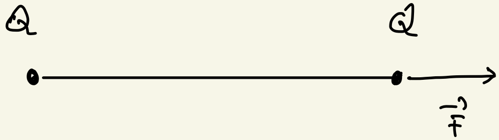
22 Decembrie 2021.

13. Two small plastic spheres each have a mass of 2.0 g and a charge of −50.0 nC. They are placed 2.0 cm apart (center to center).

a. What is the magnitude of the electric force on each sphere?

b. By what factor is the electric force on a sphere larger than its weight?



$k = 9\cdot 10^{9}\ \frac{Nm^2}{C^2}$

$Q = -50\ nC$

$r = 2\ cm$

a) $|\vec{F}| = F = ?$

b) $\frac{F}{G} = ?$

$$\boxed{\begin{gathered}|\vec{F}_{12}| = |\vec{F}_{21}| = k\,\frac{|Q_1Q_2|}{|\vec{r}_{12}|^2}\\ \vec{F}_{12} = k\,\frac{Q_1Q_2}{|\vec{r}_{12}|^3}\cdot\vec{r}_{12}\end{gathered}}$$

a) $$F = |\vec{F}_{12}| = k\cdot\frac{|Q\cdot Q|}{r^2} = \frac{kQ^2}{r^2} = \frac{9\cdot 10^{9}\cdot(-5\cdot 10^{-8})^2}{(2\cdot 10^{-2})^2}\ N$$

$$Q = -50\ nC = -50\cdot 10^{-9}\ C = -5\cdot 10^{-8}\ C.$$

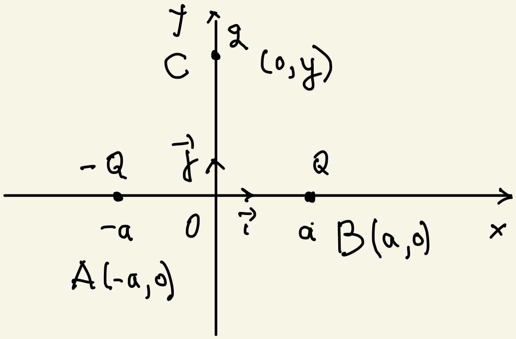
$$r = 2\ cm = 2\cdot 10^{-2}\ m.$$

$$= \frac{9\cdot 25\cdot 10^{9}\cdot 10^{-16}}{4\cdot 10^{-4}}\ N = \frac{9\cdot 25}{4}\cdot 10^{-3}\ N = \frac{225}{4}\cdot 10^{-3}\ N =$$

$$= 56{,}25 \cdot 10^{-3}\ N .$$

b) $$\frac{F}{G} = \frac{F}{mg} = \frac{56{,}25 \cdot 10^{-3}}{2 \cdot 10^{-3} \cdot 9{,}8} = \frac{56{,}25}{2 \cdot 9{,}8} \cong 2{,}9$$

45. A positive point charge Q is located at $x = a$ and a negative point charge $-Q$ is at $x = -a$. A positive charge q can be placed anywhere on the y-axis. Find an expression for $(F_{\text{net}})_x$, the x-component of the net force on q.



$$\vec{F}_q = \vec{F}_{-Qq} + \vec{F}_{Qq}$$

$$\vec{F}_{-Qq} = k \frac{-Qq}{|\vec{AC}|^3} \vec{AC}$$

$$\vec{F}_{12} = k \frac{Q_1 Q_2}{|\vec{r}_{12}|^3} \cdot \vec{r}_{12}$$

$$\begin{cases} A(-a,0) \\ C(0,y) \end{cases} \Rightarrow \vec{AC}(a,y)$$

$$\vec{AC} = a\vec{i} + y\vec{j}$$

$$|\vec{AC}| = \sqrt{a^2+y^2}$$

$$\vec{F}_{-Qq} = k \frac{-Qq}{(a^2+y^2)^{3/2}} (a\vec{i} + y\vec{j})$$

$$\vec{F}_{Qq} = k \frac{Qq}{|\vec{BC}|^3} \vec{BC}$$

$$\begin{cases} B(a,0) \\ C(0,y) \end{cases} \Rightarrow \vec{BC}(-a,y)$$

$$\vec{BC} = -a\vec{i} + y\vec{j}$$

$$|\vec{BC}| = \sqrt{(-a)^2 + y^2} = \sqrt{a^2 + y^2}$$

$$\vec{F}_{Qq} = k \frac{Qq}{(a^2+y^2)^{3/2}} \cdot \left(-a\vec{i} + y\vec{j}\right)$$

$$\vec{F}_{q} = k \frac{-Qq}{(a^2+y^2)^{3/2}} \left(a\vec{i} + y\vec{j}\right) + k \frac{Qq}{(a^2+y^2)^{3/2}} \cdot \left(-a\vec{i} + y\vec{j}\right)$$

$$= k \frac{Qq}{(a^2+y^2)^{3/2}} \left(-a\vec{i} - \underbrace{y\vec{j}}_{} - a\vec{i} + \underbrace{y\vec{j}}_{}\right) = -k \frac{Qq\,2a}{(a^2+y^2)^{3/2}} \vec{i}$$

$$= F_{qx}\,\vec{i}$$

$$\Rightarrow \boxed{F_{qx} = -\frac{2kQqa}{(a^2+y^2)^{3/2}}}$$

47. ||| **FIGURE P25.47** shows four charges at the corners of a square of side L. What is the magnitude of the net force on q?

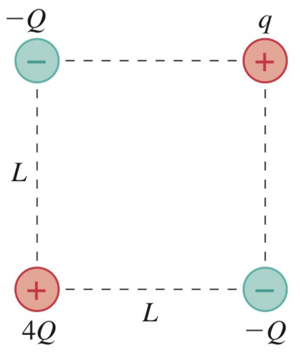


FIGURE P25.47

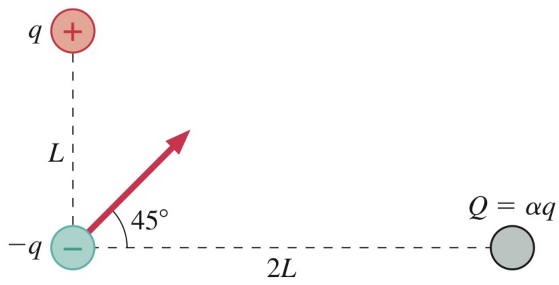
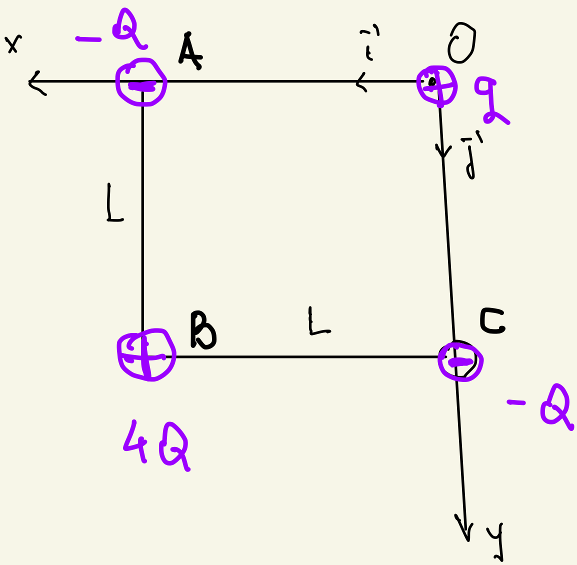


FIGURE P25.48



$$\vec{F}_q = \vec{F}_{AO} + \vec{F}_{BO} + \vec{F}_{CO}$$

$$\vec{AO} = -L\vec{i}$$

$$|\vec{AO}| = L$$

$$\vec{BO} = -L\vec{i} - L\vec{j}$$

$$|\vec{BO}| = L\sqrt{2}$$

$$\vec{CO} = -L\vec{j}$$

$$|\vec{CO}| = L$$

$$\vec{F}_q = k \cdot \frac{-Qq}{|\vec{AO}|^3}\vec{AO} + k\frac{4Qq}{|\vec{BO}|^3}\vec{BO} + k\frac{-Qq}{|\vec{CO}|^3}\vec{CO} =$$

$$\vec{F}_q = k\frac{-Qq}{L^3}(-L\vec{i}) + k\frac{4Qq}{(L\sqrt{2})^3}(-L\vec{i} - L\vec{j}) + k\frac{-Qq}{L^3}(-L\vec{j})$$

$$= k\frac{Qq}{L^3}\left[L\vec{i} + \frac{4}{2\sqrt{2}}\left(-L\vec{i} - L\vec{j}\right) + L\vec{j}\right]$$

$$= k\frac{Qq}{L^3}\cdot L\left(\vec{i} + \frac{2}{\sqrt{2}}\left(-\vec{i} - \vec{j}\right) + \vec{j}\right) =$$

$$= k\frac{Qq}{L^2}\left(\vec{i} - \sqrt{2}\,\vec{i} - \sqrt{2}\,\vec{j} + \vec{j}\right)$$

$$= k\frac{Qq}{L^2}\left[\vec{i}\left(1-\sqrt{2}\right) + \vec{j}\left(1-\sqrt{2}\right)\right]$$

$$= \frac{kQq\left(1-\sqrt{2}\right)}{L^2}\left(\vec{i} + \vec{j}\right) = F_{qx}\,\vec{i} + F_{qy}\,\vec{j}$$

$$F_{qx} = \frac{kQq\left(1-\sqrt{2}\right)}{L^2} < 0$$

$$F_{qy} = \frac{kQq\left(1-\sqrt{2}\right)}{L^2} < 0$$

$$|\vec{F}_q| = \sqrt{F_{qx}^2 + F_{qy}^2} = \sqrt{2\left(\frac{kQq\left(1-\sqrt{2}\right)}{L^2}\right)^2} =$$

$$= \frac{kQq}{L^2}\sqrt{2}\sqrt{\left(1-\sqrt{2}\right)^2} = \frac{kQq}{L^2}\sqrt{2}\left(\sqrt{2}-1\right) \Rightarrow$$

$$\Rightarrow |\vec{F}_q| = \frac{kQq}{L^2}\left(2-\sqrt{2}\right)$$

<u>Obs.</u> $$\vec{F}_q = \frac{kQq\left(1-\sqrt{2}\right)}{L^2}\left(\vec{i}+\vec{j}\right)$$

Dacă ni se cere modulul direct (fără proiecții) putem face astfel:

$$|\vec{F}_q| = \left|\frac{kQq\left(1-\sqrt{2}\right)}{L^2}\left(\vec{i}+\vec{j}\right)\right|$$

$$= \frac{kQq\left(\sqrt{2}-1\right)}{L^2}\,|\underbrace{\vec{i}+\vec{j}}_{1\cdot\vec{i}+1\cdot\vec{j}}| = \frac{kQq}{L^2}\left(\sqrt{2}-1\right)\sqrt{2} = \frac{kQq}{L^2}\left(2-\sqrt{2}\right)$$

$$\sqrt{a^2} = |a|$$