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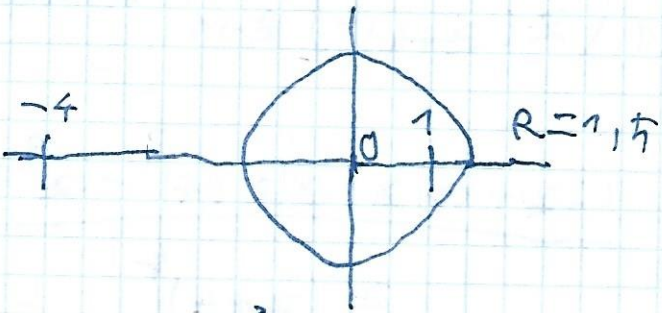
Grupa 154

2) $\int_{|z|=1,5} \frac{z^2+\sin z}{z(z-1)^2(z+4)^2}\,dz = 1$

$\|$

Fie $D$, $D = S(0, 1,5)$

$= \int_{|z|=1,5}$




$f: \mathbb{C} - \{0, 1, -4\} \to \mathbb{C}$, $f(z) = \frac{z^2+\sin z}{z(z-1)^2(z+4)^2}$

$f \in O(\mathbb{C} - \{0, 1, -4\})$

$z = 0 \in \text{Int } D$, $z = 1 \in \text{Int } D$, $z = -4 \notin \text{Int } D$

$\Rightarrow I = 2\pi i \{ \text{Rez}(f, 0) + \text{Rez}(f, 1) \}$

$\text{Rez}(f, 0)$: $\lim_{z \to 0} |f(z)| = \infty$

$\Rightarrow z = 0$ pol al lui $f$ de ordin 1

$\Rightarrow \text{Rez}(f, 0) = \lim_{z \to 0} \frac{1}{(1-1)!} \cdot (z-0) f(z) =$

$= \lim_{z \to 0} \frac{z(z^2+\sin z)}{z(z-1)^2(z+4)^2} = \lim_{z \to 0} \frac{(z^2+\sin z)}{(z-1)^2(z+4)^2} = 0$ ($z^2+\sin z \to 0$)

$\Rightarrow \text{Rez}(f, 0) = 0$

$\operatorname{Rez}(f,1): \lim_{z\to 1} |f(1)| = \infty$

$\Rightarrow z=1$ pol de ordin 2 pentru $f$

$\Rightarrow \operatorname{Rez}(f,1) = \lim_{z\to 1} \frac{1}{(2-1)!}\left((z-1)^2 f(z)\right)' = \lim_{z\to 1}\left(\frac{(z-1)^2(z^2+\sin z)}{z(z-1)^2(z+4)^2}\right)'$

$= \lim_{z\to 1}\left(\frac{z^2+\sin z}{z(z+4)^2}\right)' = \lim_{z\to 1} \frac{(z^2+\sin z)'(z(z+4)^2) - (z(z+4)^2)'(z^2+\sin z)}{z^2(z+4)^4} =$

$= \lim_{z\to 1} \frac{(2z+\cos z)z(z+4)^2 - (z^3+16z+8z^2)'(z^2+\sin z)}{z^2(z+4)^4}$

$= \lim_{z\to 1} \frac{(2z+\cos z)z(z+4)^2 - (3z^2+16+16z)(z^2+\sin z)}{z^2(z+4)^4}$

$= \frac{(2+\cos 1)\cdot 25 - 35(1+\sin 1)}{625} = \frac{50+25\cos 1 - 35 - 35\sin 1}{625}$

$= \frac{15+25\cos 1 - 35\sin 1}{625} = \frac{3+5\cos 1 - 7\sin 1}{125}$

$\Rightarrow \operatorname{Rez}(f,1) = \frac{3+5\cos 1 - 7\sin 1}{125}$

$\Rightarrow I = 2\pi i(\operatorname{Rez}(f,0) + \operatorname{Rez}(f,1)) = 2\pi i\left(0 + \frac{3+5\cos 1 - 7\sin 1}{125}\right)$

$= 2\pi i\left(\frac{3+5\cos 1 - 7\sin 1}{125}\right)$