EXAM MØA290 spring 2018

Solutions in italics

Problem 1

a) What is a strategy? *Watson p. 22*
b) What is a dominated strategy? *Watson p. 50*
c) What does it mean to play best response? *Watson p. 54*
d) Consider the game below. Is Player 1's strategy M dominated? If yes, describe a strategy that dominates M. If no, describe a belief where M is best response.

1 \ 2	X	Y
K	9, 2	1, 0
L	1, 0	6, 1
M	3, 2	4, 2

M is dominated by (1/3, 2/3, 0)

Problem 2

a) What is a Nash equilibrium? *Watson p. 97*
b) Find the Nash equilibria in the following game: *(M,L) and (U,R)*

1 \ 2	L	C	R
U	2, 0	1, 1	4, 2
M	3, 4	1, 2	2, 3
D	1, 3	0, 2	3, 0

c) What is a rationalizable strategy? *Watson p. 70*
d) Do there exist rationalizable strategy profiles in the game that are not Nash equilibria? *Yes, (U,L) and (M,R) are rationalizable, but not Nash equilibria (NE).*

Problem 3

Consider the following game:

1 \ 2	L	M	R
U	x, x	$x, 0$	$x, 0$
C	$0, x$	$2, 0$	$0, 2$
D	$0, x$	$0, 2$	$2, 0$

Find Nash equilibria in pure and mixed strategies, and show how they depend on *x*. Show the difference between $x>1$ and $x<1$.

When $x < 1$, $((0, 1/2, 1/2), (0, 1/2, 1/2))$ is a mixed strategy Nash equilibrium. Further, for $0 < x < 1$, the Nash equilibria are (U, L) and there is an equilibrium of $((1 - x, x/2, x/2), (1 - x, x/2, x/2))$. When $x > 1$, Nash equilibrium is (U, L).

Problem 4

Consider the following game, describing a contractual relationship:

1 \ 2	I	N
I	z_1, z_2	y_1, x_2
N	x_1, y_2	$0, 0$

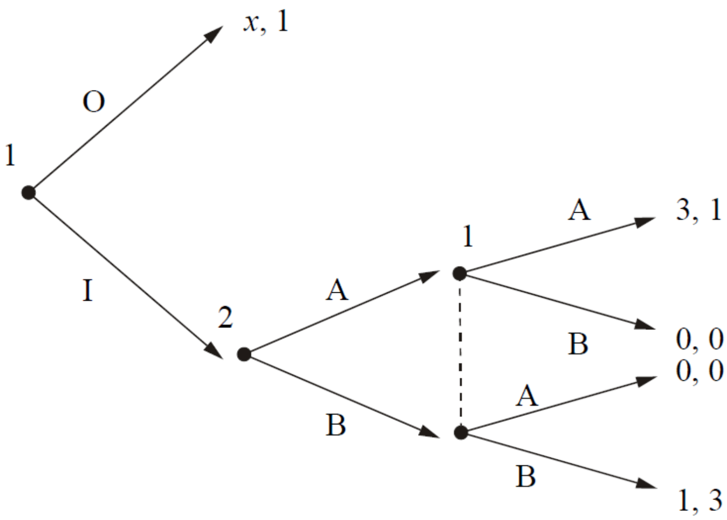
a) Use the game to explain the concepts of expectation damages and reliance damages.
b) Show when and why expectation damages are efficient.

See Watson p. 156-164

Problem 5

Consider a game in which player 1 first selects between I and O. If player 1 selects O, then the game ends with payoff vector (4,1), i.e. 4 to player 1 and 1 to player 2. If player 1 selects I, then player 2 can observe this move, and the players then play a “battle of the sexes” game in which they simultaneously and independently choose between A and B. If they both play A, then the payoff vector is (3,1). If they both play B, then the payoff vector is (1,3). If one of the players play A, while the other play B, then the payoff vector is (0,0).

a) Represent this game in the extensive and normal forms.



1 \ 2	A	B
OA	x, 1	x, 1
OB	x, 1	x, 1
IA	3, 1	0, 0
IB	0, 0	1, 3

for *x = 4*

b) Find the Nash equilibrium in pure strategies. *(OA, A), (OB, A), (OA,B) og (OB, B).*

c) Represent the proper subgame (battle of the sexes) on normal form and find equilibria in pure and mixed strategies.

1 \ 2	A	B
A	1, 3	0, 0
B	0, 0	3, 1

NE in pure strategies: (A,A) og (B,B). Mixed strategies: (¾ , ¼; ¼, ¾)

d) What are the subgame perfect Nash equilibria in pure strategies (for the whole game). Are there any Nash equilibria that are not subgame perfect? *(OB, A) og (OA,B) are not subgame perfect, while (OA, A) og (OB, B) are subgame perfect*

Problem 6

Pia is a clever accountant, as well as a skilled leader. And now she is negotiating the possibility of a new position with the CEO of the company. The contract specifies the salary w, as well as her task:

accounting or management. If she works as accountant, she gets a payoff of w- 5.000 and the company gets 100.000 – w. If Pia works as a manager, she gets a payoff of w- 30.000, while the company gets x-w. If the parties do not agree, the company gets 0, while Pia gets u in another job.

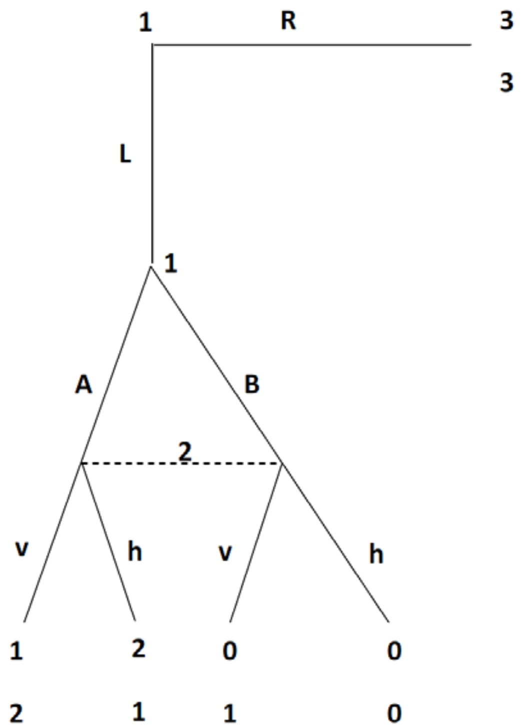
Assume x>150.000 and u<95.000. Pia's bargaining power is given by π_{P}, while the company's bargaining power is given by π_{C}.

a) Solve the bargaining problem by the use of standard bargaining solution: What should Pia work with and what will be paid?
Total surplus as accountant: 95 000. Total surplus as manager: x-30.000, which is larger than 95.000 since x>150.000. Hence, she should work as manager. Bargaining solution: Pia gets u + π_{P} (x-30.000-u) = w-30.000. That is w= (*1-* π_{P}) (u+30.000)+ $\pi_{\mathrm{P}} x$

b) Suppose Pia can invest in human capital, i.e. choose between two courses, A and B. Course A improves her management skills, thereby increasing x. Course B strengthen her outside options as an accountant, thereby increase u. What should Pia choose?
One sees that increasing u is better than increasing x if $\pi_P < \frac{1}{2}$*, while increasing x is better if* $\pi_P > \frac{1}{2}$

Problem 7

Consider the following game:



a) How many subgames are there in the game? *Two subgames*
b) Describe the set of possible strategies for each player

c) Find all pure strategy Nash equilibria.

d) Find all sub games and the pure strategy sub-game perfect Nash equilibria of the game.

	v	*h*
L,A	*1,2*	*2,1*
L,B	*0,1*	*0,0*
R,A	***3,3***	***3,3***
R,B	***3,3***	***3,3***

b) Strategies are described in the normal form representation above.

c) NE are market with bold face in the normal form representation.

d) A subgame starts after player 1 plays L. The normal form representation of the subgame:

	v	*h*
A	***1,2***	*2,1*
B	*0,1*	*0,0*

The NE in the subgame is market with bolds face. There is only one subgame perfect equilibrium: (R,A),v (the other 3 NE are not subgame perfect).

Problem 8

The stage game below, where 1 < x < 4, is played twice, with the outcome in the first stage observed before the second stage begins. The players move simultaneously. There is no discounting.

	L	M	R
U	1,1	5,0	0,0
M	0,5	4,4	0,0
D	0,0	0,0	x,x

a) For which x can the outcome (M,M) be achieved in the first stage in a pure-strategy subgame perfect Nash equilibrium? Give the strategies that do so.

Note that there are two Nash equilibria (NE) in the stage game; (D,R) and (U,L). One can potentially be a credible punishment, the other a credible reward.
Strategy combination: "Play (M,M) in first stage. If (M,M) observed in first stage, play (D,R) in 2nd stage, otherwise play (U,L) in 2nd stage." 2nd period strategies are NE, so no single-player deviations are profitable in that period. Neither will any single-player 1st period deviation (from M) be profitable if 5 + 1 ≤ 4 + x, i.e. if x ≥ 2. The strategy combination is then a subgame perfect equilibrium (SPE)

b) Consider an infinitely-repeated game with stage game given by the payoff matrix in (a). Payoffs are discounted with a discount factor $\delta \epsilon (0,1)$. For which discount factors can the outcome (M,M) each period be obtained as a subgame perfect equilibrium outcome, assuming the players use trigger strategies?

Strategy combination: "Play (M,M) in first stage. Continue to play (M,M) as long as no deviations have occurred, otherwise play (U,L). This is SPE if $5 + \frac{\delta}{1-\delta} \leq \frac{1}{1-\delta} 4$ *which is true for* $\delta \geq \frac{1}{4}$