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Grupa 164

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Examen scris

Calcul Numeric

1 a)
$$\begin{cases} x + y + 2z = 6 \\ x + 2y + 3z = 4 \\ x + y + z = 7 \end{cases}$$

cu GPT

$$\bar{A} = \left(\begin{array}{ccc|c} 1 & 1 & 2 & 6 \\ 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 7 \end{array}\right) \quad \max_{i,j=\overline{1,3}} |a_{i,j}| = |a_{23}| = 3$$

$L_1 \leftrightarrow L_2$

$$\bar{A} \leftrightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 6 \\ 1 & 1 & 1 & 7 \end{array}\right) \begin{matrix} \\ \\ x \end{matrix} \quad \begin{matrix} C_1 \leftrightarrow C_3 \\ \leftrightarrow \end{matrix} \left(\begin{array}{ccc|c} 3 & 2 & 1 & 4 \\ 2 & 1 & 1 & 6 \\ 1 & 1 & 1 & 7 \end{array}\right) \begin{matrix} \\ \\ z \end{matrix} \quad \begin{matrix} L_1/3 \\ \leftrightarrow \end{matrix} \left(\begin{array}{ccc|c} 1 & \frac{2}{3} & \frac{1}{3} & \frac{4}{3} \\ 2 & 1 & 1 & 6 \\ 1 & 1 & 1 & 7 \end{array}\right) \begin{matrix} \\ \\ z \end{matrix}$$

$$\begin{matrix} L_2 - 2L_1 \\ L_3 - L_1 \\ (-) \end{matrix} \left(\begin{array}{ccc|c} 1 & \frac{2}{3} & \frac{1}{3} & \frac{4}{3} \\ 0 & -\frac{1}{3} & \frac{1}{3} & \frac{10}{3} \\ 0 & \frac{1}{3} & \frac{2}{3} & \frac{17}{3} \end{array}\right) \begin{matrix} \\ y \\ x \end{matrix} \quad \max_{i,j=\overline{2,3}} |a_{ij}| = |a_{33}| = \frac{2}{3}$$

$L_2 \leftrightarrow L_3$

$$\bar{A} \leftrightarrow \left(\begin{array}{ccc|c} 1 & \frac{2}{3} & \frac{1}{3} & \frac{4}{3} \\ 0 & \frac{1}{3} & \frac{2}{3} & \frac{17}{3} \\ 0 & -\frac{1}{3} & \frac{1}{3} & \frac{10}{3} \end{array}\right) \begin{matrix} \\ z \\ y \end{matrix} \quad \begin{matrix} C_1 \leftrightarrow C_3 \\ \leftrightarrow \end{matrix} \left(\begin{array}{ccc|c} 1 & \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ 0 & \frac{2}{3} & \frac{1}{3} & \frac{17}{3} \\ 0 & \frac{1}{3} & -\frac{1}{3} & \frac{10}{3} \end{array}\right) \begin{matrix} \\ z \\ y \end{matrix}$$

$$\begin{matrix} L_2 / \frac{2}{3} \\ (-) \end{matrix} \left(\begin{array}{ccc|c} 1 & \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ 0 & 1 & \frac{1}{2} & \frac{17}{2} \\ 0 & \frac{1}{3} & -\frac{1}{3} & \frac{10}{3} \end{array}\right) \begin{matrix} \\ z \\ y \end{matrix} \quad \begin{matrix} L_3 - \frac{1}{3} L_2 \\ (-) \end{matrix} \left(\begin{array}{ccc|c} 1 & \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ 0 & 1 & \frac{1}{2} & \frac{17}{2} \\ 0 & 0 & -\frac{1}{2} & \frac{1}{2} \end{array}\right) \begin{matrix} \\ z \\ y \end{matrix}$$

$L_3 \mid -\frac{1}{2}$

$\leadsto \left(\begin{array}{ccc|c} 1 & \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ 0 & 1 & \frac{7}{2} & \frac{17}{2} \\ 0 & 0 & 1 & -1 \end{array}\right)$

$z \quad x \quad y$

1) $z + \frac{x}{3} + \frac{2y}{3} = \frac{4}{3} \Rightarrow z + \frac{9}{3} - \frac{2}{3} = \frac{4}{3} \Rightarrow z = -1$

$x + \frac{y}{2} = \frac{17}{2} \Rightarrow x - \frac{1}{2} = \frac{17}{2} \Rightarrow x = 9$

$y = -1$

$\Rightarrow S = \{9, -1, -1\}$

2) $A = \begin{bmatrix} 2 & 0 & 1 \\ -4 & 1 & 2 \\ 1 & 4 & 1 \end{bmatrix}$ factorizabile LU cu GEP

$k=1: \left(\begin{array}{ccc} \textcircled{2} & 0 & 1 \\ -4 & 1 & 2 \\ 1 & 4 & 1 \end{array}\right)$

$m_{21} = \frac{a_{21}^{(1)}}{a_{11}^{(1)}} = \frac{-4}{2} = 2$

$m_{31} = \frac{a_{31}^{(1)}}{a_{11}^{(1)}} = \frac{1}{2}$

$L_2 = L_2 + 2L_1, \; L_3 = L_3 - \frac{1}{2} L_1$

$\sim \left(\begin{array}{ccc} \textcircled{2} & 0 & 1 \\ 0 & 1 & 4 \\ 0 & 4 & \frac{1}{2} \end{array}\right)$

$k=2: A \sim \left(\begin{array}{ccc} \textcircled{2} & 0 & 1 \\ 0 & \textcircled{1} & 4 \\ 0 & 4 & \frac{1}{2} \end{array}\right)$

$m_{32} = \frac{a_{32}^{(2)}}{a_{22}^{(2)}} = 4$

$L_3 = L_3 - 4L_2 \sim \left(\begin{array}{ccc} 2 & 0 & 1 \\ 0 & 1 & 4 \\ 0 & 0 & \frac{-31}{2} \end{array}\right) \Rightarrow L = \begin{pmatrix} 1 & 0 & 0 \\ m_{21} & 1 & 0 \\ m_{31} & m_{32} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ \frac{1}{2} & 4 & 1 \end{pmatrix}, \; U = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 1 & 4 \\ 0 & 0 & \frac{-31}{2} \end{pmatrix}$

$$\Rightarrow A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ \frac{1}{2} & 4 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 0 & 1 \\ 0 & 1 & 4 \\ 0 & 0 & \frac{-31}{2} \end{pmatrix} = LU$$

3) $f: [0,5] \to \mathbb{R}$, $f(x) = 2^x + 3 \cdot 4^x - 15 \cdot x^2$

? $P_2(x)$ folosind metoda Newton cu diferențe divizate relative la diviziunea $(0, 3, 5)$

$x_1 = 0, x_2 = 3, x_3 = 5$

$P_2(x) = f[x_1] + f[x_1, x_2] \cdot (x - x_1) + f[x_1, x_2, x_3](x - x_1)(x - x_2)$

x_i	$f[x_i]$	$f[x_{i-1}, x_i]$	$f[x_1, x_2, x_3]$
0	4		
3	135	$\frac{131}{3}$	
5	2729	1297	$\frac{752}{3}$

$f[0] = f(0) = 4$

$f[3] = f(3) = 8 + 3 \cdot 64 - 15 \cdot 9 = 135$

$f[5] = f(5) = 32 + 3 \cdot 1024 - 75 \cdot 25 = 2729$

$f[0,3] = \frac{f(3) - f(0)}{3 - 0} = \frac{131}{3}$

$f[3,5] = \frac{f(5) - f(3)}{5 - 3} = 1297$

$f[0,3,5] = \frac{f[3,5] - f[0,3]}{5 - 0} = \frac{752}{3}$

$P_2(x) = 4 + \frac{131}{3}(x - 0) + \frac{752}{3}(x - 0)(x - 3) = 4 + \frac{131x}{3} + \frac{752x}{3} \cdot (x - 3)$

4) $I = \int_{-4}^{-1} 2x^8 + 4x^7 + x^5 + 6x^4 \, dx =$

$= \frac{2}{9}x^9 + \frac{4}{8}x^8 + \frac{x^6}{6} + \frac{6}{5}x^5 \Big|_{-4}^{-1} =$

$= -\frac{2}{9} + \frac{4}{8} + \frac{1}{6} - \frac{6}{5} - \left(-\frac{524288}{9} + 32768 + \frac{2048}{3} - \frac{6144}{5}\right)$

$= \frac{-80 + 180 + 60 - 432 + 20971520 - 11796480 - 2457600 + 442368}{360}$

$= \frac{9371376}{360} = 26031,6$

a) aproximare folosind formula de cuadratură numită a trapezului,

$I_{trapez}^{n=3} = \frac{h}{2}\left(f(a) + 2\cdot\sum_{k=2}^{3} f(x_k) + f(b)\right), h = \frac{b-a}{n}$

$x_{k+1} = a + k\cdot h, k = \overline{0,n}$

$h = \frac{-1-(-4)}{3} = 1$

$x_1 = -4 + 0h = -1, x_2 = -4 + 1h = -3, x_3 = -4 + 2h = -2, x_4 = -4 + 3h = -1$

$I_{trapez} = \frac{1}{2}(f(-4) + 2f(-3) + 2f(-2) + f(-1)) =$

$= \frac{1}{2}\{(131072 - 65536 - 1024 + 1536) + 2(13122 - 8748 - 243 + 486)$

$+ 2(512 - 512 - 32 + 96) + (2 - 4 - 1 + 6)\}$

$= \frac{1}{2}(66048 + 9234 + 128 + 3) = \frac{75413}{2} = 37706,5$

b) $E = |I - I_{trapez}^{3}| = |26031,6 - 37706,5| = 11674,9$ (4)