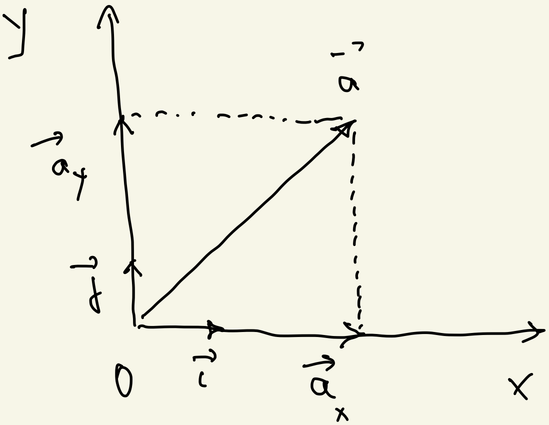
8 Decembrie 2021.

$$\vec{a} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k}$$ (scrierea analitică)

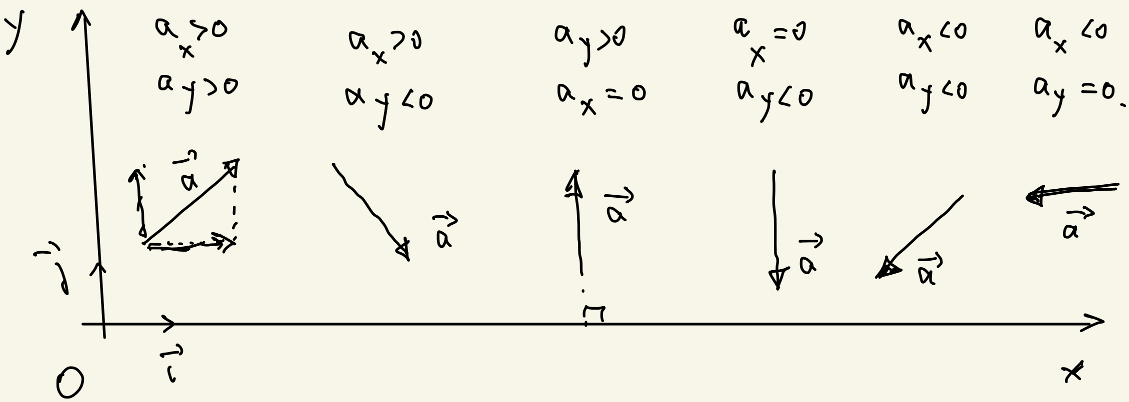
Fie un vector $\vec{a}$ situat în planul Oxy. (



$\vec{a}_x = a_x \vec{i}$

$a_x > 0$

$a_y > 0$



Unghiul dintre doi vectori

$$\vec{a}\vec{b} = a\,b\cos\alpha \Rightarrow \cos\alpha = \frac{\vec{a}\vec{b}}{a\,b}$$

$$\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$$

$$\vec{b} = b_x\vec{i} + b_y\vec{j} + b_z\vec{k}$$

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \quad ; \quad |\vec{b}| = \sqrt{b_x^2 + b_y^2 + b_z^2}$$

$$\vec{a}\vec{b} = (a_x\vec{i} + a_y\vec{j} + a_z\vec{k})(b_x\vec{i} + b_y\vec{j} + b_z\vec{k})$$

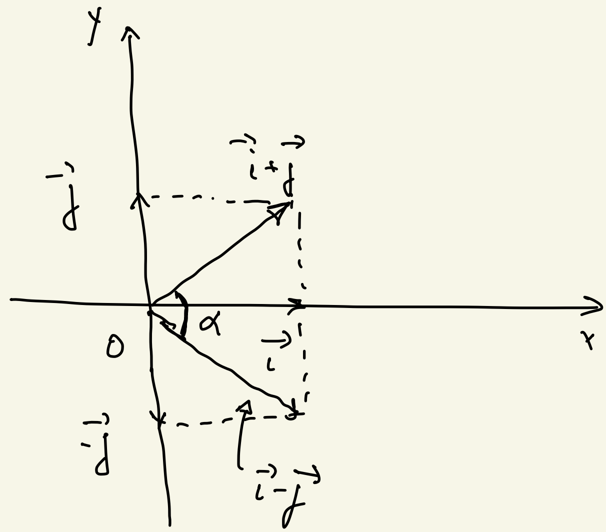
$$\vec{i}\vec{j} = \vec{j}\vec{k} = \vec{i}\vec{k} = 0$$

$$\vec{i}\cdot\vec{i} = \vec{i}^{\,2} = \underbrace{|\vec{i}|}_{1}\cdot\underbrace{|\vec{i}|}_{1}\underbrace{\cos 0}_{1} = 1 = \vec{j}^{\,2} = \vec{k}^{\,2}$$

$$\vec{a}\vec{b} = a_x b_x + a_y b_y + a_z b_z$$

$$\boxed{\cos\alpha = \frac{a_x b_x + a_y b_y + a_z b_z}{\sqrt{a_x^2 + a_y^2 + a_z^2}\,\sqrt{b_x^2 + b_y^2 + b_z^2}}}$$

Ex.



$\vec a = \vec i + \vec j + 0 \cdot \vec k$

$\vec b = \vec i - \vec j + 0 \cdot \vec k$

$a_x = 1;\ a_y = 1;\ a_z = 0$

$b_x = 1;\ b_y = -1;\ b_z = 0$

$$\cos\alpha = \frac{1\cdot 1 + 1\cdot(-1) + 0\cdot 0}{\sqrt{1^2+1^2+0^2}\cdot\sqrt{1^2+(-1)^2+0^2}} = \frac{0}{\sqrt{2}\,\sqrt{2}} = 0 \Rightarrow$$

$$\Rightarrow \alpha = 90^\circ.$$

Fie trei vectori în planul Oxy: $\vec a, \vec b, \vec c$, care satisfac relația: $\vec b + \vec c = \vec a$. Ce relație există între proiecțiile lor?

$$b_x \vec i + b_y \vec j + c_x \vec i + c_y \vec j = a_x \vec i + a_y \vec j$$

$$\vec{i}\,(b_x + c_x - a_x) + \vec{j}\,(b_y + c_y - a_y) = \vec{0} = 0$$

$$A\vec{i} + B\vec{j} = 0 \implies \begin{cases} A = 0 \\ B = 0 \end{cases}$$

$$\Rightarrow \begin{cases} b_x + c_x - a_x = 0 \\ b_y + c_y - a_y = 0 \end{cases} \Rightarrow \begin{cases} b_x + c_x = a_x \\ b_y + c_y = a_y \end{cases}$$

$$\vec{b} + \vec{c} = \vec{a}$$

Aplicatie

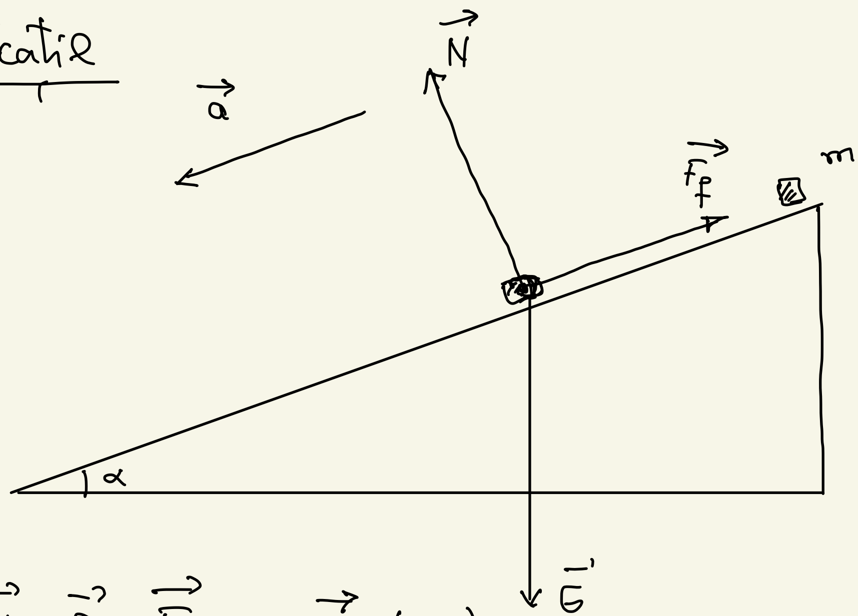
α

m

g

μ

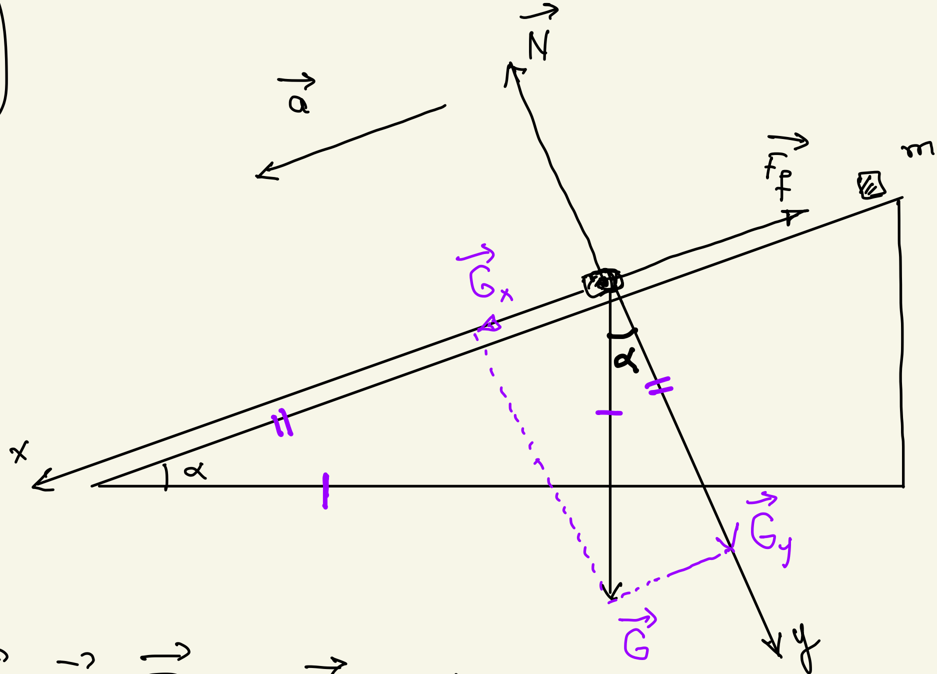
$a = |\vec{a}| = ?$



$$\vec{N} + \vec{G} + \vec{F_f} = m\vec{a} \quad (\text{Pr. II})$$

$$F_f = \mu N \quad (\text{Legea frecării})$$

I)



$$\vec{N} + \vec{G} + \vec{F_f} = m\vec{a} \quad (P_{r.}\,\underline{II})$$

$$F_f = \mu N \quad (\text{Legea frecării})$$

$$\begin{cases} 0 + G_x + (-F_f) = ma & (Ox) \\ -N + G_y + 0 = 0 & (Oy) \\ F_f = \mu N. \end{cases} \Rightarrow \begin{cases} G_x - F_f = ma \\ \boxed{N = G_y} \\ F_f = \mu N \end{cases}$$

$$\begin{cases} G_x - F_f = ma \\ \boxed{F_f = \mu G_y} \end{cases} \longrightarrow \left.\begin{array}{l} G_x - \mu G_y = ma \\ G_x = G\sin\alpha \\ G_y = G\cos\alpha \end{array}\right\} \Rightarrow$$

$$\Rightarrow G\sin\alpha - \mu G\cos\alpha = ma$$

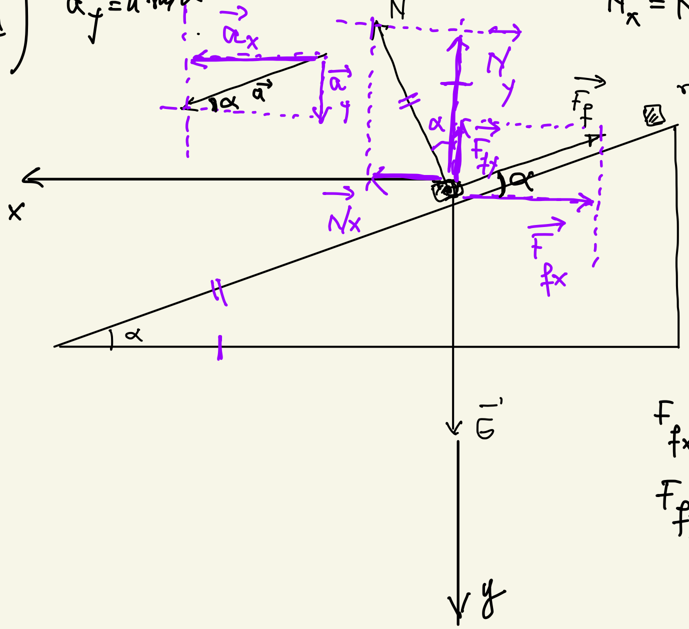
$$mg\sin\alpha - \mu\, mg\cos\alpha = ma$$

$$\boxed{g(\sin\alpha - \mu\cos\alpha) = a}$$

II)

$a_x = a\cos\alpha$ $\qquad N_y = N\cos\alpha$

$a_y = a\sin\alpha$ $\qquad N_x = N\sin\alpha$



$\vec{N} + \vec{G} + \vec{F_f} = m\vec{a}$ (Pr. II)

$F_f = \mu N$ (Legea frecării)

$F_{fx} = F_f \cdot \cos\alpha$

$F_{fy} = F_f \sin\alpha$

$$\begin{cases} N_x + 0 + (-F_{fx}) = m a_x & (Ox) \\ -N_y + G + (-F_{fy}) = m a_y & (Oy) \\ F_f = \mu N \end{cases} \Rightarrow \begin{cases} N_x - F_{fx} = m a_x \\ -N_y + G - F_{fy} = m a_y \\ F_f = \mu \cdot N \end{cases}$$

$$\begin{cases} N\sin\alpha - F_f\cos\alpha = m \cdot a\cos\alpha \\ -N\cos\alpha + G - F_f\sin\alpha = m \cdot a\sin\alpha \\ \boxed{F_f = \mu N} \end{cases} \Rightarrow \begin{cases} N\sin\alpha - \mu N\cos\alpha = m a\cos\alpha \\ -N\cos\alpha + G - \mu N\sin\alpha = m a\sin\alpha \end{cases}$$

$$\begin{cases} N(\sin\alpha - \mu\cos\alpha) = ma\cos\alpha \\ -N(\cos\alpha + \mu\sin\alpha) = ma\sin\alpha - G \end{cases}$$

$$\frac{\sin\alpha - \mu\cos\alpha}{-(\cos\alpha + \mu\sin\alpha)} = \frac{ma\cos\alpha}{ma\sin\alpha - G} \quad ; G = mg$$

$$\frac{\mu\cos\alpha - \sin\alpha}{\cos\alpha + \mu\sin\alpha} = \frac{a\cos\alpha}{a\sin\alpha - g} \quad \Big| \; \frac{1}{(\;)}$$

$$\frac{\cos\alpha + \mu\sin\alpha}{\mu\cos\alpha - \sin\alpha} = \frac{a\sin\alpha - g}{a\cos\alpha} = \frac{a\sin\alpha}{a\cos\alpha} - \frac{g}{a\cos\alpha}$$

$$\frac{\cos\alpha + \mu\sin\alpha}{\mu\cos\alpha - \sin\alpha} = tg\alpha - \frac{g}{a\cos\alpha}$$

$$\frac{g}{a\cos\alpha} = tg\alpha - \frac{\cos\alpha + \mu\sin\alpha}{\mu\cos\alpha - \sin\alpha} = \frac{\sin\alpha}{\cos\alpha} - \frac{\cos\alpha + \mu\sin\alpha}{\mu\cos\alpha - \sin\alpha}.$$

$$\frac{g}{a\cos\alpha} = \frac{\cancel{\mu\sin\alpha\cos\alpha} - \sin^2\alpha - \cos^2\alpha - \cancel{\mu\sin\alpha\cos\alpha}}{\cos\alpha(\mu\cos\alpha - \sin\alpha)}$$

$$\frac{g}{a\cos\alpha} = \frac{-1}{\cos\alpha(\mu\cos\alpha - \sin\alpha)}$$

$$\frac{g}{a} = \frac{-1}{\mu\cos\alpha - \sin\alpha}$$

$$\frac{g}{a} = \frac{1}{\sin\alpha - \mu\cos\alpha} \Rightarrow \boxed{a = g(\sin\alpha - \mu\cos\alpha)}$$

Concluzie: Relațiile de tip $\vec{b} + \vec{c} = \vec{a}$ sunt valide în orice sistem de coordonate (sunt independente de sistemul de coordonate).