24 Noiembrie 2021

Vectori. Operații cu vectori.
Scrierea analitică a vectorilor

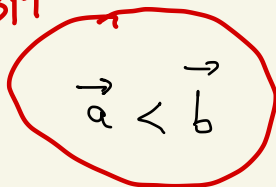**Vectori**

vârful

B

A

punct de aplicație
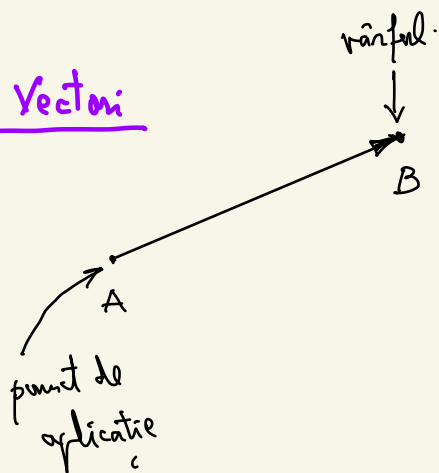
$\overrightarrow{AB}, \vec{a}, \vec{b}, \overrightarrow{MN} \ldots,$

$|\overrightarrow{AB}|, AB, |\vec{a}|, a, |\overrightarrow{MN}|, \ldots$

vector $\begin{cases} - \text{direcție} \\ - \text{sens} \\ - \text{modul} \end{cases}$

scalar $\}$ - număr real.

GREȘIT

$\vec{a} < \vec{b}$

1,2   5,6

1,2 < 5,6

$\vec{a}$

$\vec{b}$

$|\vec{a}| > |\vec{b}|$ 😊

$\vec{a} > \vec{b}$ ☹
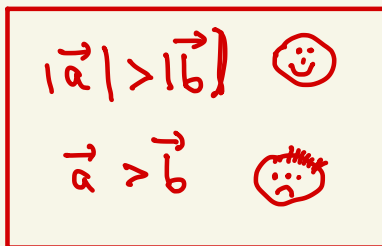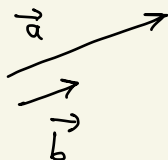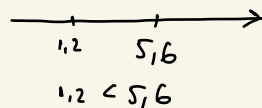
$\vec{F}$, $\vec{F}$

scalar   vector

Operații cu vectori

1) Adunarea vectorilor

$\vec{a}$ $\vec{c} = \vec{a} + \vec{b}$ $\vec{b}$

$\vec{a}$ $\vec{b}$ $\vec{c'} = \vec{a} + \vec{b}$ $\vec{b}$

$\vec{a}$ $\vec{b}$ $\vec{a} + \vec{b} + \vec{c}$ $\vec{c}$

suma vectorilor

$\vec{a}$ $\vec{b}$ $\vec{c}$

$\vec{a}$ $\vec{b}$ $\vec{c}$ $\vec{a} + \vec{b} + \vec{c}$
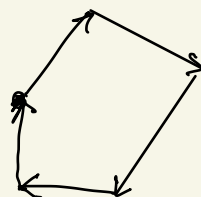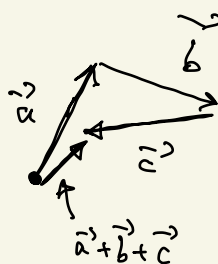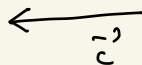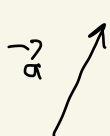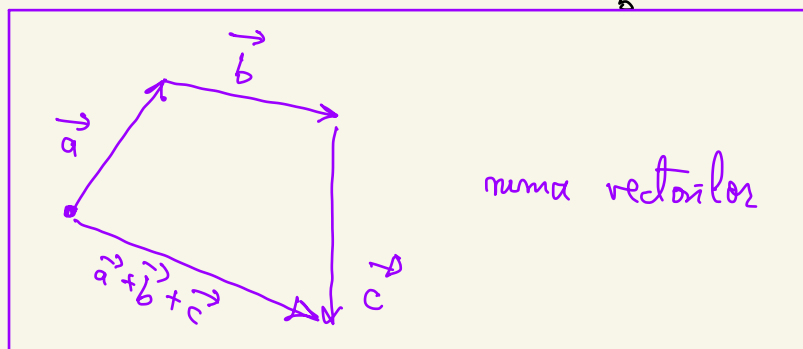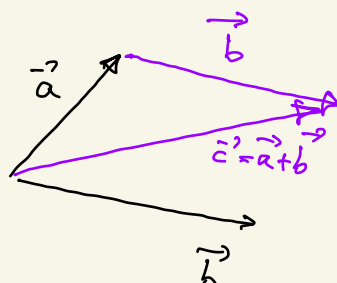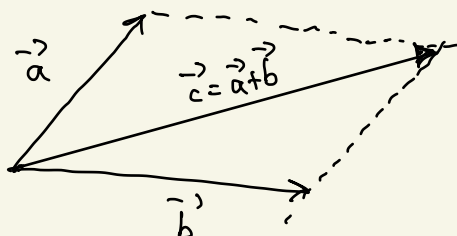
2) <u>Scăderea vectorilor</u>

$$\vec{a} + \vec{o} = \vec{o} + \vec{a} = \vec{a}$$

↑ vector nul.

$\vec{o}$, o   se acceptă ambele
scrieri.

$-\vec{b}$ ← opusul lui $\vec{b}$

$$\vec{b} + (-\vec{b}) = \vec{o} = \varnothing.$$

$$\vec{a} - \vec{b} \overset{def.}{=} \vec{a} + (-\vec{b})$$

$$\vec{a} - \vec{b} \neq \vec{b} - \vec{a}$$

$$\left|\vec{a}-\vec{b}\right| = \sqrt{a^2 + b^2}$$
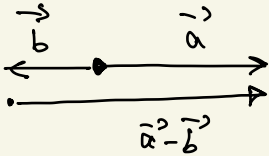
$$\left|\vec{a}-\vec{b}\right| = a + b$$
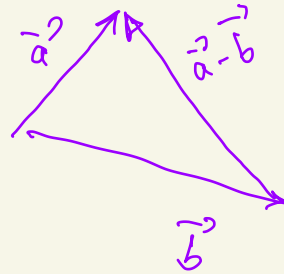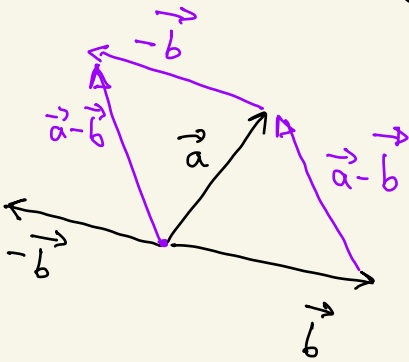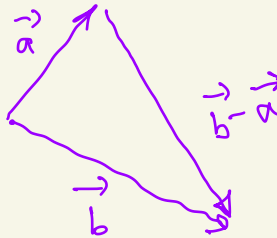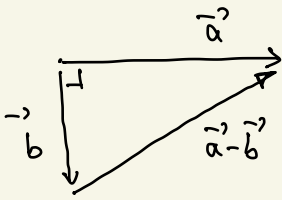

$$\left|\vec{a}-\vec{b}\right| = a - b$$

$$\left|\vec{a}-\vec{b}\right| = b - a$$

$$\vec{a}-\vec{b} = \vec{c} \implies c = a - b \leftarrow$$

$$\left|\vec{a}-\vec{b}\right| = \left|\vec{c}\right| \leftarrow$$
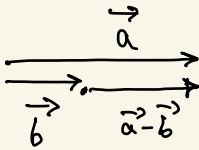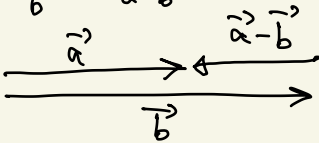
3) Înmulțirea vectorilor cu scalari

$$\alpha \in \mathbb{R}$$
$$\vec{a} \in V$$

$$\alpha \cdot \vec{a} = \alpha\,\vec{a} \overset{def.}{=} \text{un vector } \vec{b}$$

$$\boxed{\vec{b} = \alpha\,\vec{a}}$$

$\vec{b}$
$\begin{cases} \text{aceeași direcție cu } \vec{a} \\ \text{sensul} \begin{cases} \alpha > 0 \text{ același} \\ \alpha < 0 \text{ opus} \end{cases} \\ \left|\vec{b}\right| = |\alpha| \cdot \left|\vec{a}\right| \end{cases}$

$$\xrightarrow{\hspace{6cm}} \vec{a}$$

$$\xrightarrow{\hspace{1cm}} \quad \frac{1}{10}\vec{a}$$

$$\xrightarrow{\hspace{9cm}}$$
$$\frac{5}{4}\vec{a}$$

$$\xleftarrow{\hspace{1cm}} \quad -\frac{1}{10}\vec{a}$$

4) Împărțirea unui vector la un număr real.

$$\frac{\vec{a}}{2} = \frac{1}{2}\vec{a}$$

$$0,01\,\vec{a} = \frac{\vec{a}}{\frac{1}{0,01}} = \frac{\vec{a}}{100} \ .$$
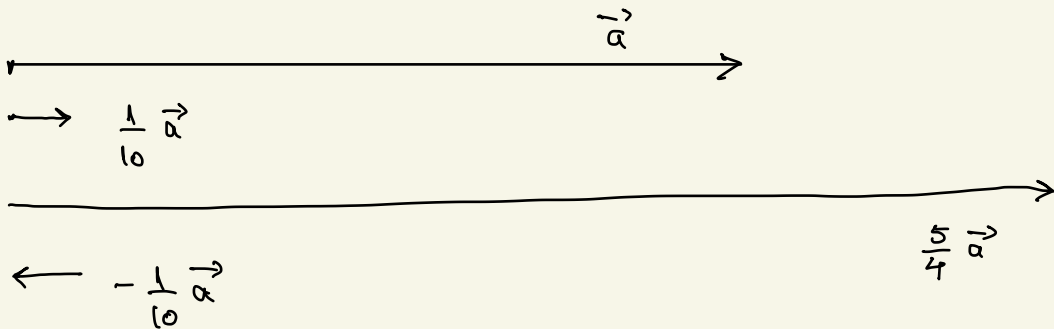
$$100\,\vec{a} = \frac{\vec{a}}{\frac{1}{100}} = \frac{\vec{a}}{0,01}$$

$$\vec{v} = \frac{d\vec{n}}{dt}$$

$$\Delta\vec{n}$$

$$\vec{v}_m = \frac{\Delta\vec{n}}{\Delta t} = \frac{1}{\Delta t}\cdot\Delta\vec{n}$$

$$\vec{v} = \lim_{\Delta t\to 0}\vec{v}_m = \lim_{\Delta t\to 0}\frac{\Delta\vec{n}}{\Delta t} = \frac{d\vec{n}}{dt} = \dot{\vec{n}}$$
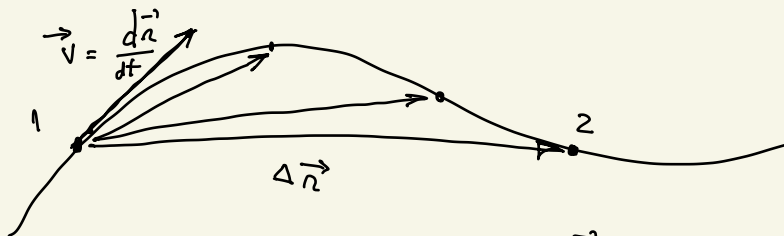
5) Produsul scalar a doi vectori.

$$\vec{a} \cdot \vec{b} = \vec{a} \vec{b} \overset{def.}{=} \text{un numar real.} \longrightarrow \vec{a}\vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos(\widehat{\vec{a},\vec{b}})$$

$$\boxed{\vec{a}\vec{b} = a\,b\,\cos\alpha}$$

$\vec{a}$

$\alpha \leq 180°$
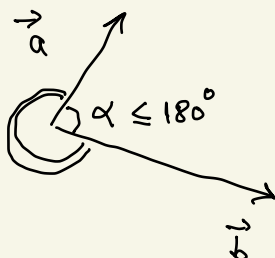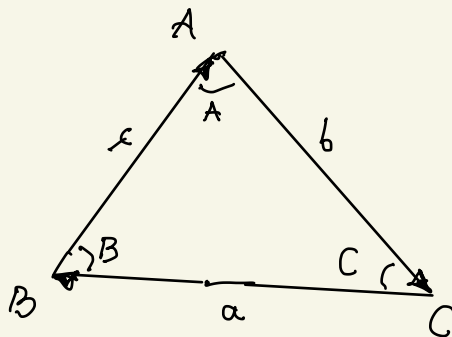
$\vec{b}$

Obs.

$\alpha = 0 \implies \overset{\vec{a}}{\underset{\vec{b}}{\rightrightarrows}} \implies \vec{a}\vec{b} = a b \cdot \underset{1}{\cos 0} = ab$

$\alpha = 90° \implies \overset{\vec{a}}{\underset{\vec{b}}{\llcorner}} \implies \vec{a}\vec{b} = a b \cdot \underset{0}{\cos 90°} = 0.$

$\alpha = 180° \implies \overset{\vec{b}}{\leftarrow} \cdot \overset{\vec{a}}{\rightarrow} \implies \vec{a}\vec{b} = ab\underset{-1}{\cos 180} = -ab$

$\alpha = 45° \implies \overset{\vec{a}}{\underset{\vec{b}}{\searrow}} 45° \implies \vec{a}\vec{b} = ab\cos 45° = ab\dfrac{\sqrt{2}}{2}$

A

c $\widehat{A}$ b

$\widehat{B}$ C $($

B a C

A

c $\widehat{A}$ b

$\widehat{B}$ C $($

B a C

$$\overrightarrow{BA} + \overrightarrow{AC} + \overrightarrow{CB} = 0 \quad \Rightarrow \quad \overrightarrow{BA} + \overrightarrow{AC} = -\overrightarrow{CB} \quad \Big|^2$$

$$\left(\overrightarrow{BA} + \overrightarrow{AC}\right)^2 = \left(-\overrightarrow{CB}\right)^2 \quad \Rightarrow \quad \overrightarrow{BA}^2 + \overrightarrow{AC}^2 + 2\,\overrightarrow{BA}\cdot\overrightarrow{AC} = \overrightarrow{CB}^2$$

<u>obs.</u> $\quad \overrightarrow{a}^2 = \overrightarrow{a}\cdot\overrightarrow{a} = a \cdot a \cdot \underbrace{\cos 0}_{1} = a^2$

$$\boxed{\overrightarrow{a}^{\textcolor{red}{2}} = a^2}$$

$$c^2 + b^2 + 2cb\,\cos\left(\overrightarrow{BA}, \overrightarrow{AC}\right) = a^2$$

$$\left(\overset{\frown}{\overrightarrow{BA}, \overrightarrow{AC}}\right) = ? \quad = \pi - A$$

$$\left(\overset{\frown}{\overrightarrow{AC}, \overrightarrow{CB}}\right) = ? \quad = \pi - C$$
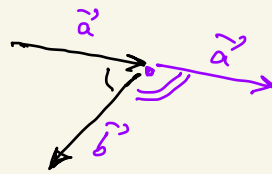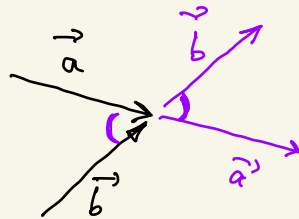
$$c^2 + b^2 + 2cb\,\cos\left(\pi - A\right) = a^2$$

$$c^2 + b^2 + 2cb\left(-\cos A\right) = a^2$$

$$c^2 + b^2 - a^2 = 2bc\cos A \quad \Rightarrow \quad \boxed{\cos A = \frac{b^2 + c^2 - a^2}{2bc}}$$

Scrierea analitică a vectorilor

O ———→————— A ————————→
$\vec{i}$ x   x
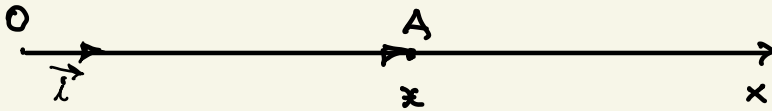
vector unitar $(|\vec{i}|=1)$
(versor al axei).

$$\vec{OA} = x\,\vec{i}$$
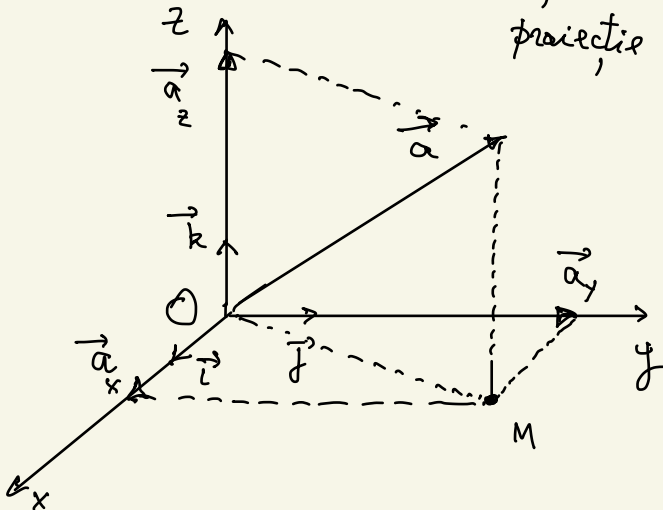

$\vec{a}$ ←———— O $\vec{i}$ → ————— $\vec{a}$ ————————→ x

$\vec{a} = -\vec{i}\cdot|\vec{a}|$

$$\vec{a} = \vec{i}\cdot|\vec{a}|$$
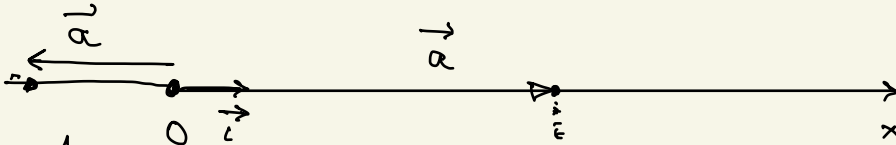
$$\vec{a} = a_x\,\vec{i}$$

proiecție a lui $\vec{a}$ pe Ox.

$\vec{a}_x,\ \vec{a}_y,\ \vec{a}_z$ ← componente
vectorului $\vec{a}$
pe cele
trei direcții

$$\vec{a} = \vec{a}_x + \vec{a}_y + \vec{a}_z$$

$$\boxed{\vec{a} = a_x\,\vec{i} + a_y\,\vec{j} + a_z\,\vec{k}}$$

$$|\vec{i}| = |\vec{j}| = |\vec{k}| = 1$$

$$\vec{a}_x = a_x \vec{i}$$

versorul direcției.

↑

componentă
vectorială
a lui $\vec{a}$ pe direcția x

proiecție a lui $\vec{a}$
pe direcția x.

$$\vec{F} = \vec{F}_x + \vec{F}_y$$

$$\vec{F}_x = F_x \vec{i}$$