

List of Model Formulations

| Index | id | model structure |
|-----------------------------------------------------------------|--------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| ## random effects only affect intrinsic growth | | |
| 1 | m.1 | $B_{t+1,i} = B_{t,i} + \beta_{s(i)} B_{t,i}^\theta$ |
| 2 | m.5 | $B_{t+1,i} = B_{t,i} + \beta_{s(i)} B_{t,i}^\theta * (1 + \varepsilon_p + \varepsilon_{ps} + \varepsilon_t + \varepsilon_{ts})$ |
| 3 | m.6 | $B_{t+1,i} = B_{t,i} + \beta_{s(i)} B_{t,i}^\theta * (1 + \varepsilon_p + \varepsilon_{ps} + \varepsilon_t + \varepsilon_{ts}) + \sum_{j \in n_j} \alpha_{ave} B_{t,j}^b$ |
| 4 | m.6.1 | $B_{t+1,i} = B_{t,i} + \beta_{s(i)} B_{t,i}^\theta * (1 + \varepsilon_p + \varepsilon_{ps} + \varepsilon_t + \varepsilon_{ts}) + \sum_{j \in n_j} \alpha_{ave} \left(\frac{B_{t,j}}{B_{t,i}}\right)^b$ |
| 5 | m.6.2 | $B_{t+1,i} = B_{t,i} + \beta_{s(i)} B_{t,i}^\theta * (1 + \varepsilon_p + \varepsilon_{ps} + \varepsilon_t + \varepsilon_{ts}) + \sum_{j \in n_j} \alpha_{ave} (B_{t,j} * B_{t,j})^b$ |
| 6 | m.7 | $B_{t+1,i} = B_{t,i} + \beta_{s(i)} B_{t,i}^\theta * (1 + \varepsilon_p + \varepsilon_{ps} + \varepsilon_t + \varepsilon_{ts}) + \sum_{j \in n_j} \alpha_{s(i),j(i)} B_{t,j}^b$ |
| 7 | m.7.1 | $B_{t+1,i} = B_{t,i} + \beta_{s(i)} B_{t,i}^\theta * (1 + \varepsilon_p + \varepsilon_{ps} + \varepsilon_t + \varepsilon_{ts}) + \sum_{j \in n_j} \alpha_{s(i),j(i)} \left(\frac{B_{t,j}}{B_{t,i}}\right)^b$ |
| 8 | m.7.2 | $B_{t+1,i} = B_{t,i} + \beta_{s(i)} B_{t,i}^\theta * (1 + \varepsilon_p + \varepsilon_{ps} + \varepsilon_t + \varepsilon_{ts}) + \sum_{j \in n_j} \alpha_{s(i),j(i)} (B_{t,j} * B_{t,j})^b$ |
| ## random effects affect both intrinsic growth and interactions | | |
| 9 | m.8 | $B_{t+1,i} = B_{t,i} + \left(\beta_{s(i)} B_{t,i}^\theta + \sum_{j \in n_j} \alpha_{ave} B_{t,j}^b \right) * (1 + \varepsilon_p + \varepsilon_{ps} + \varepsilon_t + \varepsilon_{ts})$ |
| 10 | m.8.1 | $B_{t+1,i} = B_{t,i} + \left(\beta_{s(i)} B_{t,i}^\theta + \sum_{j \in n_j} \alpha_{ave} \left(\frac{B_{t,j}}{B_{t,i}}\right)^b \right) * (1 + \varepsilon_p + \varepsilon_{ps} + \varepsilon_t + \varepsilon_{ts})$ |
| 11 | m.8.2 | $B_{t+1,i} = B_{t,i} + \left(\beta_{s(i)} B_{t,i}^\theta + \sum_{j \in n_j} \alpha_{ave} (B_{t,j} * B_{t,j})^b \right) * (1 + \varepsilon_p + \varepsilon_{ps} + \varepsilon_t + \varepsilon_{ts})$ |
| 12 | m.9 | $B_{t+1,i} = B_{t,i} + \left(\beta_{s(i)} B_{t,i}^\theta + \sum_{j \in n_j} \alpha_{s(i),j(i)} B_{t,j}^b \right) * (1 + \varepsilon_p + \varepsilon_{ps} + \varepsilon_t + \varepsilon_{ts})$ |
| 13 | m.9.1 | $B_{t+1,i} = B_{t,i} + \left(\beta_{s(i)} B_{t,i}^\theta + \sum_{j \in n_j} \alpha_{s(i),j(i)} \left(\frac{B_{t,j}}{B_{t,i}}\right)^b \right) * (1 + \varepsilon_p + \varepsilon_{ps} + \varepsilon_t + \varepsilon_{ts})$ |
| 14 | m.9.2 | $B_{t+1,i} = B_{t,i} + \left(\beta_{s(i)} B_{t,i}^\theta + \sum_{j \in n_j} \alpha_{s(i),j(i)} (B_{t,j} * B_{t,j})^b \right) * (1 + \varepsilon_p + \varepsilon_{ps} + \varepsilon_t + \varepsilon_{ts})$ |
| ## multiplicative formulation | | |
| 15 | m.10 | $B_{t+1,i} = B_{t,i} + \beta_{s(i)} B_{t,i}^\theta * (1 + \varepsilon_p + \varepsilon_{ps} + \varepsilon_t + \varepsilon_{ts} + \sum_{j \in n_j} \alpha_{s(i),j(i)} B_{t,j}^b)$ |
| 16 | m.10.1 | $B_{t+1,i} = B_{t,i} + \beta_{s(i)} B_{t,i}^\theta * (1 + \varepsilon_p + \varepsilon_{ps} + \varepsilon_t + \varepsilon_{ts} + \sum_{j \in n_j} \alpha_{s(i),j(i)} \left(\frac{B_{t,j}}{B_{t,i}}\right)^b)$ |

| Index | id | model structure |
|-------|--------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 17 | m.10.2 | $B_{t+1,i} = B_{t,i} + \beta_{s(i)} B_{t,i}^{\theta} * (1 + \varepsilon_p + \varepsilon_{ps} + \varepsilon_t + \varepsilon_{ts} + \sum_{j \in n_j} \alpha_{s(i),j(i)} (B_{t,j} * B_{t,j})^b)$ |
| 18 | m.11 | $B_{t+1,i} = B_{t,i} + \beta_{s(i)} B_{t,i}^{\theta} * \left(1 + \sum_{j \in n_j} \alpha_{s(i),j(i)} B_{t,j}^b\right) * (1 + \varepsilon_p + \varepsilon_{ps} + \varepsilon_t + \varepsilon_{ts})$ |
| 19 | m.11.1 | $B_{t+1,i} = B_{t,i} + \beta_{s(i)} B_{t,i}^{\theta} * (1 + \sum_{j \in n_j} \alpha_{s(i),j(i)} \left(\frac{B_{t,j}}{B_{t,i}}\right)^b) * (1 + \varepsilon_p + \varepsilon_{ps} + \varepsilon_t + \varepsilon_{ts})$ |
| 20 | m.11.2 | $B_{t+1,i} = B_{t,i} + \beta_{s(i)} B_{t,i}^{\theta} * \left(1 + \sum_{j \in n_j} \alpha_{s(i),j(i)} (B_{t,j} * B_{t,j})^b\right) * (1 + \varepsilon_p + \varepsilon_{ps} + \varepsilon_t + \varepsilon_{ts})$ |