3) Să se determine punctele de extrem local ale funcției $f: \mathbb{R}^2 \to \mathbb{R}$ date de

$$f(x, y) = xy + \frac{60}{x} + \frac{40}{y}$$

Căutăm punctele în care

$$\frac{df}{dx}(x, y) = 0 \Rightarrow y + -\frac{60}{x^2} = 0$$

$$\frac{df}{dy}(x, y) = 0 \Rightarrow x - \frac{40}{y^2} = 0 \Rightarrow x = \frac{40}{y^2}$$

$$\Rightarrow y - \frac{60}{\left(\frac{40}{y^2}\right)^2} = y - \frac{60}{\frac{1600}{y^4}} = y^5 - \frac{3}{8} = 0 \Rightarrow y = \sqrt[5]{\frac{3}{80}}$$

$$x - \frac{40}{y^2} = 0 \Rightarrow x - \frac{40}{\left(\left(\frac{3}{80}\right)^{\frac{2}{5}}\right)^2} = 0 \Rightarrow x = \frac{40}{\left(\frac{3}{80}\right)^{\frac{4}{5}}}$$

$$\frac{df^2}{dx^2}(x, y) = \frac{120}{x^3} \quad ; \frac{df^2}{dy^2}(x, y) = \frac{80}{x^3}, \quad \frac{df^2}{dx\,dy}(x, y) = \frac{df^2}{dy\,dx}(x, y) = 1$$

$$H(x, y) = \begin{pmatrix} \frac{df^2}{dx^2}(x, y) & \frac{df^2}{dy\,dx}(x, y) \\ \frac{df^2}{dx\,dy}(x, y) & \frac{df^2}{dy^2}(x, y) \end{pmatrix}$$

$$H\left(\frac{40}{\left(\frac{3}{80}\right)^{\frac{4}{5}}}, \sqrt[5]{\frac{3}{80}}\right) = \begin{pmatrix} \frac{120}{\left(\frac{40}{\left(\frac{3}{80}\right)^{\frac{4}{5}}}\right)^3} & 1 \\ 1 & \frac{80}{\left(\sqrt[5]{\frac{3}{\ }}\right)^3} \end{pmatrix}$$

$D_1 > 0$

$D_2 =$