COMS30121: Image Processing and Computer Vision

Motion Worksheet 1 : Understanding 2-D Motion Fields

1. Draw a plan view of the X-Z plane in a 3-D imaging system illustrating a pin hole camera model when a 3-D point $\mathbf{P}$ is projected to a 2-D point $\mathbf{p}$. The camera centre of projection is at coordinate $(0, 0, 0)$ and the principal axis of the camera lies along the Z-axis. The focal length of the camera is significantly less than the depth of the point $\mathbf{P}$.

2. On the same drawing, illustrate the relationship between a 3-D motion of the point $\mathbf{P}$ and its corresponding 2-D motion in the image plane.

3. By reference to your drawing, what ranges of 3-D motions of $\mathbf{P}$ would not be distinguishable in the 2-D motion field?

4. In a 2-D world with coordinate axes X and Z, a camera has its centre of projection at (0,0) and its principal axis lies along the Z axis. The camera produces 1-D 'images' as points in the 2-D world are projected onto the 'image line'. It has a focal length of 1. A line in the 2-D world is defined by its endpoints (-1,3) and (-2,3). Draw a diagram showing the camera and the line. Find the new position of the line after its undergoes a 2-D rigid body transformation defined by the equation $\mathbf{P}' = R\mathbf{P} + \mathbf{T}$, where R is a 2×2 matrix representing a clockwise rotation by 30^o and $\mathbf{T}$ is a 2×1 vector $[1, 1]^T$. Determine the 2-D motion of the endpoints and their corresponding 1-D motion in the image line.

5. By reference to your drawing, compare the 1-D motion of the endpoints of two lines $A = \{(-1, 3); (-2, 3)\}$ and $B = \{(-3, 9); (-6, 9)\}$ when they both undergo a 2-D motion corresponding to a rotation only. What principle of projected motion fields does this illustrate?

6. Using the motion equations below (slide 16 of lecture Motion I), describe the form of the motion fields produced by a camera moving with only translational motion whilst viewing a rigid scene. Note from the slides that (T_X, T_Y, T_Z) and $(\theta_X, \theta_Y, \theta_Z)$ denote the rectilinear and angular velocity of *scene points with respect to the camera coordinate frame*. If the component of the motion in the direction of the optical axis is non-zero, then there is a point in the field with zero motion. Determine an expression for the point.

$$\begin{aligned} v_x &= (fT_X - T_Z x)/Z + f\theta_Y - \theta_Z y - (\theta_X xy - \theta_Y x^2)/f \\ v_y &= (fT_Y - T_Z y)/Z - f\theta_X + \theta_Z x + (\theta_Y xy - \theta_X y^2)/f \end{aligned}$$

7. Figure 1 overleaf shows two 2-D motion fields resulting from moving a camera in front of a flat wall whilst keeping the camera image plane parallel with the wall. For each case, describe how the camera is moving and explain your answer by reference to the motion field equations. What is it about the form of the motion fields which suggests that the camera is moving in front of a flat wall?

8. Figure 2 overleaf shows the plan (top down) view of a rigid scene consisting of a vertical pole in front of a wall. In each case the scene is viewed by a moving camera with the motion shown by the arrow. For each case sketch the form of motion field that you would expect to observe in the image plane of the camera (NB: the 2D motion field, not the optic flow field, which you can't know without information about the appearance of the scene).

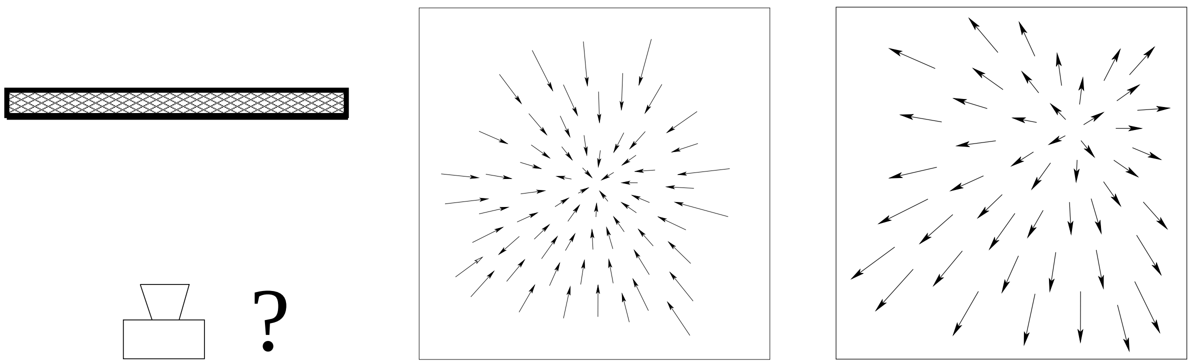

Figure 1: Two motion fields resulting from a camera moving in front of a flat wall - how is the camera moving?

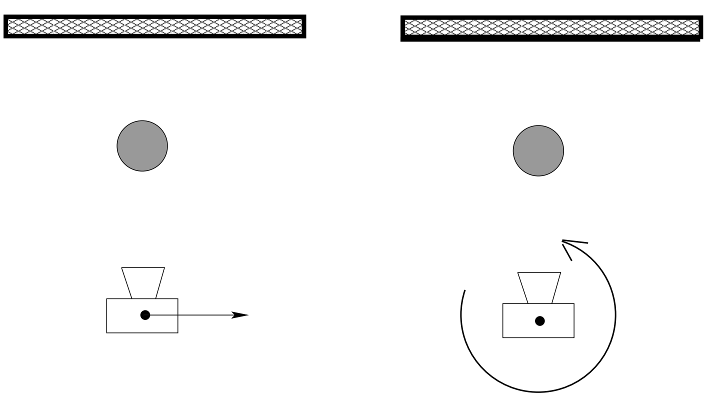

Figure 2: Camera moving in front of pole and wall - sketch the corresponding 2-D motion fields.