COMS30121
Image Processing and Computer Vision

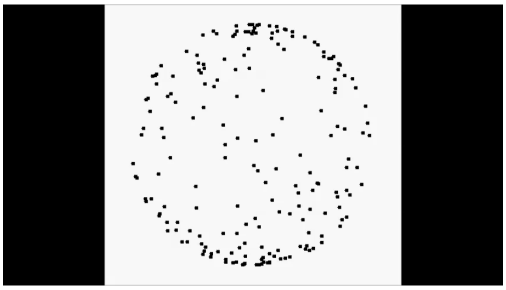
Lecture 7 : Motion I - Modelling

motion very important

(video of dogs)

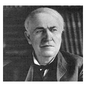
Can infer vast amount of info just with small dots!

(earth)



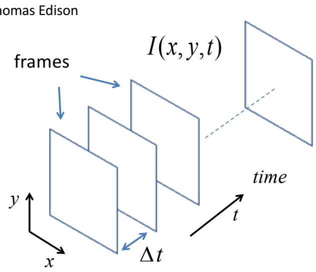
(person)

Video Sequences



Thomas Edison

Kinetograph



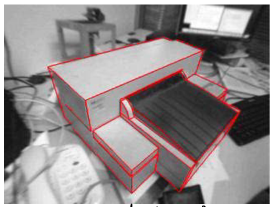
Frame rate : $1/\Delta t$
frames per second
e.g. 25 fps

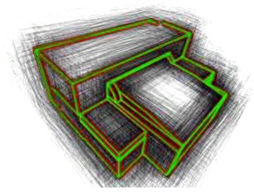
2-D Tracking

(2D tracking of pen) difficult to isolate pen 'end'.

(2D tracking of leaf) Random motion makes this tricky.

3-D Tracking – Model Based





(video footage)
wireframe model is given beforehand.

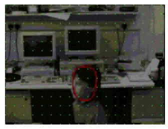
3-D Tracking and Mapping

SLAM (simult. localisation + mapping)

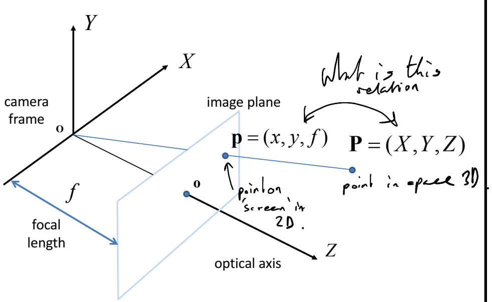
Building of a 3D map.

We are going to look at …..

- Understanding 2-D motion fields
- The optical flow equation (OFE)
- Motion estimation
  - Lucas and Kanade method (common implementation)
- Motion segmentation



Perspective Pin Hole Camera Model

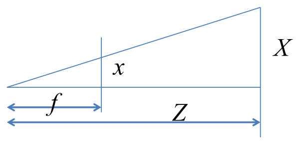


Perspective Projection Equations

- 3D point: $\mathbf{P} = (X, Y, Z)$ (on surface of object)
- Projects to 2D point: $\mathbf{p} = (x, y, f)$ (in image)
- Then using similar triangles:

$$x = \frac{fX}{Z}$$

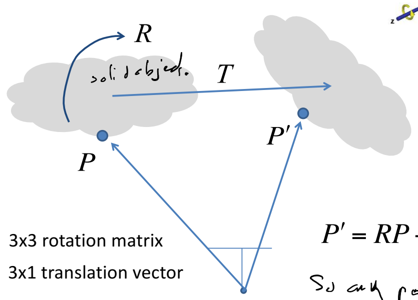
$$y = \frac{fY}{Z}$$



This ignores distortion of lens.

Order doesn't matter as long as consistent

3-D Rigid Motion



R : 3x3 rotation matrix
T : 3x1 translation vector

$$P' = RP + T$$

So any point on object is subject to same trans. + rot.

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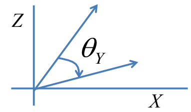
Rotation Matrices

$R = R_X R_Y R_Z$: Rotations about X, Y and Z axes

$$R_Y P = \begin{bmatrix} \cos\theta_Y & 0 & \sin\theta_Y \\ 0 & 1 & 0 \\ -\sin\theta_Y & 0 & \cos\theta_Y \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X\cos\theta_Y + Z\sin\theta_Y \\ Y \\ Z\cos\theta_Y - X\sin\theta_Y \end{bmatrix}$$



NB : for small θ_Y

$$R_Y \approx \begin{bmatrix} 1 & 0 & \theta_Y \\ 0 & 1 & 0 \\ -\theta_Y & 0 & 1 \end{bmatrix}$$

Small angle approximation!

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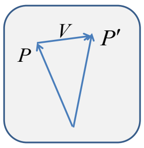
Often assume that framerate high enough that rotation is minimal between frames.

3-D Motion Field

$$V = \lim_{\Delta t \to 0} \{P' - P = (R - I)P + T\}$$



For small angles:

$$R \approx \begin{bmatrix} 1 & -\theta_Z & \theta_Y \\ \theta_Z & 1 & -\theta_X \\ -\theta_Y & \theta_X & 1 \end{bmatrix}$$

Hence:

$$V_X = \theta_Y Z - \theta_Z Y + T_X$$
$$V_Y = \theta_Z X - \theta_X Z + T_Y$$
$$V_Z = \theta_X Y - \theta_Y X + T_Z$$

$(\theta_X, \theta_Y, \theta_Z) \equiv$ Angular velocity

$(T_X, T_Y, T_Z) \equiv$ Rectilinear velocity

in direction.

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2-D Motion Field Equations

For image point $\mathbf{p} = (x, y, f)$

$$V_X = \frac{dX}{dt}$$

for perspective model.

$$v_x = \frac{dx}{dt} = \frac{d}{dt}\left(\frac{fX}{Z}\right) = f\,\frac{V_X Z - X V_Z}{Z^2}$$

Quotient rule

Look up to remind yourself

Substituting for V_X, V_Y, V_Z gives

$$v_x = (fT_X - xT_Z)/Z + f\theta_Y - \theta_Z y - (\theta_X xy - \theta_Y x^2)/f$$
$$v_y = (fT_Y - yT_Z)/Z - f\theta_X + \theta_Z x + (\theta_Y xy - \theta_X y^2)/f$$

Our fundamental motion equ.s

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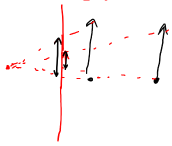
Two Components

$$v_x = (fT_X - xT_Z)/Z + f\theta_Y - \theta_Z y - (\theta_X xy - \theta_Y x^2)/f$$
$$v_y = (fT_Y - yT_Z)/Z - f\theta_X + \theta_Z x + (\theta_Y xy - \theta_X y^2)/f$$

Translational – dependent on scene depth Z

Makes sense.



Rotational – independent of scene depth Z

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