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Grupa 164

$$(3) \quad y'' - 5y' + 4y = e^{-x}, \quad y(0) = 0, \quad y'(0) = -1$$

$$y(x) = y_0 + y_p$$

$$\text{Pt } y_0: \text{ ec. omogenă: } y'' - 5y' + 4y = 0$$

$$\text{ec. caracteristică: } \lambda^2 - 5\lambda + 4 = 0$$

$$\Delta = (-5)^2 - 4 \cdot 4 = 9$$

$$\lambda_{0,2} = \frac{5 \pm \sqrt{9}}{2} \in \{1, 4\}$$

$$\lambda_{01} = 1, \lambda_{02} = 4 \Rightarrow y_0(x) = (c_1 e^x + c_2 e^{4x})$$

$$y_0(0) = 0 \Rightarrow (c_1 e^0 + c_2 e^{4 \cdot 0} = 0 \Rightarrow c_1 + c_2 = 0$$

$$y_0'(0) = -1 \Rightarrow (c_1 e^0 + 4c_2 e^{4 \cdot 0} = -1 \Rightarrow c_1 + 4c_2 = -1$$

$$\begin{cases} c_1 + c_2 = 0 \\ c_1 + 4c_2 = -1 \end{cases} \Rightarrow c_2 = -1 \Rightarrow c_1 = \frac{1}{3}$$

$$\Rightarrow y_0(x) = \frac{1}{3} e^x - \frac{1}{3} e^{4x}$$

$$y_p(x) = a e^{-x}$$

$$y_p'(x) = -a e^{-x}$$

$$y_p''(x) = a e^{-x}$$

$$\Rightarrow a e^{-x} + 5a e^{-x} + 4a e^{-x} = e^{-x} \Rightarrow 10a e^{-x} = e^{-x} \Rightarrow a = \frac{1}{10}$$

$$\Rightarrow y_p(x) = \frac{e^{-x}}{10}$$

$$\Rightarrow y(x) = y_0(x) + y_p(x) = \frac{e^x}{3} - \frac{e^{4x}}{3} + \frac{e^{-x}}{10}$$

(6)