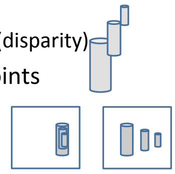
COMS30121
Image Processing and Computer Vision

Stereo I

3-D Vision using Two Eyes

Stereo Vision

- Stereo - 3-D structure from two (or more) images taken from different viewpoints.
- Position of object in each image depends on its depth.
  - 1/depth ∝ position difference (disparity)
- If we know disparity & viewpoints
  - 3-D scene structure



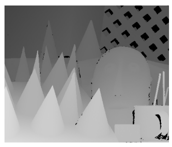
Stereo Examples

Three Problems of Stereo

- **Calibration** – determine relative position and orientation of the cameras
- **Correspondence** – determine matching points in the stereo views
- **Reconstruction** – determine 3D location in scene of matched points via triangulation

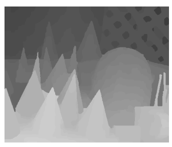
all inter-related

State of the Art Stereo Matching



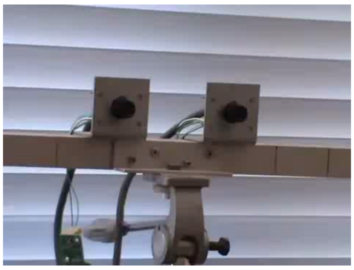
Ground truth

Estimated disparity



Global Stereo Matching Leveraged by Sparse Ground Control Points
Liang Wang and Ruigang Yang,
IEEE CVPR, 2011

Example – Real-Time Stereo

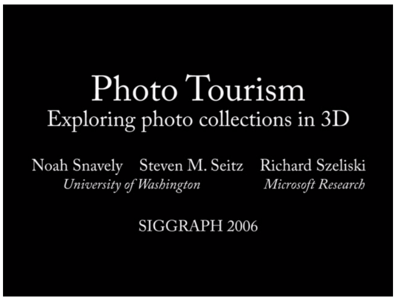


Singels & Brink, Real-time stereo reconstruction through hierarchical DP and LULU filtering, *Proc Annual Symposium PRASA* 2009

Multi-View Stereo

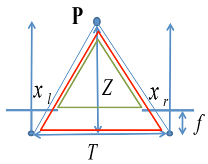
Pollefeys and Van Gool, 2002

Multi-View Stereo - Application

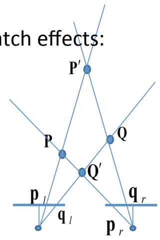


Triangulation

Simple stereo
– parallel z and x axes



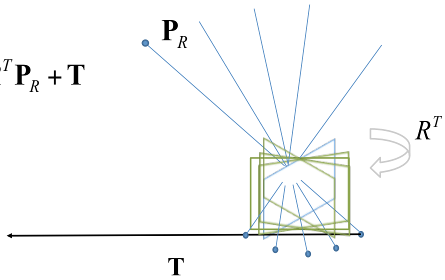
Mismatch effects:



Similar triangles : $\boxed{\frac{T}{Z}} = \boxed{\frac{T - x_l + x_r}{Z - f}} \Rightarrow Z = \frac{f\,T}{x_l - x_r} = \frac{f\,T}{d}$

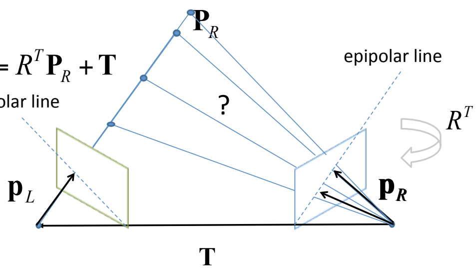
Epipolar Geometry

$\mathbf{P}_L = R^T \mathbf{P}_R + \mathbf{T}$

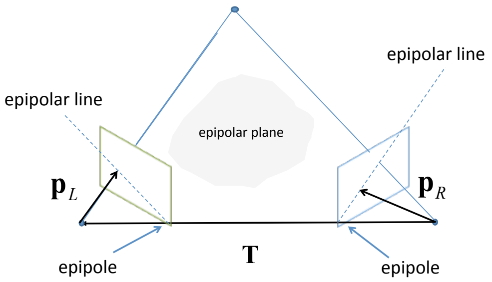


Epipolar Geometry

$\mathbf{P}_L = R^T \mathbf{P}_R + \mathbf{T}$



Epipolar Geometry



Epipolar Geometry

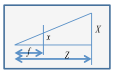
- Epipolar geometry defines relationship between two stereo views
- For known viewpoints:
  - it constrains matches to lie along epipolar lines
- For unknown viewpoints:
  - given matching points ....
  - it enables estimation of viewpoints

Epipolar Geometry - Maths

Rigid transformation between cameras:

$$\mathbf{P}_L = R^T \mathbf{P}_R + \mathbf{T} \quad \Longrightarrow \quad \mathbf{P}_R = R(\mathbf{P}_L - \mathbf{T})$$

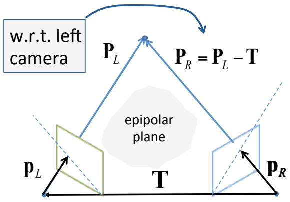
Perspective projection:



$$\mathbf{P}_L = \begin{bmatrix} X_L \\ Y_L \\ Z_L \end{bmatrix} \qquad \mathbf{p}_L = \begin{bmatrix} x_L \\ y_L \\ f \end{bmatrix} = \frac{f\mathbf{P}_L}{Z_L} \qquad \mathbf{p}_R = \begin{bmatrix} x_R \\ y_R \\ f \end{bmatrix} = \frac{f\mathbf{P}_R}{Z_R}$$

Epipolar Geometry - Maths

w.r.t. left camera



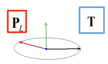
Vectors $\mathbf{P}_L$, $\mathbf{T}$ and $\mathbf{P}_L - \mathbf{T}$ all lie in epipolar plane

in plane

perpendicular to plane

$$\underbrace{(\mathbf{P}_L - \mathbf{T})}^{T}\underbrace{(\mathbf{T} \otimes \mathbf{P}_L)} = 0$$

$\otimes$ cross product :



Inner product = 0 for **perpendicular** vectors

Epipolar Geometry - Maths

$$(\mathbf{P}_L - \mathbf{T})^T (\mathbf{T} \otimes \mathbf{P}_L) = 0$$

$$(\mathbf{T} \otimes \mathbf{P}_L) = S\,\mathbf{P}_L$$

$$S = \begin{bmatrix} 0 & -T_Z & T_Y \\ T_Z & 0 & -T_X \\ -T_Y & T_X & 0 \end{bmatrix}$$

w.r.t. right camera

$$\mathbf{P}_R = R(\mathbf{P}_L - \mathbf{T})$$

$$R^T = R^{-1}$$ Rotation matrix

$$\Downarrow$$

$$R^T \mathbf{P}_R = (\mathbf{P}_L - \mathbf{T})$$
$$\mathbf{P}_R^T R = (\mathbf{P}_L - \mathbf{T})^T$$

$$\boxed{\mathbf{P}_R^T \, R \; S \; \mathbf{P}_L = 0}$$

The Essential Matrix

$$\mathbf{P}_R^T \underbrace{R \; S} \mathbf{P}_L = 0 \quad \Longrightarrow \quad \mathbf{P}_R^T \, E \; \mathbf{P}_L = 0$$

$$E = RS \quad \Longrightarrow$$ the essential matrix

$$\mathbf{p}_L = \frac{f\mathbf{P}_L}{Z_L} \qquad \mathbf{p}_R = \frac{f\mathbf{P}_R}{Z_R} \quad \Longrightarrow \quad \cancel{\frac{Z_R}{f}}\mathbf{p}_R^T \, E \, \cancel{\frac{Z_L}{f}}\mathbf{p}_L = 0$$

$$\Longrightarrow \quad \mathbf{p}_R^T \, E \; \mathbf{p}_L = 0$$