# Mathematical Model for the Interleaved Bridgeless SEPIC PFC Converter

A Comprehensive Analysis from Fundamental Physics to Control Design

Presented To: Ts. Vinukumar Luckose

October 10, 2025

# Introduction and Objectives

## Document Objective

This document presents the complete set of mathematical equations that describe the dynamic behavior of the Interleaved Bridgeless SEPIC PFC Converter. The goal is to provide a rigorous, step-by-step derivation that bridges fundamental circuit physics and practical control design.

## Three Levels of Mathematical Abstraction

### Level 1: The "Frame-by-Frame" Physics

**Per-Topology State Equations**

Exact differential equations for each of the four switching states. This is the raw, unaveraged physics of the circuit.

### Level 2: The "Big Picture" Averaged Model

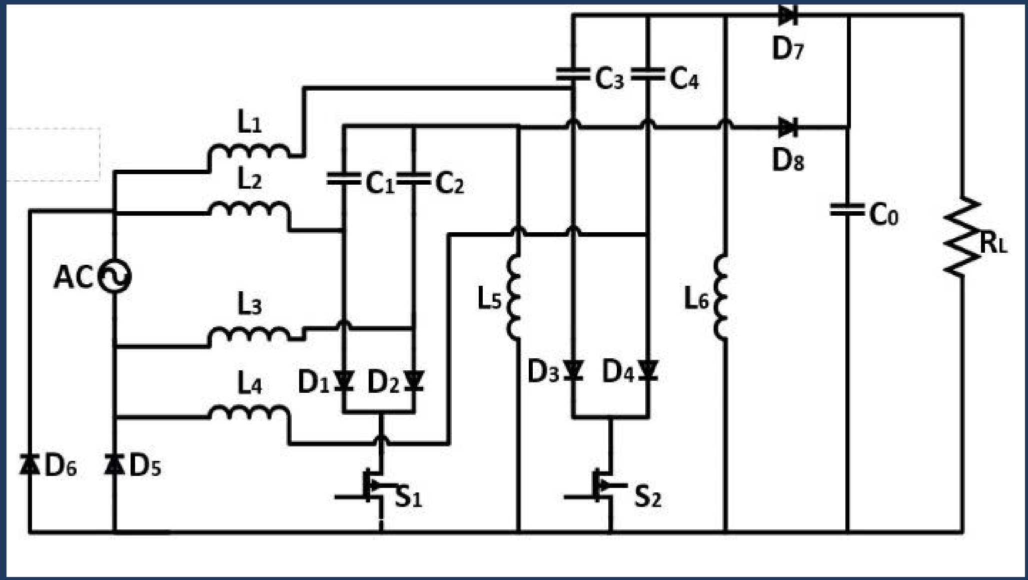
**Nonlinear State-Space Averaged Model**

A single, unified equation that blends the four topologies using duty cycle weights. This model captures the average behavior over a switching cycle.

### Level 3: The "Nudge Test" Linearized Model

**Small-Signal Linearized State-Space Model**

A linearized approximation around a steady-state operating point. This is the foundation for transfer function derivation and controller design.



# Deconstructing the System's Complexity

## 1. Higher-Order System (9th-Order)

### Six Inductors + Three Capacitors

The combination of **six independent inductors** and **three independent capacitive elements** creates a **9th-order system**. This high order results in intricate internal dynamics with multiple energy storage pathways and complex state interactions.

## 2. Time-Varying Nature

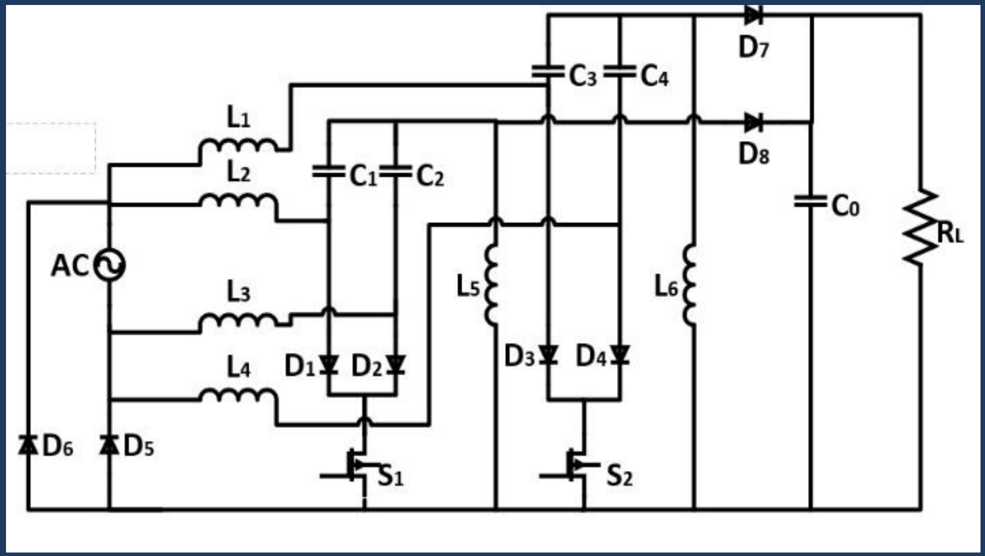
### Four Switching Topologies

The interleaved operation of two switches creates **four distinct circuit topologies** within each switching cycle. The system continuously transitions between these configurations, making the governing equations fundamentally time-varying and piecewise-defined.

## 3. Nonlinear Behavior

### Multiple Sources of Nonlinearity

The system exhibits nonlinearity from two primary sources: the **duty cycle modulation** (control inputs multiply state variables) and the **constant power load** (output power divided by bus voltage). These nonlinearities prevent direct application of linear control theory.



# Key Modeling Assumptions

To manage the complexity of the **9th-order system** and derive the core dynamic model, we begin with a set of standard, simplifying assumptions. These assumptions allow us to focus on the fundamental energy transfer mechanisms while maintaining mathematical tractability.

### 1. Continuous Conduction Mode (CCM)

All inductor currents remain positive throughout the switching cycle. This eliminates the need to model discontinuous conduction mode (DCM) boundary conditions.

### 2. Ideal Switches and Diodes

Switching devices have zero on-state resistance and zero switching time. Diodes have zero forward voltage drop and instantaneous reverse recovery.

### 3. Ideal Passive Components

Inductors and capacitors are lossless with no parasitic resistances. This assumption isolates the dynamic behavior from dissipative effects.

### 4. Symmetrical Interleaved Phases

Both phases (A and B) have identical component values. The two phases operate with a 180° phase shift for optimal ripple cancellation.

### 5. High Switching Frequency

The switching frequency is much higher than the AC line frequency, allowing the input voltage to be treated as quasi-static within each switching cycle.

### 6. Constant Power Load

The output load draws constant power P, independent of the bus voltage. This represents the worst-case scenario for stability analysis.

# The Analytical Strategy

To systematically tackle the complexity of this 9th-order, time-varying, nonlinear system, we employ a three-step analytical process. Each step builds upon the previous one, progressively transforming the raw circuit physics into a form suitable for control design.

## Step 1

### Per-Topology Analysis

**The "Frame-by-Frame" Physics**

Derive the exact differential equations for each of the four switching states using Kirchhoff's Voltage and Current Laws (KVL/KCL). This captures the instantaneous, unaveraged behavior of the circuit.

**Output:** Four sets of 9 differential equations (one set per topology)

→

## Step 2

### State-Space Averaging

**The "Big Picture" Model**

Combine the four topology equations into a single, unified model by taking a weighted average based on duty cycle fractions. This eliminates the time-varying nature and produces a continuous-time nonlinear model.

**Output:** Single nonlinear averaged state-space equation

→

## Step 3

### Small-Signal Linearization

**The "Nudge Test" Model**

Linearize the averaged model around a steady-state operating point using first-order Taylor expansion. This removes the nonlinearity and enables the use of linear control theory and transfer function analysis.

**Output:** Linearized state-space model ready for transfer function derivation

# Level 1: The "Frame-by-Frame" Physics

## The Concept

At this level, we analyze the converter as if we're watching a high-speed video of its operation, pausing at each distinct switching state. Each "frame" represents a unique circuit topology with its own set of governing differential equations.

### Feynman Method Analogy: The Engine Cycle

*"Think of a four-stroke engine. You wouldn't try to understand the entire engine cycle at once. Instead, you'd study each stroke separately: intake, compression, power, exhaust. Each stroke has different valve positions and different physics. Our converter is similar—we have four distinct 'strokes' (topologies) that repeat every switching cycle."*

## The Four Topologies

### Topology 11

Both switches ON: Energy storage phase

### Topology 10

S1 ON, S2 OFF: Phase A stores, Phase B transfers

### Topology 01

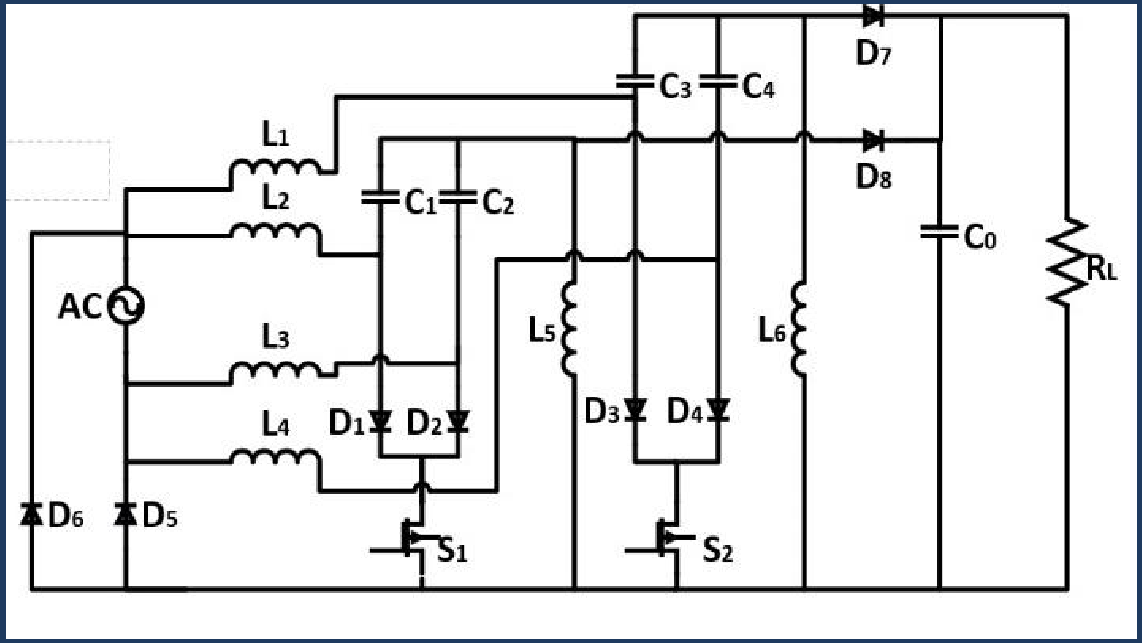
S1 OFF, S2 ON: Phase A transfers, Phase B stores

### Topology 00

Both switches OFF: Energy transfer phase

### Key Insight

Each topology produces a set of 9 first-order differential equations (one for each state variable). These equations are derived directly from Kirchhoff's Voltage and Current Laws applied to the specific circuit configuration.



# Level 1: Per-Topology State Equations

By applying Kirchhoff's Voltage and Current Laws (KVL/KCL) to each of the four simplified circuit topologies for the positive AC half-cycle ($V_{in} > 0$), we derive the following sets of first-order differential equations. Each set represents the exact, "raw" physics of the converter during that specific state.

## Topology 11 (Both Switches ON)

*Physical State: Both sub-circuits store energy from $V_{in}$. The output bus is isolated.*

$$\frac{di_{L1}}{dt} = V_{in} / L1$$
$$\frac{di_{L2}}{dt} = V_{in} / L2$$
$$\frac{di_{L3}}{dt} = 0$$
$$\frac{di_{L4}}{dt} = 0$$
$$\frac{di_{L5}}{dt} = v_{C12} / L5$$
$$\frac{di_{L6}}{dt} = v_{C34} / L6$$
$$\frac{dv_{C12}}{dt} = 0$$
$$\frac{dv_{C34}}{dt} = 0$$
$$\frac{dv_{Co}}{dt} = -P / (Co \cdot v_{Co})$$

## Topology 10 (S1 ON, S2 OFF)

*Physical State: S1-Side stores energy. S2-Side transfers its stored energy to the output bus.*

$$\frac{di_{L1}}{dt} = (V_{in} - v_{C34} - v_{Co}) / L1$$
$$\frac{di_{L2}}{dt} = V_{in} / L2$$
$$\frac{di_{L3}}{dt} = 0$$
$$\frac{di_{L4}}{dt} = 0$$
$$\frac{di_{L5}}{dt} = v_{C12} / L5$$
$$\frac{di_{L6}}{dt} = -v_{Co} / L6$$
$$\frac{dv_{C12}}{dt} = 0$$
$$\frac{dv_{C34}}{dt} = i_{L1} / (C3+C4)$$
$$\frac{dv_{Co}}{dt} = (i_{L1} + i_{L6}) / Co - P / (Co \cdot v_{Co})$$

## Topology 01 (S1 OFF, S2 ON)

*Physical State: S2-Side stores energy. S1-Side transfers its stored energy to the output bus.*

$$\frac{di_{L1}}{dt} = V_{in} / L1$$
$$\frac{di_{L2}}{dt} = (V_{in} - v_{C12} - v_{Co}) / L2$$
$$\frac{di_{L3}}{dt} = 0$$
$$\frac{di_{L4}}{dt} = 0$$
$$\frac{di_{L5}}{dt} = -v_{Co} / L5$$
$$\frac{di_{L6}}{dt} = v_{C34} / L6$$
$$\frac{dv_{C12}}{dt} = i_{L2} / (C1+C2)$$
$$\frac{dv_{C34}}{dt} = 0$$
$$\frac{dv_{Co}}{dt} = (i_{L2} + i_{L5}) / Co - P / (Co \cdot v_{Co})$$

## Topology 00 (Both Switches OFF)

*Physical State: Both sub-circuits simultaneously transfer their stored energy to the output bus.*

$$\frac{di_{L1}}{dt} = (V_{in} - v_{C34} - v_{Co}) / L1$$
$$\frac{di_{L2}}{dt} = (V_{in} - v_{C12} - v_{Co}) / L2$$
$$\frac{di_{L3}}{dt} = 0$$
$$\frac{di_{L4}}{dt} = 0$$
$$\frac{di_{L5}}{dt} = -v_{Co} / L5$$
$$\frac{di_{L6}}{dt} = -v_{Co} / L6$$
$$\frac{dv_{C12}}{dt} = i_{L2} / (C1+C2)$$
$$\frac{dv_{C34}}{dt} = i_{L1} / (C3+C4)$$
$$\frac{dv_{Co}}{dt} = (i_{L1} + i_{L2} + i_{L5} + i_{L6}) / Co - P / (Co \cdot v_{Co})$$

**Note:** These equations represent the ground truth of the circuit's behavior for the positive AC half-cycle. Each topology has 9 differential equations, corresponding to the 9th-order system. They form the building blocks for the averaged model.

# Handling AC Polarity: The $s = \mathrm{sign}(V_{in})$ Variable

## The Challenge of Bridgeless Topology

The bridgeless topology behaves like two different circuits: one for the positive AC half-cycle and one for the negative. Deriving separate models for each would be redundant and inefficient.

## The Solution: A Polarity Switch Variable

We introduce a simple variable **s** to represent the polarity of the input voltage:

$$s = \mathrm{sign}(V_{in})$$

| When $V_{in} > 0$ | When $V_{in} < 0$ |
| --- | --- |
| **$s = +1$** | **$s = -1$** |

### Key Insight

The **s** variable only multiplies the input matrix $\mathbf{B}_{in,avg}$, not the internal state matrix $\mathbf{A}_{avg}$.

$$B_{in,avg} = s \cdot (w_{11} \cdot B_{in,11} + w_{10} \cdot B_{in,10} + w_{01} \cdot B_{in,01} + w_{00} \cdot B_{in,00})$$

## Physical Meaning

**Internal Physics ($A_{avg}$):**

The way components exchange energy internally remains the same regardless of input polarity. The state matrix $A_{avg}$ is independent of the AC polarity.

**Input's Effect ($B_{in,avg}$):**

The direction of energy "push" from the input voltage depends on polarity. The sign variable s flips the direction of the input forcing function.

**Result: A single, unified model valid for the entire AC cycle**

This elegant approach eliminates the need for separate positive and negative half-cycle models. By incorporating the polarity switch variable, we maintain a single set of equations that automatically adapts to the instantaneous polarity of the AC input voltage.

The mathematical beauty of this formulation is that it preserves the fundamental symmetry of the bridgeless topology while accounting for the asymmetry introduced by the AC source polarity.

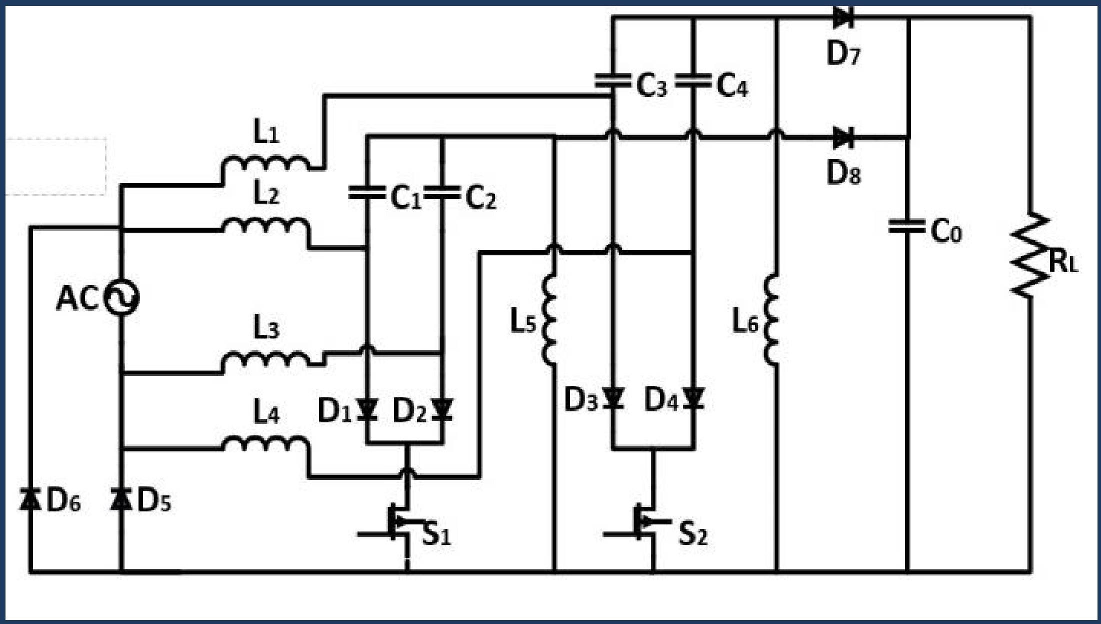
# Level 2: The "Big Picture" Averaged Model

Analyzing the four separate sets of equations is impractical. The State-Space Averaging technique allows us to combine them into a single, unified model that describes the converter's overall, average behavior over a full switching cycle.

### Feynman Method Analogy: Average Horsepower

*"Looking at the frame-by-frame physics is too complicated to understand the converter's performance. It's like trying to understand a car's speed by analyzing the exact pressure on one piston at every microsecond.*

***Instead, we want the average performance. We don't care about the individual strokes; we care about the average horsepower the engine produces. The Averaged Model is the 'average horsepower' equation for our converter."***




## The Averaging Formula

The process involves taking a weighted average of the state matrices from each of the four topologies. The final averaged state matrix $A_{avg}$ is calculated as:

$$A_{avg} = w_{11} \cdot A_{11} + w_{10} \cdot A_{10} + w_{01} \cdot A_{01} + w_{00} \cdot A_{00}$$

$A_k$: System matrices derived from the Level 1 equations

$w_k$: Duty Cycle Weights, representing the fraction of time spent in each state

### Key Insight

The resulting averaged model is a single, continuous-time representation of the converter. However, because the weights ($w_k$) are functions of the duty cycles ($d_1$, $d_2$), this model is **nonlinear**.

**Important:** $A_{avg}$ and $B_{in,avg}$ are nonlinear functions of duty cycles ($d_1$, $d_2$).

**Purpose:** This model allows us to solve for the converter's steady-state DC operating point.

# Visualizing the Interleaving Weights ($w_k$)

## What They Represent

The weights $w_{11}$, $w_{10}$, $w_{01}$, and $w_{00}$ represent the fraction of time the converter spends in each of its four possible switching states within a single switching period. They are determined by the duty cycles $d_1$ and $d_2$.

## The Two Main Operating Regions

The calculation of these weights depends on whether the ON-times of the two switches overlap.

### Case 1: Non-Overlapping Mode ($d_1 + d_2 < 1$)

- Occurs at lower duty cycles
- Switching Sequence: S1 ON → Both OFF → S2 ON → Both OFF

### Case 2: Overlapping Mode ($d_1 + d_2 > 1$)

- Occurs at higher duty cycles
- Switching Sequence: Only S1 ON → Both ON → Only S2 ON

### The Mathematical Formulas

The weights are calculated using the following piecewise formulas:

$$w_{11} = \max(0,\ d_1 + d_2 - 1)$$

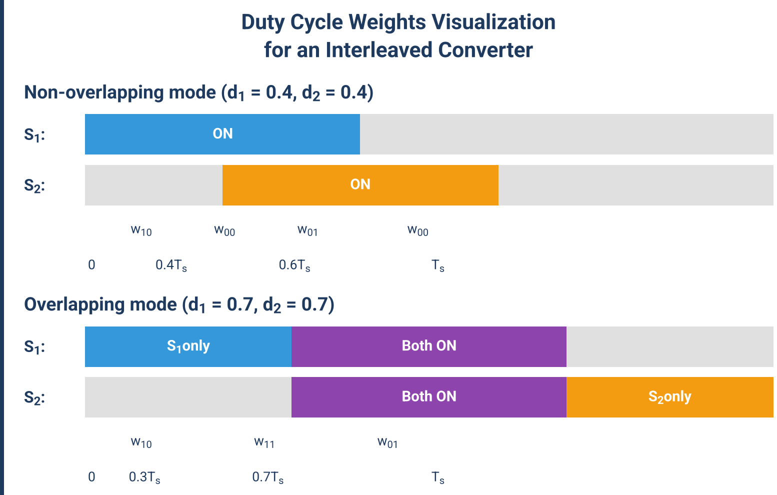
$$w_{00} = \max(0,\ 1 - d_1 - d_2)$$

$$w_{10} = d_1 - w_{11}$$

$$w_{01} = d_2 - w_{11}$$

### Key Insights

- The weights always sum to 1: $w_{11} + w_{10} + w_{01} + w_{00} = 1$
- In non-overlapping mode ($d_1 + d_2 < 1$), $w_{11} = 0$ and $w_{00} > 0$
- In overlapping mode ($d_1 + d_2 > 1$), $w_{11} > 0$ and $w_{00} = 0$
- At the boundary ($d_1 + d_2 = 1$), both $w_{11} = 0$ and $w_{00} = 0$



# Level 2: The Averaged Equation of Motion

## The Global Averaged Model

This single, unified matrix equation describes the converter's overall average behavior. It is the complete, nonlinear "equation of motion" for the converter before linearization.

$$\dot{x} = A_{avg}(d_1, d_2) \cdot x + B_{in,avg}(d_1, d_2, s) \cdot V_{in} + B_{load}(x)$$

### Component Definitions

**Averaged State Matrix $A_{avg}$:**

$$A_{avg} = w_{11} \cdot A_{11} + w_{10} \cdot A_{10} + w_{01} \cdot A_{01} + w_{00} \cdot A_{00}$$

Represents the blended "Internal Physics" of the converter.

**Averaged Input Matrix $B_{in,avg}$:**

$$B_{in,avg} = s \cdot (w_{11} \cdot B_{in,11} + w_{10} \cdot B_{in,10} + w_{01} \cdot B_{in,01} + w_{00} \cdot B_{in,00})$$

Represents the blended "External Force" from the input, including the polarity switch s.

**Nonlinear Load Vector $B_{load}(x)$:**

$$B_{load}(x) = [0, 0, 0, 0, 0, 0, 0, 0, -P/(C_o \cdot v_{Co})]^T$$

Isolates the nonlinear effect of the Constant Power Load on the 9th state variable (bus voltage $v_{Co}$).

## Duty Cycle Weights

Fraction of time spent in each interleaved state:

$$w_{11} = \max(0, d_1 + d_2 - 1)$$
$$w_{00} = \max(0, 1 - d_1 - d_2)$$
$$w_{10} = d_1 - w_{11}$$
$$w_{01} = d_2 - w_{11}$$

### Key Characteristics of this Model

**Unified:** Accounts for all four switching states in a single equation, eliminating the need to track individual topologies.

**Nonlinear:** $A_{avg}$ and $B_{in,avg}$ are functions of the control inputs ($d_1$, $d_2$), and the load term is a nonlinear function of the state $v_{Co}$.

**9th-Order:** The state vector x contains 9 elements (iL1, iL2, iL3, iL4, iL5, iL6, vC12, vC34, vCo), and all matrices are dimensioned accordingly.

**Purpose:** The primary use of this equation is to solve for the converter's steady-state DC operating point ($x_0$) by setting $\dot{x} = 0$.

# Level 3: The "Nudge Test" Linearized Model

The averaged model from Level 2 is nonlinear due to the duty cycle modulation and the constant power load. To design linear controllers and derive transfer functions, we must linearize this model around a steady-state operating point using small-signal analysis.

### Feynman Method Analogy: The Tangent Line

*"Imagine you're driving on a winding mountain road. If you want to predict where you'll be in the next second, you don't need to know the entire road—you just need to know your current position and the direction you're heading right now. That's linearization: replacing a complex curve with a simple straight line that's accurate for small movements around your current position."*

## The Linearization Process

### Step 1: Find the Operating Point

**Solve for Steady-State**

Set $\dot{x} = 0$ in the averaged model and solve for the DC values ($x_0$, $d_{10}$, $d_{20}$, $V_{in0}$). This is the "current position" around which we linearize.

### Step 2: Define Small Perturbations

**Introduce AC Variables**

Express each variable as DC + small AC: $x = x_0 + \tilde{x}$, $d_1 = d_{10} + \tilde{d}_1$, $d_2 = d_{20} + \tilde{d}_2$. The tilde (~) represents small deviations from steady-state.

### Step 3: Apply First-Order Taylor Expansion

**Compute the Jacobian Matrices**

Calculate partial derivatives of the averaged model with respect to states and inputs, evaluated at the operating point. This gives us constant-coefficient matrices.

## The Linearized State-Space Form

After linearization, the nonlinear averaged model transforms into a linear time-invariant (LTI) system:

**Small-Signal Dynamics**

$$d(\tilde{x})/dt = A_{linear} \cdot \tilde{x} + B_d \cdot [\tilde{d}_1, \tilde{d}_2]^T + B_{in} \cdot \tilde{V}_{in} + B_p \cdot \tilde{P}$$

where all matrices ($A_{linear}$, $B_d$, $B_{in}$, $B_p$) are now constant numerical matrices evaluated at the operating point.

### Why Linearization Matters

The linearized model enables us to use the powerful tools of linear control theory: transfer functions, Bode plots, pole-zero analysis, and classical controller design techniques. It provides the mathematical foundation for designing voltage and current control loops that stabilize the converter and regulate its output.

The accuracy of this linear approximation is excellent for small perturbations around the operating point, which is precisely the regime where feedback controllers operate.

# Deriving the Control Input Matrix ($B_d$)

The control input matrix $B_d$ captures how small changes in the duty cycles ($\tilde{d}_1$, $\tilde{d}_2$) affect the state dynamics. This is the most mathematically intricate part of the linearization because the duty cycles appear inside the averaged matrices themselves.

## The "Nudge Test" Concept

### Physical Interpretation

Imagine you're at steady-state and you give the duty cycle $d_1$ a tiny "nudge" upward. How does each state variable respond? The $B_d$ matrix quantifies exactly this sensitivity. Each column of $B_d$ represents the instantaneous rate of change of all states with respect to one duty cycle input.

## The Mathematical Challenge

In the averaged model, the duty cycles appear in two places:

### Averaged Model Structure

$$\dot{x} = A_{avg}(d_1,\ d_2) \cdot x + B_{in,avg}(d_1,\ d_2) \cdot V_{in} + B_{load}(x)$$

Both $A_{avg}$ and $B_{in,avg}$ depend on $d_1$ and $d_2$ through the weights $w_k$. This creates a product-rule situation when we take derivatives.

### Step 1: Compute Weight Derivatives

Calculate $\partial w_k/\partial d_1$ and $\partial w_k/\partial d_2$ for all four weights. These derivatives are piecewise-defined based on whether the system is in overlapping or non-overlapping mode.

### Step 2: Apply the Chain Rule

Use the chain rule to propagate these weight derivatives through $A_{avg}$ and $B_{in,avg}$. This gives us $\partial A_{avg}/\partial d_1$, $\partial A_{avg}/\partial d_2$, $\partial B_{in,avg}/\partial d_1$, and $\partial B_{in,avg}/\partial d_2$.

### Step 3: Evaluate at Operating Point

Substitute the steady-state values ($x_0$, $d_{10}$, $d_{20}$, $V_{in0}$) into all partial derivatives to obtain numerical matrices.

## The Chain Rule Implementation

### Detailed Derivative Calculation

The first column of $B_d$ (corresponding to $\tilde{d}_1$) is computed as:

$$B_{d,col1} = (\partial A_{avg}/\partial d_1) \cdot x_0 + (\partial B_{in,avg}/\partial d_1) \cdot V_{in0}$$

where the partial derivatives are expanded using:

$$\partial A_{avg}/\partial d_1 = \Sigma_k\ (\partial w_k/\partial d_1) \cdot A_k$$

Similarly, the second column of $B_d$ (corresponding to $\tilde{d}_2$) is:

$$B_{d,col2} = (\partial A_{avg}/\partial d_2) \cdot x_0 + (\partial B_{in,avg}/\partial d_2) \cdot V_{in0}$$

### Final Result: The $B_d$ Matrix

$B_d$ is a **9×2 matrix** where each column represents the sensitivity of all 9 state variables to one duty cycle input. This matrix is the key to understanding how control actions (duty cycle modulation) influence the converter's dynamic response.

The $B_d$ matrix forms the foundation for control-to-output transfer functions, which are essential for designing the voltage and current control loops. It reveals which states are most strongly affected by each duty cycle and helps identify potential control limitations.

In practice, numerical computation of $B_d$ is straightforward once the operating point is known, but the underlying mathematics requires careful application of multivariable calculus and matrix differentiation rules.

# Level 3: The Final Linearized Equation of Motion

This is the complete small-signal linearized state-space model of the 9th-order Interleaved Bridgeless SEPIC PFC Converter. All matrices are constant numerical values evaluated at the steady-state operating point, making this model suitable for transfer function derivation and linear controller design.

**Small-Signal Linearized Dynamics**

$$d(\tilde{x})/dt = A_{linear} \cdot \tilde{x} + B_d \cdot [\tilde{d}_1, \tilde{d}_2]^T + B_{in} \cdot \tilde{V}_{in} + B_p \cdot \tilde{P}$$

## $A_{linear}$: Linearized State Matrix

### 9×9 matrix of constant numbers

Obtained by evaluating the Jacobian ∂f/∂x at the operating point. This matrix captures how small changes in each state variable affect the rates of change of all other states.

**Special consideration:** The constant power load (CPL) introduces a negative resistance term in element (9,9) of the Jacobian:

$$\partial(-P/v_{Co})/\partial v_{Co} = P/v^2_{Co,0}$$

## $B_d$: Control Input Matrix

### 9×2 matrix of constant numbers

Represents how small changes in duty cycles $\tilde{d}_1$ and $\tilde{d}_2$ affect the state dynamics. Derived using the chain rule through the duty cycle weights:

$$B_d = [\partial f/\partial d_1 \mid \partial f/\partial d_2]$$
$$= \Sigma\ [(\partial f/\partial w_k) \cdot (\partial w_k/\partial d_i)]$$

This is the primary control input for regulating the converter's behavior through pulse-width modulation.

## $B_{in}$: Input Disturbance Matrix

### 9×1 vector of constant numbers

Describes how small variations in the input voltage $\tilde{V}_{in}$ propagate through the system. Evaluated as:

$$B_{in} = \partial f/\partial V_{in}\ |_{\text{operating point}}$$

This matrix is essential for analyzing input voltage disturbance rejection and designing feedforward compensation.

## $B_p$: Load Disturbance Vector

### 9×1 vector of constant numbers

Captures how small changes in load power $\tilde{P}$ affect the system dynamics. Since the CPL term only appears in the output capacitor equation:

$$B_p = [0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ -1/(C_o \cdot v_{Co,0})]^T$$

The non-zero element is in the 9th position, corresponding to the output voltage state variable.

## Significance of the 9th-Order Model

This linearized 9th-order model fully captures the dynamics of all six inductors (iL1 through iL6) and three capacitors (vC12, vC34, vCo). The higher order compared to simplified models provides a more accurate representation of the converter's transient behavior, particularly important for high-bandwidth control design and stability analysis under constant power load conditions.

# The Ultimate Goal: Transfer Functions for Control

The linearized state-space model enables us to derive transfer functions that relate control inputs and disturbances to system outputs. These transfer functions are the foundation for designing robust feedback controllers using classical control theory techniques.

## Key Transfer Functions

### 1. Control-to-Output Transfer Function

$$G_{vd}(s) = \tilde{v}_{Co}(s) / \tilde{d}_i(s)$$

Describes how the output voltage responds to changes in duty cycle. This is the primary transfer function for voltage loop controller design.

### 2. Input Impedance Transfer Function

$$Z_{in}(s) = \tilde{V}_{in}(s) / \tilde{i}_{in}(s)$$

Characterizes the converter's input impedance as seen from the AC source. Critical for ensuring compliance with power quality standards.

### 3. Output Impedance Transfer Function

$$Z_{out}(s) = \tilde{v}_{Co}(s) / \tilde{i}_{load}(s)$$

Quantifies how output voltage changes in response to load current variations. Essential for analyzing load regulation.

## Two-Loop Control Architecture

The transfer functions enable the design of a cascaded two-loop control structure:

### Inner Current Loop (Fast)

Regulates the input current to follow a sinusoidal reference in phase with the input voltage, achieving power factor correction. Bandwidth typically 1-5 kHz.

### Outer Voltage Loop (Slow)

Maintains constant output voltage by adjusting the amplitude of the current reference. Bandwidth typically 10-50 Hz.

## Applications of Transfer Functions

- → Bode plot analysis to determine gain and phase margins
- → Pole-zero placement for desired transient response
- → Compensator design using lead-lag, PI, or PID controllers
- → Stability analysis under constant power load conditions
- → Sensitivity analysis to component parameter variations
- → Disturbance rejection performance evaluation

## From Physics to Control

The journey from Kirchhoff's laws (Level 1) through state-space averaging (Level 2) to linearization (Level 3) culminates in these transfer functions. They transform the complex 9th-order nonlinear physics into tractable mathematical tools that control engineers can use to design, analyze, and optimize the converter's closed-loop performance.

# Conclusion and Next Steps

## Key Milestones Achieved

### Complete Mathematical Framework

Captured the complete dynamics of a **9th-order system** through three levels of abstraction: per-topology physics, state-space averaging, and small-signal linearization.

### Rigorous Derivation Process

Established a systematic methodology from fundamental circuit laws (KVL/KCL) to control-ready transfer functions, ensuring mathematical rigor at every step.

### Unified AC Polarity Handling

Developed a single model valid for both positive and negative AC half-cycles using the polarity switch variable $s = \text{sign}(V_{in})$.

### CPL Stability Foundation

Explicitly modeled the destabilizing effect of the constant power load, providing the analytical basis for stability analysis and compensator design.

## Next Steps

### 1. Transfer Function Derivation

Extract control-to-output and line-to-output transfer functions from the linearized state-space model using $G(s) = C(sI - A)^{-1}B + D$.

### 2. Controller Design

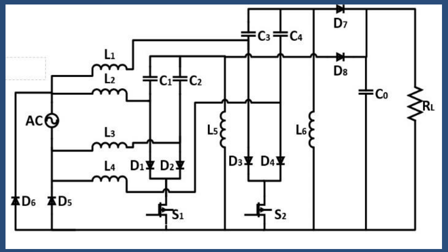
Design cascaded voltage and current control loops with appropriate compensators to achieve desired bandwidth, phase margin, and disturbance rejection.

### 3. Stability Analysis

Perform frequency-domain analysis (Bode plots, Nyquist diagrams) and time-domain simulations to verify closed-loop stability under CPL conditions.

### 4. Experimental Validation

Build a hardware prototype and validate the model predictions through experimental measurements of transient response and frequency response.




### From Physics to Control: A Complete Journey

This document provides the complete mathematical foundation for understanding, analyzing, and controlling the Interleaved Bridgeless SEPIC PFC Converter. The rigorous derivation ensures that every equation is traceable back to fundamental circuit physics, giving confidence in the model's validity and applicability.