Să se calculeze următoarele limite:

1) $\lim_{n \to \infty} \sqrt{n+2} - \sqrt{n}$

2) $\lim_{n \to \infty} \sqrt[3]{n^3+n^2} - \sqrt[3]{n^3-n^2}$

3) $\lim_{n \to \infty} \sqrt[3]{6n^3+1} - \sqrt{6n^2+2}$

4) $\lim_{n \to \infty} \frac{a^n n!}{n^n}$, $a > 0$

5) $\lim_{n \to \infty} \left(1 + \sqrt{n+1} - \sqrt{n}\right)^{2\sqrt{n}}$

6) $\lim_{n \to \infty} \sqrt[n]{a^n+b^n+c^n}$ $a, b, c > 0$

7) $\lim_{n \to \infty} \left(\frac{a^{\frac{1}{n}} + b^{\frac{1}{n}}}{2}\right)^n$ $a, b > 0$

8) $\lim_{n \to \infty} \sqrt[n]{n!}$

9) $\lim_{n \to \infty} \frac{\sqrt[n]{n!}}{n}$

10) $\lim_{n \to \infty} \sqrt[n]{C_{2n}^{n}}$

2

11) $\lim_{n\to\infty} n(\sqrt[n]{n}-1)$

12) $\lim_{n\to\infty} \frac{1+\frac{1}{2}+\dots+\frac{1}{n}}{\ln n}$

13) $\lim_{n\to\infty} 1+\frac{1}{\sqrt{2}}+\dots+\frac{1}{\sqrt{n}}$

14) $\lim_{n\to\infty} \frac{1+\sqrt{2}+\dots+\sqrt{n}}{n\sqrt{n}}$

15) $\lim_{n\to\infty} \frac{1^p+2^p+\dots+n^p}{n^{p+1}}$ $p>1$

16) $\lim_{n\to\infty} \frac{10^n+n^2+1}{6^{n+1}+3} \cdot \frac{21^n+1}{14^n+7}$

17) $\lim_{n\to\infty} n\sqrt{n}\left(\sqrt{n+1}+\sqrt{n-1}-2\sqrt{n}\right)$

18) $\lim_{n\to\infty} \sqrt[n+1]{(n+1)!}-\sqrt[n]{n!}$

19) $\lim_{n\to\infty} \frac{n}{\ln n!}$

20) $\lim_{n\to\infty} \frac{\ln n!}{n \ln n}$

Să se calculeze limitele următoarelor șiruri definite prin recurență:

(3)

1) $x_{n+1} = a x_n + b \qquad |a|<1 \qquad x_0 \in \mathbb{R}$

2) $x_{n+1} = x_n - x_n^2 \qquad x_0 \in (0,1) \qquad \lim\limits_{n\to\infty} n x_n = ?$

3) $x_{n+1} = x_n - x_n^3 \qquad x_0 \in (0,1)$

4) $x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right) \qquad a>0,\ x_0>0.$

5) $x_{n+1} = \sqrt{2+x_n} \qquad x_0>0$

6) Arătați cu ajutorul definiției că $\frac{n^3}{n^3+2n^2+7n+4} \to 1.$

$\frac{n^2}{2n^2+1} \to \frac{1}{2}, \quad \sqrt{n+1}-\sqrt{n} \to 0,$

7) Dacă $x_n \to a \in \mathbb{R} \Rightarrow$

$\frac{x_1+x_2+\ldots+x_n}{n} \to a$ și $\frac{C_n^0 x_0 + C_n^1 x_1 + \ldots + C_n^n x_n}{2^n} \to a$