Faculty of Engineering Design and Mathematics

Academic Year:	**16/17**
Examination Period:	**January**

Module Leader:	**Sanja Dogramadzi**
Module Code:	**UFMF4X-15-M**
Module Title:	**Robotic Fundamentals**
Work Item Code:	**Exam**

Duration: **3 Hours**

Standard materials required for this examination:

Examination Answer Booklet		Yes
Multiple Choice Answer Sheet		No
Graph Paper	Type of paper e.g. G3, G14	N/A
	Number of sheets per student	0

Additional materials required for this examination:

Details of additional material **supplied by UWE**: To be collected with Answer Booklet (please delete as appropriate) N/A	
Details of approved material **supplied by Student**: To be collected with Answer Booklet (please delete as appropriate) N/A	
University approved Calculator	Yes
Candidates are permitted to keep Examination Question Paper	No

Candidates are NOT permitted to turn the page over until the exam starts

Instructions to Candidates:

Candidates must answer all FOUR questions.

Question 1

a) Describe the different types of workspaces for a manipulator. [5]

b) For the two frames in the figure below (Fig. 1) do the following:

i. Determine the position and orientation of $\{C\}$ with respect to $\{A\}$ as a 4x4 homogeneous transformation matrix. [4]

ii. If a point G has coordinates (1,2,3) in $\{C\}$, what are its coordinates in $\{A\}$? [4]

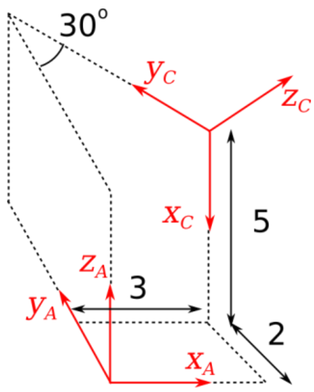

Figure 1

c) For the manipulator below (Fig. 2) place the frames and provide the respective table of DH parameters. [12]

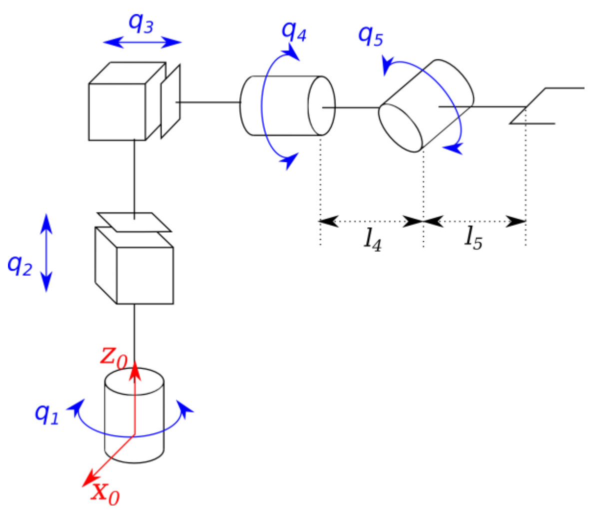

Figure 2

Question 2

a) What are singularities for a manipulator? [5]

b) For the **Planar** mechanism shown below (Fig. 3) do the following:

   i. Calculate the Forward Kinematic equations (<u>position only</u>) for the End Effector *{3}* (you can do this geometrically if you prefer). [4]

   ii. Calculate the *2x2* Jacobian of the Manipulator and give its singularities (if any) [8]

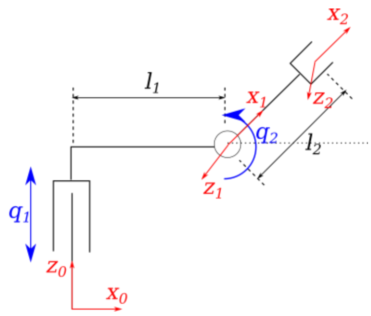

Figure 3

c) Give the inverse kinematics equations of the manipulator below (Fig. 4), if the end-effector's position and orientation are known. [8]

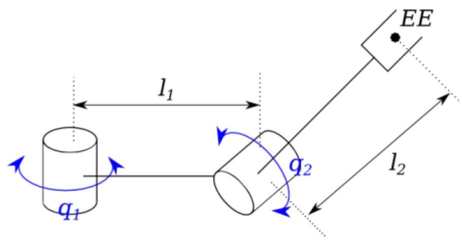

Figure 4

Question 3

a) Parallel Robots are used for various applications due to their characteristics. Give their Pros and Cons compared to Serial mechanisms. [4]

b) For the parallel manipulator given below (Fig. 5) do the following:

   i. Calculate its mobility (explain each parameter of the equation) [3]

   ii. Give the loop-closure equation for the point {C} if the world reference is at point {O}. [3]

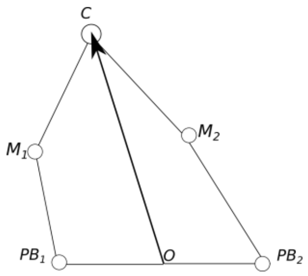

Figure 5

c) For calculating the trajectory of a joint we can use polynomial of different orders. What orders are commonly used and why? What is the alternative? [5]

d) A revolute joint is at time moment *t=0* at $\theta(0) = 10^o$ and stationary (no velocity). If the joint must move in 5 sec to 45^o and maintain an acceleration of 30 deg/sec^2, select the appropriate polynomial function to achieve this trajectory and calculate its coefficients. [10]

Question 4

a) Describe the two methods to calculate manipulator dynamics. What are their differences and when is best to use each? [7]

b) Give the state-space equation for the dynamics of a manipulator and explain each term. [6]

c) Using the Recursive Newton-Euler approach calculate the torques required by joints 1 and 2 of the manipulator below (Fig. 6) if gravity is 9.81 m/sec^2, the external force is 2N and the external torque is 5Nm. [12]

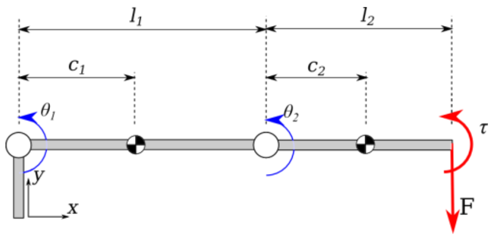

Figure 6

The parameters for each link of the manipulator, and external forces are given in the table:

Parameter	Link 1	Link 2
Joint Length	0.7m	0.6m
Centroid Location	0.5m	0.3m
Mass	2.5kg	1.5kg
Joint position	0 rad	0 rad
Joint velocity	2 rad/s	-4 rad/s
Joint acceleration	10 rad/s^2	-5 rad /s^2
Inertia about centroid	0.05 kg/m^2	0.02 kg/m^2

END OF QUESTION PAPER

Formulae Sheet

Transformation matrices

$$\mathrm{Rotx}(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{Roty}(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{Rotz}(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{Trans}(a,b,c) = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

DH parameters (standard DH)

ai is the distance between z_{i-1} and z_i w.r.t. x_i
α_i is the angle between z_{i-1} and zi w.r.t. clockwise rotation around x_i
di is the distance between x_{i-1} and xi w.r.t. z_{i-1}
θ_i is the angle between x_{i-1} and xi w.r.t. clockwise rotation around z_{i-1}

Distal

$${}^{i-1}_{i}\mathrm{T} = \begin{pmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

DH parameters (modified DH)

a_i is the distance between z_i and z_{i+1} w.r.t. x_i
α_i is the angle between z_i and z_{i+1} w.r.t. clockwise rotation around x_i
d_i is the distance between x_{i-1} and xi w.r.t. z_i
θ_i is the angle between x_{i-1} and x_i w.r.t. clockwise rotation around z_i

Proximal

$$ {}^{i-1}_{i}\mathrm{T} = \begin{pmatrix} \cos\theta_i & -\sin\theta_i & 0 & a_{i-1} \\ \sin\theta_i\cos\alpha_{i-1} & \cos\theta_i\cos\alpha_{i-1} & -\sin\alpha_{i-1} & -\sin\alpha_{i-1}d_i \\ \sin\theta_i\sin\alpha_{i-1} & \cos\theta_i\sin\alpha_{i-1} & \cos\alpha_{i-1} & \cos\alpha_{i-1}d_i \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

Euler Angles

ZYZ Euler angles

$$\beta = A\tan 2(\sqrt{r_{31}^2 + r_{32}^2}, r_{33})$$

$$\alpha = A\tan 2(\frac{r_{23}}{s\beta}, \frac{r_{13}}{s\beta})$$

$$\gamma = A\tan 2(\frac{r_{32}}{s\beta}, -\frac{r_{31}}{s\beta})$$

RPY Euler angles

$$\beta = A\tan 2(-r_{31}, \sqrt{r_{11}^2 + r_{21}^2})$$

$$\alpha = A\tan 2(\frac{r_{21}}{c\beta}, \frac{r_{11}}{c\beta})$$

$$\gamma = A\tan 2(\frac{r_{32}}{c\beta}, \frac{r_{33}}{c\beta})$$

Chain mobility formulae extended version

$$M = d(n - g - 1) + \sum_{i=1}^{g} f_i + R_c - R_M$$

d – 3 for planar/6 for spatial manipulators, n – number of moving links, g – number of joints, f_i - degrees of freedom