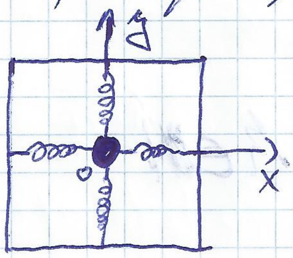
6. Compuneți două oscilații armonice perpendiculare. Deduceți forma eliptică a traiectoriei în cazul frecvențelor egale. Comentați formele specifice ale elipsei, în funcție de fazele inițiale.



$$\begin{cases} x = A\cos(\omega t+\alpha) \\ y = B\cos(\omega t+\beta) \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} \dfrac{x}{A} = \sin\omega t\cos\alpha + \cos\omega t\sin\alpha \quad \Big| \cdot \cos\beta \\ \dfrac{y}{B} = \sin\omega t\cos\beta + \cos\omega t\sin\beta \quad \Big| \cos\alpha \end{cases} \Rightarrow$$

$$\Rightarrow \frac{x}{A}\cos\beta - \frac{y}{B}\cos\alpha = -\cos\omega t\sin(\beta-\alpha)$$

$$\begin{cases} \dfrac{x}{A} = \sin\omega t\cos\alpha + \cos\omega t\sin\alpha \quad \Big| \sin\beta \\ \dfrac{y}{B} = \sin\omega t\cos\beta + \cos\omega t\sin\beta \quad \Big| \sin\alpha \end{cases} \Rightarrow$$

$$\Rightarrow \frac{x}{A}\sin\beta - \frac{y}{B}\sin\alpha = \sin\omega t\cdot\sin(\beta-\alpha)$$

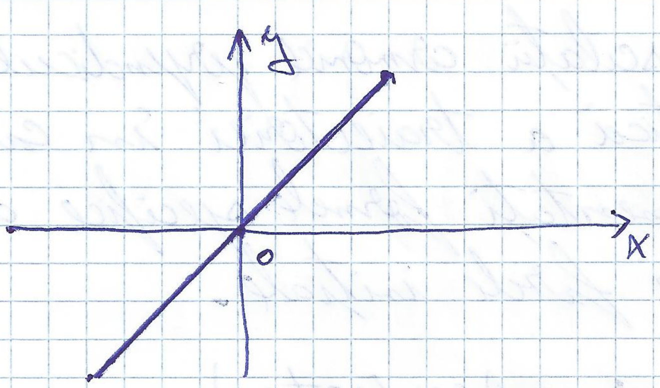
$$\begin{cases} \dfrac{x}{A}\cos\beta - \dfrac{y}{B}\cos\alpha = -\cos\omega t\sin(\beta-\alpha) \\ \dfrac{x}{A}\sin\beta - \dfrac{y}{B}\sin\alpha = \sin\omega t\cdot\sin(\beta-\alpha) \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} \dfrac{x^2}{A^2}\cos^2\beta - \dfrac{2xy}{AB}\cos\alpha\cos\beta + \dfrac{y^2}{B^2}\cos^2\alpha = \cos^2\omega t\sin^2(\beta-\alpha) \\ \dfrac{x^2}{A^2}\sin^2\beta - \dfrac{2xy}{AB}\sin\alpha\sin\beta + \dfrac{y^2}{B^2}\sin^2\alpha = \sin^2\omega t\sin^2(\beta-\alpha) \end{cases}$$

$$\Rightarrow \frac{x^2}{A^2} - \frac{2xy}{AB}\cos(\beta-\alpha) + \frac{y^2}{B^2} = \sin^2(\beta-\alpha)$$

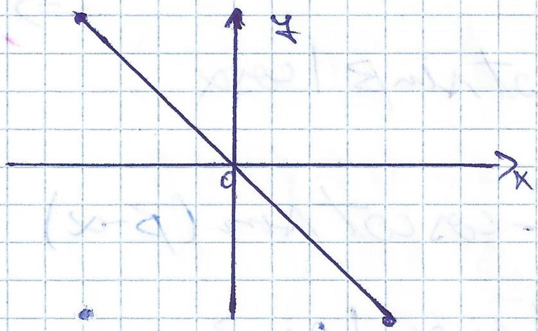
a) $\beta - \alpha = 2k\pi \quad k\in\mathbb{N} \Rightarrow$

$$\Rightarrow \frac{x}{A} - \frac{y}{B} = 0 \Rightarrow y = \frac{B}{A}\cdot x$$



b) $\beta - \alpha = (2k+1)\pi \qquad k \in \mathbb{N}$

$$\frac{x}{A} + \frac{y}{B} = 0 \Rightarrow y = -\frac{B}{A}x$$



c) $\beta - \alpha = \frac{\pi}{2} \text{ sau } \frac{3\pi}{2} \Rightarrow \frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$