I. Exerciții

1. Să se studieze natura următoarelor serii:

(a)
$$\sum_{n=1}^{\infty} \frac{1}{n}(\sqrt{n^2+n+1} - \sqrt{n^2-n-1}).$$

(b)
$$\sum_{n=1}^{\infty} \left(\frac{-2n+a}{-2n+b}\right)^{-2n}, a, b \in \mathbb{R}$$

(c)
$$\sum_{n=1}^{\infty} \frac{\ln n}{n^3} x^n, x \in (0, \infty).$$

(d)
$$\sum_{n=1}^{\infty} \frac{a^n \cdot n!}{n^n}, a > 0.$$

(e)
$$\sum_{n=1}^{\infty} \left(\frac{xn^2+7n+8}{n^2+5n+2}\right)^n, x \in (0, \infty).$$

(f)
$$\sum_{n=1}^{\infty} \frac{n! \cdot (n+3)!}{(2n+1)! x^n}, x \in (0, \infty).$$

(g)
$$\sum_{n=1}^{\infty} 4^n \cdot \tan\left(\frac{n^2+1}{4^n(n^3+5)}\right).$$

2. Să se studieze convergența și absolut convergența următoarelor serii:

(a)
$$\sum_{n=1}^{\infty} (-1)^n \cdot \frac{1}{n \cdot 2^n}.$$

(b)
$$\sum_{n=1}^{\infty} (-1)^n \cdot \frac{1}{n - \ln n}.$$

(c)
$$\sum_{n=1}^{\infty} x^n \cdot \arctan \frac{1}{n^\alpha}, \alpha \in \mathbb{R}.$$