Light Transport Techniques for Tensor Field Visualization

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Outline

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Motivation

- visualization in general is needed to generate a more readable, explorable and intuitive representation
- tensor representations are needed to describe a directional distribution for each point in space, when:
  - e.g. for vector fields: to describe the directionally dependent spatial gradient called Jacobian-matrix,
  - e.g. for fluid and solid continuum mechanics: to describe a whole distribution of stresses
  - e.g. for DT-MRI: diffusion tensor - magnetic resonance imaging: to describe the diffusion characteristics of water molecules within tissue

Objectives

- a light transport model (propagation scheme) following basic but crucial physical principles,
- application of this model for tensor field visualization interpreting tensors as light transmission properties,
- a FTLE (Finite-time Lyapunov exponents)-related approach called light transport gradient (LTG) for visualizing key structures, namely LCS (Lagrangian coherent structures) in 2D second-order tensor fields, and
- application of our approach to both synthetic and real data involving brain and heart datasets.

Related Work - Global Illumination Methods

- Discrete Ordinates Method: discretizes RTE in both spatial and angular domain
- Lattice-Boltzmann method: light propagation modeled as a diffusion process
- Light Propagation Volumes: light exchanged between neighboring cells and stored locally in capacities

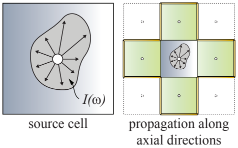


Fig.: Light Propagation Volumes, *Source:* 1

Related Work - Symmetric Tensor Field Visualization

- Glyphs: represent anisotropy with shape and orientation
- Tensor Field Lines (TFLs): follow the eigenvector along tensor field lines
- Tensorlines: introduce artificial inertia on TFLs to increase stability
- HyperLIC: use Line Integral Convolution from Vector Field Visualization on TFLs
- FTLE: exploit the gradient of the flow map of TFLs to generate an FTLE field
- Scalar Measures: tensor magnitude, diffusivity, fractional anisotropy, anisotropy coefficients (measures)

Related Work - Asymmetric Tensor Field Visualization

- Dual Eigenvectors:
- Pseudo Eigenvectors:
- Scalar Measures: tensor magnitude, tensor mode, isotropy index

Outline

Fundamentals

Important Terms

$$\text{prior (data) distribution} : p_{data} = p_z(z) \sim z$$

$$\text{generator function} : G(z, \theta_g) \qquad \theta_g : \text{generator parameters}$$

$$\text{discriminator function} : D(x, \theta_d) \qquad \theta_d : \text{discriminator parameters}$$

whereas $D(x)$ represents the propability that x is drawn from the training samples (prior data distribution) rather than p_g.

Mathematical Formulation

Formal Definition

Two-Player Minimax (specific: zero-sum) game with value function $V(G, D)$:

$$\min_G \max_D V(D, G) = \mathbb{E}_{x \sim p_{data}(x)}[\log D(x)] + \mathbb{E}_{z \sim p_z(z)}[\log(1 - D(G(z)))]$$

$$\Rightarrow \max V(D, G) = 0 \qquad \min V(D, G) = -\infty$$

outer loop: minimize $V(D, G)$ for 1 step | inner loop: maximize $V(D, G)$ for k steps

$\rightarrow$ prevents the network from overfitting the training data

$\rightarrow$ stop when gradient of G is very small $\Rightarrow$ slowly changing ($\ddot{G} \approx 0$)

Precautions

Formal Definition

Two-Player Minimax (specific: zero-sum) game with value function $V(G, D)$:

$$\min_D \max_G V(D, G) = \mathbb{E}_{x \sim p_{data}(x)}[\log D(x)] + \mathbb{E}_{z \sim p_z(z)}[\log(1 - D(G(z))]$$

$$\Rightarrow \max V(D, G) = 0 \qquad \min V(D, G) = -\infty$$

Caveat: Early Learning

$\rightarrow$ Gradients of G might be poor/vanishing:

$\Rightarrow$ D can reject samples with high confidence ($\log(1 - D(G(z))$ saturates in this case)

$\Rightarrow$ instead of minimizing $\min \log(1 - D(G(z))$, we can assign G to maximize $\log D(G(z)$ $\rightarrow$ same fixed point of dynamics, but yields much stronger gradients

SGD: Stochastic Gradient Descent

Gradients

Discriminator Gradient:

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^{m} [\log D(x^{(i)}) + \log(1 - D(x^{(i)}))]$$

Generator Gradient:

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^{m} [\log(1 - D(x^{(i)}))]$$

Outline

Architecture

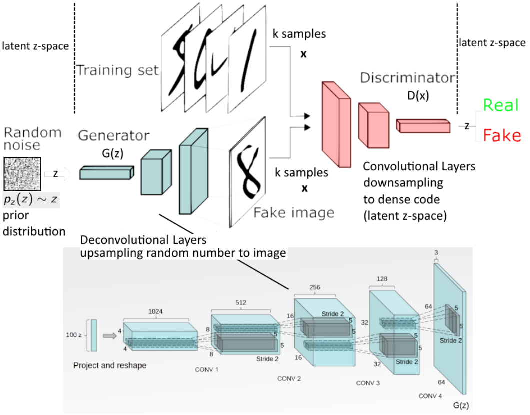


Fig.: Visualization of Architecture (general/DCGAN), *Source:* 2

Global Optimum

Global Optima

Discriminator Optimum:

$$D_G^*(\mathbf{x}) = \frac{p_{data}(\mathbf{x})}{p_{data}(\mathbf{x}) + p_g(\mathbf{x})}$$

Generator Optimum:

$$p_g = p_{data} = p_z \Rightarrow D_G^*(\mathbf{x}) = \frac{p_{data}(\mathbf{x})}{p_{data}(\mathbf{x}) + p_g(\mathbf{x})} = \frac{1}{2}$$

Convergence of the Algorithm

Adversarial Nets

Adversarial Nets represent a limited family of generative p_g distributions via the function $G(z, \theta_g)$ with p_g being indirectly optimized through optimization of θ_g. Lack of theoretical validity of multilayer perceptrons, but excellent performance in practice.

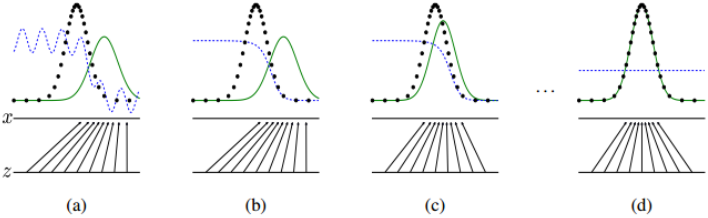


Fig.: Visualization of distributions/mapping $z \rightarrow x$, *Source:* 3

Outline

Training

Datasets

- MNIST
- TFD: toronto face database
- CIFAR-10 [21]

Experiments

Test scenario: Estimation of propability of "test set data" under (behind) $p_g \rightarrow$ fit a Gaussian Parzen window to the samples generated with G and report the log-likelihood considering test data samples obtaining variance σ from cross-validation with the test dataset.

Model	MNIST	TFD
DBN [3]	138 ± 2	1909 ± 66
Stacked CAE [3]	121 ± 1.6	2110 ± 50
Deep GSN [3]	214 ± 1.1	1890 ± 29
GAN	225 ± 2	2057 ± 26

Tab.: Log-likelihood estimates of p_g

Sampling from the Model - 1

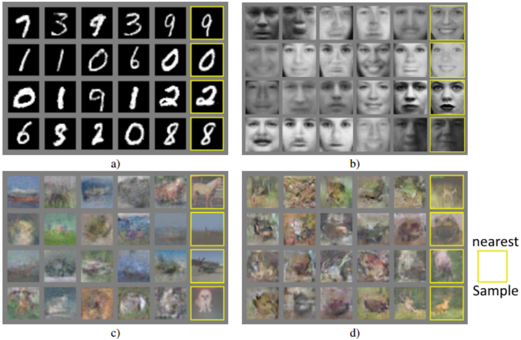


Fig.: Visualization of Samples from the model, *Source:* 3

Sampling from the Model - 2

MNIST dataset

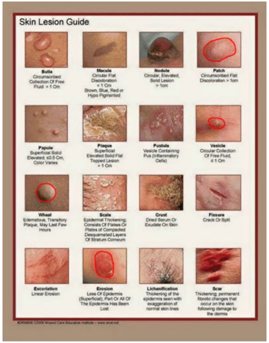
MNIST digits linearly interpolated to evaluate diversity of generative distribution p_g of GAN approach..

Fig.: Linearly interpolating digits in z-space of G after training, *Source:* 3

Application in Biomedical Image Analysis

Application Example: Skin Lesion Segmentation

- Generator G: Fully (de-)convolutional neural network to synthesize (generate) valid segmentation masks
- Discriminator D: another convolutional neural network to distinguish synthetic (fake) from real masks



Skin lesions 101

Fig.: Segmented Skin Lesions, *Source:* [1]

[1] https://suprememedical.com/Product/skin-lesion-guide

Benefits and Drawbacks

Advantages and Disadvantages

⊕ Advantages	⊖ Disadvantages
• no need for Markov Chains (MCMC-approaches)	• no explicit representation of $p_g(x)$
• training through standard backpropagation	• D needs to be sufficiently synchronized with G (D must stay inline)
• incorporation of wide variety of functions possible → applicability	→ avoidance of Helvetica scenario:
• able to represent sharp, degenerate distributions	G collapsing too many latent z's (e.g. random variables) to the same sample x, leading to insufficient generative diversity

Tab.: Advantages and Disadvantages of GANs

Outline

Summary

- GANs: new framework for estimating generative models defined by multilayer perceptrons (Convolutional Layers) trained by standard backpropagation $\rightarrow$ no need for any Markov chains (MCMC-approaches), which can have problems Mixing (Converging)
- Results Samples considered to be competitive with those of the state-of-the-art generative models
- Empirical Evaluation (Fitting a Gaussian Parzen window to measure distribution similarity of p_g as log-likelihood metric) indicates comparable scores than achieved for state-of-the-art methods like DBNs, Stacked CAE, GSN, Adversarial nets and comparable validity/representativity

Conclusion

- approach is, besides DGMs (directed graphical models), the only one, inducing no problems or further elaborations for sampling (generating samples) or training $\rightarrow$ rather simple implementation
- experiments show comparable similarities against state-of-the-art methods in log-likelihood score and variance in matching the prior distribution p_z with the generative one (p_g), but the method proposed is regarded to be more simple than others
- the synchronization of D is yet an effortful factor, since sufficient reasoning for the number of steps of the inner loop is needed to avoid the previously named "Helvetica scenario"!
- future applications might involve the synthetic generation of morphologically correct segmentation masks of seperatable objects, fake images (databases), keys/passwords (cryptography), image processing

Outlook

Straighforward extensions:

1. Conditional generative Model $p(\mathbf{x}|c)$ w. adding condition c
2. Learned approximate inference: Predict prior-z w. given latent x
3. Modeling of multiple Conditionals: $p(\mathbf{x}_S|\mathbf{x}_{\not S})$
4. Semi-Supervised Learning: Better Performance w. partially labeled training data
5. Efficiency improvements: Better methods to coordinate G and D, better distributions to sample z from during training

Questions?

Questions?

Sources and further reading

Literature

1. Goodfellow, Ian, et al. "Generative adversarial nets."
2. Izadi, Saeed & Mirikharaji, Zahra & Kawahara, Jeremy & Hamarneh, Ghassan. Generative adversarial networks to segment skin lesions.

Images

1. https://www.sevendaysvt.com/vermont/some-counterfeiters-still-do-it-old-school/Content?oid=3276910
2. https://www.altoros.com/blog/the-diversity-of-tensorflow-wrappers-gpus-generative-adversarial-networks-etc/ & https://medium.freecodecamp.org/an-intuitive-introduction-to-generative-adversarial-networks-gans-7a2264a81394
3. Goodfellow, Ian, et al. "Generative adversarial nets."