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Grupa 164

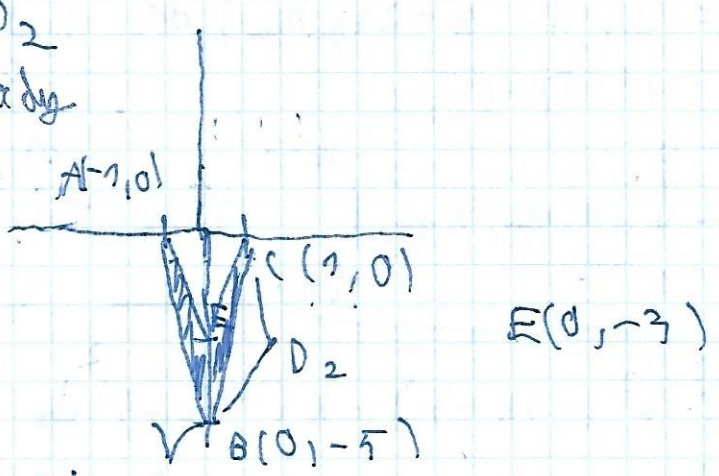
Examen Matematici Speciale

① $I = \iint_D (x^2 - 2*y)\, dx\, dy$

D patrulaterul cu vârfurile $A(-1,0), B(0,-5), C(1,0), E(0,-3)$

Fie ΔAEB D_1 și ΔBEC D_2

$\Rightarrow I = \iint_{D_1} (x^2 - 2*y)\, dx\, dy + \iint_{D_2} (x^2 - 2*y)\, dx\, dy$



Găsim ecuațiile dreptelor:

$AE: y = mx + b$

$m = \frac{-3-0}{0-(-1)} = -3 \quad \left(= \frac{y_B - y_A}{x_B - x_A}\right)$

$A: 0 = -3 \cdot -1 + b \Rightarrow b = -3 \Rightarrow$ ne AE, $y = -3x - 3$

$AB: y = mx + b$, $m = \frac{y_B - y_A}{x_B - x_A} = \frac{-5-(0)}{0-(-1)} = -5$

$A: 0 = -5 \cdot -1 + b \Rightarrow b = -5 \Rightarrow$ ne AB, $y = -5x - 5$

$EC: y = mx + b$, $m = \frac{y_C - y_E}{x_C - x_E} = \frac{0-(-3)}{1-0} = 3$

$C: 0 = 1 \cdot 3 + b \Rightarrow b = -3 \Rightarrow$ ne EC, $y = 3x - 3$

$BC: y = mx + b$, $m = \frac{y_C - y_B}{x_C - x_B} = \frac{0-(-5)}{1-0} = 5$

$C: 0 = 1 \cdot 5 + b \Rightarrow b = -5 \Rightarrow$ ne BC, $y = 5x - 5$

1) $AE: y = -3x - 3$

$AB: y = -5x - 5$

$EC: y = 3x - 3$

$BC: y = 5x - 5$

$D_1: x \in [-1, 0] \quad y \in [-5x - 5, -3x - 3]$

$D_2: x \in [0, 1], \; y \in [5x - 5, 3x - 3]$

2) $I = \iint_{[-1,0]\cdot[-5x-5,\,-3x-3]} (x^2 - 2xy)\,dx\,dy + \iint_{[0,1]\cdot[5x-5,\,3x-3]} (x^2 - 2xy)\,dx\,dy =$

$= \int_{-1}^{0} \left(\int_{-5x-5}^{-3x-3} x^2 - 2xy \, dy \right) dx + \int_{0}^{1} \left(\int_{5x-5}^{3x-3} x^2 - 2xy \, dy \right) dx =$

$= \int_{-1}^{0} \left(x^2 y - x y^2 \Big|_{-5x-5}^{-3x-3} \right) dx + \int_{0}^{1} \left(x^2 y - x y^2 \right) \Big|_{5x-5}^{3x-3} =$

$= \int_{-1}^{0} x^2(-3x-3) - x(-5x-5)^2 \, dx + \int_{0}^{1} x^2(3x-3) - x(5x-5)^2 \, dx$

$= \int_{-1}^{0} -3x^3 - 3x^2 - 25x^3 - 25x - 25x^2 \, dx + \int_{0}^{1} (3x^3 - 3x^2 - 25x^3 - 25x + 25x^2) \, dx$

$= \int_{-1}^{0} -28x^3 - 28x^2 - 25x \, dx + \int_{0}^{1} -22x^3 + 22x^2 - 25x \, dx$

$= \left(-7x^4 - \frac{28}{3}x^3 - \frac{25}{2}x^2 \right) \Big|_{-1}^{0} + \left(-\frac{11}{2}x^4 + \frac{22}{3}x^3 - \frac{25}{2}x^2 \right) \Big|_{0}^{1} =$

$= 7 - \frac{28}{3} + \frac{25}{2} - \frac{11}{2} + \frac{22}{3} - \frac{25}{2} = 7 - \frac{6}{3} - \frac{11}{2} = \frac{9}{2}$

$\Rightarrow \boxed{I = \frac{9}{2}}$

$$= \int_{-1}^{0} x^2(-3x-3) - x(-3x-3)^2 - x^2(-5x-5) + x(-5x-5)^2 \, dx$$

$$+ \int_{0}^{1} x^2(3x-3) - x(3x-3)^2 - x^2(5x-5) + x(5x-5)^2 \, dx$$

$$= \int_{-1}^{0} -3x^3 - 3x^2 - 9x^3 - 9x + 18x^2 + 5x^3 + 5x^2 + 25x^3 + 25x + 50x^2 \, dx$$

$$+ \int_{0}^{1} 3x^3 - 3x^2 - 9x^3 - 9x - 18x^2 - 5x^3 + 5x^2 + 25x^3 + 25x - 50x^2 \, dx$$

$$= \int_{-1}^{0} 18x^3 + 70x^2 + 16x \, dx + \int_{0}^{1} 14x^3 - 66x^2 + 16x \, dx =$$

$$= \frac{18}{4}x^4 + \frac{70}{3}x^3 + 8x^2 \Big|_{-1}^{0} + \frac{14x^4}{4} - \frac{66}{3}x^3 + 8x^2 \Big|_{0}^{1}$$

$$= \frac{-18}{4} + \frac{70}{3} - 8 + \frac{14}{4} - \frac{66}{3} + 8 = 2 + \frac{4}{3} = \frac{10}{3} \Rightarrow \boxed{I = \frac{10}{3}}$$