# Applied Game Theory

Lecture 6

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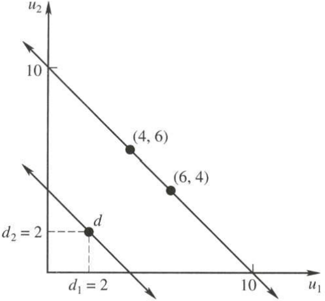
# Bargaining I

- Bargaining is about value creation and value division.
- Bargaining outcomes can be described in terms of *payoff vectors*:
    - E.g. $(4,6)$ after agreement and $(2,2)$ after disagreement
- Let $V$ denote the Bargaining set, i.e. the set of payoff vectors that contains all outcomes for a given bargaining problem.
    - $V = \{(4,6),(2,2)\}$
- Call $d$ the payoff vector associated with the no trade/no agreement bargaining outcome a.k.a. the *default outcome* or *disagreement point*.

# Bargaining II




- The players' payoffs can be written $u_1 = v_1(z) + t$ and $u_2 = v_2(z) - t$ where $z$ is a bargaining outcome, $v$ is the value of the bargaining outcome and $t$ is a transfer from player 2 to player 1 ($t < 0$ if transfer from 1 to 2).

- If $z = 0$ is disagreement and $z = 1$ is agreement, we have $v_1(0) = v_2(0) = 2$ and $v_1(1) = 4$, $v_2(1) = 6$.

- All vectors of the form $(4 + t, 6 - t)$ and $(2 + t, 2 - t)$ are in the bargaining set.

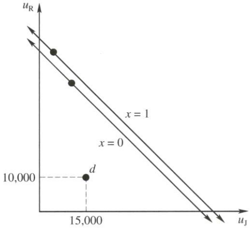
# Bargaining III

- The joint value is given by:
  - $[v_1(z) + t] + [v_2(z) - t] = v_1(z) + v_2(z)$
- The set of efficient outcomes contains all the maximal points of the players' joint value:
  - $z^* = \underset{z}{\operatorname{argmax}} \, (v_1(z) + v_2(z))$
- The *Surplus* from bargaining is the difference between the joint value of the bargaining outcome and the disagreement point:
  - $(v_1(z) + v_2(z)) - d_1 - d_2$

# Bargaining: Example

- Bargaining over an employment contract between employer R and potential employee J.
- The job may include en extra task. If extra task, then $x = 1$, if not then $x = 0$. The salary is given by $t$.
- R values J's labor to be 40.000 and the extra task to be 5.000
  - $v_R(x) = 40.000 + 5000x$
- J values the job to the amount 10.000, but the extra task has a negative value of 3.000
  - $v_J(x) = 10.000 - 3000x$
- If the parties disagree then J gets 15.000 at another job, while R gets 10.000 by hiring somebody else.
  - $d_R = 10.000$ and $d_J = 15.000$

# Bargaining: Example II




- Joint value after agreement is $u_J(x) + u_R(x) = [v_J(x) + t] + [v_R(x) - t]$.
- $10.000 - 3.000x + 40.000 + 5.000x = 50.000 + 2000x$.
- We see that $x = 1$ is better than $x = 0$, i.e. an efficient bargaining solution implies that J performs the extra task. Joint value is then $v^* = 52.000$.
- Surplus from agreement: $52.000 - d_J - d_R = 52.000 - 15.000 - 10.000 = 27.000$.

# The standard bargaining solution (Nash)

- Nash derived a solution to the bargaining problem that builds on the principle of proportionality and efficiency.
- The solution implies that each player gets the default value $d$ plus a share of the surplus, where the share is being determined by *bargaining power*.
- The players bargain over the largest possible surplus, i.e. over $v^* - d_1 - d_2$
- Player $i$'s bargaining power is $\pi_i \geq 0$, which is simply the player's proportion of the surplus. Note that $\sum_i^n \pi_i = 1$.

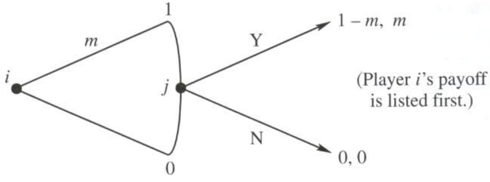
# Bargaining: Example III

- In the example, suppose $\pi_J = 1/3$, $\pi_R = 2/3$. The standard bargaining solution then implies:
  - $u_J^* = d_J + \pi_J(v^* - d_J - d_R) = 15.000 + \frac{1}{3} * 27.000 = 24.000$
  - $u_R^* = d_R + \pi_R(v^* - d_J - d_R) = 10.000 + \frac{2}{3} * 27.000 = 28.000$
- We know that J's payoff is $u_J^* = v_J(1) + t = 24.000$ i.e. $10.000 - 3.000 + t = 24.000$ i.e. salary is $t = 17.000$

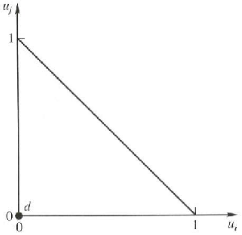
# Calculating the standard bargaining solution

- Calculate the maximized joint value $v^*$ by determining the value $x^*$ that maximizes $v_1(x) + v_2(x)$
- Set up the equations that determines the transfers $t$ :
  - $d_1 + \pi_1(v^* - d_1 - d_2) = v_1(x^*) + t$
  - $d_2 + \pi_2(v^* - d_1 - d_2) = v_2(x^*) - t$
- Solve one of these equations for $t$ to find the transfer that achieves the required split of the surplus.

# Ultimatum game



Bargaining game




- Player $j$ has two sequentially rational strategies:
    - $s_j^*$, accept all offers.
    - $\widehat{s}_j$, accept all offers $m > 0$ and reject the offer $m = 0$.
- There is no Nash equilibrium for which player $j$ plays $\widehat{s}_j$ because player $i$ will then select the lowest possible $m > 0$, and this is not well defined.
- The strategy profile where player $i$ plays $m = 0$ and player $j$ plays $s_j^*$ is a Nash equilibrium.

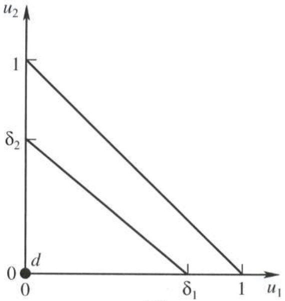
# Alternating offer- two periods bargaining I

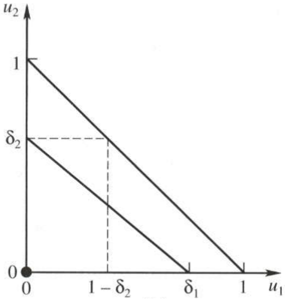
- P1 offers $m^1$. P2 accepts or rejects the offer.
- If P2 accepts, payoffs are $1 - m^1$ and $m^1$ to P1 and P2, respectively.
- If P2 rejects, he gives a counteroffer $m^2$ to P1.
- If P1 accepts the counteroffer, payoffs are $\delta_1 m^2$ and $\delta_2(1 - m^2)$ to P1 and P2, respectively, where $\delta_1$ and $\delta_2$ are discount factors.
- If P1 rejects the counteroffer, they both get 0.
- The lower the $\delta$, the less they value future payoffs, i.e. the more impatient the players are.

# Alternating offer- two periods bargaining II

- Solve by backward induction.
- The game that starts after P2 has rejected the offer is identical with the ultimatum game. P2 offers $m^2 = 0$ and will thus receive payoff $\delta_2(1 - 0) = \delta_2$
- In period 1, P2 will thus reject $m^1 < \delta_2$, accept $m^1 > \delta_2$ and is indifferent to $m^1 = \delta_2$.
- In equilibrium, P1 will thus offer $m^1 = \delta_2$ which P2 accepts. Payoffs are then $1 - \delta_2$ and $\delta_2$ to P1 and P2, respectively.
- The discount factor determines the players' bargaining power.
- The bargaining solution is efficient (no delay in equilibrium).

# Alternating offer- two periods bargaining III



# Alternating offer - infinite period bargaining I

- We look for a stationary equilibrium where a player makes the same offer when ever she is on the move.
- Take a given period. Player $j$ can accept an offer $m_j$, or give a counteroffer $m_i$ in the next period that $i$ will accept, and thus achieve a discounted payoff $\delta_j(1-m_i)$.
- For $j$ to accept one must have $\delta_j(1-m_i) = m_j$
- This goes for both players: $\delta_2(1-m_1) = m_2$ and $\delta_1(1-m_2) = m_1$
- Solving this we get $m_1 = \frac{\delta_1(1-\delta_2)}{1-\delta_1\delta_2}$ and $m_2 = \frac{\delta_2(1-\delta_1)}{1-\delta_2\delta_1}$

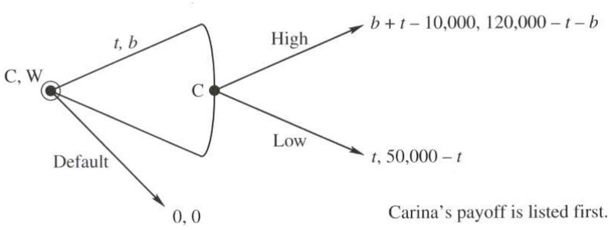
# Alternating offer - infinite period bargaining II

- In equilibrium, the offer is accepted in the first period. This yields
    - $\frac{(1-\delta_2)}{1-\delta_1\delta_2}$ to player 1 and $\frac{\delta_2(1-\delta_1)}{1-\delta_1\delta_2}$ to player 2.
- Note that there is first mover advantage.
- And the higher $\delta$, the more equal the split.

# Joint decisions

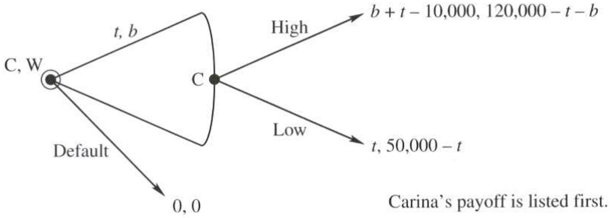
- Contractual relations consists of i) bargaining and ii) implementation.
- When focusing on implementation, one may want to describe the bargaining phase simply by referring the standard bargaining solutions.
- In the game tree we do this by including "joint decision nodes". On this node, the parties agree on a contract.

# Joint decisions: Example I




- The parties (C and W) agrees on a fixed salary $t$ and a bonus $b$ for high effort.
- High effort has cost 10.000 for C, but gives revenues 120.000 for W (against 50.000 for low effort)

# Joint decisions: Example II




- At the joint decision node we assume standard bargaining solution.
- Efficient bargaining implies $b \geq 10.000$ such that C exerts high effort
- Joint value is then $(b + t - 10.000) + (120.000 - t - b) = 110.000$
- The division of 110.000 is determined by bargaining power. 50/50 bargaining power implies $(b + t - 10.000) = 55.000$, e.g. $b = 10.000$ and $t = 55.000$.

# Regime and negotiation equilibrium

- Given an extensive-form game with joint decisions, a specification of behavior at every information set is called a regime. This is simply a generalization of the strategy concept to include joint decisions.
- A regime is called a negotiation equilibrium if its descriptions of behavior at individual decision nodes is consistent with sequential rationality and its specification of joint decisions is consistent the standard bargaining solution, for given bargaining weights.