Mathematical Model for the Interleaved Bridgeless SEPIC PFC Converter

A Comprehensive Analysis from Fundamental Physics to Control Design

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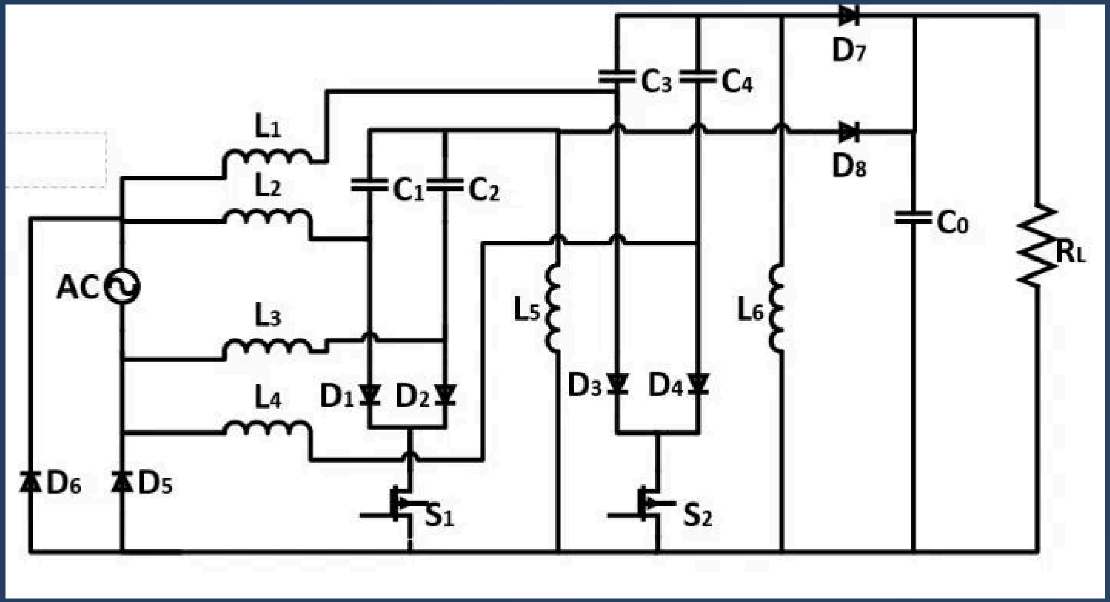
Introduction and Objectives

Document Objective

This presentation outlines the complete set of mathematical equations that describe the physical behavior of the proposed Interleaved Bridgeless SEPIC PFC Converter.

These equations form the necessary foundation for all subsequent stability analysis and control system design.



Three Levels of Mathematical Abstraction

This analysis is presented in three distinct levels of detail:

Level 1: The Fundamental Physics

Detailed "frame-by-frame" differential equations for each of the four primary switching states.

Level 2: The Global Averaged Model

A single, unified nonlinear equation that describes the converter's average behavior over a full switching cycle.

Level 3: The Linearized Small-Signal Model

The final, linear "nudge-test" model, which is the essential tool for control system design.

Deconstructing the System's Complexity

The proposed Interleaved Bridgeless SEPIC PFC Converter is a sophisticated topology. A rigorous mathematical model is necessary because the design presents three significant analytical challenges that prevent the use of simpler modeling techniques.

1 Higher-Order System

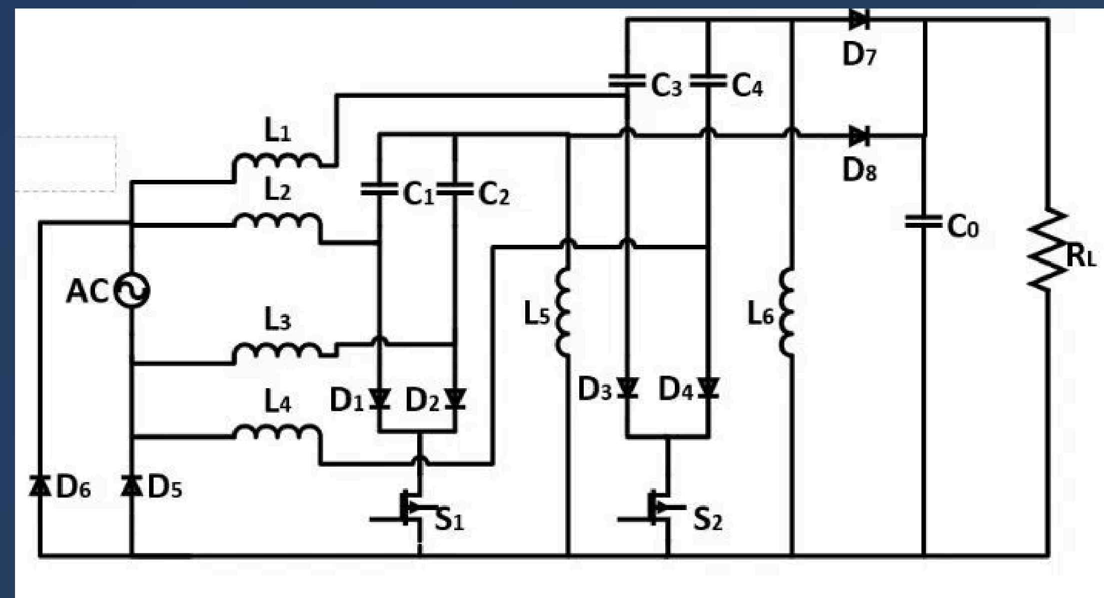
The combination of six inductors and three independent capacitive elements creates a **7th-order system**. This high order results in intricate internal dynamics and multiple potential resonant modes that must be carefully analyzed.

2 Time-Varying Nature

The bridgeless design means the circuit's physical current path changes depending on the AC input's polarity (positive vs. negative half-cycle). This makes the system's behavior time-variant, requiring special handling for each half-cycle.

3 Nonlinear Behavior

Like all switching converters, the relationship between the control input (the MOSFETs' duty cycles) and the system's response is inherently nonlinear. This is because the duty cycle multiplies the state variables, meaning linear control theory cannot be directly applied without a linearization step.



Key Modeling Assumptions

To manage the complexity of the 7th-order system and derive the core dynamic model, we begin with a set of standard, simplifying assumptions. These establish a solid, manageable baseline for the analysis. Parasitic effects and non-idealities can be added in later refinement stages.

- **Continuous Conduction Mode (CCM):** The analysis assumes all inductors operate exclusively in CCM, with no discontinuous current intervals.
- **Ideal Switches and Diodes:** Switches have zero on-resistance, infinite off-resistance, and instantaneous switching. Diodes have zero forward voltage drop.
- **Uncoupled Magnetics:** All inductors are treated as independent magnetic components with no mutual inductance.
- **Ideal Capacitors:** All capacitors are ideal, with no Equivalent Series Resistance (ESR) or leakage current.
- **No Input EMI Filter:** The model neglects any input filtering stage to isolate the converter's intrinsic dynamics.
- **Constant Power Load (CPL):** The output load is modeled as a Constant Power Load, where the current drawn is $I_{load} = P / v_{bus}$.
- **Quasi-Static Input:** Within a single high-frequency switching cycle, the low-frequency AC input voltage V_{in} is treated as a constant DC value.
- **Symmetrical Phases:** The two interleaved phases (Phase A and Phase B) are assumed to be perfectly identical in their component values.

Note: These assumptions are standard practice for deriving the fundamental small-signal model. The model's primary purpose is to analyze dynamic stability and design the feedback control system.

The Analytical Strategy

To solve this high-order, time-varying, and nonlinear system, we will employ a systematic, three-step analytical process. This methodology transforms the complex physical circuit into a final, linear model suitable for control design.

Step 1

Per-Topology Analysis

Action: Deconstruct the converter into its four fundamental switching states ("frames").

Method: Apply KVL and KCL to derive the exact, linear state-space model for each individual state.

Outcome: A set of A_k and B_k matrices representing the "ground truth" physics.

Step 2

State-Space Averaging

Action: Combine the four "frames" into a single, unified model.

Method: Blend the per-topology matrices using duty-cycle-dependent weights (w_k).

Outcome: A single, nonlinear averaged equation ($\dot{x} = A_{avg}(D)\cdot x + \dots$) that describes the converter's overall, low-frequency behavior.

Step 3

Small-Signal Linearization

Action: Create a linear "nudge-test" model valid for small ripples around a fixed DC operating point.

Method: Apply small AC perturbations and a Taylor series expansion to the averaged model.

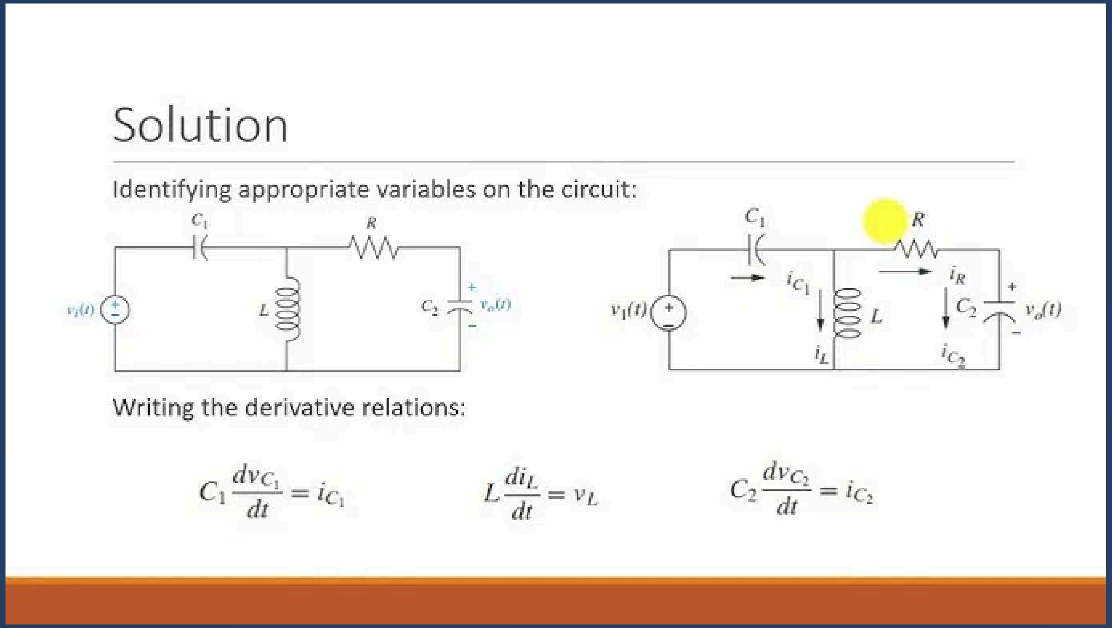
Outcome: The final, linearized state-space model ($\hat{x}' = A_{linear}\cdot\hat{x} + \dots$), which is the plant model used to derive transfer functions.

Level 1: The "Frame-by-Frame" Physics

The first step in our analysis is to derive the "ground truth" equations for the converter. We do this by analyzing the circuit's behavior in each of its four distinct switching topologies during the positive AC half-cycle.

The "Engine Cycle" Analogy

"Think of the converter's switching cycle like a four-stroke engine: ***Intake, Compression, Power, Exhaust****. In each stroke, the pistons and valves are in a different configuration, and the physics is different. Our 'Per-Topology' analysis is the same—we are examining the circuit's physics in each of its distinct operational states."*



The Four Topologies (Positive AC Half-Cycle)

Topology 11 (S1 ON, S2 ON)
Both phases are storing energy from the input.

Topology 10 (S1 ON, S2 OFF)
Phase A stores energy, while Phase B transfers its stored energy to the output.

Topology 01 (S1 OFF, S2 ON)
Phase B stores energy, while Phase A transfers its stored energy to the output.

Topology 00 (S1 OFF, S2 OFF)
Both phases are simultaneously transferring their stored energy to the output.

Level 1: Per-Topology State Equations

By applying Kirchhoff's Voltage and Current Laws (KVL/KCL) to each of the four simplified circuit topologies, we derive the following sets of first-order differential equations. Each set represents the exact, "raw" physics of the converter during that specific state.

Topology 11 (Both Switches ON)

Both phases store energy from input V_{in}. Output bus is isolated.

```
diL1a/dt = Vin / L1a
diL2a/dt = vCsa / L2a
dvCsa/dt = -iL2a / Csa
diL1b/dt = Vin / L1b
diL2b/dt = vCsb / L2b
dvCsb/dt = -iL2b / Csb
dvbus/dt = -P / (Cbus · vbus)
```

Topology 01 (S1 OFF, S2 ON)

Phase A transfers energy, Phase B stores energy.

```
diL1a/dt = (Vin - vCsa - vbus) / L1a
diL2a/dt = -vbus / L2a
dvCsa/dt = iL1a / Csa
diL1b/dt = Vin / L1b
diL2b/dt = vCsb / L2b
dvCsb/dt = -iL2b / Csb
dvbus/dt = (iL1a + iL2a) / Cbus - P / (Cbus · vbus)
```

Topology 10 (S1 ON, S2 OFF)

Phase A stores energy, Phase B transfers energy to output.

```
diL1a/dt = Vin / L1a
diL2a/dt = vCsa / L2a
dvCsa/dt = -iL2a / Csa
diL1b/dt = (Vin - vCsb - vbus) / L1b
diL2b/dt = -vbus / L2b
dvCsb/dt = iL1b / Csb
dvbus/dt = (iL1b + iL2b) / Cbus - P / (Cbus · vbus)
```

Topology 00 (Both Switches OFF)

Both phases transfer stored energy to the output bus.

```
diL1a/dt = (Vin - vCsa - vbus) / L1a
diL2a/dt = -vbus / L2a
dvCsa/dt = iL1a / Csa
diL1b/dt = (Vin - vCsb - vbus) / L1b
diL2b/dt = -vbus / L2b
dvCsb/dt = iL1b / Csb
dvbus/dt = (iL1a + iL2a + iL1b + iL2b) / Cbus - P / (Cbus · vbus)
```

These equations represent the ground truth of the circuit's behavior and form the building blocks for the averaged model.

Handling AC Polarity: The s = sign(V_{in}) Variable

The Challenge of Bridgeless Topology

The bridgeless topology behaves like two different circuits: one for the positive AC half-cycle and one for the negative. Deriving separate models for each would be redundant and inefficient.

The Solution: A Polarity Switch Variable

We use a simple variable **s** to represent the polarity of the input voltage:

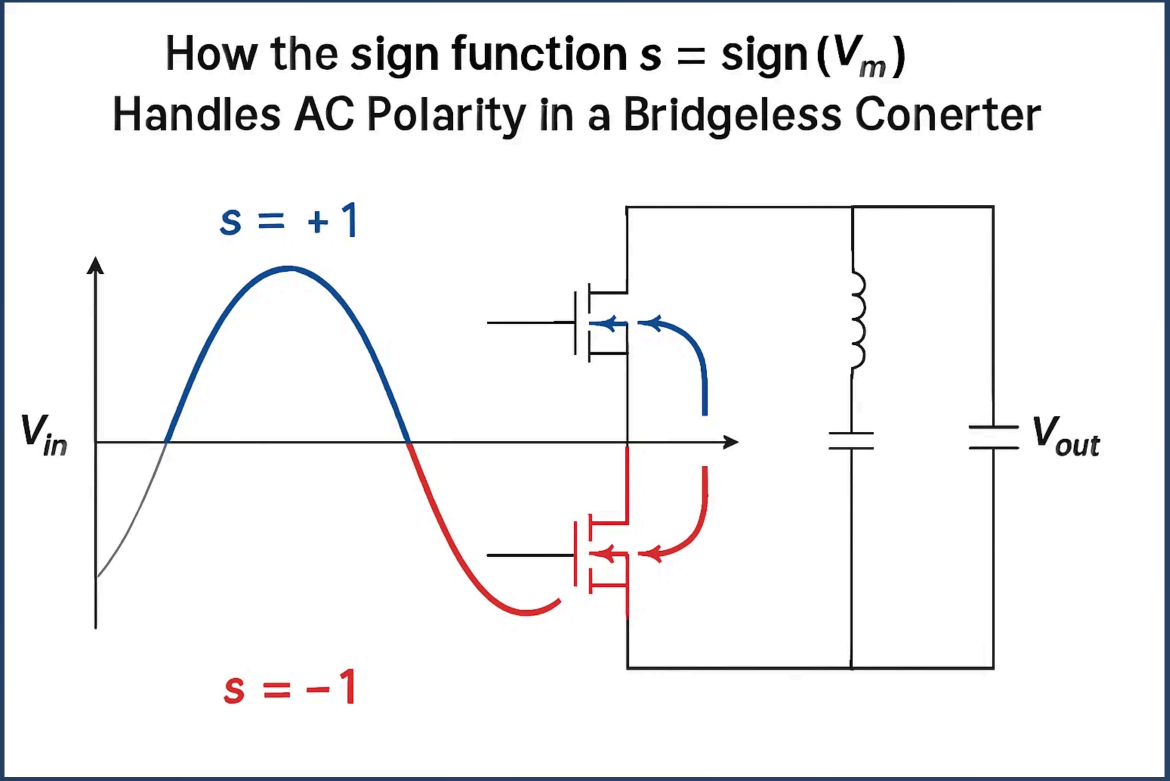
$$s = \text{sign}(V_{in})$$

When $V_{in} > 0$, **s = +1**
When $V_{in} < 0$, **s = -1**

Key Insight

The **s** variable only multiplies the **input matrix $B_{in,avg}$**, not the internal state matrix A_{avg}:

$$B_{in,avg} = s \cdot (w_{11} \cdot B_{in,11} + w_{10} \cdot B_{in,10} + w_{01} \cdot B_{in,01} + w_{00} \cdot B_{in,00})$$



Physical Meaning

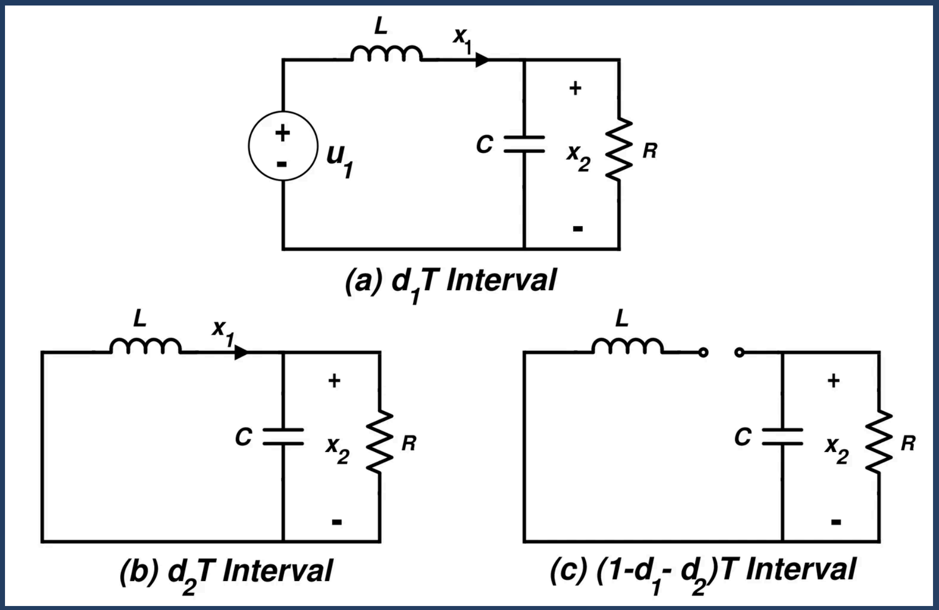
Internal Physics (A_{avg}): The way components exchange energy internally remains the same regardless of input polarity

Input's Effect ($B_{in,avg}$): The direction of energy "push" from the input voltage depends on polarity

Result: A single, unified model valid for the entire AC cycle

Level 2: The "Big Picture" Averaged Model

Analyzing the four separate sets of equations is impractical. The State-Space Averaging technique allows us to combine them into a single, unified model that describes the converter's overall, average behavior over a full switching cycle.



Feynman Method Analogy: Average Horsepower

"Looking at the frame-by-frame physics is too complicated to understand the converter's performance. It's like trying to understand a car's speed by analyzing the exact pressure on one piston at every microsecond.

*Instead, we want the **average performance**. We don't care about the individual strokes; we care about the average horsepower the engine produces. The Averaged Model is the 'average horsepower' equation for our converter."*

The Averaging Formula

The process involves taking a weighted average of the state matrices from each of the four topologies. The final averaged state matrix A_{avg} is calculated as:

$$A_{avg} = w_{11} \cdot A_{11} + w_{10} \cdot A_{10} + w_{01} \cdot A_{01} + w_{00} \cdot A_{00}$$

A_k: System matrices derived from the Level 1 equations

w_k: Duty Cycle Weights, representing the fraction of time spent in each state

Key Insight

The resulting averaged model is a single, continuous-time representation of the converter. However, because the weights (w_k) are functions of the duty cycles (d_1, d_2), this model is **nonlinear**.

Important: A_{avg} and $B_{in,avg}$ are nonlinear functions of duty cycles (d_1, d_2).

Purpose: This model allows us to solve for the converter's steady-state DC operating point.

Visualizing the Interleaving Weights (w_k)

What They Represent

The weights w_{11}, w_{10}, w_{01}, and w_{00} represent the fraction of time the converter spends in each of its four possible switching states within a single switching period. They are determined by the duty cycles d_1 and d_2.

The Two Main Operating Regions

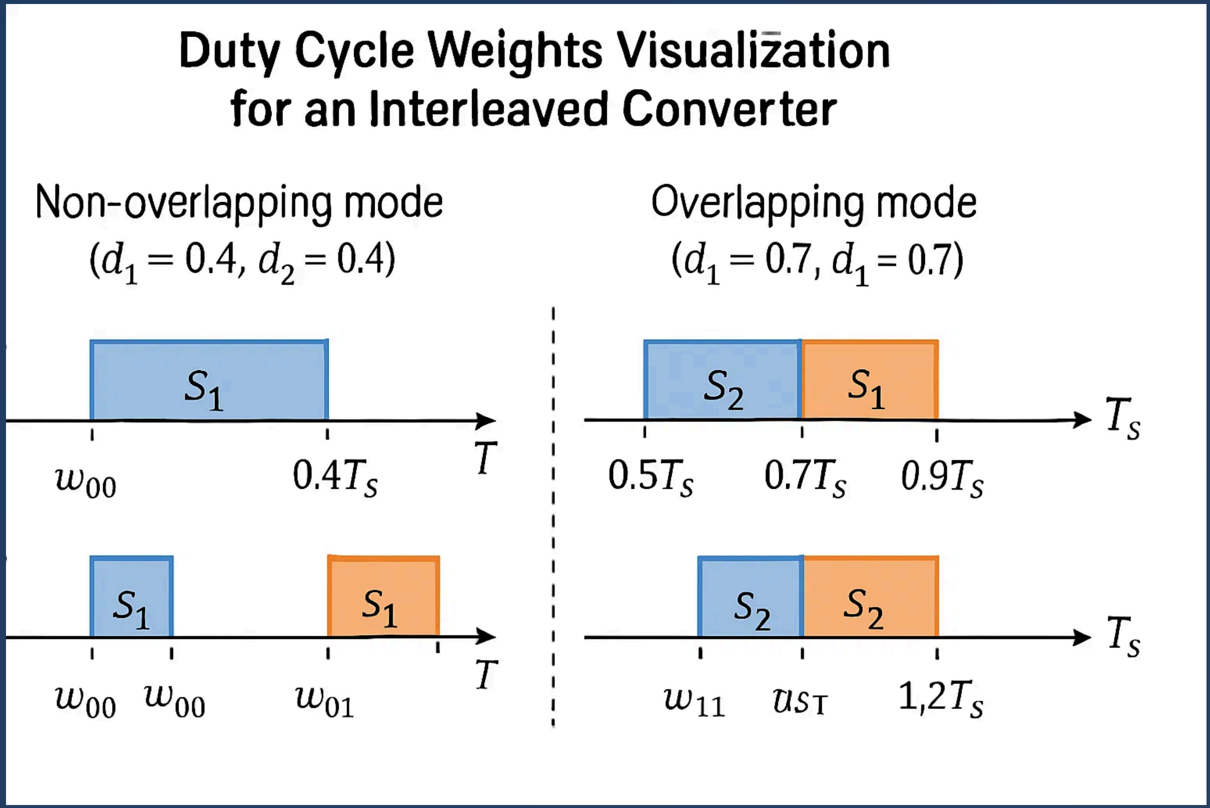
The calculation of these weights depends on whether the ON-times of the two switches overlap.

Case 1: Non-Overlapping Mode ($d_1 + d_2 < 1$)

- Occurs at lower duty cycles
- Switching Sequence: S1 ON → Both OFF → S2 ON → Both OFF

Case 2: Overlapping Mode ($d_1 + d_2 > 1$)

- Occurs at higher duty cycles
- Switching Sequence: Only S1 ON → Both ON → Only S2 ON



The Mathematical Formulas

The weights are calculated using the following piecewise formulas:

$$w_{11} = \max(0,\ d_1 + d_2 - 1)$$
$$w_{00} = \max(0,\ 1 - d_1 - d_2)$$
$$w_{10} = d_1 - w_{11}$$
$$w_{01} = d_2 - w_{11}$$

Key Insights

- The weights always sum to 1: $w_{11} + w_{10} + w_{01} + w_{00} = 1$
- In non-overlapping mode ($d_1 + d_2 < 1$), $w_{11} = 0$ and $w_{00} > 0$
- In overlapping mode ($d_1 + d_2 > 1$), $w_{11} > 0$ and $w_{00} = 0$
- At the boundary ($d_1 + d_2 = 1$), both $w_{11} = 0$ and $w_{00} = 0$

Level 2: The Averaged Equation of Motion

The Global Averaged Model

This single, unified matrix equation describes the converter's overall average behavior. It is the complete, nonlinear "equation of motion" for the converter before linearization.

$$\dot{x} = A_{avg}(d_1, d_2) \cdot x + B_{in,avg}(d_1, d_2, s) \cdot V_{in} + B_{load}(x)$$

Component Definitions

Averaged State Matrix A_{avg}:

$$A_{avg} = w_{11} \cdot A_{11} + w_{10} \cdot A_{10} + w_{01} \cdot A_{01} + w_{00} \cdot A_{00}$$

Represents the blended "Internal Physics" of the converter.

Averaged Input Matrix $B_{in,avg}$:

$$B_{in,avg} = s \cdot (w_{11} \cdot B_{in,11} + w_{10} \cdot B_{in,10} + w_{01} \cdot B_{in,01} + w_{00} \cdot B_{in,00})$$

Represents the blended "External Force" from the input, including the polarity switch s.

Nonlinear Load Vector $B_{load}(x)$:

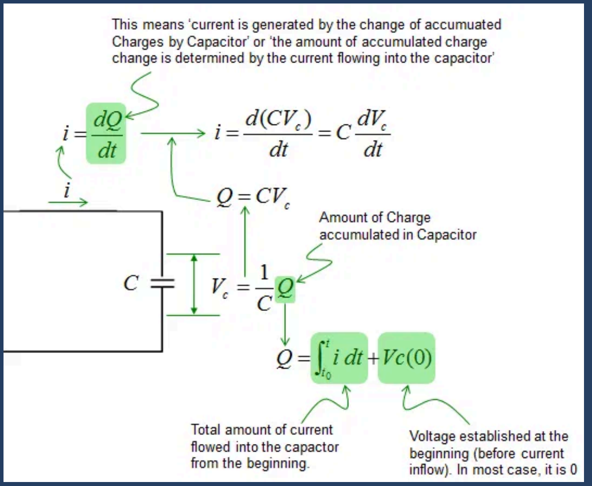
$$B_{load}(x) = [0, 0, 0, 0, 0, 0, -P/(C_{bus} \cdot v_{bus})]^T$$

Isolates the nonlinear effect of the Constant Power Load on the 7th state variable (bus voltage).

Duty Cycle Weights

Fraction of time spent in each interleaved state:

$$w_{11} = \max(0, d_1 + d_2 - 1)$$
$$w_{00} = \max(0, 1 - d_1 - d_2)$$
$$w_{10} = d_1 - w_{11}$$
$$w_{01} = d_2 - w_{11}$$



Key Characteristics of this Model

Unified: Accounts for all four switching states in a single equation.

Nonlinear: A_{avg} and $B_{in,avg}$ are functions of the control inputs (d_1, d_2), and the load term is a nonlinear function of the state v_{bus}.

Purpose: The primary use of this equation is to solve for the converter's steady-state DC operating point (x_0) by setting $\dot{x} = 0$.

Level 3: The "Nudge Test" Linearized Model

The Goal

The averaged model is still nonlinear and too complex for standard control design tools. Our final step is to create a linear approximation that is valid for small "ripples" or "nudges" around a fixed DC operating point.

Feynman Method Analogy: The Tangent Line

"Imagine you're standing at one specific point on a very curvy road. If you only look at a one-meter section right at your feet, it looks almost perfectly **straight**. This straight-line approximation is the Linearized Model.

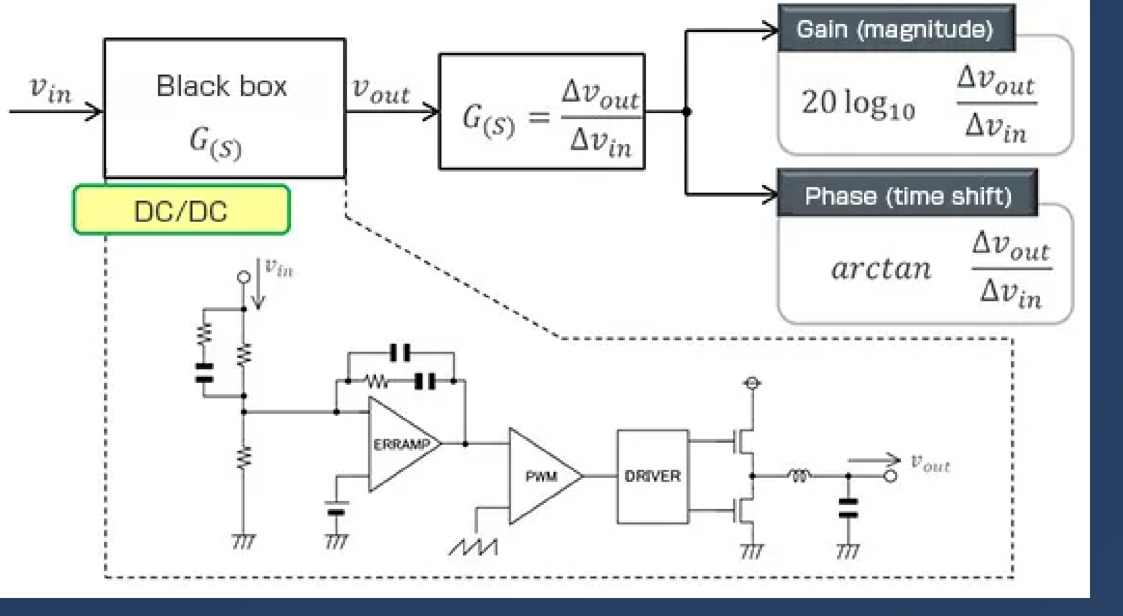
We mathematically 'stand' at a single, fixed operating point (e.g., 170V input, 1500W load) and perform a **'nudge test'**. We ask, 'If I nudge the duty cycle a tiny bit ($\hat{d}$), exactly how will the output voltage ripple ($\hat{v}_{out}$) respond?'"

The Process: Small-Signal Linearization

We take the nonlinear averaged model from Level 2 and apply a Taylor series expansion around a specific DC operating point (V_{in0}, d_1, d_2, x_0). By assuming the AC perturbations ("hat" variables) are very small, we can neglect all the higher-order terms. This transforms our nonlinear model into a Linear, Time-Invariant (LTI) model.

Why This Matters

This final linear equation is the ultimate goal of the modeling process. Because this model is now truly linear, we can finally apply the full power of linear control theory (Laplace transforms, Bode plots) to design a stable and robust PID controller.



Deriving the Control Input Matrix (B_d)

The "Nudge Test" in Detail

The B_d matrix answers the critical question for control design: "If I make a small change to the duty cycle ($\hat{d}$), how does the system's state ($\hat{x}'$) respond?" It represents the total sensitivity of the system to our control input.

> **Feynman Method Analogy: The Complex Machine**
>
> *"Imagine a complex machine with interconnected gears. The main control lever (d) does two things at once:*
>
> - It directly pushes on a fuel injector, adding energy. This is the **Direct Effect**.
> - It also slightly changes the gear ratios between the internal gears. This is the **Indirect Effect**.
>
> *The B_d matrix represents the TOTAL effect of nudging the control lever."*

The Concept

The derivation requires calculus to find the total sensitivity. The chain rule provides a precise and computationally efficient method:

$$B_d = (\partial A/\partial D)\cdot X + (\partial B/\partial D)\cdot V_{in}$$

Note: All matrices are evaluated at the DC operating point (V_{in0}, d_1, d_2, x_0).

The inverter is modeled as a gain with a time lag by

$$G_r(s) = \frac{K_{in}}{1 + sT_{in}} \qquad (9.118)$$

where

$$\left.\begin{aligned} K_{in} &= 0.65\,\frac{V_{dc}}{V_{cm}} \\ T_{in} &= \frac{1}{2f_c} \end{aligned}\right\} \qquad (9.119)$$

where V_{dc} is the dc-link voltage input to the inverter, V_{cm} is the maximum control voltage, and f_c is the switching (carrier) frequency of the inverter.

Implementation Using the Chain Rule

For the interleaved converter, B_d is calculated by:

Step 1: Calculate "Fixed-State Dynamics" (g_k)

Calculate the $\dot{x}$ vector for each topology at DC operating point x_0.

$$g_k = A_k\cdot x_0 + s\cdot B_{in,k}\cdot V_{in0}$$

Step 2: Calculate Derivatives of Weights

Find the partial derivatives of the interleaving weights ($\partial w_k/\partial d_1$ and $\partial w_k/\partial d_2$). This tells us how much the time spent in each state changes for a small nudge in each duty cycle.

Step 3: Sum the Contributions

Sum the contributions from each state to get the total sensitivity:

$$B_{d_i} = \Sigma\,(\partial w_k/\partial d_i)\cdot g_k$$

"The total change in the system's average behavior is the sum of (how much the time in each state changes) multiplied by (what the system's behavior was in that state)."

Level 3: The Final Linearized Equation of Motion

The Linearized Small-Signal Model

This is the final and most practical form of the equation for control design. It is a linear approximation valid for small "ripples" or "nudges" around a fixed DC operating point (V_{in0}, d_1, d_2, x_0).

$$\hat{x}' = A_{linear} \cdot \hat{x} + B_{in} \cdot \hat{v}_{in} + B_d \cdot \hat{u}_d + B_P \cdot \hat{P}$$

Component Definitions

A_{linear} (The System Matrix):

A 7×7 matrix of constant numbers. It is the A_{avg} matrix evaluated at the DC operating point, with the linearized "slope" of the nonlinear load added to element (7,7).

```
A_linear(7,7) = A_avg(7,7) + P_0 / (C_bus · v_bus0^2)
```

B_{in} (The Input Voltage Matrix):

A 7×1 vector of constant numbers that describes how input voltage changes ($\hat{v}_{in}$) affect the system.

B_d (The Control Input Matrix):

A 7×2 matrix of constant numbers that describes how duty cycle changes ($\hat{u}_d = [\hat{d}_1, \hat{d}_2]^T$) affect the system.

B_P (The Load Power Matrix):

A 7×1 vector of constant numbers that describes how load power changes ($\hat{P}$) affect the system.

```
B_P = [0, 0, 0, 0, 0, 0, -1/(C_bus·v_bus0)]^T
```

The inverter is modeled as a gain with a time lag by

$$G_r(s) = \frac{K_{in}}{1 + sT_{in}} \qquad (9.118)$$

where

$$\left.\begin{aligned} K_{in} &= 0.65\frac{V_{dc}}{V_{cm}} \\ T_{in} &= \frac{1}{2f_c} \end{aligned}\right\} \qquad (9.119)$$

where V_{dc} is the dc-link voltage input to the inverter, V_{cm} is the maximum control voltage, and f_c is the switching (carrier) frequency of the inverter.

Important: All matrices (A_{linear}, B_{in}, B_d, B_P) are evaluated at the DC operating point (V_{in0}, d_1, d_2, x_0).

Purpose

This final, linear equation is the **"plant model."** It is the definitive mathematical description of the converter's dynamics that will be used to derive the transfer functions needed for the PID controller design.

Key Characteristics

- **Linear:** All matrices are constant numbers
- **Time-Invariant:** Valid at the specified operating point
- **Small-Signal:** Accurate for small perturbations
- **Control-Ready:** Can be directly used with Laplace transforms and Bode plots

The Ultimate Goal: Transfer Functions for Control

From State-Space to Transfer Functions

The linearized state-space model we derived is the foundation, but control engineers need transfer functions in the Laplace domain. These transfer functions directly relate the control inputs to the system outputs, making controller design straightforward.

The Key Transfer Functions

$G_{id}(s)$: Current-to-Duty Transfer Function

Describes how changes in duty cycle affect the input current. This is critical for the inner current control loop, which ensures the converter draws a sinusoidal current from the AC mains for power factor correction.

$$G_{id}(s) = \hat{i}_{in}(s) / \hat{d}(s)$$

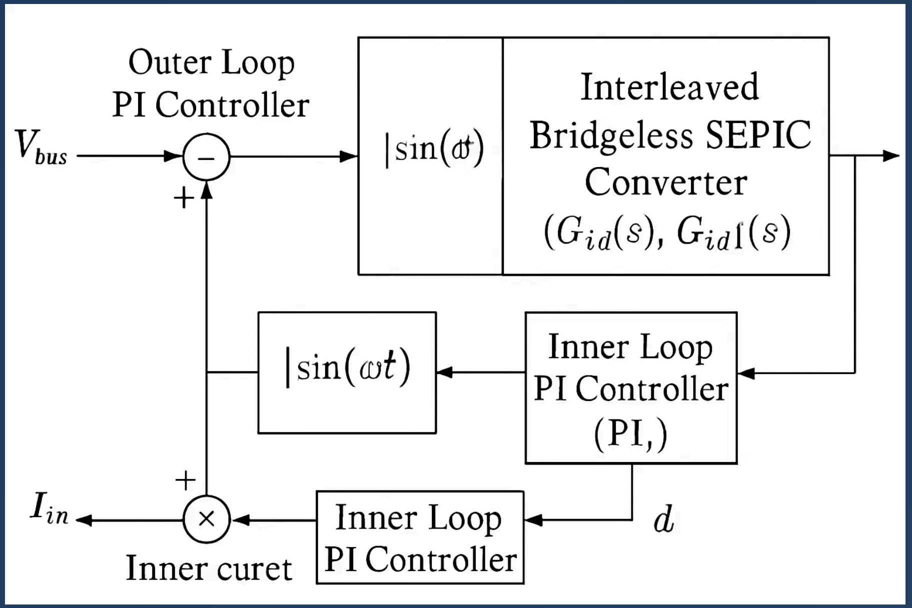
$G_{vd}(s)$: Voltage-to-Duty Transfer Function

Describes how changes in duty cycle affect the output bus voltage. This is essential for the outer voltage control loop, which regulates the DC bus voltage and maintains stability under load variations.

$$G_{vd}(s) = \hat{v}_{bus}(s) / \hat{d}(s)$$

Derivation Process

These transfer functions are obtained by applying the Laplace transform to the linearized state-space equation and solving for the desired output-to-input ratio. The process involves matrix algebra (using $(sI - A_{linear})^{-1}$) and selecting the appropriate rows and columns.



Two-Loop Control Architecture

The transfer functions enable the design of a cascaded two-loop control system:

Inner Current Loop: Uses $G_{id}(s)$ to design a fast PI controller that shapes the input current to follow a sinusoidal reference (multiplied by $|\sin(\omega t)|$), achieving near-unity power factor.

Outer Voltage Loop: Uses $G_{vd}(s)$ to design a slower PI controller that regulates the DC bus voltage to the desired setpoint and provides disturbance rejection against load changes.

Key Insight: The mathematical model we developed through all three levels culminates in these transfer functions, which are the essential tools for designing a stable, high-performance PFC controller.

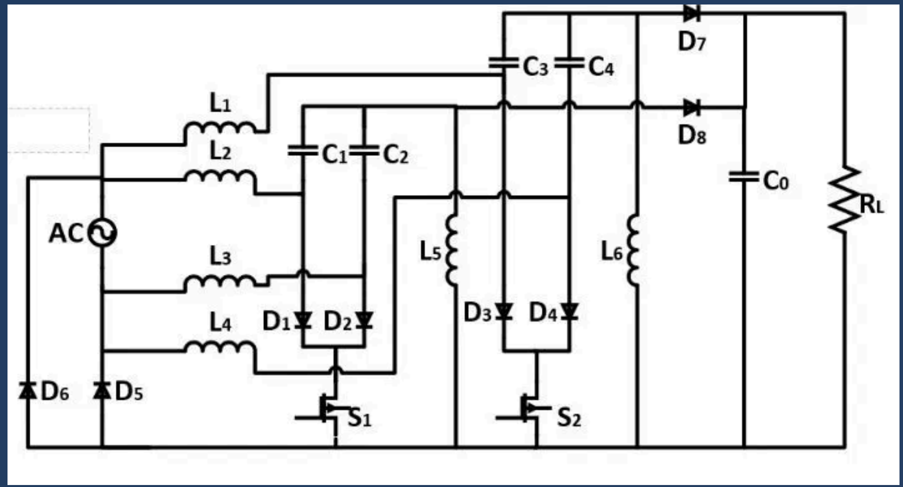
Conclusion and Next Steps

What We Have Achieved

We have successfully completed the mathematical modeling of the proposed Interleaved Bridgeless SEPIC PFC Converter. This rigorous, three-level analysis has transformed a complex, 7th-order, time-varying, nonlinear system into a practical, linear model ready for control design.

Key Milestones

- **Level 1 - Per-Topology Analysis:** Derived the exact differential equations for all four switching states using KVL and KCL, establishing the fundamental physics of the converter.
- **Level 2 - State-Space Averaging:** Combined the four sets of equations into a single, unified nonlinear model that describes the converter's average behavior over a switching cycle.
- **Level 3 - Small-Signal Linearization:** Created the final linear model valid for small perturbations around a DC operating point, enabling the use of standard linear control theory.
- **Bridgeless Topology Handling:** Implemented the sign function $s = \text{sign}(V_{in})$ to elegantly handle both AC half-cycles in a single unified model.
- **Interleaving Dynamics:** Developed the duty cycle weight formulas (w_k) to accurately model the interleaved operation in both overlapping and non-overlapping modes.



Next Steps in the Design Process

1 Derive Transfer Functions

Apply Laplace transforms to the linearized model to obtain $G_{id}(s)$ and $G_{vd}(s)$ for control design.

2 Design Two-Loop Controller

Design the inner current-loop PI controller and outer voltage-loop PI controller using frequency-domain techniques (Bode plots, gain/phase margins).

3 Validate via Simulation

Implement the controller in MATLAB/Simulink or PLECS and verify stability, transient response, and steady-state performance.

4 Hardware Prototype Testing

Build and test the physical converter prototype to validate the model and controller performance under real operating conditions.