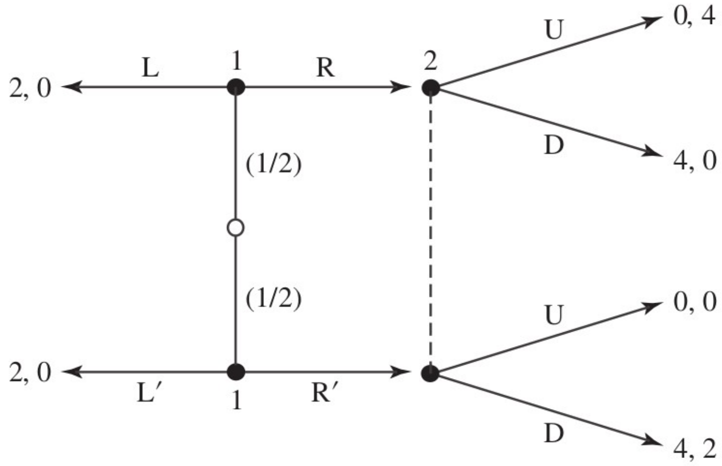
   With the fence, the gain in total surplus is 200. This is divided equally between F and R. So F pays 200 to R, and R builds the fence. This yields a payoff of 600 for F and a payoff of 1,100 for R.

b) Now suppose that R is legally required to prevent his cattle from entering F's cornfield unless allowed to do so by F. The players negotiate from the legal rule (property right), but the legal rule determines the default outcome. Assume the outcome of negotiation is given by the standard bargaining solution with equal bargaining weights. What is the outcome of negotiation?

Building the fence costs R 100. R not building the fence would cost F 300. Since 100 < 300, R cannot/will not pay enough to F to not have to build the fence. So the fence will be built and there will be no payment between them. F's payoff is 800, and R's payoff is 900.

Problem 6

Convert this game into the normal form and find all pure Bayesian Nash equilibria.



The normal form is:

1 \ 2	U	D
LL′	2, 0	2, 0
LR′	1, 0	3, 1
RL′	1, 2	3, 0
RR′	0, 2	4, 1

The BNE is (LL', U).

Problem 7

A game-theoretic model can be used to illustrate the strategic aspects of tariffs. Suppose there are two countries that are labeled 1 and 2. Let x_i be the tariff level of country *i* (in percent), for *i* = 1, 2. If country *i* picks x_i and the other country *j* selects x_j, then country *i* gets a payoff of: $\Pi_i(x_i, x_j) = 2000 + 60x_i + x_i x_j - x_i^2 - 90x_j$ (measured in billions of dollars). Assume that x_i and x_j must be between 0 and 100 and that the countries set tariff levels simultaneously and independently.

a) Find the best-response functions for the two countries.

The best-response function of player i is given by $BR_i(x_j)=30+\frac{x_j}{2}$**, for player j it is** $BR_j(x_i)=30+\frac{x_i}{2}$

b) Compute the Nash equilibrium of the tariff game. What is the equilibrium payoff to each country in equilibrium?

Solving for equilibrium, we find that $x_i=30+\frac{1}{2}\left[30+\frac{x_i}{2}\right]$, **which implies that** $x_1=x_2=60$**. The payoff to each player is equal to 2, 000 − 30(60) = 200.**

c) Show that the countries would be better off if they made a binding agreement to set lower tariffs (than in equilibrium). You do not need to speculate how such an agreement could be enforced.

Under zero tariffs, the payoff to each country is 2,000. A deviation by player i yields a payoff of 2, 000 + 60(30) − 30(30) = 2, 900.

d) Assume now that the game if played repeatedly over infinitely many periods. Find conditions on the discount factor δ such that zero tariffs can be sustained each period by a subgame perfect equilibrium. Use a grim-trigger strategy profile.

$\frac{2000}{(1-\delta)}\geq 2900+\frac{\delta}{(1-\delta)}200$ Solving for δ yields $\delta\geq 1/3$