What is a Transfer Function?

The Bridge Between Mathematical Models and Control Design

Simple Definition

A **transfer function** is a mathematical tool that describes how a system responds to an input at different frequencies.

$$G(s) = \text{Output}(s) / \text{Input}(s)$$

where s is the complex frequency variable ($s = \sigma + j\omega$)

In the Laplace domain: Differential equations become algebraic equations, making analysis much simpler.

$G_{vd1}(s)$: Control-to-Output

$$G_{vd1}(s) = \Delta V_{out}(s) / \Delta d1(s)$$

"How does output voltage change for a small change in duty cycle d1?"

- **Use:** Design voltage regulation controller
- **Measure:** DC gain, bandwidth, phase margin
- **Goal:** Fast response, no overshoot

$G_{id1}(s)$: Duty-to-Input Current

$$G_{id1}(s) = \Delta i_{L1}(s) / \Delta d1(s)$$

"How does input current change for a small change in duty cycle d1?"

- **Use:** Design PFC inner current loop
- **Measure:** Current tracking accuracy
- **Goal:** Follow AC reference sinusoid

How Transfer Functions Enable Controller Design

Step 1: Derive G(s) from linearized state-space model

$$G(s) = C \cdot (sI - A_{linear})^{-1} \cdot B_d$$

Step 2: Design controller H(s) using Bode plots

- Ensure loop gain $L(s) = G(s)\cdot H(s)$ has sufficient phase margin (typically > 45°)
- Set crossover frequency for desired bandwidth (e.g., 1-5 kHz for PFC)
- Add integrator for zero steady-state error

Step 3: Verify closed-loop stability

- Check all poles of closed-loop TF are in left-half plane (LHP)
- Simulate step response for overshoot and settling time

Practical Example: For a PFC converter, $G_{id1}(s)$ might have a DC gain of 5 A/duty and bandwidth of 10 kHz. A PI controller $H(s) = K_p + K_i/s$ is designed to achieve: (1) Unity gain crossover at 2 kHz, (2) Phase margin of 60°, and (3) Tracking error < 1% for 50 Hz AC input. This ensures high power factor (PF > 0.99) and low THD (< 5%).