$$\sum_{n\geq 1} \underbrace{x^n \cdot \left(1+\frac{1}{n}\right)^{n^2+n}}_{a_n} \qquad x>0$$

$$\sqrt[n]{a_n} = x\left(1+\frac{1}{n}\right)^{n+1} \longrightarrow x\cdot e$$

c1) $x\cdot e>1 \iff x>\frac{1}{e}$ div

c2) $x\cdot e<1 \iff x<\frac{1}{e}$ conv

c3) $x\cdot e=1 \quad x=\frac{1}{e}$.

$$\sum_{n\geq 1} \frac{1}{e^n}\cdot\left(1+\frac{1}{n}\right)^{n^2+n} =$$

$$= \sum_{n\geq 1}\left[\frac{\left(1+\frac{1}{n}\right)^{n+1}}{e}\right]^n \geq \sum_{n\geq 1} 1 = \infty$$

div

$$\left(1+\frac{1}{n}\right)^{n+1} \searrow e \quad \nearrow \left(1+\frac{1}{n}\right)^{n}$$

$$\frac{\left(1+\frac{1}{n}\right)^{n+1}}{e} \geq$$

$$\sum_{n\geq 1} x^n \cdot \frac{1}{n^\alpha} \qquad x > 0$$

$$\frac{a_{n+1}}{a_n} = \frac{\cancel{x^{n+1}}^{\,x}}{(n+1)^\alpha} \cdot \frac{n^\alpha}{\cancel{x^n}}$$

$$= x \cdot \frac{n^\alpha}{(n+1)^\alpha} \Rightarrow x$$

(1) $x > 1$ n div

(2) $x < 1$ n conv

(3) $x = 1$ $\displaystyle\sum_{n\geq 1} \frac{1}{n^\alpha}$ $\quad \alpha > 1$ conv, $\alpha \leq 1$ div

$$\sum_{n\geq 1} x^n \cdot \sin\frac{1}{\sqrt{n}} \qquad x>0$$

$$\frac{a_{n+1}}{a_n} = \frac{\cancel{x^{n+1}} \cdot \sin\frac{1}{\sqrt{n+1}}}{\cancel{x^n}\cdot \sin\frac{1}{\sqrt{n}}}$$

$$\frac{\sin x}{x} \xrightarrow[x\to 0]{} 1 \qquad \lim_{n\to\infty} \frac{\sin\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = 1 \qquad \sqrt{\frac{n}{n+1}} \to 1$$

$$\frac{a_{n+1}}{a_n} = x \cdot \frac{\sin\frac{1}{\sqrt{n+1}}}{\frac{1}{\sqrt{n+1}}} \cdot \frac{\frac{1}{\sqrt{n}}}{\sin\frac{1}{\sqrt{n}}} \cdot \frac{\frac{1}{\sqrt{n+1}}}{\frac{1}{\sqrt{n}}} \to x$$

$\downarrow 1 \qquad \downarrow 1$

$$\frac{a_{n+1}}{a_n} \to x$$

(

c1) $x > 1$ n. div

c2) $x < 1$ n. conv

c3) $x = 1$

$$\sum_{n\geq 1} \sin\frac{1}{\sqrt{n}} \sim \sum_{n\geq 1} \frac{1}{\sqrt{n}} \quad \text{div}$$

$$\sum_{n\geq 1} \frac{1}{n^{\frac{1}{2}}}$$

$\alpha = \frac{1}{2} \leq 1$ div

$$\frac{\sin\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} \to 1$$

$x > 0$

$$\sum_{n\geq 1} x^n \frac{\sqrt{n+1}-\sqrt{n}}{n} = \sum_{n\geq 1} x^n \frac{1}{n(\sqrt{n+1}+\sqrt{n})} = a_n$$

$$\frac{a_{n+1}}{a_n} = \frac{\cancel{x^{n+1}}}{\cancel{x^n}} x \cdot \frac{n \,(\sqrt{n+1}+\sqrt{n})}{(n-1)(\sqrt{n+2}+\sqrt{n+1})} \to x$$

$n/(n-1) \to 1$, $\frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+2}+\sqrt{n+1}} \to 1$

c1) $x > 1$ n div c2) $x < 1$ n conv c3) $\underline{x = 1}$ conv

$$\sum_{n\geq 1} \frac{1}{\underbrace{n(\sqrt{n+1}+\sqrt{n})}_{b_n}} \sim \sum_{n\geq 1} \frac{1}{\underbrace{n\sqrt{n}}_{c_n \text{ conv}}} \qquad \frac{b_n}{c_n} \to \frac{1}{2}$$

$$\sum_{n\geq 1}\frac{x^n}{\sqrt[n]{C_{2n}^n}} \qquad a_n=\frac{x^n}{\sqrt[n]{C_{2n}^n}}\ ;\ \sqrt[n]{a_n}=\frac{x}{\left(C_{2n}^n\right)^{\frac{1}{n^2}}}$$

$$1\leq\left(C_{2n}^n\right)^{\frac{1}{n^2}}=\left(\frac{(2n)!}{(n!)^2}\right)^{\frac{1}{n^2}}\leq\left(\frac{(n+1)(n+2)\cdots(2n)}{1\cdot 2\cdots n}\right)^{\frac{1}{n^2}}$$

$$\frac{n+k}{k}\leq n$$

$$\leq\left(n^n\right)^{\frac{1}{n^2}}=\sqrt[n]{n}\to 1 \qquad \sqrt[n]{a_n}\to x$$

(1) $x>1$ s. div (2) $x<1$ s. conv (3) $x=1$

$$\sum_{n\geq 1}\frac{1}{\sqrt[n]{C_{2n}^n}}\sim\sum_{n\geq 1}\frac{1}{4}\ \text{div}$$

$$\lim_{n\to\infty}\sqrt[n]{x_n}=\lim_{n\to\infty}\frac{x_{n+1}}{x_n} \qquad \lim_{n\to\infty}\sqrt[n]{C_{2n}^n}=\lim_{n\to\infty}\frac{C_{2n+2}^{n+1}}{C_{2n}^n}=$$

$$=\lim_{n\to\infty}\frac{\overset{(2n+1)(2n+2)}{\cancel{(2n+2)!}}}{\underset{(n+1)^2}{\cancel{\left((n+1)!\right)^2}}}\cdot\frac{\cancel{(n!)^2}}{\cancel{(2n)!}}=\lim_{n\to\infty}\frac{(2n+1)(2n+2)}{(n+1)^2}=4$$

$\sum x_n$ conv $\Rightarrow x_n\to 0$

$x_n\not\to 0 \Rightarrow \sum x_n$ div

$x>0$ $\displaystyle\sum_{n\geq 1} x^n \cdot \frac{\sqrt{n+1}}{n+4}$ $\quad \displaystyle\sum_{n\geq 1} x^n \cdot \ln\left(1+\frac{1}{n}\right)$ $\quad \displaystyle\sum_{n\geq 1} x^n \operatorname{arctg}\frac{1}{\sqrt{n}}$

$\displaystyle\sum_{n\geq 1} x^n \operatorname{tg}\frac{1}{n^2}$ $\quad \displaystyle\sum_{n\geq 1} x^n \frac{\sqrt{n^2+1}-n}{n}$ $\quad \displaystyle\sum_{n\geq 1} x^n \cdot \frac{n^3+2}{n^6+1}$

$\displaystyle\sum_{n\geq 1} x^n \sin\frac{1}{n^\alpha}$ $\quad x>0, \alpha>0$ $\quad \displaystyle\sum_{n\geq 1} x^n \sin^2 \cdot \frac{1}{\sqrt{n+1}}$

$\displaystyle\sum_{n\geq 1} x^n \ln\left(1+\frac{1}{n^\alpha}\right)$ $\quad \displaystyle\sum_{n\geq 1} x^n \operatorname{tg}^{\cancel{\alpha}}\frac{1}{\sqrt{n}}$

$\alpha>0 \quad x>0$

$\displaystyle\sum_{n\geq 1} x^n \left(1-\cos\frac{1}{n^\alpha}\right)$

$\alpha>0$