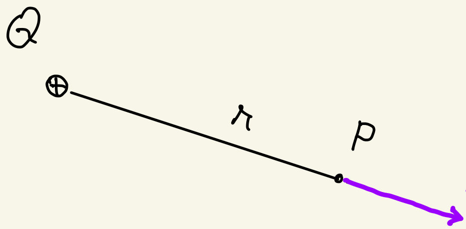
BAZELE ELECTROTEHNICII

16 Februarie 2022

Reprezentarea scalară a câmpului electrostatic.

Reprezentarea câmpului electrostatic
- → vectorială $\vec{E}$ $\left(\frac{N}{C}\right)$
- → scalară V (V) ← potențial electric



$$\vec{E} = \frac{kQ}{r^3}\vec{r}$$

Anexă matematică

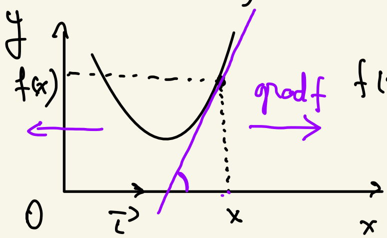
$f(x, y, z)$

$$\left.\begin{aligned} x &\to x + dx \\ y &\to y + dy \\ z &\to z + dz \end{aligned}\right| \Rightarrow df = ?$$

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$$

Operatorul gradient :
$$\operatorname{grad} f(x,y,z) \overset{def.}{=} \frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j} + \frac{\partial f}{\partial z}\vec{k}$$

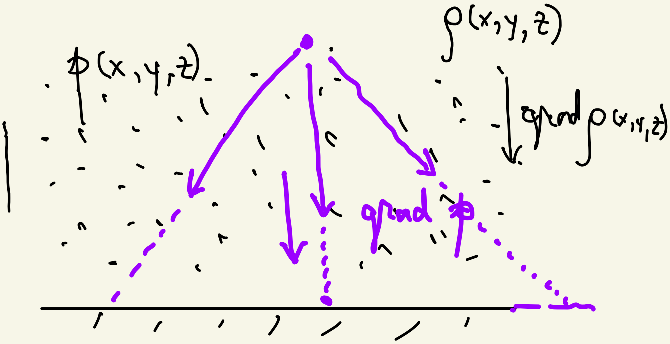
Ex.

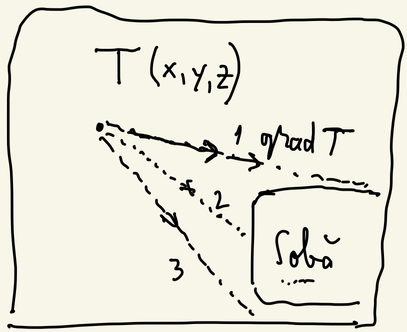


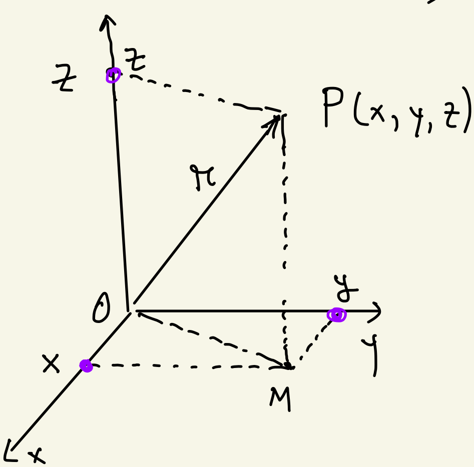
$f(x) = ax^2 + bx + c$

$$\operatorname{grad} f(x) = \frac{df}{dx}\vec{i} = f'(x)\cdot\vec{i}$$

Gradientul unei funcții este un vector orientat în sensul în care funcția crește cel mai rapid.







$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$r = \sqrt{x^2+y^2+z^2} = r(x,y,z)$$

$$\operatorname{grad} r = ??$$

$$\operatorname{grad} r = \frac{\partial r}{\partial x}\vec{i} + \frac{\partial r}{\partial y}\vec{j} + \frac{\partial r}{\partial z}\vec{k}$$

$$\frac{\partial r}{\partial x} = \frac{\partial}{\partial x}\sqrt{x^2+y^2+z^2} = \frac{\partial}{\partial x}(x^2+y^2+z^2)^{1/2} =$$

$$= \frac{1}{2}(x^2+y^2+z^2)^{\frac{1}{2}-1}\cdot(x^2+y^2+z^2)' = \frac{1}{2}(x^2+y^2+z^2)^{-1/2}\cdot 2x$$

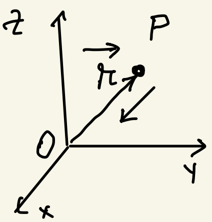
$$= \frac{x}{\sqrt{x^2+y^2+z^2}} = \frac{x}{r}$$

$$\operatorname{grad} r = \frac{x}{r}\vec{i} + \frac{y}{r}\vec{j} + \frac{z}{r}\vec{k} = \frac{x\vec{i}+y\vec{j}+z\vec{k}}{r} = \frac{\vec{r}}{r}$$

$$\operatorname{grad} r = \frac{\vec{r}}{r} \qquad |\operatorname{grad} r| = \left|\frac{\vec{r}}{r}\right| = \frac{|\vec{r}|}{r} = \frac{r}{r} = 1$$

$\frac{\vec{r}}{r} \to$ versorul direcției $\vec{r}$

$$\operatorname{grad}\frac{1}{r} = ?$$



$$\operatorname{grad}\frac{1}{r} = \operatorname{grad}\frac{1}{\sqrt{x^2+y^2+z^2}} = \frac{\partial}{\partial x}\frac{1}{\sqrt{x^2+y^2+z^2}}\vec{i} + \frac{\partial}{\partial y}\frac{1}{\sqrt{\ }}\vec{j} + \frac{\partial}{\partial z}\frac{1}{\sqrt{\ }}\vec{k}$$

$$\frac{\partial}{\partial x}\frac{1}{r} = \frac{\partial}{\partial x}\frac{1}{\sqrt{x^2+y^2+z^2}} = \frac{\partial}{\partial x}(x^2+y^2+z^2)^{-1/2} =$$

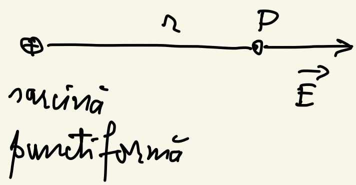
$$= -\frac{1}{2}\left(x^2+y^2+z^2\right)^{-\frac{3}{2}} \cdot 2x = -\frac{x}{(x^2+y^2+z^2)^{3/2}} = -\frac{x}{r^3}$$

$$\text{grad}\,\frac{1}{r} = -\frac{x}{r^3}\vec{i} - \frac{y}{r^3}\vec{j} - \frac{z}{r^3}\vec{k} = -\frac{x\vec{i}+y\vec{j}+z\vec{k}}{r^3} = -\frac{\vec{r}}{r^3}$$

$$\text{grad}\,\frac{1}{r} = -\frac{1}{r^3}\vec{r} \quad ; \quad \left|\text{grad}\,\frac{1}{r}\right| = \frac{1}{r^2}$$

Obs.

$$\vec{E} = \frac{kQ}{r^3}\vec{r} \quad ,$$



$$\vec{E}(\vec{r}) = -kQ\left(-\frac{\vec{r}}{r^3}\right)$$

$$\vec{E}(\vec{r}) = -kQ\,\text{grad}\,\frac{1}{r}$$

$$(3x^2)' = 3(x^2)'$$

$$\vec{E}(\vec{r}) = -\,\text{grad}\left(\frac{kQ}{r}\right)$$

$$(3x^2)' = 6x$$

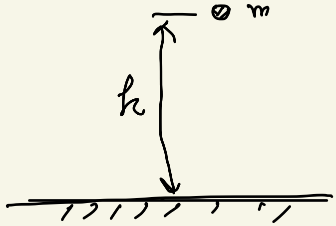
$$(3x^2+1)' = 6x$$

$$(3x^2+C)' = 6x.$$

Se definește funcția POTENȚIAL ELECTRIC al unei sarcini punctiforme prin relația

$$\boxed{V(r) = \frac{kQ}{r} + \text{const.}}$$

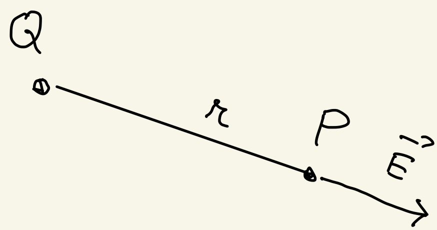
$$\boxed{\vec{E}_P = -\operatorname{grad} V_P}$$



Dacă $r \to \infty \Rightarrow V(r) \to \text{const.}$

<u>Convenim</u> ca la ∞ potențialul unei sarcini punctiforme să fie nul. Cu această convenție putem scrie că $V(r) = \dfrac{kQ}{r}$.

$$\boxed{\begin{gathered} V(r) = \frac{kQ}{r} \\ \vec{E} = \frac{kQ}{r^3}\vec{r} \\ \vec{E} = -\operatorname{grad} V \end{gathered}}$$



<u>Obs</u> $\vec{E} = \text{grad } V$

$$|\vec{E}| = |\text{grad } V| = \left| \frac{\partial V}{\partial x}\vec{i} + \frac{\partial V}{\partial y}\vec{j} + \frac{\partial V}{\partial z}\vec{k} \right|$$

$$\langle E \rangle_{SI} = \frac{N}{C} = \frac{V}{m}$$