Problem 1

Consider a game in normal form with a limited number of players and strategies. Evaluate and comment on the following statements in terms of their correctness:

a) If there is only one rationalizable strategy for each player I the game , then there can only be one Nash equilibrium.

b) If a game has a unique Nash equilibrium, then each player must have a unique rationalizable strategy.

c) If a strategy profile survives elimination of strictly dominated strategies, then this strategy profile is also a Nash equilibrium.

d) If a strategy profile is a Nash equilibrium, then there cannot be any other strategy profile that makes every player better off.

Problem 2

Consider a game in which player 1 first selects between I and O. If player 1 selects O, then the game ends with the payoff vector (x, 1) (x for player 1), where x is some positive number. If player 1 selects I, then this selection is revealed to player 2 and then the players play the battle-of-the-sexes game in which they simultaneously and independently choose between A and B. If they coordinate on A, then the payoff vector is (3, 1). If they coordinate on B, then the payoff vector is (1, 3). If they fail to coordinate, then the payoff vector is (0, 0).

a) Represent this game in the extensive and normal forms.

b) Find the pure-strategy Nash equilibria of this game. Note how they depend on the value of x

c) Calculate the mixed-strategy Nash equilibria. Note how they depend on the value of x.

d) Represent the proper subgame in the normal form and find its equilibria.

e) What are the pure-strategy subgame perfect equilibria of the game? Can you find any Nash equilibria that are not subgame perfect?

f) What are the mixed-strategy subgame perfect equilibria of the game?

Problem 3

Consider a contractual setting in which the technology of the relationship is given by the following underlying game:

1 \ 2	I	N
I	6, 5	–1, 1
N	8, –1	0, 0

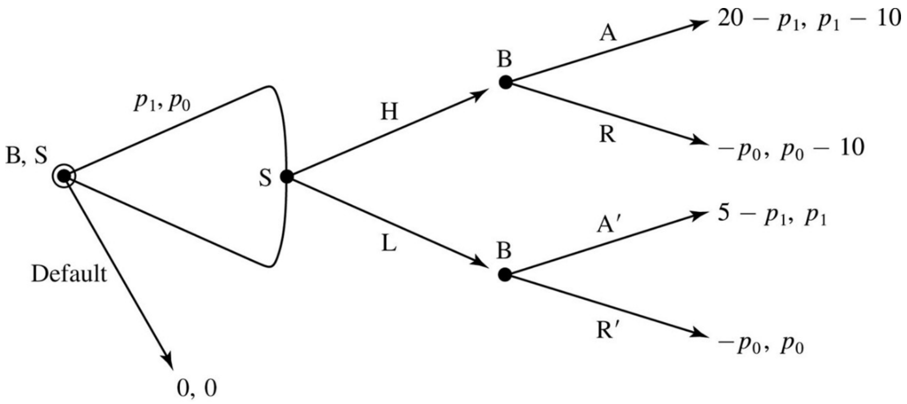
Suppose an external enforcer will compel transfer α from player 2 to player 1 if (N, I) is played, transfer β from player 2 to player 1 if (I, N) is played, and transfer γ from player 2 to player 1 if (N, N) is played. The players wish to support the investment outcome (I, I).

a) Suppose there is limited verifiability, so that α = β = γ is required. Assume that this number is set by the players' contract. Determine whether (I, I) can be enforced. Explain your answer.

b) Suppose there is full verifiability, but that α, β, and γ represent reliance damages imposed by the court. Determine whether (I, I) can be enforced. Explain your answer.

Problem 4

Consider the following game:



The buyer (B) and the seller (S) agrees on prices p_1 or p_0 depending on whether the buyer accepts delivery (A) or not (R). Prior to trade, the seller can choose high (H) or low (L) quality.

a) What is the efficient outcome of the game?

b) Fully describe the negotiation equilibrium of the game, under the assumption that the parties have equal bargaining weights.

Problem 5

Consider a two-player Cournot type of interaction between two firms. Each firm chooses quantity $q_i \in [0, \infty]$ and bears a cost of producing quantity $q_i \in [0, \infty]$ that is given by $c_i(q_i) = 0$ They produce identical goods and sell in the same market, which has an inverse demand curve of $p = 8 - q_1 - q_2$.

a) Suppose the firms make their production decisions simultaneously and independently with no scope for collusion or contracting between them. Find each firm's best-response function.

b) If the players could write an externally enforced contract that conditioned on their choices of quantities (the court observes the quantity selected by each player), what quantities would their contract specify? In the absence of an externally-enforced contract, does either player have an incentive to unilaterally deviate from the jointly optimal quantity? Explain. Describe a contract that implements the jointly optimal quantities.

Problem 6

a) Assume that the following stage game is played twice ($T = 2$) and that payoffs are not discounted. Describe a subgame perfect Nash equilibrium for which the strategy profile (U, L) is being played in the first period ($t = 1$).

1 \ 2	L	M	R
U	8, 8	0, 9	0, 0
C	9, 0	0, 0	3, 1
D	0, 0	1, 3	3, 3

b) Assume now that the following stage game is played over two rounds ($T = 2$). Is there a subgame perfect Nash equilibrium for which (C, C) is played in the first period? Explain.

1 \ 2	C	D
C	2, 2	0, 3
D	3, 0	1, 1

c) How would your answer in b) change if the horizon of the game is extended to ten periods ($T = 10$).