Formulae Sheet

Transformation matrices

$$\text{Rotx}(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{Roty}(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{Rotz}(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{Trans}(a,b,c) = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

DH parameters (standard DH)

ai is the distance between z_{i-1} and z_i w.r.t. x_i
α_i is the angle between z_{i-1} and zi w.r.t. clockwise rotation around x_i
di is the distance between x_{i-1} and xi w.r.t. z_{i-1}
θ_i is the angle between x_{i-1} and xi w.r.t. clockwise rotation around z_{i-1}

Distal

$$\overset{i-1}{i}\mathrm{T} = \begin{pmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

DH parameters (modified DH)

a_i is the distance between z_i and z_{i+1} w.r.t. x_i
α_i is the angle between z_i and z_{i+1} w.r.t. clockwise rotation around x_i
d_i is the distance between x_{i-1} and xi w.r.t. z_i
θ_i is the angle between x_{i-1} and x_i w.r.t. clockwise rotation around z_i

Proximal

$$ {}^{i-1}_{i}\mathrm{T} = \begin{pmatrix} \cos\theta_i & -\sin\theta_i & 0 & a_{i-1} \\ \sin\theta_i \cos\alpha_{i-1} & \cos\theta_i \cos\alpha_{i-1} & -\sin\alpha_{i-1} & -\sin\alpha_{i-1} d_i \\ \sin\theta_i \sin\alpha_{i-1} & \cos\theta_i \sin\alpha_{i-1} & \cos\alpha_{i-1} & \cos\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

Euler Angles

ZYZ Euler angles

$$\beta = A\tan 2(\sqrt{r_{31}^2 + r_{32}^2}, r_{33})$$

$$\alpha = A\tan 2(\frac{r_{23}}{s\beta}, \frac{r_{13}}{s\beta})$$

$$\gamma = A\tan 2(\frac{r_{32}}{s\beta}, -\frac{r_{31}}{s\beta})$$

RPY Euler angles

$$\beta = A\tan 2(-r_{31}, \sqrt{r_{11}^2 + r_{21}^2})$$

$$\alpha = A\tan 2(\frac{r_{21}}{c\beta}, \frac{r_{11}}{c\beta})$$

$$\gamma = A\tan 2(\frac{r_{32}}{c\beta}, \frac{r_{33}}{c\beta})$$

Chain mobility formulae extended version

$$M = d(n - g - 1) + \sum_{i=1}^{g} f_i + R_c - R_M$$

d – 3 for planar/6 for spatial manipulators, n – number of moving links, g – number of joints, f_i - degrees of freedom