

EXAMINATION PAPER

Subject number: MØA290

Subject: Applied game theory

Examination day and date: Friday May 25

Hours: 09.00-13.00

Number of pages: 5 (including this one)

Number of assignments: 8

Number of enclosures:

Allowed remedies: Non-programmable calculator

Comment:

Good luck!!

**THE CANDIDATE MUST CONTROL THAT THE EXAMINATION
PAPERS ARE COMPLETE.**

Problem 1

- a) What is a strategy?
- b) What is a dominated strategy?
- c) What does it mean to play best response?
- d) Consider the game below. Is Player 1's strategy M dominated? If yes, describe a strategy that dominates M. If no, describe a belief where M is best response.

An extensive form game tree. Player 1 chooses between K, L, and M. Choosing K leads to Player 2 choosing between X and Y. Choosing L leads to Player 2 choosing between X and Y. Choosing M leads to Player 2 choosing between X and Y. Payoffs are (Player 1, Player 2).

	2	
1	X	Y
K	9, 2	1, 0
L	1, 0	6, 1
M	3, 2	4, 2

Problem 2

- a) What is a Nash equilibrium?
- b) Find the Nash equilibria in the following game:

An extensive form game tree. Player 1 chooses between U, M, and D. Choosing U leads to Player 2 choosing between L, C, and R. Choosing M leads to Player 2 choosing between L, C, and R. Choosing D leads to Player 2 choosing between L, C, and R. Payoffs are (Player 1, Player 2).

	2		
1	L	C	R
U	2, 0	1, 1	4, 2
M	3, 4	1, 2	2, 3
D	1, 3	0, 2	3, 0

- c) What is a rationalizable strategy?
- d) Do there exist rationalizable strategy profiles in the game that are not Nash equilibria?

Problem 3

Consider the following game:

	2		
1	L M R		
U	x, x	$x, 0$	$x, 0$
C	$0, x$	$2, 0$	$0, 2$
D	$0, x$	$0, 2$	$2, 0$

Find Nash equilibria in pure and mixed strategies, and show how they depend on x . Show the difference between $x > 1$ and $x < 1$.

Problem 4

Consider the following game, describing a contractual relationship:

	2	
1	I N	
I	z_1, z_2	y_1, x_2
N	x_1, y_2	$0, 0$

- a) Use the game to explain the concepts of expectation damages and reliance damages.
- b) Show when and why expectation damages are efficient.

Problem 5

Consider a game in which player 1 first selects between I and O. If player 1 selects O, then the game ends with payoff vector $(4,1)$, i.e. 4 to player 1 and 1 to player 2. If player 1 selects I, then player 2 can observe this move, and the players then play a “battle of the sexes” game in which they simultaneously and independently choose between A and B. If they both play A, then the payoff vector is $(3,1)$. If they both play B, then the payoff vector is $(1,3)$. If one of the players play A, while the other play B, then the payoff vector is $(0,0)$.

- a) Represent this game in the extensive and normal forms.
 - b) Find the Nash equilibrium in pure strategies.
 - c) Represent the proper subgame (battle of the sexes) on normal form and find equilibria in pure and mixed strategies.
 - d) What are the subgame perfect Nash equilibria in pure strategies (for the whole game). Are there any Nash equilibria that are not subgame perfect?

Problem 6

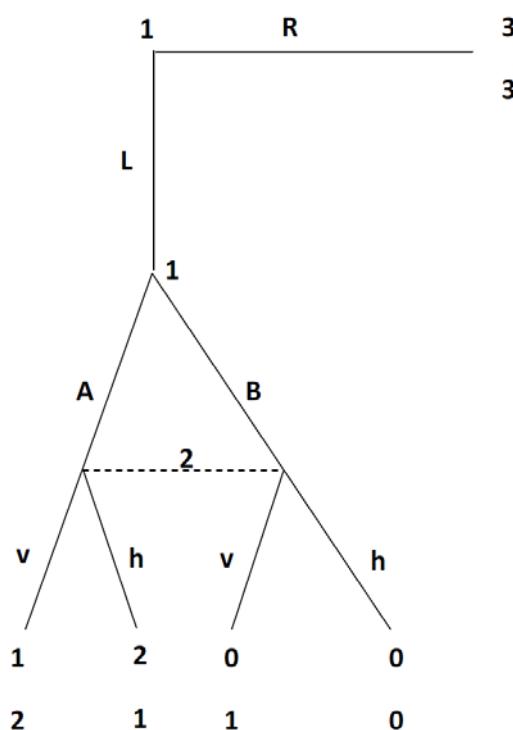
Pia is a clever accountant, as well as a skilled leader. And now she is negotiating the possibility of a new position with the CEO of the company. The contract specifies the salary w , as well as her task: accounting or management. If she works as accountant, she gets a payoff of $w - 5.000$ and the company gets $100.000 - w$. If Pia works as a manager, she gets a payoff of $w - 30.000$, while the company gets $x - w$. If the parties do not agree, the company gets 0, while Pia gets u in another job.

Assume $x > 150.000$ and $u < 95.000$. Pia's bargaining power is given by π_P , while the company's bargaining power is given by π_C .

- a) Solve the bargaining problem by the use of standard bargaining solution: What should Pia work with and what will be paid?
 - b) Suppose Pia can invest in human capital, i.e. choose between two courses, A and B. Course A improves her management skills, thereby increasing x . Course B strengthens her outside options as an account, thereby increase u . What should Pia choose?

Problem 7

Consider the following game:



- a) How many subgames are there in the game?
- b) Describe the set of possible strategies for each player
- c) Find all pure strategy Nash equilibria.
- d) Find all sub games and the pure strategy sub-game perfect Nash equilibria of the game.

Problem 8

The stage game below, where $1 < x < 4$, is played twice, with the outcome in the first stage observed before the second stage begins. The players move simultaneously. There is no discounting.

	L	M	R
U	1,1	5,0	0,0
M	0,5	4,4	0,0
D	0,0	0,0	x,x

- a) For which x can the outcome (M,M) be achieved in the first stage in a pure-strategy subgame perfect Nash equilibrium? Give the strategies that do so.
- b) Consider an infinitely-repeated game with stage game given by the payoff matrix in (a). Payoffs are discounted with a discount factor $\delta \in (0,1)$. For which discount factors can the outcome (M,M) each period be obtained as a subgame perfect equilibrium outcome, assuming the players use trigger strategies?