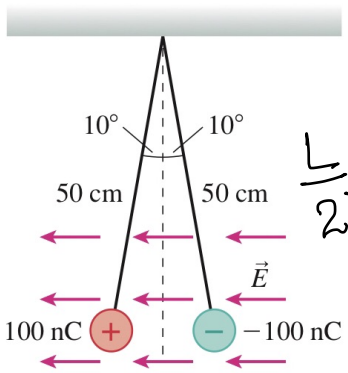
5 Ianuarie 2022



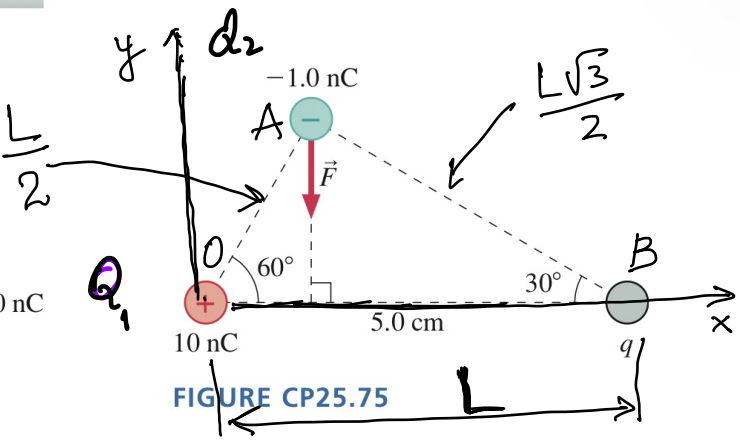


FIGURE CP25.74 FIGURE CP25.75

75. The force on the −1.0 nC charge is as shown in FIGURE CP25.75. What is the magnitude of this force?

$Q_1 = 10\,nC$

$Q_2 = -1\,nC$

$F = ?$

$$\vec{F}_2 = \vec{F}_{12} + \vec{F}_{q2}$$

$$\vec{F}_2 = k\frac{Q_1 Q_2}{|\vec{OA}|^3}\cdot\vec{OA} + k\frac{qQ_2}{|\vec{BA}|^3}\vec{BA}$$

$$\vec{OA} = \frac{L}{4}\vec{i} + \frac{L\sqrt{3}}{4}\vec{j}$$

$$|\vec{OA}| = \sqrt{\frac{L^2}{16} + \frac{3L^2}{16}} = \sqrt{\frac{L^2}{4}} = \frac{L}{2}$$

$$\vec{BA} = -\frac{L\sqrt{3}}{2}\cos 30^0\,\vec{i} + \frac{L\sqrt{3}}{2}\sin 30^0\,\vec{j}$$

$$= -\frac{3L}{4}\vec{i} + \frac{L\sqrt{3}}{4}\vec{j}$$

$$|\vec{BA}| = \sqrt{\left(-\frac{3L}{4}\right)^2 + \left(\frac{L\sqrt{3}}{4}\right)^2} = \sqrt{\frac{9L^2}{16} + \frac{3L^2}{16}} =$$

$$= \sqrt{\frac{12L^2}{16}} = \sqrt{\frac{3L^2}{4}} = L\frac{\sqrt{3}}{2}$$

$$\vec{F_2} = k\frac{Q_1Q_2}{\left(\frac{L}{2}\right)^3}\cdot\left(\frac{L}{4}\vec{i} + \frac{L\sqrt{3}}{4}\vec{j}\right) +$$

$$+ k\frac{qQ_2}{\left(\frac{L\sqrt{3}}{2}\right)^3}\cdot\left(-\frac{3L}{4}\vec{i} + \frac{L\sqrt{3}}{4}\vec{j}\right) = F_{2x}\vec{i} + F_{2y}\vec{j}$$

$$\begin{cases} F_{2x} = \dfrac{kQ_1Q_2}{\left(\frac{L}{2}\right)^3}\cdot\dfrac{L}{4} + \dfrac{kqQ_2}{\left(\frac{L\sqrt{3}}{2}\right)^3}\cdot\left(-\dfrac{3L}{4}\right) = 0 \\ F_{2y} = \dfrac{kQ_1Q_2}{\left(\frac{L}{2}\right)^3}\cdot\dfrac{L\sqrt{3}}{4} + \dfrac{kqQ_2}{\left(\frac{L\sqrt{3}}{2}\right)^3}\cdot\dfrac{L\sqrt{3}}{4} \end{cases}$$

$$\frac{kQ_1Q_2}{\left(\frac{L}{2}\right)^3}\cdot\frac{L}{4}+\frac{kqQ_2}{\left(\frac{L\sqrt{3}}{2}\right)^3}\cdot\left(-\frac{3L}{4}\right)=0$$

$$\frac{kQ_1Q_2}{\frac{L^3}{8}}\cdot\frac{L}{4}-\frac{kqQ_2}{\frac{3\sqrt{3}L^3}{8}}\frac{3L}{4}=0$$

$$\frac{kQ_1Q_2\ 8L}{4L^3}-\frac{kqQ_2\ \ 8\cdot 3L}{4\cdot 3\sqrt{3}\,L^3}=0$$

$$\frac{kQ_2}{L^2}\left(\frac{Q_1\cdot 8}{4}-\frac{q\cdot 8\cdot 3}{4\cdot 3\sqrt{3}}\right)=0$$

$$\Rightarrow 2Q_1-\frac{2q}{\sqrt{3}}=0 \Rightarrow Q_1-\frac{q}{\sqrt{3}}=0 \Rightarrow$$

$$\Rightarrow \boxed{q=\sqrt{3}\,Q_1}$$

$$\vec{F_2}=F_{2x}\vec{i}+F_{2y}\vec{j}=F_{2y}\vec{j}$$

$$\|\vec{F_2}\|=\|F_{2y}\vec{j}\|=|F_{2y}|$$

$$F_{2y} = \frac{kQ_1Q_2}{\left(\frac{L}{2}\right)^3} \cdot \frac{L\sqrt{3}}{4} + \frac{kqQ_2}{\left(\frac{L\sqrt{3}}{2}\right)^3} \cdot \frac{L\sqrt{3}}{4}$$

$$F_{2y} = \frac{kQ_1Q_2 \cdot 8}{L^3} \cdot \frac{L\sqrt{3}}{4} + \frac{k\sqrt{3}Q_1Q_2 \cdot 8}{3\sqrt{3}L^3} \cdot \frac{L\sqrt{3}}{4} =$$

$$= \frac{kQ_1Q_2}{L^2} 2\sqrt{3} + \frac{kQ_1Q_2}{L^2} \cdot \frac{3 \cdot 8}{3\sqrt{3} \cdot 4} =$$

$$= \frac{kQ_1Q_2}{L^2}\left(2\sqrt{3} + \frac{2}{\sqrt{3}}\right) = \frac{kQ_1Q_2}{L^2} \frac{8}{\sqrt{3}}$$

$$|\vec{F_2}| = |F_{2y}| = \left|\frac{kQ_1Q_2}{L^2} \cdot \frac{8}{\sqrt{3}}\right| = \frac{k|Q_1||Q_2|}{L^2} \cdot \frac{8}{\sqrt{3}} =$$

$$= \frac{9 \cdot 10 \cdot 10^{-9} \cdot 10^{-9} \cdot 8}{\left(5 \cdot 10^{-2}\right)^2 \cdot \sqrt{3}} N = \frac{9 \cdot 10^{-8} \cdot 8}{25 \cdot 10^{-4} \cdot \sqrt{3}} N$$

$$= \frac{9 \cdot 8 \cdot 10^{-4}}{25 \cdot \sqrt{3}} N = \frac{9 \cdot 8 \cdot \sqrt{3} \cdot 10^{-4}}{25 \cdot 3} N = \frac{3 \cdot 8 \cdot \sqrt{3}}{25} \cdot 10^{-4} N$$

$$= \frac{3 \cdot 8 \cdot \sqrt{3} \cdot 4}{100} \cdot 10^{-4} N = 3 \cdot 8 \cdot 4 \cdot \sqrt{3} \cdot 10^{-6} N =$$

$$\simeq 24 \cdot 4\sqrt{3} \cdot 10^{-6} N = 96 \cdot \sqrt{3} \cdot 10^{-6} N \simeq$$

$$\simeq 96 \cdot 1{,}73 \cdot 10^{-6} N = 166 \cdot 10^{-6} N = 1{,}66 \cdot 10^{-4} N$$

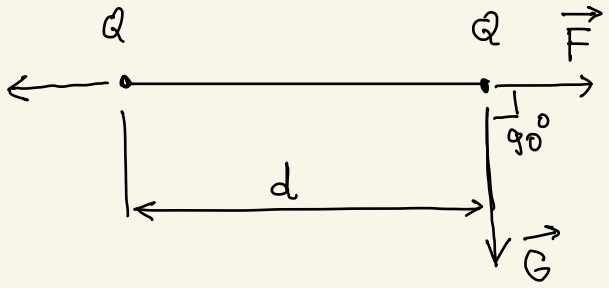
31. ‖ Two 1.0 g spheres are charged equally and placed 2.0 cm apart. When released, they begin to accelerate at 150 m/s^2. What is the magnitude of the charge on each sphere?

$m = 1g = 10^{-3}\,kg$

$d = 2\,cm = 2\cdot 10^{-2}\,m$

$a = 150\ \frac{m}{s^2}$

$Q = ?$



$$\vec{F} + \vec{G} = m\vec{a}$$

$$|\vec{F} + \vec{G}| = m|\vec{a}|$$

$$\sqrt{F^2 + G^2} = ma$$

$$F^2 + G^2 = m^2 a^2$$

$$F = k\frac{|QQ|}{d^2} = \frac{kQ^2}{d^2}$$

$$\left(\frac{kQ^2}{d^2}\right)^2 + m^2 g^2 = m^2 a^2 \Rightarrow \left(\frac{kQ^2}{d^2}\right)^2 = m^2(a^2 - g^2)$$

$$\frac{kQ^2}{d^2} = m\sqrt{a^2 - g^2} \Rightarrow Q^2 = \frac{md^2}{k}\sqrt{a^2 - g^2}$$

$$|Q| = \sqrt{\frac{mg^2}{k}\sqrt{a^2-g^2}} = g\cdot\sqrt{\frac{m}{k}\sqrt{a^2-g^2}} =$$

$$= 2\cdot 10^{-2}\sqrt{\frac{10^{-3}}{9\cdot 10^{9}}\sqrt{150^2-9{,}8^2}}\ C \simeq 2\cdot 10^{-2}\sqrt{\frac{10^{-12}}{9}\cdot 150}\ C$$

$$= 2\cdot 10^{-2}\cdot 10^{-6}\sqrt{\frac{150}{9}}\ C = 2\cdot 10^{-8}\frac{\sqrt{150}}{3}\ C =$$

$$= \frac{2\cdot 10^{-8}}{3}\sqrt{25\cdot 6}\ C = \frac{10\cdot 10^{-8}}{3}\sqrt{6}\ C = \frac{10^{-7}\cdot\sqrt{6}}{3}C$$

$$\simeq 0{,}82\cdot 10^{-7}\,C = 8{,}2\cdot 10^{-8}\,C = 82\cdot 10^{-9}\,C = 82\,nC.$$