ROBOTIC FUNDAMENTALS (UFMF4X-15-M)

Dynamics

(Intro & Lagrange)

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Previously on ROBOTIC FUNDAMENTALS

Cartesian trajectories and Via points

–Via points have different calculations for different parts of the trajectory

–Linear motion achieved only in Cartesian Space

–Cartesian Space might lead manipulator to singular configurations

–Cartesian Space calculations are happening in discrete and continuous methods

Questions?

Today's Lecture

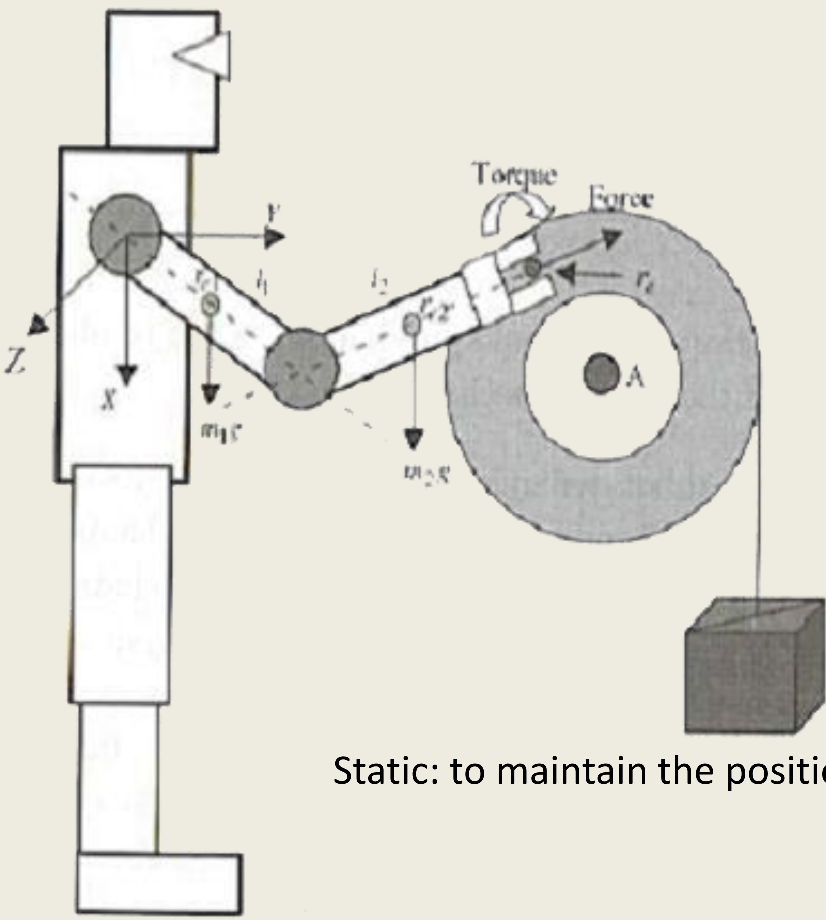
Dynamics Description

Focus on Euler-Lagrange

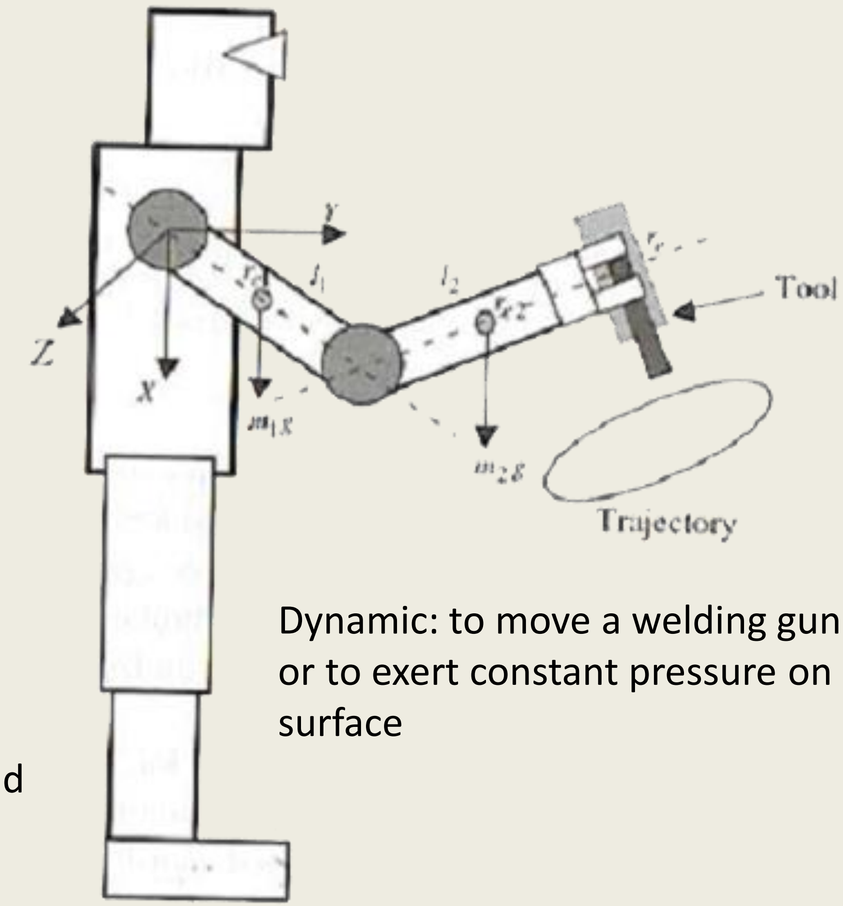
WHY DYNAMICS?

What I need to know?

https://www.youtube.com/watch?v=M7nLQpWiy1o



Static: to maintain the position of a load



Dynamic: to move a welding gun or to exert constant pressure on a surface

What are the force and torque needed to apply to each joint?
An inverse dynamics problem

Dynamics

- Manipulator dynamics - relationship between the forces/torques acting in robot's joints and the resulting motion of robot's links – forward dynamics

- As in kinematics, inverse problem of finding <u>input torques to obtain desired output motion</u> – inverse dynamics

- Links/joint torques are controlled so that the end-effecter can trace a desired trajectory. <u>Controller compensates accelerations and deceleration</u> by adding or reducing torque in joints. – torque control

Forward/Inverse Dynamics

Forward dynamics: from given torques calculate the resultant manipulator motions. This calculation is used for the dynamic simulation of manipulators.

Inverse dynamics: this is the calculation of joint torques from joint positions, velocities and accelerations. This calculation is useful in robot control.

Torque Control

Joint actuators balance torques from four sources:

- **Dynamic torques** – arise from motion
  - Inertial (proportional to acceleration – Newton’s II law)
  - Centripetal (constrain body to rotate about the point, directed towards the centre,~ ω^2)
  - Coriolis (result of interaction between 2 rotating links)
- **Static torques** – arise from external load
- **Gravity torques** – arise from link’ s mass
- **External forces/torques** – arise from the task

Approaches to Dynamic modeling

Newton Euler balance of forces and torques	**Lagrange** Energy-based approach
• Equations written for each link • Inverse dynamics in real time • Best suited for synthesis (implementation) of model based control	• Multi-body robot seen as a whole • Internal reaction forces between the links are eliminated • Closed-form equations are directly obtained • Best suited for study of dynamic properties and analysis of control

Dynamics Formulations

Newton Euler – forces and moments acting on the individual arm links, including coupling forces and moments between links. Equations also include constrain forces acting between adjacent links.

Lagrange – dynamic behaviour in terms of work done by the system and energy stored in the system. Robot is a black box that has energy balance.

State – Space Equation

The equation of motion from both approaches can be described as:

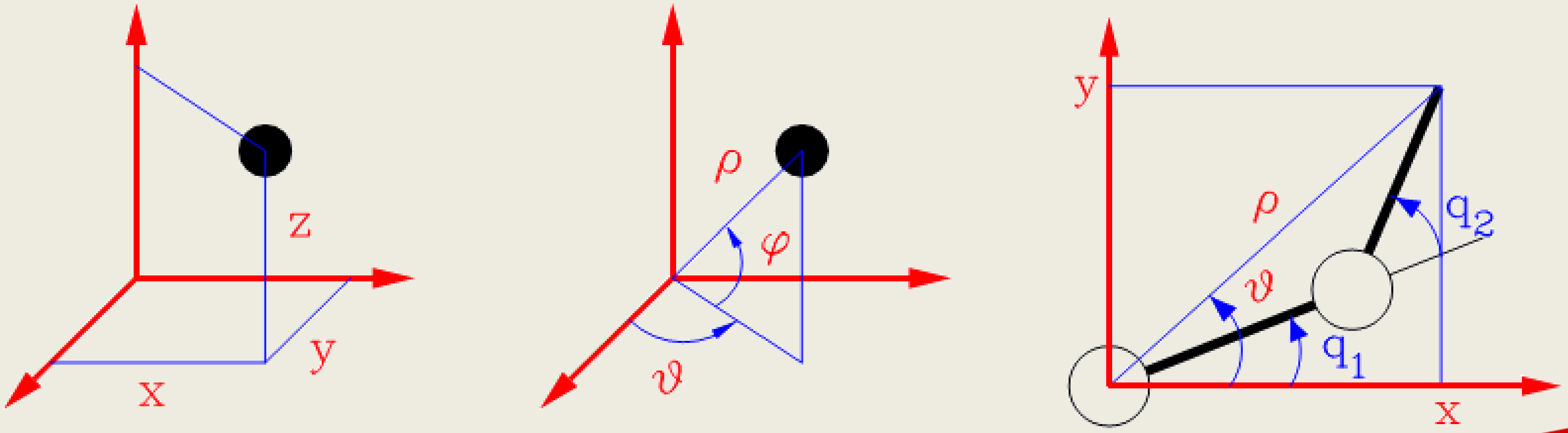
$$\tau = \underbrace{M(\theta)}_{\text{Mass matrix}}\ddot{\theta} + \underbrace{V(\theta,\dot{\theta})}_{\text{Centripetal and Coriolis matrix}} + \underbrace{G(\theta)}_{\text{Gravity term}}$$

LAGRANGE

General Lagrange model

Generalized variables, or Lagrange coordinates:

- Independent variables used to describe the position of rigid bodies in the space.
- For the same physical system, more choices for the Lagrangian coordinates are usually possible.



The Lagrangian function

Lagrangian (*L*) is the scalar function that connects Kinetic energy (*K*) with potential energy (*P*):

$$L = K - P$$

The Kinetic Energy depends on position and velocity while the Potential Energy on position only:

$$L(\theta, \dot{\theta}) = K(\theta, \dot{\theta}) - P(\theta)$$

Euler – Lagrange Equation

Non-conservative generalised force performing work on q_i

$$\psi_i = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} \quad i = 1 \ldots n$$

In robotics is:

$$\psi_i = \tau_i + J^T Fc - d_i \dot{q}_i$$

where

τ_i Joint actuator torque

$J^T Fc$ Term due to external forces

$d_i \dot{q}_i$ Joint friction torque

Lagrange – Kinetic & Potential Energy

$$L(\theta, \dot{\theta}) = K(\theta, \dot{\theta}) - P(\theta)$$

And it is easy to calculate from the geometric properties of the manipulator:

$$K = \sum_{i=1}^{n} K_i \quad P = \sum_{i=1}^{n} P_i$$

where *n* is the number o f links

Lagrange – Kinetic Energy

The kinetic energy of a manipulator can be determined for each link from:

- The link mass $\boldsymbol{m}$
- The inertia matrix $\boldsymbol{I}$ computed from a frame fixed to the centre of mass in which it has a constant expression
- The linear velocity of the centre of mass $\boldsymbol{v}$ and the rotational velocity $\boldsymbol{\omega}$ of the link both expressed in respect to the base frame.
- The rotation matrix $\boldsymbol{R}$ between the frame fixed to the link and the base frame

Lagrange – Kinetic Energy (2)

$$I = \begin{matrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{matrix}$$

$$K = \frac{1}{2} m_i v_{ci}^T v_{ci} + \frac{1}{2} \omega_{ci}^T I_i \omega_{ci}$$

Kinetic energy by angular velocity

Kinetic energy by linear velocity

Which are connected to θ , $\dot{\theta}$ and thus kinetic energy can be:

$$K(\theta, \dot{\theta}) = \frac{1}{2} \dot{\theta}^T M(\theta) \dot{\theta}$$

Lagrange – Potential Energy

Rigid bodies, only have potential energy gravity:

$$P_i = \int_{L_i} \mathbf{g}^T \mathbf{p} dm = \mathbf{g}^T \int_{L_i} \mathbf{p} dm = \mathbf{g}^T \mathbf{p}_{Ci} m_i$$

Depend on the joint angle θ:

$$P(\theta) = -\mathbf{g}^T \mathbf{p}_{Ci}(\theta) m_i + P_{ref}$$

Note: P_{ref} is the potential energy of reference

Compute the Dynamic model

- From the Euler-Lagrange equation:

$$\psi_i = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i}$$

- Substituting *L* with *K* and *P* will give us:

$$\psi_i = \frac{d}{dt}\frac{\partial K}{\partial \dot{\theta}} - \frac{\partial K}{\partial \theta} + \frac{\partial \mathcal{P}}{\partial \theta}$$

Which lead us to the state-space equation:

$$\tau + J^T F_C = M(\theta)\ddot{\theta} + V(\theta, \dot{\theta}) + G(\theta)$$

Simple Pendulum – Example

$$x = l\sin\theta$$

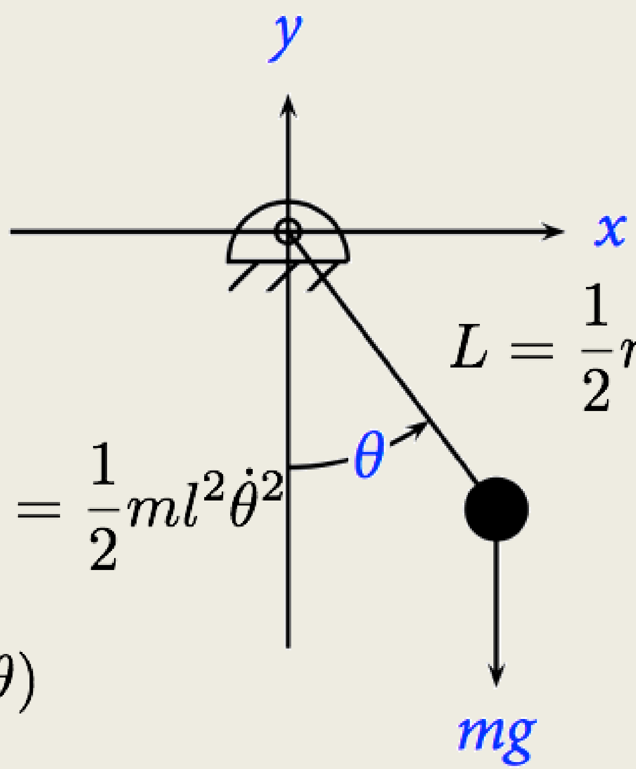
$$y = -l\cos\theta$$

$$\dot{x} = l\cos\theta\cdot\dot{\theta}$$

$$\dot{y} = \quad l\sin\theta\cdot\dot{\theta}$$

$$K(\theta,\dot{\theta}) = \frac{1}{2}m\left(\dot{x}^2+\dot{y}^2\right) = \frac{1}{2}ml^2\dot{\theta}^2$$

$$P = mgl(1-\cos\theta)$$



$$L = K - P$$

$$L = \frac{1}{2}ml^2\dot{\theta}^2 + mgl(1-\cos\theta)$$

$$\frac{\partial L}{\partial\dot{\theta}} = ml^2\dot{\theta}$$

$$\frac{\partial L}{\partial\theta} = \quad mgl\sin\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}}\right) - \frac{\partial L}{\partial\theta} = \tau \Rightarrow \boxed{ml^2\ddot{\theta} - mgl\sin\theta = \tau}$$

2 link manipulator – Example

In class

Other Considerations

- Actuation System Modeling
  - Motors, Reduction Gears, transmission system etc.
- Impact on the system's dynamics
  - If motors are installed on the links, then masses and inertia are changed
  - Non-linear effects such as backlash, friction elasticity etc.
  - Introduces its own dynamics (electromechanical, pneumatic, hydraulic etc.) that may be non negligible
- For complete system-wide modeling these must be taken into account (and can be by introducing suitable terms in the dynamic model)

Conclusions

Dynamics is about the state of the manipulator

Two main approaches

Euler-Lagrange is a systemic approach