Light Transport Techniques for Tensor Field Visualization

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Master’s Thesis Presentation
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19. Juli 2019

Outline

1. Introduction
2. Fundamentals
3. Method
4. Results and Evaluation
5. Conclusion and Future Work

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Motivation

- visualization in general is needed to generate a more readable, explorable and intuitive representation
- tensor representations are needed to describe a directional distribution for each point in space, when:
  - e.g. for vector fields: to describe the directionally dependent spatial gradient called Jacobian-matrix,
  - e.g. for fluid and solid continuum mechanics: to describe a whole distribution of stresses
  - e.g. for DT-MRI: diffusion tensor - magnetic resonance imaging: to describe the diffusion characteristics of water molecules within tissue

Objectives

- a light transport model (propagation scheme) following basic but crucial physical principles,
- application of this model for tensor field visualization interpreting tensors as light transmission properties,
- a FTLE (Finite-time Lyapunov exponents)-related approach called light transport gradient (LTG) for visualizing key structures, namely LCS (Lagrangian coherent structures) in 2D second-order tensor fields, and
- application of our approach to both synthetic and real data involving brain and heart datasets.

Related Work - Global Illumination Methods

- Discrete Ordinates Method: discretizes RTE in both spatial and angular domain
- Lattice-Boltzmann method: light propagation modeled as a diffusion process
- Light Propagation Volumes: light exchanged between neighboring cells and stored locally in capacities

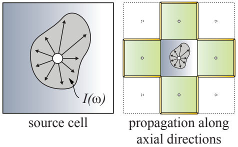

Fig.: Light Propagation Volumes, *Source:* ①

Related Work - Symmetric Tensor Field Visualization

- Glyphs: represent anisotropy with shape and orientation
- Tensor Field Lines (TFLs): follow the eigenvector along tensor field lines
- Tensorlines: introduce artificial inertia on TFLs to increase stability
- HyperLIC: use Line Integral Convolution from Vector Field Visualization on TFLs
- FTLE: exploit the gradient of the flow map of TFLs to generate an FTLE field
- Scalar Measures: tensor magnitude, diffusivity, fractional anisotropy, anisotropy coefficients (measures)

Related Work - Asymmetric Tensor Field Visualization

- Dual Eigenvectors: use complex conjugate eigenvectors as co-visualization for the complex domain along with ordinary eigenvectors to represent the real domain
- Pseudo Eigenvectors: extension for dual eigenvectors to a full set or graph
- Scalar Measures: tensor magnitude, tensor mode, isotropy index

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Fundamentals

Tensor Fields

- order-o generalization of a matrix with indices of run length n for $n \times n$ matrices
- number arrays following covariant or contravariant transformation rules
- represenation needed for a whole directional distribution for each point in space

Tab.: Tensor Shapes

order	0	1	2	3	...	o
shape	scalar	vector	matrix	“3D matrix”	...	order-o matrix

Fundamentals

Cauchy stress tensor

- classical physical example of a tensor
- consistent of 3 stress vectors arranged in row-major order
- these represent the magnitude and orientation of the resulting stress at plane x, y, z in direction x, y, z
- maps an incoming direction vector $\mathbf{n}$ a resulting stress vector $\mathbf{T}^{(\mathbf{n})} = \mathbf{n} \cdot \boldsymbol{\sigma}$ per transformation

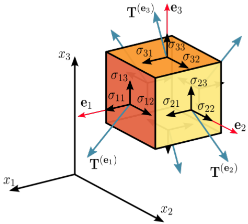


Fig.: Cauchy stress tensor, *Source:* ②

Polar Coordinates

Conversion Formulas

Polar↦Cartesian

$$x = r\cos\omega,$$
$$y = r\sin\omega.$$

Cartesian↦Polar

$$r = \sqrt{x^2 + y^2},$$
$$\omega = \operatorname{atan2}(y, x).$$

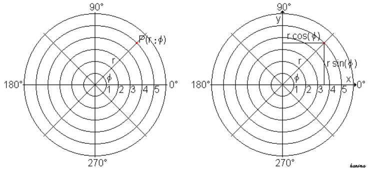


Fig.: Polar coordinates, *Source:* ③

- polar function $f : \omega \mapsto r$ maps each angle ω a magnitude $r(\omega)$

Principal Component Analysis

Matrix Decompositions

Eigenvalue Decomposition: $\mathbf{A} = \mathbf{RSR}^*$
Singular Value Decomposition (SVD): $\mathbf{A} = \mathbf{U\Sigma V}^*$

- used to determine the main directions of variance (principal components) in stochastic data
- used to determine the subsequent base transformations composing affine transformations

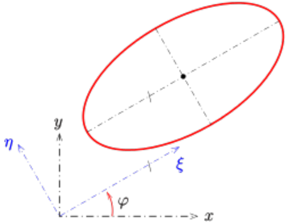


Fig.: Principal component analysis, *Source:* ④

Principal Component Analysis

Finite-Time Lyapunov exponents

Definition

Local: $\sigma = \frac{1}{|T|} \ln \frac{\Delta}{\delta}$

Global: $\sigma(\mathbf{x}) = \frac{1}{|T|} \ln \|\nabla \varphi(\mathbf{x})\|_2$

whereas $\|A\|_2 = \sqrt{\lambda_{max}(A^T A)}$ is the spectral norm of matrix A

- measure for separation abilities of time-dependent dynamical systems concerning massless tracer particles
- in particular concerning Lagrangian view in vector fields: placing two close-by particles into and moving with the flow

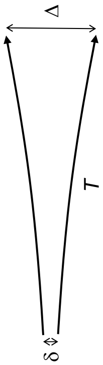


Fig.: Path line separation, *Source:* [4]

Glyphs

- glyphs are used to represent the principal component ellipsoid, i.e., the anisotropy characteristics and/or tensor magnitude
- glyphs can be found in varying shapes, we define ellipsoids here for simplicity

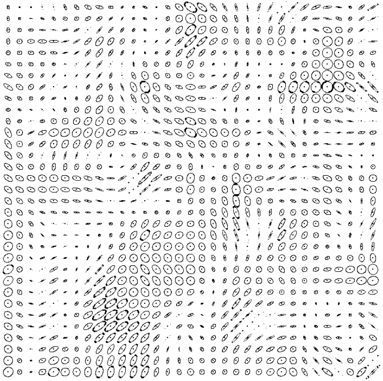


Fig.: 2*D* glyphs for Random field

Tensor Field Lines

Procedure

follow major/minor eigenvectors along pathlines

- ambiguity for nearly isotropic cases: small changes in magnitude effect large changes in glyph orientation
- therefore susceptive to noise in data

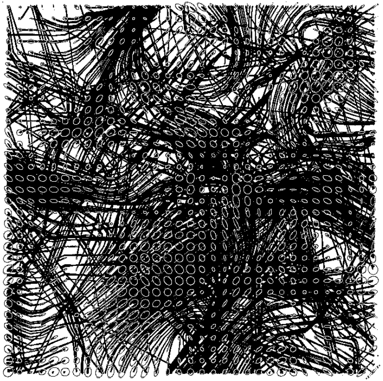


Fig.: $2D$ glyphs and tensor field lines

Asymmetric Tensor Fields

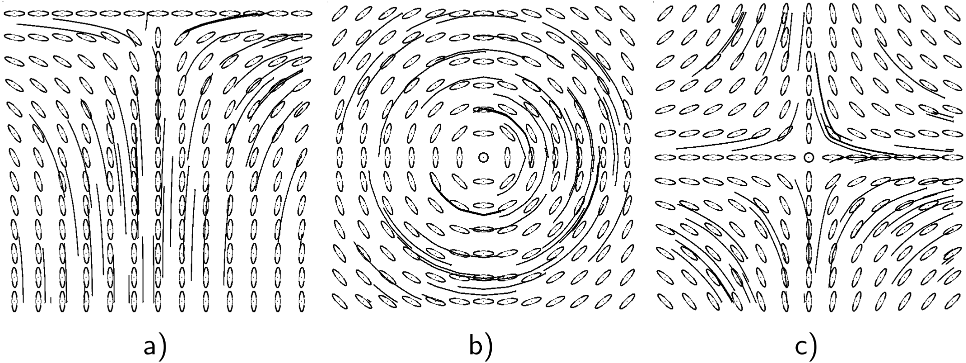


Fig.: Several test fields: a)Drain, Rings, Inverse

Symmetric Tensor Fields

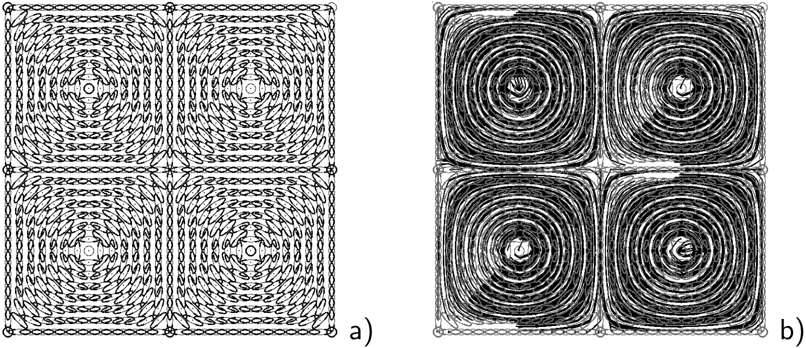
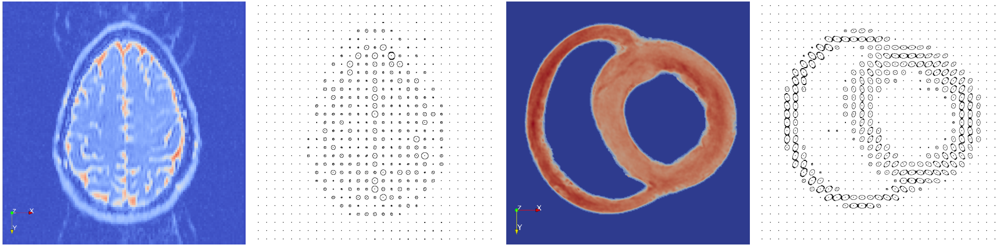


Fig.: Gyre test field: a) glyphs, b) tensor field lines for a)

Asymmetric Tensor Fields



a) b) a) b)

Fig.: Brain and Heart dataset: a) tensor magnitude, b) ellipsoid glyphs

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Propagation Scheme

- light propagation scheme exchanging energies between neighboring cells
- propagates intensities from initially set cells to corresponding neighbors
- eventually to model anisotropy characteristics of tensor field on top

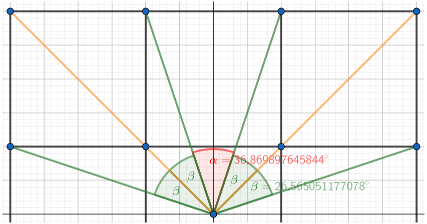


Fig.: Propagation Scheme

Propagation Scheme

$$\varepsilon_\alpha = \frac{\frac{\beta}{\alpha+2\beta}}{\frac{\beta}{\alpha+2\beta}+\frac{\beta}{2\beta}} \approx 0.362291$$

$$\varepsilon_\beta = \frac{\frac{\beta}{2\beta}}{\frac{\beta}{\alpha+2\beta}+\frac{\beta}{2\beta}} \approx 0.63771$$

$$\Phi_t = \varepsilon_k \int I(\omega)\, d\omega$$

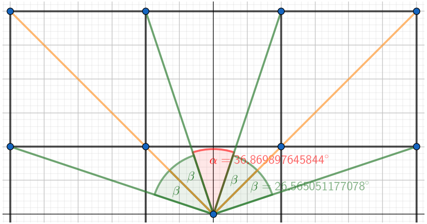


Fig.: Propagation Scheme

Propagation Scheme

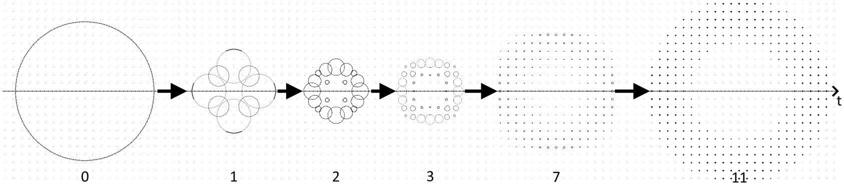


Fig.: Impulse response

$$\sum_{\cos_+} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos_+(\omega)\, d\omega \approx 2,$$

$$\cos_k(\omega) = \frac{\Phi_t}{\sum_{\cos_+}} cos_+(\omega - k \cdot \frac{\pi}{4}) \qquad with\ k \in [0,7]$$

Propagation Scheme - Procedure

Steps

1. Accumulation: Evaluating and Accumulating the Polar Profiles
2. Applying the linear combination (partition) weights
3. Injection: Scaling and Placing of a Cosine Lobe

For now, we are able to propagate intensities as circular waves in a 2D Cartesian grid.

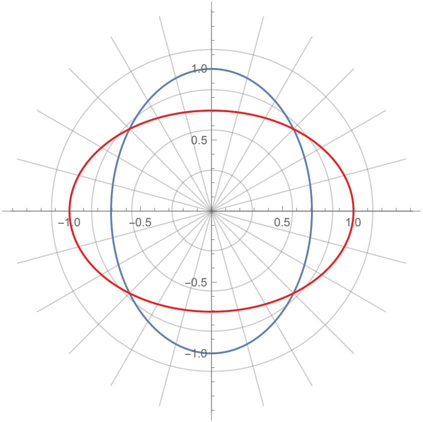
Transmission Profiles

Redefinition

$$\Phi_t = n_f \varepsilon_k \int T(\omega) I(\omega)\, d\omega$$

$$n_f = \frac{\bar{T}\bar{I}}{\overline{TI}}$$

- transmission profiles (cyan) are weighted with the intensity profiles (red) as a window function
- a normalization factor n_f is required to respect energy conservation principles



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Summary

- GANs: new framework for estimating generative models defined by multilayer perceptrons (Convolutional Layers) trained by standard backpropagation → no need for any Markov chains (MCMC-approaches), which can have problems Mixing (Converging)
- Results Samples considered to be competitive with those of the state-of-the-art generative models
- Empirical Evaluation (Fitting a Gaussian Parzen window to measure distribution similarity of p_g as log-likelihood metric) indicates comparable scores than achieved for state-of-the-art methods like DBNs, Stacked CAE, GSN, Adversarial nets and comparable validity/representativity

Conclusion

- approach is, besides DGMs (directed graphical models), the only one, inducing no problems or further elaborations for sampling (generating samples) or training → rather simple implementation
- experiments show comparable similarities against state-of-the-art methods in log-likelihood score and variance in matching the prior distribution p_z with the generative one (p_g), but the method proposed is regarded to be more simple than others
- the synchronization of D is yet an effortful factor, since sufficient reasoning for the number of steps of the inner loop is needed to avoid the previously named "Helvetica scenario"!
- future applications might involve the synthetic generation of morphologically correct segmentation masks of seperatable objects, fake images (databases), keys/passwords (cryptography), image processing

Outlook

Straighforward extensions:

1. Conditional generative Model $p(\mathbf{x}|c)$ w. adding condition c
2. Learned approximate inference: Predict prior-z w. given latent x
3. Modeling of multiple Conditionals: $p(\mathbf{x}_S|\mathbf{x}_{\not S})$
4. Semi-Supervised Learning: Better Performance w. partially labeled training data
5. Efficiency improvements: Better methods to coordinate G and D, better distributions to sample z from during training

Questions?

Questions?

Sources and further reading

Literature

1. Goodfellow, Ian, et al. "Generative adversarial nets."
2. Izadi, Saeed & Mirikharaji, Zahra & Kawahara, Jeremy & Hamarneh, Ghassan. Generative adversarial networks to segment skin lesions.

Images

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3. Goodfellow, Ian, et al. "Generative adversarial nets."