Applied Game Theory

Lecture 1

Sebastian Fest

Some motivating examples

Two people enter a bus. Two adjacent cramped seats are free. Each person must decide whether to sit or stand. Sitting alone is more comfortable than sitting next to the other person, which is more comfortable than standing.

Suppose that each person cares only about her own comfort. Will one/both persons end up standing or sitting?

Suppose that each person only cares about the other persons comfort, and, out of politeness, prefers to stand than to sit if the other person stands. Will one/both persons end up standing or sitting?

Some motivating examples

Sherlock Holmes, pursued by his opponent, Moriarty, leaves London for Dover. The train stops at Canterbury on the way, and he disembarks there rather than travelling on to Dover. He has seen Moriarty at the London railway station, recognizes that he is very clever and expects that Moriarity will take a faster special train in order to catch him in Dover. Holmes' anticipation may turn out to be correct. But what if Moriarity had been still more clever, had estimated Holmes' mental abilities better and had foreseen his actions accordingly? Then, obviously, he would have traveled to Canterbury. Holmes again would have had to calculate that, and he himself would have decided to go on to Dover, whereupon, Moriarity would again have "reacted" differently. What should Holmes do?

Some motivating examples

Each one of two local bars charges its own price for a beer, either \$2, \$4, or \$5. The cost of obtaining and serving the beer can be neglected. It is expected that 6000 beers per month are drunk in a bar by tourists, who choose one of the two bars randomly, and 4000 beers per month are drunk by natives who go to the bar with the lowest price, and split evenly in case both bars offer the same price. What prices would the bars select?

What is a game?

- Games are formal descriptions of strategic settings.
- A strategic setting is characterized by interdependence, i.e. where a person's behavior affects another person's well-being.
- Game theory therefore is a methodology of formally studying situations of interdependence.
- Goal is to gain a better understanding of individual behavior and social interactions including competition, cooperation and collaboration.
    - e.g. pricing, auctions, contract strategies, negotiations, trade, incentives and economic organization.

The content of a game

- A list of players.
- A complete description of what the players can do.
- A description of what the players know when they act.
- A specification of how the players' actions lead to outcomes.
- A specification of the players' preferences over outcomes.

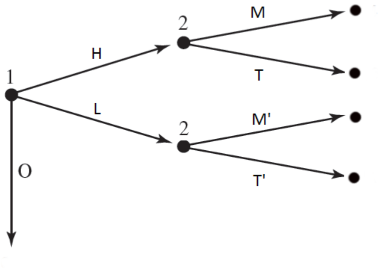
"Cash-in-a-hat game"

- Two players, 1 and 2
- Player 1 can put 0\$, 2\$ or 6\$ in a hat.
- Then, the hat is passed to player 2
- Player 2 sees what is inside the hat and can either match (i.e., add the same amount of money in the hat) or take the cash.
- Player 1 earns nothing if he puts nothing inside the hat; earns a net profit of 2\$ if he puts 2\$ and player 2 also puts 2\$, loses 2\$ otherwise; earns a net profit of 6\$ if he puts 6\$ and player 2 also puts 6\$, loses 6\$ otherwise;
- Player 2 earns nothing if player 1 puts nothing inside the hat; earns a net profit of 3\$ if he matches 2\$, gains 2\$ otherwise; earns a net profit of 4\$ if he matches 6\$, gains 6\$ otherwise;

Extensive form: Concepts I

- A game tree consists of nodes and branches.
- *Decision nodes*: Indicate that decisions are to be made.
- *Terminal nodes*: Represent outcomes of the game, i.e. where the game ends.

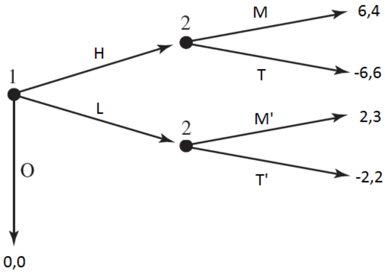
Extensive form for "Cash in a hat game"



Extensive form: Concepts II

- *Payoffs* at a terminal node reflect a player's preferences over outcomes.
- *Payoff-vector* shows the players' payoff at a given terminal node. Player 1's payoff is written first, then Player 2's, etc.

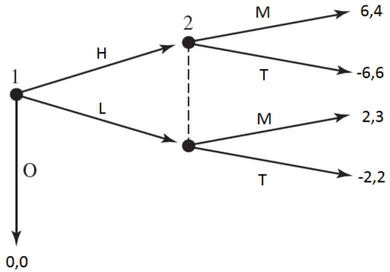
Extensive form for "Cash in a hat game"



Extensive form: Concepts IIi

- *Information sets*: Specify the players' information at a decision node.
    - Nodes belonging to the same information set are connected by dotted lines.
    - The information set belongs to one player only.
    - By using information sets we can indicate that players move simultaneously.

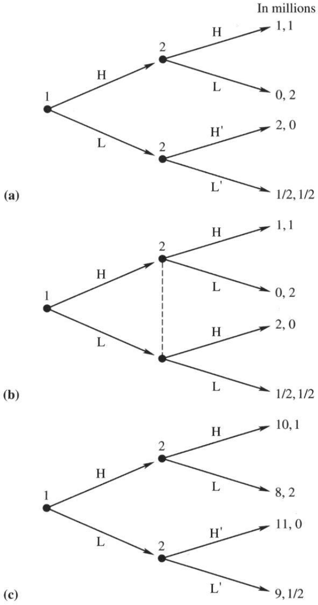
Changing what player 2 knows



The market game: Extensive form

- Two competing firms produce and sell the same product.
- Each firm either selects a high (H) or low (L) price for the product.

The market game: Extensive form



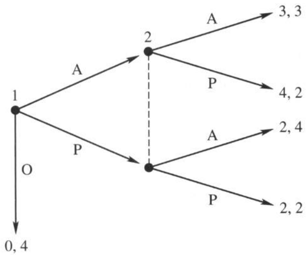
Strategy I

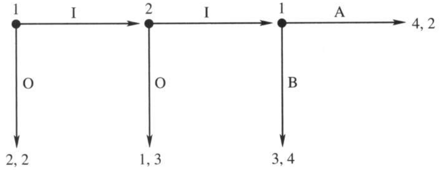
- A *strategy* is a complete contingent plan for a player in the game.
- A player's strategy describes a player's actions at all possible decisions nodes, i.e. at all information sets for that player.
- *A strategy set*, S_i, is the set of all possible strategies for player i.
  - In the market game (a): $S_1 = \{H, L\}$, $S_2 = \{HH', HL', LH', LL'\}$
- A given strategy is written: $s_i \in S_i$
  - In the market game (a): $s_1 = L$, $s_2 = LH'$

Strategy II

- The (cartesian) product of all strategy sets constructs the strategy space $S = S_1 \times S_2 \times ... \times S_n$
- A *strategy profile* describes the strategies for all the players in the game.
  - Denoted as $s = (s_1, s_2, ..., s_n)$ where n is the number of players, and s_i is the strategy for player i where $i = 1, 2, ..., n$
  - A strategy profile is a vector in the strategy space.
- $s_{-i} = (s_1, s_2, ..., s_{i-1}, s_{i+1}, ..., s_n)$ describes the strategies to all the players, except player i. A strategy profile can thus be written $s = (s_i, s_{-i})$

Strategies: Examples I



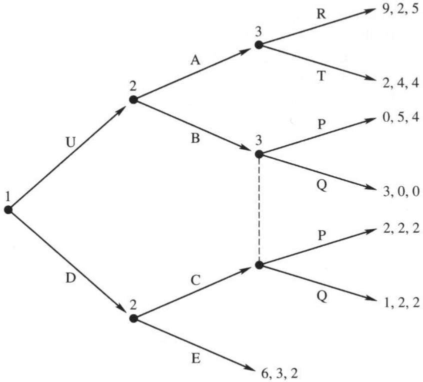


Strategies: Counterintuitive?

A strategy for player 1 requires a specification of the choice at his second information set even in the situation in which he plans to opt out. Why?

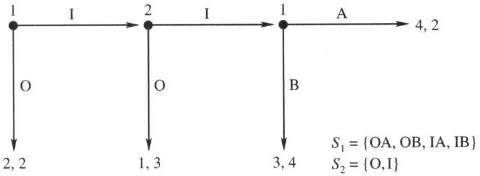
- Each move in the game needs to adhere to our version of rationality that we impose on players. This requires players to hold a belief about what other players would do at their information set.
- Players can make mistakes, therefore we need to specify what happens if players accidentally deviate.

Strategies: Examples II



Normal form: Concepts I

- Player i's payoff can be written $u_i(s)$ where $s \in S$ is a strategy profile.



- In the above figure, the set of strategy profiles is given by

$$S = \{(OA, O), (OA, I), (OB, O), (OB, I), (IA, O), (IA, I), (IB, O), (IB, I)\}$$

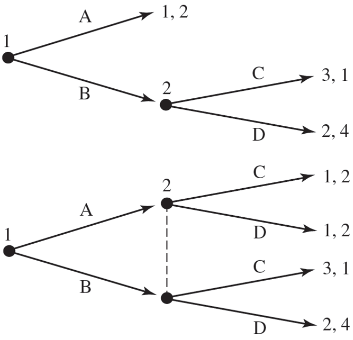
- What is $u_1(OA, O)$, $u_1(IA, I)$, $u_2(IA, O)$?

Normal form: Concepts II

1 \ 2	I	O
OA	2, 2	2, 2
OB	2, 2	2, 2
IA	4, 2	1, 3
IB	3, 4	1, 3

- A game in normal form (with 2 players) can be written as a matrix where each row represents a strategy for player 1, and each column represents a strategy for player 2. Each cell thus constitute a strategy profile, and within the cells we write the payoffs associated with the strategy profile.
- Formal definition: A game in normal form consists of players $\{1, 2, ..., n\}$, the strategy sets of all players, $S_1, S_2, ..., S_n$, and the pay-off functions of all the players $u_1, u_2, ..., u_n$

Normal form vs. extensive form



1 \ 2	C	D
A	1, 2	1, 2
B	3, 1	2, 4

Normal form: "Classic" games

1 \ 2	H	T
H	1, –1	–1, 1
T	–1, 1	1, –1

Matching Pennies

1 \ 2	C	D
C	2, 2	0, 3
D	3, 0	1, 1

Prisoners' Dilemma

1 \ 2	Opera	Movie
Opera	2, 1	0, 0
Movie	0, 0	1, 2

Battle of the Sexes

1 \ 2	H	D
H	0, 0	3, 1
D	1, 3	2, 2

Hawk-Dove/Chicken

1 \ 2	A	B
A	1, 1	0, 0
B	0, 0	1, 1

Coordination

1 \ 2	A	B
A	2, 2	0, 0
B	0, 0	1, 1

Pareto Coordination

D \ S	P	D
P	4, 2	2, 3
D	6, –1	0, 0

Pigs