Network of Thrones

Author(s): Andrew Beveridge and Jie Shan

Source: *Math Horizons*, Vol. 23, No. 4 (April 2016), pp. 18–22

Published by: Taylor & Francis, Ltd. on behalf of the Mathematical Association of America

Stable URL: https://www.jstor.org/stable/10.4169/mathhorizons.23.4.18

Network of Thrones

Andrew Beveridge and Jie Shan

The international hit HBO series *Game of Thrones,* adapted from George R. R. Martin's epic fantasy novel series *A Song of Ice and Fire,* features interweaving plotlines and scores of characters. With so many people to keep track of in this sprawling saga, it can be a challenge to fully understand the dynamics between them.

To demystify this saga, we turn to *network science,* a new and evolving branch of applied graph theory that brings together traditions from many disciplines, including sociology, economics, physics, computer science, and mathematics. It has been applied broadly across the sciences, the social sciences, the humanities, and in industrial settings.

In this article we perform a network analysis of *Game of Thrones* to make sense of the intricate character relationships and their bearing on the future plot (but we promise: no spoilers!).

Helen Sloan/HBO

First, a quick introduction to *Game of Thrones*: Westeros and Essos, separated by the Narrow Sea, are homes of several noble houses (figure 1). The narrative starts at a time of peace, with all the houses unified under the rule of King Robert Baratheon, who holds the Iron Throne. Early on, King Robert dies in a hunting accident, and the young, cruel Prince Joffrey ascends the throne, backed by his mother's house, Lannister. However, the prince's legitimacy, and even his identity, are seriously questioned across the kingdom. As a result, war breaks out, with multiple claimants to the Iron Throne.

Driven by cause or circumstance, characters from the many noble families launch into arduous and intertwined journeys. Among these houses are the honorable Stark family (Eddard, Catelyn, Robb, Sansa, Arya, Bran, and Jon Snow), the pompous Lannisters (Tywin, Jaime, Cersei, Tyrion, and Joffrey), the slighted Baratheons (led by Robert's brother Stannis) and the exiled Daenerys, the last of the once-powerful House Targaryen.

The Social Network

Our first task is to turn the *Game of Thrones* world into a social network. Our network, shown in figure 2, has sets of vertices V and edges E. The 107 vertices represent the characters, including ladies and lords, guards and mercenaries, councilmen and consorts, villagers and savages. The vertices are joined by 353 integer-weighted edges, in which higher weights correspond to stronger relationships between those characters.

We generated the edges using *A Storm of Swords,* the third book in the series. We opted for this volume because the main narrative has matured, with the characters scattered geographically and enmeshed in

Figure 1. The *Game of Thrones* world: Westeros, the Narrow Sea, and Essos (from left to right). Sigils represent the locations of the noble houses at the beginning of the saga.
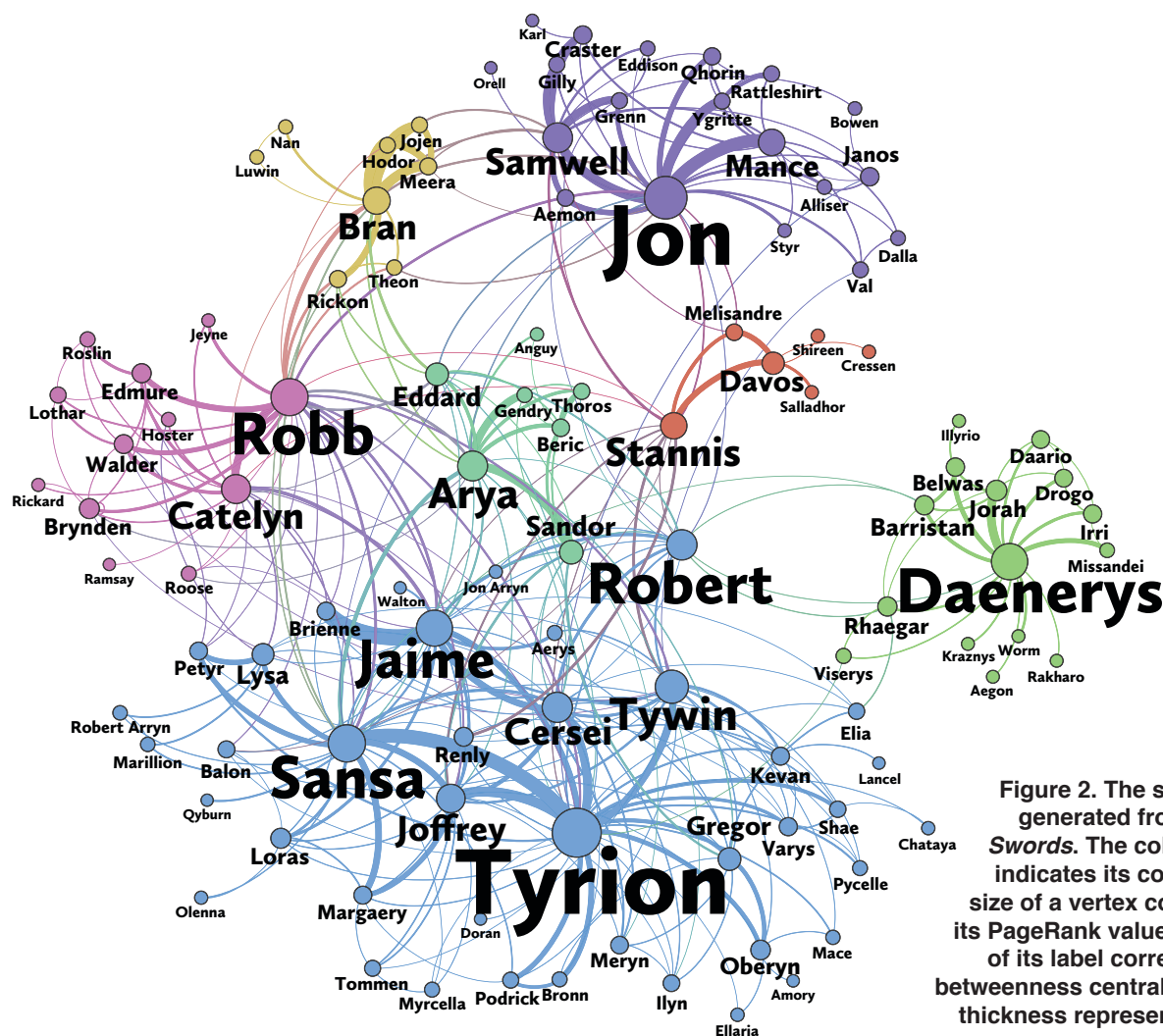
Figure 2. The social network generated from *A Storm of Swords*. The color of a vertex indicates its community. The size of a vertex corresponds to its PageRank value, and the size of its label corresponds to its betweenness centrality. An edge's thickness represents its weight.

their own social circles. We parsed the ebook, incrementing the edge weight between two characters when their names (or nicknames) appeared within 15 words of one another. Afterward, we performed some manual validation and cleaning. Note that an edge between two characters doesn't necessarily mean that they are friends—it simply means that they interact, speak of one another, or are mentioned together.

The complex structure of our network reflects the interweaving plotlines of the story. Notably, we observe two characteristics found in many real-world networks. First, the network contains multiple denser subnetworks, held together by a sparser global web of edges. Second, it is organized around a subset of highly influential people, both locally and globally. We now describe how to quantify these observations using the analytical tools of network science.

Community Detection

The network layout and colors in figure 2 clearly identify seven communities: the Lannisters and King's Landing, Robb's army, Bran and friends, Arya and

companions, Jon Snow and the far North, Stannis's forces, and Daerenys and the exotic people of Essos. Remarkably, these communities were identified from only the network structure, as we explain below.

We want to divide the network into coherent communities, meaning that there are many edges within communities and few edges between communities. We detect our network communities by using a global metric called *modularity*. Let $w_{ij} \geq 0$ denote the weight of the edge between vertices i and j, where $w_{ij} = 0$ when there is no edge. Let $k_i = \sum_{j \in V} w_{ij}$ denote the weighted degree of vertex i. Intuitively, the modularity Q compares our given network to a network with the same weighted degrees, but in which all edges are rewired at random. This random network should be community-free, so it makes a good baseline for comparison.

Suppose that vertices i and j belong to the same community C. We would expect that w_{ij} is at least as large as the number of edges between them in our randomly rewired network. A touch of combinatorial probability shows that the expected number of such random edges is $k_i k_j / 2m$, where m is the total number of edges in
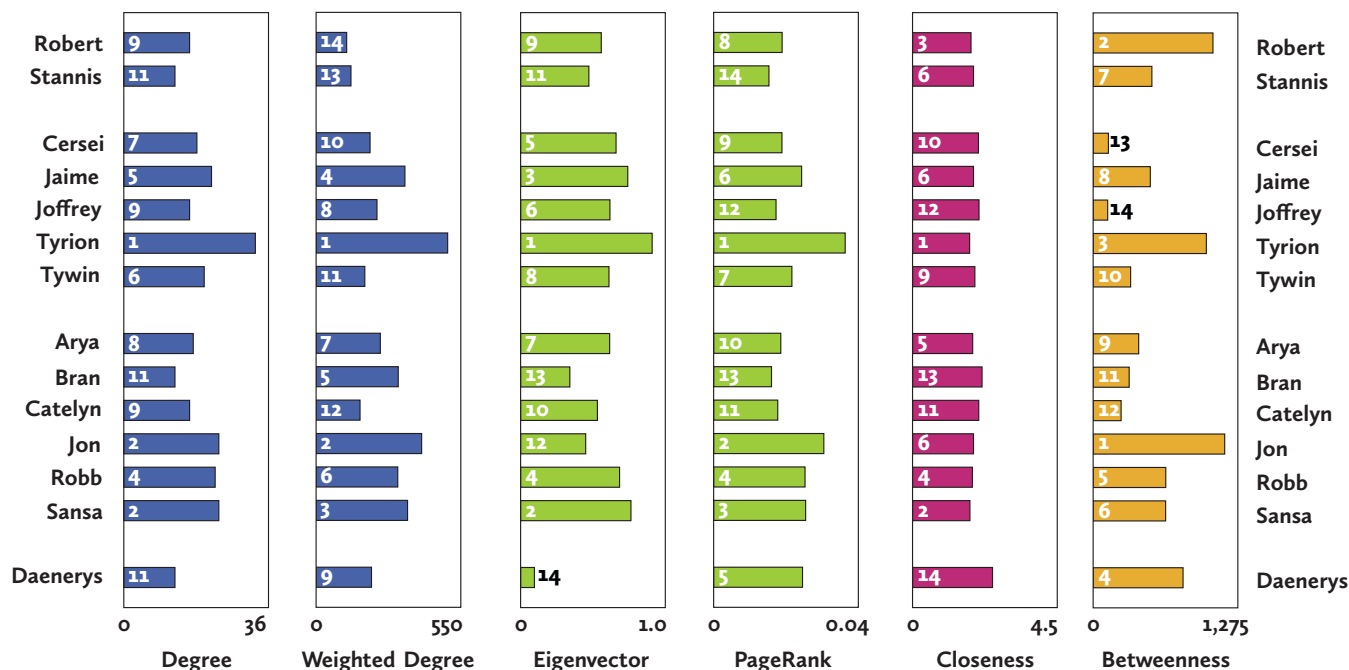
Figure 3. Centrality measures for the network. Larger values correspond to greater importance, except for closeness centrality, where smaller values are better. Numbers in the bars give the rankings of these characters.

the network. Summing over all vertices in a community C, we have

$$\sum_{i,j \in C}\left(w_{ij} - \frac{k_i k_j}{2m}\right) \geq 0.$$

Meanwhile, if C is not actually a community, then this quantity may be negative. The modularity Q of a vertex partition $C_1,...,C_l$ of the network is

$$Q = \frac{1}{2m}\sum_{k=1}^{\ell}\sum_{i,j \in C_k}\left(w_{ij} - \frac{k_i k_j}{2m}\right),$$

where we have normalized this quantity so that $-1 \leq Q \leq 1$.

Our goal is to partition the vertices into communities so as to maximize Q. Finding this partition is computationally difficult, so we use a fast approximation algorithm called the Louvain method.

Crucially, the algorithm determines the number of communities; it is not an input. In our case, we discover the seven communities in figure 2. The King's Landing community accounts for 37 percent of the network. When we perform community detection on this major subnetwork, we obtain four communities. A high resolution version of figure 2 and the network of subcommunities of King's Landing can be found at *maa.org/math-horizons-supplements.*

Centrality Measures

Network science can also identify important vertices. A person can play a central role in multiple ways. For example, she could be well connected, be centrally located, or be uniquely positioned to help disperse information or influence others. Figure 3 displays the importance of 14 prominent characters, according to six centrality measures, which we explain below.

Degree centrality is the number of edges incident with the given vertex. *Weighted degree centrality* is defined similarly by summing the weights of the incident edges. In our network, degree centrality measures the number of connections to other characters, while weighted degree centrality measures the number of interactions.

Eigenvector centrality is weighted degree centrality with a feedback loop: A vertex gets a boost for being connected to important vertices. The importance x_i of vertex i is the weighted sum of the importance of its neighboring vertices: $x_i = \sum_{j \in V} w_{ji} x_j$ for each $i \in V$. Solving the resulting linear system gives the eigenvector centrality. (This name comes from linear algebra: We actually find an eigenvector for eigenvalue $\lambda = 1$ of the matrix W with entries w_{ij}.)

Let's compare the weighted degree and eigenvector centralities for our network. The late King Robert receives a huge boost: He has only 18 connections,

but half of them are to other prominent players! Most leading characters also benefit from the feedback loop, being directly involved in the political intrigue and sweeping military turmoil that grips the realm. The exceptions are isolated from the main action: Bran (presumed dead and on the run), Jon Snow (marginalized in the far North), and Daenerys (exiled across the Narrow Sea).

PageRank is another variation on this theme. This measure was the founding idea behind Brin and Page's Google search engine. Each vertex has an inherent importance $\beta \geq 0$, along with an importance acquired from its neighbors. Unlike eigenvector centrality, a vertex does not get full credit for the total importance of its neighbors. Instead, that neighbor's importance is divided equally among its direct connections. In other words, a vertex of very high degree passes along only a small fraction of its importance to each neighbor.

The PageRank y_i of vertex i is given by

$$y_i = \alpha \sum_{j \in V} \frac{w_{ji}}{k_j} y_j + \beta,$$

where $\alpha + \beta = 1$ and $\alpha, \beta \geq 0$. Researchers typically use $\beta = 0.15$ to find an effective balance between inherent importance and the neighborhood boost.

PageRank does not penalize our three far-flung characters and actually has the opposite effect on Daenerys. In fact, the PageRank ordering is nearly identical to the degree centrality ordering, except Daenerys jumps from 12th place to fifth place. So PageRank correctly identifies the charismatic Daenerys as one of the most important players, even though she has relatively few connections.

This brings us to two centrality measures whose definitions take a more global view of the network. The *closeness centrality* of a vertex is the average distance from the vertex to all other vertices. (Unlike the other centrality measures, lower values correspond to greater importance.) The closeness values for our list of main characters is quite compressed, except for the faraway Daenerys. However, Tyrion and Sansa have a slight edge over everyone else.

The final centrality measure is the most subtle. The *betweenness centrality* of a vertex measures how frequently that vertex lies on short paths between other pairs of vertices. Mathematically, the betweenness z_i of vertex i is

$$z_i = \sum_{j,k \in V} \frac{\sigma_{jk}(i)}{\sigma_{jk}},$$

where σ_{jk} is the number of (j,k)-shortest paths and $\sigma_{jk}(i)$ is the number of these (j,k)-shortest paths that go through vertex i. A vertex that appears on many short paths is a broker of information in the network: Efficient communication between different parts of the network will frequently pass through such a vertex. Such connectors have the potential to be highly influential by inserting themselves into the dealings of other parties.

Betweenness centrality gives a distinctive ranking of the characters. This is the only measure in which Tyrion does not come out on top: He places third, behind Jon Snow (thanks to his ties to both House Stark and the remote denizens of the North) and Robert Baratheon (the only person directly connected to all four noble houses of the leading characters). Meanwhile, Daenarys rises to fourth place (her best showing) because of the hub-and-spoke nature of the eclectic Essos community.

There is no single "right" centrality measure for a network. Each measure gives complementary

Jon Snow is uniquely positioned in the network, with connections to highborn lords, the Night's Watch militia, and the savage wildlings beyond the Wall.

information, and taking them in concert can be quite revealing. In our network, three characters stand out consistently: Tyrion, Jon, and Sansa. Acting as the Hand of the King, Tyrion is thrust into the center of the political machinations of the capitol city. Our analysis suggests that he is the true protagonist of the book.

Meanwhile, Jon Snow is uniquely positioned in the network, with connections to highborn lords, the Night's Watch militia, and the savage wildlings beyond the Wall. The real surprise may be the prominence of Sansa Stark, a de facto captive in King's Landing. However, other players are aware of her value as a Stark heir and they repeatedly use her as a pawn in their plays for power. If she can develop her cunning, then she can capitalize on her network importance to dramatic effect.

Meanwhile, Robert and Daenarys stand out by overperforming in certain centrality measures. They

Directions to Be Read, Then Ignored

Gary Gordon and Rebecca Gordon

In writing worksheets, homework assignments, quizzes, and exams, we (the authors) often take a few liberties with the standard "Do all of these problems to the best of your ability" instructions at the top of the paper. Here are a few of the ones we've used, often in reference to the topic of the day, but also referring to current events, pop culture, and students in the class—and always just plain silly.

» Like snowflakes, no two of these problems are exactly the same. Also like snowflakes, they will melt if you touch them. Solve all of these problems without touching anything.

» There is an invisible dog in the room. It will bark uncontrollably if you make a mistake. Good luck!

» One of these problems was not approved for human consumption. If you eat that one by mistake, the antidote is to eat the next problem. (Note: This tells you something about which problem might be toxic.)

» You walk into a doctor's office and say, "I need an injection." He says, "$f(x) = 2x + 3$." Comment.

» Why does calculus keep getting harder? Come to think of it, why does everything keep getting harder? Classes, friendships, life—seriously, it's everything! These are some intense philosophical questions that won't be answered on this test. Good luck trying to concentrate now!

» As we sit here in this classroom, a large comet is speeding toward the earth. Scientists expect it to hit this building in exactly one hour. Plan your test-taking strategy accordingly.

» This is the beginning of a long, pointless exercise—like college. Enjoy!

» You have only 50 minutes for this one—don't waste time reading the directions. ∎

Gary and Rebecca Gordon are a father-daughter math team. Gary teaches at Lafayette College and is the Math Horizons *problems editor. Rebecca teaches mathematics at Newark Academy. The family is publishing a book on the card game SET, called* The Joy of SET.
Email: gordong@lafayette.edu
Email: rgordon@newarka.edu

---

provide a clear counterpoint to one another and return our attention to the Iron Throne itself. Robert's memory unifies the crumbling network of the recent past, while Daenarys will surely upend the current network when she returns to Westeros in pursuit of the throne.

A Networked Life

We have visited the realms of Westeros and Essos to tour the basic tools of network science. We performed an empirical analysis of our network, finding communities and identifying influential people. Our network analysis confirmed some expectations and provided new insights into this richly imagined saga. We have considered a fanciful application of network science to give an enticing taste of its capabilities. More serious applications abound, and network science promises to be invaluable in understanding our modern networked life. ∎

Further Reading

[1] A.-L. Barabási, *Network Science. barabasilab.neu. edu/networksciencebook/downlPDF.html.*

[2] D. Easley, J. Kleinberg. *Networks, Crowds and Markets.* Cambridge University Press, Cambridge, 2010.

[3] S. Fortunato. Community detection in graphs. *Physics Reports* **486** nos. 3–5 (2010) 75–174.

[4] M. Newman, *Networks: An Introduction.* Oxford University Press, Oxford, 2010.

Andrew Beveridge is an associate professor of mathematics at Macalester College. He plays bass guitar in a band called Math Emergency and owns the world's only velvet Erdős painting.
Email: abeverid@macalester.edu

Jie Shan graduated from Macalester College in 2014 with majors in computer science, applied math and stats, and economics. When away from computers and numbers, he is a board game enthusiast, wannabe comic artist, and recreational soccer player.
Email: jieshan22@gmail.com