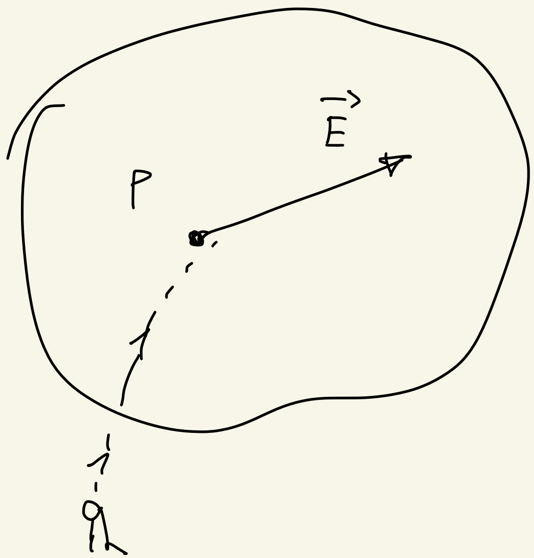
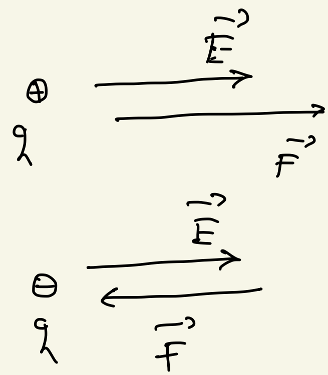
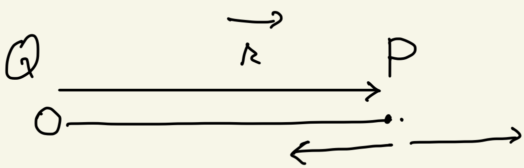
12 Ianuarie 2022



$$\vec{F} = q \cdot \vec{E}$$



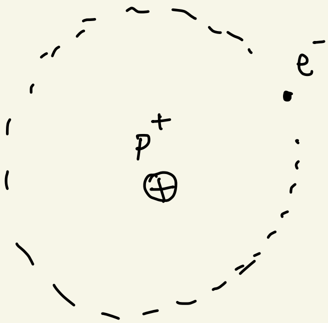


$$\vec{E} = k\frac{Q}{r^3}\vec{r}$$

formula intensității câmpului electric produs de o sarcină punctiformă Q la distanța r de ea.

Ex.

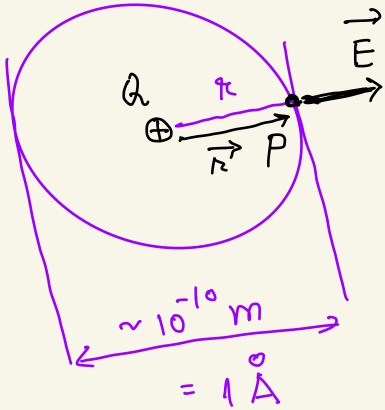
1)



$q_p = e = 1,6 \cdot 10^{-19}\,C$

$q_e = -e = -1,6 \cdot 10^{-19}\,C.$

atomul de hidrogen.



$r \simeq 0,5 \cdot 10^{-10}\,m$

$$\vec{E}_P = k\,\frac{Q}{r^3}\,\vec{r}$$

$$|\vec{E}_P| = \left|k\,\frac{Q}{r^3}\,\vec{r}\right| =$$

$$= \frac{k \cdot e}{r^3}\,|\vec{r}| \;\; \left.\begin{matrix} \\ \end{matrix}\right\} \Rightarrow$$

$$|\vec{r}| = r$$

$$|\vec{E}_P| = \frac{ke}{r^2} = \frac{9 \cdot 10^{9} \cdot 1,6 \cdot 10^{-19}}{(0,5 \cdot 10^{-10})^2}\,\frac{N}{C} =$$

$$= \frac{9 \cdot 1,6 \cdot 10^{-10}}{0,25 \cdot 10^{-20}}\,\frac{N}{C} = \frac{9 \cdot 1,6}{0,25} \cdot 10^{10}\,\frac{N}{C}$$

$$= 9 \cdot 1,6 \cdot 4 \cdot 10^{10} \frac{N}{C} = 14,4 \cdot 4 \cdot 10^{10} \frac{N}{C} = 57,6 \cdot 10^{10} \frac{N}{C}$$

$$\simeq 5,8 \cdot 10^{11} \frac{N}{C}$$

$$\vec{F}_e = (-e)\vec{E}$$

$$|\vec{F}_e| = |(-e)\vec{E}| = eE = 1,6 \cdot 10^{-19} \cdot 5,8 \cdot 10^{11} N =$$

$$\simeq 1,6 \cdot 6 \cdot 10^{-8} N = 9,6 \cdot 10^{-8} N \simeq 10^{-7} N$$

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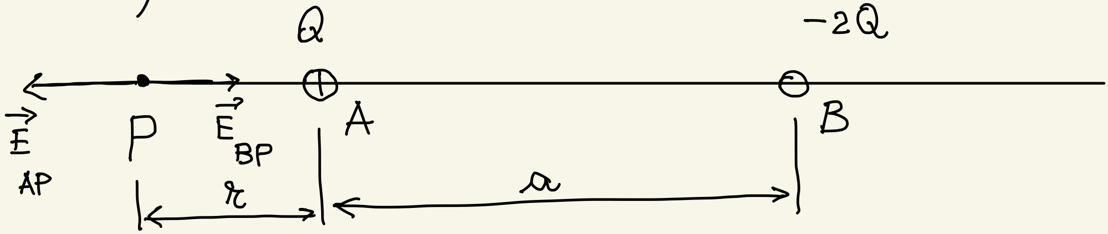
2)



Metoda I (cu module, fără versori)

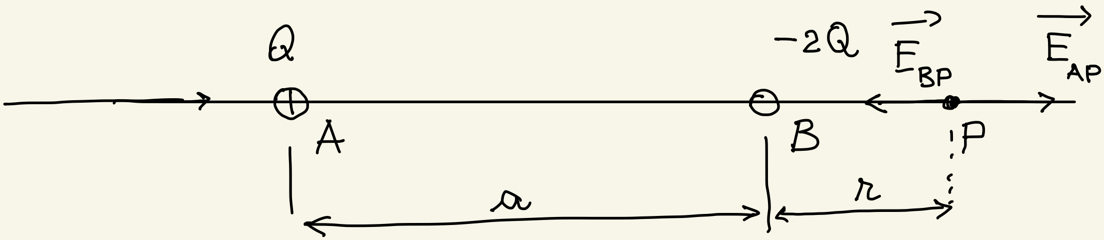
$$|\vec{E}_{AP}| = |\vec{E}_{BP}| \Rightarrow \left| k \frac{Q}{AP^3} \cdot \vec{AP} \right| = \left| k \cdot \frac{-2Q}{BP^3} \cdot \vec{BP} \right|$$

$$k \cdot \frac{|Q|}{AP^2} = k \frac{2|Q|}{BP^2} \Rightarrow \frac{1}{AP^2} = \frac{2}{BP^2}$$

$$\frac{1}{AP} = \frac{\sqrt{2}}{BP} \Rightarrow \frac{1}{r} = \frac{\sqrt{2}}{r+a} \Rightarrow r + a = r\sqrt{2}$$

$$a = r\left(\sqrt{2} - 1\right) \Rightarrow r = \frac{a}{\sqrt{2} - 1} = a\left(1 + \sqrt{2}\right) > 0$$

$$\simeq 2{,}41 \cdot a$$



$$|\vec{E}_{BP}| = |\vec{E}_{AP}| \Rightarrow \left| k \frac{-2Q}{r^3} \overrightarrow{BP} \right| = \left| k \frac{Q}{(a+r)^3} \overrightarrow{AP} \right|$$

$$k \frac{2|Q|}{r^3} r = k \frac{|Q|}{(a+r)^3} (a+r)$$

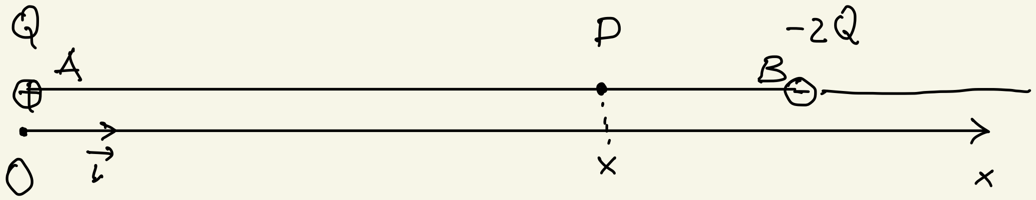
$$\frac{2}{r^2} = \frac{1}{(a+r)^2} \Longrightarrow \frac{\sqrt{2}}{r} = \frac{1}{a+r} \Rightarrow$$

$$\Rightarrow r = a\sqrt{2} + r\sqrt{2} \Rightarrow r\left(1 - \sqrt{2}\right) = a\sqrt{2} \Rightarrow$$

$$\Rightarrow r = \frac{a\sqrt{2}}{1 - \sqrt{2}} = \frac{a\sqrt{2}\left(1 + \sqrt{2}\right)}{-1} = -a\left(\sqrt{2} + 2\right) < 0$$

Absurd.

Metoda a II-a



$$\vec{E}_P = \vec{E}_{AP} + \vec{E}_{BP}$$

$A(0)$

$P(x)$

$B(a)$

$$\vec{E}_P = k \cdot \frac{Q}{AP^3} \cdot \overrightarrow{AP} + k\, \frac{-2Q}{BP^3}\, \overrightarrow{BP}$$

$$\overrightarrow{\underset{0}{A}\underset{x}{P}} = (x_P - x_A)\,\vec{i} = (x-0)\,\vec{i} = x\,\vec{i}$$

$$|\overrightarrow{AP}| = AP = |x\,\vec{i}\,| = |x|$$

$$\overrightarrow{\underset{a}{B}\underset{x}{P}} = (x_P - x_B)\,\vec{i} = (x-a)\,\vec{i}$$

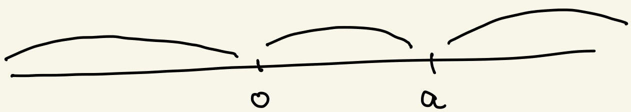
$$|\overrightarrow{BP}| = BP = |(x-a)\,\vec{i}\,| = |x-a|$$

$$\vec{E}_p = k\frac{Q}{|x|^3}\, x\,\vec{i} + k\frac{-2Q}{|x-a|^3}(x-a)\,\vec{i} = 0$$

$$kQ\,\vec{i}\left(\frac{x}{|x|^3} - \frac{2(x-a)}{|x-a|^3}\right) = 0 \Rightarrow$$

$$\Rightarrow \boxed{\frac{x}{|x|^3} - \frac{2(x-a)}{|x-a|^3} = 0}$$

$$|x| \longrightarrow 0 \qquad |x-a| \longrightarrow a$$



a) $\underline{x < 0}$

$$\frac{x}{(-x)^3} - \frac{2(x-a)}{(a-x)^3} = 0$$

$$\frac{x}{-x^3} - \frac{2(x-a)}{(a-x)(x-a)^2} = 0$$

$$-\frac{1}{x^2} - \frac{2}{(a-x)(x-a)} = 0$$

$$-\frac{1}{x^2} + \frac{2}{(x-a)^2} = 0 \Rightarrow \frac{1}{x^2} = \frac{2}{(x-a)^2}$$

$$\sqrt{\frac{1}{x^2}} = \sqrt{\frac{2}{(x-a)^2}} \Rightarrow \frac{1}{-x} = \frac{\sqrt{2}}{a-x} \Rightarrow$$

$$\Rightarrow a - x = -x\sqrt{2} \Rightarrow a = x(1-\sqrt{2})$$

$$x = \frac{a}{1-\sqrt{2}} = \frac{a(1+\sqrt{2})}{-1} = -a(1+\sqrt{2}) \simeq \underline{\underline{-2{,}41a < 0}}$$

b) $\underline{0 < x < a}$

$$\frac{x}{x^3} - \frac{2(x-a)}{(a-x)^3} = 0 \Rightarrow \frac{1}{x^2} - \frac{2(x-a)}{(a-x)(x-a)^2} = 0$$

$$\frac{1}{x^2} - \frac{2}{(a-x)(x-a)} = 0 \Rightarrow \frac{1}{x^2} + \frac{2}{(x-a)^2} = 0 \Rightarrow x \in \emptyset$$

c) $\underline{x > a}$

$$\frac{x}{x^3} - \frac{2(x-a)}{(x-a)^3} = 0 \Rightarrow \frac{1}{x^2} - \frac{2}{(x-a)^2} = 0$$

$$\frac{1}{x^2} = \frac{2}{(x-a)^2} \Rightarrow \frac{1}{|x|} = \frac{\sqrt{2}}{|x-a|} \Rightarrow$$

$$\Rightarrow \frac{1}{x} = \frac{\sqrt{2}}{x-a} \Rightarrow x - a = \sqrt{2}\,x$$

$$x(1-\sqrt{2}) = a \Rightarrow x = \frac{a}{1-\sqrt{2}} < 0 \Rightarrow$$ nu are soluție

Singura soluție este $x = -a(1+\sqrt{2})$!

Consultații pe 2 februarie ora 10^{00}