COMS30121
Image Processing and Computer Vision

Motion II

Motion II

- True and apparent motion
- Optical flow
- Optical flow equation
- Aperture problem
- Motion estimation
- Lucas and Kanade algorithm

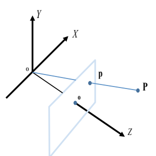
Projected 2-D Motion Field

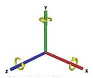
Last lecture

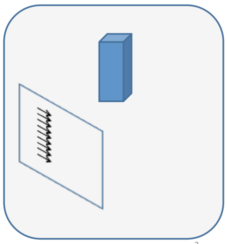
$$\mathbf{v} = tran(x, y, \mathbf{T}, \mathbf{Z}) + rot(x, y, \boldsymbol{\theta})$$

Translation component

Rotation component

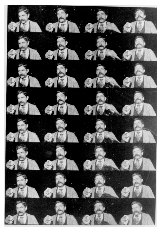






Motion Estimation

- The estimation of the 2-D motion field from frames in an image sequence
- Using spatial and temporal variation of pixel values
- **BUT**- relationship between variation in pixel values – known as **apparent motion** or **optical flow** – and the true motion is not straightforward.



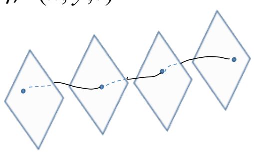
Optical Flow

Image sequence : $I(x, y, t)$

Variation of pixel values between frames
– **apparent motion** or **optical flow**

Assumption : along trajectory, $I(x, y, t)$ constant

Hence : $\frac{d}{dt} I(x, y, t) = 0$



Optical Flow Equation (OFE)

Hence : $\frac{d}{dt} I(x, y, t) = 0$

x, y are also functions of t, hence use chain rule to give total derivative:

OFE

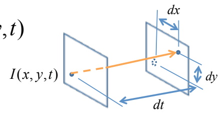
$$\frac{\partial I}{\partial x}\frac{dx}{dt} + \frac{\partial I}{\partial y}\frac{dy}{dt} + \frac{\partial I}{\partial t} = 0 \Rightarrow \boxed{I_x v_x + I_y v_y + I_t = 0}$$

Gradients : (I_x, I_y, I_t) Motion : (v_x, v_y)

Optical Flow II

Model : pixel value at (x, y, t) will move by dx, dy and dt

⇨ $I(x+dx, y+dy, t+dt) = I(x,y,t)$

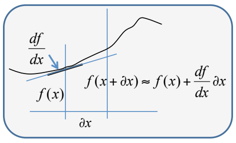


Linear approximation :

$$I(x+dx, y+dy, t+dt) \approx$$

$$I(x,y,t) + \frac{\partial I}{\partial x}dx + \frac{\partial I}{\partial y}dy + \frac{\partial I}{\partial t}dt$$

= 0



Optical Flow Equation II

From linear approximation : $\frac{\partial I}{\partial x}dx + \frac{\partial I}{\partial y}dy + \frac{\partial I}{\partial t}dt = 0$

Dividing both sides by dt : $\frac{\partial I}{\partial x}\frac{dx}{dt} + \frac{\partial I}{\partial y}\frac{dy}{dt} + \frac{\partial I}{\partial t} = 0$

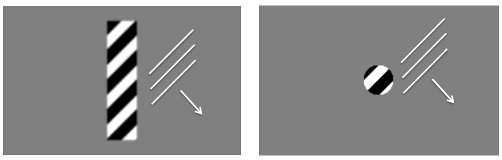
Gradients : $\left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}, \frac{\partial I}{\partial t}\right) = (I_x, I_y, I_t)$

Motion : $\left(\frac{dx}{dt}, \frac{dy}{dt}\right) = (v_x, v_y)$

OFE

$$I_x v_x + I_y v_y + I_t = 0$$

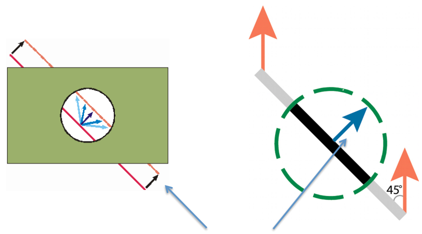
Aperture Problem



See also :

http://web.mit.edu/persci/demos/Motion&Form/demos/one-square/one-square.html

Normal Flow



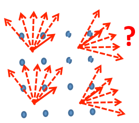
Constraining the OFE

$$I_x v_x + I_y v_y + I_t = 0$$

At one pixel, OFE is under constrained – can only estimate normal flow

Hence, need to add extra constraint(s)

Example : assume parametric form of motion field in regions



Example : constant velocity

Constraining the OFE

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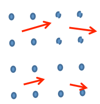
Example : assume parametric form of motion field in regions



Example : linear in x and y, e.g. $v_x = ax + by + c$

Constant Velocity Model

For a region, find the velocity $\mathbf{v} = (v_x, v_y)$ which minimises :

$$\varepsilon(v_x, v_y) = \sum_{region} (\underbrace{I_x v_x + I_y v_y + I_t}_{\text{OFE} \longrightarrow 0})^2$$

Solution : take derivatives w.r.t v_x and v_y , set to zero, and solve for v_x and v_y .

Lucas and Kanade Algorithm

Solve for $\mathbf{v} = (v_x, v_y)$ given that :

$$v_x \sum I_x^2 + v_y \sum I_x I_y = -\sum I_t I_x$$

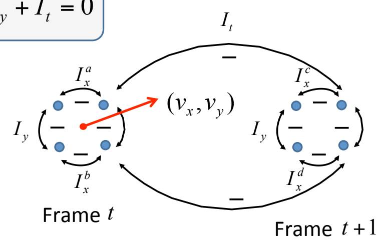
$$v_x \sum I_x I_y + v_y \sum I_y^2 = -\sum I_t I_y$$

$$\mathbf{A} = \sum \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \quad \mathbf{b} = -\sum \begin{bmatrix} I_t I_x \\ I_t I_y \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} v_x \\ v_y \end{bmatrix} = \mathbf{A}^{-1}\mathbf{b}$$

Spatial & Temporal Gradients

$$\boxed{I_x v_x + I_y v_y + I_t = 0}$$

$$I_x = \frac{\partial I}{\partial x} \qquad I_y = \frac{\partial I}{\partial y} \qquad I_t = \frac{\partial I}{\partial t}$$



$$I_x \approx (I_x^a + I_x^b + I_x^c + I_x^d)/4$$