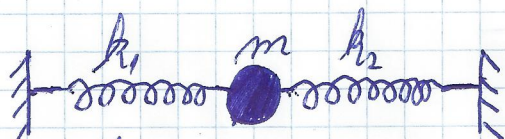


4. Compuneți două oscilații armonice paralele prin metodele: factorială, trigonometrică, a numerelor complexe.



a) Trigonometric
 $x = A \cos(\omega t + \alpha)$

$$\begin{cases} x_1 = A_1 \cos(\omega t + \alpha_1) \\ x_2 = A_2 \cos(\omega t + \alpha_2) \end{cases}$$

$$k_1 = k_2$$

$$x_1 + x_2 = A \cos(\omega t + \alpha) = x \quad A = ? \quad \alpha = ?$$

$$x_1 + x_2 = A_1 \cos \omega t \cos \alpha_1 - A_1 \sin \omega t \sin \alpha_1 + A_2 \cos \omega t \cos \alpha_2 - A_2 \sin \omega t \sin \alpha_2 = A \cos \omega t \cos \alpha - A \sin \omega t \sin \alpha \Rightarrow$$

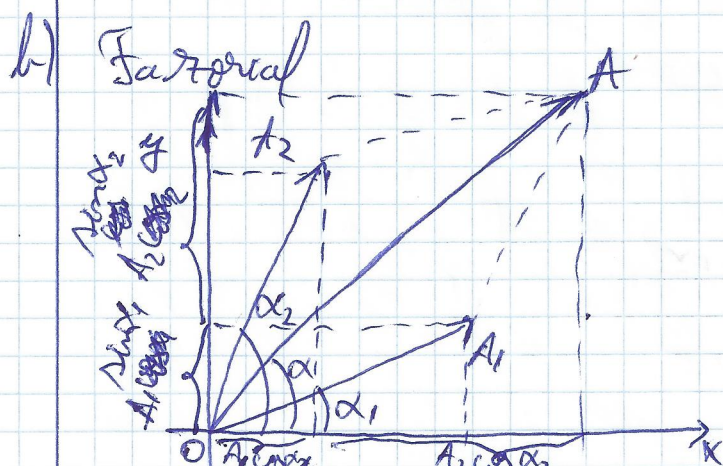
$$\Rightarrow \begin{cases} A \cos \alpha = A_1 \cos \alpha_1 + A_2 \cos \alpha_2 \\ A \sin \alpha = A_1 \sin \alpha_1 + A_2 \sin \alpha_2 \end{cases} \Rightarrow \begin{cases} A^2 \cos^2 \alpha = A_1^2 \cos^2 \alpha_1 + A_2^2 \cos^2 \alpha_2 + 2A_1 A_2 \cos \alpha_1 \cos \alpha_2 \\ A^2 \sin^2 \alpha = A_1^2 \sin^2 \alpha_1 + A_2^2 \sin^2 \alpha_2 + 2A_1 A_2 \sin \alpha_1 \sin \alpha_2 \end{cases}$$

$$\Rightarrow A^2 = A_1^2 + A_2^2 + 2A_1 A_2 (\cos \alpha_1 \cos \alpha_2 + \sin \alpha_1 \sin \alpha_2)$$

$$\Rightarrow A^2 = A_1^2 + A_2^2 + 2A_1 A_2 \cos(\alpha_1 - \alpha_2)$$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos(\alpha_1 - \alpha_2)}$$

$$\tan \alpha = \frac{A_1 \sin \alpha_1 + A_2 \sin \alpha_2}{A_1 \cos \alpha_1 + A_2 \cos \alpha_2} \Rightarrow \alpha = \arctan \frac{A_1 \sin \alpha_1 + A_2 \sin \alpha_2}{A_1 \cos \alpha_1 + A_2 \cos \alpha_2}$$



$$\vec{A}_1 + \vec{A}_2 = \vec{A} \Rightarrow (\vec{A}_1 + \vec{A}_2) \cdot (\vec{A}_1 + \vec{A}_2) = \vec{A} \cdot \vec{A}$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos(\widehat{a, b})$$

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos(\alpha_1 - \alpha_2) \Rightarrow$$

$$\Rightarrow A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos(\alpha_1 - \alpha_2)}$$

$$\tan \alpha = \frac{A_1 \sin \alpha_1 + A_2 \sin \alpha_2}{A_1 \cos \alpha_1 + A_2 \cos \alpha_2}$$