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Grupa 164

Examen Analiză Matematică

1) Să se studieze convergența seriei

$$\sum_{n\geq 1} \left(\frac{3n+a}{3n+b}\right)^{3n}$$

Criteriul radicalului

$$l = \lim_{n\to\infty} \sqrt[n]{\left(\frac{3n+a}{3n+b}\right)^{3n}} = \lim_{n\to\infty}\left(\frac{3n+a}{3n+b}\right)^{3} = \lim_{n\to\infty}\left(\frac{n\left(3+\frac{a}{n}\right)}{n\left(3+\frac{b}{n}\right)}\right)^{3}$$

$$= \lim_{n\to\infty}\left(\frac{3}{3}\right)^{3} = 1$$

Criteriul suficient de divergență:

$$\lim_{n\to\infty}\left(\frac{3n+a}{3n+b}\right)^{3n} = \lim_{n\to\infty}\left(\frac{3n+b}{3n+b}+\frac{a-b}{3n+b}\right)^{3n}$$

$$= \lim_{n\to\infty}\left(1+\frac{a-b}{3n+b}\right)^{\frac{3n+b}{a-b}\cdot\frac{a-b}{3n+b}\cdot 3n} =$$

$$= \lim_{n\to\infty} e^{\frac{3n(a-b)}{3n+b}} = \lim_{n\to\infty} e^{\frac{n(3(a-b))}{n\left(3+\frac{b}{n}\right)}} = e^{\frac{3(a-b)}{3}} = e^{a-b}$$

$e^{a-b} \neq 0$ pentru $(a,b)\in\mathbb{R}^2$

$\Rightarrow$ Seria divergentă