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Control oriented modelling of four wheeled electric vehicles

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Sommario

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Summary

Abstract, in English.

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Introduction

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Chapter 2

Tyres

Tyres are the main source of external forces and moments acting upon any road vehicle[cita milliken race car VD], for this reason, understanding their behaviour is key in making accurate predictions about vehicle dynamics. The tyre is a very complex system exhibiting high non linearities, especially in the wide operating range imposed by racing[trova una fonte]. A realistic model of the road-vehicle interaction is clearly necessary in order to obtain significant results from simulations but the detailed study of tyres is far beyond the objectives of this work. The present chapter provides only the fundamental concepts behind tire modelling needed to introduce the numerical model that was used.

2.1 Tyre modelling basics and definitions

All forces acting between the vehicle and the road are distributed over the surfaces of the so called *tyre contact patches*. As the tyre contact patches size and shape are dependent on the operating conditions [trova fonte], for each wheel, we consider a *tyre contact point*, C, as the point at which an applied vector can approximate all forces acting between the tyre and the road.

Various definitions for this point may be used to include more or less complicated effects due to tyre compliance, such as dynamic caster or centrifugal deformation.[cita un articolo su questi effetti] Modelling of such phenomena would require extensive experimental data and is considered to be outside the scope of this research, thus, the following simplified definition for the tyre contact point was deemed satisfactory. The road is assumed to be a flat and planar surface and the tyre is to be considered as an infinitely thin rigid disk

tangent to this plane. The tangent point will be considered as the tyre contact point.

The wheel co-ordinate system is required for the definition of all the related physical quantities. We consider a reference system [figura] with origin in C, whose x axis lies along the intersection between the road plane and the wheel plane, such that it is generally oriented in the direction of motion, and whose z axis is pointing down, perpendicular to the road plane, the y axis is determined by asserting $Cxyz$ to be a right-handed cartesian coordinate system.

The angle between the xz plane and the tyre plane is known as the camber angle [trova definizione SAE].

The x, y and z axes are respectively associated with the *longitudinal*, *lateral* and *vertical* dimensions of the wheel.

As tyre compliance is neglected in this work, the vertical force F_z acting through the tyre and its contact point is considered as indipendent of tyre behaviour. This choice also excludes all transient responses of the rubber. The longitudinal and lateral friction forces, F_x and F_y, are to be computed since these are what effectively make the vehicle accelerate and corner and are of primary interest.

Consider a wheel [figura] having radius R and angular velocity ω about it's spin axis, passing through the wheel center and orthogonal to the wheel plane. We consider ω as positive when the upper half of the wheel is moving in the positive x direction. The lowest point of the wheel is then moving with velocity $v_w = -\omega R$ in the wheel x direction. By considering the velocity vector of the point C in a road-fixed reference system and letting v_x and v_y be its components along the wheel x and y axes the longitudinal velocity of the road with respect to the wheel system is $v_{rx} = -v_x$. It is therefore possible to define the longitudinal slip as the difference between these two velocities at the contact point:

$$v_{sx} = v_w - v_{rx} = v_x - \omega R.$$

We further define the slip ratio as [SAE J670]

$$\kappa = -\frac{v_{sx}}{v_x} = \frac{\omega R}{v_x} - 1.$$

The slip angle α is the angle formed between the contact point velocity vector in the road frame and the wheel x axis. It can be formally defined by the equation

$$\tan \alpha = \frac{v_y}{v_x}.$$

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F_x and F_y scale almost linearly with the vertical load exerted on the tyre, for this reason it makes sense to define the longitudinal and lateral friction coefficients as

$$\mu_x = \frac{F_x}{F_z}$$

and

$$\mu_y = \frac{F_y}{F_z}$$

κ and α are the primary variables affecting the friction coefficients and are used to estimate μ_x and μ_y. However, there are other factors that define the steady state tyre behaviour, some of which are relatively straight-forward, such as camber angle, vertical load, wheel rotational velocity, tyre pressure and temperature and others which are extremely difficult to quantify, such as asphalt characteristics, and rubber aging.

The typical correlation between friction coefficients and slip ratio and slip angle is shown in [figure dei coeff di attrito tipo dal Kiencke] , ideally, all curves pass through the origin, due to the fact that without slip the speed of the wheel is the same as that of the contact point, and no friction forces can be generated. In reality, there is a slight offset caused by rolling resistance and asymmetric tyre contructions [letto in qualcosa di pacejka].

It is also clearly noticeable that in order to obtain maximum longitudinal traction their will be an optimal amount of slip ratio. Limiting the tractive torque to not exceed this value is the exact purpouse of launch control systems.

Another effect of the pneumatic tyre is the so called self aligning torque, this moment is exerted about the tyre z axis and is an important contributor to the steering effort which is applied to the steering wheel, this will be relevant to the steering dynamics introduced in the 12 DoF model presented in Chapter 5.

2.2   Magic Formula tyre models

A series of semi-empirical tyre models was developed during the past 30 years by Hans B. Pacejka of Delft University.

These models are based on the same parametric mathematical expression, nicknamed *Magic Formula* because of the good fitting properties it offers despite the lack of a physical foundation. The general form of these functions is

$$R(k) = d\sin(c\arctan(b(1-e)k + e\arctan(bk)))$$

where R is the physical quantity to be estimatated and k is the most relevant variable whilst d, c, b, e and k are fitting coefficients, which may in turn depend on other variables. The set of equations released in 1996, called *Deft Tyre 96* [riferisci all'articolo 96] was the first to include combined effects of longitudinal and lateral slip, subsequent editions of the model have been conceived to model tyre pressure changes [riferisci all'articolo 2004] and more. The version which has been chosen in this work is the PAC2002 model [riferisci all'URL del pdf online], formalized by MSC Software and released as part of their Adams CAR multibody simulation package, which includes a set of tools for obtaining the fitting coefficients from experimental datasets. The standard file format used for the storage of the tyre coefficients is text based and can also be interpreted by an appropriate MATLAB script [refer to implementation paragraph]. The input variables of this tyre model are the slip ratio, slip angle, vertical force and camber angle. Outputs are the longitudinal and lateral forces and the self aligning torque.

2.3 Tyre Data

Experimental tyre data that is specific to Formula SAE and Formula Student tyres has been gathered by the FSAE Tyre Test Consortium [link al TTC] at the Calspan test facility. The expensive tests are funded by the one-time fee the student teams pay to access the information. As access to experimental data was not possible during the preparation of this work, a sample tyre model was used. As the coefficients were given, the fitting process was not necessary. To verify that the tyre model was consistent with the physical intuitions developed so far, some plots were drawn (shown in [figure fatte in matlab]), these both exhibit a plausible correlation between the slip quantities and the friction forces.

Chapter 3

Common models

3.1 Single Track Model

3.2 *some other model*

Chapter 4

Six degree-of-freedom vehicle model

4.1 6DoF Model structure

The state of the car is identified by the six degrees of freedom required to define the spacial position and orientation of its chassis. The car interacts with the road through a simplified suspension system. The model outputs are the vertical forces on each of the four wheels, obtained as the forces acting through each of the suspesion systems, and the velocities of the respective ground contact points, all of which may be used to calculate friction by an external tyre model. Inputs to the model are the longitudinal and lateral forces acting on each wheel and the angle of the front wheels which is used to rotate the force vectors acting on them, in order to obtain steering control. The block diagram in [figura] shows the intended flow of information for this model working in conjuction with a tyre model.

4.2 Chassis description

The road is assumed to be a horizontal plane, so we can define the xyz inertial reference system where z is pointed downwards, in the direction of gravity, x and y lie on the road surface and may be arbitrarily chosen as long as the right-handedness of the reference frame is guaranteed. The chassis is represented by a rigid body whose mass m and rotational inertia matrix I resemble those of the entire vehicle, inclusive of the driver and the wheels. The body reference system $x'y'z'$ is fixed to the chassis and originates

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at the Center of Gravity. An unambiguous definition of the axes is made in the static equilibrium orientation when the suspension system is bearing the weight of the car and no other forces are at play. In this condition z' is defined to be parallel to z and likewise pointed downwards, x' is directed forwards, parallel to the road plane, and the y' axis is oriented to the right of the car as seen by the driver. The attitude of the chassis is defined by the Tait-Bryan angles mapping the inertial system axes to the chassis axes [appendix] [figura SAE J670]. Note that the given definition for the chassis reference system implies zero roll and pitch at static equilibrium. The chassis is assumed to be symmetrical about the $x'z'$ plane, this identifies the y' axis as a principle axis of inertia and eliminates all the related product of inertia terms, yielding

$$I = \begin{bmatrix} I_{xx} & 0 & I_{xz} \\ 0 & I_{yy} & 0 \\ I_{xz} & 0 & x_{zz} \end{bmatrix}.$$

[usare molteplici figure: 1. sistemi di riferimento e coordinate lagrangiane - 2. sospensioni su un solo asse - 3. vista 3d delle sospensioni su una sola ruota - 4. vista 3d completa ma senza molle]

4.3 Suspension

Suspension is represented by four vertical spring-damper systems associated with each wheel of the car. Quantities relating to each of the systems will be distinguished by subscripts according to Table 4.1, the w subscript is used when referring generically to any one of them.

Table 4.1: Wheel and suspension systems subscripts

Subscript	Pertaining wheel or suspension system
FR	Front Right
FL	Front Left
RR	Rear Right
RL	Rear Left

The upper end of each suspension system is attached to an appropriate point P_w statically defined with respect to the chassis, the lower end point, W_w, is obtained by projecting P_w onto the road plane.

The correct weight transfer effects are obtained by assigning the corresponding wheel contact point x and y coordinates to each of the P_w points, these are easily calculated from the front and rear track lengths (t_f and t_r) and the distances of the front and rear axles from the CoG.

The height of the points P_w are finally chosen in order to obtain realistic roll dynamics.

All automotive suspension systems may be characterized by a roll center for each axle. This is the point at which lateral forces acting on the wheels are reacted to the chassis. The roll center is typically below the center of mass, causing vehicles to lean towards the outside of turns when driving, this is in line with the situation shown in [image], where the lateral force at the center of mass is the centrifugal force acting in the non inertial reference frame of the car, balanced by friction acting on the wheel contact points. The roll center height is determined by suspension kinematics and dynamic load distribution so it is not always constant. The SAE definition of roll center is

"The point in the transverse vertical plane through any pair of wheel centers at which lateral forces may be applied to the sprung mass without producing suspension roll"

This definition directly relates the P_w points to the roll center height.

This can be explained by modifying a generic equilibrium situation such as the one described in figure [la stessa di prima] and applying a new side force F' acting on the midpoint M_F of P_{FR} and P_{FL}, if the tyres are capable of generating the necessary friction forces (F_1 and F_2), the system will settle in a new equilibrium state. The sum of vertical forces acting on the tyres does not change as it is equal to

$$F_z := F_{z1} + F_{z2} = mg.$$

As F_z does not change, the increase in side friction force is entirely due to changes in the lateral friction coefficients, which are both taken as equal to μ_y.

The fact that the spring systems are constrained in the vertical orientation means the horizontal forces acting at the ground contact points effect the chassis directly at the points P_{FR} and P_{FL} respectively.

The horizontal force balance is given by

$$F + F' = F_1 + F_2 = \mu_y(F_{z1} + F_{z2}) = \mu_y mg$$

To find the equilibrium roll angle the torque balance about point M_F must be solved

$$\frac{t_F}{2} \sin \phi (F_1 - F_2) - \frac{t_F}{2} \cos \phi (F_{z1} - F_{z2}) - F h_F \cos \phi = 0$$

substituting the friction forces yields

$$\frac{t_F}{2}(F_{z1} - F_{z2})[\mu_y \sin\phi - \cos\phi] - Fh_F \cos\phi = 0$$

The vertical forces act through the suspension springs. As Hooke's law is linear, the difference between them is proportional to the difference between the spring lengths which is in turn obtained as a function of the roll angle.

$$F_{z1} - F_{z2} = -kt_F \sin\phi$$

The moment balance then becomes

$$\mu_y k \frac{t_F^2}{2} \sin^2\phi - k\frac{t_F^2}{2}\sin\phi\cos\phi - Fh_F\cos\phi = 0$$

[find source for roll angles in passenger cars being a few ° max] The equation may be linearized with respect to ϕ.

$$-k\frac{t_F^2}{2}\phi - Fh_F = 0$$

Doing so makes the solution independent of μ_y, the only quantity influenced by F'. The height of point M_F is then effectively the roll center height according to the SAE definition.

The roll rate K_ϕ is defined as roll angle per unit of lateral force applied to the CG, applying this definition to the linearized equation gives

$K_\phi = \frac{\phi}{F} = \frac{2h}{Kt_F^2}$

The spring rates of the suspension model can be derived in order to match the roll-rate of the car.

can be by needed in the equation above, so

$$F_{z1} - F_{z2}$$

Table 4.2: Coordinates of the suspension force application points

Chassis system co-ordinate	$P_F R$	$P_F L$	$P_R R$	$P_R L$
x	l_f	l_f	$-l_r$	$-l_r$
y	$\frac{t_f}{2}$	$-\frac{t_f}{2}$	$\frac{t_r}{2}$	$-\frac{t_r}{2}$
z	$h_{CG} - q_f$	$h_{CG} - q_f$	$h_{CG} - q_r$	$h_{CG} - q_r$

4.4   Limitations

While this model benefits from the simplicity of it's description The main limitation of this model is the lack of the wheel rotational dynamics, which leads to missing reaction torques present during acceleration and braking.

4.5   6DoF Dynamics equation set

[districare il groviglio che mi lascia Matlab]

Chapter 5

The twelve degree-of-freedom Model

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5.5   12DoF Dynamics equation set

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Appendix A

Appendix A - Math

Appendix B

Appendix B - 6DoF Matlab Code

Appendix C

Appendix C - 12DoF Matlab code

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