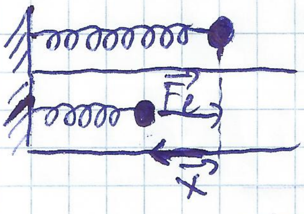
8. Tratați problema oscilațiilor amortizate pseudo-periodice

$$\vec{R} = -r\dot{\vec{x}}$$

$$m\ddot{\vec{x}} = \vec{R} + \vec{F_e} \Rightarrow m\ddot{\vec{x}} = -r\dot{\vec{x}} - k\vec{x}$$



$$\ddot{\vec{x}} + \frac{r}{m}\dot{\vec{x}} + \frac{k}{m}\vec{x} = 0$$

$$\begin{cases} \frac{r}{m} = 2b \\ \frac{k}{m} = \omega^2 \end{cases}$$

$$\ddot{x} + 2b\dot{x} + \omega^2 x = 0 \qquad x = Ae^{st}$$

$$s^2 Ae^{st} + \frac{r}{m} s Ae^{st} + \frac{k}{m} Ae^{st} = 0 \Rightarrow$$

$$\Rightarrow s^2 x + 2bsx + \omega^2 x = 0 \Rightarrow x(s^2 + 2bs + \omega^2) = 0 \Rightarrow$$

$$\Rightarrow s^2 + 2bs + \omega^2 = 0$$

$$\Delta = 4b^2 - 4\omega^2 \qquad s_{1,2} = \frac{-2b \pm 2\sqrt{b^2 - \omega^2}}{2} =$$

$$= -b \pm \sqrt{b^2 - \omega^2}$$

$$x = A_1 e^{(-b + \sqrt{b^2 - \omega^2})t} + A_2 e^{(-b - \sqrt{b^2 - \omega^2})t} \Rightarrow$$

$$\Rightarrow x = e^{-bt}\left(A_1 e^{t\sqrt{b^2 - \omega^2}} + A_2 e^{-t\sqrt{b^2 - \omega^2}}\right)$$

$$b < \omega \qquad \omega^2 - b^2 = \omega'^2$$

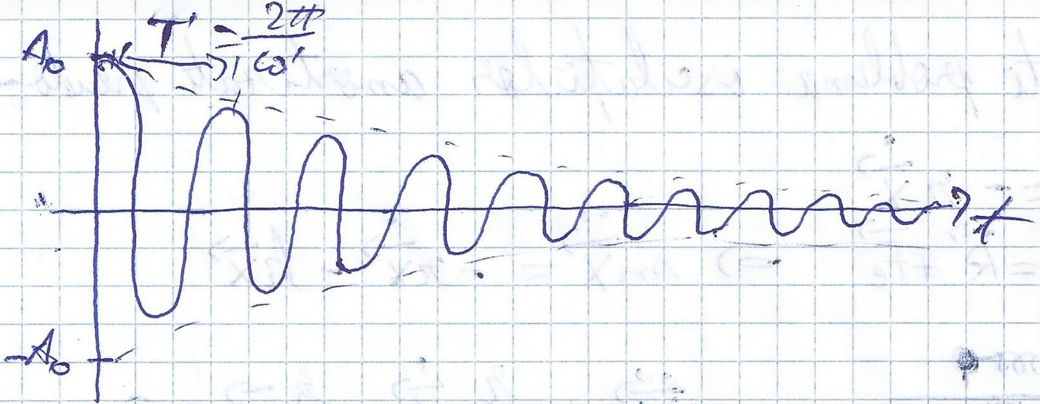
$$\sqrt{b^2 - \omega^2} = \sqrt{-\omega'^2} = \pm i\omega'$$

$$x(t) = e^{-bt}\left(A_1 e^{i\omega' t} + A_2 e^{-i\omega' t}\right)$$

$$\begin{cases} A_1 = \frac{1}{2} A_0 e^{i\alpha} \\ A_2 = \frac{1}{2} A_0 e^{-i\alpha} \end{cases}$$

$$x(t) = \frac{A_0}{2} e^{-bt}\left(e^{i(\omega' t + \alpha)} + e^{-i(\omega' t + \alpha)}\right) =$$

$$= A_0 e^{-bt} \cos(\omega' t + \alpha)$$



$$\frac{X(t)}{X(t+T')} = \frac{A_0 e^{-bt}\cos(\omega' t+\alpha)}{A_0 e^{-b(t+T')}\cos[\omega'(t+T')+\alpha]}$$

$$\omega'(t+T') = \omega' t + 2\pi$$

$$\frac{X(t)}{X(t+T')} = e^{+bT'} \Rightarrow \ln\frac{X(t)}{X(t+T')} = bT' = D$$

$$A(\tau) = A_0 e^{-1} = A_0 e^{-b\tau} \Rightarrow -1 = -b\tau \Rightarrow \tau = \frac{1}{b}$$

τ este timp de relaxare
D este decrementul