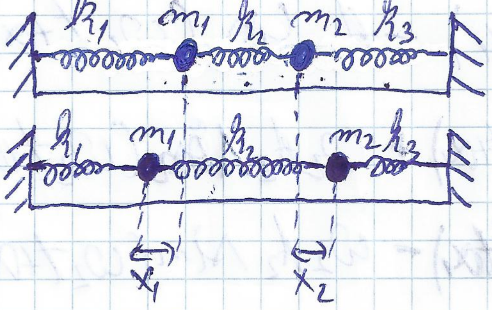
7. Rezolvați problema oscilațiilor armonice în cazul a două mișcări oscilatorii liniar armonice cuplate (oscilațiile și perna de aer liniară).



$$\begin{cases} m_1 \cdot \ddot{\vec{x}}_1 = -k_1\vec{x}_1 + k_2\vec{x}_2 - k_2\vec{x}_1 \\ m_2 \cdot \ddot{\vec{x}}_2 = -k_3\vec{x}_2 - k_2\vec{x}_2 + k_2\vec{x}_1 \end{cases}$$

$$\begin{cases} k_2 = k_c \\ m_1 = m_2 = m \\ k_1 = k_3 = k \end{cases} \qquad \begin{cases} \ddot{x}_1 + \frac{k+k_c}{m}x_1 - \frac{k_c}{m}x_2 = 0 \\ \ddot{x}_2 + \frac{k+k_c}{m}x_2 - \frac{k_c}{m}x_1 = 0 \end{cases}$$

$$\begin{cases} \frac{k+k_c}{m} = \omega_0^2 \\ \frac{k_c}{m} = \omega_c^2 \end{cases} \qquad \begin{cases} \ddot{x}_1 + \omega_0^2 x_1 - \omega_c^2 x_2 = 0 \\ \ddot{x}_2 + \omega_0^2 x_2 - \omega_c^2 x_1 = 0 \end{cases} \Rightarrow$$

$$\begin{cases} x_1 + x_2 = q_1 \\ x_1 - x_2 = q_2 \end{cases} \Rightarrow \begin{cases} x_1 = \frac{q_1+q_2}{2} \\ x_2 = \frac{q_1-q_2}{2} \end{cases} \qquad \begin{cases} \ddot{q}_1 + (\omega_0^2 - \omega_c^2)q_1 = 0 \\ \ddot{q}_2 + (\omega_0^2 + \omega_c^2)q_2 = 0 \end{cases}$$

$$\begin{cases} \omega_1^2 = \omega_0^2 - \omega_c^2 = \frac{k}{m} \\ \omega_2^2 = \omega_0^2 + \omega_c^2 = \frac{k+2k_c}{m} \end{cases}$$

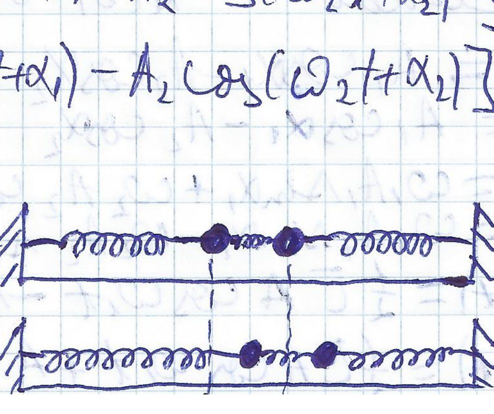
$$\begin{cases} q_1(t) = A_1\cos(\omega_1 t + \alpha_1) \\ q_2(t) = A_2\cos(\omega_2 t + \alpha_2) \end{cases}$$

$$\begin{cases} \dot{x}_1 = A_1 s\, e^{st} \\ \ddot{x} = A_1 s^2 e^{st} \\ x = A_1 e^{st} \end{cases}$$

$$\begin{cases} x_1(t) = \frac{1}{2}\left[A_1\cos(\omega_1 t + \alpha_1) + A_2\cos(\omega_2 t + \alpha_2)\right] \\ x_2(t) = \frac{1}{2}\left[A_1\cos(\omega_1 t + \alpha_1) - A_2\cos(\omega_2 t + \alpha_2)\right] \end{cases}$$

a) Oscilații simetrice

$x_1(0) = x_2(0) = A$



$$\dot{x}_1(0) = \dot{x}_2(0) = 0$$

$$x_1(t) = \frac{1}{2}\left[A_1 \cos(\omega_1 t + \alpha_1) + A_2 \cos(\omega_2 t + \alpha_2)\right]$$

$$x_2(t) = \frac{1}{2}\left[A_1 \cos(\omega_1 t + \alpha_1) - A_2 \cos(\omega_2 t + \alpha_2)\right]$$

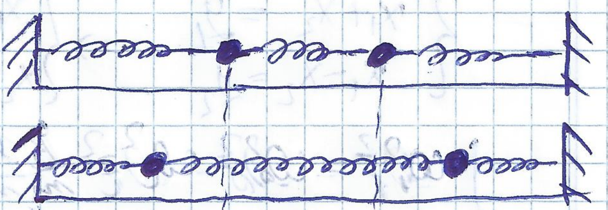
$$\dot{x}_1(t) = -\frac{1}{2}\left[\omega_1 A_1 \sin(\omega_1 t + \alpha_1) + \omega_2 A_2 \sin(\omega_2 t + \alpha_2)\right]$$

$$\dot{x}_2(t) = -\frac{1}{2}\left[\omega_1 A_1 \sin(\omega_1 t + \alpha_1) - \omega_2 A_2 \sin(\omega_2 t + \alpha_2)\right]$$

$$\begin{cases} 2A = A_1 \cos\alpha_1 + A_2 \cos\alpha_2 \\ 2A = A_1 \cos\alpha_1 - A_2 \cos\alpha_2 \\ 0 = \omega_1 A_1 \sin\alpha_1 + \omega_2 A_2 \sin\alpha_2 \\ 0 = \omega_1 A_1 \sin\alpha_1 - \omega_2 A_2 \sin\alpha_2 \end{cases} \Rightarrow \begin{cases} A_2 = 0 \\ \alpha_1 = \alpha_2 = 0 \\ A_1 = 2A \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x_1(t) = A \cos\omega_1 t \\ x_2(t) = A \cos\omega_1 t \end{cases}$$

b) Oscilații antisimetrice



$$x_1(0) = A \quad x_2(0) = -A$$

$$\dot{x}_1(0) = 0 \quad \dot{x}_2(0) = 0$$

$$\begin{cases} 2A = A_1 \cos\alpha_1 + A_2 \cos\alpha_2 \\ -2A = A_1 \cos\alpha_1 - A_2 \cos\alpha_2 \\ 0 = \omega_1 A_1 \sin\alpha_1 + \omega_2 A_2 \sin\alpha_2 \\ 0 = \omega_1 A_1 \sin\alpha_1 - \omega_2 A_2 \sin\alpha_2 \end{cases} \Rightarrow \begin{cases} A_1 = 0 \\ A_2 = 2A \\ \alpha_1 = \alpha_2 = 0 \end{cases} \Rightarrow$$

$$\cancel{\Rightarrow \begin{cases} x_1(t) = \frac{A}{2}\left[\cos\omega_1 t - \cos\omega_2 t\right] \\ x_2(t) = \frac{A}{2}\left[\cos\omega_1 t + \cos\omega_2 t\right] \end{cases}} \Rightarrow \begin{cases} x_1(t) = A\cos(\omega_2 t) \\ x_2(t) = -A\cos(\omega_2 t) \end{cases}$$

c) Bătăi $\quad x_1(0) = 0 \quad x_2(0) = A \quad \dot{x}_1(0) = \dot{x}_2(0) = 0$

$$\begin{cases} 0 = A_1 \cos\alpha_1 + A_2 \cos\alpha_2 \\ 2A = A_1 \cos\alpha_1 - A_2 \cos\alpha_2 \\ 0 = \omega_1 A_1 \sin\alpha_1 + \omega_2 A_2 \cos\alpha_2 \\ 0 = \omega_1 A_1 \sin\alpha_1 - \omega_2 A_2 \sin\alpha_2 \end{cases} \Rightarrow \begin{cases} A_1 = A \\ A_2 = -A \\ \alpha_1 = \alpha_2 = 0 \end{cases}$$

$$\begin{cases} x_1(t) = \frac{1}{2}\left[A \cos\omega_1 t - A \cos\omega_2 t\right] \\ x_2(t) = \frac{1}{2}\left[A \cos\omega_1 t + A \cos\omega_2 t\right] \end{cases}$$