$\lim_{x\to a} f(x)=l \Leftrightarrow \tilde{f}$ cont in l

Teoremă Fie $A\subset\mathbb{R}^m$, $f:A\to\mathbb{R}^m$, $a\in A'$, $l\in\mathbb{R}^m$. A.U.A.S.E

1) $\lim_{x\to a} f(x)=l$

2) $\forall\varepsilon>0\ \exists\delta_\varepsilon>0$ a.î. $\forall x\in A$ cu $\underline{0<d(x,a)}<\delta_\varepsilon \Rightarrow d(f(x),f(a))<\varepsilon$

$x\neq a$

3) $\forall (x_n)_n\subset A$ $x_n\to a$, $\boxed{x_n\neq a}$ $\forall n\geq 1 \Rightarrow f(x_n)\to l$

Șiruri de funcții

$x\in\mathbb{R}$ $\lim_{n\to\infty}\frac{x^2+2e^{nx}}{1+e^{nx}}$

$\lim_{n\to\infty} nx=\begin{cases}\infty & x>0\\ 0 & x=0\\ -\infty & x<0\end{cases}$ $\lim_{n\to\infty} e^{nx}=\begin{cases}\infty=e^\infty & x>0\\ 1=e^0 & x=0\\ 0=e^{-\infty} & x<0\end{cases}$

c1 $x>0$ $\lim_{n\to\infty}\frac{x^2+2e^{nx}}{1+e^{nx}}=\lim_{n\to\infty}\frac{\cancel{e^{nx}}\left(\frac{x^2}{e^{nx}}+2\right)}{\cancel{e^{nx}}\left(\frac{1}{e^{nx}}+1\right)}=2$

c2 $x=0$ $\lim_{n\to\infty}\frac{2}{2}=1$

c3 $x<0$ $\lim_{n\to\infty}\frac{x^2+2e^{nx}}{1+e^{nx}}=x^2$

$f_n,f:\mathbb{R}\to\mathbb{R}$

$f_n(x)=\frac{x^2+2e^{nx}}{1+e^{nx}}$ $f(x)=\begin{cases}2 & x>0\\ 1 & x=0\\ x^2 & x<0\end{cases}$

$f_n\xrightarrow{s} f$

cont dist.

Def Fie A o mulțime, (X,d) s.m. $f_n,f:A\to X$.

Spunem că $f_n\xrightarrow{s} f$ dacă $\forall x\in A$ $\lim_{n\to\infty}f_n(x)=f(x)$ ($f_n(x)\xrightarrow{n} f(x)$)

$\forall x\in A\ \forall\varepsilon>0\ \exists n_{\varepsilon,x}$ a.î. $\forall n\geq n_{\varepsilon,x}\Rightarrow d(f_n(x),f(x))<\varepsilon$

$f_n\xrightarrow{u} f$

$\forall\varepsilon>0\ \exists n_\varepsilon$ a.î. $\forall n\geq n_\varepsilon\Rightarrow d(f_n(x),f(x))\leq\varepsilon$ $\forall x\in A$

$a_n=\sup_{x\in A} d(f_n(x),f(x))\leq\varepsilon$

$f_n\xrightarrow{u} f\Leftrightarrow a_n\to 0$

Obs $f_n\xrightarrow{u} f\Rightarrow f_n\xrightarrow{s} f$ ($\not\Leftarrow$)

$f_n:[0,1]\to\mathbb{R}$ $f_n(x)=x^n$

$\lim_{n\to\infty}\underset{x^n}{f_n(x)}=\begin{cases}0 & x<1\\ 1 & x=1\end{cases}=f(x)$ $f:[0,1]\to\mathbb{R}$

$f_n\xrightarrow{s} f$

$a_n=\sup_{x\in[0,1]}|f_n(x)-f(x)|=\sup_{x\in[0,1)} x^n=1\not\to 0$

$f_n\not\xrightarrow{u} f$

$x=1$ $|f_n(1)-f(1)|=0$, $f_n(1)=1$, $f(1)=1$

$g_n:[0,1]\to\mathbb{R}$

$g_n(x)=x^n(1-x)^3$

$\lim_{n\to\infty}g_n(x)=0$ $g_n\xrightarrow{s} g=0$

$a_n=\sup_{x\in[0,1]}|$

$\lim\limits_{x\to a} f(x) = l \iff \tilde{f}$ cont în l

Teoremă Fie $A \subset \mathbb{R}^n$, $f: A \to \mathbb{R}^m$, $a \in A'$ și $l \in \mathbb{R}^m$. A.U.A.S.E.

1) $\lim\limits_{x\to a} f(x) = l$

2) $\forall \varepsilon > 0\ \exists \delta_\varepsilon > 0$ a.î. $\forall x \in A$ cu $\underbrace{0 < d(x,a)}_{x \neq a} < \delta_\varepsilon \Rightarrow d(f(x), f(a)) < \varepsilon$

3) $\forall (x_n)_n \subset A$, $x_n \to a$, $\boxed{x_n \neq a}\ \forall n \geq 1 \Rightarrow f(x_n) \to l$

Șiruri de funcții

$x \in \mathbb{R}$ $\lim\limits_{n\to\infty} \frac{x^2 + 2e^{nx}}{1 + e^{nx}}$

$\lim\limits_{n\to\infty} nx = \begin{cases} \infty & x > 0 \\ 0 & x = 0 \\ -\infty & x < 0 \end{cases}$ $\lim\limits_{n\to\infty} e^{nx} = \begin{cases} \infty = e^{\infty} & x > 0 \\ 1 = e^0 & x = 0 \\ 0 = e^{-\infty} & x < 0 \end{cases}$

c1 $x > 0$ $\lim\limits_{n\to\infty} \frac{x^2 + 2e^{nx}}{1 + e^{nx}} = \lim\limits_{n\to\infty} \frac{\cancel{e^{nx}}\left(\frac{x^2}{e^{nx}} + 2\right)}{\cancel{e^{nx}}\left(\frac{1}{e^{nx}} + 1\right)} = 2$

c2 $x = 0$ $\lim\limits_{n\to\infty} \frac{2}{2} = 1$

c3 $x < 0$ $\lim\limits_{n\to\infty} \frac{x^2 + 2e^{nx}}{1 + e^{nx}} = x^2$

$f_n, f: \mathbb{R} \to \mathbb{R}$

$f_n(x) = \frac{x^2 + 2e^{nx}}{1 + e^{nx}}$ $f(x) = \begin{cases} 2 & x > 0 \\ 1 & x = 0 \\ x^2 & x < 0 \end{cases}$

$f_n \xrightarrow{s} f$

f_n: cont, f: dist.

Def Fie A o mulțime, (X, d) și $f_n, f: A \to X$.

Spunem că $f_n \xrightarrow{s} f$ dacă $\forall x \in A$ $\lim\limits_{n\to\infty} f_n(x) = f(x)$ ($f_n(x) \xrightarrow{n} f(x)$)

$\forall x \in A\ \forall \varepsilon > 0\ \exists n_{\varepsilon, x}$ a.î. $\forall n \geq n_{\varepsilon, x} \Rightarrow d(f_n(x), f(x)) < \varepsilon$

$f_n \xrightarrow{u} f$

$\forall \varepsilon > 0\ \exists n_\varepsilon$ a.î. $\forall n \geq n_\varepsilon \Rightarrow d(f_n(x), f(x)) \leq \varepsilon\ \ \forall x \in A$

$a_n = \sup\limits_{x \in A} d(f_n(x), f(x)) \leq \varepsilon$

$f_n \xrightarrow{u} f \iff a_n \to 0$

Obs $f_n \xrightarrow{u} f \Rightarrow f_n \xrightarrow{s} f$ ($\not\Leftarrow$)

$f_n: [0,1] \to \mathbb{R}$ $f_n(x) = x^n$

$\lim\limits_{n\to\infty} \underbrace{f_n(x)}_{x^n} = \begin{cases} 0 & x < 1 \\ 1 & x = 1 \end{cases} = f(x)$ $f: [0,1] \to \mathbb{R}$

$f_n \xrightarrow{s} f$

$a_n = \sup\limits_{x \in [0,1)} |f_n(x) - f(x)| = \sup\limits_{x \in [0,1)} x^n = 1 \not\to 0$

$x = 1$ $|\underbrace{f_n(1)}_{1} - \underbrace{f(1)}_{1}| = 0$ $f_n \not\xrightarrow{u} f$

$g_n: [0,1] \to \mathbb{R}$

$g_n(x) = x^n(1-x)^3$

$\lim\limits_{n\to\infty} g_n(x) = 0$ $g_n \xrightarrow{s} g = 0$

$a_n = \sup\limits_{x \in [0,1]} |g_n(x) - \underbrace{g(x)}_{0}|$

$= \sup\limits_{x \in [0,1]} g_n(x)$

$g_n'(x) = nx^{n-1}(1-x)^3 - x^n 3(1-x)^2$

$= x^{n-1}(1-x)^2(n - nx - 3x) = 0$

$x = 0$ $x = 1$ $x = \frac{n}{n+3}$

	0		$\frac{n}{n+3}$		1
g_n'	0	$+ +$	0	$-$	0
g_n	$\boxed{0}$	$\nearrow$	$\boxed{g_n\left(\frac{n}{n+3}\right)}$	$\searrow$	$\boxed{0}$

$g_n\left(\frac{n}{n+3}\right) = a_n$

$a_n = \underbrace{\left(\frac{n}{n+3}\right)^n}_{\leq 1}\left(1 - \frac{n}{n+3}\right)^3 \leq \left(\frac{3}{n+3}\right)^3 \to 0$

$g_n \xrightarrow{u} g$

Teoremă Fie (X_1, d_1) și (X_2, d_2), $f_n, f: X_1 \to X_2$ și $a \in X_1$.
Dacă $f_n \xrightarrow{u} f$ și f_n cont în a $\forall n \geq 1 \Rightarrow f$ cont în a

---

$f, f_n: (a,b) \to \mathbb{R}$ $f_n \xrightarrow{u} f$

$c \in (a,b)$ f_n cont în c

$f_n \xrightarrow{u} f$

$\forall \varepsilon > 0\ \exists n_\varepsilon$ a.î. $\forall n \geq n_\varepsilon \Rightarrow |f_n(x) - f(x)| < \varepsilon\ \forall x \in (a,b)$

Fixăm $n \geq n_\varepsilon$.

f_n cont în c

$\forall \varepsilon > 0\ \exists \delta_\varepsilon > 0$ a.î.

Profesor Blaga Mirela-Gabriela

Tabel de integrale nedefinite

1.	$\int dx = x + C$	21.	$\int \frac{dx}{\sin^2 x} = -ctg\, x + C$
2.	$\int x^n dx = \frac{x^{n+1}}{n+1} + C$	22.	$\int \frac{dx}{\cos^2 x} = tg\, x + C$
3.	$\int x\, dx = \frac{x^2}{2} + C$	23.	$\int (1 + tg^2 x)\, dx = tg\, x + C$
4.	$\int \sqrt{x}\, dx = \frac{2}{3} x\sqrt{x} + C$	24.	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln\left\|\frac{x-a}{x+a}\right\| + C$
5.	$\int \frac{dx}{\sqrt{x}} = 2\sqrt{x} + C$	25.	$\int \frac{dx}{x^2 - 1} = \frac{1}{2} \ln\left\|\frac{x-1}{x+1}\right\| + C$
6.	$\int \frac{dx}{x} = \ln\|x\| + C$	26.	$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} arctg \frac{x}{a} + C$
7.	$\int \frac{dx}{ax+b} = \frac{1}{a} \ln\|ax+b\| + C$	27.	$\int \frac{dx}{x^2 + 1} = arctg\, x + C$
8.	$\int \frac{dx}{x^2} = -\frac{1}{x} + C$	28.	$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln(x + \sqrt{x^2 + a^2}) + C$
9.	$\int \ln x dx = x\ln x - x + C$	29.	$\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln\left\|x + \sqrt{x^2 - a^2}\right\| + C$
10.	$\int e^x dx = e^x + C$	30.	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
11.	$\int e^{-x} dx = -e^{-x} + C$	31.	$\int \frac{dx}{\sqrt{1 - x^2}} = \arcsin x + C$
12.	$\int a^x\, dx = \frac{a^x}{\ln a} + C$	32.	$\int \frac{x}{\sqrt{x^2 \pm a^2}} dx = \sqrt{x^2 \pm a^2} + C$
13.	$\int \sin x\, dx = -\cos x + C$	33.	$\int \frac{x}{\sqrt{a^2 - x^2}} dx = -\sqrt{a^2 - x^2} + C$
14.	$\int \cos x\, dx = \sin x + C$	34.	$\int \sqrt{x^2 + a^2}\, dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2} \ln(x + \sqrt{x^2 + a^2}) + C$
15.	$\int tg\, x\, dx = -\ln\|\cos x\| + C$	35.	$\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2} \ln\left\|x + \sqrt{x^2 - a^2}\right\| + C$
16.	$\int ctg\, x\, dx = \ln\|\sin x\| + C$	36.	$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \arcsin \frac{x}{a} + C$
17.	$\int f'(x)\, dx = f(x) + C$	37.	$\int f(x)\, dx = F(x) + C$
18.	$\int f(x)f'(x)\, dx = \frac{f^2(x)}{2} + C$	38.	$\int f(x)F(x)\, dx = \frac{F^2(x)}{2} + C$
19.	$\int \frac{f'(x)}{f(x)} dx = \ln\|f(x)\| + C$	39.	$\int \frac{f(x)}{F(x)} dx = \ln\|F(x)\| + C$
20.	$\int f^n(x)f'(x)\, dx = \frac{f^{n+1}(x)}{n+1} + C$	40.	$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$

149. $\sum_{n=1}^{\infty}\left(\frac{n!}{n^n}\right)^n$

150. $\sum_{n=1}^{\infty}\left(\frac{3n+2}{5n-1}\right)^{2n^2+1}$

151. $\sum_{n=1}^{\infty}\frac{\sqrt{n^2+1}}{\sqrt{n^3+1}}$

152. $\sum_{n=1}^{\infty}\left(\frac{6n^2+7n+5}{2n^2+5n+9}\right)^n$

153. $\sum_{n=1}^{\infty}\frac{1}{3^{\ln n}}$

154. $\sum_{n=1}^{\infty}\left(\frac{1}{n}-\sin\frac{1}{n}\right)$

155. $\sum_{n=2}^{\infty}\frac{1}{(\ln a)^{\ln n}}, a>0$

156. $\sum_{n=1}^{\infty}\frac{(n+1)^{n^2}}{n^{n(n+1)}(n+2)^n}$

157. $\sum_{n=1}^{\infty}\frac{1}{3^n-n}$

158. $\sum_{n=1}^{\infty}\frac{(2n-1)!!}{(2n)!}\cdot\frac{1}{n}$

159. $\sum_{n=1}^{\infty}\frac{n^a+1}{n^b+1};\ a,b\in R$

160. $\sum_{n=1}^{\infty}\arcsin\frac{2n-1}{5n^3+7n+4}$

161. $\sum_{n=1}^{\infty}\frac{3^n+2^n}{n!}$

162. $\sum_{n=1}^{\infty}\frac{1}{\sqrt[n]{n+5}}$

163. $\sum_{n=1}^{\infty}(-1)^n\frac{1}{n\cdot 3^n}$

164. $\sum_{n=1}^{\infty}(-1)^{n-1}\ln\frac{3n^2+2}{n^2+1}$

165. $\sum_{n=1}^{\infty}\frac{n^{2n}}{(2n)!}$

166. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\sqrt{2n+1}}$

167. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{e^{-n^2}}{n+1}$

168. $\sum_{n=0}^{\infty}a^n\cdot tg\frac{a}{2^n},\ a>0$

169. $\sum_{n=1}^{\infty}\frac{(n!)^2}{(2n)!}$

170. $\sum_{n=1}^{\infty}\frac{3n-7}{n(n+1)(n+2)}$

171. $\sum_{n=1}^{\infty}\frac{n^2+n-1}{(n+1)!}$

172. $\sum_{n=1}^{\infty}\frac{n!\cdot a^n}{n^n\cdot 2^n}, a>0$

173. $\sum_{n=2}^{\infty}\frac{1}{n^2\ln n}$

174. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{2+(-1)^n}{n^2}$

175. $\sum_{n=1}^{\infty}\frac{2^n+5^n}{2^{n+1}+5^{n+1}}$

176. $\sum_{n=1}^{\infty}(-1)^{n-1}\left(\frac{1}{\sqrt{2}}\right)^n$

177. $\sum_{n=1}^{\infty}\frac{3^n\cdot n!}{n^n}$

178. $\sum_{n=1}^{\infty}\frac{(-1)^n(2n+1)}{3^n}$

179. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n^4+1}+\sqrt{n^2+1}}$

180. $\sum_{n=1}^{\infty}\frac{3n+5}{2n+3}\cdot a^n, a>0$

181. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}+2^n}$

182. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n+a^n}}, a>-1$

183. $\sum_{n=1}^{\infty}\frac{a^n\cdot n^n}{n!}, a>0$

184. $\sum_{n=1}^{\infty}\frac{2\cdot 7\cdot 12\cdot\ldots\cdot(5n-3)}{5\cdot 9\cdot 13\cdot\ldots\cdot(4n+1)}$

185. $\sum_{n=1}^{\infty}\ln\left(\frac{n^4+1}{n^4}\right)$

186. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n+1}{(n+1)\sqrt{n+1}-1}$

187. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n(n+1)}(\sqrt{n}+\sqrt{n+1})}$

188. $\sum_{n=1}^{\infty}\frac{4^{n+3}-(-1)^{n+1}3^{n+1}}{5^{n+2}}$

189. $\sum_{n=1}^{\infty}(-1)^{n-1}\left[\frac{(2n-1)!!}{(2n)!!}\right]^3$

190. $\sum_{n=1}^{\infty}\left(a\frac{n^2+n+1}{3n^2}\right)^n, a>0$

191. $\sum_{n=1}^{\infty}\left(\sqrt{(n+1)(n+a)}-n\right)^n, a>0$

192. $\sum_{n=1}^{\infty}\left(\frac{n^2+3n}{n^2+3}\right)^{n^2}\cdot a^n,\ a>0$

193. $\sum_{n=1}^{\infty}e^{-(a\ln n+\ln n^2)},\ 0<a<1$

194. $\sum_{n=1}^{\infty}\frac{n!}{\alpha(\alpha+1)(\alpha+2)\ldots(\alpha+n-1)}, \alpha>0$

195. $\sum_{n=1}^{\infty}\left(\frac{3^n+4^n}{3^{n+1}+4^{n+1}}\right)^n\cdot a^n,\ a>0$

196. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1\cdot 3\cdot 5\cdot\ldots\cdot(2n-1)}{2\cdot 4\cdot 6\cdot\ldots\cdot(2n)}$

197. $\sum_{n=1}^{\infty}\frac{a(a+1)(a+2)\ldots(a+n)}{b(b+1)(b+2)\ldots(b+n)};0<a<b$

198. $\sum_{n=1}^{\infty}\left(\frac{n^2+n+1}{n^2+1}\right)^{n^2+1}\cdot a^n,\ a>0$

199. $\sum_{n=1}^{\infty}\left(\frac{3^n+4^n}{3^{n+1}+4^{n+1}}\right)^n\cdot a^n,\ a>0$

200. $\sum_{n=1}^{\infty}\frac{(a+1)(2a+1)\ldots(na+1)}{(b+1)(2b+1)\ldots(nb+1)};\ a,b\in R_+$

Să se studieze natura următoarelor serii:

19. $\sum_{n=1}^{\infty}\left[\frac{1\cdot 5\cdot 9\cdot\ldots\cdot(4n-3)}{5\cdot 9\cdot 13\cdot\ldots\cdot(4n+1)}\right]^2$ 20. $\sum_{n=1}^{\infty}\left(\frac{n+1}{n}\right)^n$ 21. $\sum_{n=1}^{\infty} n\sin\frac{1}{n}$ 22. $\sum_{n=1}^{\infty}(-1)^n\frac{5n-3}{3n+5}$ 23. $\sum_{n=1}^{\infty}\frac{(-2)^n+3^n}{(-2)^{n+1}+3^{n+1}}$

24. $\sum_{n=1}^{\infty}\frac{1}{\ln(2n+1)}$ 25. $\sum_{n=1}^{\infty}\frac{1}{\ln(n^2+2)}$ 26. $\sum_{n=1}^{\infty}\frac{n^n}{n!\cdot 3^n}$ 27. $\sum_{n=1}^{\infty}\frac{6n^2+5}{5n^3-1}$ 28. $\sum_{n=1}^{\infty}(3n^3-n)^{-\frac{1}{4}}$

29. $\sum_{n=1}^{\infty}\frac{\sqrt[5]{3n^7+n^2+1}+n+2}{6n^2-2n+1}$ 30. $\sum_{n=1}^{\infty}\sin\frac{1}{n^4}$ 31. $\sum_{n=1}^{\infty}\ln\left(1+\frac{1}{\sqrt{n}}\right)$ 32. $\sum_{n=1}^{\infty}\frac{1}{n+5^n}$ 33. $\sum_{n=1}^{\infty}\frac{1}{\sqrt[2n+1]{4n+2}}$

34. $\sum_{n=1}^{\infty}\frac{1}{2n^3+5n+7}$ 35. $\sum_{n=1}^{\infty}\frac{\sqrt[3]{n^2+1}}{\sqrt[4]{n^3+2}+1}$ 36. $\sum_{n=1}^{\infty}\frac{2n-1}{n^3+4n+3}$ 37. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n^4+2}+3\sqrt{n^2+1}+7}$

38. $\sum_{n=1}^{\infty}\frac{1\cdot 5\cdot 9\cdot\ldots\cdot(4n-3)}{1\cdot 6\cdot 11\cdot\ldots\cdot(5n-4)}$ 39. $\sum_{n=1}^{\infty}\frac{n!}{4^n}$ 40. $\sum_{n=1}^{\infty}\frac{1}{n+a^n}, a>0$ 41. $\sum_{n=1}^{\infty}\frac{n!}{(a+1)(a+2)\ldots(a+n)}, a>-1$

42. $\sum_{n=1}^{\infty}\left[\frac{1.3.5.\ldots.(2n-1)}{2.4.6.\ldots(2n)}\right]^3$ 43. $\sum_{n=1}^{\infty}\frac{1}{n}\cdot\left(\frac{3}{5}\right)^n$ 44. $\sum_{n=1}^{\infty}\left(\frac{4n-1}{3n+2}\right)^n$ 45. $\sum_{n=1}^{\infty}\left(n(\sqrt[n]{2}-1)\right)^n$ 46. $\sum_{n=1}^{\infty}\frac{n^n}{n!}$

47. $\sum_{n=1}^{\infty}\left(\frac{5n+3}{5n+4}\right)^{2n^2-3}$ 48. $\sum_{n=1}^{\infty}\left(\frac{3n-1}{3n+2}\right)^n$ 49. $\sum_{n=1}^{\infty}(-1)^n\left(\frac{5n+3}{5n+2}\right)^n$ 50. $\sum_{n=1}^{\infty}\frac{4n-1}{3n+2}$ 51. $\sum_{n=1}^{\infty}a^{\ln n}, a>0$

R: convergente: **26, 28, 30, 32, 34, 36, 38** (dacă $a>1$), **40, 41** (dacă $a>1$), **42, 43, 45, 46, 47, 50, 51** (dacă $a<1/e$)

Să se determine natura următoarelor serii:

52. $\sum_{n=1}^{\infty}\frac{n!}{2^n\cdot n^n}a^n,\ a>0$ 53. $\sum_{n=1}^{\infty}\left(\frac{n^2+3n+5}{n^2+2n+3}\right)^{n^2}\cdot a^n, a>0$ 54. $\sum_{n=1}^{\infty}\frac{2^n+3^n}{n!}$

55. $\sum_{n=1}^{\infty}\frac{(n+1)!(n+3)!}{2^n\cdot(2n-1)!}a^n,\ a>0$ 56. $\sum_{n=1}^{\infty}\frac{(n!)^2}{(2n)!}$ 57. $\sum_{n=1}^{\infty}\left(\sqrt{n^2+3n+2}-\sqrt{n^2-2n+3}\right)^n$

Studiaţi convergenţa şi absolut convergenţa seriilor:

58. $\sum_{n=1}^{\infty}(-1)^n\frac{1}{2n+1}$ 59. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$ 60. $\sum_{n=1}^{\infty}(-1)^n\frac{3n-1}{2n^2}$ 61. $\sum_{n=1}^{\infty}(-1)^n\frac{1}{n\cdot 2^n}$

62. $\sum_{n=1}^{\infty}(-1)^n\frac{3n-1}{2n^3}$ 63. $\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt{n+1}}$ 64. $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt[n]{n}}$ 65. $\sum_{n=1}^{\infty}(-1)^n\left(\frac{2n+3}{2n-1}\right)^n$

66. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n!}$ 67. $\sum_{n=1}^{\infty}\frac{n!}{(-3)^n}$ 68. $\sum_{n=1}^{\infty}(-1)^n\frac{1}{n-\ln n}$ 69. $\sum_{n=1}^{\infty}(-1)^{n-1}\sin\frac{1}{n}$

Atunci când este posibil, calculaţi suma următoarelor serii:

70. $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$ **R:**1 71. $\sum_{n=1}^{\infty}\frac{1}{n(n+1)(n+2)(n+3)}$ **R:**1/18 72. $\sum_{n=1}^{\infty}\frac{1}{(2n-1)(2n+1)(2n+3)}$ **R:**1/12

73. $\sum_{n=1}^{\infty}\frac{4n}{4n^4+1}$ **R:**1 74. $\sum_{n=0}^{\infty}\frac{1}{n^2+5n+6}$ **R:**1/2 75. $\sum_{n=0}^{\infty}\frac{an^2+bn+c}{n!}; a,b,c\in R$ **R:** $e(2a+b+c)$

76. $\sum_{n=1}^{\infty}\frac{n^2+n-1}{(n+2)!}$ **R:**1/2 77. $\sum_{n=1}^{\infty}\frac{4^{n+3}-(-1)^n 3^{n+1}}{5^{n+2}}$ **R:** $\frac{2057}{200}$ 78. $\sum_{n=1}^{\infty}\frac{5n-1}{n^3+4n^2+3n}$ **R:**17/9

79. $\sum_{n=1}^{\infty}\frac{(-1)^n+n}{(-4)^n}$ **R:**13/75 80. $\sum_{n=1}^{\infty}\frac{n+2}{n!+(n+1)!+(n+2)!}$ **R:**1/2 81. $\sum_{n=1}^{\infty}\frac{1}{(n+1)\sqrt{n}+n\sqrt{n+1}}$ **R:**1

82. $\sum_{n=1}^{\infty}\frac{n^2+n+1}{2^n}$ **R:** 9

85. $\sum_{n=1}^{\infty}\frac{2n^2+3n+4}{5^n}$ **R:**23/8

88. $\sum_{n=1}^{\infty}\frac{n}{a^n}, |a|>1$ **R:** $\frac{a}{(a-1)^2}$

91. $\sum_{n=0}^{\infty}\frac{(-2)^{n+3}+3^{2n+1}}{10^{n+2}}$

94. $\sum_{n=1}^{\infty}\frac{1}{3^n}\sin\frac{n\pi}{3}$

98. $\sum_{n=1}^{\infty}\frac{(-1)^n(2n+1)}{3^n}$

100. $\frac{1}{2^1}+\frac{1}{3^2}+\frac{1}{2^3}+\frac{1}{3^4}+\ldots..+$

Stabiliţi natura seriilor:

101. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n+1}$

105. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n^n}{(n+1)^n}$

109. $\sum_{n=1}^{\infty}(-1)^{n+1}\cos(n+1)$

113. $\sum_{n=1}^{\infty}\frac{7^n}{n!}$

117. $\sum_{n=1}^{\infty}\left(\frac{4n-3}{7n+1}\right)^n$

121. $\sum_{n=1}^{\infty}n\ln\left(1-\frac{1}{n}\right)$

125. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n!(1+2^n)}$

129. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{2n^3+n-1}}$

133. $\sum_{n=1}^{\infty}\left(\frac{n+a}{n+b}\right)^n$, a,

137. $\sum_{n=1}^{\infty}\frac{1}{n}\cdot\left(\frac{6}{5}\right)^n$

141. $\sum_{n=1}^{\infty}n^2 e^{-\sqrt{n}}$

145. $\sum_{n=1}^{\infty}\left(\frac{n}{3n-1}\right)^{2n}$

- $D \in \tau$ mulțime deschisă
- (X, τ) un spațiu topologic
- F o mulțime închisă dacă $X \setminus F \in \tau$
- $\mathcal{V}_a = \{V \mid \exists D \in \tau$ a.î. $a \in D \subset V\}$

$x_n \to a$ $\forall V \in \mathcal{V}_a \Rightarrow \exists n_V$ a.î. $\forall n \geq n_V \Rightarrow x_n \in V$

Ex $\tau = P(X)$

$(X, \tau = P(X))$

1) $\emptyset, X \in P(X)$
2) $D_1, D_2 \in P(X) \Rightarrow D_1 \cap D_2 \in P(X)$
3) $\{D_i\}_{i \in I} \subset P(X) \Rightarrow \bigcup_{i \in I} D_i \in P(X)$

$\mathcal{F} = P(X)$

$\mathcal{V}_a = \{V \mid \exists D \in \underset{\tau}{\overset{\shortparallel}{P(X)}}$ a.î. $a \in \underset{\{a\}}{\overset{\shortparallel}{D}} \subset V\}$

$= \{V \mid a \in V\}$

$x_n \to a$ $\forall V \in \mathcal{V}_a \Rightarrow \exists n$ a.î. $\forall n \geq n_V \Rightarrow x_n \in V$

$V = \{a\}$ $\exists n_a$ a.î. $\forall n \geq n_0 \Rightarrow x_n \in \{a\}$ $x_n = a$

În (X, τ) un șir convergent de la un rang este const

(X, d) $d(x,y) = \begin{cases} 1 & x \neq y \\ 0 & x = y \end{cases}$

$\tau_d = P(X)$

$\tau = \{\emptyset, X\}$

1) $\emptyset, X \in \tau$
2)

$\cap$	$\emptyset$	X
$\emptyset$	$\emptyset$	$\emptyset$
X	$\emptyset$	X

3) $(D_i)_i \in \{\emptyset, X\}$

c1) $\exists j$ a.î. $D_j = X \Rightarrow \bigcup D_i = X$

c2) $D_i = \emptyset$ $\forall i \in I \Rightarrow \bigcup D_i = \emptyset$

$\mathcal{F} = \{X, \emptyset\}$ $X = X \setminus \emptyset$ închisă

$\mathcal{V}_a = \{V \mid \exists D \in \tau$ $a \in D \subset V\} = \{X\}$

$D \neq \emptyset$ $D = X$

$\Rightarrow V = X$

$x_n \to a \Rightarrow x_n \in \underset{V}{\overset{\shortparallel}{X}}$ $\forall n$

limita nu este unică

(X, d) $x_n \to a \Leftrightarrow d(x_n, a) \to 0$

$x_n \to b \Leftrightarrow d(x_n, b) \to 0$

$0 = d(a,b) \leq d(x_n, a) + d(b, x_n) \to 0$

$\Rightarrow \boxed{a = b}$

$\mathbb{R}$ $\tau = \{(a, \infty)$ $a \in \overline{\mathbb{R}}\}$

1) $\emptyset = (\infty, \infty)$ $\mathbb{R} = (-\infty, \infty) \in \tau$
2) $(a, \infty) \cap (b, \infty) = (\max(a,b), \infty)$
3) $\bigcup_{i \in I} (a_i, \infty) = (\inf_{i \in I} a_i, \infty) \in \tau$

$\mathcal{F} = \{\emptyset, \mathbb{R}\} \cup \{(-\infty, a]\}_{a \in \mathbb{R}}$

$\mathcal{V}_a = \{V \mid \exists (b, \infty)$ a.î. $a \in \underset{\Rightarrow b < a}{(b, \infty)} \subset V\}$

$= \{V \mid \exists \varepsilon > 0$ a.î. $(a - \varepsilon, \infty) \subset V\}$

$x_n \to a$ $\forall \varepsilon > 0 \; \exists n_\varepsilon$ a.î. $\forall n \geq n_\varepsilon \Rightarrow x_n \in (a - \varepsilon, \infty)$

$\forall V \in \mathcal{V}_a \Rightarrow \exists n_V$

$V = (a - \varepsilon, \infty)$

$x_n > a - \varepsilon$ $n \geq n_\varepsilon$

$\boxed{\underline{\lim} \, x_n \geq a}$

$x_n \xrightarrow{\tau} a$

(X, τ) $A \subset X$

$\tau_A = \{A \cap D \mid D \in \tau\}$ - topologia indusă

1) $\emptyset = A \cap \emptyset$ $A = A \cap X \in \tau_A$
2) $G_1, G_2 \in \tau_A \Rightarrow \begin{matrix} G_1 = A \cap D_1 \\ G_2 = A \cap D_2 \\ D_1, D_2 \in \tau \end{matrix} \Rightarrow G_1 \cap G_2 = A \cap \underset{\in \tau}{(D_1 \cap D_2)} \in \tau_A$
3)

$\mathcal{V}_a^A = \{A \cap V \mid V \in \mathcal{V}_a^X\}$

$(x_n)_n \subset A$ $a \in A$ $\boxed{x_n \xrightarrow{\tau_A} a \Leftrightarrow x_n \xrightarrow{\tau} a}$