Problem 1

Consider a game in normal form with a limited number of players and strategies. Evaluate and comment the following statements in terms of their correctness:

a) If there is only one rationalizable strategy for each player, then there can only be one Nash equilibrium.

Correct. Nash equilibria must allways be rationalizable.

b) If a game has a unique Nash equilibrium, then each player must have a unique rationalizable strategy.

False. Nash equilibria are a subset of the set of rationalizable strategies, but they do not need to be propper (strict) subsets.

c) If a strategy profile survives elimination of strictly dominated strategies, then this strategy profile is also a Nash equilibrium.

False. All strategies that survive elimination of strictly dominated strategies are rationalizable. These are not necessarily also Nash equilibria (see answer in b)).

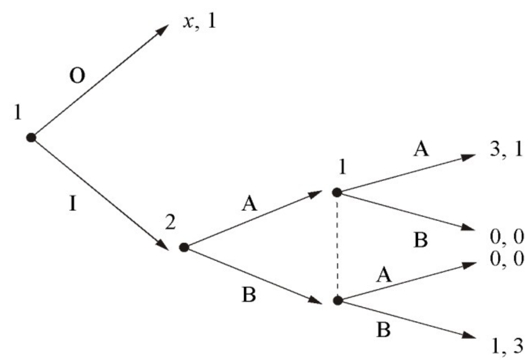
d) If a strategy profile is a Nash equilibrium, then there cannot be any other strategy profile that makes every player better off.

False. The efficiency of a strategy profile has no bearing on whether or not it also represents a Nash equilibrium, and vice versa.

Problem 2

Consider a game in which player 1 first selects between I and O. If player 1 selects O, then the game ends with the payoff vector (x, 1) (x for player 1), where x is some positive number. If player 1 selects I, then this selection is revealed to player 2 and then the players play the battle-of-the-sexes game in which they simultaneously and independently choose between A and B. If they coordinate on A, then the payoff vector is (3, 1). If they coordinate on B, then the payoff vector is (1, 3). If they fail to coordinate, then the payoff vector is (0, 0).

a) Represent this game in the extensive and normal forms.



1 \ 2	A	B
OA	x, 1	x, 1
OB	x, 1	x, 1
IA	3, 1	0, 0
IB	0, 0	1, 3

b) Find the pure-strategy Nash equilibria of this game. Note how they depend on the value of x

If $x > 3$, the equilibria are (OA,A), (OB,A), (OA,B), (OB,B). If $x = 3$, add (IA, A) to this list. If $1 < x < 3$, the equilibria are (IA,A), (OA,B), (OB,B). If $x = 1$, add (IB, B) to this list. If $x < 1$, the equilibria are (IA,A), (IB,B).

c) Calculate the mixed-strategy Nash equilibria. Note how they depend on the value of x.

If $x > 3$ any mixture with positive probabilities over OA and OB for player 1, and over A and B for player 2.

If $1 < x < 3$, then IB is dominated. Any mixture (with positive probabilities) over OA and OB will make player 2 indifferent. Player 2 plays A with a probability that does not exceed x/3.

Next consider the case in which $3/4 \leq x \leq 1$. Let p denote the probability that player 1 plays IA, let q denotes the probability with which she plays IB, and let $1 - p - q$ denote the probability that player 1 plays OA or OB. There is a mixed strategy equilibrium in which $p = q = 0$. Here, player 2 mixes so that player 1 does not want to play IA or IB, implying that player 2 can put no more than probability x/3 on A and no more than x on B. There is not an equilibrium with p and/or q positive. To see this, note that for player 2 to be indifferent, we need $p = 3q$. We also need player 2 to mix so that player 1 is indifferent between IA and IB, but (for $x > 3/4$) this mixture makes player 1 strictly prefer to select OA or OB.

For $x < 3/4$, OA and OB are dominated. In equilibrium, player 1 chooses IA with probability 3/4 and IB with probability 1/4. In equilibrium, player 2 chooses A with probability 1/4, and B with probability 3/4.

d) Represent the proper subgame in the normal form and find its equilibria.

1 \ 2	A	B
A	1, 3	0, 0
B	0, 0	3, 1

The pure strategy equilibria are (A, A) and (B, B). There is also a mixed equilibrium {(3/4, 1/4) (1/4, 3/4)}.

e) What are the pure-strategy subgame perfect equilibria of the game? Can you find any Nash equilibria that are not subgame perfect?

The Nash equilibria that are not subgame perfect include (OB, A), (OA, B), and the above mixed equilibria in which, once the proper sub-game is reached, player 1 does not play A with probability 3/4 and/or player 2 does not play A with probability 1/4.

f) What are the mixed-strategy subgame perfect equilibria of the game?

The subgame perfect mixed equilibria are those in which, once the proper subgame is reached, player 1 does play A with probability ¾ and player 2 does plays A with probability 1/4.

Problem 3

Consider a contractual setting in which the technology of the relationship is given by the following underlying game:

1 \ 2	I	N
I	6, 5	–1, 1
N	8, –1	0, 0

Suppose an external enforcer will compel transfer α from player 2 to player 1 if (N, I) is played, transfer β from player 2 to player 1 if (I, N) is played, and transfer γ from player 2 to player 1 if (N, N) is played. The players wish to support the investment outcome (I, I).

a) Suppose there is limited verifiability, so that $\alpha = \beta = \gamma$ is required. Assume that this number is set by the players' contract. Determine whether (I, I) can be enforced. Explain your answer.

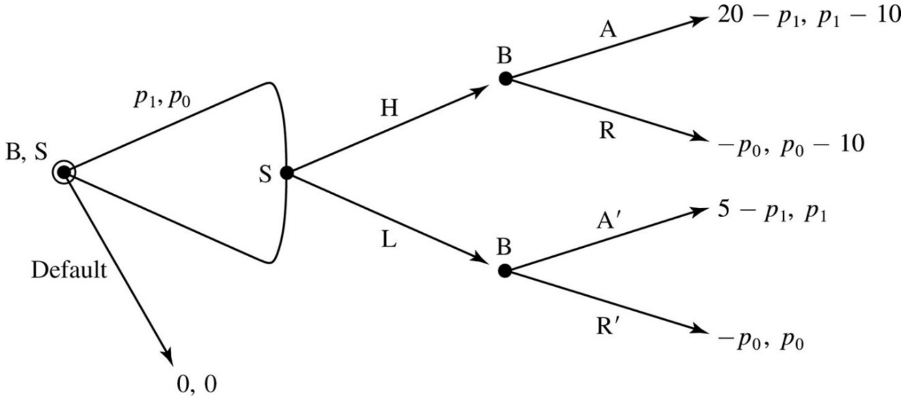
For (I, I) to be a Nash equilibrium, we need $6 \geq 8 + \alpha$ and $5 \geq 1 - \alpha$. Any $\alpha \in [-4, -2]$ works.

b) Suppose there is full verifiability, but that α, β, and γ represent reliance damages imposed by the court. Determine whether (I, I) can be enforced. Explain your answer.

Reliance damages implies returning the defendant to the position of getting 0, which is the equilibrium outcome with no contract. Consider (N, I). Reliance damages yield a payoff of 7 to player 1, resulting in an incentive to deviate from (I, I). Therefore (I, I) cannot be supported as a NE under reliance damages.

Problem 4

Consider the following game:



The buyer (B) and the seller (S) agrees on prices p_1 or p_0 depending on whether the buyer accepts delivery (A) or not (R). Prior to trade, the seller can choose high (H) or low (L) quality.

a) What is the efficient outcome of the game?

S plays H, B plays A

b) Fully describe the negotiation equilibrium of the game, under the assumption that the parties have equal bargaining weights.

The surplus is 10. Each player gets 5. Thus, p_1 is 15 and p_0 between -5 and 5. The seller chooses H. The buyer chooses A if H and R if L.

Problem 5

Consider a two-player Cournot type of interaction between two firms. Each firm chooses quantity $q_i \in [0, \infty]$ and bears a cost of producing quantity $q_i \in [0, \infty]$ that is given by $c_i(q_i) = 0$ They produce identical goods and sell in the same market, which has an inverse demand curve of $p = 8 - q_1 - q_2$.

a) Suppose the firms make their production decisions simultaneously and independently with no scope for collusion or contracting between them. Find each firm's best-response function.

Player i chooses q_i to maximize $\Pi_i = (8 - q_i - q_j) q_i$, which has a first-order condition of $8 - 2q_i - q_j = 0$, yielding $BR_i(q_j) = 4 - \frac{q_j}{2}$.

b) If the players could write an externally enforced contract that conditioned on their choices of quantities (the court observes the quantity selected by each player), what quantities would their contract specify? In the absence of an externally-enforced contract, does either player have an incentive to unilaterally deviate from the jointly optimal quantity? Explain. Describe a contract that implements the jointly optimal quantities.

The players would "share the monopoly" by each producing half of the $Q = q_i + q_j$ that maximizes $[8 - Q]Q$, yielding $Q^M = 4$ and the players each produce half of this monopoly quantity. This yields $\pi_i = 8$.

Note that $BR_i(2) = 3$, which yields $\pi_i = 9$. Contract: The self enforced part is to play (2, 2), and the externally enforced part is there are zero transfers unless someone unilaterally deviates from (2, 2). In that case, impose a transfer of at least 1 from the deviating player to the non-deviating player.

Problem 6

a) Assume that the following stage game is played twice ($T=2$) and that payoffs are not discounted. Describe a subgame perfect Nash equilibrium for which the strategy profile (U, L) is being played in the first period ($t=1$).

1 \ 2	L	M	R
U	8, 8	0, 9	0, 0
C	9, 0	0, 0	3, 1
D	0, 0	1, 3	3, 3

The game has three Nash equilibria (NE = {(D, M), (C, R), (D, R)}). The efficient strategy profile which is not a Nash equilibrium is (U, L). Because there is more than one Nash equilibrium in this game, reputational equilibria might exist in which the efficient strategy profile is played in the first period. Notice the following strategy description for example:

Strategy for player 1: Play U in t = 1 and D in t = 2 if player 2 played L in t = 1, otherwise play C in t = 2.

Strategy for player 2: Play L in t = 1 and R in t = 2 if player 1 played U in t = 1, otherwise play M in t = 2.

To see that this constitutes a SPNE, notice that if each player adheres to the described strategy, his or her payoff from the game is equal to 8 + 3 = 11. If either player decides to deviate in t = 1 and play his or her best response in the first period, then the payoff from the game equals 9 + 1 = 10, which is strictly less than adhering to the prescribed strategy. Thus, (UD, LR) is a subgame perfect Nash equilibrium.

b) Assume now that the following stage game is played over two rounds ($T=2$). Is there a subgame perfect Nash equilibrium for which (C, C) is played in the first period? Explain.

1 \ 2	C	D
C	2, 2	0, 3
D	3, 0	1, 1

(D, D) is the only one Nash equilibrium in this prisoners' dilemma (stage) game. Since the only Nash equilibrium has to be played in the final round, (D, D) will also be played in the first (previous) period. This can be easily seen by noticing that, if one player plays C

in the first period, the other player can gain 3 by playing D in the same period. This strategy yields a higher final payoff than playing C as well (2 + 1 = 3 by playing C in t=1 and D in t = 2; 3 + 1 = 4 by playing D in both periods). Cooperation cannot be sustained in the finitely repeated prisoners' dilemma game because there are no other Nash equilibria that either player can induce after observing that the other player did not cooperate.

c) How would your answer in b) change if the horizon of the game is extended to ten periods ($T=10$).

Changing the horizon of the game to ten periods has no effect on the possibility to sustain mutual cooperation (C, C) in the prisoners dilemma game. The explanation given in b) is also valid for the last two periods of the ten period game. By the logic of backward induction, deviation will then also occur in the 8th period, and subsequently in all previous periods such that mutual defection is the outcome for each round.