SEBASTIAN PAPANICOLAU | PARCIAL B ANÁLISIS MATEMÁTICO I | HOJA N° | FECHA

1) $$\int_1^{e^x} f(\ln(t))\,dt = \frac{1}{2}x^2$$

$$\left(\int_1^{e^x} f(\ln(t))\,dt\right)' = x$$

$$f(\ln(e^x)) \cdot e^x = x$$

$$f(x) \cdot e^x = x$$

$$f(x) = \frac{x}{e^x}$$

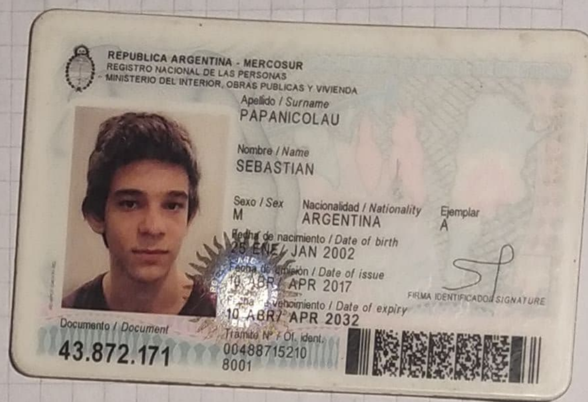
$$f'(x) = \frac{e^x - x \cdot e^x}{e^{2x}}$$

$$0 = \frac{e^x(1-x)}{e^x \cdot e^x} = \frac{1-x}{e^x}$$

$$0 = 1 - x$$

$$x = 1$$

f TIENE EXTREMO EN $x = 1$ => $\boxed{\text{FALSO}}$



NOTA

2) AREA ENTRE CURVAS

$f(x) = \frac{1}{2}x + 2$

$g(x) = |x-2|$

$x \in$ $2 = |x| \Rightarrow x = 2 \wedge x = -2$

$F \cap g: \frac{1}{2}x + 2 = |x-2| : \frac{1}{2}x + 2 = x - 2 \quad \vee \quad -\frac{1}{2}x + 2 = x - 2$

$x > 0$

$\frac{1}{2}x + 2 = x - 2$

$-\frac{1}{2}x + 4 = 0$

$-\frac{x}{2} = -4$

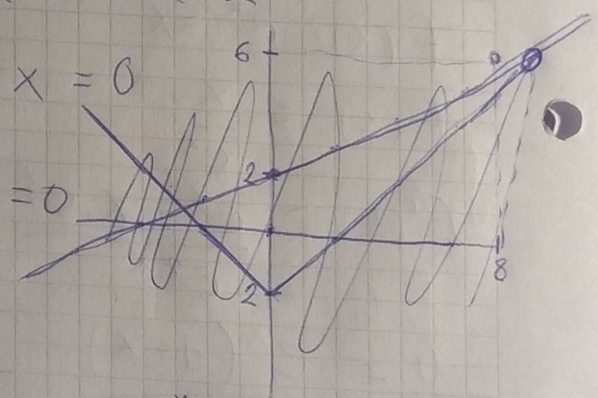
$x = +8$

$x < 0$

$-\frac{1}{2}x - 2 = x - 2$

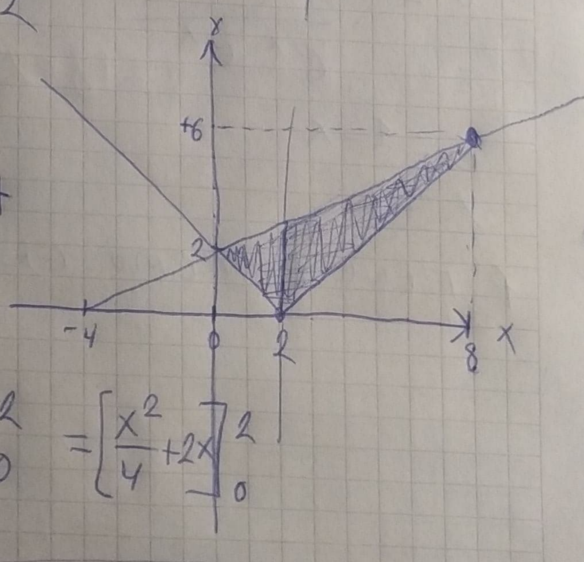
$-\frac{3}{2}x = 0$

$x = 0$



$f(8) = 4 + 2 = 6$ $\quad F(0) = 2$

$(8,6)$ $\quad (0,2)$



$AREA = \int_0^2 \left(\frac{1}{2}x + 2\right) dx - \int_0^2 (-x + 2) dx +$

$\int_2^8 \frac{1}{2}x + 2 - \int_2^8 x - 2$

$\int_0^2 \left(\frac{1}{2}x + 2\right) dx = \left[\frac{1}{2} \cdot \frac{x^2}{2} + 2x\right]_0^2 = \left[\frac{x^2}{4} + 2x\right]_0^2$

$\frac{4}{4} + 4 = 1 + 4 = 5 - 0 = 5$

$\int_0^2 (-x + 2) dx = \left[-\frac{x^2}{2} + 2x\right]_0^2 = -\frac{4}{2} + 4 = -2 + 4 = 2$

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$$\int_2^8 \frac{1}{2}x+2\,DX = \left[\frac{x^2}{4}+2x\right]_2^8 = \frac{64}{4}+16-\left[\frac{4}{4}+8\right] = 32-9 = 2$$

$$\int_2^8 x-2\,DX = \left[\frac{x^2}{2}-2x\right]_2^8 = \frac{64}{2}-16-\left[\frac{4}{2}-4\right] = 16+2 = 18$$

$$AREA = 5-2+23-18 = 8$$

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3) $h(x) = \int_0^{\ln(x^2+1)} f(T)\,DT + x$

$h(0) = \int_0^0 f(T)\,DT + 0 = 0$

$\boxed{h(0) = 0}$

$h'(x) = \left(\int_0^{\ln(x^2+1)} f(t)\,Dt\right)' + 1$

$h'(x) = f(\ln(x^2+1)) \cdot \frac{2x}{x^2+1} + 1$

$h'(0) = f(\ln(1)) \cdot \frac{0}{1} + 1$

$\boxed{h'(0) = \cancel{0}\ 1}$

$h''(x) = f'(\ln(x^2+1)) \cdot \frac{2x}{x^2+1} \cdot \frac{2x}{x^2+1} + f(\ln(x^2+1)) \cdot$

$\frac{2 \cdot (x^2+1) - 2x \cdot 2x}{(x^2+1)^2}$

$h''(0) = f'(0) \cdot 0 + f(0) \cdot 2$

$f(x) = -2x + 5$

$f(0) = 5$

$f'(x) = -2$

$\boxed{h''(0) = 10}$

NOTA

$$T(x) = h(0) + h'(0) \cdot x + \frac{h''(0)}{2} \cdot x^2$$

$$\boxed{T(x) = x + 5x^2}$$

HOJA N°

FECHA

4) $f(x) = \dfrac{1}{(x-8)^{2/3}}$; $y=0$; $0 \leq x \leq 8$

$$\int_0^8 \frac{1}{(x-8)^{2/3}}\,dx = \int_0^8 (x-8)^{-2/3}\,dx \qquad u = x-8 \quad du = dx$$

$$\int_0^8 u^{-2/3}\,du = u^{1/3} \cdot 3 = \left[\boxed{\sqrt[3]{x-8} \cdot 3}\right]_0^8 =$$

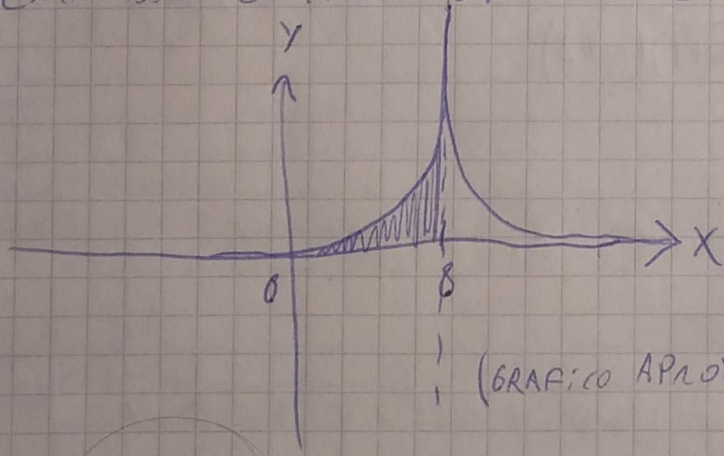
$$\sqrt[3]{0} \cdot 3 - \sqrt[3]{-8} \cdot 3 = 0 - (-2) \cdot 3 = 6$$

$(x-8)^{2/3} \neq 0$

$x - 8 \neq 0$

$x \neq 0$

f PRESENTA UNA DISCONTINUIDAD EN $x=8$, POR LO QUE NO PODEMOS DEFINIR EL AREA CONTEMPLADA ENTRE LAS CURVAS



(GRAFICO APROXIMADO)

NOTA

5) $$\sum_{N=0}^{\infty} \frac{3^N \cdot (x-7)^N}{(N+1)(N+3)}$$

POR CRITERIO DE D'ALAMB~~ERT~~

$$\lim_{N\to\infty} \left| \frac{3^N \cdot (x-7)^N}{(N-1)(N-3)} \cdot \frac{(N-2)(N-4)}{3^{N-1} \cdot (x-7)^{N-1}} \right| =$$

$$\lim_{N\to\infty} \left| \frac{\cancel{3^N} \cdot \cancel{(x-7)^N} \cdot (N-2) \cdot (N-4)}{(N-1)(N-3) \cdot \cancel{3^N} \cdot 3^{-1} \cdot \cancel{(x-7)^N} \cdot (x-7)^{-1}} \right|$$

$$\lim_{N\to\infty} \left| \frac{(N^2-6N+8) \cdot (x-7)}{(N^2-4N+3) \cdot} \right| = \left| \frac{1}{1} \cdot x-7 \right| = |x-7|$$

$|x-7| < 1 \Rightarrow x-7 < 1 \qquad x-7 > -1$

$x < 8 \qquad x > 6$

INTERVALO DE CONV: $(8;6)$ $R = 1 = 8-6 = 1$

$$\sum_{N=0}^{\infty} \frac{3^N \cdot (-1)^N}{(N+1)(N+3)}$$

ANALIZO CV. EN SERIE CON TERMINOS EN VAL. ABS

$$\sum_{N=0}^{\infty} \frac{3^N}{(N+1)(N+3)}$$

$$\lim_{N\to\infty} \frac{\cancel{3^N} \cdot N \cdot (N+2)}{\cancel{3^N} \cdot 3^{-1} \cdot (N+1)(N+3)} = \frac{N^2+2N}{1/3 \cdot N^2+4N+3} = 3 > 1$$

LA SERIE DIVERGE

$x=8$

$$\sum_{N=0}^{\infty} \frac{3^N \cdot (8-7)^N}{(N+1)(N+3)} = \sum_{N=0}^{\infty} \frac{3^N \cdot 1^N}{(N+1)(N+3)} = \sum_{N=0}^{\infty} \frac{3^N}{(N+1)(N+3)}$$

$$\lim_{N\to\infty} \frac{\cancel{3^N} \cdot N \cdot (N+2)}{\cancel{3^N} \cdot 3^{-1} (N+1)(N+3)} = \lim_{N\to\infty} \frac{3N^2+6N}{N^2+4N+3} = \boxed{3}$$

LA SERIE DIVERGE

INTERVALO DE CONVERGENCIA

$x \in (6, 8)$ (SIN LOS EXTREMOS 6 Y 8)

$R = 1$