Applied Game Theory

Lecture 8

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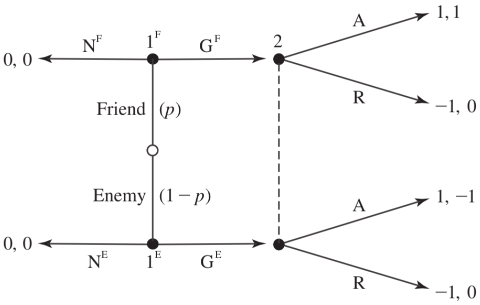
Incomplete information I

- Many strategic situation are characterized by *private* or *asymmetric information*, i.e. where one player knows something that other players do not know:
  - e.g. bargaining, contract, auctions
- We represent private information through random events or moves of nature.
- *Incomplete information* refers to games having moves of nature that generate asymmetric information between the players.

Incomplete information II



- The chance node determines P1's type with probability p (friend) and $1-p$ (enemy).
- P2 does not observe P1's type.
- P1's best response will require him to think of how he would act if he were the other type, because P2 will try to infer from P1s act what type he is.

Incomplete information III

1 \ 2	A	R
$G^F G^E$	$1, 2p-1$	$-1, 0$
$G^F N^E$	p, p	$-p, 0$
$N^F G^E$	$1-p, p-1$	$p-1, 0$
$N^F N^E$	$0, 0$	$0, 0$

- Due to the uncertainty of payoffs, these games are also called *Bayesian games*.
- E.g. the strategy profile $(G^F G^E, A)$ yields payoff $p + (1-p) = 1$ to P1 and $p + (1-p) * (-1) = 2p - 1$ to P2.

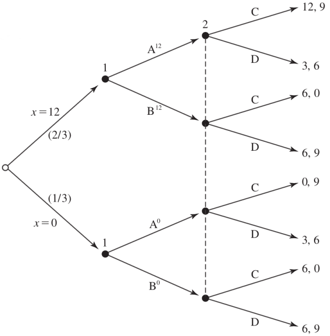
Bayesian game: Example I

1 \ 2	C	D
A	x, 9	3, 6
B	6, 0	6, 9

$$x = \begin{cases} 12 & \text{with probability } 2/3 \\ 0 & \text{with probability } 1/3 \end{cases}$$

- P1 observes x, then engages in the resulting simultaneous move game, i.e. P1 has private information when playing with P2.

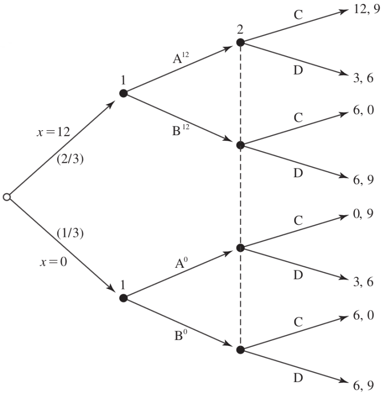
Bayesian game: Example II



1 \ 2	C	D
$A^{12}A^{0}$	8, 9	3, 6
$A^{12}B^{0}$	10, 6	4, 7
$B^{12}A^{0}$	4, 3	5, 8
$B^{12}B^{0}$	6, 0	6, 9

- Superscripts denote whether P1 has observed $x = 0$ or $x = 12$.
- The payoffs in the normal form are determined by the probability distribution over x.

Bayesian game: Example III



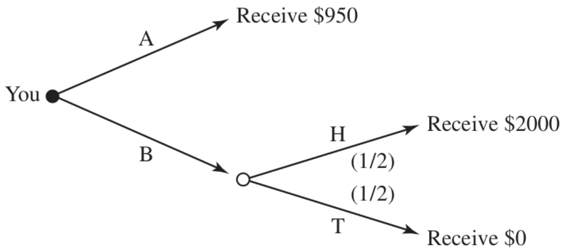
1 \ 2	C	D
$A^{12}A^{0}$	8, 9	3, 6
$A^{12}B^{0}$	10, 6	4, 7
$B^{12}A^{0}$	4, 3	5, 8
$B^{12}B^{0}$	6, 0	6, 9

- For example, the strategy profile $(B^{12}A^{0}, D)$ yields payoff $\frac{2}{3} * 6 + \frac{1}{3} * 3 = 5$ to P1 and $\frac{2}{3} * 9 + \frac{1}{3} * 6 = 8$ to P2.
- Through rationalization we find the *Bayesian Nash equilibrium* $\{B^{12}B^{0}, D\}$.

Risk and Incentives

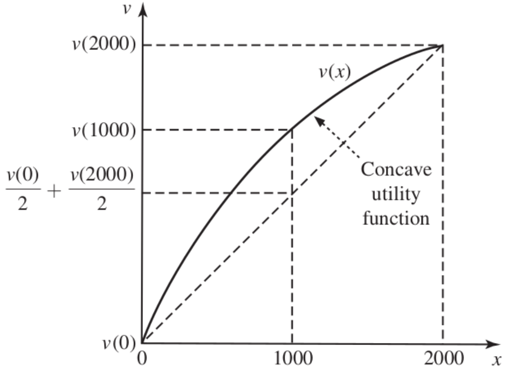
- Random events might also occur at the end of the game, i.e. after some action took place.
- A prominent example involves contracting between principal and agent.
  - Agents effort is not verifiable.
  - We can only write contracts dependent on the observed output.
  - The output is determined by effort and chance.
  - Risk preferences will determine the shape of the incentive contract.

Risk aversion I



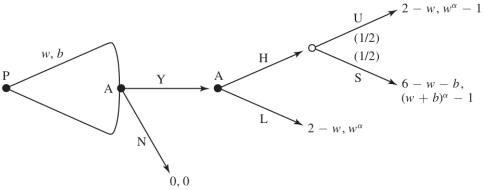
- The player prefers A over B if $v(950) > \frac{1}{2} * v(0)\frac{1}{2} * v(2000)$.
- In general, a person is risk avers if he prefers a sure payment over an uncertain payment that has the same expected payoff i.e. if $v(E[x]) > E[v(x)])$ with x being the same lottery over uncertain payoffs.

Risk aversion II



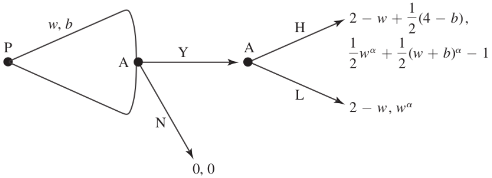
- Arrow-Pratt measure of relative risk aversion is $-\frac{xv''(x)}{v'(x)}$
- For $v(x) = x^\alpha$ this is $\frac{-x\alpha(\alpha-1)x^{\alpha-2}}{\alpha x^{\alpha-1}} = (1-\alpha)$

Principal-Agent: Example I



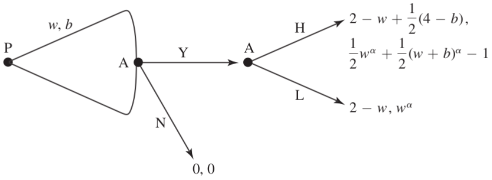
- Agent gets expected wage w and a bonus b in case of success success.
- High effort costs 1 and gives success with probability $1/2$.
  Low effort costs nothing and gives no success with certainty.
- Success yields revenue 6 to principal, while no success yields 2.

Principal-Agent: Example II



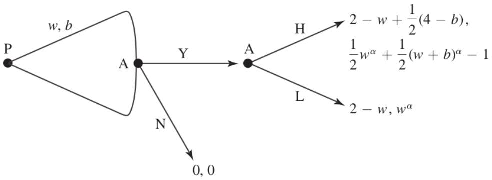
- Assume that the agent is risk averse with utility function $v(x) = x^{\alpha}$ and that the principal is risk neutral with utility function $v(x) = x$.
- High effort is efficient: It costs 1 and gives an expected revenue increase of 2.
- Solve game by using backward induction.

Principal-Agent: Example III



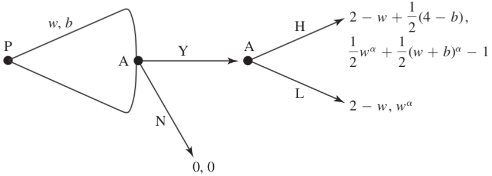
- A selects H if $\frac{1}{2}(w)^\alpha + \frac{1}{2}(w + b)^\alpha - 1 \geq w^\alpha$.
- This is called the *incentive compatibility constraint* (IC-constraint).
- For A being able to select H, he also must chose Y in the first place, i.e. be better-off than choosing N.
- $\frac{1}{2}(w)^\alpha + \frac{1}{2}(w + b)^\alpha - 1 \geq 0$
- This is called the *participation constraint* (PC-constraint).

Principal-Agent: Example IV



- P offers contract that maximizes his payoff given both constraints.
- In optimum, both constraints have to hold with equality.
- This implies $w^\alpha = 0$, and thus $w = 0$. Solving for b yields $b = 2^{1/\alpha}$

Principal-Agent: Example V



- P's expected payoff is $4 - 2^{(1-\alpha)/\alpha}$
- If A is risk neutral, i.e. $\alpha = 1$, then P's payoff is $4 - 2^0 = 3$
- The difference in payoffs for P for $\alpha = 1$ and $\alpha < 1$ is the risk premium that P must pay to induce A to exert high effort.
- The higher the risk aversion, the larger the risk premium.
- For P to offer a bonus contract to A, we must have $4 - 2^{(1-\alpha)/\alpha} > 2 \iff \alpha \geq \frac{1}{2}$
- If the risk premium is too big, then P will not offer a bonus contract to A, only a fixed wage.