ROBOTIC FUNDAMENTALS (UMFM4X-15-M)

Forward Kinematics

UWE Bristol University of the West of England

Previously on

ROBOTIC FUNDAMENTALS

- Kinematics and Reference frames – Basis of the Analysis
- Connecting Frames – Translation Vectors and Rotation Matrices
- Unified Representation – Homogeneous Transformations – Compound Transformations
- Euler Angles
- Quaternions
- HT in Matlab – plotting vectors and CS

Questions?

Today's Lecture

The Denavit Hartenberg (DH) convention

Placing of frames

Examples

Joint, Cartesian, Actuator space

What is the goal of FK?

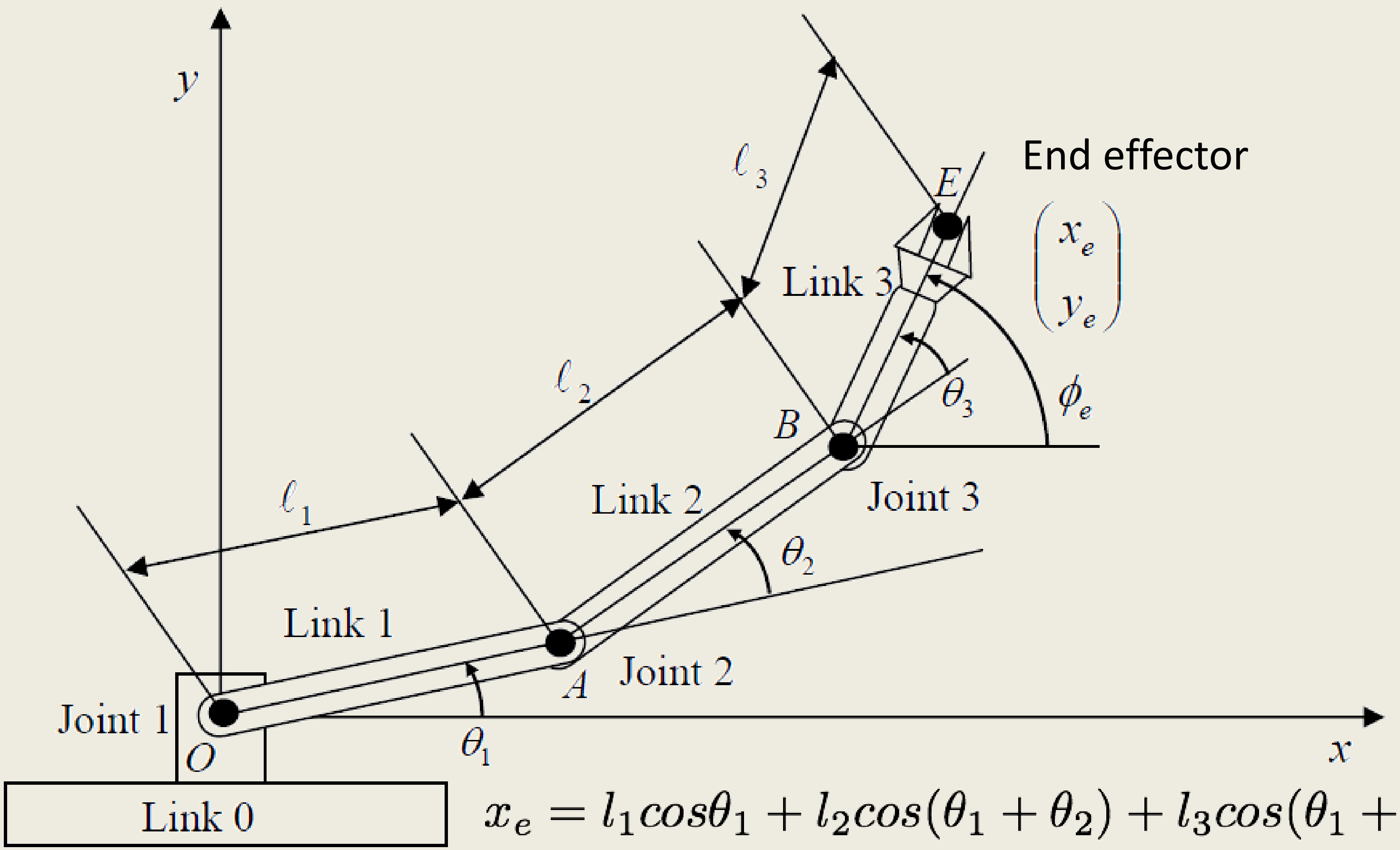
- To give the **Cartesian coordinates** of the end effector of a manipulator in terms of *joint parameters*

$$x, y, z = f(q_1, q_2, \ldots, q_n)$$

- Give a set of **orientation values** for the end-effector of a manipulator in terms of *joint parameters*

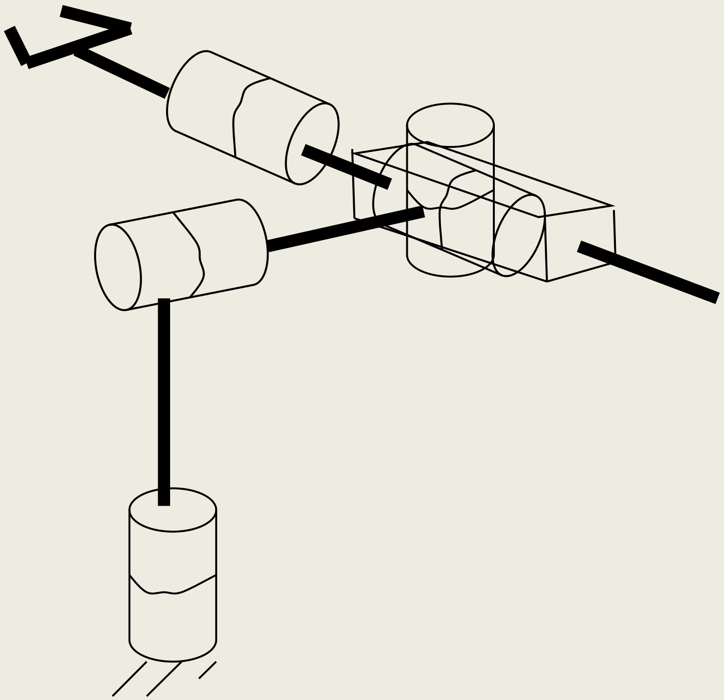
$$RPY \text{ or } ZYZ \text{ or } ZXZ = f(q_1, q_2, \ldots, q_n)$$

Planar RRR



$$x_e = l_1 cos\theta_1 + l_2 cos(\theta_1 + \theta_2) + l_3 cos(\theta_1 + \theta_2 + \theta_3)$$
$$y_e = l_1 sin\theta_1 + l_2 sin(\theta_1 + \theta_2) + l_3 sin(\theta_1 + \theta_2 + \theta_3)$$
$$\phi_e = \theta_1 + \theta_2 + \theta_3$$

What about complex problems?



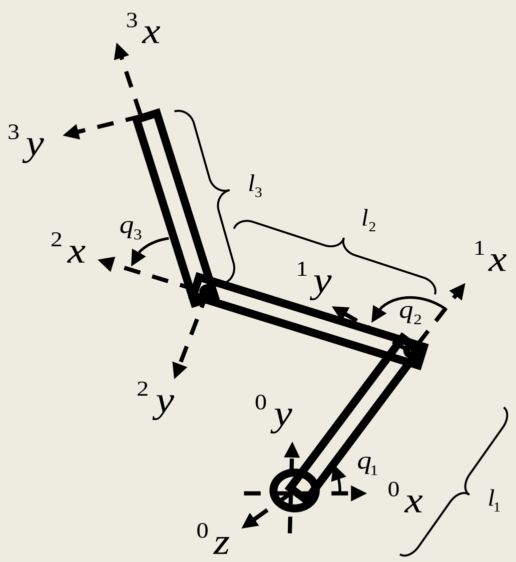
DENAVIT HARTENBERG

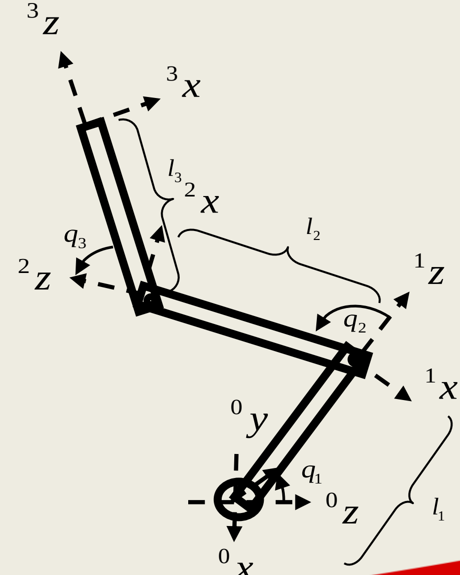
Denavit Hartenberg (DH)

- Principle of use
- Proximal vs Distal
- Examples
- Joint, Cartesian, Actuator space

DH Approach

- There is a large number of ways to encode kinematics using HT
- We will sacrifice some of this flexibility for a more systematic approach: **DH (Denavit-Hartenberg) parameters.**
- DH parameters is a standard for describing a series of transforms for arbitrary mechanisms.





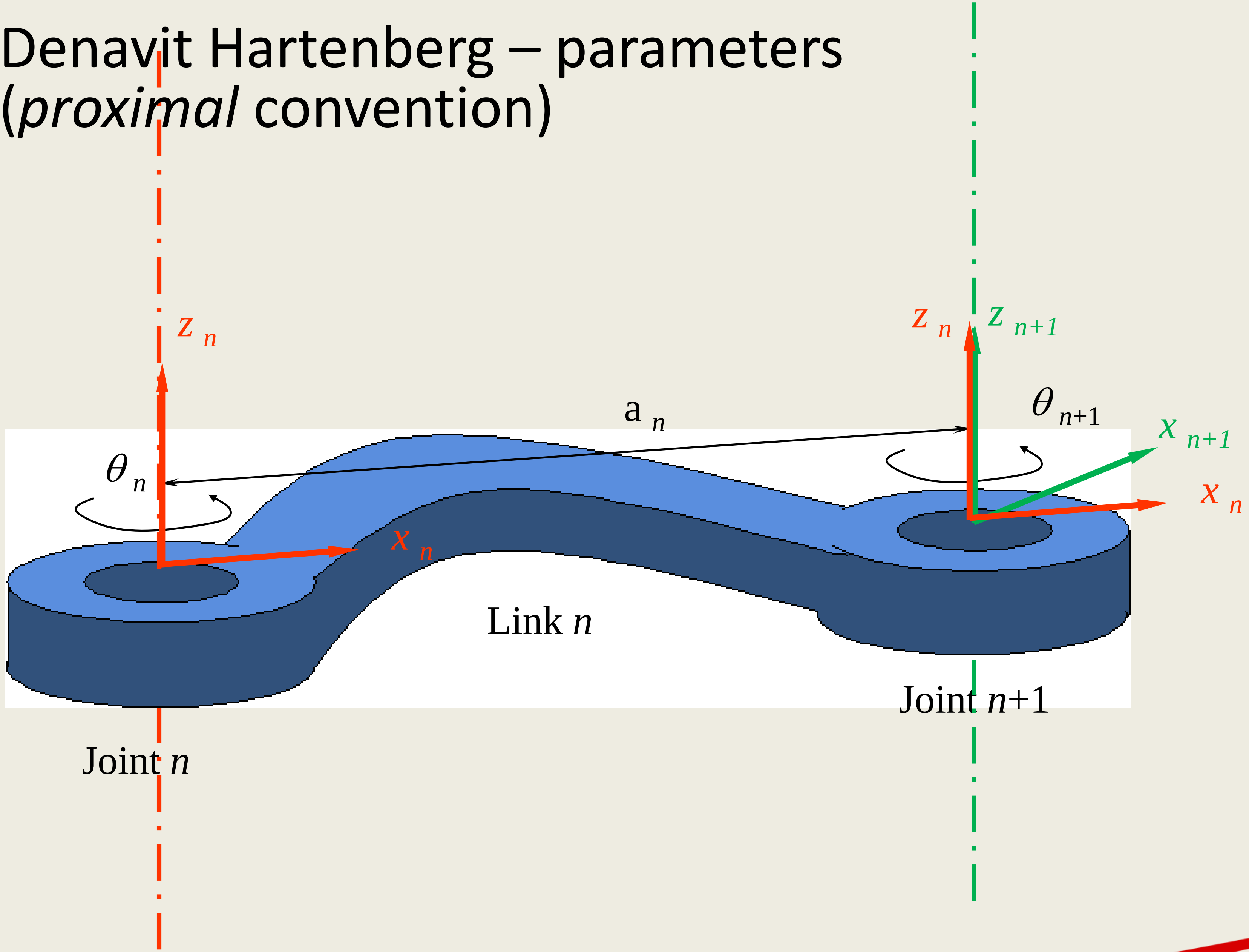
Denavit Hartenberg notation

- Uniquely defines the architecture of a robot manipulator (kinematic chain) for any configuration

- Link shape is not important; joints and links are numbered from base forward, eg. joint 1 refers to the connection point between the base and link 1, etc.

- Proximal or Distal (standard) convention

Denavit Hartenberg – parameters (*proximal* convention)

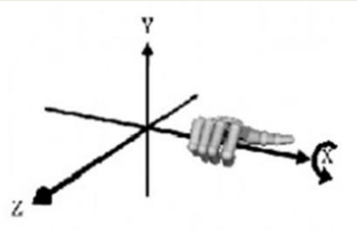


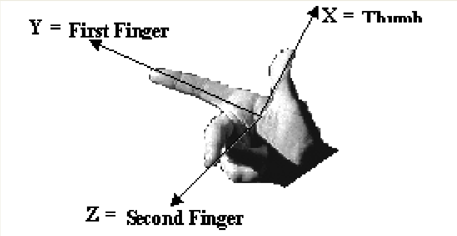
Denavit Hartenberg – how to fix frames (*proximal* convention)

For all the links we set:

- Z_n axis to lie along the axis of motion of n^{th} joint (or n+1 in case of standard DH)
- X_n axis to be along a_n (perpendicular) pointing from Z_n to Z_{n-1} axis

  If a_n=0: X_n is normal to the plane of Z_n to Z_{n-1}.
- Y_i completes the right hand rule





For **first** and **last** links

- Link 0: reference/static frame. For simplification, choose z_0 to be aligned with z_1 and origin of frame {0} to be the same as {1} when q_1=0.
- Link n: for simplification, choose x_n to be aligned with x_{n-1}

Four DH parameters (*proximal* convention)

Four parameters can be defined which uniquely specify the link and joint geometry. Each parameter for link *n* can be thought of as a successive movement required to map frame *{n} to* ~~*{i+1}*~~:
n+1

1. Link **length a_n**: length of the perpendicular - displacement in the x_n direction to bring the origin of frame *{n}* coincident with that of frame {*n+1*}

2. Link **twist α_n**: rotation about x_n to make z_n coincident with z_{n+1} (defined from z_n to z_{n+1})

3. Link **offset d_n**: displacement along z_n to go from x_{n-1} to the link x_n

4. Joint **angle θ_n**: rotation about z_n required to align x_{n-1} with x_n (defined from x_{n-1} to x_n)

DH parameters – Proximal Table form

n	a_{n-1}	a_{n-1}	d_n	θ_n
1				
2				

Link Number
(or joint No)

Four DH parameters (*proximal* convention)

Displacements or rotations only along/around **x** or **z** axes!

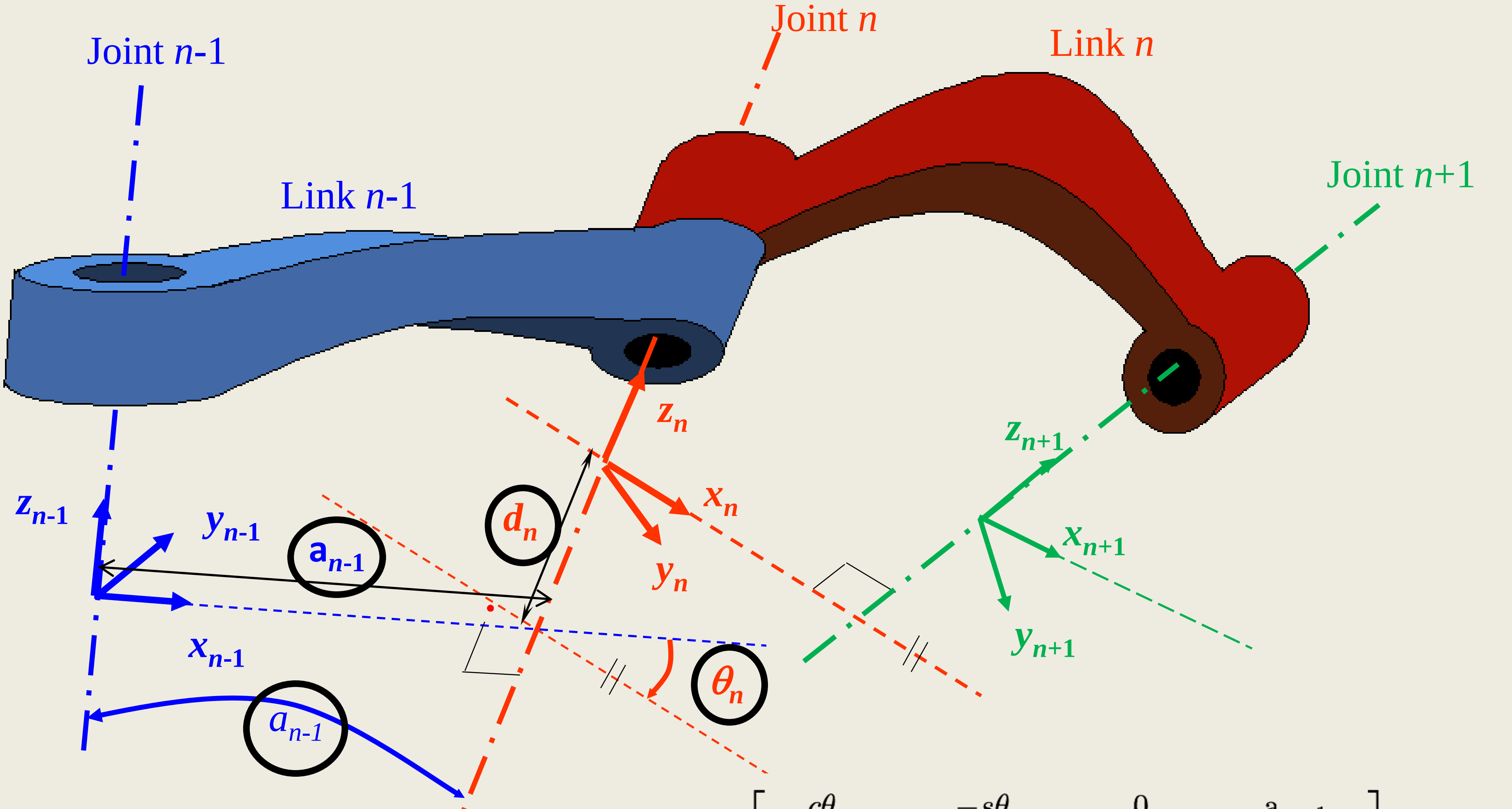
1. **a_n**: distance along **x_n, n -> n+1**
2. **α_n**: rotation about **x_n, n -> n+1**
3. **d_n**: distance along **z_n, n-1 -> n**
4. **θ_n**: rotation about **z_n, n-1 -> n**

OR

1. **a_{n-1}**: distance along **x_{n-1}, n-1 -> n**
2. **α_{n-1}**: rotation about **x_{n-1}, n-1 -> n**
3. **d_n**: distance along **z_n, n-1 -> n**
4. **θ_n**: rotation about **z_n, n-1 -> n**

Proximal convention (modified DH)

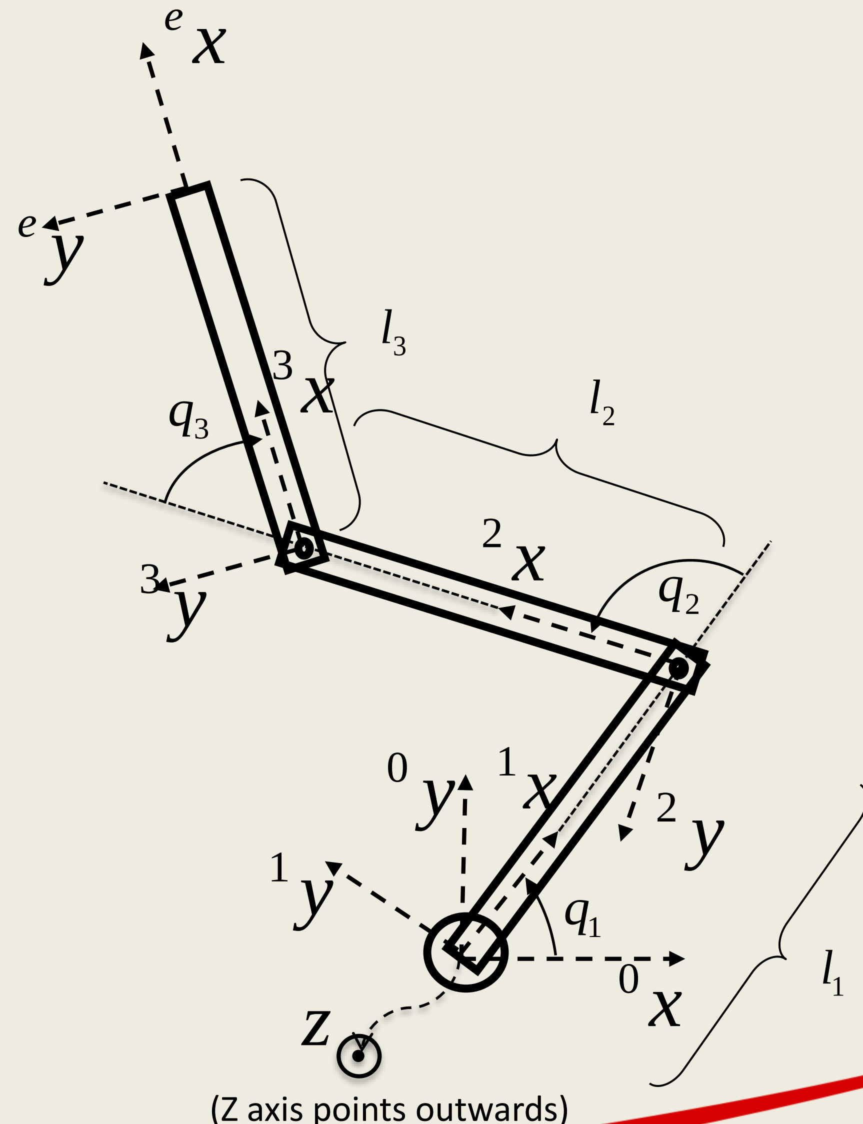
a_n = the distance from $\boldsymbol{z_n}$ to $\boldsymbol{z_{n+1}}$ measured along $\boldsymbol{x_n}$
α_n = the angle between $\boldsymbol{z_n}$ and $\boldsymbol{z_{n+1}}$ measured about $\boldsymbol{x_n}$
d_n = the distance from $\boldsymbol{x_{n-1}}$ to $\boldsymbol{x_n}$ measured along $\boldsymbol{z_n}$
θ_n = the angle between $\boldsymbol{x_{n-1}}$ to $\boldsymbol{x_n}$ measured about $\boldsymbol{z_n}$



$$
{}^{n-1}_{n}T = R_{x_{n-1}}(\alpha_{n-1})T_{x_{n-1}}(\mathrm{a}_{n-1})R_{z_n}(\theta_n)T_{z_n}(d_n) = \begin{bmatrix} c\theta_n & -s\theta_n & 0 & \mathrm{a}_{n-1} \\ s\theta_n c\alpha_{n-1} & c\theta_n c\alpha_{n-1} & -s\alpha_{n-1} & -s\alpha_{n-1}d_n \\ s\theta_n s\alpha_{n-1} & c\theta_n s\alpha_{n-1} & c\alpha_{n-1} & c\alpha_{n-1}d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Example 1: Planar RRR – **Proximal** convention

n	a_{n-1}	α_{n-1}	d_n	θ_n
1				
2				
3				
e				



Example 1: Planar RRR

$$ {}^{n-1}_{\ n}T = \begin{bmatrix} c\theta_n & -s\theta_n & 0 & a_{n-1} \\ s\theta_n c\alpha_{n-1} & c\theta_n c\alpha_{n-1} & -s\alpha_{n-1} & -s\alpha_{n-1}d_n \\ s\theta_n s\alpha_{n-1} & c\theta_n s\alpha_{n-1} & c\alpha_{n-1} & c\alpha_{n-1}d_n \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^0T_1 = \begin{pmatrix} c_{q_1} & -s_{q_1} & 0 & 0 \\ s_{q_1} & c_{q_1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^1T_2 = \begin{pmatrix} c_{q_2} & -s_{q_2} & 0 & l_1 \\ s_{q_2} & c_{q_2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

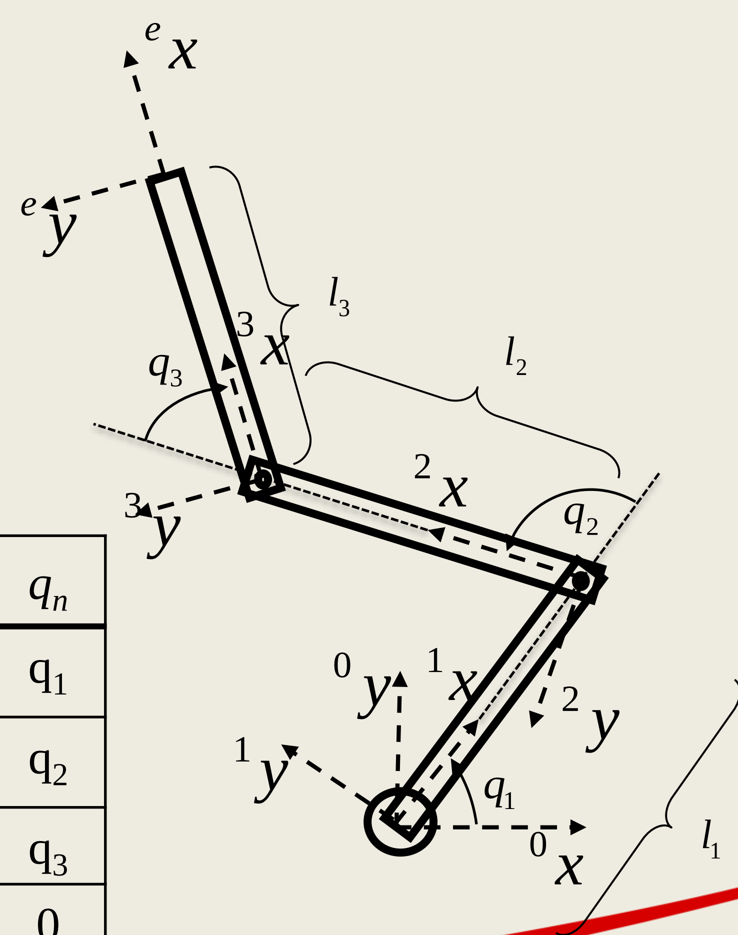
$$ {}^2T_3 = \begin{pmatrix} c_{q_3} & -s_{q_3} & 0 & l_2 \\ s_{q_3} & c_{q_3} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^3T_e = \begin{pmatrix} 1 & 0 & 0 & l_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^0T_3 = {}^0T_1\,{}^1T_2\,{}^2T_3 $$

$$ \boxed{{}^0T_e = {}^0T_3\,{}^3T_e} $$

n	a_{n-1}	a_{n-1}	d_n	q_n
1	*0*	0	0	q_1
2	l_1	0	0	q_2
3	l_2	0	0	q_3
e	l_3	0	0	0



Distal convention – frames & parameters

Let us define a coordinate frame $o_i x_i y_i z_i$ attached to link i as follows:

1. z - axis is along the rotation direction for revolute joints, along the translation direction for prismatic joints.
2. The z_{i-1} axis lies along the axis of motion of the ith joint.
3. The origin o_i is located at the intersection of joint axis z_i with the common normal to z_i and z_{i-1}.
4. The x_i axis is taken along the common normal and points from joint i to joint $i+1$.
5. The y_i axis is selected to complete right-hand frame. The y_i axis is defined by the cross product $\boldsymbol{y_i} = \boldsymbol{z_i} \times \boldsymbol{x_i}$.

Between z_{i-1} and z_i

Showing only z and x axes is sufficient, drawing is made clearer by ***NOT*** showing y axis.

1. Link **length a_i**: offset distance from o_i to the intersection of the z_{i-1} and x_i axes <u>along x_i</u>
2. Link **twist α_i**: angle <u>about x_i</u> from z_{i-1} axis to the z_i
3. Link **offset d_i**: distance from o_{i-1} to the intersection of z_{i-1} with x_i <u>along z_{i-1}</u>
4. Joint **angle θ_i**: angle <u>about z_{i-1}</u> from x_{i-1} to x_i

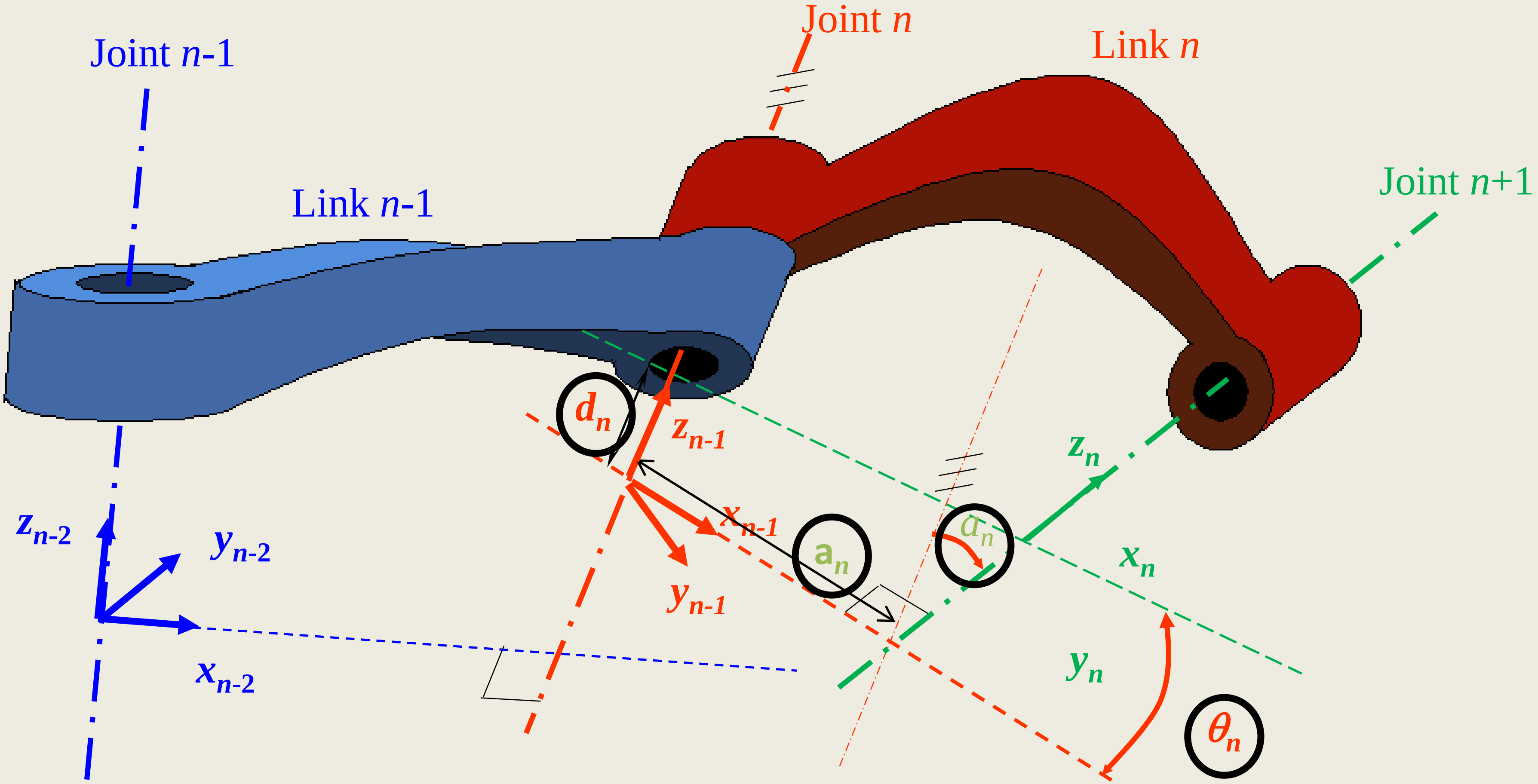
DH parameters – Distal Table form

i	a_i	a_i	d_i	θ_i
1				
2				

Link Number

Distal convention (standard DH)

a_n = the distance from $\boldsymbol{z_{n-1}}$ to $\boldsymbol{z_n}$ measured along $\boldsymbol{x_n}$
α_n = the angle between $\boldsymbol{z_{n-1}}$ and $\boldsymbol{z_n}$ measured about $\boldsymbol{x_n}$
d_n = the distance from $\boldsymbol{x_{n-1}}$ to $\boldsymbol{x_n}$ measured along $\boldsymbol{z_{n-1}}$
θ_n = the angle between $\boldsymbol{x_{n-1}}$ to $\boldsymbol{x_n}$ measured about $\boldsymbol{z_{n-1}}$



$$
{}^{n-1}_{\ \ n}T = R_{z_{n-1}}(\theta_n)T_{z_{n-1}}(d_n)T_{x_n}(\mathrm{a}_n)R_{x_n}(\alpha_n) = \begin{bmatrix} c\theta_n & -c\alpha_n s\theta_n & s\alpha_n s\theta_n & \mathrm{a}_n c\theta_n \\ s\theta_n & c\alpha_n c\theta_n & -s\alpha_n c\theta_n & \mathrm{a}_n s\theta_n \\ 0 & s\alpha_n & c\alpha_n & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Comparison of DH conventions

Proximal (modified)

1. a_n: distance along x_n, $z_n \to z_{n+1}$
2. α_n: rotation about x_n, $z_n \to z_{n+1}$
3. d_n: distance along z_n, $x_{n-1} \to x_n$
4. θ_n: rotation about z_n, $x_{n-1} \to x_n$

n	a_{n-1}	α_{n-1}	d_n	θ_n
1				
2				

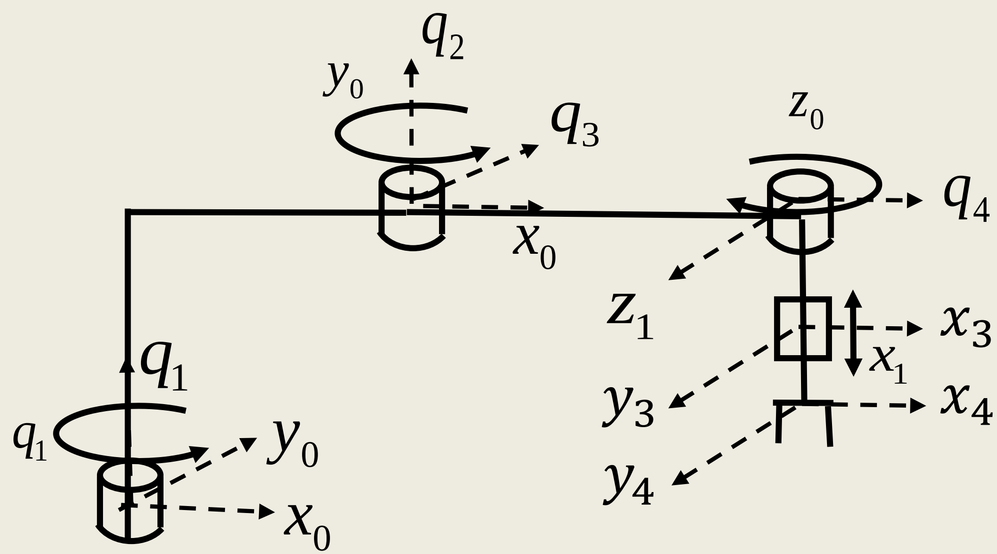
Distal (standard)

1. a_n: distance along x_n, $z_{n-1} \to z_n$
2. α_n: rotation about x_n, $z_{n-1} \to z_n$
3. d_n: distance along z_{n-1}, $x_{n-1} \to x_n$
4. θ_n: rotation about z_{n-1}, $x_{n-1} \to x_n$

n	a_n	α_n	d_n	θ_n
1				
2				

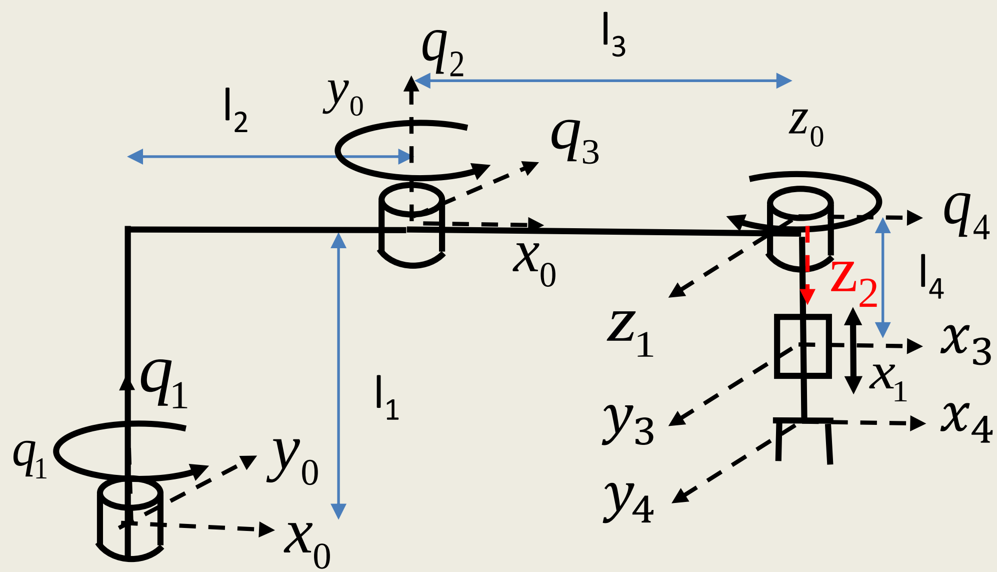
Example 2: RRRP – **Distal** convention

Practice proximal convention on RRRP in class



Example 2: RRRP – **Distal** convention

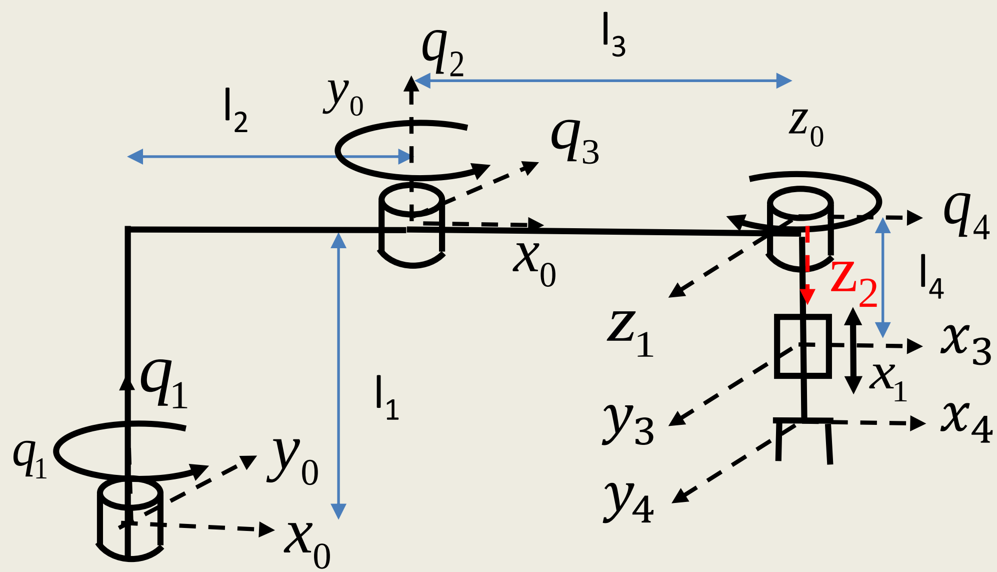
n	a_n	α_n	d_n	θ_n
1				
2				
3				
4				



a_n = the distance from $\boldsymbol{z_{n-1}}$ to $\boldsymbol{z_n}$ measured along $\boldsymbol{x_n}$
α_n = the angle between $\boldsymbol{z_{n-1}}$ and $\boldsymbol{z_n}$ measured about $\boldsymbol{x_n}$
d_n = the distance from $\boldsymbol{x_{n-1}}$ to $\boldsymbol{x_n}$ measured along $\boldsymbol{z_{n-1}}$
θ_n = the angle between $\boldsymbol{x_{n-1}}$ to $\boldsymbol{x_n}$ measured about $\boldsymbol{z_{n-1}}$

Example 2: RRRP – **Distal** convention

n	a_n	a_n	d_n	θ_n
1	l_2	0	l_1	q_1
2	l_3	π	0	q_2
3	0	0	l_4	q_3
4	0	0	q_4	0



Joint space, Actuator space and Cartesian space

- Kinematic joints actuated directly by actuator ⇨

Joint Space = Actuator Space

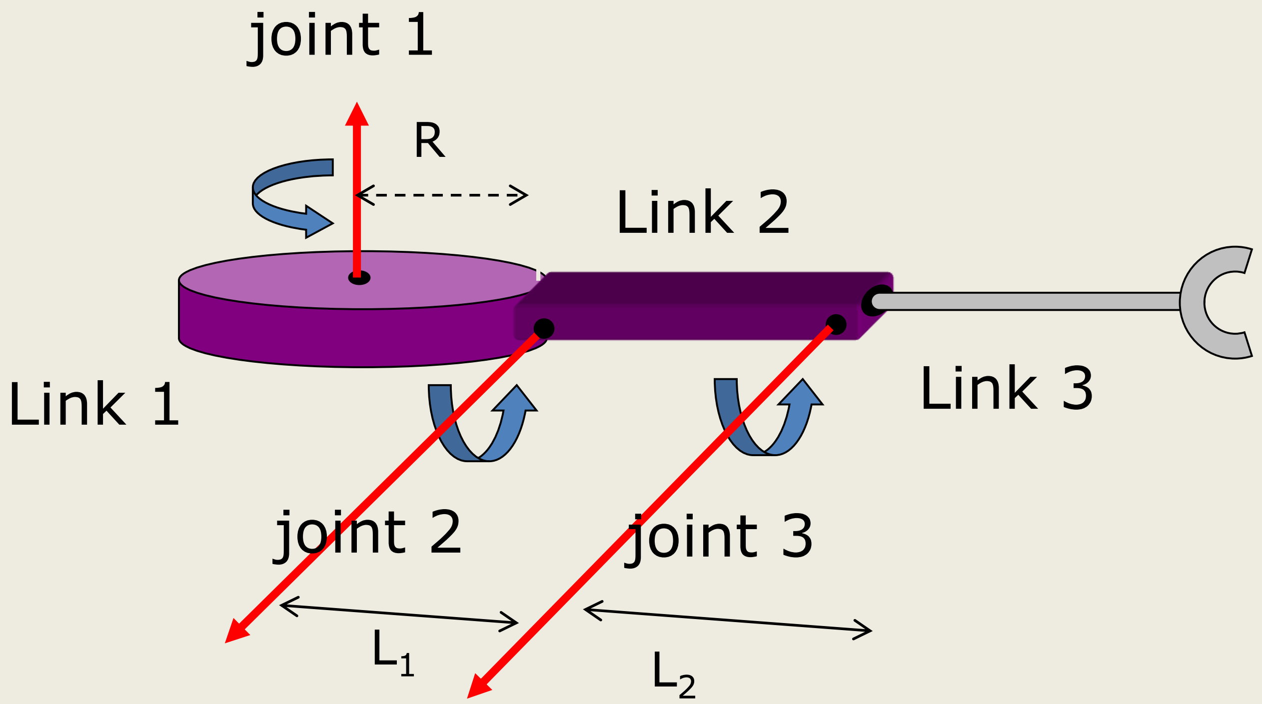
- Actuator + linkage ⇨ Joint Space ≠ Actuator Space
- We can compute the Cartesian space from joint space

Cartesian Space = Task Space or Operational Space

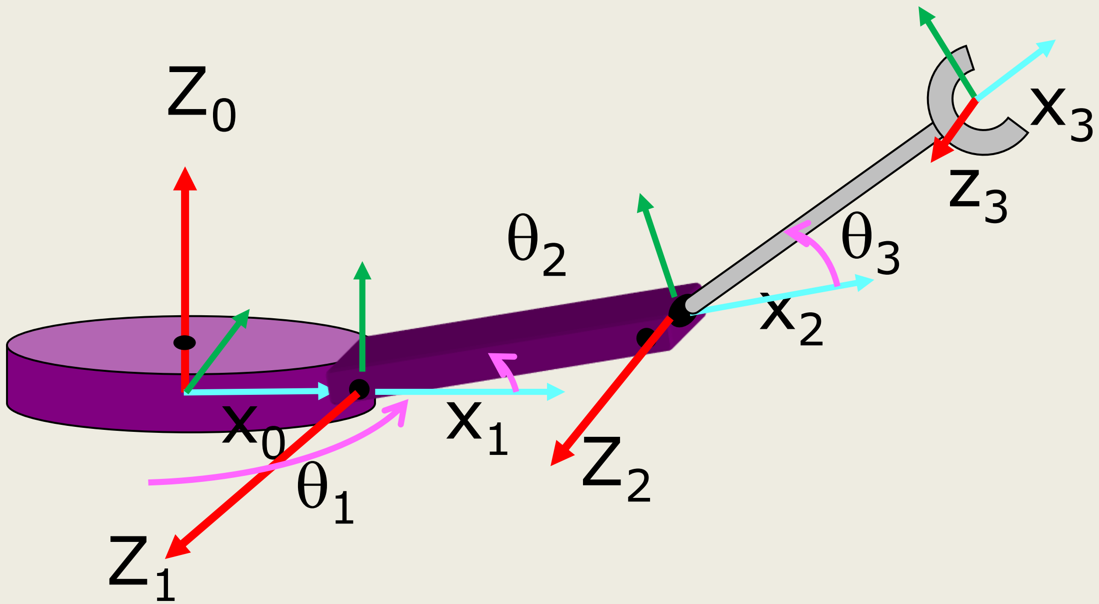
- Example: End-effector representation in joint space and Cartesian space

$$q = \begin{bmatrix} p_x \\ p_y \\ \phi \end{bmatrix} = \begin{bmatrix} a_1 \cos\theta_1 + a_2 \cos\theta_{12} + a_3 \cos\theta_{123} \\ a_1 \sin\theta_1 + a_2 \sin\theta_{12} + a_3 \sin\theta_{123} \\ \theta_1 + \theta_2 + \theta_3 \end{bmatrix}$$

Example 3 – 3D Space RRR: Home Position

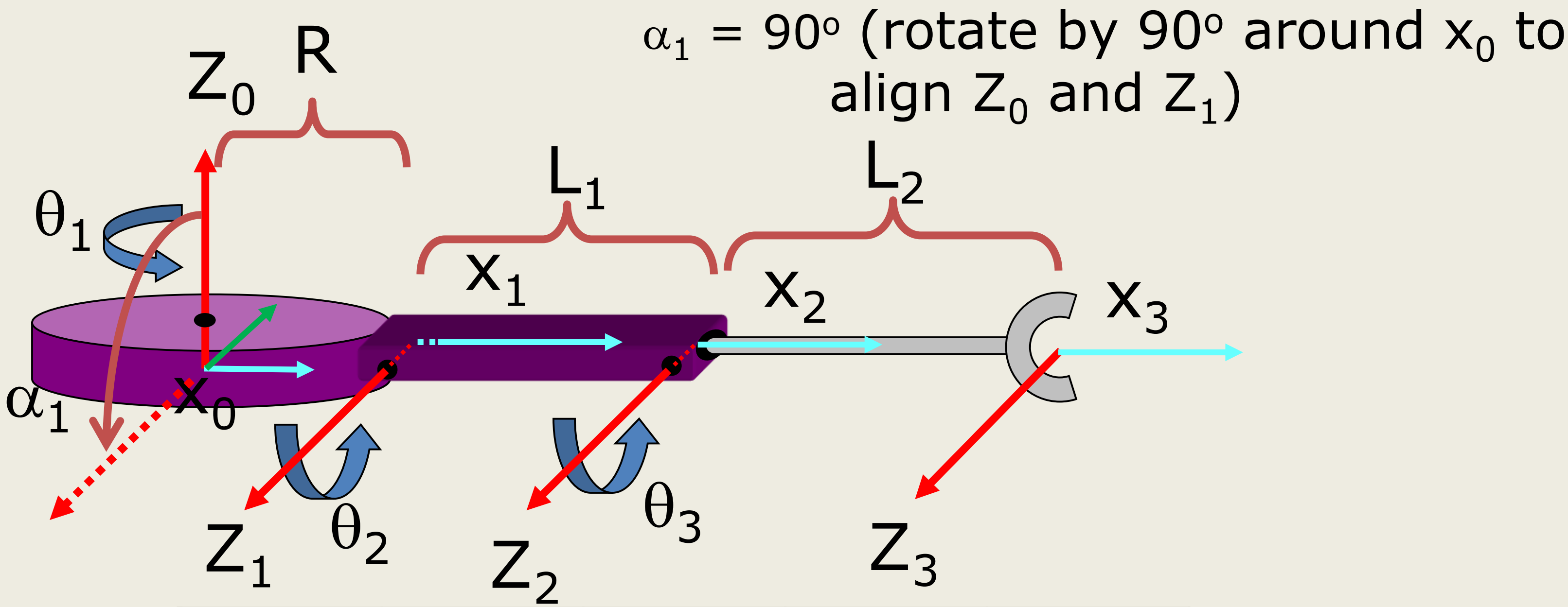


Example 3 – 3D Space RRR: Off-Home Position



Observe that frame i moves with link i

Example 3 – 3D Space RRR: Table of DH parameters



n	a_n	α_n	d_n	θ_n
1	R	90°	0	θ_1
2	L_1	0	0	θ_2
3	L_2	0	0	θ_3

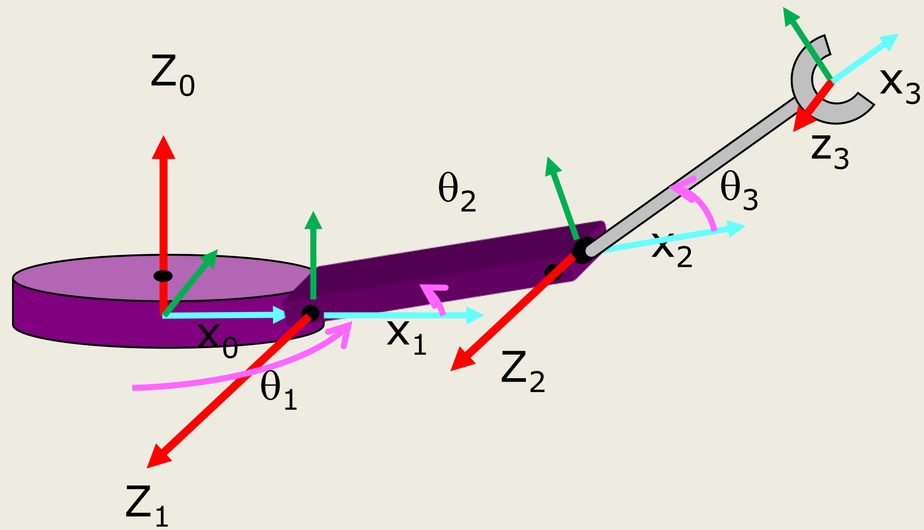
Example 3 – 3D Space RRR: Transformation Matrices

$$
{}^{0}_{1}T = \begin{bmatrix} c\theta_1 & 0 & s\theta_1 & Rc\theta_1 \\ s\theta_1 & 0 & -c\theta_1 & Rs\theta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{1}_{2}T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & L_1 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & L_1 s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{2}_{3}T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & L_2 c\theta_3 \\ s\theta_3 & c\theta_3 & 0 & L_2 s\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Example 3 – 3D Space RRR (1)



x_1 axis expressed wrt {0}

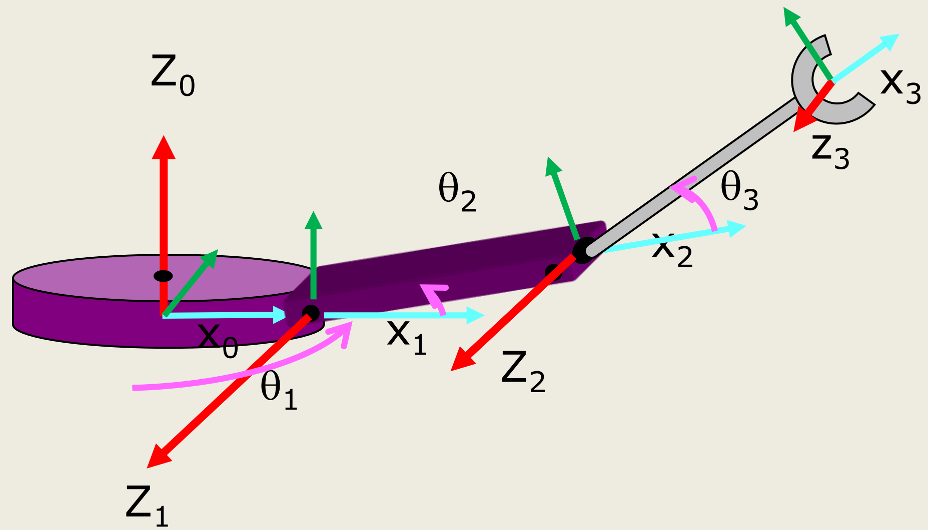
y_1 axis expressed wrt {0}

z_1 axis expressed wrt {0}

Origin of {1} w.r.t. {0}

$$ {}^0_1T = \begin{bmatrix} c\theta_1 & 0 & s\theta_1 & Rc\theta_1 \\ s\theta_1 & 0 & -c\theta_1 & Rs\theta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Example 3 – 3D Space RRR (2)



x_2 axis expressed wrt {1}

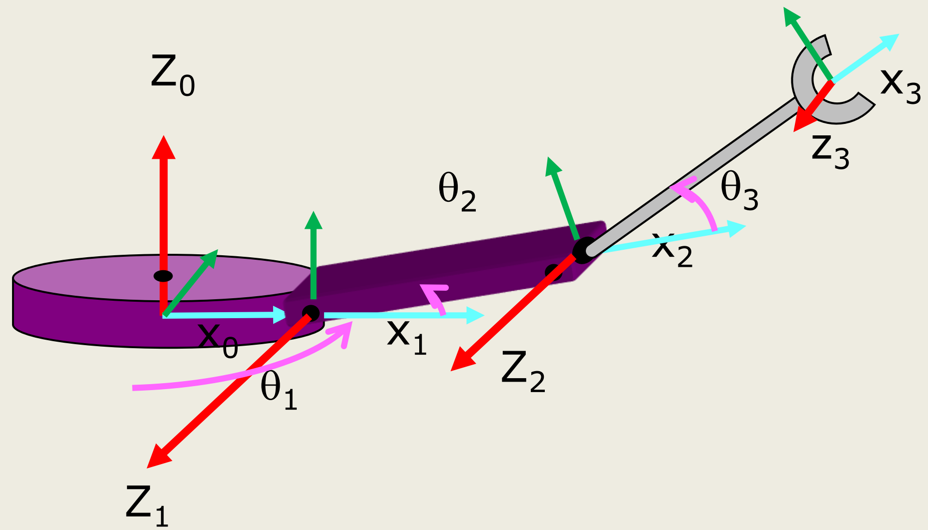
y_2 axis expressed wrt {1}

z_2 axis expressed wrt {1}

$$
{}^1_2T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & L_1 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & L_1 s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Origin of {2} w.r.t. {1}

Example 3 – 3D Space RRR (3)



x_3 axis expressed wrt {2}

y_3 axis expressed wrt {2}

z_3 axis expressed wrt {2}

$$ {}^2_3T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & L_2 c\theta_3 \\ s\theta_3 & c\theta_3 & 0 & L_2 s\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Origin of {3} w.r.t. {2}

Example 3 – 3D Space RRR: Compound Transformation

$$ {}^{0}_{3}T = {}^{0}_{1}T \; {}^{1}_{2}T \; {}^{2}_{3}T $$

$$ {}^{0}_{3}T = $$

$$ \begin{bmatrix} c\theta_1 c\theta_{23} & -c\theta_1 s\theta_{23} & s\theta_1 & c\theta_1(L_2 c\theta_{23} + L_1 c\theta_2 + R) \\ s\theta_1 c\theta_{23} & -s\theta_1 s\theta_{23} & -c\theta_1 & s\theta_1(L_2 c\theta_{23} + L_1 c\theta_2 + R) \\ s\theta_{23} & c\theta_{23} & 0 & L_2 s\theta_{23} + L_1 s\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

where

$$ \begin{aligned} c\theta_i &= \cos(\theta_i) \\ s\theta_i &= \sin(\theta_i) \\ c\theta_{23} &= \cos(\theta_2 + \theta_3) \\ s\theta_{23} &= \sin(\theta_2 + \theta_3) \end{aligned} $$

Example 3 – 3D Space RRR: HT to FK

$$ {}^0_3T = $$

$$\begin{bmatrix} c\theta_1 c\theta_{23} & -c\theta_1 s\theta_{23} & s\theta_1 & c\theta_1(L_2 c\theta_{23} + L_1 c\theta_2 + R) \\ s\theta_1 c\theta_{23} & -s\theta_1 s\theta_{23} & -c\theta_1 & s\theta_1(L_2 c\theta_{23} + L_1 c\theta_2 + R) \\ s\theta_{23} & c\theta_{23} & 0 & L_2 s\theta_{23} + L_1 s\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = \left[\begin{array}{ccc:c} & Rotation & & Translation \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right]$$

Example 3 – 3D Space RRR: Cartesian Coordinates

$$
{}^{0}_{3}T =
$$

$$
\begin{bmatrix}
c\theta_1 c\theta_{23} & -c\theta_1 s\theta_{23} & s\theta_1 & \boxed{c\theta_1(L_2 c\theta_{23} + L_1 c\theta_2 + R)} \\
s\theta_1 c\theta_{23} & -s\theta_1 s\theta_{23} & -c\theta_1 & s\theta_1(L_2 c\theta_{23} + L_1 c\theta_2 + R) \\
s\theta_{23} & c\theta_{23} & 0 & L_2 s\theta_{23} + L_1 s\theta_2 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

$$
x_e = c\theta_1(L_2 c\theta_{23} + L_1 c\theta_2 + R)
$$

$$
y_e = s\theta_1(L_2 c\theta_{23} + L_1 c\theta_2 + R)
$$

$$
z_e = L_2 s\theta_{23} + L_1 s\theta_2
$$

Example 3 – 3D Space RRR: RPY Angles

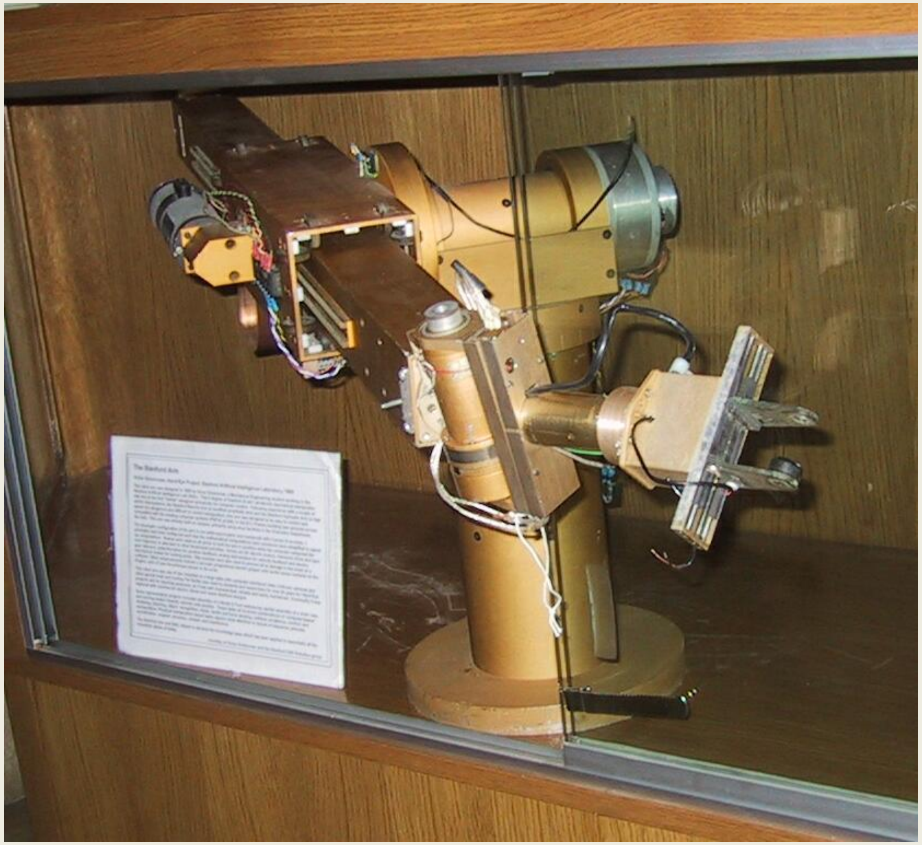
$$\beta = A\tan 2(-r_{31}, \sqrt{r_{11}^2 + r_{21}^2})$$
$$\alpha = A\tan 2(\frac{r_{21}}{c\beta}, \frac{r_{11}}{c\beta})$$
$$\gamma = A\tan 2(\frac{r_{32}}{c\beta}, \frac{r_{33}}{c\beta})$$

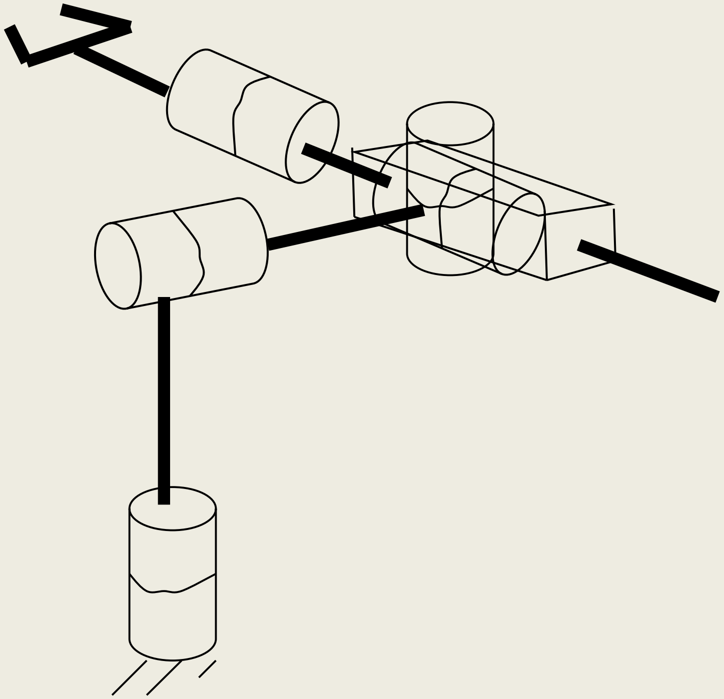
$${}^0_3T =$$

$$\begin{bmatrix} c\theta_1 c\theta_{23} & -c\theta_1 s\theta_{23} & s\theta_1 & c\theta_1(L_2 c\theta_{23} + L_1 c\theta_2 + R) \\ s\theta_1 c\theta_{23} & -s\theta_1 s\theta_{23} & -c\theta_1 & s\theta_1(L_2 c\theta_{23} + L_1 c\theta_2 + R) \\ s\theta_{23} & c\theta_{23} & 0 & L_2 s\theta_{23} + L_1 s\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\beta = \theta_2 + \theta_3$$
$$\alpha = \theta_1$$
$$\gamma = 90^o$$

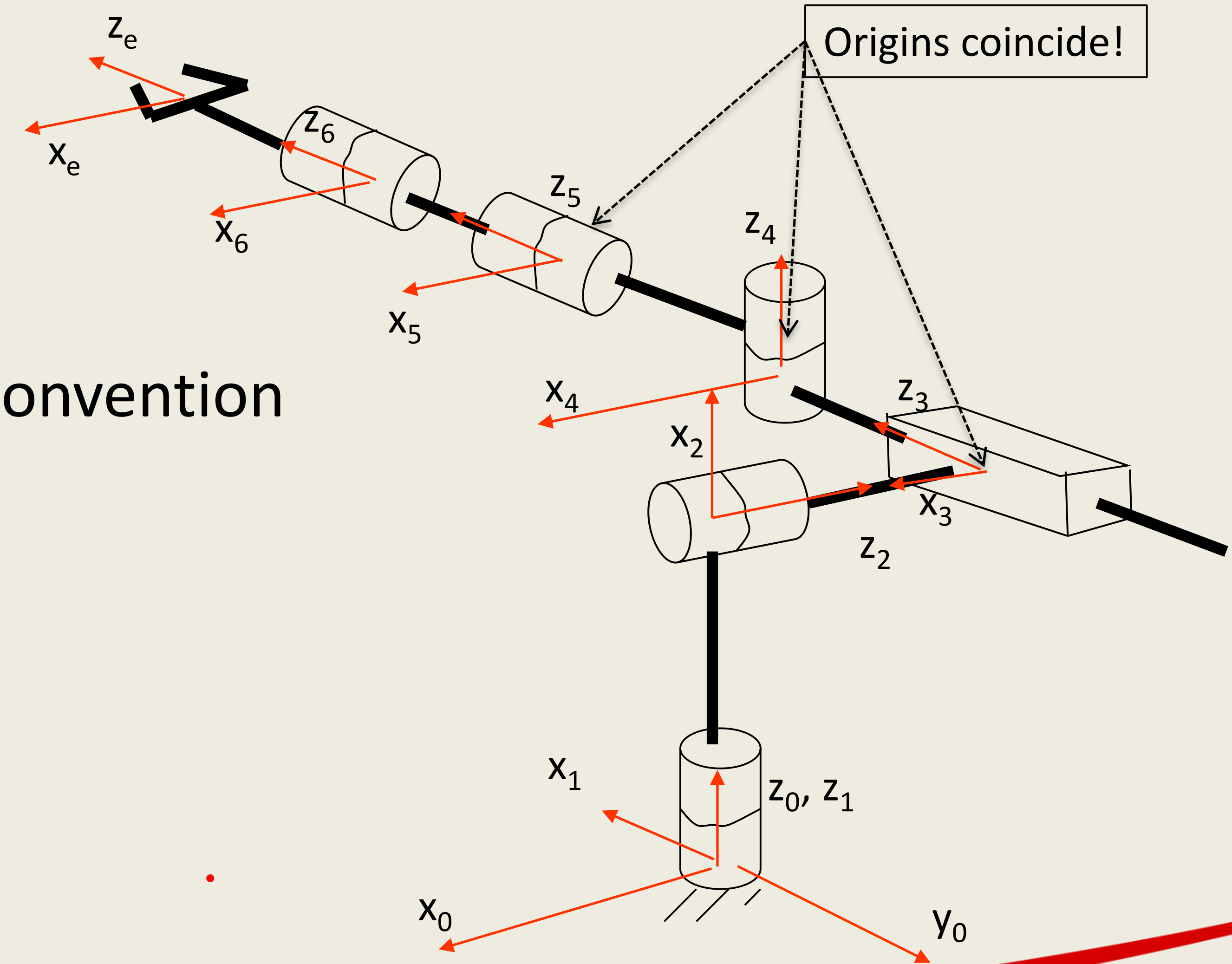
Example – the Stanford Arm





Example – the Stanford Arm

Proximal DH convention



Example – the Stanford Arm

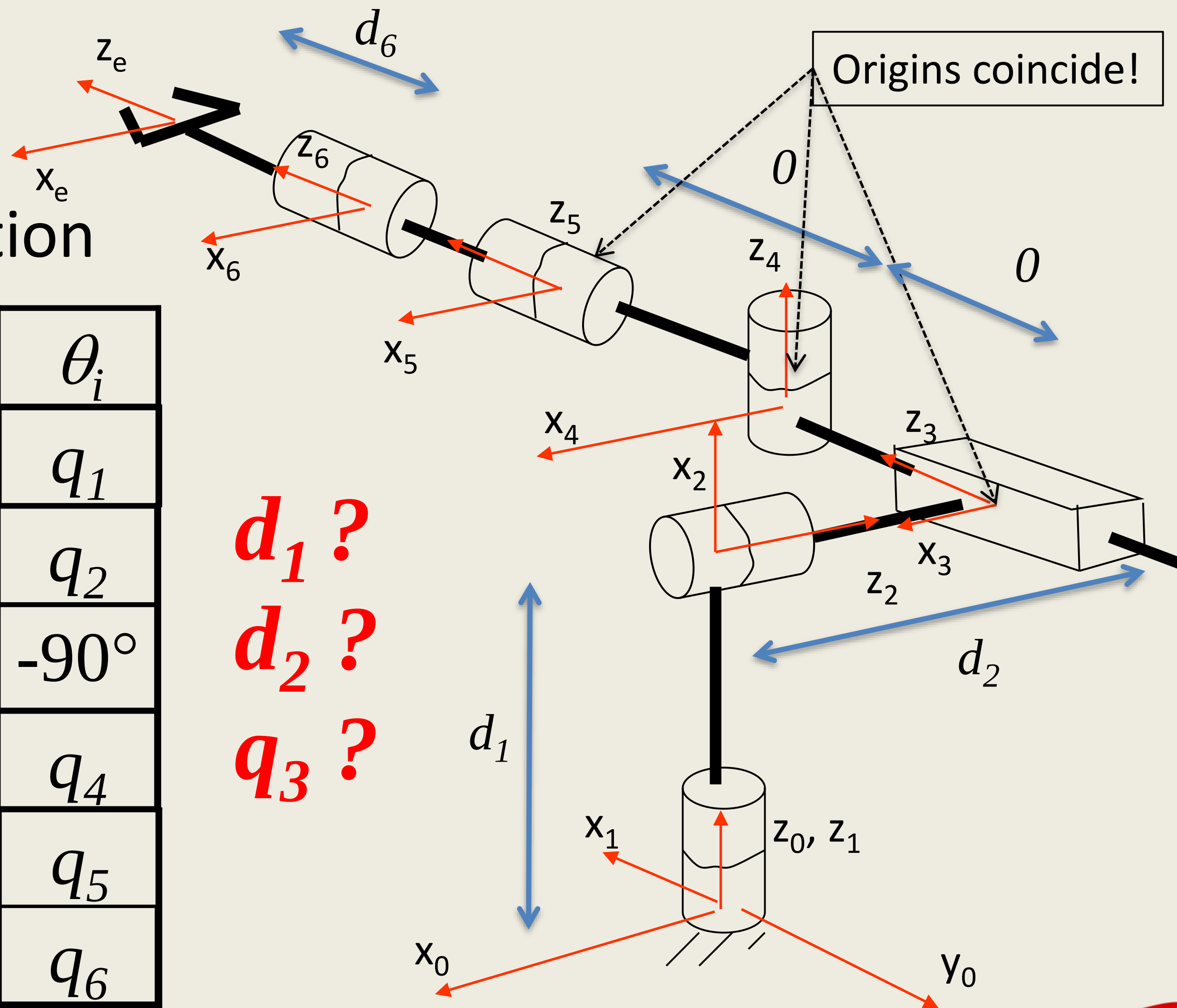
Proximal DH convention

i	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	*0*	q_1
2	0	90°	*0*	q_2
3	0	90°	*0*	-90°
4	0	-90°	*0*	q_4
5	0	90°	*0*	q_5
6	0	0	d_6	q_6

d_1 ?

d_2 ?

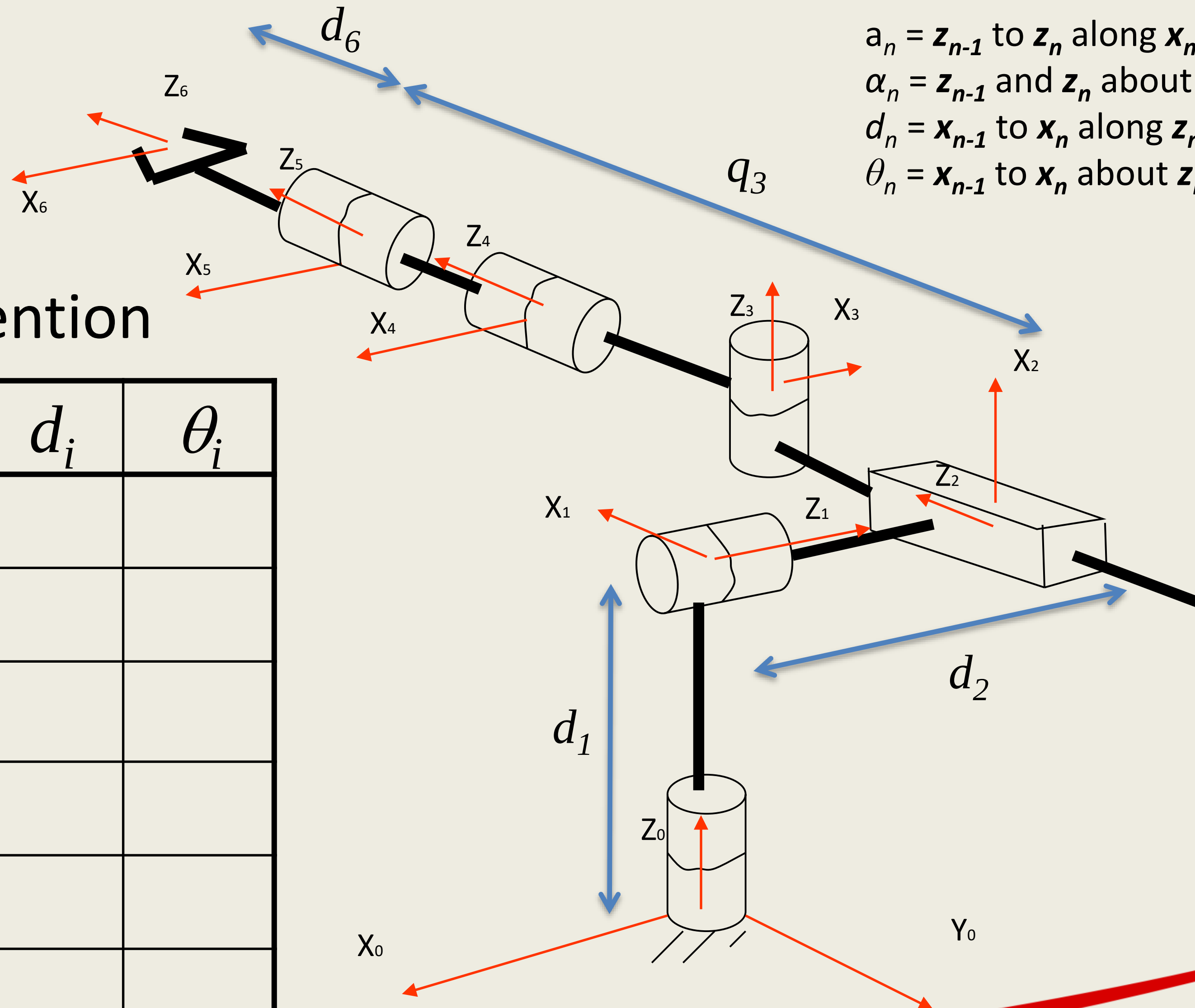
q_3 ?



We have to use the distal convention!

Example – the Stanford Arm

a_n = $\boldsymbol{z_{n-1}}$ to $\boldsymbol{z_n}$ along $\boldsymbol{x_n}$
α_n = $\boldsymbol{z_{n-1}}$ and $\boldsymbol{z_n}$ about $\boldsymbol{x_n}$
d_n = $\boldsymbol{x_{n-1}}$ to $\boldsymbol{x_n}$ along $\boldsymbol{z_{n-1}}$
θ_n = $\boldsymbol{x_{n-1}}$ to $\boldsymbol{x_n}$ about $\boldsymbol{z_{n-1}}$

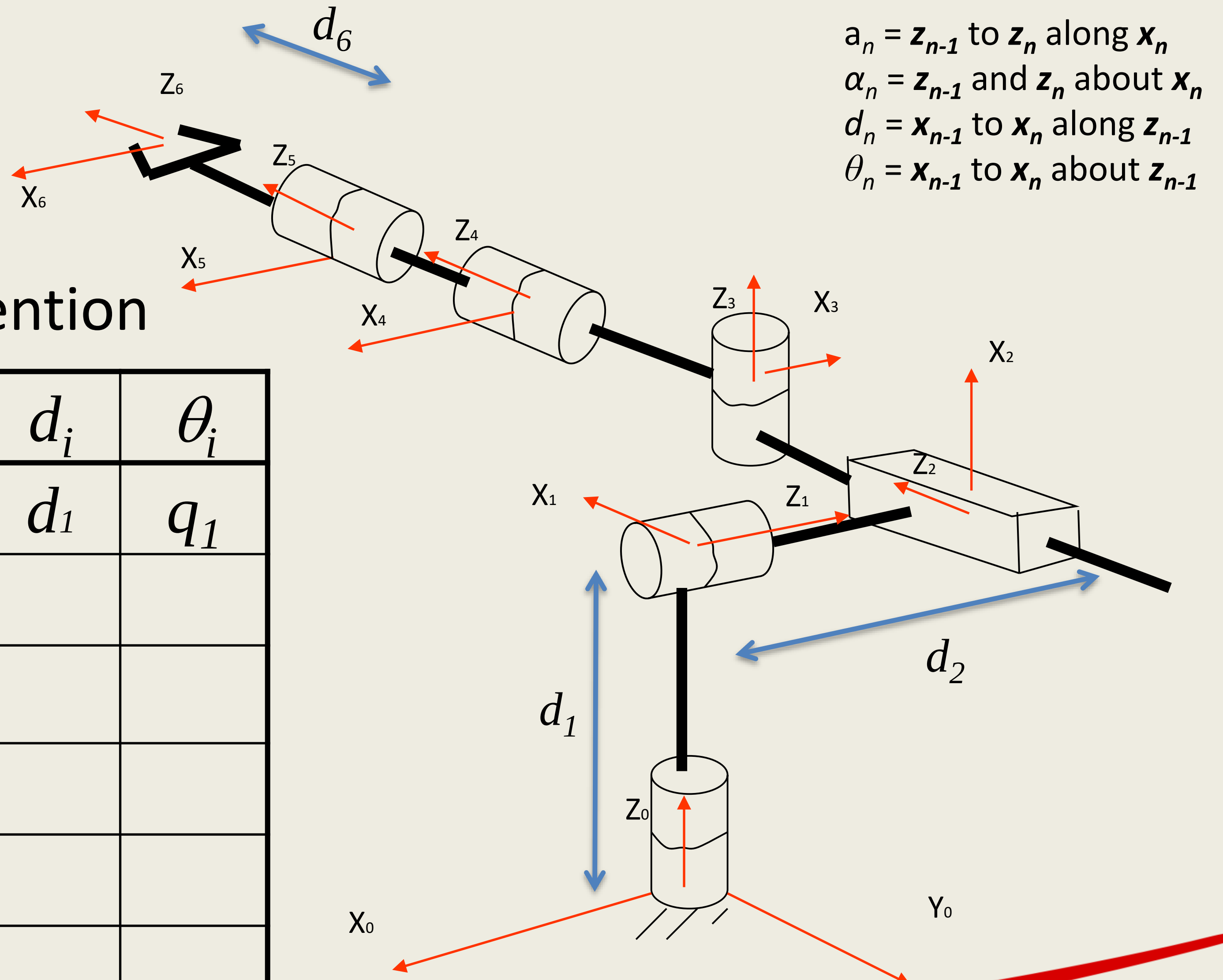


Distal DH convention

i	a_i	α_i	d_i	θ_i
1				
2				
3				
4				
5				
6				

Example – the Stanford Arm

a_n = $\boldsymbol{z_{n-1}}$ to $\boldsymbol{z_n}$ along $\boldsymbol{x_n}$
α_n = $\boldsymbol{z_{n-1}}$ and $\boldsymbol{z_n}$ about $\boldsymbol{x_n}$
d_n = $\boldsymbol{x_{n-1}}$ to $\boldsymbol{x_n}$ along $\boldsymbol{z_{n-1}}$
θ_n = $\boldsymbol{x_{n-1}}$ to $\boldsymbol{x_n}$ about $\boldsymbol{z_{n-1}}$

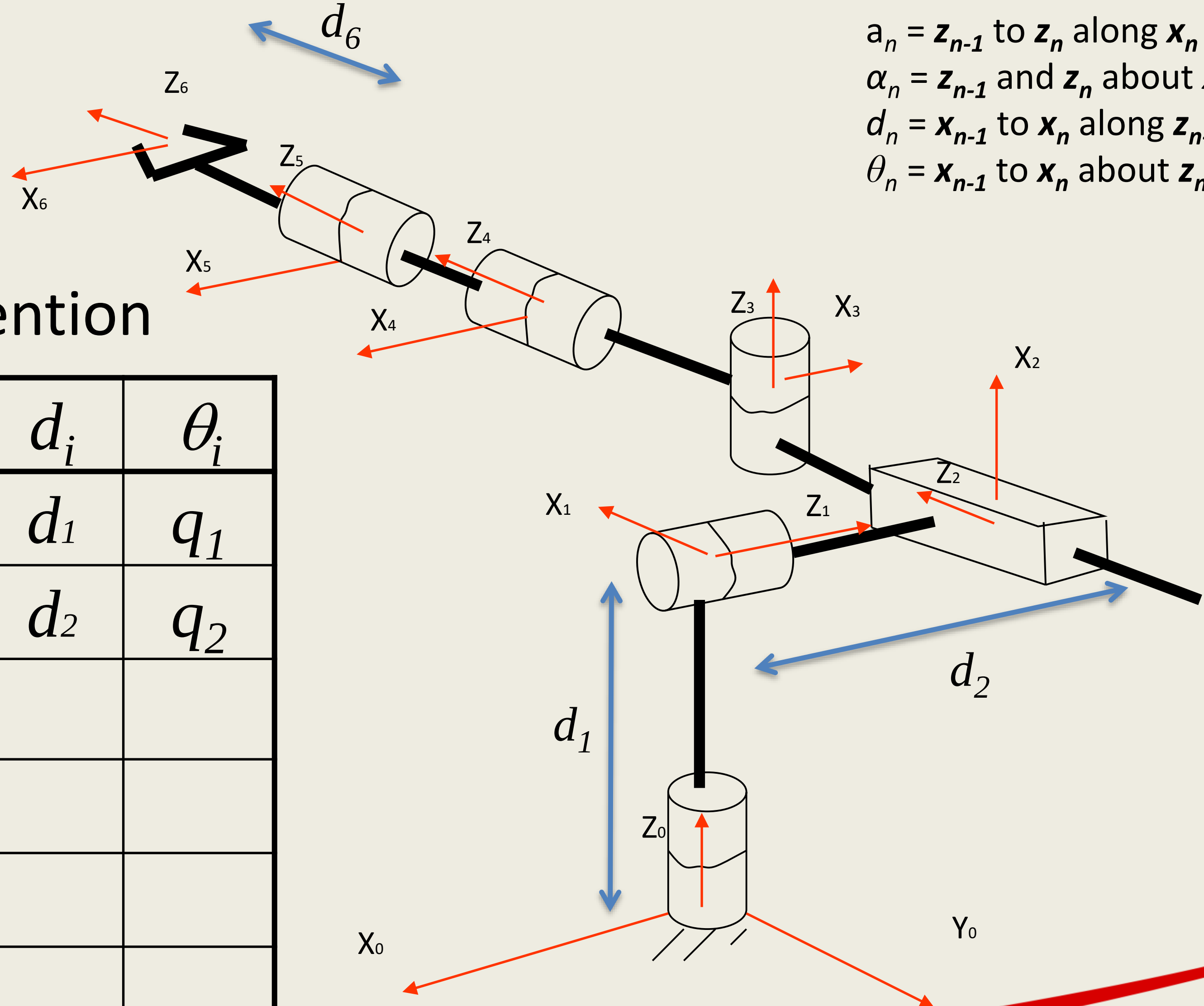


Distal DH convention

i	a_i	α_i	d_i	θ_i
1	0	90°	d_1	q_1
2				
3				
4				
5				
6				

Example – the Stanford Arm

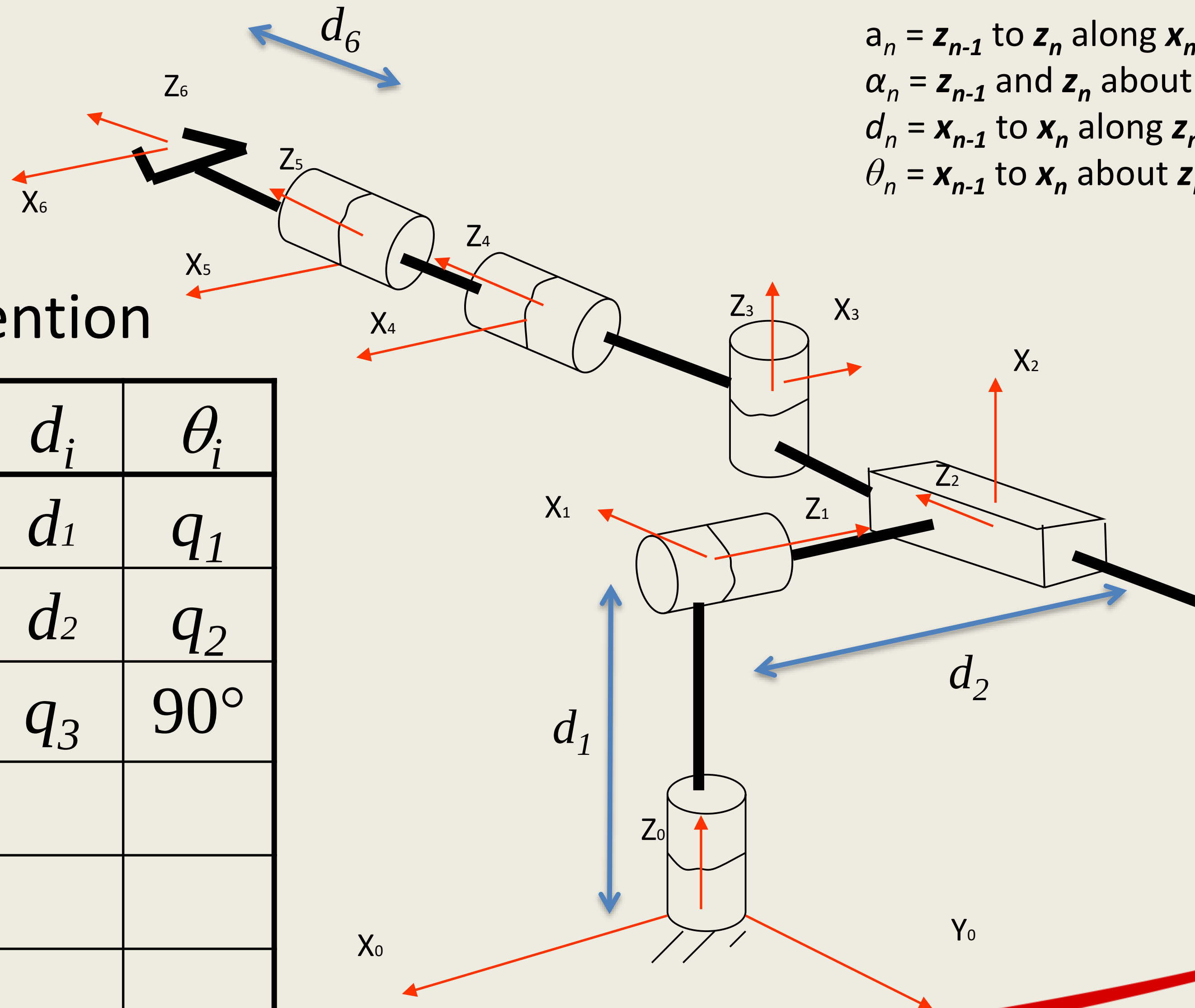
a_n = $\boldsymbol{z_{n-1}}$ to $\boldsymbol{z_n}$ along $\boldsymbol{x_n}$
α_n = $\boldsymbol{z_{n-1}}$ and $\boldsymbol{z_n}$ about $\boldsymbol{x_n}$
d_n = $\boldsymbol{x_{n-1}}$ to $\boldsymbol{x_n}$ along $\boldsymbol{z_{n-1}}$
θ_n = $\boldsymbol{x_{n-1}}$ to $\boldsymbol{x_n}$ about $\boldsymbol{z_{n-1}}$



Distal DH convention

i	a_i	α_i	d_i	θ_i
1	0	90°	d_1	q_1
2	0	90°	d_2	q_2
3				
4				
5				
6				

Example – the Stanford Arm



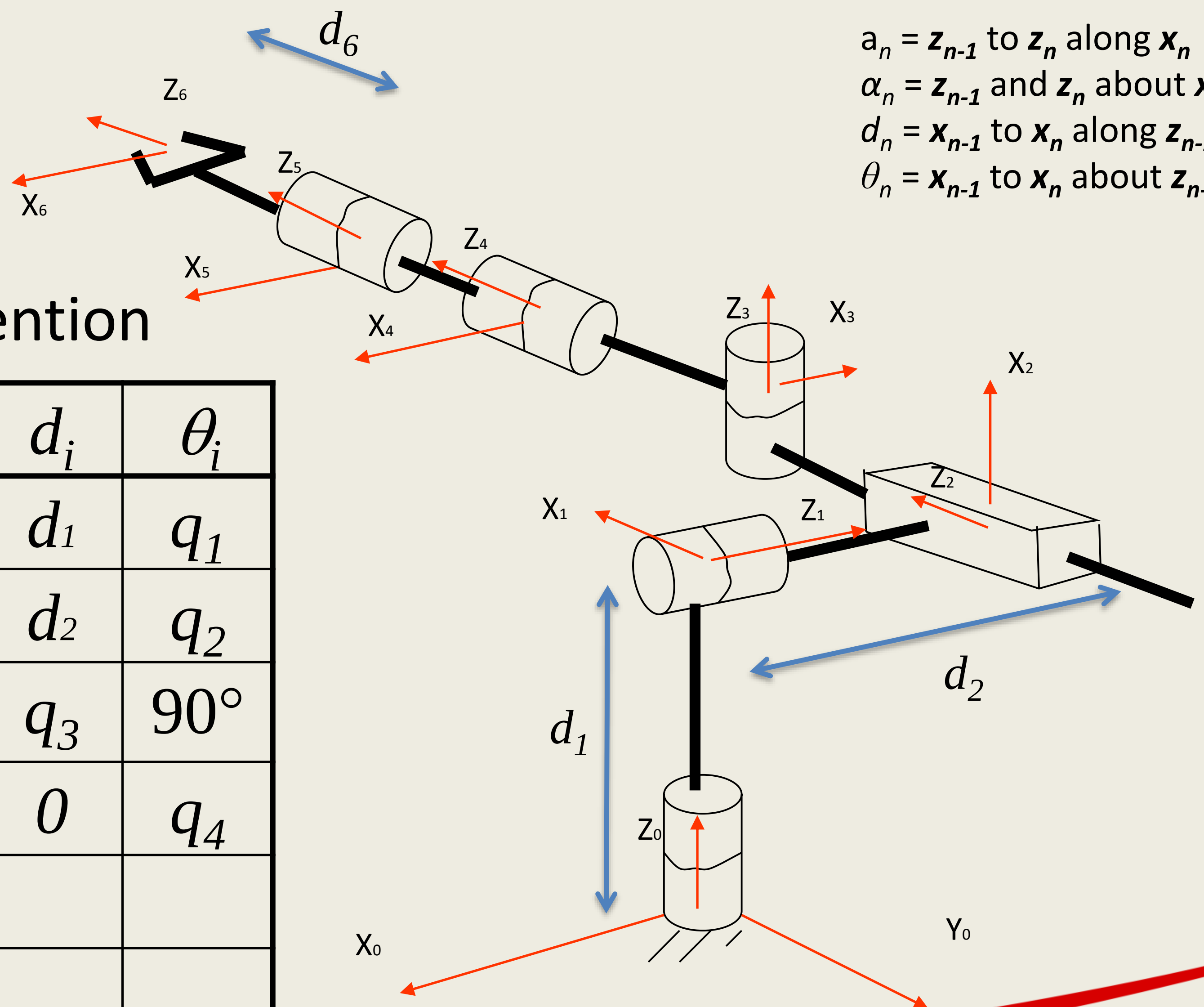
a_n = $\boldsymbol{z_{n-1}}$ to $\boldsymbol{z_n}$ along $\boldsymbol{x_n}$
α_n = $\boldsymbol{z_{n-1}}$ and $\boldsymbol{z_n}$ about $\boldsymbol{x_n}$
d_n = $\boldsymbol{x_{n-1}}$ to $\boldsymbol{x_n}$ along $\boldsymbol{z_{n-1}}$
θ_n = $\boldsymbol{x_{n-1}}$ to $\boldsymbol{x_n}$ about $\boldsymbol{z_{n-1}}$

Distal DH convention

i	a_i	α_i	d_i	θ_i
1	0	90°	d_1	q_1
2	0	90°	d_2	q_2
3	0	90°	q_3	90°
4				
5				
6				

Example – the Stanford Arm

a_n = $\boldsymbol{z_{n-1}}$ to $\boldsymbol{z_n}$ along $\boldsymbol{x_n}$
α_n = $\boldsymbol{z_{n-1}}$ and $\boldsymbol{z_n}$ about $\boldsymbol{x_n}$
d_n = $\boldsymbol{x_{n-1}}$ to $\boldsymbol{x_n}$ along $\boldsymbol{z_{n-1}}$
θ_n = $\boldsymbol{x_{n-1}}$ to $\boldsymbol{x_n}$ about $\boldsymbol{z_{n-1}}$

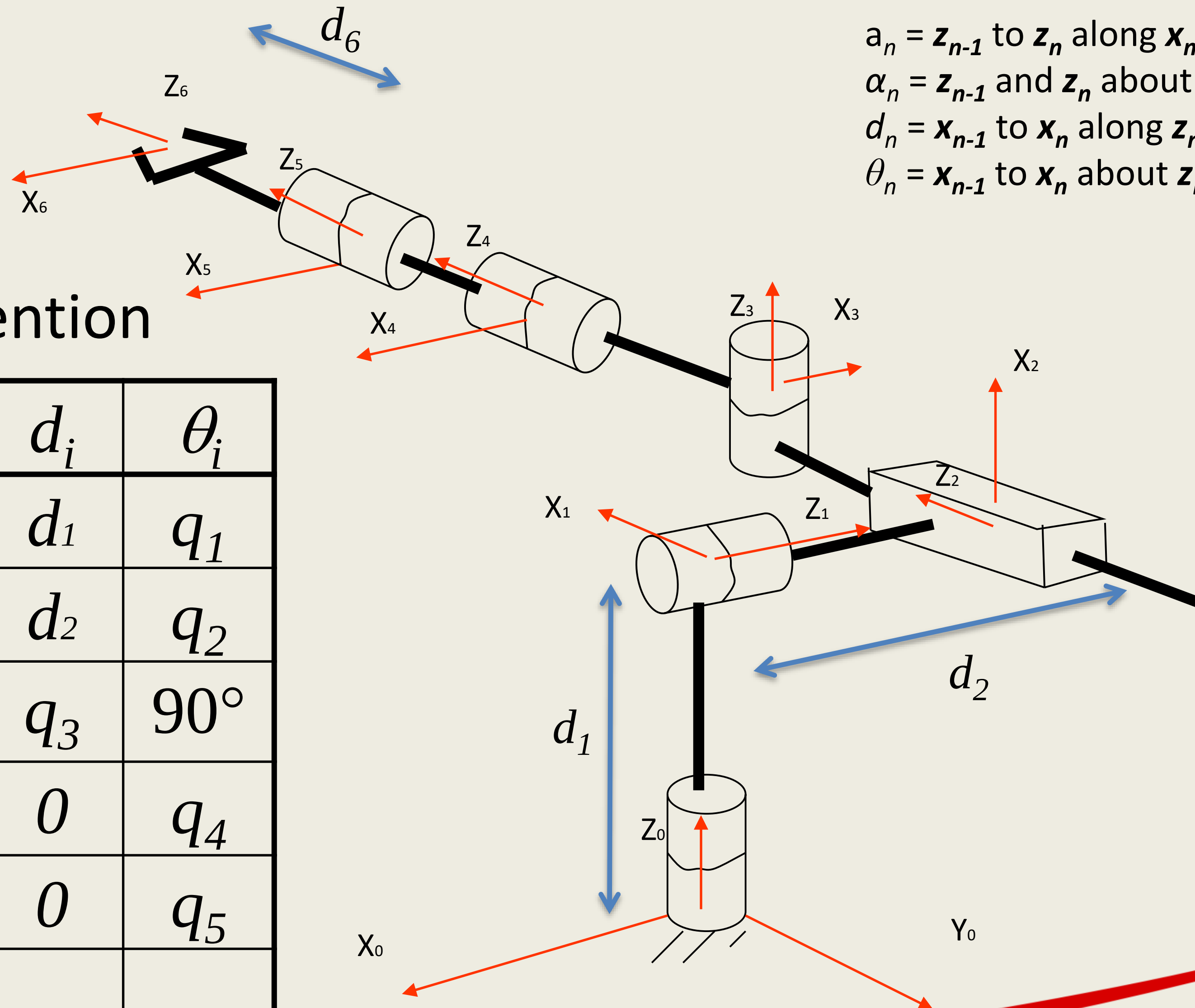


Distal DH convention

i	a_i	α_i	d_i	θ_i
1	0	90°	d_1	q_1
2	0	90°	d_2	q_2
3	0	90°	q_3	90°
4	0	90°	0	q_4
5				
6				

Example – the Stanford Arm

a_n = $\boldsymbol{z_{n-1}}$ to $\boldsymbol{z_n}$ along $\boldsymbol{x_n}$
α_n = $\boldsymbol{z_{n-1}}$ and $\boldsymbol{z_n}$ about $\boldsymbol{x_n}$
d_n = $\boldsymbol{x_{n-1}}$ to $\boldsymbol{x_n}$ along $\boldsymbol{z_{n-1}}$
θ_n = $\boldsymbol{x_{n-1}}$ to $\boldsymbol{x_n}$ about $\boldsymbol{z_{n-1}}$

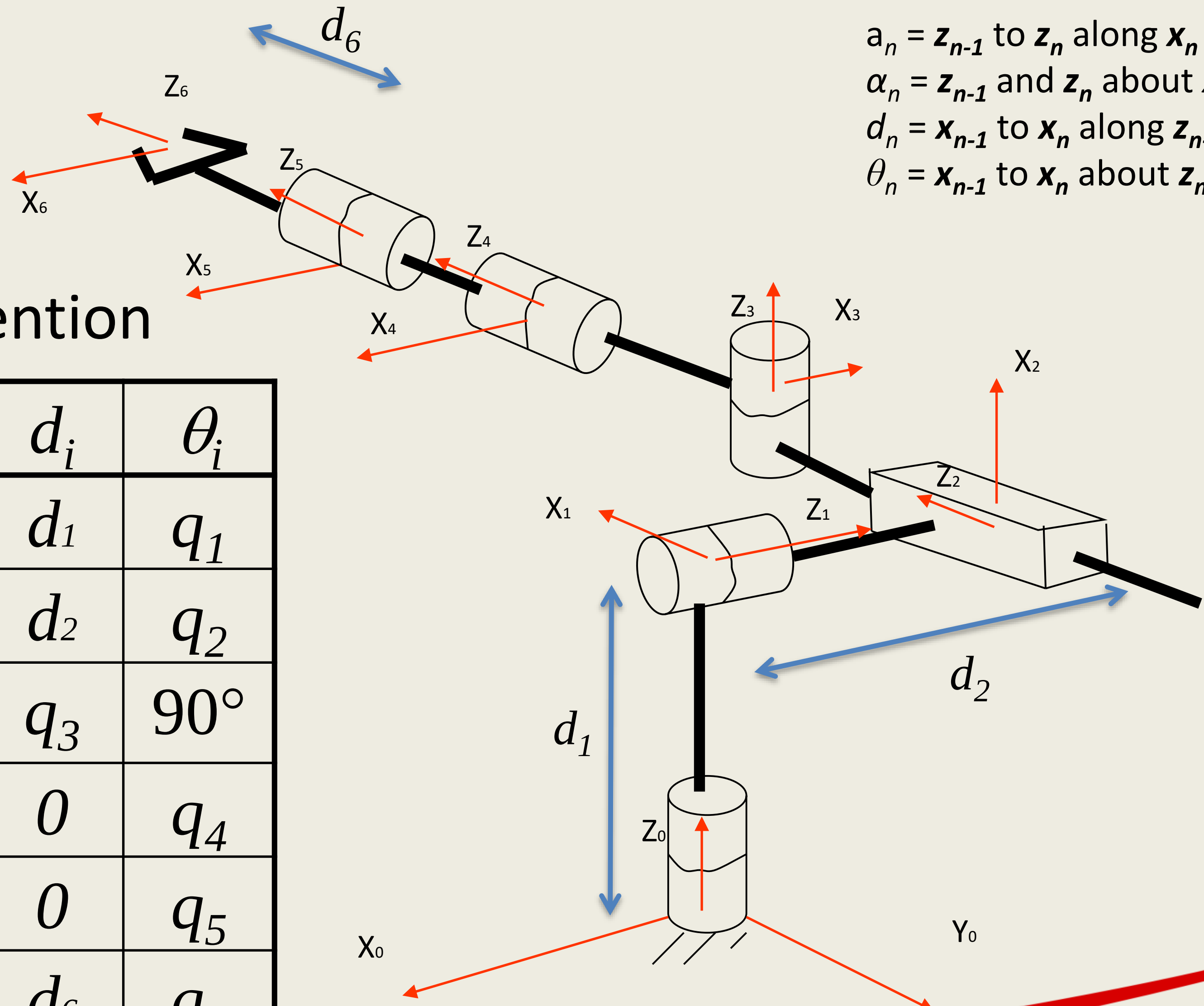


Distal DH convention

i	a_i	α_i	d_i	θ_i
1	0	90°	d_1	q_1
2	0	90°	d_2	q_2
3	0	90°	q_3	90°
4	0	90°	0	q_4
5	0	0°	0	q_5
6				

Example – the Stanford Arm

a_n = $\boldsymbol{z_{n-1}}$ to $\boldsymbol{z_n}$ along $\boldsymbol{x_n}$
α_n = $\boldsymbol{z_{n-1}}$ and $\boldsymbol{z_n}$ about $\boldsymbol{x_n}$
d_n = $\boldsymbol{x_{n-1}}$ to $\boldsymbol{x_n}$ along $\boldsymbol{z_{n-1}}$
θ_n = $\boldsymbol{x_{n-1}}$ to $\boldsymbol{x_n}$ about $\boldsymbol{z_{n-1}}$



Distal DH convention

i	a_i	α_i	d_i	θ_i
1	0	90°	d_1	q_1
2	0	90°	d_2	q_2
3	0	90°	q_3	90°
4	0	90°	0	q_4
5	0	0°	0	q_5
6	0	0°	d_6	q_6

Summary

Denavit-Hartenberg is a systematic way of **setting reference frames**.

Forward kinematics using the Denavit-Hartenberg approach:

- Fix frames and axes on each joint
- Find the 4 DH parameters and fill the DH table
- Use the formula to calculate each transformation matrix ${}^{i-1}T_i$
- Multiply consecutive transformation matrices to derive ${}^{0}T_n$

Next week: Inverse Kinematics